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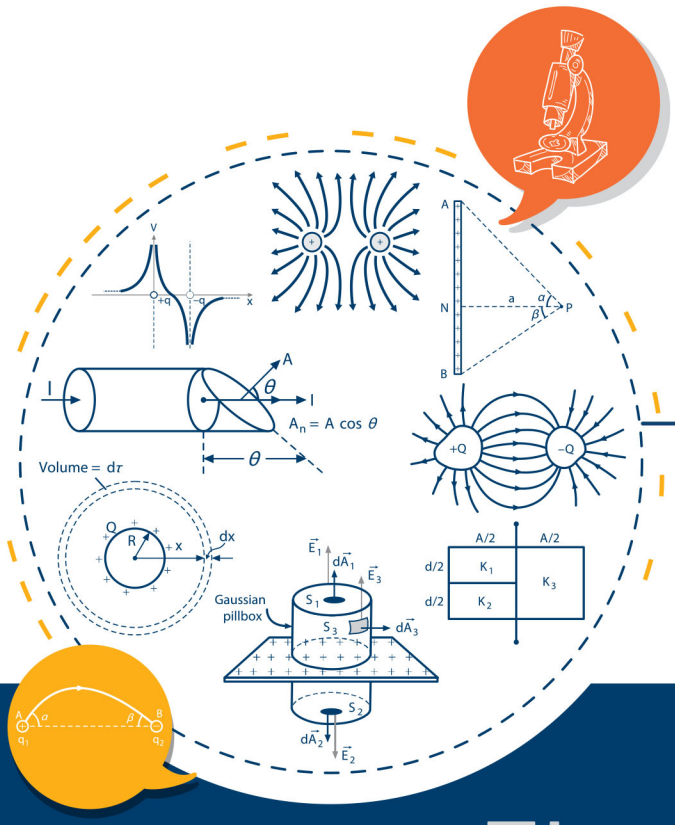

JEE ADVANCED PHYSICS

Electrostatics and Current Electricity

Rahul Sardana

Pearson

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JEE Advanced Physics: Electrostatics and Current Electricity

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JEE Advanced Physics: Electrostatics and Current Electricity

Rahul Sardana

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CONTENTS

Chapter Insight xiv

Preface xix

About the Author xx

CHAPTER 1 ELECTROSTATICS 1.1

Coulomb's Law, Electric Field, Dipole and Applications 1.1

Introduction 1.1

Electric Charge 1.1

Conductors and Insulators 1.2

Methods of Charging 1.2

Properties of Electric Charge 1.3

Concept of Charge Distribution(s) 1.3

Coulomb's Law 1.4

Important Points Regarding Coulomb's Law 1.5

Coulomb's Law in Vector Form 1.5

Coulomb's Law in Position Vector Form 1.6

Principle of Superposition 1.14

Equilibrium of Three Charges 1.16

Electrostatic Field ($\vec{E}$) 1.20

Electrostatic Lines of Force: Properties 1.23

Electric Field at the Axis of a Circular Uniformly Charged Ring 1.24

Electric Field Due to a Large Thick Charged Sheet 1.33

Motion of a Charged Particle in an Electric Field 1.43

Electric Dipole and Dipole Moment 1.45

Electric Field Due to a Dipole at a Point Lying on the Axial Line (End on Position) 1.45

Electric Field Due to a Dipole at a Point Lying on the Equitorial Line (Broad Side on Position) 1.46

Electric Field Due to a Dipole at Any Point $P(r, \theta)$ 1.46

Torque on a Dipole Placed in a Uniform Electric Field 1.47

Potential Energy of a Dipole Placed in a Uniform Electric Field 1.48

Small Oscillations of a Dipole Placed in a Uniform Electric Field 1.49

Force on an Electric Dipole in Non-uniform Electric Field 1.50
Force on a Dipole in the Surrounding of a Long Charged Wire 1.51
Electric Force Between Two Dipoles. 1.51
Concept of Distributed Dipole 1.53

Gauss's Law and Applications 1.57

Area as a Vector. 1.57
Electric Flux 1.57
Concept of Solid Angle 1.61
Solid Angle Subtended Due to a Random Surface at a Given Point 1.61
Solid Angle Subtended by a Surface Not Normal to Axis of Cone or Not Lying on Surface of Sphere 1.62
Relation Between Half Angle of Cone and Solid Angle at Vertex 1.62
Electric Flux Produced by a Point Charge 1.63
Electric Flux Calculation Due to a Point Charge Using Solid Angle 1.65
Gauss's Law 1.67
Electric Field Due to a Point Charge q at a Point P at Distance x 1.69
Electric Field Strength of a Charged Conducting Sphere or Conducting Shell . 1.70
Electric Field Due to a Non-conducting Uniformly Charged Sphere. 1.71
Electric Field Due to an Infinitely Long Thin Charged Wire 1.73
Electric Field Due to a Long Uniformly Charged Conducting Cylinder . . . 1.74
Electric Field Due to a Long Uniformly Charged Non-conducting Cylinder . . 1.74
Electric Field Due to an Infinite Non-conducting Thin Uniformly Charged Sheet 1.76
Electric Field Due to an Infinite Charged Conducting Sheet 1.76

Electrostatic Potential, Potential Energy, Conductors and Applications 1.82

Electrostatic Potential Energy and Potential 1.82
Electrostatic Interaction Energy of a System of Two Charged Particles 1.82
Electrostatic Interaction Energy for a System of Particles 1.83
Electrostatic Potential (V) 1.84
Electrostatic Potential Difference (ΔV) 1.84
Potential Due to an Assembly of Charges 1.85
Points with Zero Potential Due to Two Point Charges 1.85
Graphical Representation of Potential of a System of Two Point Charges . . . 1.86
Electrostatic Potential at the Axis of a Uniformly Charge Rod 1.87
Electrostatic Potential at Point P Lying on Perpendicular Bisector of Rod. . . 1.88
Electrostatic Potential Due to a Charged Ring at Its Centre 1.89

Electrostatic Potential at a Point P on the Axis of the Ring at Distance x from Its Centre . . . 1.90
Variation of Electrostatic Potential on the Axis of a Charged Ring . . . 1.90
Electrostatic Potential Due to a Uniformly Charged Disc at a Point P on Its Axis . . . 1.91
Electric Potential of an Annulus . . . 1.93
Electrostatic Potential on the Edge of a Uniformly Charged Disc . . . 1.94
Electrostatic Potential Due to a Charged Conducting Sphere/Charged Shell . . . 1.95
Electrostatic Potential Due to a Non-conducting Uniformly Charged Sphere . . . 1.96
Potential Due to a Dipole . . . 1.97
Potential at Point $P(r, \theta)$ Due to a Small Dipole . . . 1.98
Binding Energy of a Dipole . . . 1.98
Equipotential Surfaces: Introduction . . . 1.101
Equipotential Surface: Properties . . . 1.101
Relation Between Electrostatic Field and Potential . . . 1.104
Motion of Charged Particles and Conservation Laws . . . 1.107
Conductors and Their Properties . . . 1.113
Charge Distribution on a System of Parallel Plates . . . 1.118
Method of Images . . . 1.123
Charge Distribution on Metal Object . . . 1.125
Electric Pressure on a Charged Surface Due to External Electric Field . . . 1.126
Mechanical Force on a Charged Conductor . . . 1.126
Electrostatic Energy Density (u_e) . . . 1.127
Electrostatic Field and Potential Due to Induced Charges . . . 1.130
Earthing of Charged or Uncharged Metal Bodies . . . 1.131
Current Due to Movement of Charges Through Earth . . . 1.131
Field Energy Density of Electric Field . . . 1.135
Electrostatic Self Energy of a Charged Conducting Sphere . . . 1.136
Electrostatic Self Energy of a Uniformly Charged Non-conducting Sphere . . . 1.137
Total Electrostatic Energy of a System of Charged Bodies . . . 1.138
Electrostatic Energy of a System of Concentric Shells . . . 1.138
Solved Problems . . . *1.141*
Practice Exercises . . . *1.160*
Single Correct Choice Type Questions . . . *1.160*
Multiple Correct Choice Type Questions . . . *1.184*
Reasoning Based Questions . . . *1.192*
Linked Comprehension Type Questions . . . *1.194*

Matrix Match/Column Match Type Questions 1.201
Integer/Numerical Answer Type Questions 1.208
Archive: JEE Main 1.213
Archive: JEE Advanced 1.221
Answer Keys–Test Your Concepts and Practice Exercises 1.232

CHAPTER 2 CAPACITANCE AND APPLICATIONS 2.1

Capacitors: Introduction 2.1
Calculation of Capacitance 2.2
Definition of C 2.2
Energy Stored in a Charged Capacitor 2.2
Capacitance of an Isolated Sphere 2.3
Charge Sharing Between Two Charged Conductors 2.3
Sharing of Charges Between Capacitors and Common Potential 2.5
Principle of a Parallel Plate Capacitor 2.7
Parallel Plate Capacitor 2.8
Electrostatic Force Between the Plates of a Parallel Plate Capacitor 2.9
Capacitors in Series 2.10
Capacitors in Parallel. 2.11
Electrostatic Energy Density (u_E). 2.14
Energy for Series and Parallel Combinations 2.15
Calculating the Net Capacitance of Circuits 2.15
Simple Circuits 2.15
Circuits With Extra Wires 2.17
Concept of Line of Symmetry 2.18
Balanced Wheatstone Bridge 2.18
Extended Wheatstone Bridge 2.19
Infinite Chain of Capacitors 2.19
Network With More Than One Cell 2.20
Case of Compound Dielectrics 2.20
Dielectric Slab Inserted in a Parallel Plate Capacitor 2.26
Conducting Slab Inserted in a Parallel Plate Capacitor 2.27
Charge Induced on a Dielectric and Gauss's Law for Dielectrics 2.27
Effect of Insertion of Dielectric in a Parallel Plate Capacitor 2.29
Force on a Dielectric Slab Being Inserted Between Capacitor Plates 2.32
Dielectric Breakdown 2.34
Electric Energy Density of Dry Air 2.35
Kirchhoff's Laws for Capacitor Circuits 2.38

Flow of Charge . . . 2.40
Energy Supplied/Consumed by the Battery . . . 2.40
Generation of Heat . . . 2.40
Spherical Capacitor . . . 2.45
Cylindrical Capacitor . . . 2.50
Energy Stored in a Spherical Shell . . . 2.51
Solved Problems . . . *2.53*
Practice Exercises . . . *2.68*
Single Correct Choice Type Questions . . . *2.68*
Multiple Correct Choice Type Questions . . . *2.91*
Reasoning Based Questions . . . *2.98*
Linked Comprehension Type Questions . . . *2.99*
Matrix Match/Column Match Type Questions . . . *2.105*
Integer/Numerical Answer Type Questions . . . *2.110*
Archive: JEE Main . . . *2.112*
Archive: JEE Advanced . . . *2.117*
Answer Keys–Test Your Concepts and Practice Exercises . . . *2.121*

CHAPTER 3 ELECTRIC CURRENT AND CIRCUITS . . . 3.1

Electric Current . . . 3.1
Different Situations Producing Current . . . 3.2
Current Density ($\vec{J}$) . . . 3.5
Ohm's Law and Electrical Resistance . . . 3.5
Resistivity (ρ), Conductivity (σ) and Conductance (G) . . . 3.5
Recasting a Wire of Given Mass . . . 3.7
Fractional Change in Resistance . . . 3.8
Current Density and Drift Velocity . . . 3.8
Ohm's Law: Revisited . . . 3.9
Mobility . . . 3.10
Variation of Resistance with Temperature . . . 3.12
Colour Code for Carbon Resistances . . . 3.14
Thermistor . . . 3.14
Electrical Energy and Power . . . 3.15
Electromotive Force (EMF) . . . 3.17
Resistors in Series and in Parallel . . . 3.18
Network Analysis . . . 3.21
Earthing or Grounding in an Electrical Circuit . . . 3.22
Delta to Star or Delta-star Transformation . . . 3.23

Star to Delta or Star-delta Transformation . . . 3.24
Short and Open Circuits . . . 3.28
Principle of Symmetry . . . 3.30
Symmetrical Circuits . . . 3.30
Identical Potential Points . . . 3.32
Infinite Ladder and Grid . . . 3.32
Balanced Wheatstone Bridge Like Situations . . . 3.34
Kirchhoff's Circuit Rules . . . 3.39
The Principle of Superposition or Superposition Theorem . . . 3.46
Nodal Analysis in Circuits . . . 3.47
Nodal Analysis Method . . . 3.48
Primary and Secondary Cells/Batteries . . . 3.52
Ideal Voltage Source . . . 3.52
Practical Voltage Source . . . 3.52
Discharging of Cell . . . 3.53
Charging of Cell . . . 3.53
Short-circuiting of a Cell . . . 3.53
Internal Resistance of Cell (r) . . . 3.53
Terminal Potential Difference Across a Cell . . . 3.54
Maximum Power Transfer Theorem . . . 3.54
Identical Cells in Series . . . 3.55
Non-identical Cells in Series . . . 3.56
Identical Cells in Parallel . . . 3.57
Non-identical Cells in Parallel . . . 3.57
Mixed Grouping of Identical Cells . . . 3.59
Ammeter . . . 3.61
Voltmeter . . . 3.63
Error in the Measurement by Ammeter . . . 3.66
Error in the Measurement by Voltmeter . . . 3.66
Heating Effect of Current . . . 3.71
Power . . . 3.71
Electric Energy . . . 3.71
Specification of a Bulb or Other Electric Appliances . . . 3.72
Power Transformation Rule . . . 3.73
Combination of Bulbs . . . 3.73
Time Relation for Series and Parallel Combination . . . 3.75
Characteristics of a Fuse . . . 3.75

Wheatstone Bridge: Condition of Balance . . . 3.79
The Metre Bridge . . . 3.81
End Corrections in Metre Bridge . . . 3.82
Post Office Box . . . 3.82
Potentiometer . . . 3.84
Applications of Potentiometer . . . 3.87
Difference Between Voltmeter and Potentiometer . . . 3.90

RC Circuit and Applications . . . 3.95
RC Circuit . . . 3.95
Leakage Current Through a Capacitor . . . 3.101
Solved Problems . . . *3.108*
Practice Exercises . . . *3.122*
Single Correct Choice Type Questions . . . *3.122*
Multiple Correct Choice Type Questions . . . *3.142*
Reasoning Based Questions . . . *3.150*
Linked Comprehension Type Questions . . . *3.152*
Matrix Match/Column Match Type Questions . . . *3.156*
Integer/Numerical Answer Type Questions . . . *3.159*
Archive: JEE Main . . . *3.163*
Archive: JEE Advanced . . . *3.174*
Answer Keys–Test Your Concepts and Practice Exercises . . . *3.183*

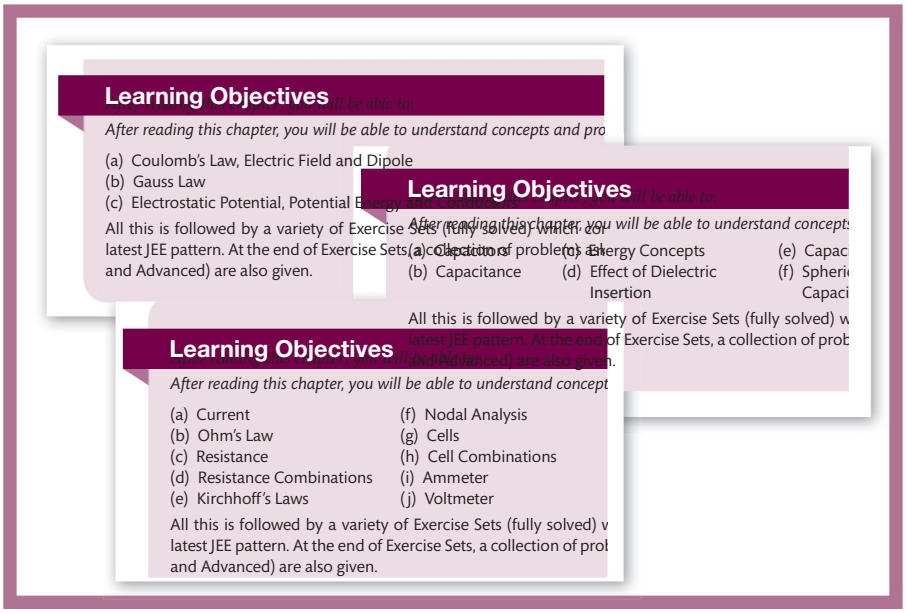
HINTS AND EXPLANATIONS

Chapter 1: Electrostatics . . . H.3
Chapter 2: Capacitance and Applications . . . H.126
Chapter 3: Electric Current and Circuits . . . H.198

CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.


Learning Objectives
After reading this chapter, you will be able to understand concepts and pro
(a) Coulomb's Law, Electric Field and Dipole
(b) Gauss Law
(c) Electrostatic Potential, Potential Energy and Conservative
All this is followed by a variety of Exercise Sets (fully solved) which cor
latest JEE pattern. At the end of Exercise Sets, a collection of problems
and Advanced) are also given.
Learning Objectives
After reading this chapter, you will be able to understand concepts
(a) Capacitors
(b) Capacitance
(c) Energy Concepts
(d) Effect of Dielectric Insertion
(e) Capac
(f) Spheri
Capaci
All this is followed by a variety of Exercise Sets (fully solved) w
latest JEE pattern. At the end of Exercise Sets, a collection of prob
and Advanced) are also given.
Learning Objectives
After reading this chapter, you will be able to understand concept
(a) Current
(b) Ohm's Law
(c) Resistance
(d) Resistance Combinations
(e) Kirchhoff's Laws
(f) Nodal Analysis
(g) Cells
(h) Cell Combinations
(i) Ammeter
(j) Voltmeter
All this is followed by a variety of Exercise Sets (fully solved) v
latest JEE pattern. At the end of Exercise Sets, a collection of prob
and Advanced) are also given.

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

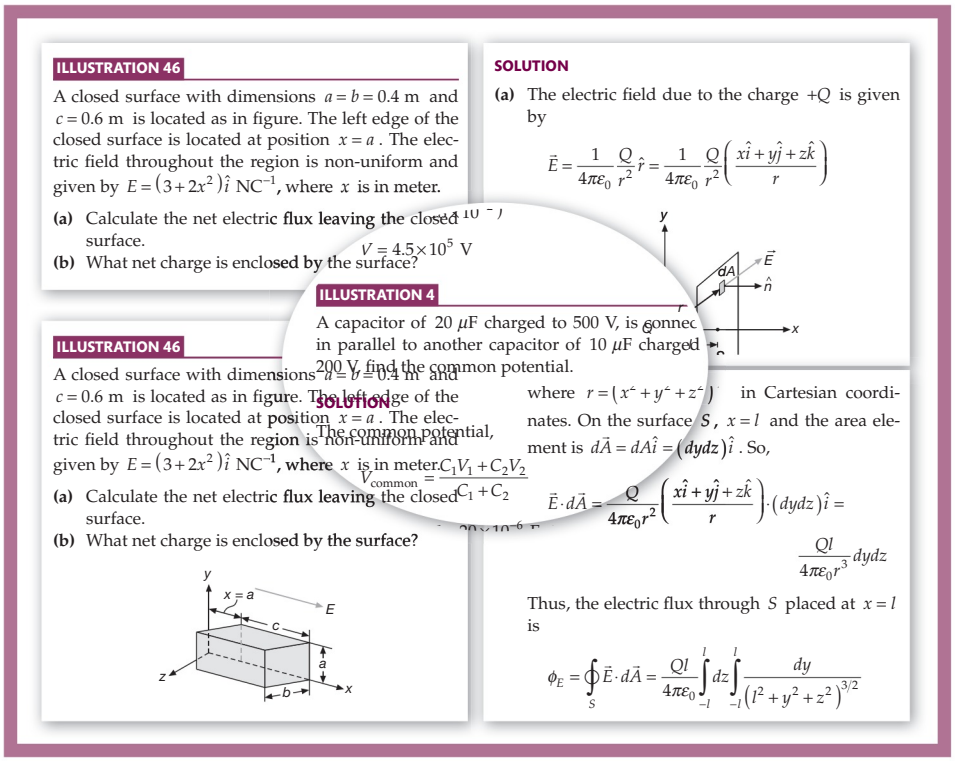


ILLUSTRATION 46
A closed surface with dimensions $a = b = 0.4$ m and $c = 0.6$ m is located as in figure. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is non-uniform and given by $E = (3 + 2x^2)\hat{i}$ NC^{-1}, where x is in meter.
(a) Calculate the net electric flux leaving the closed surface.
(b) What net charge is enclosed by the surface?
SOLUTION
(a) The electric field due to the charge $+Q$ is given by
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}\right)$$
where $r = (x^2 + y^2 + z^2)$ in Cartesian coordinates. On the surface S, $x = l$ and the area element is $d\vec{A} = dA\hat{i} = (dydz)\hat{i}$. So,
$$\vec{E}\cdot d\vec{A} = \frac{Q}{4\pi\varepsilon_0 r^2}\left(\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}\right)\cdot(dydz)\hat{i} = \frac{Ql}{4\pi\varepsilon_0 r^3}dydz$$
Thus, the electric flux through S placed at $x = l$ is
$$\phi_E = \oint_S \vec{E}\cdot d\vec{A} = \frac{Ql}{4\pi\varepsilon_0}\int_{-l}^{l}dz\int_{-l}^{l}\frac{dy}{(l^2+y^2+z^2)^{3/2}}$$
ILLUSTRATION 4
A capacitor of 20 μF charged to 500 V, is connec
in parallel to another capacitor of 10 μF charged
200 V, find the common potential.
SOLUTION
The common potential,
$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$
$V = 4.5\times10^5$ V
ILLUSTRATION 46
A closed surface with dimensions $a = b = 0.4$ m and $c = 0.6$ m is located as in figure. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is non-uniform and given by $E = (3 + 2x^2)\hat{i}$ NC^{-1}, where x is in meter.
(a) Calculate the net electric flux leaving the closed surface.
(b) What net charge is enclosed by the surface?

Test Your Concepts-I

Based on General Capacitance

(Solutions on page H.126)

1. A capacitor of 20 μF charged to 500 V, is connected in parallel to another capacitor of 10 μF charged to 200 V, find the common potential.
2. Calculate the heat generated when a condenser of 100 μF capacity and charged to 200 V is discharged through a 2 Ω resistance.
5. Consider two conducting spheres with radii R_1 and R_2 separated by a distance much greater than radius of either. A total charge Q is shared between the spheres, subject to the condition that the electric potential energy of the system is a minimum. Find the energy of this system. Connect the two spheres by a conducting wire and find the energy of the system. Where has the energy gone?

Test Your Concepts-II

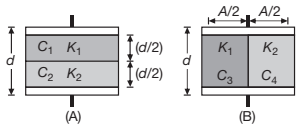
Based on Series & Paral

1. Two identical parallel-plate capacitors, each wi capacitance C, are charged to potential differen ΔV and connected in parallel. Then the plate sep ration in one of the capacitors is doubled.
   - **(a)** Find the total energy of the system of tw capacitors before the plate separation doubled.
   - **(b)** Find the potential difference across eac capacitor after the plate separation is double
   - **(c)** Find the total energy of the system after th plate separation is doubled.

Test Your Concepts-III

Based on Diele

1. A capacitor is filled with two dielectrics of sam dimensions but of dielectric constant 2 and respectively. Find the ratio of capacities in the tw arrangements.



Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

Applying Kirchhoff's Rules

In this chapter we have seen how Kirchhoff's rules can be used to analyze multiloop circuits. The steps are summarized below:

- **(a)** Draw a circuit diagram, and label all the quantities, both known and unknown. The number of unknown quantities is equal to the number of linearly independent equation
- **(b)** Assign a direction to the c of the circuit. (If the actua what you have assumed, be a negative number).
- **(c)** Apply *KCL* to each ju more convenient to a currents in the branch of unknowns to be solv

Problem Solving Technique(s)

How to Remember (Star to Delta)?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divided by the third star resistance.

Problem Solving Technique(s)

Since $q_p = q\left(1-\frac{1}{K}\right)$

For a conductor $K \to \infty$. Hence,

$q_p = q$, $\sigma_p = \sigma$ and $E = 0$

Hence, we may conclude the above discussion as under

(a) $E_{vacuum} = E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$

{K = dielectric constant}

Problem Solving Technique(s)

While solving problems of this type always keep in mind that, whenever a battery is disconnected then,

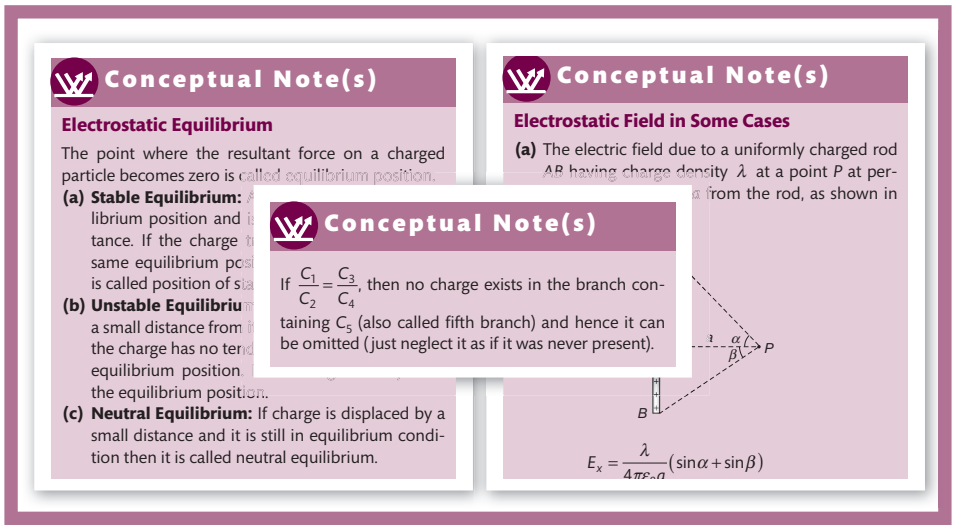
q = constant

and if battery remains attached, then V = constant

Problem Solving Technique(s)

systematic way of applying s (*KCL*) at each essential node ents the branch current in terms of the . This will give us a set of equations that we solve together to find the node voltages. Once we find the node voltages, we can use them to calculate any other currents or voltages of interest.

STEP-1: Identify all the essential nodes and select one of them as a reference node i.e. the node at zero poten-



Conceptual Note(s)

Electrostatic Equilibrium

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

(a) Stable Equilibrium: A librium position and i tance. If the charge t same equilibrium posi is called position of sta

(b) Unstable Equilibriu a small distance from t the charge has no ten equilibrium position the equilibrium position.

(c) Neutral Equilibrium: If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

Conceptual Note(s)

If $\frac{C_1}{C_2}=\frac{C_3}{C_4}$, then no charge exists in the branch containing C_5 (also called fifth branch) and hence it can be omitted (just neglect it as if it was never present).

Conceptual Note(s)

Electrostatic Field in Some Cases

(a) The electric field due to a uniformly charged rod AB having charge density λ at a point P at perpendicular distance a from the rod, as shown in

$$E_x=\frac{\lambda}{4\pi\varepsilon_0 a}(\sin\alpha+\sin\beta)$$

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

SOLVED PROBLEMS

PROBLEM 1

A parallel plate capacitor is placed in a cylindrical tank filled with a liquid of dielectric constant K. The area of cross-section of the tank is A and height of the liquid is equal to the length of the square plate of plate area l^2. The separation between the plates is d. A small orifice of area a is opened at the bottom of the tank at $t=0$. If the capacitor in the process remains connected with a battery of emf E and assuming the level of liquid in the capacitor remains same as outside, find the current in the circuit as a function of time

SOLUTION

According to Equation of Continuity, Rate of Change of volume is av, where, v is the velocity of efflux

$$\Rightarrow \quad y=\left(\sqrt{l}-\frac{a\sqrt{2g}}{2A}t\right)^2 \qquad \ldots(1)$$

Capacitance of the capacitor at time t is,

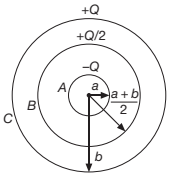
$$C=\frac{\varepsilon_0 l(l-y)}{d}+\frac{K\varepsilon_0 yl}{d}$$

PROBLEM 4

The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively. Find capacitance of the resultant capacitor.

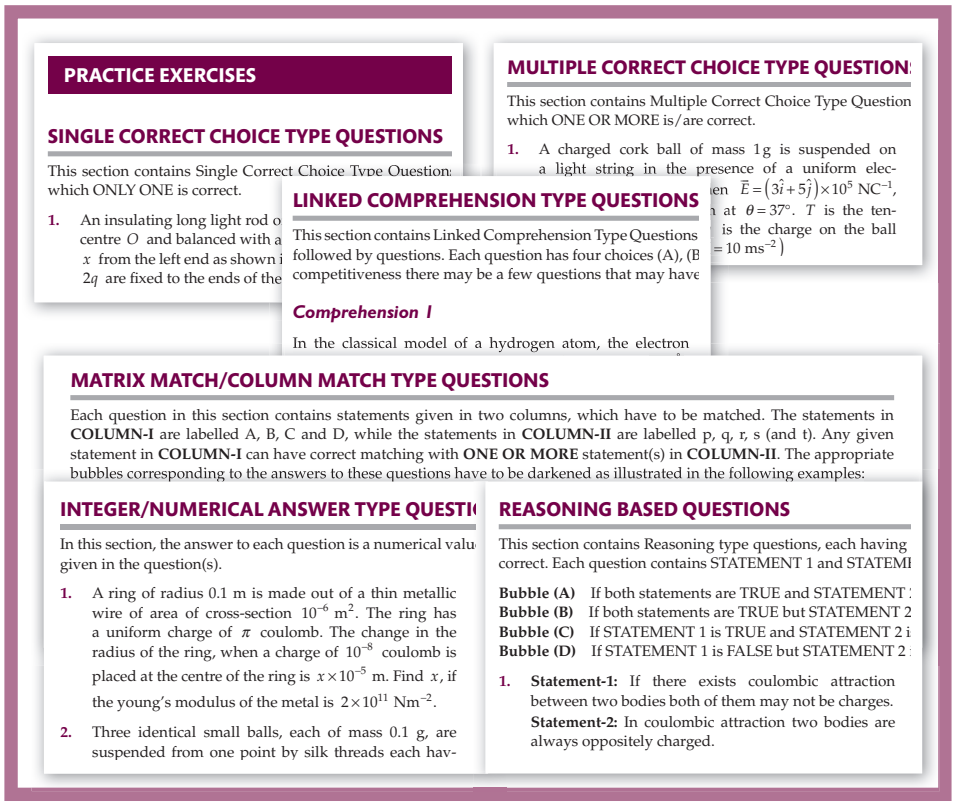
PROBLEM 8

Figure shows three conducting spherical shells A, B and C with charges $-Q$, $+\frac{Q}{2}$ and $+Q$ respectively. Calculate the capacitance of the system between points A and C.



SOLUTION

Potential at A, V_A is given by



PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Question which ONLY ONE is correct.

1. An insulating long light rod o centre O and balanced with a x from the left end as shown i $2q$ are fixed to the ends of the

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions followed by questions. Each question has four choices (A), (B competitiveness there may be a few questions that may have

Comprehension I

In the classical model of a hydrogen atom, the electron

MULTIPLE CORRECT CHOICE TYPE QUESTION

This section contains Multiple Correct Choice Type Question which ONE OR MORE is/are correct.

1. A charged cork ball of mass 1g is suspended on a light string in the presence of a uniform elec- en $\vec{E}=(3\hat{i}+5\hat{j})\times10^5\ \text{NC}^{-1}$, at $\theta=37°$. T is the ten- is the charge on the ball $=10\ \text{ms}^{-2}$)

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

INTEGER/NUMERICAL ANSWER TYPE QUESTI

In this section, the answer to each question is a numerical valu given in the question(s).

1. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross-section $10^{-6}\ \text{m}^2$. The ring has a uniform charge of π coulomb. The change in the radius of the ring, when a charge of 10^{-8} coulomb is placed at the centre of the ring is $x\times10^{-5}$ m. Find x, if the young's modulus of the metal is $2\times10^{11}\ \text{Nm}^{-2}$.
2. Three identical small balls, each of mass 0.1 g, are suspended from one point by silk threads each hav-

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having correct. Each question contains STATEMENT 1 and STATEMI

Bubble (A) If both statements are TRUE and STATEMENT
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 i
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2

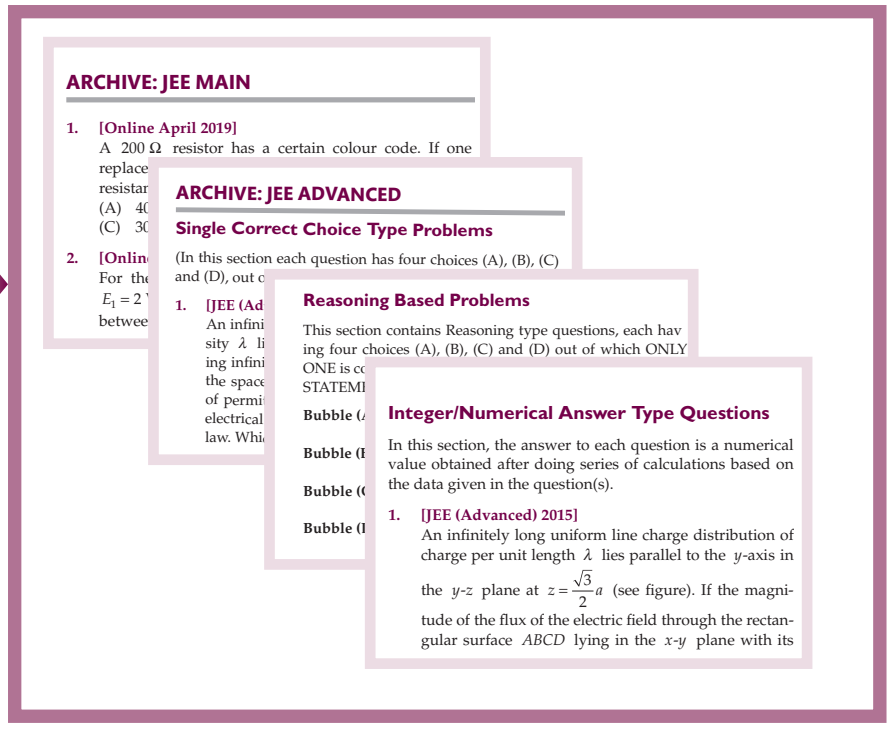
1. **Statement-1:** If there exists coulombic attraction between two bodies both of them may not be charges.
   Statement-2: In coulombic attraction two bodies are always oppositely charged.

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advaned

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.



ARCHIVE: JEE MAIN

1. [Online April 2019]
   A 200 Ω resistor has a certain colour code. If one replace resistar
   (A) 4(
   (C) 3(
2. [Onlin
   For th
   $E_1=2$
   betwee

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out c

1. [JEE (Ad
   An infini
   sity λ li
   ing infini
   the space
   of permi
   electrical
   law. Whi

Reasoning Based Problems

This section contains Reasoning type questions, each hav ing four choices (A), (B), (C) and (D) out of which ONLY ONE is c STATEMI

Bubble (
Bubble (
Bubble (
Bubble (

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. [JEE (Advanced) 2015]
   An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z=\frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its

CHAPTER 1: ELECTROSTATICS

Test Your Concepts-I (Based on Coulomb's Law)

1. $F_{air} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

Let us now replace the slab of thickness t with an equivalent amount of air such that two charges placed in air at separation x will experience the same force if

$\Rightarrow \quad q = ne = \frac{6\times10^{24}}{63500}\times1.6\times10^{-19}$ C

$\Rightarrow \quad q = \frac{1920}{127}$ C

Now, one piece of Cu has positive charge and t other has a negative charge, so both will attract ea other with a force of

Single Correct Choice Type Questions

1. By Law of Conservation of Moments about O,

$$\begin{pmatrix}\text{Total Clockwise}\\(\text{CW})\text{ Moments}\end{pmatrix} = \begin{pmatrix}\text{Total Counter}\\\text{Clockwise (CCW)}\\\text{Moments}\end{pmatrix}$$

$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right) + W\left(x-\frac{L}{2}\right) = \frac{2qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right)$

$\Rightarrow \quad W\left(x-\frac{L}{2}\right) = \frac{QqL}{8\pi\varepsilon_0 h^2} \quad \Rightarrow \quad x = \frac{QqL + 4\pi\varepsilon_0 h^2 LW}{8\pi\varepsilon_0 h^2 W}$

Hence, the correct answer is (D).

3. The transfer of electrons from one body to the other produces charge on the body

Hence,

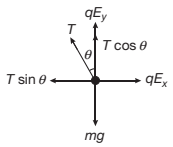
$$\begin{pmatrix}\text{number of electrons}\\\text{given by one body}\end{pmatrix} = \begin{pmatrix}\text{number of electrons}\\\text{obtained by the other}\end{pmatrix}$$

So, mass of negatively charged body slightly increases while mass of positively charged body slightly increases

Multiple Correct Choice Type Questions

1. $T\cos\theta + qE_y = mg$ ...(1)

$T\sin\theta = qE_x$ ...(2)



$\Rightarrow \quad T(0.6) = 3\times10^5 q$

$\Rightarrow \quad q = \frac{6}{30}\times10^{-5}\,T$

$\Rightarrow \quad q = 2\times10^{-6}T$

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

CHAPTER 1

Electrostatics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Coulomb's Law, Electric Field and Dipole
(b) Gauss Law
(c) Electrostatic Potential, Potential Energy and Conductors

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

COULOMB'S LAW, ELECTRIC FIELD, DIPOLE AND APPLICATIONS

INTRODUCTION

Life without electricity and its effects would have been hard to imagine. The electromagnetic force between charged particles is one of the fundamental forces of nature. Here we shall begin by describing some of the basic properties of electrostatic forces and then smoothly moving on to the basic electrical phenomenon and applications along with a rigorous mathematical treatment through ILLUSTRATIVE EXAMPLES.

The branch, known as electromagnetism is concerned with the nature of electric and magnetic phenomenon and the connection between them. I shall be presenting this together with some applications of interest to you from IIT-JEE view point exclusively.

When a glass rod is rubbed with silk, it acquires power to attract light bodies such as small pieces of paper. The bodies which acquire this power are said to be charged. If these charges do not move they are called **static charges** and the branch of Physics which deals with static charges is called **electrostatics**.

ELECTRIC CHARGE

Charge is a scalar quantity which is categorised into two types.

(a) Positive charge (anciently called **Vitreous**)
(b) Negative charge (anciently called **Resinous**)

A body having no charge, is said to be neutral in nature i.e., on a neutral body the sum of positive charges is equal to the sum of negative charges.

The positive charge means deficiency of electrons, whereas the negative charge on a body implies excess of electrons.

The S.I. unit of charge is coulomb (C).

Units of Charge

(a) S.I. unit of charge is coulomb (C). One coulomb of charge is that charge which when placed at rest in vacuum at a distance of one metre from an equal and similar stationary charge repels it and is repelled by it with a force of 9×10^9 N.

(b) In the Cgs system there are two units of charge statcoulomb (SC) or esu, abcoulomb or emu

$1 \text{ coulomb} = 3\times10^9 \text{ esu} = \frac{1}{10} \text{ emu}.$

(c) $1 \text{ coulomb} = 3\times10^9 \text{ SC} = \frac{1}{10} \text{ abcoulomb}$

(d) Practical units of charge are ampere hour with 1 Ahr = 3600 C and faraday (F) with 1 F = 96500 C

(e) Smallest unit of charge is statcoulomb (SC) and largest is faraday (F).

CONDUCTORS AND INSULATORS

Objects can broadly be classified as

(a) Conductors: Objects which allow charges to flow through them are called **conductors**. This category generally comprises metals with few exceptions from non-metals too (like graphite).

(b) Insulators: Objects which do not allow charges to flow through them are called **insulators**.

METHODS OF CHARGING

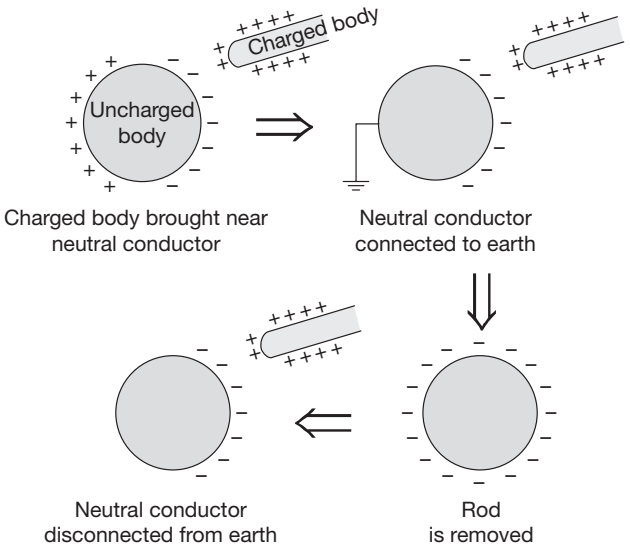
(a) By Friction: In this method, when two bodies are rubbed together some electrons are transferred from one body to other. As a result, one body becomes positively charged while the other becomes negatively charged. Electricity so obtained by rubbing or friction is known as frictional electricity.

EXAMPLE:
When glass rod is rubbed with silk, the rod becomes positively charged while the silk becomes negatively charged. Clouds also become charged by friction

Remark(s)

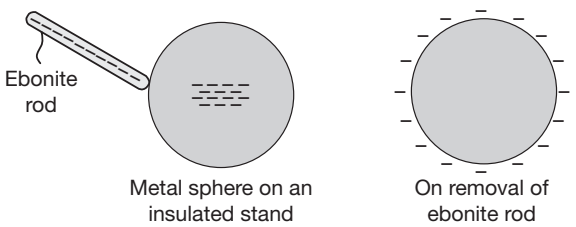
When two bodies have to be charged through friction, one of them has to be an insulator.

(b) By Induction: Whenever a charged body (conductor or insulator) is brought near a neutral conductor, the charged body will attract opposite charge and repel similar charge present in neutral conductor. This process is called **charging by induction**. Charging of body by Induction is shown here. Assume the neutral conductor to be placed on an insulated stand, not shown.



This process of induction is not applicable when the sphere were made of an insulated material such as plastic.

(c) By Conduction (charging by contact): When a charged body is in direct contact with an uncharged body, charge flows from former to latter till both are at same potential. This flow of charges is due to mutual repulsions between same kind of charges on charged body. When a negatively charged ebonite rod is rubbed on a metal object (say sphere) placed on an insulated stand (not shown), some of the electrons that are already in excess on the rod are transferred to the sphere. As soon as the electrons reach the metal sphere they start repelling each other and get spread over the surface of the sphere.



Remark(s)

(a) Nature of induced charge is always opposite to inducing charge.

(b) Induced charge can be lesser than or equal to inducing charge

$q_{\text{induced}} = -q_{\text{inducing}}\left(1-\frac{1}{K}\right)$, where K is the dielectric constant.

(c) If a charged body is brought near uncharged gold leaf electroscope or similar charged body brought near charged electroscope, leaves diverge. If oppositely charged body brought near charged electroscope, leaves converge.

PROPERTIES OF ELECTRIC CHARGE

(a) Like charges repel each other and unlike charges attract each other.

(b) Charge Conservation
The algebraic sum of all the charges in an isolated system is a constant. In crude language we can say that charge can neither be created nor be destroyed, however it can simply be transferred from one body to the other.

(c) Relativistic Invariance
Charge on a body is relativistically invariant. i.e., charge on the body at rest equals the charge on the body at relativistic speeds (speeds comparable to the speed of light). However charge density is not relativistically invariant. Mathematically,

$$(q)_{\text{at rest}} = (q)_{\text{in motion}}$$

or $(q)_{\text{at rest}} = (q)_{\text{at relativistic speeds}} = (q)_{\text{at non relativistic speeds}}$

Physically, we just want to say that the charge on a body is simply independent of the velocity of the body. However, this may not be the case with charge density.

(d) Charge Quantisation
Charge on a body q must always exist as an integral multiple of some fundamental unit of charge (called electronic charge) e, where $e = 1.6\times10^{-19}$ C.
Mathematically, $q = \pm ne$, $n = 1, 2, 3, ...$
From here we conclude that a neutral body can have +1 C of charge when it falls deficient of 6.25×10^{18} electrons.

Remark(s)

Quarks are particles which have fractional charges (e.g. $\frac{2e}{3}$, $-\frac{e}{3}$ etc) but they never exist independently i.e., they always exist in groups so as to make total charge on a body as an integral multiple of e. So existence of quarks does not violate charge quantisation.

(e) A charged body can attract light uncharged body. (Due to charging by induction)

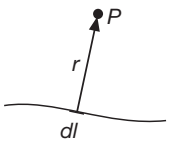
(f) Charges are always added algebraically.

EXAMPLE:
If a neutral body is first given a charge of +5 C and subsequently a charge of –7 C, then it will finally have a charge of +5 C – 7 C = –2 C.

CONCEPT OF CHARGE DISTRIBUTION(S)

Sometimes we observe that the charge, instead of being pointlike, is distributed over the entire body. In such a case we shall be using the concept of charge distributions as discussed.

CASE-1: Charge distributed on thin ropes, threads, or thin rods.



Line charge, λ

In this kind of situation(s), we shall be using the concept of Linear Charge Density λ, defined as charge per unit length. Mathematically, we have

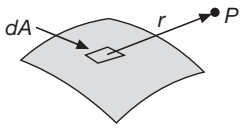
$$\lambda = \frac{dq}{dl} = \frac{dq}{dx} \quad \text{i.e.} \quad dq = \lambda dx$$

If the charge is distributed uniformly over the thin rod of length L, then we have $Q = \lambda L$.
However, if the charge is distributed non-uniformly over the length of the rod, then we have

$$Q = \int \lambda dx$$

This expression has to be integrated within suitable limits for getting the total charge on the rod.

CASE-2: Charge distributed on thin sheets, or thin surfaces



Surface charge, σ

In this kind of situation(s), we shall be using the concept of Surface Charge Density σ, defined as charge per unit area. Mathematically, we have

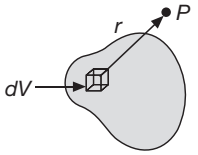
$$\sigma = \frac{dq}{dA} \quad \text{i.e.} \quad dq = \sigma dA$$

If the charge is distributed uniformly over the entire surface of area A, then we have $Q = \sigma A$.
For example, for a uniformly charged shell of radius R, the charge on the shell is $Q = \sigma A = \sigma(4\pi R^2)$.
However, if the charge is distributed non-uniformly over the area of the surface, then we have

$$Q = \int \sigma dA$$

This expression has to be integrated within suitable limits for getting the total charge on the surface.

CASE-3: Charge distributed on thin sheets, or thin surfaces



Volume charge, ρ

In this kind of situation(s), we shall be using the concept of Surface Charge Density σ, defined as charge per unit area. Mathematically, we have

$$\sigma = \frac{dq}{dA} \quad \text{i.e.} \quad dq = \sigma dA$$

If the charge is distributed uniformly over the entire surface of area A, then we have $Q = \sigma A$.
For example, for a uniformly charged shell of radius R, the charge on the shell is $Q = \sigma A = \sigma(4\pi R^2)$.
However, if the charge is distributed non-uniformly over the area of the surface, then we have

$$Q = \int \sigma dA$$

This expression has to be integrated within suitable limits for getting the total charge on the surface.

ILLUSTRATION 1

A spherical region of space has a distribution of charge such that the volume charge density varies with the radial distance from the centre r as $\rho = \rho_0 r^2$, $0 \le r \le R$ where ρ_0 is a positive constant. Calculate the total charge Q on sphere of radius R.

SOLUTION

Consider an infinitesimal concentric spherical element of thickness dr, radius r. If dq is the charge on this element, then

$$dq = \rho dV \qquad \{\text{where } dV = 4\pi r^2\, dr\}$$

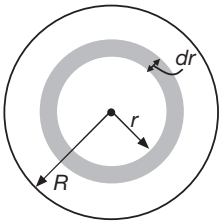
$$\Rightarrow \quad dq = (\rho_0 r^2)(4\pi r^2 dr)$$

$$\Rightarrow \quad dq = 4\pi\rho_0 r^4 dr$$

$$\Rightarrow \quad Q = \int_0^Q dq = 4\pi\rho_0 \int_0^R r^4 dr$$

$$\Rightarrow \quad Q = 4\pi\rho_0 \left[\frac{r^5}{5}\Big|_0^R\right]$$

$$\Rightarrow \quad Q = \frac{4}{5}\pi\rho_0 R^5$$



COULOMB'S LAW

The magnitude of the force (F) of attraction or repulsion between two point charges q_1 and q_2 placed in vacuum at separation r is

(a) directly proportional to the product of the magnitude of the two charges.

$$F \propto q_1 q_2 \qquad \ldots(1)$$

(b) inversely proportional to the square of the distance of separation between them.

$$F \propto \frac{1}{r^2} \text{ (called Inverse Square Law)} \qquad \ldots(2)$$

Combining (1) and (2), we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad ...(3)$$

where $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$

and $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$ is the absolute permittivity of free space or vacuum.

If the charges are placed in a medium, then

$$F_{\text{med}} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad ...(4)$$

where ε (read as Epsilon) is the permittivity of the medium given by,

$$\varepsilon = \varepsilon_0\varepsilon_r$$

where ε_r = Relative permittivity of the medium, called the **dielectric constant** (K). So

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = K$$

$$\Rightarrow \quad F_{\text{med}} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad F_{\text{med}} = \frac{F_{\text{vacuum/air}}}{K}$$

IMPORTANT POINTS REGARDING COULOMB'S LAW

(a) It is applicable only for point charges.

(b) The constant of proportionality K in SI units in vacuum is expressed as $\frac{1}{4\pi\varepsilon_0}$ and in any other medium expressed as $\frac{1}{4\pi\varepsilon}$. If charges are dipped in a medium then electrostatic force on one charge is $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$. ε_0 and ε are called permittivity of vacuum and absolute permittivity of the medium respectively. The ratio $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$ is called relative permittivity of the medium, which is a dimensionless quantity.

(c) The value of relative permittivity ε_r is constant for medium and can have values between 1 to ∞. For vacuum, by definition it is equal to 1. For air it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals the value of ε_r is ∞ and for water is 81. The material in which more charge can induce ε_r will be higher.

(d) The value of $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$.

$$\Rightarrow \quad \varepsilon_0 = 8.855\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$$

(e) Dimensional formula of ε is $M^{-1}L^{-3}T^4A^2$

(f) The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium, in which they lie.

(g) The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a close loop of any shape is zero.

(h) Since the force is a central force, in the absence of any other external force, angular momentum of one particle w.r.t. the other particle (in two particle system) is conserved.

(i) In vector form formula can be given as below.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{|\vec{r}|^3}\vec{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

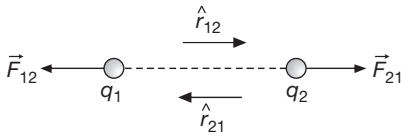
(q_1 & q_2 are to be substituted with sign.)

Here $\vec{r}$ is position vector of the test charge (on which force is to be calculated) with respect to the source charge (due to which force is to be calculated).

COULOMB'S LAW IN VECTOR FORM

Consider two point charges q_1 and q_2 separated by a distance r. If $q_1q_2 > 0$ i.e., if both q_1 and q_2 are positive or both q_1 and q_2 are negative then the charges repel each other.

$\vec{F}_{12}$ is the force exerted on charge q_1 by charge q_2 and $q_1q_2 > 0$



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

If $\vec{F}_{21}$ is the force exerted on q_2 due to q_1

then $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Both $\hat{r}_{21}$ and $\hat{r}_{12}$ have same magnitude i.e., unity and are oppositely directed

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}(-\hat{r}_{21}) = -\vec{F}_{12}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

So the forces exerted by charges on each other are equal in magnitude opposite in direction

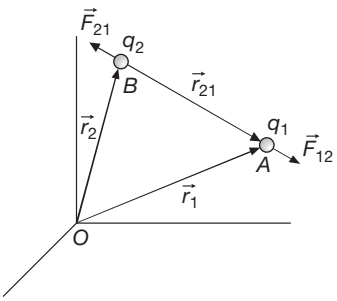
COULOMB'S LAW IN POSITION VECTOR FORM

Consider two point charges q_1 and q_2 located at points A and B respectively. According to Coulomb's Law the force $\vec{F}_{12}$ exerted on charge q_1 by q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

Since, $\hat{r}_{21} = \dfrac{\vec{r}_{21}}{|\vec{r}_{21}|}$

$$\Rightarrow \quad \vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\frac{\vec{r}_{21}}{r_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21}$$



Applying Triangle Law of vectors to vector triangle OBA

$$\vec{r}_2 + \vec{r}_{21} = \vec{r}_1$$

$$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

Similarly $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

If the charges are attracting each other i.e., are of opposite nature, then

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

and similarly for $\vec{F}_{12}$

So, if charge q_1, is lying at position vector $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and a charge q_2 is lying at position vector $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, then

(a) For like charges, we have

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \text{ and}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

(b) For unlike charges, we have

$$\vec{F}_{12} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \text{ and}$$

$$\vec{F}_{21} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Problem Solving Technique(s)

(a) The electrostatic force (F) is central (it is directed along the line joining the centres of two charges) and spherically symmetric (it is a function only of r).

(b) F is a force obeying the inverse Square Law.

(c) While applying Coulomb's Law, *I* must remind you to remember the following two points.

Firstly, the charges are assumed to be at rest. (As we shall see later that charges in motion also produce and experience magnetic forces).

Secondly, the charges are assumed to be point particles. However, when the charge is distributed **uniformly over a spherical surface**, then force on a point charge outside the surface may be computed from Coulomb's Law by treating the charge on the sphere as if it were concentrated at its centre and so

Furthermore, if the dimensions of two charged bodies are small in comparison to their separation, then too Coulomb's Law (as applied for point charges) will provide an approximate value for the force between them. In all other cases, integral calculus finds its applications by suitably applying the concepts of charge distributions (linear charge distributions, surface charge distributions or volume charge distributions, discussed later).

(d) While calculating the force between the two charges from Coulomb's Law, never take into account, the sign of the two charges. The sign just indicates the nature of the force. However it is advisable to take the sign when writing Coulomb's Law in vector form.

(e) When we are to calculate the force on charge 1 due to charge 2, then we assume charge 2 to be fixed and vice versa, unless and otherwise stated.

$$|\vec{F}_{12}| = m_1 a_1$$

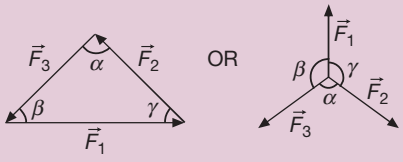
$$|\vec{F}_{21}| = m_2 a_2$$

i.e., $m_1 a_1 = m_2 a_2$ (by Newton's Third Law)

where m_1 and m_2 are the masses of particles having charges q_1 and q_2 respectively.

(f) When two identical bodies having charges q_1 and q_2 respectively are brought in contact and separated, then the charge on each body is $\frac{q_1 + q_2}{2}$.

(g) Some problems can also be solved by using the concept of Lami's Theorem, according to which "if a body is in equilibrium under the influence of three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ i.e., $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$, then



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

(h) For two charges q_1 and q_2 having position vectors $\vec{r}_1$ and $\vec{r}_2$ the electrostatic force $\vec{F}_{12}$ is given by the expression

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

and $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$

with q_1 and q_2 substituted in the above expressions along with the nature of the charge signified by the signs on them.

(i) If we are to find the electrostatic force on a charge q_0 at a point P, due to a charge distribution (may be uniform or non uniform), then we calculate the force on q_0 due to an infinitesimal element of the distribution and then integrate it within appropriate limits.

ILLUSTRATION 2

Two point charges Q_1 and Q_2 are 3 m apart and their combined charge is 20 μC.

(a) If one repels other with force of 0.075 N. Calculate the two charges.

(b) If one attracts the other with force of 0.525 N. Find the magnitude of the charges.

SOLUTION

(a) Since combined charge of Q_1 and Q_2 is

$$Q_1 + Q_2 = 20\ \mu C \qquad \ldots(1)$$

The force is repulsive, so according to Coulomb's Law

$$0.075 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = 9\times10^9\left[\frac{Q_1 Q_2}{(3)^2}\right]$$

$$\Rightarrow \quad Q_1 Q_2 = 75\times10^{-12}\,C^2 = 75\ \mu C^2$$

Substituting value of Q_2 from (1)

$$Q_1(20 - Q_1) = 75$$

$$\Rightarrow \quad Q_1^2 - 20Q_1 + 75 = 0$$

$$\Rightarrow \quad Q_1 = 15, 5$$

So the Q's are 5 μC and 15 μC

(b) Force is attractive, so one charge is negative so force equation is

$$-0.525 = 9\times10^9 \frac{Q_1Q_2}{9}$$

$$\Rightarrow \quad Q_1Q_2 = -525\ \mu C^2$$

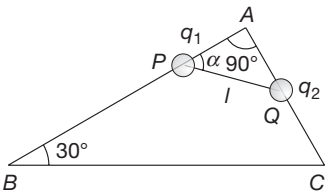
$$\Rightarrow \quad Q_1(20 - Q_1) = -525$$

$$\Rightarrow \quad Q_1^2 - 20Q_1 - 525 = 0$$

From here Q values are $35\ \mu C$ and $-15\ \mu C$ or $-35\ \mu C$ and $15\ \mu C$

ILLUSTRATION 3

A rigid insulated wire frame in the form of a right-angled triangle ABC is set in a vertical plane as shown in the figure. Two beads of equal masses m each and carrying the charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. Considering the case when the beads are stationary determine



(a) the angle α
(b) the tension in the cord and
(c) the normal reaction on the beads. If the cord is now cut, what are the values of the charges for which the beads continue to remain stationary?

SOLUTION

(a) The forces acting on bead at P having charge q_1 are

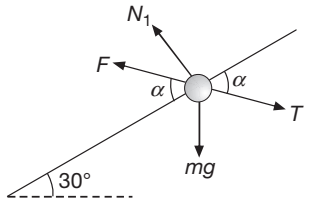
(i) Weight mg acting vertically downward

(ii) Tension T in the string along the length PQ

(iii) The electric force $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{l^2}$ between the beads along the length PQ

(iv) Normal reaction N_1 of wire on bead.

For P



For bead P to be in equilibrium,

$$mg\cos60° = (T - F)\cos\alpha \qquad \ldots(1)$$

and $$N_1 = mg\cos30° + (T - F)\sin\alpha \qquad \ldots(2)$$

For bead Q to be in equilibrium,

$$mg\sin60° = (T - F)\sin\alpha \qquad \ldots(3)$$

and $$N_2 = mg\cos60° + (T - F)\cos\alpha \qquad \ldots(4)$$

Dividing (3) by (1)

$$\tan60° = \tan\alpha$$

Hence, $\alpha = 60°$

(b) Hence, From (3),

$$T - F = mg$$

$$\Rightarrow \quad T = mg + F = mg + \frac{q_1q_2}{4\pi\varepsilon_0 l^2}$$

From (2) and (4), we have

$$N_1 = mg\cos30° + (T - F)\sin60°$$

$$\Rightarrow \quad N_1 = mg\cos30° + mg\sin60° = \sqrt{3}\ mg$$

and $$N_2 = mg\sin30° + mg\cos60° = mg$$

(c) If string is cut, Tension $T = 0$

Hence we have, $\frac{-q_1q_2}{4\pi\varepsilon_0 l^2} = mg$

$$q_1q_2 = -4\pi\varepsilon_0 mgl^2$$

Hence if the beads are to remain in equilibrium after the string is cut, q_1 and q_2 should have opposite charges satisfying the above condition.

ILLUSTRATION 4

A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r. Calculate the period of revolution and its speed also.

SOLUTION

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}=mr\omega^2=\frac{4\pi^2mr}{T^2}$$

$$\Rightarrow \quad T^2=\frac{(4\pi\varepsilon_0)r^2(4\pi^2mr)}{q_1q_2}$$

$$\Rightarrow \quad T=4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

Since, $\frac{q_1q_2}{4\pi\varepsilon_0 r^2}=\frac{mv^2}{r}$

$$\Rightarrow \quad v=\sqrt{\frac{q_1q_2}{4\pi\varepsilon_0 mr}}$$

ILLUSTRATION 5

What would be the interaction force between two copper spheres, each of mass 1 g, separated by the distance 1 m, if the total electronic charge in them differed from the total charge of the nuclei by one percent?

SOLUTION

Total number of atoms in the sphere of mass $1\text{ g}=\frac{1}{63.54}\times6.023\times10^{23}$

So the total nuclear charge

$$Q=\frac{6.023\times10^{23}}{63.54}\times1.6\times10^{-19}\times29$$

Now the charge on the sphere (q) = Total nuclear charge − Total electronic charge

$$\Rightarrow \quad q=\frac{6.023\times10^{23}}{63.54}\times1.6\times10^{-19}\times\frac{29\times1}{100}=4.298\times10^{2}\text{ C}$$

Hence force of interaction between these two spheres,

$$F=\frac{1}{4\pi\varepsilon_0}\frac{(4.398\times10^{2})^2}{1^2}\text{N}$$

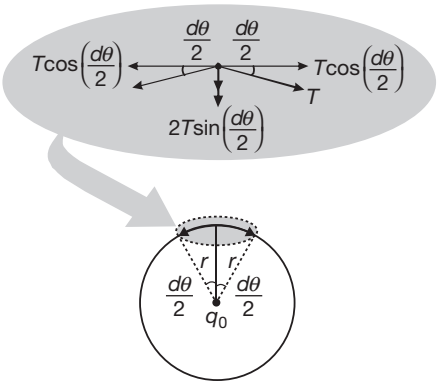
$$F=9\times10^{9}\times19.348\text{ N}=1.74\times10^{15}\text{ N}$$

ILLUSTRATION 6

A thin wire ring of radius r has an electric charge q. What will be the increment of the force stretching the wire if a point charge q_0 is placed at the ring's centre?

SOLUTION

Let us consider an element of arc length dl having a charge dq. Then $dq=\left(\frac{q}{2\pi r}\right)dl$.



If dF is the force of repulsion between the element and the charge q_0 at the centre, then

$$dF=\frac{1}{4\pi\varepsilon_0}\frac{q_0dq}{r^2} \qquad \text{(radially outwards)}$$

For equilibrium to be there

$$dF=2T\sin\left(\frac{d\theta}{2}\right)$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_0dq}{r^2}\simeq2T\left(\frac{d\theta}{2}\right)$$

$$\left\{\because \text{ for small angle, } \sin\left(\frac{d\theta}{2}\right)\simeq\frac{d\theta}{2}\right\}$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_0\left(\frac{q}{2\pi r}\right)dl}{r^2}=T\left(\frac{dl}{r}\right)$$

$$\Rightarrow \quad T=\frac{qq_0}{8\pi^2\varepsilon_0 r^2}$$

ILLUSTRATION 7

Two small equally charged spheres, each of mass m, are suspended from the same point by light silk threads of length l. The separation between the spheres is $x\ll l$. Calculate the rate $\frac{dq}{dt}$ with which the charge leaks off each sphere if their velocity of approach varies as $v=\frac{\alpha}{\sqrt{x}}$, where α is a positive constant.

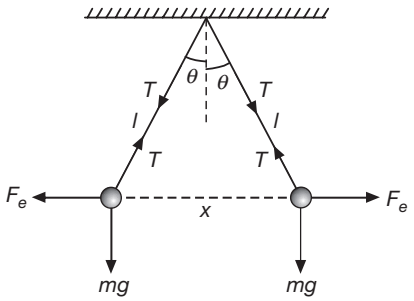
SOLUTION

In this problem, let us first calculate the value of x in terms of other known parameters. Let us consider the particles to be in equilibrium, then

$$T\cos\theta = mg \qquad \ldots(1)$$

$$T\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$



Since $x \ll l$

$\therefore$ $\tan\theta$ is very small and hence $\tan\theta \simeq \theta = \dfrac{x}{2l}$

$$\Rightarrow \quad \frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mg\, x^2}$$

$$\Rightarrow \quad x^3 = \frac{q^2 l}{2\pi\varepsilon_0 mg} \qquad \ldots(3)$$

$$\Rightarrow \quad x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$$

Take derivative of (3) w.r.t. time, we get

$$3x^2\frac{dx}{dt} = \left(\frac{l}{2\pi\varepsilon_0 mg}\right)2q\left(\frac{dq}{dt}\right)$$

But according to the problem, $v = \dfrac{dx}{dt} = \dfrac{\alpha}{\sqrt{x}}$

$$\Rightarrow \quad 3x^2\frac{\alpha}{\sqrt{x}} = \left(\frac{x^3}{q^2}\right)2q\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{3\alpha}{2}\left(\frac{q}{x^{3/2}}\right)$$

But $\dfrac{x^3}{q^2} = \dfrac{l}{2\pi\varepsilon_0 mg}$

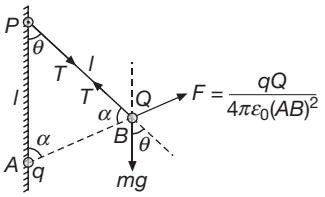
$$\Rightarrow \quad \frac{dq}{dt} = \frac{3\alpha}{2}\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$$

ILLUSTRATION 8

A particle A having a charge q is fixed on a vertical (insulated) wall. A second particle B of mass m, charge Q is suspended by a silk thread of length l from a point P on the wall that is at a distance l above the A. Calculate the angle made by the thread with the vertical, when B stays in equilibrium.

SOLUTION

Before starting the problem, let us draw a diagram that gives a visualisation of the problem.



Let the angle of suspension be θ. If $\angle PAB = \alpha$, then $\angle PBA = \alpha$ (because angles opposite to equal sides are equal). Also, in ΔPAB

$$2\alpha + \theta = \pi$$

$$\Rightarrow \quad \alpha = \frac{\pi}{2} - \frac{\theta}{2} \qquad \ldots(1)$$

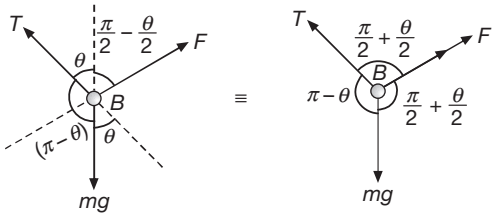
Further from the diagram, we observe three forces to be acting on B,

(a) Weight mg (acting vertically downwards)

(b) Tension T (acting along QP)

(c) Coulombic force $F = \dfrac{Qq}{4\pi\varepsilon_0 (AB)^2}$ along AB

Free body diagram of B is shown here



Now, we can make the use of Lami's Theorem and get

$$\frac{mg}{\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)}=\frac{F}{\sin(\pi-\theta)}$$

$$\Rightarrow \quad \frac{mg}{\cos\left(\frac{\theta}{2}\right)}=\frac{F}{\sin\theta} \qquad \ldots(2)$$

where $F=\dfrac{Qq}{4\pi\varepsilon_0(AB)^2}$

In triangle PAB

$$\frac{l}{\sin\alpha}=\frac{AB}{\sin\theta}$$

$$\Rightarrow \quad \frac{l}{\cos\left(\frac{\theta}{2}\right)}=\frac{AB}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} \qquad \left\{\because\ \alpha=\frac{\pi}{2}-\frac{\theta}{2}\right\}$$

$$\Rightarrow \quad AB=2l\sin\left(\frac{\theta}{2}\right) \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{mg}{\cos\left(\frac{\theta}{2}\right)}=\frac{Qq}{4\pi\varepsilon_0\left(4l^2\sin^2\left(\frac{\theta}{2}\right)\right)\sin\theta}$$

$$\Rightarrow \quad mg=\frac{Qq}{32\pi\varepsilon_0 l^2\sin^3\left(\frac{\theta}{2}\right)}$$

$$\left\{\because\ \sin\theta=2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right\}$$

$$\Rightarrow \quad \sin^3\left(\frac{\theta}{2}\right)=\frac{Qq}{32\pi\varepsilon_0 mgl^2}$$

$$\Rightarrow \quad \sin\left(\frac{\theta}{2}\right)=\left(\frac{Qq}{32\pi\varepsilon_0 mgl^2}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \theta=2\sin^{-1}\left[\left(\frac{Qq}{32\pi\varepsilon_0 mgl^2}\right)^{\frac{1}{3}}\right]$$

ILLUSTRATION 9

Three particles, each of mass 1 g and carrying a charge q are suspended from a common point by three insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g=10\ \text{ms}^{-2}$).

SOLUTION

After drawing the diagram and applying Coulomb's Law, we get

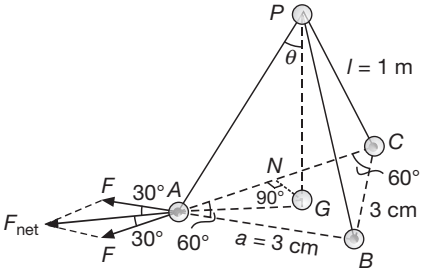
$$F=F_{AB}=F_{BC}=F_{AC}=\frac{q^2}{4\pi\varepsilon_0 a^2}$$

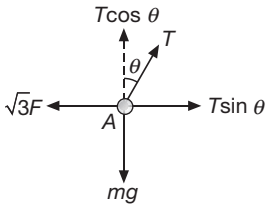
Also,

$$F_{\text{net}}=\sqrt{F^2+F^2+2F^2\cos 60}$$

$$\Rightarrow \quad F_{\text{net}}=\sqrt{3}F=\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2}$$

Let us now draw the free body diagram for A. The following forces are acting on A (see figure).





(a) Tension T in the thread AP acting upwards (as shown)

(b) Force $\sqrt{3}F$, where $F=\dfrac{q^2}{4\pi\varepsilon_0 a^2}$

(c) Weight mg (vertically down)

For A to be in equilibrium, we get

$$T\sin\theta=\sqrt{3}F$$

$$T\cos\theta=mg$$

$$\Rightarrow \quad \tan\theta=\frac{\sqrt{3}F}{mg} \qquad \ldots(1)$$

But since, we observe from the data provided in the problem, that $AP \gg AB$. So θ must be very small and hence

$$\tan\theta\simeq\theta=\frac{AG}{AP}$$

So, now we need to find AG (where G denotes the centroid of the equilateral triangle ABC).

In ΔAGN,

$$\cos 30 = \frac{AN}{AG}$$

$$\Rightarrow \quad AG = \frac{\left(\frac{a}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{a}{\sqrt{3}} \qquad \left\{\because\ AN = \frac{a}{2}\right\}$$

So, from equation (1), we get

$$\frac{\frac{a}{\sqrt{3}}}{l} = \frac{\sqrt{3}\frac{q^2}{4\pi\varepsilon_0 a^2}}{mg}$$

$$\Rightarrow \quad q^2 = \frac{4\pi\varepsilon_0 mga^3}{3l}$$

$$\Rightarrow \quad q^2 = \frac{1}{9\times10^9} \times \frac{\left[\frac{1}{1000}\times10\times\left(\frac{3}{100}\right)^3\right]}{3(1)} \qquad \left\{\because\ m = \frac{1}{1000}\ \text{kg},\ a = \frac{3}{100}\ \text{m}\right\}$$

$$\Rightarrow \quad q^2 = 10^{-17} = 10\times10^{-18}$$

$$\Rightarrow \quad q = 3.17\times10^{-9}\ \text{C}$$

Test Your Concepts-I

Based on Coulomb's Law

(Solutions on page H.3)

1. Two similar point charges q_1 and q_2 are placed at a distance r apart in air. A dielectric slab of thickness $t(\ll r)$ having dielectric constant K is placed between the charges. Calculate the Coulomb force of repulsion between the charges. Now assume that a slab of thickness half the separation between the charges is placed between the charges and the Coulomb's repulsive force is reduced in the ratio 9 : 4. Calculate K for such a slab.
2. A copper atom consists of copper nucleus that has 29 electrons surrounding it. The atomic weight of copper is 63.5 gmol^{-1}. Now take two pieces of copper each weighing 10 g and transfer an electron from one piece to other for every 1000 atoms in that piece. Calculate the Coulomb force between the two pieces after the transfer of electrons, if the pieces are 1 cm apart. Given $N_A = 6.0\times10^{23}$ mol^{-1}, $e = 1.6\times10^{-19}$ C.
3. A wire of length L is placed along x-axis with one end at the origin. The linear charge density of the wire varies with distance x from the origin as $\lambda = \lambda_0\left(\frac{x^2}{L}\right)$, where λ_0 is a positive constant. Determine the total charge Q on the rod.
4. Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result of immersion in the liquid the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.
5. Calculate the ratio of the electrostatic force to the gravitation force between two electrons, between two protons. At what value of the specific charge (ratio of charge to mass) of a particle would these forces become equal (in their absolute values) in the case of interaction of identical particles?
6. A positive point charge 50 μC is located in the plane xy at the point with radius vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find the electrostatic force $\vec{F}$ and its magnitude on a charge of 2 μC placed at the point with radius vector $\vec{r} = 8\hat{i} - 5\hat{j}$. Here $\vec{r}_0$ and $\vec{r}$ are expressed in metre.
7. A charge $q = 1\ \mu$C is placed at a point $P(1,\ 2,\ -4)$m. Find the electric force on a $-17\ \mu$C charge at point $Q(4,\ 6,\ -16)$m to the nearest approximation.
8. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross-section 10^{-6} m^2. The ring has

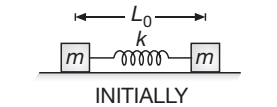
a uniform charge of π coulomb. Find the change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the centre of the ring. Young's modulus of the metal is 2×10^{11} Nm^{-2}.

9. Three identical small balls, each of mass 0.1 g, are suspended at one point on silk threads having a length of $\ell = 20$ cm. What charge should be imparted to each ball so that every thread makes an angle of $\alpha = 30°$ with the vertical?

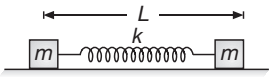
10. A small point mass m has a charge q, which is constrained to move inside a narrow frictionless cylinder. At the base of the cylinder is a point mass of charge Q having the same sign as q. Show that if the mass m is displaced by a small amount from its equilibrium position and released, it will exhibit simple harmonic motion with angular frequency $\omega = \sqrt{\frac{2g}{\ell}}$ where ℓ is the equilibrium position of charge q.

11. A small bead of mass m, charge $-q$ is constrained to move along a frictionless wire. A positive charge Q lies at a distance L from the wire. Initially the bead constrained to move along the wire is just above $+Q$. Now if the bead is displaced a distance x, where $x \ll L$, and released, it will exhibit simple harmonic motion. Obtain an expression for the time period of simple harmonic motion of the bead.

12. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having a spring constant k and an unstretched length L_0. A total charge Q is slowly placed on the system, causing the spring to stretch to an equilibrium length L, as shown. Determine the value of Q, assuming that all the charge resides on the blocks and assuming the blocks as point charges.



INITIALLY



FINALLY

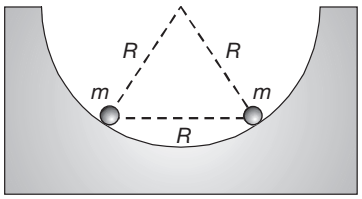
13. Two identical oppositely charged metallic spheres placed 0.5 m apart, attract each other with a force of 0.108 N. When connected to each other by a copper wire for a short while, they begin to repel each other with a force of 0.036 N. Find the initial charge on each one of them.

14. A charge of 50 μC has been distributed between two spheres, such that they repel each other with a force of 1 N with their centres 2 m apart. Find the distribution.

15. In an H atom, an electron revolves round a proton in a circular orbit of 0.53 Å. Calculate the radial acceleration, the angular velocity and the period of revolution of the electron.

16. Show that the gravitational force is negligible in comparison to the electrostatic force in an Hydrogen atom in which an electron is revolving around the proton in a circle of radius 0.53 Å.

17. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, non-conducting walls, the beads move, and at equilibrium they are a distance R apart (shown in figure). Determine the charge on each bead.

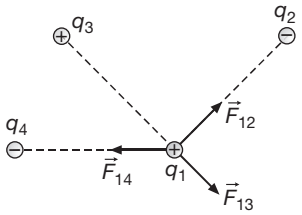


18. Two particles each of charge 10^{-7} C and mass 5 g, stay in limiting equilibrium on a horizontal surface. The particles have a separation of 10 cm between them. Assume the coefficient of friction between each particle and the table to be μ. Calculate μ.

19. A charge Q is divided into two parts. Calculate the ratio of the charges on the divided parts such that force between the divided parts is the maximum.

PRINCIPLE OF SUPERPOSITION

This principle is used to calculate the force on a charge due to an assembly of charges placed near it. According to this principle, "the net force on a charge due to the others in the assembly is calculated by taking the vector sum of all the forces acting on that charge, one at a time". Figure below shows an arrangement of four interacting charged particles.



So, net force on q_1, according to the principle is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

In general, the net force on q_1 due to assembly of N charges is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots\ldots\ldots + \vec{F}_{1N}$$

Here, we must note one thing that force between any pair of charges is simply not affected by the presence of remaining others.

Also, this principle will also be used to find the electric field due to an assembly of charges at a point.

Problem Solving Technique(s)

Suppose we wish to calculate the net force on q_1 due to q_2, q_3 and q_4 (in the arrangement already shown), then we follow the following series of steps.

STEP-1: Find out whether the force due to a given charge on q_1 is attractive or repulsive. Draw the force vector with its tail at q_1, either towards q_1 (in case of repulsion) or away from q_1 (in case of attraction).

STEP-2: Find out the magnitude of each force, using Coulomb's Law. Do not take into account the sign of the charges (the signs account for the direction of force on q_1, already that has been taken into account in Step 1). For this step to be completed, you will also require the separation between q_1 and others i.e., r_{12}, r_{13}, r_{14} etc., then

$$F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2},\ F_{13} = \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}^2},\ F_{14} = \frac{q_1 q_4}{4\pi\varepsilon_0 r_{14}^2}$$

STEP-3: Now select a convenient coordinate axis, so as to calculate the components of these forces (say x and y components). Then in 2D space (say xy plane),

$$F_{1x} = (F_{12})_x + (F_{13})_x + (F_{14})_x$$

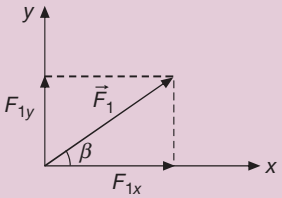
and $$F_{1y} = (F_{12})_y + (F_{13})_y + (F_{14})_y$$

Here, please note that all vectors must be expressed in terms of unit vectors $(\hat{i}, \hat{j}$ or $\hat{k})$ notations, unless otherwise indicated.

STEP-4: As a final step, if we wish to calculate the magnitude of the net force on q_1, then

$$F_1 = \sqrt{(F_{1x})^2 + (F_{1y})^2}$$

where $$\vec{F}_1 = (F_{1x})\hat{i} + (F_{1y})\hat{j}$$



This vector can be diagramatically expressed as

So, if we say that, let $\vec{F}_1$ makes an angle β with x-axis, then the direction is given by

$$\tan\beta = \frac{F_{1y}}{F_{1x}}$$

STEP-5: If need be, then we can extend the same to 3D space, where we modify the equations as

$$F_{1x} = (F_{12})_x + (F_{13})_x + (F_{14})_x$$

$$F_{1y} = (F_{12})_y + (F_{13})_y + (F_{14})_y$$

$$F_{1z} = (F_{12})_z + (F_{13})_z + (F_{14})_z$$

and then

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2 + F_{1z}^2}$$

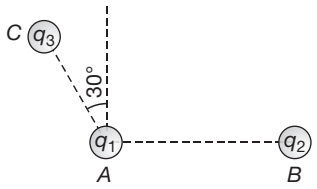
where

$$\vec{F}_1 = (F_{1x})\hat{i} + (F_{1y})\hat{j} + (F_{1z})\hat{k}$$

To have a basic understanding of all the steps, see the following Illustrative Examples.

ILLUSTRATION 10

Figure below shows three charges q_1, q_2 and q_3 placed at A, B and C respectively. If $q_1 = -1\ \mu C$, $q_2 = +3\ \mu C$, $q_3 = -2\ \mu C$, $AB = 15$ cm and $AC = 10$ cm. Calculate the net force that acts on q_1 placed at A.



SOLUTION

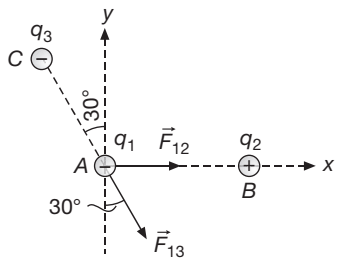
Here we shall use the concept of Principle of Superposition along with the problem solving strategy we have learnt already.

Before we start, let us consider the origin to be at A and the reconstruction of the diagram is shown.

So, we have drawn/shown all the forces that are acting on the charge q_1.

$$F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2} = \frac{(9\times10^9)(10^{-6})(3\times10^{-6})}{(15\times10^{-2})^2} = 1.2\ \text{N}$$

$\Rightarrow \quad F_{12} = 1.2\ \text{N}$ (attractive in nature)



Similarly

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}$$

$$\Rightarrow \quad F_{13} = \frac{(9\times10^9)(10^{-6})(2\times10^{-6})}{(10\times10^{-2})^2} = 1.8\ \text{N}$$

$\Rightarrow \quad F_{13} = 1.8\ \text{N}$ (repulsive in nature)

Since both the forces have different directions, so we shall apply principle of superposition in component form along x and y -axis.

$$\Rightarrow \quad F_{1x} = (F_{12})_x + (F_{13})_x = F_{12} + F_{13}\sin(30°)$$

and $\quad F_{1y} = (F_{12})_y + (F_{13})_y = 0 - F_{13}\cos(30°)$

$$\Rightarrow \quad F_{1x} = 1.2 + 1.8\sin(30°)$$

$$\Rightarrow \quad F_{1x} = 2.1\ \text{N}$$

and $\quad F_{1y} = -1.8\dfrac{\sqrt{3}}{2} = -1.6\ \text{N}$

(Negative sign shows that F_{1y} acts in the downward direction)

$$\Rightarrow \quad \vec{F}_1 = \left(2.1\hat{i} - 1.6\hat{j}\right)\ \text{N}$$

and $\quad \left|\vec{F}_1\right| = \sqrt{(2.1)^2 + (1.6)^2} = \sqrt{4.41 + 2.56} = 2.64\ \text{N}$

ILLUSTRATION 11

Four identical free charges each of value q are located at the corners of a square of side a. What must be the charge Q that has to be placed at the centre of the system so that the system stays in equilibrium?

SOLUTION

Let us here consider the equilibrium of any charge q at 1 (say).

For equilibrium of charge at 1, we have

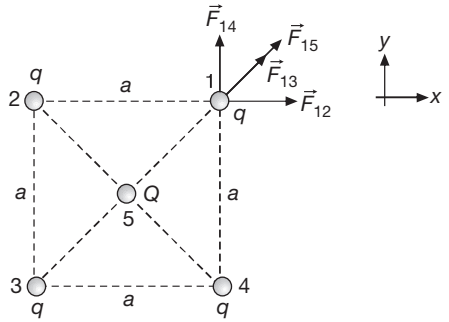
$$\sum \vec{F}_1 = \vec{0}$$

$$\Rightarrow \quad \sum F_{1x} = 0 \qquad \ldots(1)$$

$$\Rightarrow \quad \sum F_{1y} = 0 \qquad \ldots(2)$$

Out of these two equations any one can be selected as both of these will give identical results. So, lets take

$$\sum F_{1x} = 0$$



Now, $F_{1x} = \sum F_{1x} = (F_{12})_x + (F_{13})_x + (F_{14})_x + (F_{15})_x$

$$\Rightarrow \quad F_{1x} = F_{12} + F_{13}\cos(45°) + 0 + F_{15}\cos(45°)$$

$$\Rightarrow \quad F_{1x} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a^2} + \frac{q^2}{(\sqrt{2}a)^2}\frac{1}{\sqrt{2}} + \frac{Qq}{\left(\frac{a}{\sqrt{2}}\right)^2}\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad F_{1x} = \frac{1}{4\pi\varepsilon_0 a^2}\left(q^2 + \frac{q^2}{2\sqrt{2}} + \sqrt{2}Qq\right)$$

So, for $F_{1x} = 0$, we get

$$q\left(1 + \frac{1}{2\sqrt{2}}\right) + \sqrt{2}Q = 0$$

$$\Rightarrow \quad Q = -(1 + 2\sqrt{2})\frac{q}{4}$$

We shall get the same result even if we would have taken $F_{1y} = 0$ $\left(\text{or } \sum F_{1y} = 0\right)$

EQUILIBRIUM OF THREE CHARGES

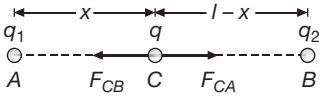
Equilibrium of q when q_1 and q_2 are fixed at a separation l from each other.

CASE-1: q_1 and q_2 similar in nature

Here, the point C, where the net force due to q_1 and q_2 on q will be zero must lie between the charges q_1 and q_2 on the line joining them, towards the charge of smaller magnitude.

If $AC = r_1$, $CB = r_2$ and $AB = l$ (given)

then $|\vec{F}_{CA}| = |\vec{F}_{CB}|$



$$\Rightarrow \quad \frac{q_1 q}{4\pi\varepsilon_0 r_1^2} = \frac{q_2 q}{4\pi\varepsilon_0 r_2^2}$$

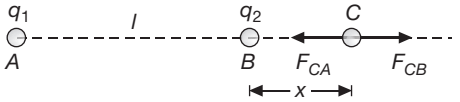
$$\Rightarrow \quad \frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{x}{l-x}\right)^2$$

CASE-2: q_1 and q_2 opposite in nature and $|q_2| < |q_1|$

Here the point C, where the net force due to q_1 and q_2 on q will be zero must lie on the line joining the two charges q_1 and q_2 but outside them towards the charge of smaller magnitude i.e. q_2.

If again $AC = r_1$, $BC = r_2$ and $AB = l$ (given), then

$$|\vec{F}_{CA}| = |\vec{F}_{CB}|$$



$$\Rightarrow \quad \frac{q_1 q}{4\pi\varepsilon_0 r_1^2} = \frac{q_1 q}{4\pi\varepsilon_0 r_2^2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{l+x}{x}\right)^2$$

Conceptual Note(s)

Electrostatic Equilibrium

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

(a) Stable Equilibrium: A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

(b) Unstable Equilibrium: If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

(c) Neutral Equilibrium: If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

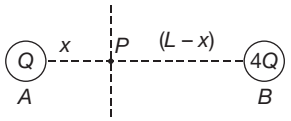
*However if, q_1 and q_2 are not fixed, then equilibrium of q will exactly be the same as done before, but if we have to keep q_1 and q_2 also in equilibrium, then we have to make the net force on q_1 (at A) due to q_2 (at B) and q (at C) to be zero and then repeat the same for q_2 also to get the desired results. See **Illustration(s)** below to learn how to perform in these kind of problems.*

ILLUSTRATION 12

Two free charges $+Q$ and $+4Q$ are placed at a separation L. Find the magnitude, sign and the location of the third charge that makes the system to stay in equilibrium.

SOLUTION

Since both the charges have similar nature, so irrespective of a nature of third charge, the location of the third charge must be somewhere between $+Q$ and $+4Q$ on the line joining them. Let the net force be zero at P at a distance x from $+Q$.



$$\Rightarrow \quad |\vec{F}_{PA}| = |\vec{F}_{PB}|$$

$$\Rightarrow \quad \frac{Qq}{4\pi\varepsilon_0 x^2} = \frac{(4Q)q}{4\pi\varepsilon_0 (L-x)^2}$$

$$\Rightarrow \quad \frac{L-x}{x} = \pm 2$$

$\frac{L-x}{x} = 2$	$\frac{L-x}{x} = -2$
$\Rightarrow \quad L = 3x$	$\Rightarrow \quad L - x = -2x$
$\Rightarrow \quad x = \frac{L}{3}$	$\Rightarrow \quad x = -L$
	But this coordinate lies outside A and B so, this value of x is REJECTED.

$\Rightarrow$ q must be placed at $\frac{L}{3}$ from Q or $\frac{2L}{3}$ from $4Q$.

To find the charge q, such that the system is in equilibrium, we proceed further by taking either

$$\sum F_A = 0 \quad \text{OR} \quad \sum F_B = 0$$

So let us take

$$\sum F_A = 0$$

$$\Rightarrow \quad F_{AP} + F_{AB} = 0$$

$$\Rightarrow \quad \frac{Qq}{4\pi\varepsilon_0 \left(\frac{L}{3}\right)^2} + \frac{(4Q)Q}{4\pi\varepsilon_0 L^2} = 0$$

$$\Rightarrow \quad 9q + 4Q = 0$$

$$\Rightarrow \quad q = -\frac{4Q}{9}$$

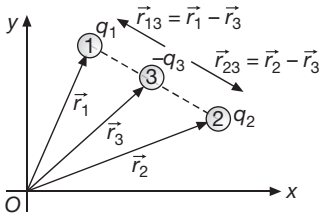
- Please note that we could also have taken $\sum F_B = 0$, to get the same result.
- Also, note that even if $q = +\frac{4Q}{9}$, then still it will be in equilibrium, but then the system would not be in equilibrium.

ILLUSTRATION 13

Two positive charges q_1 and q_2 are located at points 1 and 2 with radius vectors $\vec{r}_1$ and $\vec{r}_2$. Find a negative charge q_3 and the radius vector $\vec{r}_3$ of a point 3 at which it has to be placed for the force acting on each of the three charges to be equal to zero.

SOLUTION

Before we start the problem, we must keep in mind that the radius vector of any point P is actually the position vector (PV) of the point P which is the vector drawn from the origin to the point P.
So, let the origin be at O (see figure)



For equilibrium of $-q_3$, we have

$$\vec{F}_{31} + \vec{F}_{32} = \vec{0}$$

$$\Rightarrow \quad \frac{q_1 q_3}{4\pi\varepsilon_0 |\vec{r}_1 - \vec{r}_3|^3}(\vec{r}_1 - \vec{r}_3) + \frac{q_2 q_3}{4\pi\varepsilon_0 |\vec{r}_2 - \vec{r}_3|^3}(\vec{r}_2 - \vec{r}_3) = \vec{0} \qquad \ldots(1)$$

Also, from figure, we see that

$$\hat{r}_{13} = -\hat{r}_{23}$$

$$\Rightarrow \quad \frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|} = -\frac{\vec{r}_2 - \vec{r}_3}{|\vec{r}_2 - \vec{r}_3|} \qquad \ldots(2)$$

Put (2) in (1), we get

$$\frac{q_1}{|\vec{r}_1 - \vec{r}_3|^2} = \frac{q_2}{|\vec{r}_2 - \vec{r}_3|^3}$$

$$\Rightarrow \quad \pm\sqrt{q_1}\,(\vec{r}_2-\vec{r}_3)=\sqrt{q_2}\,(\vec{r}_1-\vec{r}_3)$$

Taking positive sign, we get

$$\sqrt{q_1}\vec{r}_2-\sqrt{q_2}\vec{r}_1=\left(\sqrt{q_1}-\sqrt{q_2}\right)\vec{r}_3$$

$$\Rightarrow \quad \vec{r}_3=\frac{\sqrt{q_1}\vec{r}_2-\sqrt{q_2}\,\vec{r}_1}{\sqrt{q_1}-\sqrt{q_2}}$$

This must be REJECTED as it will lie outside q_1 and q_2.

Taking negative sign, we get

$$\sqrt{q_1}\vec{r}_2+\sqrt{q_2}\vec{r}_1=\left(\sqrt{q_1}+\sqrt{q_2}\right)\vec{r}_3$$

$$\Rightarrow \quad \vec{r}_3=\frac{\sqrt{q_1}\vec{r}_2+\sqrt{q_2}\vec{r}_1}{\sqrt{q_1}+\sqrt{q_2}}$$

Clearly, this position vector or radius vector will lie on the line joining q_1 and q_2, between them.

So, $\vec{r}_3=\dfrac{\sqrt{q_1}\vec{r}_2+\sqrt{q_2}\vec{r}_1}{\sqrt{q_1}+\sqrt{q_2}}$

For q_1 to be in equilibrium, let charge placed on 3 be q_3, then

$$\vec{F}_{12}+\vec{F}_{13}=\vec{0}$$

$$\Rightarrow \quad \frac{q_1q_2}{4\pi\varepsilon_0|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)+\frac{q_1q_3}{4\pi\varepsilon_0|\vec{r}_1-\vec{r}_3|^3}(\vec{r}_1-\vec{r}_3)=\vec{0}$$

While writing the above expression, we have not taken any nature of q_3. The answer will have its sign attached to q_3. So, as of now q_3 is assumed to be positive and then lets see what happens. Also, as per the assumption made

$$\hat{r}_{12}=\hat{r}_{13}$$

$$\Rightarrow \quad \frac{q_2}{|\vec{r}_1-\vec{r}_2|^2}=-\frac{q_3}{|\vec{r}_1-\vec{r}_3|^2} \qquad \left\{\because \frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}=\frac{\vec{r}_1-\vec{r}_3}{|\vec{r}_1-\vec{r}_3|}\right\}$$

$$\Rightarrow \quad q_3=-q_2\left|\frac{\vec{r}_1-\vec{r}_3}{\vec{r}_1-\vec{r}_2}\right|^2 \qquad \ldots(3)$$

Now, $\vec{r}_1-\vec{r}_3=\vec{r}_1-\left(\dfrac{\sqrt{q_1}\vec{r}_2+\sqrt{q_2}\vec{r}_1}{\sqrt{q_1}+\sqrt{q_2}}\right)$

$$\Rightarrow \quad \vec{r}_1-\vec{r}_3=\frac{\sqrt{q_1}\vec{r}_1+\cancel{\sqrt{q_2}\vec{r}_1}-\sqrt{q_1}\vec{r}_2-\cancel{\sqrt{q_2}\vec{r}_1}}{\sqrt{q_1}+\sqrt{q_2}}$$

$$\Rightarrow \quad \vec{r}_1-\vec{r}_3=\frac{\sqrt{q_1}(\vec{r}_1-\vec{r}_2)}{\sqrt{q_1}+\sqrt{q_2}}$$

$$\Rightarrow \quad \left|\frac{\vec{r}_1-\vec{r}_3}{\vec{r}_1-\vec{r}_2}\right|^2=\frac{q_1}{\left(\sqrt{q_1}+\sqrt{q_2}\right)^2}$$

Using (3), we get

$$\Rightarrow \quad q_3=-\frac{q_1q_2}{\left(\sqrt{q_1}+\sqrt{q_2}\right)^2}$$

ILLUSTRATION 14

Two identical positive point charges, each having a charge Q are fixed at a separation $2a$. A point charge q lies midway between the fixed charges. Show that for small

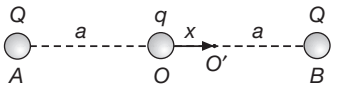
(a) displacement (compared to a) along the line joining the fixed charges, the charge q executes SHM, if it is positive in nature.

(b) lateral displacement, the charge q executes SHM, if it is negative in nature.

Compare the periods of oscillations in the above two cases.

SOLUTION

(a) When q is displaced along the line joining the fixed charges. Let the displacement of q be $x(\ll a)$.



The restoring force F acting on q is

$$F=-\frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{(a-x)^2}-\frac{1}{(a+x)^2}\right]$$

(Negative sign in the above expression indicates that F is directed towards the mean position)

$$F=-\frac{Qq}{4\pi\varepsilon_0}\frac{4ax}{\left(a^2-x^2\right)^2}$$

Since $x\ll a$

$$\therefore \quad F=-\frac{Qq}{4\pi\varepsilon_0}\frac{4x}{a^3}$$

$$\left\{\because \text{for } x\ll a,\ \left(a^2-x^2\right)^2\cong a^4\right\}$$

$$\Rightarrow \quad m\ddot{x}+\left(\frac{Qq}{\pi\varepsilon_0a^3}\right)x=0$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Qq}{\pi m \varepsilon_0 a^3}\right)x = 0$$

Compare with the standard equation of SHM, that is

$$\ddot{x} + \omega_1^2 x = 0$$

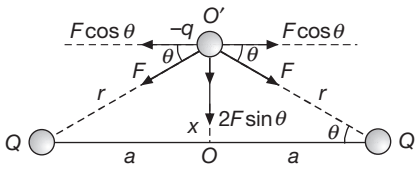
$$\Rightarrow \quad \omega_1 = \sqrt{\frac{Qq}{\pi \varepsilon_0 m a^3}}$$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{\pi \varepsilon_0 m a^3}{Qq}} \qquad \ldots(1)$$

(b) When q is replaced by $-q$ and is given a lateral displacement $OO' = x (\ll a)$. Then the forces acting on $-q$ are shown in figure.

where $F = \dfrac{Qq}{4\pi\varepsilon_0 r^2}$ and $r = \sqrt{a^2 + x^2}$

The charge $-q$ is attracted towards Q with a force F (given above) on resolution, the component $F\cos\theta$ cancels, so that the net force $2F\sin\theta$ restores $-q$ back to mean position O.



$$\Rightarrow \quad F_{net} = -2F\sin\theta$$

$$\Rightarrow \quad m\ddot{x} = -2\frac{Qq}{4\pi\varepsilon_0 (a^2 + x^2)}\sin\theta$$

But $\sin\theta = \dfrac{x}{\sqrt{a^2 + x^2}}$

$$\Rightarrow \quad F_{net} = -2\frac{Qqx}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}$$

Since $x \ll a$

$$\Rightarrow \quad (a^2 + x^2)^{3/2} \cong a^3$$

$$\Rightarrow \quad F_{net} = -2\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)x$$

$$\Rightarrow \quad m\ddot{x} + \left(\frac{Qq}{2\pi\varepsilon_0 a^3}\right)x = 0$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Qq}{2\pi\varepsilon_0 m a^3}\right)x = 0$$

Again compare with standard equation of SHM, we get

$$\ddot{x} + \omega_2^2 x = 0$$

$$\Rightarrow \quad \omega_2 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 m a^3}}$$

$$\Rightarrow \quad T_2 = 2\pi\sqrt{\frac{2\pi\varepsilon_0 m a^3}{Qq}} \qquad \ldots(2)$$

Finally, from (1) and (2), we get

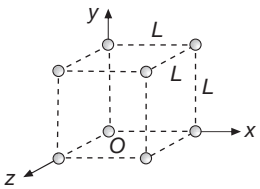
$$\frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$$

Test Your Concepts-II

Based on Principle of Superposition

(Solutions on page H.7)

1. Eight charges each of magnitude Q are located at the corners of a cube of side L (see figure). One corner is at the origin and the edges lie along the rectangular axes. Calculate the net electrostatic force on the charge at $\vec{r} = L\hat{i} + L\hat{j} + L\hat{k}$.



2. Four equal positive charges, each of charge Q are arranged at the corners of a square of side ℓ. A unit positive charge of mass m is placed at point P at height h above the centre of the square. Calculate Q, so that the unit charge is in equilibrium.
3. Two equal positive point charges Q are separated by a distance $2a$. A point test charge q_0 is located in a plane normal to the line joining the two charges and midway between them. Find the radius r of a circle of symmetry in this plane for which the force on the test charge has a maximum value. Find the direction of the force on this charge if it is assumed to be positive. Also find F_{max}.
4. Three identical spheres each having a charge $2q$ and radius R are kept such that each touches the other two. Find the magnitude of the electric force on any sphere due to the other two.
5. Five point charges each of value $+q$ are placed on five vertices of a regular hexagon of side a. Calculate the magnitude of the force on a point charge $-q$ placed at the centre of the hexagon.
6. (a) Twelve equal charges, q, are situated at the corners of a regular 12 sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge Q at the center?
   (b) Suppose one of the 12 q's is removed (say the one at "6 o'clock"). What is the force on Q? Explain your reasoning carefully.
   (Assume the distance between q and Q pair(s) to be a)
7. Four identical charges, each having a charge $+q$ are fixed at the corners of a square of side L. A fifth point charge $-Q$ lies a distance x along the line perpendicular to the plane of the square. Find the force exerted by the charges on $-Q$. Show that for $z \ll a$ the motion of $-Q$ is simple harmonic. Also calculate the time period of oscillations of $-Q$ if it has a mass m?
   (Neglect gravitational force.)
8. Consider a regular polygon with 24 sides. The distance from the center to each vertex is a. Twenty three identical charges each having a charge q are placed at the vertices of the polygon. A single charge Q is placed at the centre of the polygon experiences a force F. What is the magnitude and direction of the force experienced by the charge Q?
9. Three charges, each of magnitude $100\ \mu C$ are located in vacuum at the corners A, B and C of an equilateral triangle of side length 3 m. If charges placed at A and C are positive and the one placed at B is negative, then find the magnitude and direction of force F experienced by the charge placed at C.

ELECTROSTATIC FIELD $(\vec{E})$

The region of space around a source charge (q) in which it can exert a force on a test charge (q_0).

Mathematically, electric field is the force experience per unit test charge q_0 placed in the electrostatic influence of source charge q.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric field strength is a vector quantity directed away from a positive charge and towards the negative charge. SI unit of electric field is newton/coulomb (NC^{-1}) or volt/metre(Vm^{-1}).

The dimensional formula for E is $MLT^{-3}A^{-1}$ i.e., $[E] = MLT^{-3}A^{-1}$

Since $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$

$$\Rightarrow \quad \vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

is the electric field due to a source point charge q at a distance r from it.

Problem Solving Technique(s)

(a) For calculating the electric field due to a charge at a point P at distance r we proceed as follows.

STEP-1: Place a charge of +1 C at point P.

STEP-2: Calculate the force between q and +1 C.

STEP-3: This force is the value of E due to q at the point P.

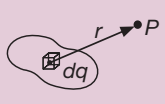
STEP-4: The direction of force experienced by a charge of +1 C is the direction of E due to q at the point P.

(b) If we are to find the electrostatic field due to assembly of charges $q_1, q_2, ..., q_n$ at a point P, then we apply Superposition Principle and then

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$$

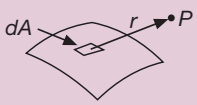
where we find $\vec{E}_1, \vec{E}_2, ... \vec{E}_n$ at point P by the technique mentioned above.

(c) If we are to find the electric field at a point P due to a uniform charge distribution, then we calculate the field due to an infinitesimal element of the distribution at the point P and then integrate it within appropriate limits.

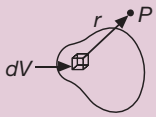


(a) Continuous distribution

(b) Line charge, λ



(c) Surface charge, σ



(d) Volume charge, ρ

$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}$$

For charge distributed on a wire with linear charge density λ,

$$E = \frac{1}{4\pi\varepsilon_0}\int_{\ell} \frac{\lambda d\ell}{r^2}$$

For charge distributed on a surface with surface charge density σ,

$$E = \frac{1}{4\pi\varepsilon_0}\int_{A} \frac{\sigma dA}{r^2}$$

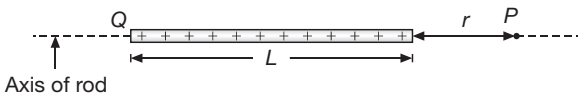
For charge distributed on a volume with volume charge density ρ,

$$E = \frac{1}{4\pi\varepsilon_0}\int_{V} \frac{\rho dV}{r^2}$$

If the charge distributions are uniform, then charge densities can be taken out of the integral to get desired results.

ILLUSTRATION 15

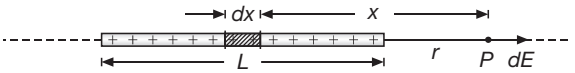
Calculate the electric field due to a uniformly charged rod of length L at a point P at distance r from one end of the rod. Assume total charge on the rod to be Q.



SOLUTION

For this we consider an infinitesimal element of length dx on rod as shown in figure. Charge on the infinitesimal elemental length dx is

$$dq = \frac{Q}{L}dx$$



This dq (an infinitesimal element) can be regarded as a point charge, hence the electric field dE due to this element at point P is dE given by

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad dE = \frac{\left(\frac{Q}{L}\right)dx}{4\pi\varepsilon_0 x^2}$$

The net electric field strength at point P can be given by integrating this expression over the whole length of rod as

$$E_P = \int dE = \int_{r}^{r+L} \frac{Qdx}{4\pi\varepsilon_0 L x^2}$$

$$\Rightarrow \quad E_P = \frac{Q}{4\pi\varepsilon_0 L}\int_r^{r+L} x^{-2}dx$$

$$\Rightarrow \quad E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[-\frac{1}{x}\right]_r^{r+L}$$

$$\Rightarrow \quad E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{r}-\frac{1}{r+L}\right]$$

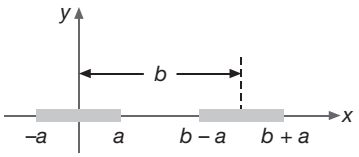
$$\Rightarrow \quad E_P = \frac{Q}{4\pi\varepsilon_0 r(r+L)}$$

SPECIAL CASE: This special case can be taken as a check point too. For $x \gg L$ i.e., for distances far away from the rod, the rod with charge Q must be have as a point charge. In this expression, when we put $r \gg L$, then $E \cong \frac{Q}{4\pi\varepsilon_0 r^2}$ (= field due to a point charge Q at a distance r from it).

ILLUSTRATION 16

Two identical thin rods of length $2a$ carry equal charges $+Q$ that is distributed uniformly along their lengths. The rods lie along the x-axis with their centers separated by a distance $b > 2a$. Show that the magnitude of the force exerted by the left rod on the right one is given by

$$F = \frac{Q^2}{16\pi\varepsilon_0 a^2}\log_e\left(\frac{b^2}{b^2-4a^2}\right)$$



SOLUTION

This problem is a very special one where we shall learn to calculate the force between two extended bodies using the concept of electric field. For this let us divide the problem in two parts.

PART I

Here we shall imagine the rod to the right to be absent and then let us calculate the field due to the rod on the left at a hypothetical point P (say) at a distance d from one end of the rod. Let us consider an element of length dx at a distance x from the point P. If dE be the electric field due to the rod at P, then

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \left(\frac{\lambda}{4\pi\varepsilon_0}\right)x^{-2}dx$$

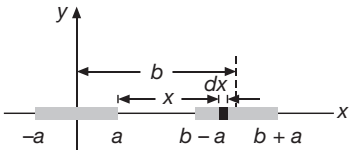
$$\Rightarrow \quad E = \frac{\lambda}{4\pi\varepsilon_0}\left(\int_d^{d+2a} x^{-2}dx\right) = \frac{\lambda(2a)}{4\pi\varepsilon_0(d+2a)}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 d(d+2a)}$$

PART II

Now after we have calculated the field due to one rod, we actually observe that the second rod will be lying in the field of the first rod. For finding the force on this rod let us again consider an infinitesimal element of length dx at a distance x from the nearest end of the left rod. If dF be the force due to this rod then

$$dF = Edq = \frac{Q(\lambda dx)}{4\pi\varepsilon_0 x(x+2a)}$$



Since $\lambda = \frac{Q}{2a}$

$$\Rightarrow \quad dF = \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{Q}{2a}\right)\left[\frac{dx}{x(x+2a)}\right]$$

$$\Rightarrow \quad F = \int dF = \frac{Q^2}{8\pi\varepsilon_0 a}\int_{b-2a}^{b}\frac{dx}{x(x+2a)}$$

$$\Rightarrow \quad F = \frac{Q^2}{8\pi\varepsilon_0 a}\left[-\frac{1}{2a}\log_e\left(\frac{2a+x}{x}\right)\right]\Bigg|_{b-2a}^{b}$$

$$\Rightarrow \quad F = \frac{Q^2}{16\pi\varepsilon_0 a^2}\left[-\log_e\left(\frac{2a+b}{b}\right)+\log_e\left(\frac{b}{b-2a}\right)\right]$$

$$\Rightarrow \quad F = \frac{Q^2}{16\pi\varepsilon_0 a^2}\log_e\left[\frac{b^2}{(b-2a)(b+2a)}\right]$$

$$\Rightarrow \quad F = \frac{Q^2}{16\pi\varepsilon_0 a^2}\log_e\left(\frac{b^2}{b^2-4a^2}\right)$$

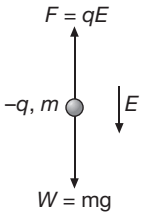
ILLUSTRATION 17

Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge $-10\ \mu C$ and mass 10 mg. (Take $g = 10\ ms^{-2}$)

SOLUTION

Force on a charge q in an electric field $\vec{E}$ is

$$\vec{F} = q\vec{E}$$



According to given problem, we have

$$|\vec{F}| = |\vec{W}|$$

$$\Rightarrow \quad |q|E = mg$$

$$\Rightarrow \quad E = \frac{mg}{|q|} = 10\ NC^{-1}, \text{ in downward direction.}$$

ELECTROSTATIC LINES OF FORCE: PROPERTIES

A line of force is an imaginary path straight or curved such that the tangent to it at any point gives the direction of electrostatic field at that point. A field line is an imaginary line along which a unit positive charge would move when set free. The lines of force are drawn such that the number of lines per unit area of cross-section, (area held normally to the field lines) is proportional to magnitude of $\vec{E}$.

Pattern of Field Lines

The pattern of electric field lines can be obtained by considering the following points.

(a) **Symmetry:** For every point above the line joining the two charges there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

(b) **Near Field:** Very close to a charge, the field due to that charge predominates. Therefore, the lines are radial and spherically symmetric.

(c) **Far Field:** Far from the system of charges, the pattern should look like that of a single point charge of value $Q = \Sigma Q_i$. Thus, the lines should be radially outward, unless $Q = 0$.

(d) **Null Point:** This is a point at which $\vec{E} = \vec{0}$, and no field lines should pass through it.

Properties of Field Lines

(a) Field lines always come (emanate) out of positive charge and enter the negative charge.

(b) The number of lines per unit area through a surface held normally to the line is observed to be proportional to the magnitude of the electric field existing in a given region.

(c) Field lines never cross each other; otherwise the field would be pointing in two different direction at the same point.

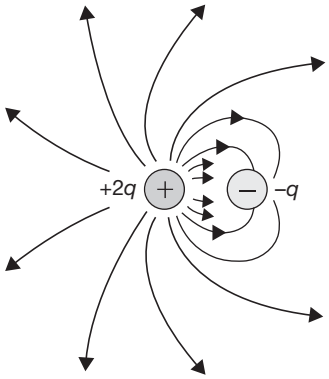
(d) Field lines never form closed loops.

(e) Field lines are always directed from higher potential to lower potential.

(f) Field lines never exist inside a conductor.

(g) If N_1 is the number of field lines coming out of a charge q_1 and N_2 is the number of field lines entering q_2, then

$$\frac{|q_1|}{|q_2|} = \frac{N_1}{N_2}$$



The electric field lines for a point charge +2*q* and a second point charge –*q*. Note that two lines leave the charge +2*q* for every one that terminates on –*q*.

(h) If N_1 is the number of field lines coming out of a charge q_1 and N_2 is the number of field lines coming out of charge q_2, then

$$\frac{q_1}{q_2} = \frac{N_1}{N_2}$$

This relation also exists if field lines are entering both the charges

(i) Tangent to field line at a point gives the direction of field at that point.

(j) Field lines exhibit longitudinal (length wise) contraction, thus indicating that unlike charges attract each other. (See Figure 1)

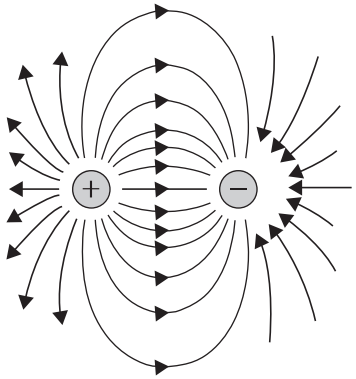


Figure 1 The electric field lines for two equal and opposite point charges an electric dipole. Note that the number of lines that leave the positive charge equals the number that terminate at the negative charge.

(k) Field lines exhibit lateral (sideways) expansion, thus indicating that like charges repel each other. (See Figure 2)

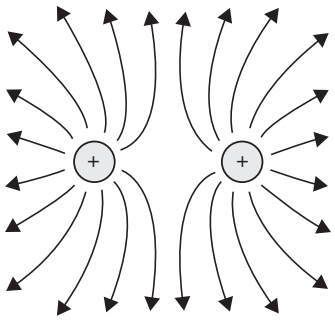


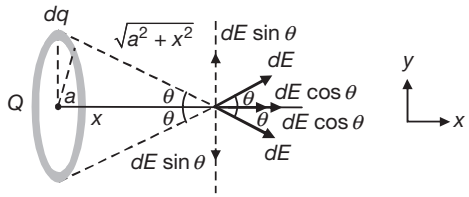
Figure 2 The electric field lines for two equal positive point charge

(l) Field lines always enter or leave a surface at right angles.

ELECTRIC FIELD AT THE AXIS OF A CIRCULAR UNIFORMLY CHARGED RING

Consider a ring of uniform charge density λ. If a is the radius of the ring and Q is the total charge on the ring, then

$$\lambda = \frac{Q}{2\pi a} \qquad ...(1)$$



Consider two elements of length dl placed symmetrically on the two diametrically opposite ends. If dE is the field due to each such element, then $dE\sin\theta$ components will cancel out such that the net field is just due to $dE\cos\theta$ components summed up over the entire ring. So,

$$E_{net} = E_x = \int dE\cos\theta = \int \frac{dq}{4\pi\varepsilon_0\left(\sqrt{a^2+x^2}\right)^2}\cos\theta$$

$$\Rightarrow \quad E_x = \frac{\lambda\cos\theta}{4\pi\varepsilon_0\left(a^2+x^2\right)}\int_0^{2\pi a} dl$$

$$\Rightarrow \quad E_x = \frac{Qx}{4\pi\varepsilon_0\left(a^2+x^2\right)^{3/2}} \text{ (along } +x \text{ axis)}$$

$$\left\{\because \cos\theta = \frac{x}{\sqrt{a^2+x^2}}\right\}$$

Remark(s)

(a) For the point P to lie at far off distance from the centre of the ring i.e., for $x >> a$ we have $\left(x^2+a^2\right)^{3/2} \cong x^3$.

$$\Rightarrow \quad E_x \simeq \frac{Q}{4\pi\varepsilon_0 x^2}$$

This result under the specified condition just matches with the field due to a point charge at a distance x from it.

(b) For this field to be a MAXIMUM

$$\frac{dE}{dx} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left[x\left(x^2+a^2\right)^{-3/2}\right] = 0$$

$$\Rightarrow \quad \left(x^2+a^2\right)^{-3/2} + x\left(-\frac{3}{2}\right)\left(x^2+a^2\right)^{-5/2}(2x) = 0$$

$$\Rightarrow \quad 1=\frac{3x^2}{x^2+a^2}$$

$$\Rightarrow \quad x^2+a^2=3x^2$$

$$\Rightarrow \quad x=\pm\frac{a}{\sqrt{2}}$$

i.e., the field will attain a maximum value at $x=\pm\frac{a}{\sqrt{2}}$ along the axis of the ring and the maximum value equals E_{max} given by

$$E_{max}=\frac{Q\frac{a}{\sqrt{2}}}{4\pi\varepsilon_0\left(a^2+\frac{a^2}{2}\right)^{3/2}}$$

$$\Rightarrow \quad E_{max}=\frac{2Q}{4\pi\varepsilon_0 3\sqrt{3}a^2}$$

$$\Rightarrow \quad E_{max}=\frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$$

Problem Solving Technique(s)

For **symmetrical charge distributions,** if we are asked to calculate the electric field at a point which lies symmetrically with respect to the charge distribution, then we proceed as follows :

STEP-1: Consider an infinitesimal element having a charge dq (say) that lies at a distance r from the point P. If dE is the electric field due to this element then

$$dE=\frac{dq}{4\pi\varepsilon_0 r^2}$$

STEP-2: Now consider another identical mirror infinitesimal element having same charge and located at the same distance from P. Then field due to this element will also be dE.

STEP-3: On resolving dE due to both the elements along suitable axis we observe that one set of components cancels and the net electric field will then be calculated by taking the integral of the component of the electric field due to the contribution of the single element.

$$E_{net}=\int(\text{contribution due to a single element})$$

However, in the case of **unsymmetrical charge distributions**, if we have to calculate the electric field at any point P, then we proceed as follows :

STEP-1: Consider an infinitesimal element having a charge dq (say) that lies at a distance r from the point P. If dE is the electric field due to this element then

$$dE=\frac{dq}{4\pi\varepsilon_0 r^2}$$

STEP-2: Resolve dE along the selected axes (say x and y) so as to get the components dE_x and dE_y.

STEP-3: Integrate dE_x and dE_y separately to get E_x and E_y.

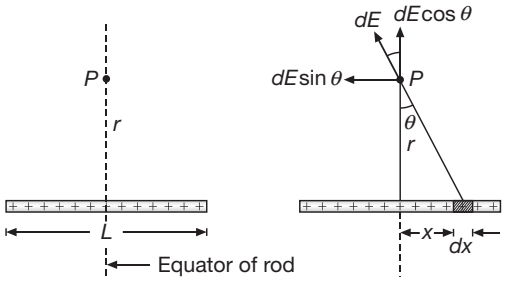
STEP-4: Then $\vec{E}=E_x\hat{i}+E_y\hat{j}$ such that $|\vec{E}|=E=\sqrt{E_x^2+E_y^2}$ and if $\vec{E}$ makes an angle β with x-axis, then $\tan\beta=\frac{E_y}{E_x}$.

ILLUSTRATION 18

Calculate the electric field due to a uniformly charged rod of length L at a point P that lies at a distance r on its perpendicular bisector. Assume charge on the rod to be Q. Discuss the result when $r\gg L$ and $L\gg r$. In the case when $L\gg r$, also draw a plot of $\frac{r}{L}$ (along x-axis) vs $\frac{E}{E_0}$ (on y-axis) where $E_0=\frac{Q}{4\pi\varepsilon_0 L^2}$.

SOLUTION

For this we consider an element of length dx at a distance x from centre of rod as shown in Figure.



The charge on this element is

$$dq = \frac{Q}{L}dx$$

If the strength of electric field at point P due to this infinitesimal charge dq is dE, then

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+x^2)}$$

Let us again take a symmetrical element such that it is just the mirror image of this element. Then we can see that the component $dE\sin\theta$ will get cancelled and net electric field at point P will be due to integration of $dE\cos\theta$ only.

Thus net electric field strength at point P can be given as

$$E = E_P = \int dE\cos\theta$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\int_{-L/2}^{+L/2}\frac{Qdx}{L(r^2+x^2)}\times\frac{r}{\sqrt{r^2+x^2}}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{L}\int_{-L/2}^{+L/2}\frac{dx}{(r^2+x^2)^{3/2}}$$

Let the integral be $I = \int\frac{dx}{(r^2+x^2)^{3/2}}$

To evaluate this let us substitute

$$x = r\tan\theta$$

$$\Rightarrow \quad dx = r\sec^2\theta d\theta$$

$$\Rightarrow \quad I = \int\frac{r\sec^2\theta d\theta}{r^3\sec^3\theta}$$

$$\Rightarrow \quad I = \int\cos\theta d\theta$$

$$\Rightarrow \quad I = \sin\theta$$

Since $x = r\tan\theta$

$$\Rightarrow \quad \sin\theta = \frac{x}{\sqrt{x^2+r^2}}$$

So, $E = E_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{Lr}\left(\frac{x}{x^2+r^2}\right)_{-\frac{L}{2}}^{+\frac{L}{2}}$

$$\Rightarrow \quad E = E_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{Lr}\left(\frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+r^2}}+\frac{\frac{L}{2}}{\sqrt{\frac{L^2}{4}+r^2}}\right)$$

$$\Rightarrow \quad E = \frac{2Q}{4\pi\varepsilon_0 r\sqrt{L^2+4r^2}}$$

In the limit where $r \gg L$ the above expression reduces to the "point-charge" limit and we get

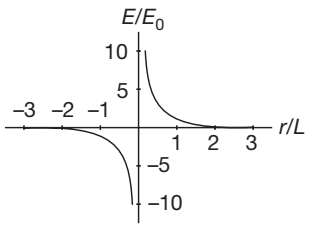
$$E \simeq \frac{Q}{4\pi\varepsilon_0 r^2}$$

On the other hand, when $L \gg r$, we have

$$E \simeq \frac{Q}{2\pi\varepsilon_0 rL} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

In this infinite length limit, the system has cylindrical symmetry. In this case, an alternative approach based on Gauss's Law will also be derived at a later stage.

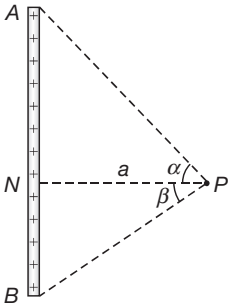
The characteristic behaviour of $\frac{E}{E_0}$ (with $E_0 = \frac{Q}{4\pi\varepsilon_0 l^2}$) as a function of $\frac{r}{L}$ is shown in figure.



Electric field of a non-conducting rod as a function of r/L

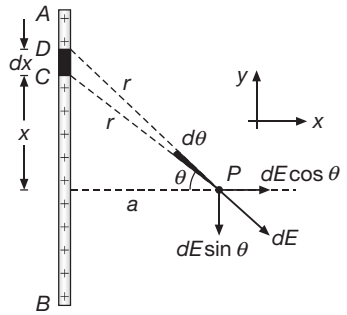
ILLUSTRATION 19

Calculate the electric field due to a uniformly charged rod AB having charge density λ at a point P at perpendicular distance a from the rod, as shown in figure.



SOLUTION

Here it is very important to note the unsymmetrical placement of the point P and hence we must calculate the fields E_x and E_y separately. For this, let us consider an infinitesimal element of length dx at a distance x as shown.



The net electric field at point P due to this infinitesimal element is dE. This dE is resolved into components.

(a) $dE_x = dE\cos\theta$

(b) $dE_y = dE\sin\theta$

The net field E can be calculated by finding E_x and E_y from the above expressions. So

$$E_x = \int dE\cos\theta \text{, where } dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}$$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dx\cos\theta}{r^2} = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dx\cos\theta}{(a^2+x^2)}$$

Also, we observe that

$$\tan\theta = \frac{x}{a}$$

$$\Rightarrow \quad x = a\tan\theta$$

$$\Rightarrow \quad dx = a\sec^2\theta d\theta$$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{a\sec^2\theta\cos\theta d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0 a}\int_{-\beta}^{\alpha} \cos\theta d\theta$$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\beta + \sin\alpha)$$

Similarly, let us calculate the value of E_y, given by

$$E_y = \int dE_y = \int dE\sin\theta$$

$$\Rightarrow \quad E_y = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dx\sin\theta}{(a^2+x^2)}$$

Since $x = a\tan\theta$

$$\Rightarrow \quad dx = a\sec^2\theta d\theta$$

$$\Rightarrow \quad E_y = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{a\sec^2\theta\sin\theta\, d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 a}\int_{-\beta}^{\alpha} \sin\theta d\theta$$

$$\Rightarrow \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 a}\left(-\cos\theta|_{-\beta}^{\alpha}\right)$$

$$\Rightarrow \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\beta - \cos\alpha)$$

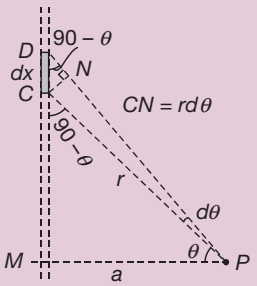
So, the electric field due to a rod of length having uniform charge density λ at a point P, that subtends an angle α at one end and β at the other is given by

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\alpha + \sin\beta)$$

and $$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\beta - \cos\alpha)$$

Misconception Removal

While attempting this problem, we may think that $dx = rd\theta$, but that would be a wrong step. From the magnified diagram of the element shown $PC = r$ and $PN = r$. So its CN that equals $rd\theta$ and not CD. Since CD is extremely small, so $\angle MCP = \angle CDP = 90 - \theta$. Now, we observe the triangle NCD, then



$$\sin(90-\theta)=\frac{CN}{CD}=\frac{r\,d\theta}{dx}$$

$$\Rightarrow \quad dx=\frac{r\,d\theta}{\cos\theta}$$

$$\Rightarrow \quad dx=\frac{a}{\cos^2\theta}d\theta$$

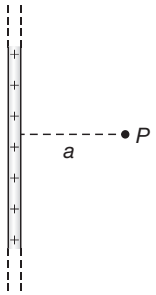
$$\Rightarrow \quad dx=a\sec^2\theta\,d\theta$$

Otherwise we would have got $dx=a\sec\theta d\theta$ which definitely would not fetch us correct result for E.

So, we can also extend the results obtained to more look alike situations as the special cases of the above discussion.

Situation 1:

For an infinite rod having uniform charge density λ, electric field at point P is calculated by taking $\alpha\longrightarrow\frac{\pi}{2}$ and $\beta\longrightarrow\frac{\pi}{2}$.



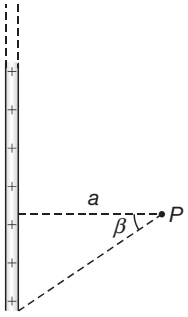
$$\Rightarrow \quad E_x=\frac{\lambda}{2\pi\varepsilon_0 a} \text{ and } E_y=0$$

Situation 2:

For a semi-infinite rod having uniform charge density λ, electric field at point P is calculated by taking

$$\alpha\longrightarrow\frac{\pi}{2} \text{ and } \beta=\beta$$

$$\Rightarrow \quad E_x=\frac{\lambda}{4\pi\varepsilon_0 a}(1+\sin\beta) \text{ and } E_y=\frac{\lambda}{4\pi\varepsilon_0}\cos\beta$$



Situation 3:

For a semi-infinite rod having uniform density λ, the electric field at point P (which lies on the line joining P to wire normally and passing through the known end of wire) is calculated by taking.

$$\alpha\longrightarrow\frac{\pi}{2} \text{ and } \beta\longrightarrow 0$$

$$\Rightarrow \quad E_x=\frac{\lambda}{4\pi\varepsilon_0 a} \text{ and } E_y=\frac{\lambda}{4\pi\varepsilon_0 a}$$

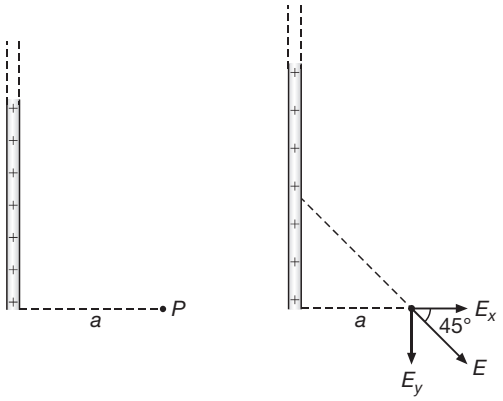


ILLUSTRATION 20

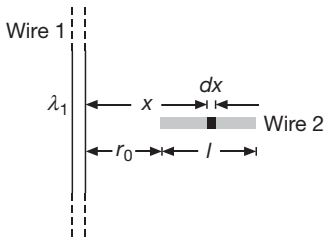
A segment of a charged wire of length l, charge density λ_2, and an infinitely long charged wire, charge density λ_1, lie in a plane at right angles to each other. The separation between the wires is r_0. Determine the force of interaction between the wires.

SOLUTION

Electric field near a long wire is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

The second wire lies in the non uniform field of first wire so that each element of second wire experiences a different magnitude of field due to the first wire.



Let us consider an infinitesimal element of length dx having a charge $dq=\lambda_2 dx$, at a distance x from the

long wire. The infinitesimal force acting on this element dF is given by

$$dF = Edq = \left(\frac{\lambda_1}{2\pi\varepsilon_0 x}\right)\lambda_2 dx$$

The force acting on each element depends on x, the separation between wire 1 and wire 2.

Integrating the expression for dF within the limits $x = r_0$ to $x = r_0 + l$, we obtain

$$F = \int_{r_0}^{r_0+l} \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\frac{dx}{x} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\log_e\left(1+\frac{l}{r_0}\right)$$

ILLUSTRATION 21

A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to Q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

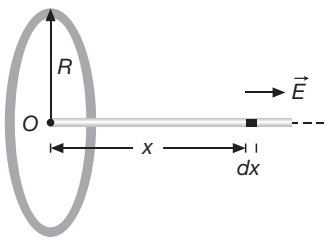
SOLUTION

The electric field strength due to ring at a point on its axis (say x-axis) at distance x from the centre of the ring is given by

$$E(x) = \frac{Qx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

And from symmetry $\vec{E}$ at every point on the axis is directed along the x-axis

Let us consider an element dx on thread which carries the charge $dq = \lambda dx$. The electric force experienced by the element in the field of ring.



$$dF = (\lambda\, dx)E(x) = \frac{\lambda Qx\, dx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

So, $$F = \int_0^\infty \frac{\lambda Qx\, dx}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

On integrating we get, $F = \dfrac{\lambda Q}{4\pi\varepsilon_0 R}$

ILLUSTRATION 22

Calculate the electric field at the centre of a uniformly charged wire in the form of an arc of radius r. The wire subtends an angle ϕ at the centre.

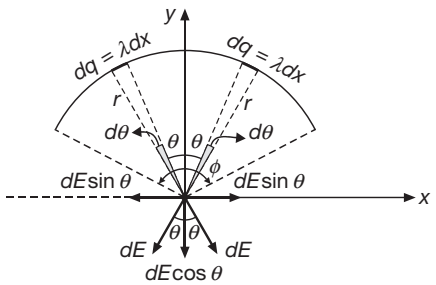
SOLUTION

Consider an element $dq(=\lambda\, dx)$ of the wire that makes an angle θ with y-axis and subtends an angle $d\theta$ at the centre. Then

$$dq = \lambda(r\, d\theta)$$

Since, $$dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda(r\, d\theta)}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad dE = \frac{\lambda\, d\theta}{4\pi\varepsilon_0 r}$$



Again take another element which is the mirror image of the element already taken. On resolution, we observe that the x components cancel, while the net field is equal to integral of contribution due to a single element. Hence

$$E = E_y = \int dE\cos\theta$$

$$\Rightarrow \quad E = \int dE\cos\theta$$

$$\Rightarrow \quad E = \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-\frac{\phi}{2}}^{+\frac{\phi}{2}} \cos\theta d\theta$$

$$\Rightarrow \quad E=\frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$\Rightarrow \quad E=\frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin\left(\frac{\phi}{2}\right)-\sin\left(-\frac{\phi}{2}\right)\right]$$

$$\Rightarrow \quad E=\frac{\lambda}{4\pi\varepsilon_0 r}2\sin\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad E=\frac{\lambda}{2\pi\varepsilon_0 r}\sin\left(\frac{\phi}{2}\right)$$

Problem Solving Technique(s)

(a) For semi-circle, $\phi=\pi$

$$\Rightarrow \quad E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

(b) For Quarter-circle $\phi=\frac{\pi}{2}$

$$\Rightarrow \quad E=\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 r}$$

(c) For circle, $\phi=2\pi$

$$\Rightarrow \quad E=0$$

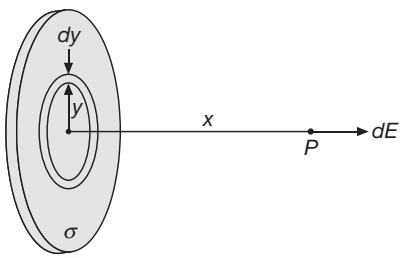
ILLUSTRATION 23

Calculate the electric field at point P that lies on the axis of a uniformly charged disc of radius R having charge density σ at a distance x from the centre.

SOLUTION

To find electric field at point P due to this disc let us consider an elemental ring of radius y and width dy. If dq be the charge on this infinitesimal element of area $dA=2\pi y dy$, then

$$dq=\sigma dA=\sigma(2\pi y dy)$$



Now we know that electric field strength due to a ring of radius R, charge Q, at a distance x from its centre on its axis can be given as

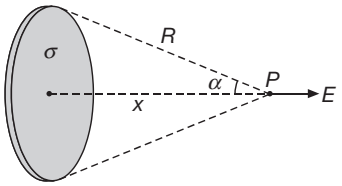
$$E_{\text{axis}}=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+R^2)^{3/2}} \qquad \text{(Derived earlier)}$$

So, due to the infinitesimal elemental ring the electric field strength dE at point P is

$$dE=\frac{1}{4\pi\varepsilon_0}\frac{(dq)x}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad dE=\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi y\,dy\,x}{(x^2+y^2)^{3/2}}$$

Please note that, here once point P is taken, then value of x remains fixed. So,



$$E=E_{\text{axis}}=\int_0^R dE$$

$$\Rightarrow \quad E=\int dE=\frac{1}{4\pi\varepsilon_0}\int_0^R\frac{\sigma 2\pi xy\,dy}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad E=\frac{\sigma\pi x}{4\pi\varepsilon_0}\int_0^R\frac{2y\,dy}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad E=\frac{\sigma x}{4\pi\varepsilon_0}\left[-\frac{2}{\sqrt{x^2+y^2}}\Big|_{y=0}^{y=R}\right]$$

$$\Rightarrow \quad E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

Also, we observe here that, if we assume the complete disc to subtend an apex angle α at P, then, the field can also be expressed as

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]=\frac{\sigma}{2\varepsilon_0}(1-\cos\alpha)$$

$$\left(\text{Because } \cos\alpha=\frac{x}{\sqrt{R^2+x^2}}\right)$$

So, finally, we get

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]=\frac{\sigma}{2\varepsilon_0}(1-\cos\alpha)$$

Remark(s)

(a) Please note that here we have used

$$dE=\frac{(dq)x}{4\pi\varepsilon_0(a^2+x^2)^{3/2}} \quad ...(1)$$

But actually the result obtained from this expression is

$$E=\frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{x}-\frac{1}{\sqrt{R^2+x^2}}\right] \quad ...(2)$$

where it would not be possible to calculate E at $x=0$. So, to be precise with the boundaries here, we just calculate $E(\text{at } x=0)$ and $E(\text{as } x\to 0)$ and both will yield different results.

$E(\text{at } x=0)=0$ {from expression (2)}

and $E(\text{as } x\to 0)=\frac{\sigma}{2\varepsilon_0}$ (i.e., very close to the disc) {from expression (1)}

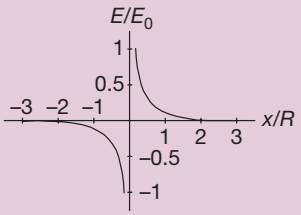
So, we can say that E has two different values i.e., at the centre and just outside the disc and hence E shows discontinuity at $x=0$.

(b) So, $E_x=\begin{cases}\frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right), & x>0\\ \frac{\sigma}{2\varepsilon_0}\left(-1-\frac{x}{\sqrt{R^2+x^2}}\right), & x<0\end{cases}$

(c) On the other hand we may also consider the limit where $R\gg x$. Physically this means that the plane is very large, or the field point P is extremely close to the surface of the plane. The electric field in this limit becomes, in unit vector notation,

$$\vec{E}=\begin{cases}\frac{\sigma}{2\varepsilon_0}\hat{i}, & x>0\\ -\frac{\sigma}{2\varepsilon_0}\hat{i}, & x<0\end{cases}$$

The plot of the electric field in this limit is shown in figure

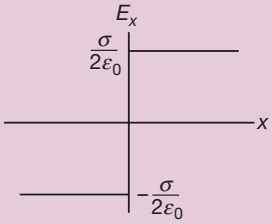


Notice the discontinuity in electric field as we cross the plane. The discontinuity is given by

$$\Delta E_x=E_{x+}-E_{x-}=\frac{\sigma}{2\varepsilon_0}-\left(-\frac{\sigma}{2\varepsilon_0}\right)=\frac{\sigma}{\varepsilon_0}$$

As we shall see during the study of Gauss's Law that if a given surface has a charge density σ, then the normal component of the electric field across that surface always exhibits a discontinuity with $\Delta E_n=\frac{\sigma}{\varepsilon_0}$.

(d) The electric field $\frac{E_x}{E_0}$, where $E_0=\frac{\sigma}{2\varepsilon_0}$ as a function of $\frac{x}{R}$ is shown in figure



Electric field of an infinitely large non-conducting plane

(e) To show that the "point-charge" limit is retained for $x\gg R$, we make use of the expansion series, in which

$$1-\frac{x}{\sqrt{x^2+R^2}}=1-\left(1+\frac{R^2}{x^2}\right)^{-1/2}=$$

$$1-\left(1-\frac{1}{2}\frac{R^2}{x^2}+....\right)\approx\frac{R^2}{2x^2}$$

This gives

$$E_x=\frac{\sigma}{2\varepsilon_0}\frac{R^2}{2x^2}=\frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^2}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

which is indeed the expected "point-charge" result.

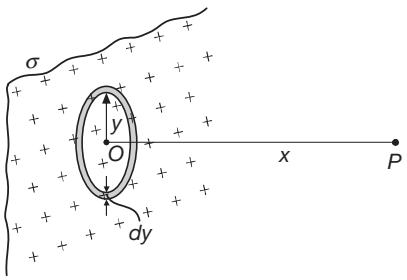
ILLUSTRATION 24

Calculate the electric field strength due to a uniformly charged infinite sheet having charge density σ at a point P at perpendicular distance x from the sheet.

SOLUTION

Let us consider an infinitesimal elemental ring of radius y having width dy with centre at O as shown. If dq be the charge on this ring, then

$$dq = \sigma(2\pi y\, dy)$$



The electric field strength due to this elemental ring at point P at distance x is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dqx}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\int_0^\infty \frac{\sigma(2\pi y\, dy)x}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad E = \frac{\pi\sigma x}{4\pi\varepsilon_0}\int_0^\infty \frac{2y\, dy}{(x^2+y^2)^{3/2}}$$

$$\Rightarrow \quad E = \frac{\sigma x}{4\varepsilon_0}\int_0^\infty (x^2+y^2)^{-3/2}\, 2y\, dy$$

Since, $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}$, $n \neq -1$

$$\Rightarrow \quad E = \frac{\sigma x}{4\varepsilon_0}\left(\frac{(x^2+y^2)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\Bigg|_{y=0}^{y\to\infty}\right)$$

$$\Rightarrow \quad E = -\frac{2\sigma x}{4\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2}}\Bigg|_{y=0}^{y\to\infty}\right)$$

$$\Rightarrow \quad E = -\frac{\sigma x}{2\varepsilon_0}\left(0-\frac{1}{x}\right)$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} \qquad \{\text{independent of value of } x\}$$

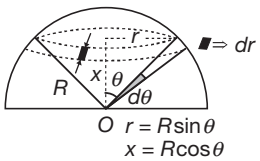
ILLUSTRATION 25

Calculate the electric field due to a thin uniformly charged hemispherical shell (of radius R having uniform charge density σ) at its centre.

SOLUTION

Let us consider an infinitesimal elemental ring on its surface having angular width $d\theta$ at an angle θ from its axis as shown. The surface area dA of this ring is

$$dA = (2\pi R\sin\theta)(Rd\theta)$$



Charge on this elemental ring is

$$dq = \sigma dA = \sigma(2\pi R^2\sin\theta d\theta)$$

Now due to this ring, electric field strength at centre C is

$$dE = \frac{1}{4\pi\varepsilon_0}\left[\frac{dq(R\cos\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}\right]$$

$$\left\{\because\ dE = \frac{(dq)x}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}\right\}$$

$$\Rightarrow \quad dE = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma(2\pi R^2\sin\theta d\theta)R\cos\theta}{R^3}\right]$$

$$\Rightarrow \quad dE = \frac{\pi\sigma}{4\pi\varepsilon_0}\sin(2\theta)d\theta$$

Net electric field at center is obtained by integrating the above expression for dE between the limits zero to $\frac{\pi}{2}$. So,

$$E = \int dE = \frac{\sigma}{4\varepsilon_0}\int_0^{\pi/2}\sin 2\theta\, d\theta$$

$$\Rightarrow \quad E = \frac{\sigma}{4\varepsilon_0}\left(-\frac{\cos 2\theta}{2}\Big|_0^{\pi/2}\right)$$

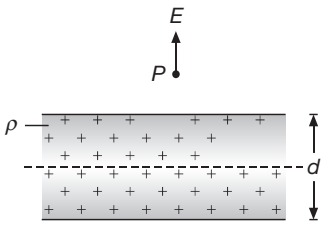
$$\Rightarrow \quad E = \frac{\sigma}{4\varepsilon_0}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$\Rightarrow \quad E = \frac{\sigma}{4\varepsilon_0}$$

ELECTRIC FIELD DUE TO A LARGE THICK CHARGED SHEET

CASE-1: OUTSIDE THE SHEET

Consider a large sheet of thickness d having uniform charge density ρ. On both sides of sheet electric field strength is directed, away from the sheet.



The electric field strength at a point P in front of the sheet is

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \ldots(1)$$

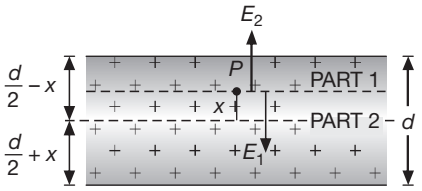
where σ is the charge per unit surface area of the sheet. Further

$$\sigma = \rho d$$

$$\Rightarrow \quad E = \frac{\rho d}{2\varepsilon_0} \qquad \ldots(2)$$

CASE-2: INSIDE THE SHEET

To find electric field strength at an interior point P of the sheet at a distance x from its centre as shown, we divide the sheet in two sheet parts. One of thickness $\left(\frac{d}{2}-x\right)$ and other of thickness $\left(\frac{d}{2}+x\right)$ as shown.



Due to the thinner sheet **(PART 1)** electric field at point P is in downward direction, say it is E_1 which is given by using equation (2).

$$E_1 = \frac{\rho\left(\frac{d}{2}-x\right)}{2\varepsilon_0}$$

Similarly due to thicker sheet **(PART 2)** electric field at P is in upward direction, say it is E_2 which is given again by using equation (2).

$$E_2 = \frac{\rho\left(\frac{d}{2}+x\right)}{2\varepsilon_0}$$

Net electric field at point P will be $E_P\,(=E)$ given by

$$E_P = E = E_2 - E_1$$

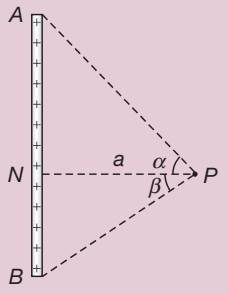
$$\Rightarrow \quad E = E_P = \frac{\rho\left(\frac{d}{2}+x\right)}{2\varepsilon_0} - \frac{\rho\left(\frac{d}{2}-x\right)}{2\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho x}{\varepsilon_0}$$

Conceptual Note(s)

Electrostatic Field in Some Cases

(a) The electric field due to a uniformly charged rod AB having charge density λ at a point P at perpendicular distance a from the rod, as shown in figure is



$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\alpha + \sin\beta)$$

and $$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\beta - \cos\alpha)$$

So, we can also extend the result obtained to more look alike situations.

(b) For an infinite rod having uniform charge density λ, electric field at point P is calculated by taking $\alpha \longrightarrow \frac{\pi}{2}$ and $\beta \longrightarrow \frac{\pi}{2}$.

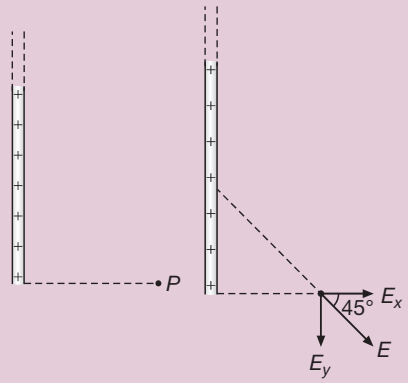
$$\Rightarrow \quad E_x = \frac{\lambda}{2\pi\varepsilon_0 a} \text{ and } E_y = 0$$

(c) For a semi-infinite rod having uniform charge density λ, electric field at point P is calculated by taking $\alpha \longrightarrow \frac{\pi}{2}$ and $\beta = \beta$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(1+\sin\beta) \text{ and } E_y = \frac{\lambda}{4\pi\varepsilon_0}\cos\beta$$

(d) For a semi-infinite rod having uniform density λ, the electric field at point P (which lies on the line joining P to wire normally and passing through the known end of wire) is calculated by taking $\alpha \longrightarrow \frac{\pi}{2}$ and $\beta \longrightarrow 0$

$$\Rightarrow \quad E_x = \frac{\lambda}{4\pi\varepsilon_0 a} \text{ and } E_y = \frac{\lambda}{4\pi\varepsilon_0 a}$$



(e) The electric field at the centre of a uniformly charged wire in the form of a circular arc of radius r subtending an angle ϕ at the centre is $E = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\left(\frac{\phi}{2}\right)$, along the angle bisector.

(i) For semi-circle, $\phi = \pi$

So, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

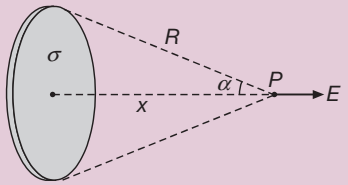
(ii) For Quarter-circle, $\phi = \frac{\pi}{2}$

$$\Rightarrow \quad E = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 r}$$

(iii) For circle, $\phi = 2\pi$

$$\Rightarrow \quad E = 0$$

(f) The electric field at point P that lies on the axis of a uniformly charged disc of radius R having charge density σ at a distance x from the centre is



$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right] = \frac{\sigma}{2\varepsilon_0}(1-\cos\alpha)$$

If Q is the total charge on the disc, then

$$E = \frac{2Q}{4\pi\varepsilon_0 R^2}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

(g) The electric field strength due to a uniformly charged infinite sheet having charge density σ at a point P at perpendicular distance x from the sheet is $E = \frac{\sigma}{2\varepsilon_0}$ (independent of value of x).

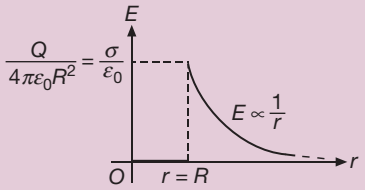
(h) The electric field due to a thin uniformly charged shell (of radius R having uniform charge density σ) at its centre is $E = \frac{\sigma}{4\varepsilon_0}$.

(i) Due to a charged spherical conductor (spherical shell) of charge Q, radius R at a distance r from its centre.

$$E = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2} & \text{for} \quad r \geq R \\ 0 & \text{for} \quad r < R \end{cases}$$

If the conducting sphere has uniform surface charge density σ, then

$$E = \begin{cases} \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2} & \text{for} \quad r \geq R \\ 0 & \text{for} \quad r < R \end{cases}$$

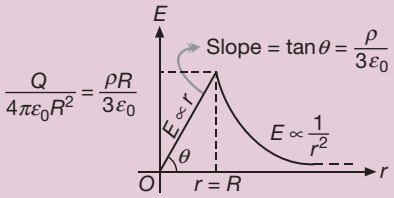


(j) Due to a uniformly charged non-conducting sphere of charge Q, radius R at a distance r from the centre of the sphere.

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{for} \quad r \geq R \\ \dfrac{Qr}{4\pi\varepsilon_0 R^3} & \text{for} \quad r < R \end{cases}$$

If the sphere has uniform volume charge density ρ, then

$$E = \begin{cases} \dfrac{\rho R^3}{3\varepsilon_0 r^2} & \text{for} \quad r \geq R \\ \dfrac{\rho r}{3\varepsilon_0} & \text{for} \quad r < R \end{cases}$$



(k) Due to an infinite thread having uniform charge distribution λ at distance r from it.

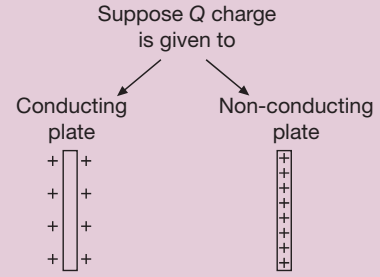
$$E = \frac{2\lambda}{4\pi\varepsilon_0 r}$$

(l) Due to an infinite thin conducting plane sheet of charge with surface charge density σ.

$$E = \frac{\sigma}{\varepsilon_0}$$

(m) Due to an infinite non conducting plane sheet of charge with surface charge density σ.

$$E = \frac{\sigma}{2\varepsilon_0}$$



Electric field for both the cases

$$E = \frac{Q}{2A\varepsilon_0}$$

$E = \frac{\sigma_{\text{conducting}}}{\varepsilon_0}$ | $E = \frac{\sigma_{\text{non-conducting}}}{2\varepsilon_0}$

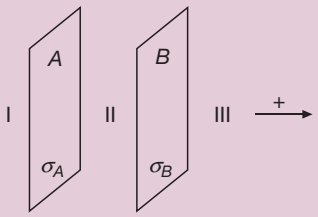
Where $\sigma_{\text{conducting}} = \frac{Q}{2A}$ | Where $\sigma_{\text{non-conducting}} = \frac{Q}{A}$

Because Q is distributed in area 2A. | Because Q is distributed in area A.

(n) Near a conductor of any shape,

$$E = \frac{\sigma}{\varepsilon_0}$$

(o) Due to two parallel plane thin sheets with surface charge density σ_A and σ_B in the regions specified in figure



$$E_{\text{I}} = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

$$E_{\text{II}} = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

$$E_{\text{III}} = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

If $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$, then

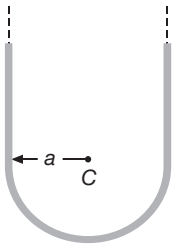
$$E_{\text{I}} = E_{\text{III}} = 0 \text{ and } E_{\text{II}} = \frac{\sigma}{\varepsilon_0}$$

(p) At the axis of a uniformly charged ring of radius R at a distance x from the centre of the ring.

$$E = \frac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}$$

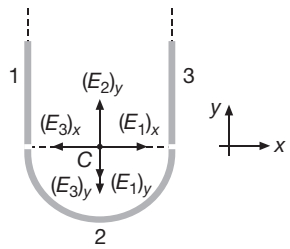
ILLUSTRATION 26

In the given arrangement find the electric field at C in the figure. Here the U-shaped wire uniformly charged with linear charge density λ.



SOLUTION

Here we shall divide the arrangement into three parts as shown and then apply the principle of superposition.



Here we have used the results obtained previously

$$(E_1)_x = \frac{\lambda}{4\pi\varepsilon_0 a} \qquad \text{(along } +x \text{ axis)}$$

$$(E_1)_y = \frac{\lambda}{4\pi\varepsilon_0 a} \qquad \text{(along } -y \text{ axis)}$$

Similarly

$$(E_3)_x = \frac{\lambda}{4\pi\varepsilon_0 a} \qquad \text{(along } -x \text{ axis)}$$

$$(E_3)_y = \frac{\lambda}{4\pi\varepsilon_0 a} \qquad \text{(along } -y \text{ axis)}$$

and $(E_2)_x = 0$

$$(E_2)_y = \frac{\lambda}{2\pi\varepsilon_0 a} \qquad \text{(along } +y \text{ axis)}$$

So, $\vec{E}_C = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$\Rightarrow \quad \vec{E}_C = \vec{0}$

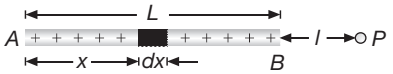
ILLUSTRATION 27

Calculate the electric field due to a rod of length L at a point P at a distance l from its end, when the charge density of the rod varies with distance x from the other end of the rod as $\lambda = \lambda_0 x$ where λ_0 is a positive constant.

SOLUTION

The rod AB shown here is of length L having linear charge density that depends on distance x from end A of rod as

$$\lambda = \lambda_0 x$$



and we have to calculate electric field strength at point P. For this let us consider an infinitesimal element of length dx at a distance x from the end A as shown. Here charge on this element is

$$dq = \lambda dx = \lambda_0 x\, dx$$

Now electric field strength E at point P due to infinitesimal charge dq is dE (say), given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(L+l-x)^2}$$

Net electric field at P due to complete rod (from A to B) is calculated by integrating the above expression within limits taken from zero to L. Thus we have

$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda_0 x\, dx}{(L+l-x)^2} = \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^L \frac{x\, dx}{(L+l-x)^2}$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^L \frac{x\, dx}{(L+l-x)^2}$$

To calculate the integral, let us denote the integral by

$$I = \int \frac{x\,dx}{(L+l-x)^2}$$

Substituting $L+l-x = y$

$$\Rightarrow \quad -dx = dy$$

$$\Rightarrow \quad I = \int \frac{-\left[(L+l)-y\right]dy}{y^2}$$

$$\Rightarrow \quad I = -(L+l)\int y^{-2}\,dy + \int \frac{y}{y^2}\,dy$$

$$\Rightarrow \quad I = -(L+l)\frac{y^{-2+1}}{-2+1} + \log_e y$$

$$\Rightarrow \quad I = \frac{(L+l)}{y} + \log_e y$$

$$\Rightarrow \quad I = \frac{L+l}{L+l-x} + \log_e (L+l-x)$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{(L+l-x)^2}$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\left[\left(\frac{L+l}{L+l-x}\right)\Big|_0^L + \log_e (L+l-x)\Big|_0^L\right]$$

$$\Rightarrow \quad E = \frac{\lambda_0 (L+l)}{4\pi\varepsilon_0}\left[\frac{1}{L+l-L} - \frac{1}{L+l}\right] + \frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{L+l-L}{L+l}\right)$$

$$\Rightarrow \quad E = \frac{\lambda_0 (L+l)}{4\pi\varepsilon_0}\left[\frac{1}{l} - \frac{1}{L+l}\right] + \frac{\lambda_0}{4\pi\varepsilon_0}\log_e\left(\frac{l}{L+l}\right)$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\frac{(L+l)L}{l(L+l)} + \frac{\lambda_0}{4\pi\varepsilon_0}\log_e\left(\frac{l}{L+l}\right)$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\left[\frac{L}{l} + \log_e\left(\frac{l}{L+l}\right)\right]$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0}\left[\frac{L}{l} - \log_e\left(1+\frac{L}{l}\right)\right]$$

ILLUSTRATION 28

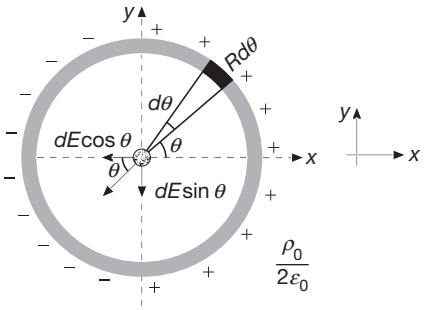
A thin non conducting ring of radius R has linear charge density λ varying as $\lambda = \lambda_0 \cos\phi$, where λ_0 is a positive constant and ϕ is the azimuthal angle. Calculate the magnitude of the electric field strength at

(a) the centre of the ring

(b) on the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at $x \gg R$.

SOLUTION

(a) Here by observing the function $\lambda = \lambda_0 \cos\theta$, we can state that the first and fourth quadrant are positively charged and second and third quadrant are negatively charged.



To find electric field strength at the centre of the ring, we consider an infinitesimal element on ring having polar width $d\theta$ and making an angle θ with x-axis. The charge on this infinitesimal element is

$$dq = \lambda\,dx$$

$$\Rightarrow \quad dq = \lambda(Rd\theta)$$

$$\Rightarrow \quad dq = (\lambda_0 \cos\theta)(Rd\theta) \qquad \ldots(1)$$

The electric field strength at the centre of the ring due to this element is

$$dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{dq}{R^2}\right) \qquad \ldots(2)$$

To find net electric field at centre of ring due to this semicircular distribution, we integrate the components of this electric field taken for the semi-circumference of ring. Here components $dE\sin\theta$ will cancel each other and the component $dE\cos\theta$ will be contributing to the net field. Thus net electric field strength at centre due to semicircular distribution will be given as

$$E = E_{\substack{\text{due to semi-circular}\\ \text{positive charge}\\ \text{distribution}}} + E_{\substack{\text{due to semi-circular}\\ \text{negative charge}\\ \text{distribution}}}$$

$$\Rightarrow \quad E = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE\cos\theta$$

$$\Rightarrow \quad E = \frac{2\lambda_0}{4\pi\varepsilon_0 R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$\Rightarrow \quad E = \frac{\lambda_0}{2\pi\varepsilon_0 R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\theta|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{\sin(2\theta)}{2}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right)$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\pi\varepsilon_0 R}(\pi + 0)$$

$$\Rightarrow \quad E = \frac{\lambda_0}{4\varepsilon_0 R}$$

(b) Take an infinitesimal element dl at an azimuthal angle ϕ. This element subtends an angle $d\phi$ at the centre O. The charge on this element

$$dq = \lambda\, dl = \lambda(R d\phi)$$

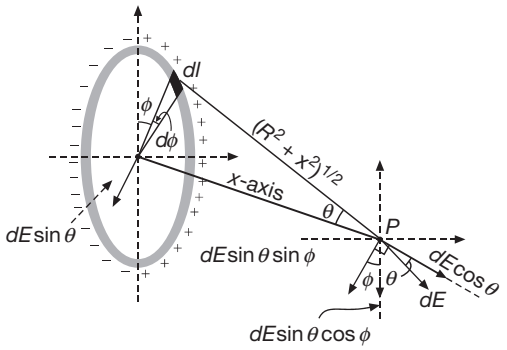
$$\Rightarrow \quad dq = (\lambda_0\cos\phi)(R d\phi)$$

$$\{\because\ \lambda = \lambda_0\cos\phi \text{ and } dl = Rd\phi\}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+x^2)}$$

Electric field due to positive half ring at P on the axis is

$$E_1 = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} dE\sin\theta\cos\phi$$



$$\Rightarrow \quad E_1 = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{dq}{(R^2+x^2)}\times\frac{R}{(R^2+x^2)^{1/2}}\cos\phi$$

$$\Rightarrow \quad E_1 = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\lambda_0 R\cos\phi d\phi\times R\cos\phi}{(R^2+x^2)^{3/2}}$$

$$\Rightarrow \quad E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda_0 R^2}{(R^2+x^2)^{3/2}}\left(2\int_0^{\frac{\pi}{2}}\left[\frac{1+\cos 2\phi}{2}\right]d\phi\right)$$

$$\Rightarrow \quad E_1 = \frac{2\lambda_0 R^2}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad E_1 = \frac{\lambda_0 R^2}{8\varepsilon_0(R^2+x^2)^{3/2}}$$

Similarly, for negative part, we have

$$E_2 = \frac{\lambda_0 R^2}{8\varepsilon_0(R^2+x^2)^{3/2}}$$

So, net electric field at point P is given by

$$E_{\text{net}} = E_1 + E_2 = \frac{\lambda_0 R^2}{4\varepsilon_0(R^2+x^2)^{3/2}}$$

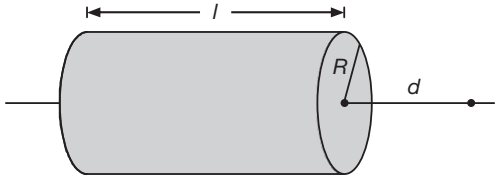
The direction will be perpendicular to the axis of ring

For $x \gg R$,

$$E_{\text{net}} = \frac{\lambda_0 R^2}{4\varepsilon_0(x^3)}$$

ILLUSTRATION 29

(a) Consider a uniformly charged thin walled right circular cylindrical shell having total charge Q, radius R, and length l. Determine the electric field at a point a distance d from the right side of the cylinder as shown in figure.

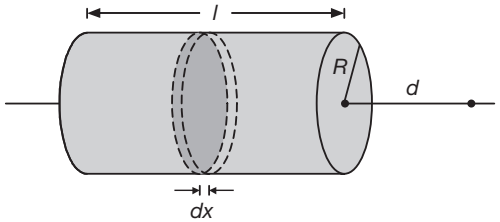


(b) Now consider a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Find the field it creates at the same point.

SOLUTION

(a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\frac{Qdx}{l}$ and produces, at the chosen point, a field

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{x}{(x^2+R^2)^{\frac{3}{2}}}\frac{Qdx}{l}\hat{i}$$



The total field is

$$E = \int dE = \frac{Q}{4\pi\varepsilon_0 l}\int_d^{d+\ell}\frac{xdx}{(x^2+R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad E = \frac{Q}{8\pi\varepsilon_0 l}\int_d^{d+\ell}(x^2+R^2)^{-\frac{3}{2}}2xdx$$

Since $\int(x^2+R^2)^{-\frac{3}{2}}2xdx = -\frac{2}{\sqrt{x^2+R^2}}$

$$\Rightarrow \quad E = -\frac{Q}{4\pi\varepsilon_0 l}\left(\frac{1}{\sqrt{x^2+R^2}}\right)\Bigg|_d^{d+\ell}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 l}\left(\frac{1}{\sqrt{d^2+R^2}} - \frac{1}{\sqrt{(d+l)^2+R^2}}\right)$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\frac{Qdx}{l}$, and charge per area $\sigma = \frac{Qdx}{\pi R^2 l}$. One disk produces a field

$$d\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)\hat{i}$$

So, the field due to the entire cylinder is given by

$$E = \int dE = \frac{Q}{2\pi\varepsilon_0 R^2 l}\int_d^{d+\ell}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)dx$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 R^2 l}\left(\int_d^{d+l}dx - \frac{1}{2}\int_d^{d+l}(x^2+R^2)^{-\frac{1}{2}}2xdx\right)$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 R^2 l}\left(x - \sqrt{x^2+R^2}\right)\Bigg|_d^{d+l}$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 R^2 l}\left(l + \sqrt{d^2+R^2} - \sqrt{(d+l)^2+R^2}\right)$$

Test Your Concepts-III

Based on Electric Field

(Solutions on page H.10)

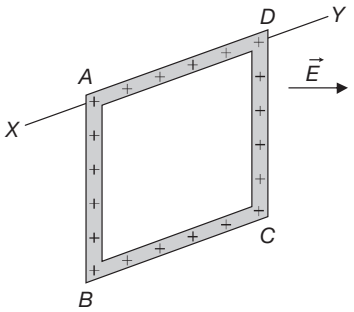
1. Four particles each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.

2. A circular wire loop of radius R carries a total charge Q distributed uniformly over its length. A small length $x(\ll R)$ of the wire is cut off. Find the electric field at the centre due to the remaining wire.

3. An electric field of 10^5 NC^{-1} points due North at a certain spot. Find the magnitude and direction of the force that acts on a charge of +2 μC and −5 μC at this spot?

4. Three identical positive charges q are arranged at the vertices of an equilateral triangle of side ℓ. Calculate the intensity of the field at the vertex of a regular tetrahedron of which the triangle is the base.

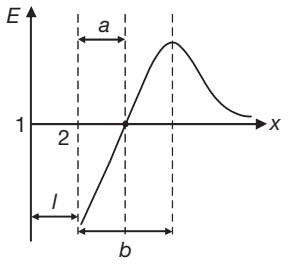
5. A square frame $ABCD$ is made of four thin rods, each of length L and mass m and charged with a charge q. The frame is hanging from one of its sides as shown in figure along a horizontal axis XY. At $t = 0$ a horizontal electric field is switched on in horizontal direction, perpendicular to the plane of the frame. Find the minimum value of E so that the square frame rotates upto horizontal level.



6. A particle of mass m and charge $-Q$ is constrained to move along the axis of a ring of radius R. The ring carries a uniform charge density $+\lambda$ along its length. Initially the particle is in the plane of the ring where the force on it is zero. Show that the period of oscillation of the particle when it is displaced slightly from its equilibrium position is given by

$$T = 2\pi\sqrt{\frac{2\varepsilon_0 mR^2}{\lambda Q}}$$

7. Two point charges are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in the figure. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is ℓ.

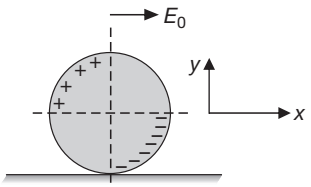


(a) Find the sign of each charge.

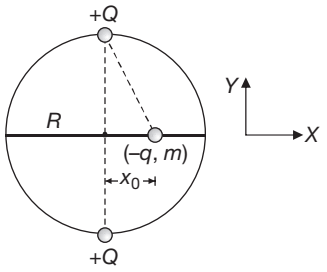
(b) Find the ratio of the absolute values of the charges $\left|\frac{Q_1}{Q_2}\right|$

(c) Find the value of b where the field intensity is maximum.

8. A non-conducting ring of mass m and radius R is charged as shown. The charge density, i.e., charge per unit length is λ. It is then placed on a rough non-conducting horizontal plane. At time $t = 0$, a uniform electric field $\vec{E} = E_0\hat{i}$ is switched on and the ring starts rolling without sliding. Determine the frictional force (magnitude and direction) acting on the ring when it starts moving.



9. A thin insulating wire is stretched along the diameter of an insulated circular hoop of radius R. A small bead of mass m and charge $-q$ is threaded onto the wire. Two small identical charges are tied to the hoop at points opposite to each other, so that the line connecting them is perpendicular to the thread (shown in figure). The bead is released at a point which is a distance x_0 from the centre of the hoop.



(a) Calculate the resultant force acting on the charged bead.

(b) Describe (qualitatively) the motion of the bead after it is released.

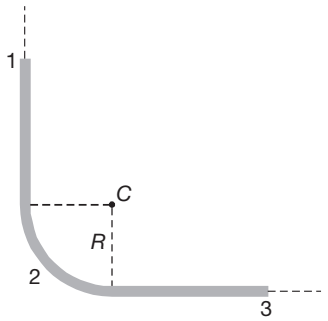
(c) Write the exact equation that governs the motion of the bead along the thread.

(d) After the assumption that $\frac{x_0}{R} \ll 1$ is used to obtain an approximate equation of motion, find the displacement and velocity of the bead as functions of time.

(e) When will the velocity of the bead vanish for the first time?

10. A non-conducting ring of mass m and radius R is lying at rest in the vertical XY plane on a smooth non-conducting horizontal XZ plane. Charge $+q$ and $-q$ are distributed uniformly on the ring, on the two sides of the vertical diameter of the ring. A constant and uniform electric field $\vec{E}$ is set up along the x direction. The ring is given a small angular rotation about an axis perpendicular to its plane and released. Find the period of oscillation of the ring.

11. In the given arrangement find electric field at C. Complete wire is uniformly charged at linear charge density λ.

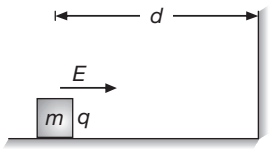


12. A pitch ball covered with tin foil having a mass m hangs by a fine silk thread of length ℓ in a horizontal electric field E. When the ball is given an electric charge q, it stands out a distance d from the vertical line. Show that the electric field is given by

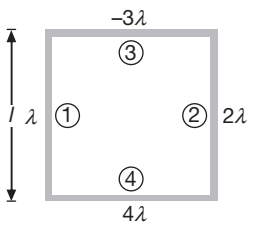
$$E = \frac{mgd}{q\sqrt{\ell^2 - d^2}}$$

13. A bob of mass m carrying a positive charge q is suspended from a light inextensible string of length ℓ inside a parallel plate condenser with its plates making an angle β with the horizontal. The upper plate of the condenser is negatively charged and the intensity of electric field inside the condenser is E. Find the period of vibration of the pendulum and the angle between the thread in equilibrium position and the vertical.

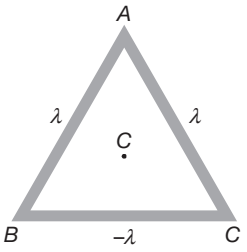
14. A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure. The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming collisions (if any) to be only of elastic nature, find the time period of resulting oscillatory motion. Is it a simple harmonic motion.



15. In the given arrangement of a charged square frame find electric field at centre. The linear charged density is as shown in figure.



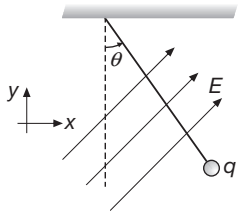
16. In the figure shown, three charged rods each of length L, having charge densities λ, $-\lambda$ and λ form a triangle. Calculate the electric field due to the arrangement at its centre.



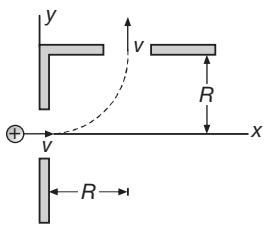
17. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in figure. When an electric field given by $E = \left(A\hat{i} + B\hat{j}\right)\text{NC}^{-1}$ is switched on (where A and B are positive numbers) the ball attains equilibrium making an angle θ with the vertical. Find

(a) the charge on the ball and

(b) the tension in the string.



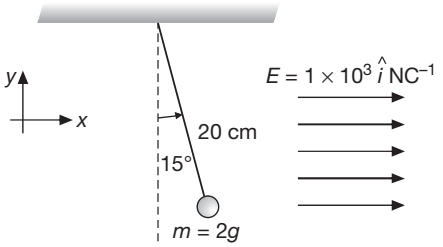
18. A physicist studying the properties of ions in the upper atmosphere wishes to construct an apparatus having the following characteristics. In the setup used, an electric field is applied such that a beam of ions, each having charge q, mass m, and initial velocity $v\hat{i}$, is turned through an angle of 90° such that each ion undergoes displacement $R\hat{i} + R\hat{j}$. The ions enter a chamber as shown in figure, and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.



(a) Suppose the electric field is produced by two concentric cylindrical electrodes (not shown in the diagram) and hence is radial. Calculate the magnitude of the electric field.

(b) If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

19. A small, 2 g plastic ball is suspended by a 20 cm long string in a uniform electric field as shown in Figure. If the ball is in equilibrium when the string makes a 15° angle with the vertical, what is the net charge on the ball?



20. Consider n equal positive point charges each of magnitude $\frac{Q}{n}$ placed symmetrically around a circle of radius R. Calculate the magnitude of the electric field at a point a distance x on the line passing through the centre of the circle and perpendicular to the plane of the circle.

21. A line of charge starts at $x = +x_0$ and extends to positive infinity. The linear charge density is $\lambda = \frac{\lambda_0 x_0}{x}$. Determine the electric field at the origin.

22. A rod lies along x-axis with one end at origin and other at $x \to \infty$. The rod has a uniform charge density of $\lambda\ \text{Cm}^{-1}$. Find the electric field at the point $x = -a$ on x-axis. Find E_x and E_y for a point on the y-axis where $y = b$.

MOTION OF A CHARGED PARTICLE IN AN ELECTRIC FIELD

CASE-1:

In uniform electric field when a positively charge particle is **released from rest**, it starts moving in the direction of electric field with uniform acceleration a given by

$$a = \frac{qE}{m}$$

Then $v = at = \left(\frac{qE}{m}\right)t$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

and kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q^2E^2}{m}\right)t = yqE$$

CASE-2:

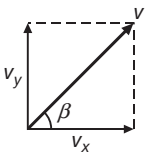
In uniform electric field when a positively charge particle is released with **non-zero initial velocity** u **perpendicular** to the electric field, then
Velocity at any instant

Also, $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow \quad \vec{v} = u\hat{i} + \left(\frac{qEt}{m}\right)\hat{j} \quad \left\{\because \vec{E} = E\hat{j}\,(\text{say}),\ \text{then } \vec{u} = u\hat{i}\right\}$$

Comparing with

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$



we have

$$v_x = u$$

$$v_y = \frac{qEt}{m}$$

Since, $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

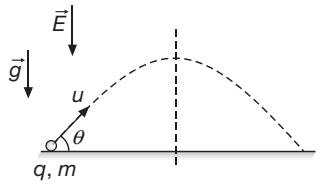
$$\Rightarrow \quad v = \sqrt{u^2 + \frac{q^2E^2t^2}{m^2}}$$

If β is the angle made by v with x-axis, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{qEt}{mu}$$

CASE-3:

If the particle is thrown in a uniform electric field in the direction different from the direction of electric field, it will follow a parabolic trajectory like projectile motion. Consider the following case.



A particle of mass m and charge q is thrown from ground at an angle θ with initial speed u. A uniform electric field $\vec{E}$ exists in downward direction as shown. Here during motion of particle we can consider the effective acceleration to be

$$a = g + \frac{qE}{m}$$

Here in above example we can use all the concepts of projectile motion by replacing g by a. If electric field in space is in upward direction, effective acceleration due to gravity will be

$$a = g - \frac{qE}{m}$$

ILLUSTRATION 30

A particle of mass 9×10^{-31} kg and a negative charge of 1.6×10^{-19} C is projected horizontally with a velocity of 10^5 ms^{-1} into a region between two infinite horizontal parallel plates of metal. The distance between the plates is 0.3 cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected respectively to the positive and negative terminals of a 30 V battery. Calculate the component of the velocity of the particle just before it hits one of the plates.

SOLUTION

$$E = \frac{V}{d}$$

$$\Rightarrow \quad E = \frac{30}{3\times10^{-3}} = 10^4 \text{ NC}^{-1} \qquad \left\{\because d = 3\times10^{-3} \text{ m}\right\}$$

Force on the particle of negative charge moving between the plates is

$$F = eE = 1.6\times10^{-19}\times10^{4} = 1.6\times10^{-15}\ \text{N}$$

The direction of force will be towards the positive plate i.e., upwards
Acceleration of the particle is

$$a = \frac{eE}{m}$$

$$\Rightarrow \quad a = \frac{(1.6\times10^{-15})}{(9\times10^{-31})}$$

$$\Rightarrow \quad a = 1.77\times10^{15}\ \text{ms}^{-2}$$

The electric intensity E is acting in the vertical direction so the horizontal velocity v of the particle remains same. If y is the displacement of the particle, in upward direction, then

$$y = \frac{1}{2}at^2$$

where $y = 0.1\ \text{cm} = 10^{-3}\ \text{m}$, $a = 1.77\times10^{15}\ \text{ms}^{-2}$

$$\Rightarrow \quad 10^{-3} = \frac{1}{2}\times(1.77\times10^{15})(t^2)$$

$$\Rightarrow \quad t = 1.063\times10^{-9}\ \text{s}$$

Component of velocity in the direction of field is given by

$$v_y = at$$

$$\Rightarrow \quad v_y = (1.77\times10^{15})(1.063\times10^{-9})$$

$$\Rightarrow \quad v_y = 1.881\times10^{6}\ \text{ms}^{-1} \text{ and } v_x = 10^{5}\ \text{ms}^{-1}$$

ILLUSTRATION 31

A projectile of mass m having charge q is thrown with initial velocity u at an angle α with the horizontal. An electric field of strength E exists, making an angle β with the downward vertical away from the point of projection. Find the time of flight and range of projectile.

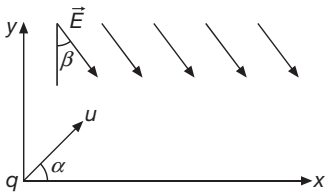
SOLUTION

Initial velocity is given by (in components)

$$u_y = u\sin\alpha,\ u_x = u\cos\alpha$$

and acceleration $a_y = \left(\frac{qE}{m}\cos\beta + g\right)$

and $a_x = \frac{qE\sin\beta}{m}$



Time of flight $T = \frac{2v_y}{a_y} = \frac{2u\sin\alpha}{\left(\frac{qE}{m}\cos\beta + g\right)}$

In x direction range of projectile is given by

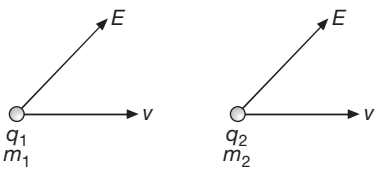
$$x = u_xt + \frac{1}{2}a_xt^2$$

$$\Rightarrow \quad R = u_xT + \frac{1}{2}a_xT^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{\frac{qE}{m}\cos\beta + g}\right) + \frac{1}{2}\left(\frac{qE\sin\beta}{m}\right)\left(\frac{2u\sin\alpha}{\frac{qE}{m}\cos\beta + g}\right)^2$$

ILLUSTRATION 32

Two balls of charge q_1 and q_2 initially have a velocities of equal magnitude and same direction. After a uniform electric field has been applied for a certain time interval, the direction of first ball changes by $60°$ and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes thereby $90°$.



Initial condition

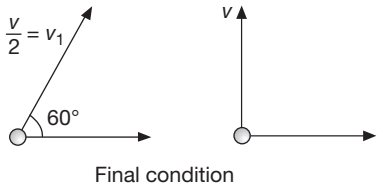
(a) Determine the magnitude of the charge to mass ratio of the second ball if it is equal to α_1 for the first ball.

(b) In what ratio will the velocity of the second ball change?
Ignore the electrostatic interaction between the balls

SOLUTION

(a) Let the electric field acting on each ball be given by

$$E = E_x\hat{i} + E_y\hat{j}$$



From Impulse Momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2.

Noting that $v_1 = \frac{v}{2}$, we have

$$q_1\left(E_x\hat{i} + E_y\hat{j}\right)\Delta t = m_1\left(\frac{v}{2}\cos 60°\hat{i} + \frac{v}{2}\sin 60°\hat{j}\right) - m_1 v\hat{i} \quad \ldots(1)$$

$$q_2\left(E_x\hat{i} + E_y\hat{j}\right)\Delta t = m_2\left(v_2\cos 90°\hat{i} + v_2\sin 90°\hat{j}\right) - m_2 v\hat{i} \quad \ldots(2)$$

On comparing the x and y-components on both sides of equation (1), we get

$$\frac{q_1}{m_1}E_x\Delta t = -\frac{3}{4}v \text{ and} \quad \ldots(3)$$

$$\frac{q_1}{m_1}E_y\Delta t = \frac{\sqrt{3}}{4}v \quad \ldots(4)$$

Similarly, from equation (2), we get

$$\frac{q_2}{m_2}E_x\Delta t = -v \text{ and} \quad \ldots(5)$$

$$\frac{q_2}{m_2}E_y\Delta t = v_2 \quad \ldots(6)$$

From equations (3) and (5), by dividing the equations expressing x-components, we get

$$\frac{\left(\frac{q_1}{m_1}\right)}{\left(\frac{q_2}{m_2}\right)} = \frac{3}{4} \quad \ldots(7)$$

$$\Rightarrow \quad \frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

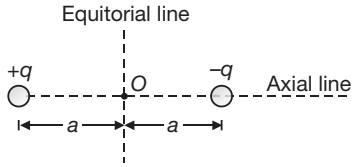
(b) Also $\dfrac{\frac{q_1}{m_1}}{\frac{q_2}{m_2}} = \dfrac{\sqrt{3}v}{4v_2}$

$$\Rightarrow \quad \frac{\sqrt{3}v}{4v_2} = \frac{3}{4}$$

$$\Rightarrow \quad v_2 = \frac{v}{\sqrt{3}}$$

ELECTRIC DIPOLE AND DIPOLE MOMENT

Two equal and opposite charges separated by a distance together constitute a dipole.



Dipole moment $(\vec{p})$ is defined as the simple product of magnitude of either charge and the distance of separation between the two charges.

$$\vec{p} = q(2\vec{a})$$

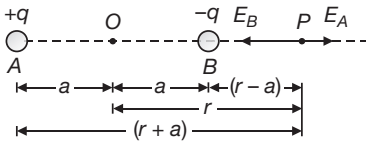
Dipole moment $\vec{p}$ always points from $-q$ to $+q$. Its SI unit is coulombmetre(Cm).

ELECTRIC FIELD DUE TO A DIPOLE AT A POINT LYING ON THE AXIAL LINE (END ON POSITION)

The electric field due to a dipole at point P at distance r from the centre of the dipole is

$$E_{\text{axial}} = E_B - E_A$$

$$\Rightarrow \quad E_{\text{axial}} = \frac{q}{4\pi\varepsilon_0(r-a)^2} - \frac{q}{4\pi\varepsilon_0(r+a)^2}$$



$$\Rightarrow \quad E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{4raq}{(r^2-a^2)^2}$$

Since, $p = q(2a)$. So,

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$$

For $r >> a$

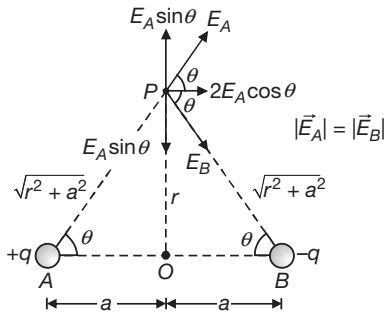
$$E_{\text{axial}} \simeq \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

ELECTRIC FIELD DUE TO A DIPOLE AT A POINT LYING ON THE EQUITORIAL LINE (BROAD SIDE ON POSITION)

The electric field due to the dipole at the point P at distance r from O is

$$E_{\text{equitorial}} = 2E_A\cos\theta \text{ where } E_A = \frac{q}{4\pi\varepsilon_0(r^2+a^2)}$$

(Because the components $E_A\sin\theta$ and $E_A\sin\theta$ cancel out)



$$\Rightarrow \quad E_{\text{equitorial}} = 2\frac{q}{4\pi\varepsilon_0(r^2+a^2)}\frac{a}{\sqrt{r^2+a^2}}$$

$$\left\{\because \cos\theta = \frac{a}{\sqrt{a^2+r^2}}\right\}$$

$$\Rightarrow \quad E_{\text{equitorial}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}} \left\{\begin{matrix}\text{from positive to}\\ \text{negative charge}\end{matrix}\right\}$$

Vectorially $\vec{E}_{\text{equitorial}} = \frac{1}{4\pi\varepsilon_0}\frac{(-\vec{p})}{(r^2+a^2)^{3/2}}$

For $r >> a$

$$E_{\text{equitorial}} \simeq \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Remark(s)

So we conclude that at points lying far away from the centre of the dipole {for $r >> a$}

$$E_{\text{axial}} \simeq 2E_{\text{equitorial}}$$

ILLUSTRATION 33

The electric field due to a short dipole at a distance r, on the axial line, from its mid-point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

SOLUTION

$$\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$$

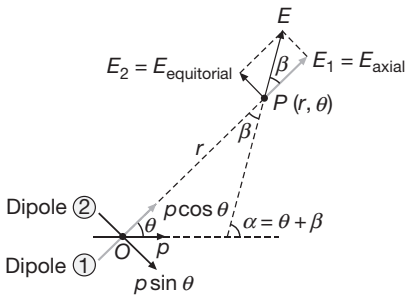
$$\Rightarrow \quad \frac{2}{r^3} = \frac{1}{r'^3}$$

$$\Rightarrow \quad \frac{r^3}{r'^3} = 2$$

$$\Rightarrow \quad \frac{r}{r'} = 2^{\frac{1}{3}}$$

ELECTRIC FIELD DUE TO A DIPOLE AT ANY POINT *P*(*r*, *θ*)

Consider a point $P(r,\theta)$ at a large distance r from the centre O of the dipole at an angle θ with $\vec{p}$ (as shown).



On resolving $\vec{p}$ in two components $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$, we observe that the point P lies at the axial line of Dipole ① (having dipole moment $p_1 = p\cos\theta$) and on the equatorial line of Dipole ② (having dipole moment $p_2 = p\sin\theta$). So,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p_1}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \text{ and}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p_2}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$\Rightarrow \quad E = \sqrt{E_1^2 + E_2^2}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

If β is the angle made by the net field E with E_1 then,

$$\tan\beta = \frac{E_2}{E_1} = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \tan\beta = \frac{1}{2}(\tan\theta)$$

So, we observe that the net field E makes an angle $\alpha = \theta + \beta$ with the dipole moment $\vec{p}$.

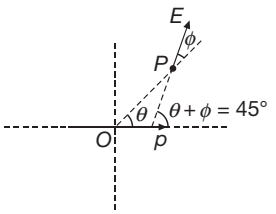
ILLUSTRATION 34

Find the locus of all the points where the resultant $\vec{E}$ field will always have a bearing of $45°$ with the axis of a short dipole.

SOLUTION

$$\theta + \phi = \frac{\pi}{4} \text{ (Given)} \qquad \ldots(1)$$

Since, $\tan\phi = \frac{1}{2}\tan\theta$ (from the theory of dipole) …(2)



So, from (1) and (2), we get

$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \tan^2\theta + 3\tan\theta - 2 = 0$$

Solving this Quadratic in $\tan\theta$, we get

$$\tan\theta = \frac{-3 \pm \sqrt{17}}{2} = 0.56 \text{ or } -3.56$$

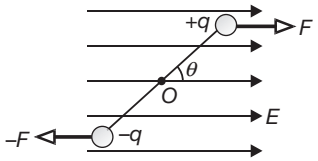
$$\Rightarrow \quad \theta = 29°18'$$

TORQUE ON A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

Suppose an electric dipole is placed in a uniform external electric field $\vec{E}$ where the dipole moment makes an angle θ with the field. The forces on the two charges are equal and opposite as shown, each having a magnitude

$$F = qE$$

Thus, we see that the net force on the dipole is zero. However, the two forces produce a net torque on the dipole, and the dipole tends to rotate such that its axis gets aligned with the field.



The torque due to the force on the positive charge about an axis through O is given by $Fa\sin\theta$ where, $a\sin\theta$ is the moment arm of F about O. This force tends to produce a clockwise rotation. Likewise, the torque on the negative charge about O is also $Fa\sin\theta$, and so the net torque τ about O is given by

$$\tau = 2Fa\sin\theta$$

Since $F = qE$ and $p = 2aq$

$$\Rightarrow \quad \tau = 2aqE\sin\theta$$

$$\Rightarrow \quad \tau = pE\sin\theta$$

It is convenient to express the torque in vector form as the cross product of the vectors $\vec{p}$ and $\vec{E}$, so vectorially,

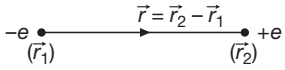
$$\vec{\tau} = \vec{p}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p_x & p_y & p_z \\ E_x & E_y & E_z \end{vmatrix}$$

ILLUSTRATION 35

A charge e is placed at (1, 2 , 1) and another charge $-e$ is placed at (0, 1, 0) such that they form an electric dipole. There exists a uniform electric field $E=\left(2\hat{i}+3\hat{j}\right)$. Calculate the dipole moment vector and torque experienced by the dipole.

SOLUTION

(i) $\vec{p}=q\vec{r}$



$\Rightarrow \quad \vec{p}=e\left(\vec{r}_2-\vec{r}_1\right)$

Now, $\vec{r}_1=\hat{j}$

$\vec{r}_2=\hat{i}+2\hat{j}+\hat{k}$

$\Rightarrow \quad \vec{r}_2-\vec{r}_1=\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow \quad \vec{p}=e\left(\hat{i}+\hat{j}+\hat{k}\right)$

and $\quad |\vec{p}|=\sqrt{3}e$

(ii) $\vec{\tau}=\vec{p}\times\vec{E}$

$\Rightarrow \quad \vec{\tau}=e\left(\hat{i}+\hat{j}+\hat{k}\right)\times\left(2\hat{i}+3\hat{j}\right)$

$$\vec{\tau}=e\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 3 & 0\end{vmatrix}=e\left(-3\hat{i}+2\hat{j}+\hat{k}\right)\text{Nm}$$

POTENTIAL ENERGY OF A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

Work must be done by an external agent to rotate the dipole through a given angle in the field. This work done is then stored as potential energy in the system, that is, the dipole and the external field. The work dW required to rotate the dipole through an angle $d\theta$ is given by

$$dW=\tau\,d\theta$$

Since,

$$\tau=pE\sin\theta$$

This work is transformed into potential energy U. We find this for a rotation from θ_0 to θ. So,

$$U=\int_{\theta_0}^{\theta}\tau\,d\theta=\int_{\theta_0}^{\theta}pE\sin\theta d\theta=pE\int_{\theta_0}^{\theta}\sin\theta d\theta$$

$$\Rightarrow \quad U=pE(-\cos\theta)\Big|_{\theta_0}^{\theta}=pE\left(\cos\theta_0-\cos\theta\right)$$

The term involving $\cos\theta_0$ is a constant that depends on the initial orientation of the dipole. It is convenient to choose $\theta_0=90°$, so that $\cos\theta_0=\cos 90°=0$. In this case, we can express U as

$$U=-pE\cos\theta$$

This is equivalent to the dot product of the vectors $\vec{p}$ and $\vec{E}$. So,

$$U=-\vec{p}\cdot\vec{E}=-\left(p_xE_x+p_yE_y+p_zE_z\right)$$

ILLUSTRATION 36

When an electric dipole is placed in a uniform electric field making angle θ with electric field, it experiences a torque τ. Calculate the minimum work done in changing the orientation to 2θ.

SOLUTION

$$\tau=pE\sin\theta$$

$$\Rightarrow \quad pE=\frac{\tau}{\sin\theta}$$

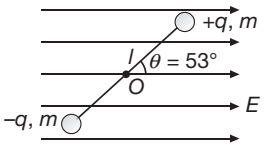
$$W=\Delta U=-pE\cos\theta_2+pE\cos\theta_1$$

$$\Rightarrow \quad W=pE(\cos\theta-\cos 2\theta) \qquad \{\because \theta_1=\theta,\ \theta_2=2\theta\}$$

$$\Rightarrow \quad W=\frac{\tau}{\sin\theta}(\cos\theta-\cos 2\theta)$$

ILLUSTRATION 37

In the arrangement shown, if the dipole is released from $\theta=53°$, then calculate



(i) Its angular acceleration just after releasing.

(ii) Its angular velocity when it passes through stable equilibrium.

(iii) The work required to rotate it by 180°

SOLUTION

(i) $\tau_{\text{net}}=pE\sin(53°)=I\alpha$

$$\Rightarrow \quad \alpha = \frac{(ql)E\left(\frac{4}{3}\right)}{\frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2}$$

$$\Rightarrow \quad \alpha = \frac{\frac{4}{3}qlE}{\frac{7}{12}ml^2} = \frac{16}{7}\left(\frac{qE}{ml}\right)$$

(ii) By Law of Conservation of Energy, we get

$$(K+U)_{\text{initial}} = (K+U)_{\text{final}}$$

$$\Rightarrow \quad 0 + \left(-pE(\cos 53°)\right) = \frac{1}{2}I\omega^2 + \left(-pE\cos 0°\right)$$

Where $I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{7}{12}ml^2$

$$\Rightarrow \quad \omega = \sqrt{\frac{4pE}{5I}} = \sqrt{\frac{4qlE}{5\left(\frac{7}{12}ml^2\right)}} = \sqrt{\frac{48qE}{35ml}}$$

(iii) $W = U_f - U_i$

$$\Rightarrow \quad W = \left(-pE\cos(180° + 53°)\right) - \left(-pE\cos 53°\right)$$

$$W = (ql)E\left(\frac{4}{3}\right) + (ql)E\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad W = \left(\frac{8}{3}\right)qlE$$

ILLUSTRATION 38

A light massless rod of length l lies in x-y plane with its centre at origin and it makes an angle θ with x-axis. A particle of mass m and charge $-q$ is attached at its left end and another particle of mass m and charge q at the other end. Write the dipole moment vector. Now an electric field of constant magnitude E and directed along x-axis is switched on. Write an expression for the torque on the rod at the initial positon. What is the angular speed of the rod at the instant when it becomes parallel to x-axis ?

SOLUTION

The dipole moment has a magnitude

$$p = q(2a) = ql$$

And it is directed along a unit vector $\cos\theta\hat{i} + \sin\theta\hat{j}$

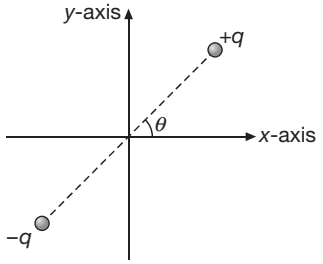
$$\Rightarrow \quad \vec{p} = ql\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right]$$

The electric field is given as

$$\vec{E} = E\hat{i}$$

Since $\vec{\tau} = \vec{p} \times \vec{E}$

$$\Rightarrow \quad \vec{\tau} = ql\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) \times E\hat{i} = -\left(qlE\sin\theta\right)\hat{k}$$



To calculate angular speed, applying Law of Conservation of Mechanical Energy, i.e.

$$U_i + K_i = U_f + K_f$$

where, $U_i = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

$$U_f = -\vec{p}\cdot\vec{E} = -pE$$

$\{\because$ rod becomes parallel to x-axis$\}$

$K_i = 0$ and

$$K_f = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2\right]\omega^2$$

$$\Rightarrow \quad -pE\cos\theta = -pE + \frac{ml^2}{4}\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4qE(1-\cos\theta)}{ml}}$$

SMALL OSCILLATIONS OF A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

Let the dipole possess a moment of inertia I about O. Since

$$\tau = -pE\sin\theta$$

where negative sign indicates that this is the restoring torque which tries to restore dipole to its mean position.

For small θ, $\sin\theta \cong \theta$

$$\Rightarrow \quad \tau = -pE\theta$$

Further $\tau = I\alpha = I\ddot{\theta}$

$$\Rightarrow \quad I\ddot{\theta} = -pE\theta$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{pE}{I}\right)\theta = 0$$

This equation shows that the dipole will execute oscillations when given a small angular displacement from the mean position. The time period of oscillations is T given by

$$T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{I}{pE}} = 2\pi\sqrt{\frac{I_{CM}}{pE}}$$

where I_{CM} is the moment of inertia of the dipole about the centre of mass of the dipole.

FORCE ON AN ELECTRIC DIPOLE IN NON-UNIFORM ELECTRIC FIELD

In a non-uniform electric field, if a dipole is placed at a point where electric field is $\vec{E}$, then the interaction energy of dipole at this point is given by

$$U = -\vec{p}\cdot\vec{E}$$

The force on dipole due to electric field is

$$\vec{F} = -\vec{\nabla}U$$

For unidirectional variation in electric field, we get

$$|\vec{F}| = -\frac{d}{dx}\left(\vec{p}\cdot\vec{E}\right)$$

If the dipole is placed in the direction of electric field, then

$$F = -p\frac{dE}{dx}$$

ILLUSTRATION 39

An electric dipole is placed at distance x from an infinitely long rod of linear charge density λ.

(a) What is net force acting on dipole?
(b) What is work done in rotating dipole through $180°$?
(c) If dipole is slightly rotated about its equilibrium position. Find the time period of oscillation.

SOLUTION

(a) We know that electric field at a distance x from the rod

$$E = \frac{\lambda}{2\pi\varepsilon_0 x} \qquad \{\text{linear charge distribution}\}$$

directed along the axis of the dipole. The direction can be determined by the fact that electric field is always perpendicular to equipotential surface.
Net force on dipole due to electric field of rod

$$F = q\frac{dE}{dx}$$

As we know potential difference $= dV = -Edr$
Change in electrostatic Potential Energy

$$= dU = \frac{dV}{q} = -Edr$$

$$dU = -qEdr$$

$$F = -\frac{dU}{dr} = \frac{pdE}{dr} \qquad \{\because U = -pE \text{ as } p \parallel E\}$$

As dipole is lying along x-axis

$$F = p\frac{dE}{dx} = 2aq\frac{d}{dx}\left[\frac{\lambda}{2\pi\varepsilon_0 x}\right]$$

$$F = \frac{\lambda aq}{\pi\varepsilon_0 x^2}$$

(b) Workdone = Difference in P.E. at two positions

$$\Rightarrow \quad W = -pE\cos 180° - \left(-pE\cos 0\right)$$

$$\Rightarrow \quad W = 2\left(2aq\right)\left(\frac{\lambda}{2\pi\varepsilon_0 x}\right) = \frac{2\lambda aq}{\pi\varepsilon_0 x}$$

(c) When the dipole is slightly disturbed from its equilibrium position by an angle θ.
Restoring torque $= -pE\sin\theta$

$$\{\sin\theta \simeq \theta \text{ for small displacement}\}$$

$$\Rightarrow \quad \tau = -pE\theta = -2aq \times \frac{\lambda\theta}{2\pi\varepsilon_0 x} = \frac{-\lambda a\theta q}{\pi\varepsilon_0 x}$$

Now torque is also given as

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia and α is angular acceleration

$$I\frac{d^2\theta}{dt^2} = -\frac{\lambda a\theta q}{\pi\varepsilon_0 x} \qquad \{\text{for dipole } I = 2ma^2\}$$

$$\Rightarrow \quad 2ma^2\frac{d^2\theta}{dt^2} = \frac{\lambda a\theta q}{\pi\varepsilon_0 x}$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \frac{\lambda q\theta}{2\pi\varepsilon_0 \max} = 0$$

Comparing it with standard equation of S.H.M.

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \text{ where } \omega = \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 \max}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\pi\varepsilon_0 \max}{\lambda q}}$$

FORCE ON A DIPOLE IN THE SURROUNDING OF A LONG CHARGED WIRE

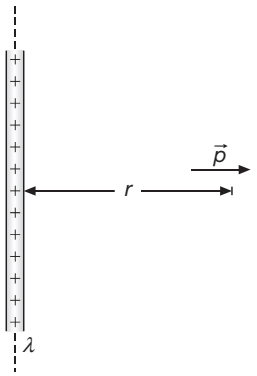
In the situation shown in figure, the electric field strength due to the wire, at the position of dipole is

$$E = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Thus force on dipole is

$$F = -p\left(\frac{dE}{dr}\right) = -p\left[-\frac{\lambda}{2\pi\varepsilon_0 r^2}\right]$$

$$\Rightarrow \quad F = \frac{\lambda p}{2\pi\varepsilon_0 r^2}$$



Here negative charge of dipole is placed close to wire and hence net force on the dipole due to wire will be attractive.

ELECTRIC FORCE BETWEEN TWO DIPOLES

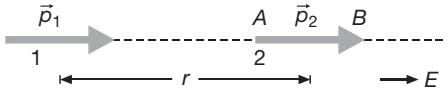
Electrostatic force between two dipoles of dipole moments p_1 and p_2 lying at a separation $r(r \gg a)$ is

$$F = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{6p_1p_2}{r^4}, & \text{when dipoles are placed coaxially to each other} \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{3p_1p_2}{r^4}, & \text{when dipoles are placed perpendicular to each other} \end{cases}$$

The above results have been derived here to give you a view of the fundamentals related to dipoles. Please keep in mind that the above results are obtained on the basis of assumption that the centres of the dipoles lie far apart from each other i.e., $r \gg 2a$ for each one of them.

CASE-1:

When the dipoles are placed coaxially



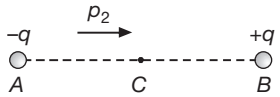
Consider two dipoles with dipole moments $\vec{p}_1$ and $\vec{p}_2$ (as shown) with their centres at distance $r(\gg 2a)$.

The electric field of the dipole 1 on the left side exerts a net force on the dipole 2 placed to its right. The electric field due to 1 at the centre of 2 is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p_1}{r^3} \qquad \{\text{from } A \text{ to } B\}$$

$$\Rightarrow \quad dE = \frac{2p_1}{4\pi\varepsilon_0}d(r^{-3})$$

$$\Rightarrow \quad dE = -\frac{6p_1}{4\pi\varepsilon_0 r^4}dr \qquad \ldots(1)$$



C is midpoint of dipole 2 with AB = dr

Let the $-q$ charge of dipole be placed at A and the $+q$ charge be placed at B. So, the electric field at A (where $-q$ lies) is

$$E_A = E + |dE|$$

{obviously field due to dipole 1 is stronger at A in comparison to B}

and $E_B = E - |dE|$

As a result of this, force on $-q$ at A is

$F_1 = qE_A$ {towards left}

and that on $+q$ at B is

$F_2 = qE_B$ {towards right}

$\Rightarrow$ Net force on the dipole is

$F = F_1 - F_2$ {towards left}

$\Rightarrow F = qE_A - qE_B$

$\Rightarrow F = q\left[(E+|dE|)-(E-|dE|)\right]$

$\Rightarrow F = 2q|dE|$

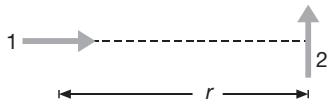
$\Rightarrow F = 2q\left(\dfrac{6p_1}{4\pi\varepsilon_0 r^4}\right)dr$

$\Rightarrow F = \dfrac{6p_1(2qdr)}{4\pi\varepsilon_0 r^4}$

$\Rightarrow F = \dfrac{6p_1p_2}{4\pi\varepsilon_0 r^4}$ $\{\because 2q(dr) = p_2\}$

CASE-2:

When the dipoles are placed perpendicular to each other.



Proceeding exactly the same way as in case-1, we get

$$F = \frac{3p_1p_2}{4\pi\varepsilon_0 r^4}$$

All assumptions here are exactly the same as before, other than this that the field due to dipole 1 at the centre of 2 will be

$$E = \frac{p_1}{4\pi\varepsilon_0 r^3}$$

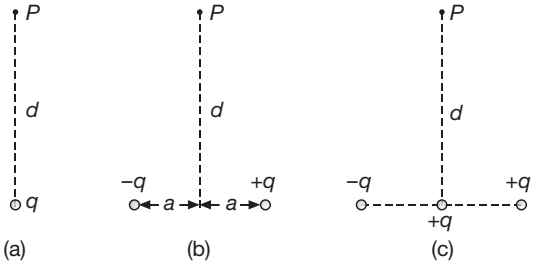
and hence we get

$$F = \frac{3p_1p_2}{4\pi\varepsilon_0 r^4}$$

as the force between the dipoles placed perpendicular to each other.

ILLUSTRATION 40

Find the magnitude of the electric field at the point P in the configuration shown in figure for $d \gg a$. Take $2qa = p$.



SOLUTION

(a) The electric field due to a point charge at P is given by

$$E_P = \frac{q}{4\pi\varepsilon_0 d^2}$$

(b) As $d \gg a$ a combination of $-q$ and $+q$ can be treated as a dipole. Hence, the electric field at P is

$$E_P = \frac{q(2a)}{4\pi\varepsilon_0 d^3}$$ (towards left)

Direction will be horizontal towards left

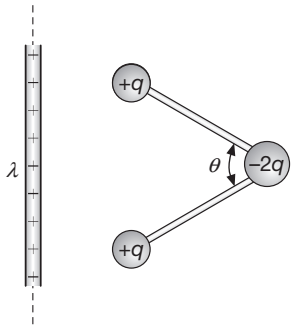
(c) This case is the super position of above two cases

Thus we have $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{q}{d^2}\hat{j} + \left(-\dfrac{p}{d^3}\right)\hat{i}\right]$

$\Rightarrow \vec{E} = \dfrac{1}{4\pi\varepsilon_0 d^2}\left(q\hat{i} - \dfrac{p}{d}\hat{j}\right)$

ILLUSTRATION 41

A water molecule is placed at a distance l from the line carrying linear charge density λ. Find the maximum force exerted on the water molecule. The shape of water molecule and the partial charges on H and O atoms are shown in figure. Assume l to be much larger than the dimensions of the molecule.



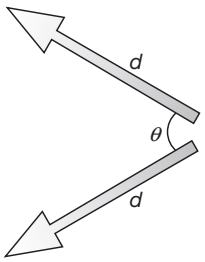
SOLUTION

The figure can be resolved as combination of 2 dipoles
Dipole moments of each $p = qd$
Here total dipole moment of system is

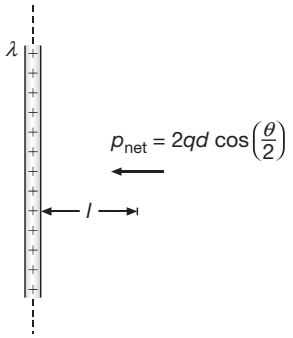
$$p_{net} = 2qd\cos\left(\frac{\theta}{2}\right)$$

Now

$$|\vec{F}| = \vec{p}_{net}\left(\frac{d\vec{E}}{dx}\right)$$



For maximum force, the angle between $\vec{p}_{net}$ and $\frac{d\vec{E}}{dx}$ is 0°



$$\Rightarrow \quad F_{max} = 2qd\cos\left(\frac{\theta}{2}\right)\left[\frac{d}{dx}\left(\frac{\lambda}{2\pi\varepsilon_0 x}\right)\right]$$

$$\Rightarrow \quad F_{max} = 2qd\cos\left(\frac{\theta}{2}\right)\times\frac{\lambda}{2\pi\varepsilon_0}\left(-\frac{1}{x^2}\right)$$

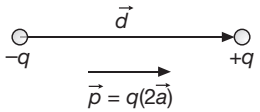
$$\Rightarrow \quad F_{max} = \frac{\lambda qd\cos\left(\frac{\theta}{2}\right)}{\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad |\vec{F}_{max}| = \frac{\lambda qd\cos\left(\frac{\theta}{2}\right)}{\pi\varepsilon_0 x^2}$$

$$F_{max} = \frac{\lambda qd\cos\left(\frac{\theta}{2}\right)}{l^2}$$

CONCEPT OF DISTRIBUTED DIPOLE

Till now we have discussed about an electric dipole that has two equal and opposite charges separated by a small distance $2\vec{a}$ as shown. Its dipole moment is defined, as $\vec{p} = q(2\vec{a})$ from $-q$ to $+q$.



Sometimes the two charges of dipole are not concentrated at its ends. The system, instead of having two charges has more than two charges.

In such a case we select a convenient origin. Then we locate the coordinates of the charges. Consider a system having charges q_1, q_2 and q_3 at points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) then

$$p_x = q_1x_1 + q_2x_2 + q_3x_3,$$

$$p_y = q_1y_1 + q_2y_2 + q_3y_3$$

and $p_z = q_1z_1 + q_2z_2 + q_3z_3$

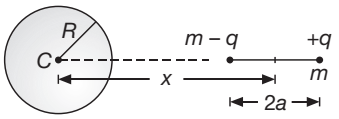
and the net dipole moment is then given by

$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

where $|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2}$

ILLUSTRATION 42

In figure shown, an electric dipole lies at a distance x from the centre of the axis of a charged ring of radius R with charge Q uniformly distributed over it.



(a) Find the net force acting on the dipole.
(b) What is the work done in rotating the dipole through 180° ?
(c) The dipole is slightly rotated about its equilibrium position. Find the time period of oscillation. Assume that the dipole is linearly restrained.

SOLUTION

(a) Field at a distance x on the axis of a ring is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}$$

The net force on the dipole is

$$F = p\frac{dE}{dx} = 2aq\frac{1}{4\pi\varepsilon_0}Q\frac{d}{dx}\left[\frac{x}{(R^2+x^2)^{3/2}}\right]$$

$$F = \frac{aQq}{2\pi\varepsilon_0}\frac{R^2-2x^2}{(R^2+x^2)^{5/2}}$$

(b) Work done in rotating a dipole is equal to change in its potential energy.

$$W = \Delta U$$

$$\Rightarrow \quad U_i = -\vec{p}\cdot\vec{E} = -pE\cos(0°) = -pE$$

and $\quad U_f = -\vec{p}\cdot\vec{E} = -pE\cos(180°) = +pE$

$$\Rightarrow \quad W = 2pE$$

$$\Rightarrow \quad W = \frac{2(2a)}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}} = \frac{aQqx}{\pi\varepsilon_0(R^2+x^2)^{3/2}}$$

(c) Restoring torque $\tau = -pE\sin\theta = -pE\theta$ (for small θ)

$$\tau = I\frac{d^2\theta}{dt^2} = 2ma^2\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{pE}{2ma^2}\theta$$

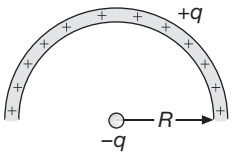
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{pE}} = 2\pi\sqrt{\frac{2ma^2}{pE}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2ma^2}{2aq}\frac{(R^2+x^2)^{3/2}\,4\pi\varepsilon_0}{Qx}}$$

$$\Rightarrow \quad T = \sqrt{\frac{16\pi^2\varepsilon ma}{qQx}(R^2+x^2)^{3/2}}$$

ILLUSTRATION 43

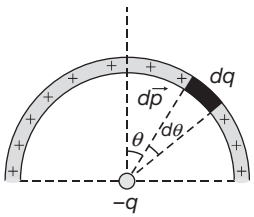
On a half ring a charge $+q$ is uniformly distributed and another equal negative charge $-q$ is placed at the centre of ring.



In this system negative charge is a point charge and positive charge is distributed on the ring. This system is called, distributed dipole. Calculate the dipole moment of this system.

SOLUTION

For this we consider a polar element of angular width $d\theta$ at an angle θ from the vertical axis as shown.



Charge on this element is

$$dq = \frac{q}{\pi}d\theta$$

Now in the point charge $-q$, we consider an element charge $-dq$ which forms a dipole with the polar element of charge $+dq$. This element dipole moment can be given as

$$dp = dqR = \frac{q}{\pi} Rd\theta$$

Total dipole moment of system can be given by integrating all such elemental dipole moments as

$$p_{\text{total}} = \int dp \cos\theta$$

{The components $dp\sin\theta$ cancel out}

$$\Rightarrow \quad p_{\text{total}} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{q}{\pi} R\cos\theta d\theta$$

$$\Rightarrow \quad p_{\text{total}} = \frac{qR}{\pi}[\sin\theta]\Big|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

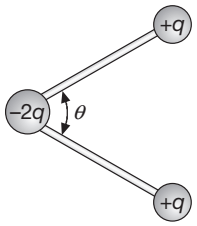
$$\Rightarrow \quad p_{\text{total}} = \frac{2qR}{\pi}$$

Test Your Concepts-IV

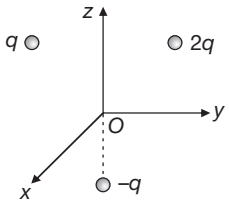
Based on Dipoles and Dipole Moment

(Solutions on page H.15)

1. Three charges $-2q$, q and q are arranged in the configuration shown. Calculate resultant dipole moment of the system.



2. Three point charges $2q$, q and $-q$ are located respectively at $(0, a, a)$, $(0, -a, a)$ and $(0, 0, -a)$ as shown. Find the dipole moment of this distribution.

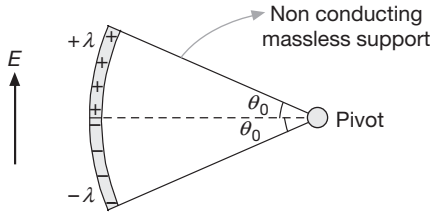


3. A small freely oriented dipole with dipole moment p lies at the centre of a uniformly charged hemisphere, charge density σ and radius R. Determine the potential energy of the dipole and the period of small oscillations about an axis perpendicular to the axis of the hemisphere. The moment of inertia of the dipole about the rotation axis is I.

4. Figure shows an assembly of two charged rods each of mass m, length L connected by non-conducting massless rods of length d. The system is free to rotate about the axis shown. The system is given an anticlockwise small angular displacement. Show that the motion is simple harmonic and determine its time period.

What is the work done required to turn the system through an angle (a) $\frac{\pi}{2}$ (b) π ?

5. Figure shows a charged rod of mass m bent in the form of an arc of a circle of radius R. The charge distribution on the rod is shown in the figure. The assembly is kept in a uniform electric field.



(a) Show that for small angular displacement the system will perform SHM. Determine its time period.

(b) Considering the size of the system to be very small, determine the work done required to turn it through 180°. Neglect any displacement of centre of charge.

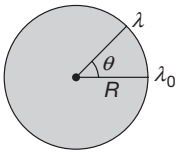
6. A dipole with an electric moment p is located at a distance r from a long thread charged uniformly with a linear density λ. Find the force $\vec{F}$ acting on the dipole if the vector $\vec{p}$ is oriented

(a) along the thread

(b) along the radius vector $\vec{r}$

(c) at right angles to the thread and the radius vector $\vec{r}$.

7. A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\theta$, where λ_0 is the value of λ at $\theta = 0$. Find the net electric dipole moment for this charge distribution.



8. A positive point charge q is fixed at origin. A small dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis. Find

(a) the kinetic energy of the dipole when it reaches a distance d from the origin, and

(b) the force experienced by the charge q at this moment.

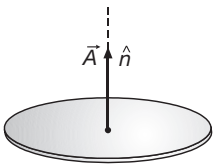
GAUSS'S LAW AND APPLICATIONS

AREA AS A VECTOR

The physical quantity, area is a pseudo vector and we treat area of a surface as a vector quantity whose direction is along the outward normal to the surface.

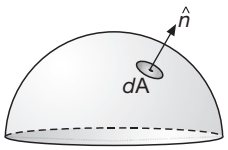
The area vector $\vec{A}$ of a surface which has surface area A can be written as

$$\vec{A} = A\hat{n}$$



Where $\hat{n}$ is the unit vector in the direction along outward normal to the surface.

When a surface is three dimensional we consider a small infinitesimal area element dA on this surface and direction of this elemental area vector is along the local outward normal to the surface at the point where elemental area is chosen as shown.



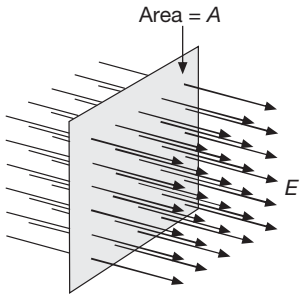
So, $\vec{A} = (dA)\hat{n}$

Here $\hat{n}$ is the unit vector in the direction along the outward normal at elemental area dA.

ELECTRIC FLUX

Consider an electric field that is uniform in both magnitude and direction, as shown in figure. The field lines penetrate a rectangular surface of area A, whose plane is oriented perpendicular to the field. Since the number of lines per unit area (in other words, the line density) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux ϕ_E (Greek phi)

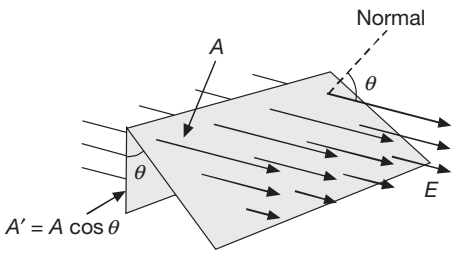
$$\phi_E = EA$$



Field lines representing a uniform electric field penetrating a plane of area A perpendicular to the field. The electric flux ϕ_E through this area is equal to EA.

From the SI units of E and A, we see that ϕ_E has SI units $\mathrm{Nm^2C^{-1}}$. Electric flux is proportional to the number of electric field lines penetrating some surface.

If the surface under consideration is not perpendicular to the field, the flux through it must be less than EA. We can understand this by considering figure, where the normal to the surface of area A is at an angle θ to the uniform electric field. Note that the number of lines that cross this area A is equal to the number that cross the area A', which is a projection of area A onto a plane oriented perpendicular to the field. From figure we see that the two areas are related by $A' = A\cos\theta$.

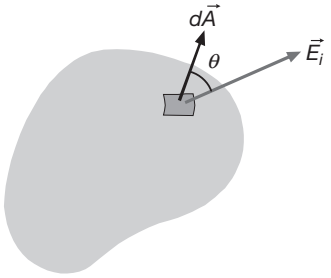


Field lines representing a uniform electric field penetrating an area A that is at an angle θ to the field. Because the number of lines that go through A' is the same as the number that go through A, the flux through A' is equal to the flux through A and is given $\Phi_E = \mathrm{EA}\cos\theta$.

Because the flux through A equals the flux through A', we conclude that the flux through A is

$$\phi_E = EA' = EA\cos\theta \quad \ldots(1)$$

From this result, we see that the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (when the normal to the surface is parallel to the field, that is, $\theta = 0°$ in figure); the flux is zero when the surface is parallel to the field (when the normal to the surface is perpendicular to the field, that is $\theta = 90°$).



A small element of surface area $d\vec{A}$. The electric field makes an angle θ with the vector $d\vec{A}$, defined as being normal to the surface element, and the flux through the element is equal to $EdA\cos\theta$.

$$\phi_E = \int_{\text{surface}} \vec{E}\cdot d\vec{A} \quad \ldots(2)$$

Equation (2) is a surface integral, which means it must be evaluated over the surface in question. In general, the value of Φ_E depends both on the field pattern and on the surface. Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux ϕ_E through a closed surface as

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E_n dA \quad \ldots(3)$$

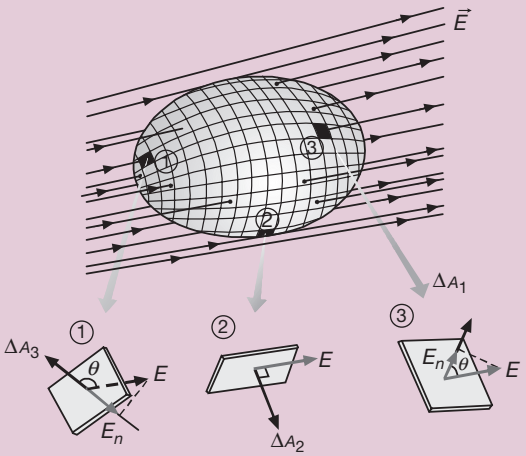
where E_n represents the component of the electric field normal to the surface. If the field is normal to the surface at each point and constant in magnitude, the calculation is straightforward.

Remark(s)

(a) We are often interested in evaluating the flux through a closed surface, which is defined as one that divides space into an inside and an outside region, so that one cannot move from one region to the other without crossing the surface. The surface of a sphere, for example, is a closed surface. Consider the closed surface in figure. The vectors $\Delta\vec{A}_i$ point in different directions for the various surface elements, but at each point they are normal to the surface and, by convention, always point outward.

For element ①, where the field lines are crossing the surface from outside to inside, $180° > \theta > 90°$ and the flux is negative because $\cos\theta$ is negative.

For element ②, the field lines graze the surface (perpendicular to the vector $\Delta\vec{A}_2$); thus, $\theta = 90°$ and the flux is zero.



A closed surface in an electric field. The area vectors ΔA_i are, by convention, normal to the surface and point outward. The flux through an area element can be negative (element ①), zero (element ②), or positive (element ③).

For element ③, the field lines are crossing the surface from the inside to the outside and $\theta < 90°$; hence, the flux $\Delta\phi_E = \vec{E}\cdot\Delta\vec{A}_1$ through this element is positive.

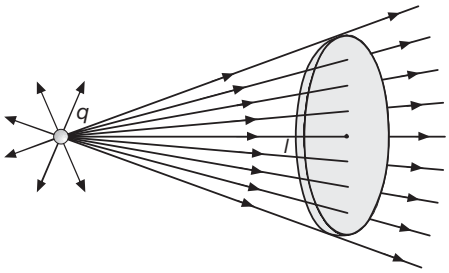
(b) The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface. If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.

ILLUSTRATION 44

Calculate the electric flux through the surface of the disc of radius R due to a point charge q placed at a point P on its axis at a distance l from its centre.

SOLUTION

Since a point charge q has electric field originating in radially outward direction.

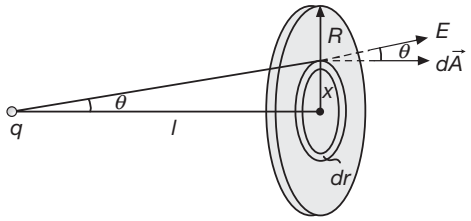


To calculate this flux, we consider an infinitesimal ring on disc surface of radius x and width dx as shown. Area of this strip is

$$dA = (2\pi x)dx$$

The electric field due to q at this elemental ring is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2+l^2)}$$



If $d\phi$ is the flux due to this elemental ring, then

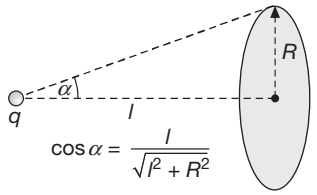
$$d\phi = \vec{E}\cdot d\vec{A} = EdA\cos\theta$$

$$\Rightarrow \quad d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{(l^2+x^2)}(2\pi x\,dx)\left(\frac{l}{\sqrt{l^2+x^2}}\right)$$

$$\Rightarrow \quad d\phi = \frac{ql}{2\varepsilon_0}\frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$\Rightarrow \quad \phi = \int d\phi = \frac{ql}{2\varepsilon_0}\int_0^R \frac{x\,dx}{(l^2+x^2)^{3/2}}$$

$$\Rightarrow \quad \phi = \frac{ql}{2\varepsilon_0}\left[\frac{1}{2}\int_0^R (l^2+x^2)^{-\frac{3}{2}}\,2x\,dx\right]$$



Since, $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}$, $n \neq -1$

So, $$\phi = \frac{ql}{2\varepsilon_0}\left[\frac{1}{2}\frac{(l^2+x^2)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\Bigg|_0^R\right]$$

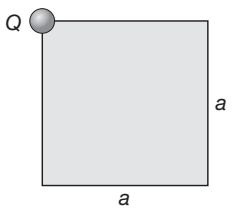
$$\Rightarrow \quad \phi = -\frac{ql}{2\varepsilon_0}\left[\frac{1}{\sqrt{l^2+x^2}}\Bigg|_0^R\right]$$

$$\Rightarrow \quad \phi = \frac{ql}{2\varepsilon_0}\left[\frac{1}{l}-\frac{1}{\sqrt{R^2+l^2}}\right]$$

$$\Rightarrow \quad \phi = \frac{q}{2\varepsilon_0}\left(1-\frac{l}{\sqrt{R^2+l^2}}\right) = \frac{q}{2\varepsilon_0}(1-\cos\alpha)$$

ILLUSTRATION 45

A point charge Q is placed at the corner of a square of side a, then find the flux through the square.



SOLUTION

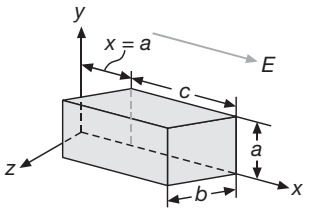
The electric field due to Q at any point of the square will be along the plane of square and the electric field line are perpendicular to square, so $\phi = 0$.
In other words we can say that no line is crossing the square so flux $= 0$.

ILLUSTRATION 46

A closed surface with dimensions $a = b = 0.4$ m and $c = 0.6$ m is located as in figure. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is non-uniform and given by $E = (3 + 2x^2)\hat{i}$ NC^{-1}, where x is in meter.

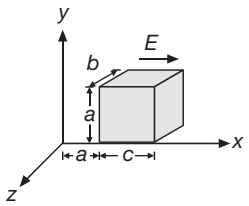
(a) Calculate the net electric flux leaving the closed surface.

(b) What net charge is enclosed by the surface?



SOLUTION

The electric field throughout the region is directed along x, therefore, E will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore



$$\phi_E = -\left(E_x|_{x=a}\right)A + \left(E_x|_{x=a+c}\right)A$$

$$\Rightarrow \quad \phi_E = -(3 + 2a^2)ab + (3 + 2(a+c)^2)ab$$

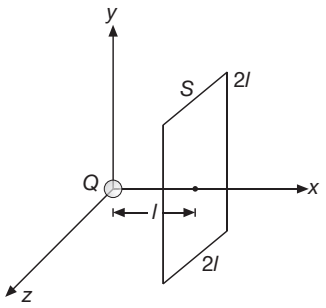
$$\Rightarrow \quad \phi_E = 2abc(2a + c)$$

Substituting the given values for a, b and c, we find $\phi_E = 0.269$ Nm^{2}C^{-1}

$$Q = \varepsilon_0 \phi_E = 2.38 \times 10^{-12} \text{ C} = 2.38 \text{ pC}$$

ILLUSTRATION 47

(a) Compute the electric flux through a square surface of edges $2l$ due to a charge $+Q$ located at a perpendicular distance l from the center of the square, as shown in figure.

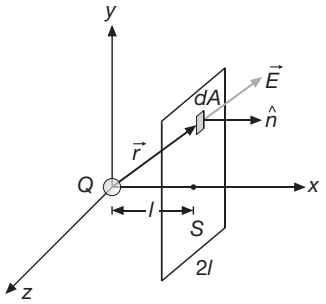


(b) Using the result obtained in (a), if the charge $+Q$ is now at the centre of a cube of side $2l$, what is the total flux emerging from all the six faces of the closed surface?

SOLUTION

(a) The electric field due to the charge $+Q$ is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{r}\right)$$



where $r = \left(x^2 + y^2 + z^2\right)^{1/2}$ in Cartesian coordinates. On the surface S, $x = l$ and the area element is $d\vec{A} = dA\hat{i} = (dydz)\hat{i}$. So,

$$\vec{E} \cdot d\vec{A} = \frac{Q}{4\pi\varepsilon_0 r^2}\left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{r}\right) \cdot (dydz)\hat{i}$$

$$\Rightarrow \quad \vec{E} \cdot d\vec{A} = \frac{Ql}{4\pi\varepsilon_0 r^3}dydz$$

Thus, the electric flux through S placed at $x = l$ is

$$\phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{Ql}{4\pi\varepsilon_0}\int_{-l}^{l} dz \int_{-l}^{l} \frac{dy}{\left(l^2 + y^2 + z^2\right)^{3/2}}$$

$$\Rightarrow \quad \phi_E = \frac{Ql}{4\pi\varepsilon_0}\int_{-l}^{l} dz \frac{y}{\left(l^2+z^2\right)\left(l^2+y^2+z^2\right)^{1/2}}\Bigg|_{-l}^{l}$$

$$\Rightarrow \quad \phi_E = \frac{Ql}{2\pi\varepsilon_0}\int_{-l}^{l} \frac{l\,dz}{\left(l^2+z^2\right)\left(2l^2+z^2\right)^{1/2}}$$

$$\Rightarrow \quad \phi_E = \frac{Q}{2\pi\varepsilon_0}\tan^{-1}\left(\frac{z}{\sqrt{z^2+2l^2}}\right)\Bigg|_{-l}^{l}$$

$$\Rightarrow \quad \phi_E = \frac{Q}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)-\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$$

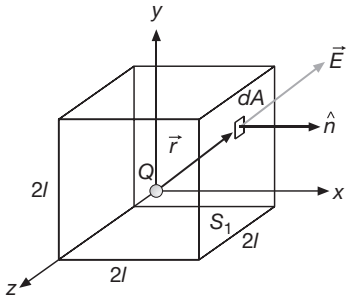
$$\Rightarrow \quad \phi_E = \frac{Q}{2\pi\varepsilon_0}\left(\frac{\pi}{6}+\frac{\pi}{6}\right)=\frac{Q}{6\varepsilon_0}$$

In evaluating the above integrals, the following results have been used.

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\left(x^2+a^2\right)^{1/2}}$$

$$\int \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)^{1/2}} =$$

$$\frac{1}{a\left(b^2-a^2\right)^{1/2}}\tan^{-1}\sqrt{\frac{b^2-a^2}{a^2\left(x^2+b^2\right)}}\,,\ b^2>a^2$$



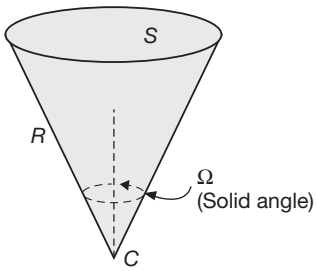
(b) From symmetry arguments, the flux through each face must be the same. Thus, the total flux through the cube is just six times that through one face. So,

$$\Phi_E = 6\left(\frac{Q}{6\varepsilon_0}\right)=\frac{Q}{\varepsilon_0}$$

The result shows that the electric flux Φ_E passing through a closed surface is proportional to the charge enclosed. In addition, the result further strengthens the notion that Φ_E is independent of the shape of the closed surface.

CONCEPT OF SOLID ANGLE

The three dimensional angle enclosed by the lateral surface of a cone at its vertex (as shown in figure) is the **Solid Angle**.

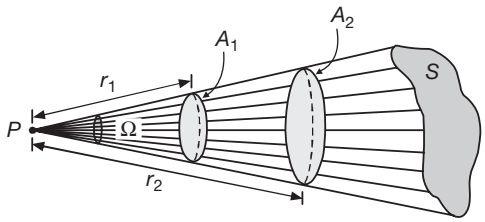


Solid angle is also defined as the three dimensional angle subtended by a spherical section at its centre of curvature. As in the figure shown point C is the centre of curvature of a spherical section S of radius R which subtends a solid angle Ω (Greek symbol omega) at point C.

SOLID ANGLE SUBTENDED DUE TO A RANDOM SURFACE AT A GIVEN POINT

Consider a random surface S as shown in figure. To find the solid angle subtended by this surface at a point P, let us join all the points of the periphery or boundary of the surface S to the point P by straight lines. This gives a cone with vertex at P.

Now by taking centre at P, we construct several spherical sections on this cone of different radii as shown. Let the area of spherical section which is of radius r_1 be A_1 and the area of section of radius r_2 be A_2.



Mathematically we observe that the ratio of area of any sphere enclosed by the cone to the square of radius of that sphere is a constant and this constant is called solid angle Ω.

In the figure shown the solid angle Ω subtended at P is given by

$$\Omega = \frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}$$

Solid angle is a dimensionless physical quantity and its S.I. unit is steradian.

One steradian is the solid angle subtended at the centre of sphere by the surface of the sphere having area equal to square of the radius of sphere. Also,

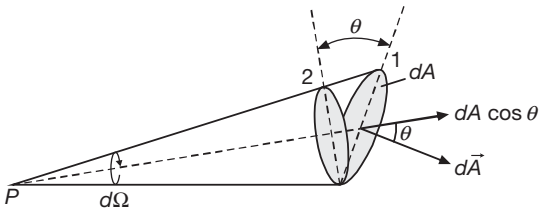
$$\Omega_{max} = \frac{\text{Area of the sphere}}{(\text{Radius of sphere})^2} = \frac{4\pi r^2}{r^2}$$

$$\Rightarrow \quad \Omega_{max} = 4\pi \text{ steradian}$$

SOLID ANGLE SUBTENDED BY A SURFACE NOT NORMAL TO AXIS OF CONE OR NOT LYING ON SURFACE OF SPHERE

Consider a small surface AB of area $d\vec{A}$ as shown. Let PQ be the axis of cone formed by this surface. Here PQ is not normal to the surface 1 (in light grey shade). The solid angle Ω subtended at point P is given by

$$d\Omega = \frac{dA\cos\theta}{r^2} \qquad \ldots(1)$$



where θ is the angle between surface area vector $d\vec{A}$ and the axis of cone PQ as shown in the Figure.

Since, the total solid angle that any closed surface can subtend is 4π steradian.

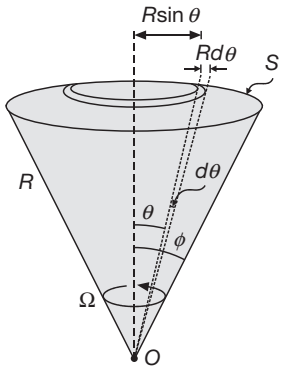
So,

$$\oint \frac{dA\cos\theta}{r^2} = 4\pi$$

RELATION BETWEEN HALF ANGLE OF CONE AND SOLID ANGLE AT VERTEX

Consider a spherical section S of radius R, which subtends a half angle ϕ (in radian) at the centre of curvature. To find the area of this section, we consider an infinitesimal circular strip on this section having angular width $d\theta$ and radius $R\sin\theta$. The surface area dA of this strip is

$$dA = (2\pi R\sin\theta)(Rd\theta)$$



The total area of spherical section is calculated by integrating the area of this elemental strip within limits from 0 to ϕ.

Total area of spherical section is

$$A = \int dA = \int_0^{\phi} 2\pi R^2 \sin\theta\, d\theta$$

$$\Rightarrow \quad A = 2\pi R^2 [-\cos\theta]_0^{\phi}$$

$$\Rightarrow \quad A = 2\pi R^2 (1-\cos\phi) \qquad \ldots(1)$$

If this section subtends a solid angle Ω at its centre O is Ω then, by definition

$$\Omega = \frac{A}{R^2} \qquad \ldots(2)$$

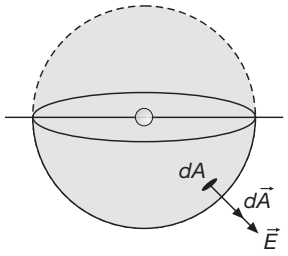
From (1) and (2), we get

$$\Omega = 2\pi(1-\cos\phi) \qquad \ldots(3)$$

This equation gives the relation in half angle of a cone ϕ and the solid angle enclosed by the lateral surface of cone at its vertex.

ELECTRIC FLUX PRODUCED BY A POINT CHARGE

The figure shows a point charge placed at the centre of a spherical surface of radius R. For convenience and picture clarity only lower half of sphere is drawn in the figure.



Since the charge q lies inside the sphere, so the total flux that originates comes out from the spherical surface. To find the total flux, let us consider an elemental area dA on surface. The electric field on the points on surface of sphere is

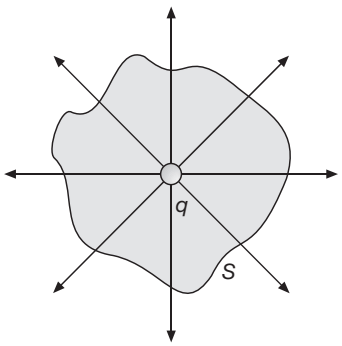
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

The electric flux coming out from the surface dA is

$$d\phi = \vec{E}\cdot d\vec{A} = EdA\cos 0$$

{because $\vec{E}$ and $d\vec{A}$ are parallel}

$$\Rightarrow \quad d\phi = \frac{q}{4\pi\varepsilon_0 R^2}dA$$



So, total flux coming out from the spherical surface is

$$\phi = \int d\phi = \int \frac{q}{4\pi\varepsilon_0 R^2}dA$$

At every point on spherical surface the magnitude of electric field remains same and hence we have

$$\phi = \frac{q}{4\pi\varepsilon_0 R^2}\oint dA$$

$$\Rightarrow \quad \phi = \frac{q}{4\pi\varepsilon_0 R^2}\left(4\pi R^2\right)$$

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0}$$

Thus total flux, originating from charge q is $\frac{q}{\varepsilon_0}$. Similarly a charge $-q$ absorbs $\frac{q}{\varepsilon_0}$ electric lines (flux) into it. Figure shows a charge q enclosed in a closed surface S of random shape. Here we can say that the total electric flux emerging out from the surface S is the flux that originates from charge q and hence flux emerging from surface is

$$\phi_S = \frac{q}{\varepsilon_0}$$

Please note that, the above result is independent of the shape of surface it only depends on the amount of charge enclosed by the surface. Also field lines leaving a surface will give positive flux and the field lines entering a surface give negative flux.

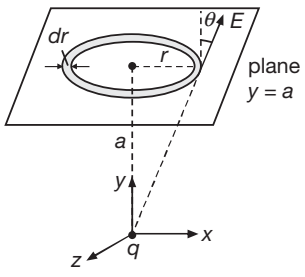
ILLUSTRATION 48

A point charge q is fixed at the origin. Calculate the electric flux through the infinite plane $y = a$.

SOLUTION

Consider a ring of radius r on the plane $y = a$. The thickness of ring is dr. Electric field on the ring due to charge q is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2 + a^2\right)}$$



Area of the ring $dA = (2\pi r)\,dr$

$$\cos\theta = \frac{a}{\sqrt{r^2 + a^2}}$$

Flux passing through the infinitesimal ring is

$$d\phi_e = EdA\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + a^2}(2\pi r\,dr)\frac{a}{\sqrt{r^2 + a^2}}$$

$$\Rightarrow \quad d\phi_e = \frac{q}{2\varepsilon_0}\frac{ar\,dr}{(r^2 + a^2)^{3/2}}$$

Total flux passing through the plane is given by

$$\phi_e = \int_0^\infty d\phi_e = \frac{qa}{2\varepsilon_0}\int_0^\infty \frac{r\,dr}{(r^2 + a^2)^{3/2}}$$

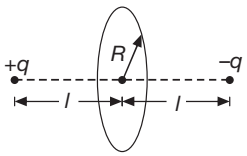
$$\Rightarrow \quad \phi_e = \frac{q}{2\varepsilon_0} \qquad \left\{\because \int_0^\infty \frac{r\,dr}{(r^2 + a^2)^{3/2}} = \frac{1}{a}\right\}$$

Test Your Concepts-V

Based on Flux

(Solutions on page H.18)

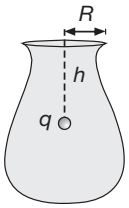
1. Two point charges q and $-q$ separated by a distance 2ℓ. Find the flux of electric field strength vector across the circle of radius R placed with its centre coinciding with the midpoint of line joining the two charges in the perpendicular plane.



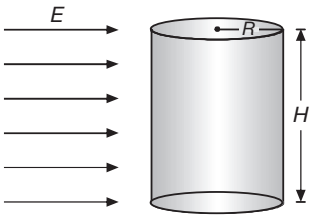
2. A point charge q is kept at the centre of the cylinder of length L and radius R. Calculate the flux through the curved surface of the cylinder.

3. A very long uniformly charged thread oriented along the axis of a circle of radius R rests on its centre with one of the ends. The charge of the thread per unit length is equal to λ. Find the flux of the vector E across the circle area.

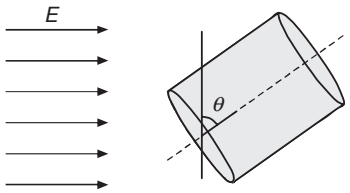
4. Consider the situation shown in figure. A point charge q is placed at a depth $h = \sqrt{3}\,R$ exactly below the centre of mouth of a vessel whose open end is circular having a radius R. Calculate the electric flux through the lateral surface of this vessel.



5. A cylinder of height H and radius R is placed in a uniform electric field as shown in figure. Find flux crossing the cylinder.



6. A cylinder of height H and radius R is placed in a uniform electric field (as shown) such that the axis of the cylinder makes an angle θ with the vertical. Calculate the flux through the cylinder.



7. A non-uniform electric field is given by the expression $E = ay\hat{i} + bz\hat{j} + cx\hat{k}$, where a, b, and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = \ell$.

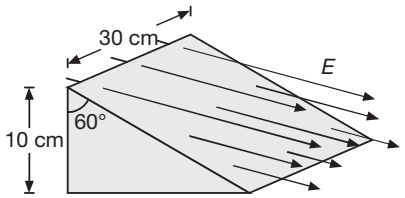
8. An electric field having a magnitude of 3.5 kNC^{-1} is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.35 m wide and 0.7 m long, assuming that the plane

(a) is parallel to the *yz* plane.
(b) is parallel to the *xy* plane.
(c) contains the *y*-axis, and its normal makes an angle of 60° with the *x*-axis.

9. On a day, above the Earth's surface, when a thunderstorm is brewing a vertical electric field of magnitude 2×10^4 NC^{-1} exists. A car having a rectangular size of 6 m by 3 m is travelling along the road sloping downward at 60°. Determine the electric flux through the bottom of the car.

10. A loop of radius 40 cm is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be 5.04×10^5 Nm^2 C^{-1}. Calculate the magnitude of the electric field.

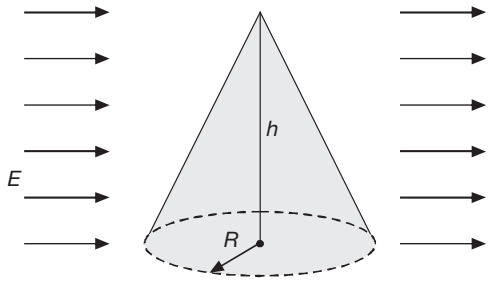
11. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80\times10^4$ NC^{-1} as shown.



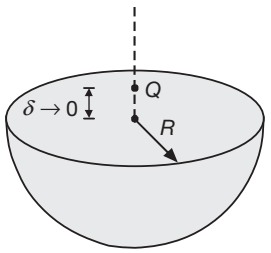
Calculate the electric flux through the
(a) vertical rectangular surface.
(b) slanted surface.
(c) entire surface of the box.

12. A pyramid with horizontal square base, 6 m on each side, and a height of 4 m is placed in a vertical electric field of 52 NC^{-1}. Calculate the total electric flux through the pyramid's four slanted surfaces.

13. A cone with base radius *R* and height *h* is located on a horizontal table. A horizontal uniform field *E* penetrates the cone, as shown in Figure. Determine the electric flux that enters the left-hand side of the cone.



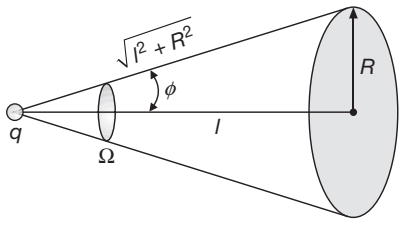
14. A point charge *Q* is located just above the center of the flat face of a hemisphere of radius *R* as shown in figure . What is the electric flux through



(a) the curved surface and
(b) the flat face?

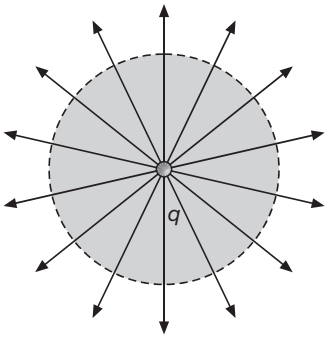
ELECTRIC FLUX CALCULATION DUE TO A POINT CHARGE USING SOLID ANGLE

Consider a point charge *q*, placed at a distance *l* from the centre of a circular disc of radius *R*. Let us find the electric flux passing through the surface of disc due to the charge *q*.



Since a total flux $\frac{q}{\varepsilon_0}$ originates from a point charge q in all directions

We can say that from a point charge q, $\frac{q}{\varepsilon_0}$ flux originates in a solid angle 4π.



The solid angle enclosed by cone subtended by the disc at the point charge is given by

$$\Omega = 2\pi(1-\cos\phi)$$

$$\Rightarrow \quad \Omega = 2\pi\left(1-\frac{l}{\sqrt{l^2+R^2}}\right) \qquad \left\{\because \cos\phi = \frac{l}{\sqrt{l^2+R^2}}\right\}$$

The flux of q associated with the surface of the disc

$$\phi = \left(\frac{\text{Total flux associated with closed surface due to the charge}}{4\pi}\right)\left(\text{Solid angle subtended}\right)$$

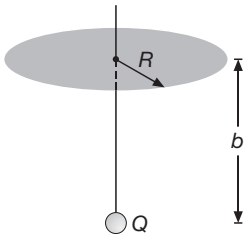
$$\phi_{\text{disc}} = \frac{\left(\frac{q}{\varepsilon_0}\right)}{4\pi}\times\Omega = \left(\frac{q}{4\pi\varepsilon_0}\right)\Omega$$

$$\Rightarrow \quad \phi_{\text{disc}} = \frac{q}{4\pi\varepsilon_0}\times 2\pi\left(1-\frac{l}{\sqrt{l^2+R^2}}\right)$$

$$\Rightarrow \quad \phi_{\text{disc}} = \frac{q}{2\varepsilon_0}\left(1-\frac{l}{\sqrt{l^2+R^2}}\right) = \frac{q}{2\varepsilon_0}(1-\cos\phi)$$

ILLUSTRATION 49

A point charge Q is located on the axis of a disk of radius R at a distance b from the plane of the disk. Show that if one fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}b$.



SOLUTION

Since $\phi_{E,\text{ disc}} = \frac{Q}{2\varepsilon_0}\left[1-\frac{b}{\sqrt{R^2+b^2}}\right]$

According to the problem, we have

$$\phi_{E,\text{ disc}} = \frac{1}{4}\left(\frac{Q}{\varepsilon_0}\right)$$

$$\Rightarrow \quad \frac{Q}{4\varepsilon_0} = \frac{Q}{2\varepsilon_0}\left(1-\frac{b}{\sqrt{R^2+b^2}}\right)$$

$$\Rightarrow \quad \frac{1}{2} = 1-\frac{b}{\sqrt{R^2+b^2}}$$

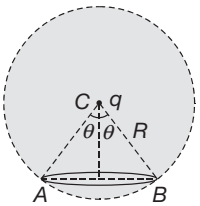
$$\Rightarrow \quad \frac{b^2}{R^2+b^2} = \frac{1}{4}$$

$$\Rightarrow \quad 4b^2 = R^2+b^2$$

$$\Rightarrow \quad R = \sqrt{3}\,b$$

ILLUSTRATION 50

A point charge q is placed on the top of a cone of semi vertex angle θ. Show that the electric flux associated with the base of the cone due to the charge is $\frac{q(1-\cos\theta)}{2\varepsilon_0}$.



SOLUTION

Consider a Gaussian surface, a sphere with its centre at the top and radius the slant length of the cone. The flux through the whole sphere is $\frac{q}{\varepsilon_0}$. Therefore, the

flux through the base of the cone can be calculated by using the following formula,

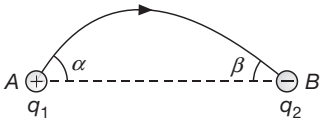
$$\phi = \frac{q}{4\pi\varepsilon_0}\,\Omega$$

Since, $\Omega = 2\pi(1-\cos\theta)$

$$\Rightarrow \quad \phi = \frac{q}{2\varepsilon_0}(1-\cos\theta)$$

ILLUSTRATION 51

Two charges $+q_1$ and $-q_2$ are placed at A and B respectively. A line of force emanates from q_1 at angle α with the line AB. At what angle will it terminate at $-q_2$?



SOLUTION

A line can leave $+q_1$ in a cone of apex angle α and then enter $-q_2$ in a cone of apex angle β.

So, flux due to the charge $+q_1$ is $\phi_1 = \frac{q_1}{2\varepsilon_0}(1-\cos\alpha)$

and that due to the charge $-q_2$ is $\phi_2 = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$.

Since, we know that only one line is leaving q_1 to enter $-q_2$. So, we can say

$$\frac{\phi_1}{\phi_2} = \frac{N_1}{N_2} = 1$$

$$\Rightarrow \quad \frac{q_1}{2\varepsilon_0}(1-\cos\alpha) = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$$

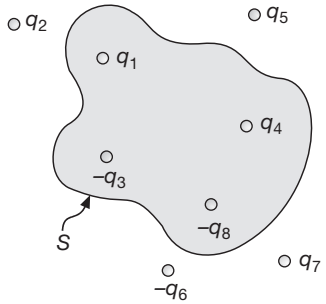
$$\Rightarrow \quad q_1\left[2\sin^2\left(\frac{\alpha}{2}\right)\right] = q_2\left[2\sin^2\left(\frac{\beta}{2}\right)\right]$$

$$\Rightarrow \quad \sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad \beta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)\right]$$

GAUSS'S LAW

This law gives the mathematical relation between the electric flux associated with a closed surface (number of field lines entering or leaving a closed surface) and the charge enclosed by the surface.



Gauss's Law states that the total flux due to a closed surface is equal to the $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

Mathematically Gauss's Law is written as

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{\Sigma q_{\text{enc}}}{\varepsilon_0}$$

Here the sign $\oint$ represents the integration over a closed surface S which encloses a total charge Σq_{enc}.

Let us consider a surface S shown which encloses four charges q_1, $-q_3$, q_4 and $-q_8$. For the surface S, if we find surface integral of electric field $\oint_S \vec{E}\cdot d\vec{A}$, it gives the total electric flux coming out from the surface, which can be given as

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}(q_1+q_4-q_3-q_8) \qquad \{\text{Gauss's Law}\}$$

Remark(s)

(a) Here electric field $\vec{E}$ is the net electric field at the points on the surface of S. Remember that the electric field we use to find the flux must be the net electric field of the system due to all the charges inside and outside the Gaussian surface, but the total flux associated with the surface is due to the charges that are enclosed inside the closed surface i.e. the Gaussian surface.

(b) Using Gauss's Law we can find electric field strength due to some symmetrical distribution of charges.

(c) For application of Gauss's Law, we choose a closed imaginary surface over which we apply Gauss's Law, called Gaussian surface.

(d) Gauss's Law can be used to calculate electric field strength, for this we first choose a proper Gaussian surface on which the electric field strength is to be calculated.

(e) Sometimes, when a random Gaussian surface is chosen then the integral $\oint \vec{E} \cdot d\vec{A}$ involves complex calculations. To make these calculations easier, we choose a Gaussian surface keeping following points in mind.

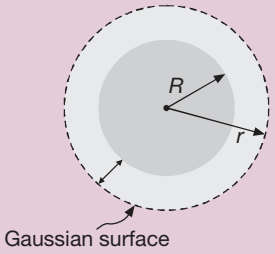
(i) The Gaussian surface should be chosen in such a way that at every point of surface the magnitude of electric field is either uniform or zero.

(ii) The surface should be chosen in such a way that at every point of surface electric field strength is either parallel or perpendicular to the surface.

Keeping the above two points e) (i) and e) (ii) in mind, let us conclude what type of Gaussian surface should be selected in different situations

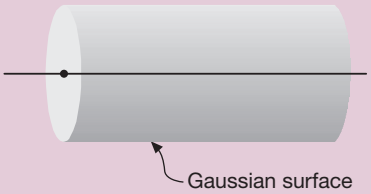
A. SPHERICAL SYMMETRY

Make the Gaussian surface a concentric sphere.



B. CYLINDRICAL SYMMETRY

Make the Gaussian surface a coaxial cylinder.



C. PLANE SYMMETRY

Use a Gaussian Pillbox which passes through the surface.

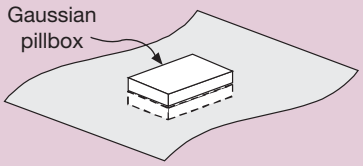
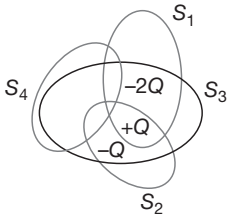


ILLUSTRATION 52

Four closed surfaces, S_1, S_2, S_3 and S_4, together with the charges $-2Q$, Q, and $-Q$ are sketched in figure. Find the electric flux through each surface.



SOLUTION

$$\phi_E = \frac{q_{enc}}{\varepsilon_0}$$

Through S_1

$$\phi_E = \frac{-2Q+Q}{\varepsilon_0} = -\frac{Q}{\varepsilon_0}$$

Through S_2

$$\phi_E = \frac{+Q-Q}{\varepsilon_0} = 0$$

Through S_3

$$\phi_E = \frac{-2Q+Q-Q}{\varepsilon_0} = -\frac{2Q}{\varepsilon_0}$$

Through S_4

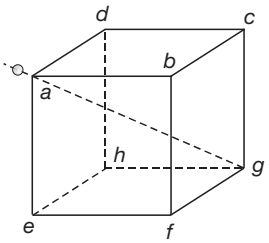
$$\phi_E = 0$$

Conceptual Note(s)

(a) Electric field intensity at the Gaussian surface is due to all the charges present inside as well as outside the Gaussian surface.
(b) Flux through Gaussian surface is independent of its shape.
(c) Flux through Gaussian surface depends only on total charge present inside Gaussian surface.
(d) Flux through Gaussian surface is independent of position of charges inside Gaussian surface.
(e) Flux due to field lines entering a closed surface is taken as negative and flux due to field lines leaving a surface is taken as positive. This is because $\hat{n}$ is taken positive in outward direction.
(f) In a Gaussian surface $\phi = 0$ does not imply $E = 0$ at every point of the surface but $E = 0$ at every point implies $\phi = 0$.

ILLUSTRATION 53

The line ag in figure is a diagonal of a cube. A point charge q is located at the vertex a of the cube. Determine the electric flux through each of the sides of the cube which meet at the point a.



SOLUTION

No charge is inside the cube. The net flux through the cube is zero. Positive flux comes out through the three faces meeting at g. These three faces together fill solid angle equal to one-eighth of a sphere as seen from q, and together pass flux $\frac{1}{8}\left(\frac{q}{\varepsilon_0}\right)$.

Each face containing a intercepts equal flux going into the cube so,

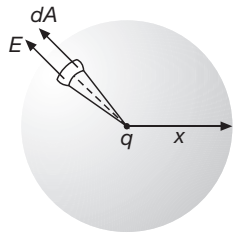
$$\Phi_{E,\text{ net}} = 0$$

$$\Rightarrow \quad 3\Phi_{E,\text{ abcd}} + \frac{q}{8\varepsilon_0} = 0$$

$$\Rightarrow \quad \Phi_{E,\text{ abcd}} = -\frac{q}{24\,\varepsilon_0}$$

ELECTRIC FIELD DUE TO A POINT CHARGE q AT A POINT P AT DISTANCE x

To find electric field strength at P, we first consider a Gaussian surface so that point P lies on its surface.



Here, the Gaussian surface taken has to be spherical such that at every point on this surface electric field due to the charge q is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} \qquad \{\text{directed radially outwards}\}$$

According to Gauss's Law

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \oint E dA\cos(0^\circ) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E\int dA = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E\left(4\pi x^2\right) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

Also, we observe that for the Gaussian surface selected the electric field vector is parallel to $d\vec{A}$ and also the magnitude of $\vec{E}$ is uniform at every point, thus the integral $\oint \vec{E}\cdot d\vec{A}$ can be easily evaluated.

Problem Solving Technique(s)

Gauss's Law provides a convenient tool for evaluating electric field. However, its application is limited only to systems that possess certain symmetry, namely, systems with cylindrical, planar and spherical symmetry. In the table below, we give some examples of systems in which Gauss's Law is applicable for determining electric field, with the corresponding Gaussian surfaces:

System	Symmetry	Gaussian Surface
Sphere, Spherical shell	Spherical	Concentric Sphere
Infinite rod	Cylindrical	Coaxial Cylinder
Infinite plane	Planar	Gaussian "Pillbox"

The following steps may be useful when applying Gauss's Law.

(a) Identify the symmetry associated with the charge distribution.

(b) Determine the direction of the electric field, and a "Gaussian surface" on which the magnitude of the electric field is constant over portions of the surface.

(c) Divide the space into different regions associated with the charge distribution. For each region, calculate q_{enc}, the charge enclosed by the Gaussian surface.

(d) Calculate the electric flux ϕ_E (or ϕ) through the Gaussian surface for each region.

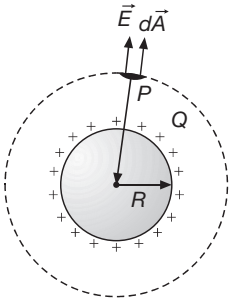
(e) Now $\phi_E = \frac{q_{enc}}{\varepsilon_0}$ (due to Gauss's Law) and deduce the magnitude of the electric field.

Let us apply this technique to the situations discussed hereafter.

ELECTRIC FIELD STRENGTH OF A CHARGED CONDUCTING SPHERE OR CONDUCTING SHELL

CASE-1: OUTSIDE ($x > R$)

To find electric field at an outer point at a distance $x(>R)$ from the centre of sphere, we consider a spherical Gaussian surface of radius x.



Due to symmetry the electric field strength at every point of this surface is E. Using Gauss's Law we have

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad \oint E dA\cos 0 = \frac{1}{\varepsilon_0}(q_{enc})$$

Here we have

$$E\oint dA = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E(4\pi x^2) = \frac{Q}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 x^2}$$

CASE-2: AT THE SURFACE

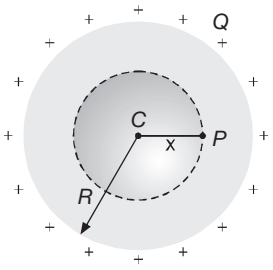
Similarly for points on the surface, we can consider a spherical Gaussian surface of radius R which gives electric field strength on the sphere surface as

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

In both these cases, it seems as if the net field is as if the charge Q is concentrated at the centre of the sphere.

CASE-3: INSIDE THE SPHERE ($x < R$)

To find electric field strength at an interior point of the sphere, we consider an inner spherical Gaussian surface of radius $x(x<R)$.



Here if we apply Gauss's Law for this surface, we have

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

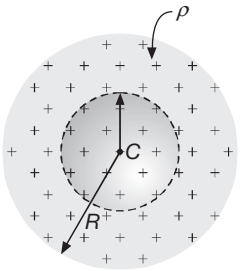
$$\Rightarrow \quad \vec{E} = \vec{0}$$

{as all the charge resides on surface so $q_{enc} = 0$}

ELECTRIC FIELD DUE TO A NON-CONDUCTING UNIFORMLY CHARGED SPHERE

CASE-1 and 2: For points outside and at the surface of sphere, the electric field strength is calculated by using Gauss's Law similar to the previous case of conducting sphere. Here too we get the net field at these points (outside and at the surface) as if the net charge is concentrated at the centre.

CASE-3: For interior points of sphere, we consider a spherical Gaussian surface of radius $x(< R)$ as shown.



If we apply Gauss's Law for this surface, then we get

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Here enclosed charge q_{enc} will be calculated as follows

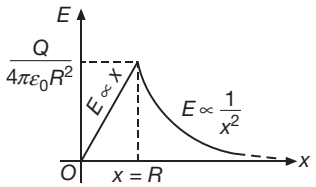
$$q_{enc} = \rho\left(\frac{4}{3}\pi x^3\right)$$

$$\text{Thus } E(4\pi x^2) = \frac{\rho\left(\frac{4}{3}\pi x^3\right)}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho x}{3\varepsilon_0} \qquad \{\text{for } x < R\}$$

One important thing that we observe regarding the electric field due to conducting sphere is that it exhibits discontinuity at $x = R$ where as the field due to the non conducting uniformly charged sphere is continuous in nature and the field due to a uniformly charged sphere is given by

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 x^2} & x \geq R \\ & \text{(outside and at the surface)} \\ \left(\dfrac{\rho}{3\varepsilon_0}\right)x & x < R \\ & \text{(inside)} \\ \text{zero} & x = 0 \\ & \text{(at the centre)} \end{cases}$$



So, to conclude, we observe that for a charged conducting sphere or a charged conducting shell, the net field is given by

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 x^2} & x \geq R \\ & \text{(outside and at the surface)} \\ \text{zero} & x < R \\ & \text{(inside)} \end{cases}$$

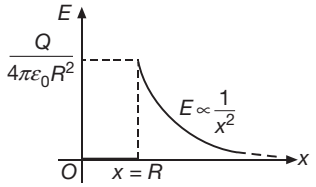


ILLUSTRATION 54

A ball of radius R carries a positive charge whose volume density depends on a separation r from the ball's centre as $\rho = \rho_0\left(1-\frac{r}{R}\right)$, where ρ_0 is a constant. Assuming the permittivities of the ball and the environment is equal to unity, find

(a) the magnitude of the electric field strength as a function of the distance r both inside and outside the ball.

(b) the maximum intensity E_{max} and the corresponding distance r_m.

SOLUTION

(a) Let the ball be divided into a number of infinitesimal spherical shells. Consider one shell at a distance r and of thickness dr. Volume (dV) of this shell is given by

$$dV = 4\pi r^2 dr$$

Also $dq = \rho\, dV$

Given that $\rho = \rho_0\left(1-\frac{r}{R}\right)$

$$\Rightarrow \quad dq = \rho_0\left(1-\frac{r}{R}\right)\times 4\pi r^2 dr$$

So, total charge enclosed by the sphere of radius r is given by

$$\int_0^q dq = 4\pi\rho_0\int_0^r\left(1-\frac{r}{R}\right)r^2 dr$$

$$\Rightarrow \quad q = 4\rho_0\int_0^r\left(1-\frac{r^3}{R}\right)dr$$

$$\Rightarrow \quad q = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]_0^r = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

By Gauss's Theorem, we have

$$E\left(4\pi r^2\right) = \frac{q_{enc}}{\varepsilon_0}$$

For $r < R$ i.e., inside

$$q_{enc} = 4\pi\rho_0\left(\frac{r^3}{3}-\frac{r^4}{4R}\right)$$

$$\Rightarrow \quad E = \frac{4\pi\rho_0}{4\pi\varepsilon_0 r^2}\left(\frac{r^3}{3}-\frac{r^4}{4R}\right)$$

$$\Rightarrow \quad E = \frac{4\pi\rho_0}{4\pi\varepsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$$

$$\Rightarrow \quad E = \frac{\rho_0 r}{3\varepsilon_0}\left(1-\frac{3r}{4R}\right) \qquad \ldots(1)$$

When $r > R$ then

$$q = \int_0^R \rho_0\left(r-\frac{r}{R}\right)4\pi r^2 dr = \frac{4\pi\rho_0 R^3}{12}$$

$$\Rightarrow \quad E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$$

(b) For electric field to be maximum we have

$$\frac{dE}{dr} = 0$$

Since $E = \frac{\rho_0}{3\varepsilon_0}\left(r-\frac{3r^2}{4R}\right)$

$$\Rightarrow \quad \frac{dE}{dr} = \frac{\rho_0 R}{9\varepsilon_0}\left(1-\frac{6r}{4R}\right) = 0$$

$$\Rightarrow \quad \left(1-\frac{6r}{4R}\right) = 0$$

$$\Rightarrow \quad r = r_m = \frac{2R}{3}$$

Thus maximum electric field will be obtained by putting

$$r = r_m = \frac{2R}{3} \text{ in equation (1)}$$

$$\Rightarrow \quad E_{max} = \frac{\rho_0}{3\varepsilon_0}\left(\frac{2R}{3}-\frac{3}{4R}\times\frac{4R^2}{9}\right) = \frac{\rho_0 R}{9\varepsilon_0}$$

$$\Rightarrow \quad E_{max} = \frac{\rho_0 R}{9\varepsilon_0}$$

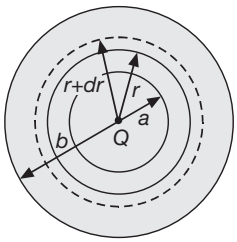
ILLUSTRATION 55

A system consists of a ball of radius R carrying a spherically symmetric charge Q and the surrounding space filled with a charge of volume density $\rho = \frac{\alpha}{r}$, where α is a constant, r the distance from

the centre of the ball. Find the ball's charge at which the magnitude of the electric field strength vector is independent of r outside the ball. How high is this strength? The permittivities of the ball are assumed to be unity.

SOLUTION

Let us draw a Gaussian surface of radius $r(a<r<b)$ concentric with the spheres of radii a and b respectively.



Let the charge enclosed between the radii r and a be q. Then

$$q=\int \rho(r)\,dV$$

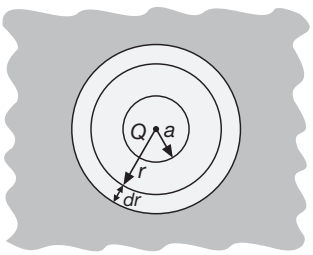
$$\Rightarrow \quad q=\int \rho(r)4\pi r^2 dr$$

$$\Rightarrow \quad q=4\pi\int_a^r \frac{\alpha}{r}r^2 dr$$

$$\Rightarrow \quad q=\frac{4\pi\alpha}{2}\left(r^2-a^2\right)$$

$$\Rightarrow \quad q=2\pi\alpha\left(r^2-a^2\right)$$

Also, we see that as per the instructions given in problem, the ball already possesses a charge Q. So, by Gauss's Theorem, total charge enclosed inside Gaussian surface is $(q+Q)$.



and $\oint_S \vec{E}\cdot d\vec{A}=\frac{1}{\varepsilon_0}(Q+q)$

$$\Rightarrow \quad E\left(4\pi r^2\right)=\frac{1}{\varepsilon_0}\left[Q+2\pi\alpha\left(r^2-a^2\right)\right]$$

$$\Rightarrow \quad E=\frac{\alpha}{2\varepsilon_0}+\frac{Q-2\pi\alpha a^2}{4\pi\varepsilon_0 r^2}$$

For E to be of constant magnitude, we have

$$Q-2\pi\alpha a^2=0$$

$$\Rightarrow \quad \alpha=\frac{Q}{2\pi a^2}$$

and $\quad E=\frac{\alpha}{2\varepsilon_0}$

ELECTRIC FIELD DUE TO AN INFINITELY LONG THIN CHARGED WIRE

Let us find electric field strength due to a long charged wire having linear charge density λ at a point P situated at a distance r from the wire.

For this application of Gauss's Law, we consider a coaxial cylindrical Gaussian surface of length l and radius x as shown. Applying Gauss's Law on this surface, we get

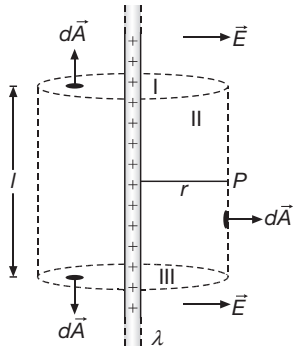
$$\oint \vec{E}\cdot d\vec{A}=\frac{q_{\text{enc}}}{\varepsilon_0} \qquad \ldots(1)$$

Now, we see the closed Gaussian surface to be made of three parts I, II and III. Two flat circular faces (I and III) called Lids and one cylindrical lateral surface I called curved surface. Let us split the closed surface integration in three parts, then

$$\oint \vec{E}\cdot d\vec{A}=\int_I \vec{E}\cdot d\vec{A}+\int_{II} \vec{E}\cdot d\vec{A}+\int_{III} \vec{E}\cdot d\vec{A}$$

Here we observe that for part I and III, electric field strength vector is perpendicular to the area vector as shown in figure hence no flux will be associated with these parts. Thus we have

$$\text{Flux due to Lids}=\int_I \vec{E}\cdot d\vec{A}=\int_{III} \vec{E}\cdot d\vec{A}=0$$



But according to Gauss's Law given in equation (1), we have

$$\int_{II} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

{Since the enclosed charged is $q_{enc} = \lambda l$}

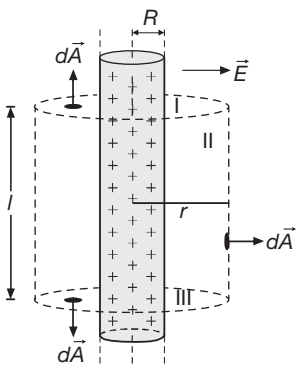
Since for lateral surface or curved surface, we observe that $\vec{E}$ is parallel to $d\vec{A}$ and is constant on the entire lateral surface, so

$$\phi = \oint \vec{E}\cdot d\vec{A} = E\int_{II} dA = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \quad E(2\pi rl) = \frac{\lambda l}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

ELECTRIC FIELD DUE TO A LONG UNIFORMLY CHARGED CONDUCTING CYLINDER

Figure shows a long cylinder of radius R which is uniformly charged on its surface having surface charge density σ.



At all the interior points of a metal/conducting body electric field strength is zero. For finding electric field strength at outer points at a distance r from the axis of the cylinder, we consider a cylindrical Gaussian surface of radius r and length l as shown in figure. Now on applying Gauss's Law we get

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

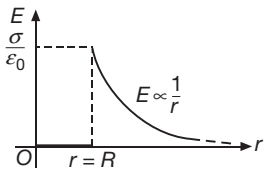
Here enclosed charge in the cylindrical Gaussian surface is

$$q_{enc} = \sigma(2\pi Rl)$$

Here also similar to previous case the electric flux through the circular faces i.e., lids is zero, hence according to Gauss's Law, we have

$$\oint \vec{E}\cdot d\vec{A} = \int_{II} \vec{E}\cdot d\vec{A} = \frac{\sigma(2\pi Rl)}{\varepsilon_0}$$

$$\Rightarrow \quad E\int_{II} dA = \frac{\sigma(2\pi Rl)}{\varepsilon_0}$$



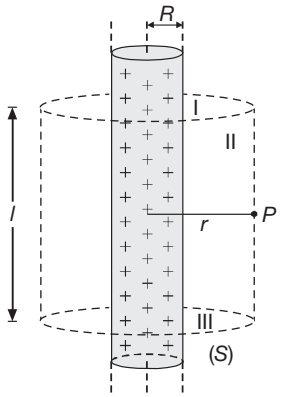
$$\Rightarrow \quad E(2\pi rl) = \frac{\sigma(2\pi Rl)}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma R}{\varepsilon_0 r} \qquad \{\text{for } r > R\}$$

$$\text{So, } E = \begin{cases} \dfrac{\sigma R}{\varepsilon_0 r} & \text{for } r \geq R \text{ (outside and at the surface)} \\ 0 & \text{for } r < R \text{ (inside)} \end{cases}$$

ELECTRIC FIELD DUE TO A LONG UNIFORMLY CHARGED NON-CONDUCTING CYLINDER

Let us take a long cylinder of radius R, charged uniformly having volume charge density ρ. To find the electric field strength at a distance r from the axis of the cylinder we again consider a cylindrical Gaussian surface (S) shown in figure.



Applying Gauss's Law on this surface, we get

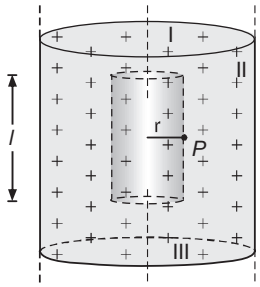
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow \quad \oint_{II} \vec{E}\cdot d\vec{A} = \frac{\rho(\pi R^2 l)}{\varepsilon_0} \qquad \{\text{Since } q_{\text{enc}} = \rho(\pi R^2 l)\}$$

(Because, as in earlier cases flux contribution due to circular faces/lids is still zero)

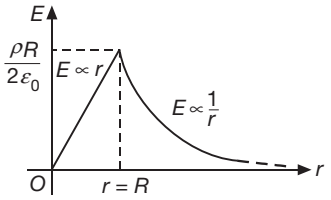
$$\Rightarrow \quad E\int dA = \frac{\rho\pi R^2 l}{\varepsilon_0}$$

$$\Rightarrow \quad E(2\pi rl) = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}$$



To find electric field inside the cylinder at a distance r from the axis, we consider a small cylindrical Gaussian surface of radius r and length l. Applying Gauss's Law for this surface, we get

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$



$$\Rightarrow \quad E\int_{II} dA = \frac{\rho\pi r^2 l}{\varepsilon_0}$$

$$\Rightarrow \quad E(2\pi rl) = \frac{\rho\pi r^2 l}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{2\varepsilon_0}$$

Again we observe that here E is continuous, where as it exhibited discontinuity in case of the conducting cylinder.

ILLUSTRATION 56

An infinitely long insulating cylinder of radius R has a volume charge density that varies with the radius as

$$\rho = \rho_0\left(a - \frac{r}{b}\right)$$

where ρ_0, a and b are positive constants and r is the distance from the axis of the cylinder. Use Gauss's Law to determine the magnitude of the electric field at radial distances

(a) $r < R$ **(b)** $r > R$

SOLUTION

In this case the charge density is not uniform, and Gauss's Law is written as $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int \rho dV$. We use a Gaussian surface which is a cylinder of radius r, length l and is coaxial with the charge distribution.

(a) When $r < R$, this becomes $E(2\pi rl) = \frac{\rho_0}{\varepsilon_0}\int_0^r \left(a - \frac{r}{b}\right)dV$.

The element of volume is a cylindrical shell of radius r, length l and thickness dr so that $dV = 2\pi rl\,dr$.

$$E(2\pi rl)=\left(\frac{2\pi r^2 l\rho_0}{\varepsilon_0}\right)\left(\frac{a}{2}-\frac{r}{3b}\right)$$

So, inside the cylinder, we get

$$E=\frac{\rho_0 r}{2\varepsilon_0}\left(a-\frac{2r}{3b}\right)$$

(b) When $r>R$, Gauss's Law becomes

$$E(2\pi rl)=\frac{\rho_0}{\varepsilon_0}\int_0^R\left(a-\frac{r}{b}\right)(2\pi rldr)$$

So, outside the cylinder, we get

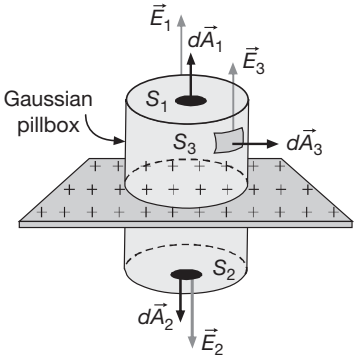
$$E=\frac{\rho_0 R^2}{2\varepsilon_0 r}\left(a-\frac{2R}{3b}\right)$$

ELECTRIC FIELD DUE TO AN INFINITE NON-CONDUCTING THIN UNIFORMLY CHARGED SHEET

To find the electric field strength at a point P at distance r infront of the charged sheet we consider a cylindrical Gaussian surface (also called as Gaussian Pill box) as shown in figure of face area A. Applying Gauss's Law for this surface, we have

$$\oint\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow\quad \int_I\vec{E}\cdot d\vec{A}+\int_{II}\vec{E}\cdot d\vec{A}+\int_{III}\vec{E}\cdot d\vec{A}=\frac{\sigma A}{\varepsilon_0}\quad\{\because\ q_{enc}=\sigma A\}$$



In this case $\int_{II}\vec{E}\cdot d\vec{A}=0$, as the lateral surface of cylinder is parallel to the direction of electric field strength. So, $\vec{E}$ becomes perpendicular to $\vec{A}$ for curved surface and hence no flux is associated with the curved surface.

$$\phi_{total}=\int_I EdA+\int_{II}EdA=\frac{\sigma A}{2\varepsilon_0}$$

$$\Rightarrow\quad 2EA=\frac{\sigma A}{\varepsilon_0}$$

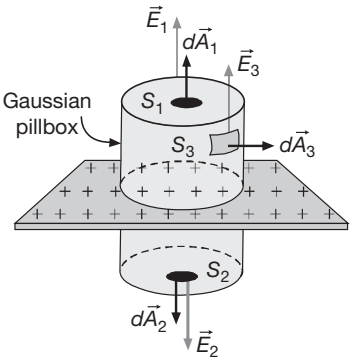
{As electric field is uniform on both sides}

$$\Rightarrow\quad E=\frac{\sigma}{2\varepsilon_0}$$

{This field is independent of the distance of point P from the sheet}

ELECTRIC FIELD DUE TO AN INFINITE CHARGED CONDUCTING SHEET

Figure shows a large charged conducting sheet, charged on both the surfaces with surface charge density σ. There is no charge within the volume of the sheet and also the electric field inside the metal sheet is zero. To find electric field strength at a point P in front of the sheet we consider a cylindrical Gaussian surface (or Gaussian Pill Box) as shown.



Applying Gauss's Law to this surface, we get

$$\oint\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow\quad \int_I\vec{E}\cdot d\vec{A}+\int_{II}\vec{E}\cdot d\vec{A}+\int_{III}\vec{E}\cdot d\vec{A}=\frac{2\sigma A}{\varepsilon_0}$$

{Here $q_{enc}=2\sigma A$}

Since the field inside the sheet is zero, so the Gaussian Pill box will enclose a charge σA at the left and σA at the right too. So that the total charge enclosed is

$$q_{enc} = 2\sigma A$$

$$\Rightarrow \quad \phi_{total} = \int_{I} \vec{E}\cdot d\vec{A} + \int_{II} \vec{E}\cdot d\vec{A} + \int_{III} \vec{E}\cdot d\vec{A} = \frac{2\sigma A}{\varepsilon_0}$$

$$\Rightarrow \quad 2EA = \frac{2\sigma A}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$

{Again here E is independent of the distance of point P from the sheet}

ILLUSTRATION 57

A sphere of radius R is charged with a non-uniform charge density which varies with the distance x from the centre as

$$\rho = \frac{\rho_0}{x}$$

where ρ_0 is a positive constant. Find the electric field strength at a point P situated at a distance r from centre of sphere

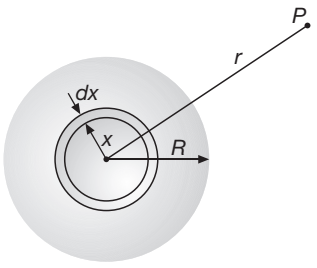
(a) outside it
(b) inside it
(c) also plot the variation of E with r.

SOLUTION

(a) $E_P = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

{where Q is the total charge of sphere}

For outer points we see that the entire charge of sphere to be concentrated at its centre. We can also prove this again by taking a spherical Gaussian surface as done in earlier cases. Let us now calculate Q. This is done by integrating the charge of an elemental shell of radius x and thickness dx as shown in figure.



The charge dq in this infinitesimal shell is

$$dq = \rho\, dV$$

$$\Rightarrow \quad dq = \rho\left(4\pi x^2\, dx\right)$$

$$\Rightarrow \quad dq = \frac{\rho_0}{x}\left(4\pi x^2\, dx\right)$$

$$\Rightarrow \quad dq = 4\pi\rho_0 x\, dx$$

$$\Rightarrow \quad Q = \int dq = \int_0^R 4\pi\rho_0 x\, dx$$

$$\Rightarrow \quad Q = 4\pi\rho_0\left[\frac{x^2}{2}\right]_0^R$$

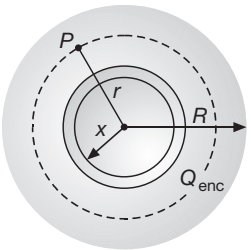
$$\Rightarrow \quad Q = 2\pi\rho_0 R^2$$

Thus electric field strength at point outside the sphere at distance r from the centre is

$$E_p = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\left(2\pi\rho_0 R^2\right)}{r^2} = \frac{\rho_0 R^2}{2\varepsilon_0 r^2}$$

$$\Rightarrow \quad E_{outside} = \frac{\rho_0 R^2}{2\varepsilon_0 r^2}$$

(b) To find electric field strength at an interior point at distance r from the centre of sphere, we first find the charge enclosed within the inner sphere of radius r such that point P is on the surface. Thus enclosed charge is



$$Q_{enc} = \int_0^r \frac{\rho_0}{R}\left(4\pi x^2 dx\right)$$

$$\Rightarrow \quad Q = 2\pi\rho_0 r^2$$

Here electric field strength at point will be due to the inner charged core having charge Q_{enc} as if it were again concentrated at the centre. So

$$E_{\text{inside}} = E_P = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{enc}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\left(2\pi\rho_0 r^2\right)}{r^2}$$

$$\Rightarrow \quad E_P = \frac{\rho_0}{2\varepsilon_0}$$

{Independent of distance r from the centre}

(c) Since

$$E = \begin{cases} \dfrac{\rho_0 R^2}{2\varepsilon_0 r^2} & \text{for } r \geq R \\ & \text{(outside and at the surface)} \\ \dfrac{\rho_0}{2\varepsilon_0} & \text{for } r < R \\ & \text{(inside the sphere)} \end{cases}$$

Also, we observe that

$$E_{\text{surface}} = \frac{\rho_0}{2\varepsilon_0} \qquad \{\text{by putting } r = R\}$$

So, the field has a continuity as shown

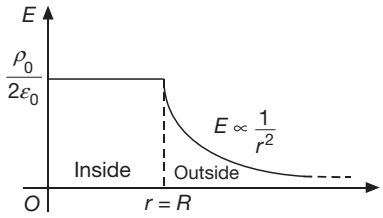


ILLUSTRATION 58

A region between yz plane and another plane parallel to yz plane at $x = d$ has charge density ρ which varies with distance x from origin O as

$$\rho = \rho_0 x^2$$

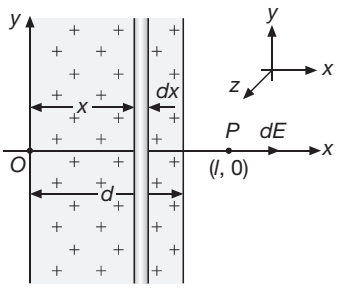
where ρ_0 is a positive constant. Calculate the electric field strength due to this slab at point $P(l, O)$.

SOLUTION

Let us consider an elemental sheet of thickness dx at a distance x from the yz plane. Since we know that due to a sheet of thickness d the electric field is $E = \dfrac{\rho d}{2\varepsilon_0}$. Electric field strength at P due to this sheet is

$$dE = \frac{\rho dx}{2\varepsilon_0}$$

$$\Rightarrow \quad dE = \frac{\rho_0 x^2}{2\varepsilon_0}dx \qquad \left\{\because \rho = \rho_0 x^2\right\}$$



Net electric field at P is given by

$$E_P = \int dE = \int_0^d \frac{\rho_0 x^2}{2\varepsilon_0}dx$$

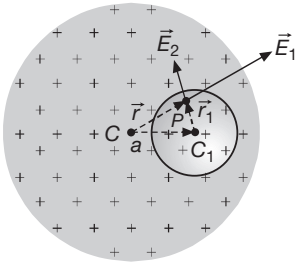
$$\Rightarrow \quad E_P = \frac{\rho_0 d^3}{6\varepsilon_0}$$

ILLUSTRATION 59

Inside a ball charged uniformly with volume density ρ, there is a spherical cavity. The centre of the cavity is displaced with respect to the centre of the ball by $\vec{a}$. Calculate the field strength inside the cavity, assuming the permittivity equal to unity.

SOLUTION

Consider a point P in the cavity at a position vector $\vec{r}$ from the centre of sphere and at a position vector $\vec{r}_1$ from the centre of cavity as shown.



If $\vec{E}$ be the electric field strength at P due to the complete charge of the sphere (inside cavity also) then we know electric field strength inside a uniformly charged sphere is given as

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

Similarly if we assume that the charge is only there in the region of cavity and hence this will also be a uniformly charged small sphere. If $\vec{E}_1$ be the electric field only due to the cavity charge then,

$$\vec{E}_1 = \frac{\rho\vec{r}_1}{3\varepsilon_0}$$

Now the electric field due to the charged sphere in the cavity at point P can be given as

$$\vec{E}_{net} = \vec{E} - \vec{E}_1$$

{As now charge of cavity is removed}

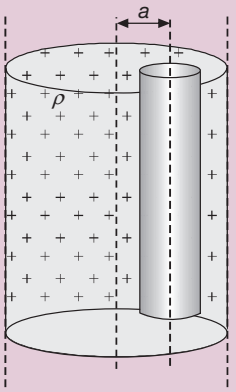
$$\Rightarrow \quad \vec{E}_{net} = \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{r}_1) \qquad \{\because\ \vec{a} + \vec{r}_1 = \vec{r}\}$$

$$\Rightarrow \quad \vec{E}_{net} = \frac{\rho\vec{a}}{3\varepsilon_0} \qquad \{\text{As } \vec{r} - \vec{r}_1 = \vec{a}\}$$

Problem Solving Technique(s)

"Cavity can be considered to be negative charge embedded in positive charge so that the common region having size of cavity has no charge on it".

This shows that the net electric field inside the cavity is uniform and in the direction of $\vec{a}$ i.e., along the line joining the centre of spheres and cavity.



Similarly we can find the electric field strength inside a cylindrical cavity of a long uniformly charged cylinder. If cavity axis is displaced from axis of cylinder by a displacement vector $\vec{a}$, then by the analysis we have already done for a sphere, we can say that the electric field strength inside the cavity is also uniform and hence will be given as

$$\vec{E} = \frac{\rho\vec{a}}{2\varepsilon_0}$$

Test Your Concepts-VI

Based on Gauss's Law

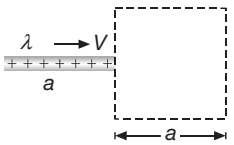
(Solutions on page H.20)

1. An electric dipole is placed at the centre of a sphere. Find the electric flux passing through the sphere.

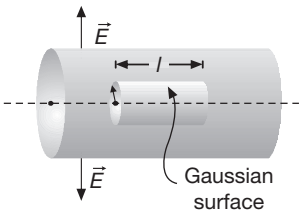
2. A point charge q is placed at the centre of a cube of edge a. Calculate the flux linked
(a) with all the faces of the cube.
(b) with each face of the cube.
If charge is not at the centre, then what will be the answers for parts (a) and (b).

3. The intensity of an electric field depends only on the coordinates x and y as $\vec{E} = \frac{\alpha(x\hat{i} + y\hat{j})}{x^2 + y^2}$, where, α is a constant and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Calculate the charge within a sphere of radius R having its centre at the origin.

4. Figure shows an imaginary cube of side a. A uniformly charged rod of length a moves towards right at a constant speed v. At $t = 0$, the right end of the rod just touches the left face of the cube. Plot a graph between electric flux passing through the cube versus time.



5. A long cylinder (shown in figure) carries a charge density that is proportional to the distance x from the axis as $\rho = kx$, for some constant k. Find the electric field inside this cylinder.

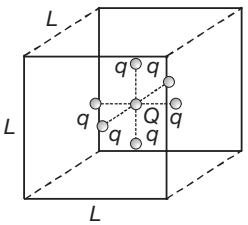


6. **(a)** A point charge q is located a distance d from an infinite plane. Determine the electric flux through the plane due to the point charge.

(b) If a point charge q is located a very small distance from the centre of a very large square on the line perpendicular to the square and going through its centre. Determine the approximate electric flux through the square due to the point charge.

(c) Explain why the answers to parts (a) and (b) are identical.

7. A positive point charge Q is located at the centre of a cube of edge L. In addition, six other identical negative point charges q are positioned symmetrically around Q as shown in figure. Determine the electric flux through one face of the cube. Also find the condition when no flux is associated with the cube.



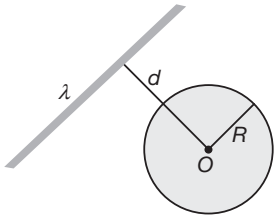
8. An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q. A spherical Gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$.

(a) Find an expression for the electric flux passing through the surface of the Gaussian sphere as a function of r for $r < a$.

(b) Find an expression for the electric flux for $r > a$.

(c) Plot the flux versus r.

9. An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in figure. Determine the total electric flux through the surface of a sphere of radius R having centre at O resulting from this line charge. Consider both cases, where $R < d$ and $R > d$.

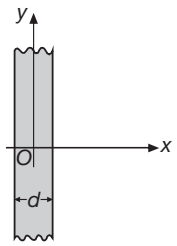


10. A solid insulating sphere of radius R has a non-uniform charge density that varies with r according to the expression $\rho = \rho_0 r^2$, where ρ_0 is a constant and $r < R$ is measured from the centre of the sphere. Show that the magnitude of the electric field

(a) outside $(r > R)$ the sphere is $E = \dfrac{\rho_0 R^5}{5\varepsilon_0 r^2}$.

(b) inside $(r < R)$ the sphere is $E = \dfrac{\rho_0 r^3}{5\varepsilon_0}$.

11. A slab of insulating material has a non-uniform positive charge density $\rho = \rho_0 x^2$, where x is measured from the centre of the slab as shown in Figure, and ρ_0 is a constant. The slab is infinite in the y and z directions. Derive expressions for the electric field in the



(a) exterior regions of the slab $\left(\text{i.e., } x > \dfrac{d}{2} \text{ and } x < -\dfrac{d}{2}\right)$

(b) interior region of the slab $\left(\text{i.e., } -\dfrac{d}{2} < x < \dfrac{d}{2}\right)$.

12. A spherically symmetric charge distribution has a charge density given by $\rho = \frac{\rho_0}{r}$, where ρ_0 is constant. Find the electric field as a function of *r*.

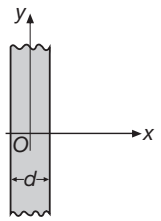
13. (a) Using the mathematical similarity between Coulomb's Law and Newton's Law of Universal Gravitation, show that Gauss's Law for gravitation expressed as

$$\oint \vec{g} \cdot d\vec{A} = -4\pi G \sum m_{enc}$$

where $\sum m_{enc}$ is the net mass enclosed by the Gaussian surface and $\vec{g} = \frac{\vec{F}_g}{m}$ represents the gravitational field at any point on the Gaussian surface.

(b) Determine the gravitational field at a distance *r* from the centre of the Earth where $r < R_E$, assuming that the Earth's mass density is uniform.

14. A slab of insulating material (infinite in two of its three dimensions) has a uniform positive charge density ρ. An edge view of the slab is shown in figure.



(a) Show that the magnitude of the electric field a distance *x* from its centre and inside the slab is $E = \frac{\rho x}{\varepsilon_0}$.

(b) Suppose an electron of charge $-e$ and mass *m* can move freely within the slab. It is released from rest at a distance *x* from the centre. Show that the electron exhibits simple harmonic motion with a period

$$T = 2\pi\sqrt{\frac{m\varepsilon_0}{\rho_e}}$$

15. Inside an infinitely long circular cylinder charged uniformly with volume density ρ there is a circular cylindrical cavity. The distance between the axes of the cylinder and the cavity is equal to $\vec{a}$. Find the electric field strength $\vec{E}$ inside the cavity. The permittivity is assumed to be equal to unity.

ELECTROSTATIC POTENTIAL, POTENTIAL ENERGY, CONDUCTORS AND APPLICATIONS

ELECTROSTATIC POTENTIAL ENERGY AND POTENTIAL

Electrostatic Potential Energy

Potential energy of a system of particles is defined as the work done in assembling the system of charges in a given configuration against the interaction forces of particles. Potential energy of a system of particles is a concept associated with conservative fields only. Since, electric field is also conservative in nature so, we associate the concept of potential energy with it.

When all particles of a system are separated far apart by infinite distance there exists no interaction between them. This state, we take as reference of Zero Potential Energy (ZPE).

Electrostatic potential energy is categorised in two ways.

(a) Electrostatic interaction energy of charged particles of a system.

(b) Electrostatic self energy of a charged object.

Electrostatic Interaction Energy

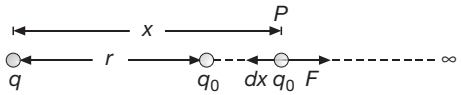
Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to a given configuration (as that required in system).

When charged particles are lying at infinity, their potential energy is taken to be zero, because no interaction exists between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and hence energy is supplied to the system so that final potential energy of system will be positive. If the force between the particles is attractive, work will be done by the system and final potential energy of system will be negative.

In giving these arguments please keep this thing in mind that work done by a conservative force equals the decrease in potential energy of the system.

ELECTROSTATIC INTERACTION ENERGY OF A SYSTEM OF TWO CHARGED PARTICLES

Figure shows two positive charges q and q_0 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing test charge q_0 from infinity to the given point P at a distance r from the source charge q. Let, at any instant the test charge q_0 be at a distance x from the source charge q. Let q_0 be displaced through dx towards q. The electrostatic force of repulsion F acts on q_0 away from q.



If dW be the work done, then

$$dW = \vec{F} \cdot d\vec{x}$$

$$\Rightarrow \quad dW = \frac{qq_0}{4\pi\varepsilon_0 x^2} dx \cos(180^\circ)$$

$$\Rightarrow \quad dW = -\frac{qq_0}{4\pi\varepsilon_0} x^{-2} dx$$

$$\Rightarrow \quad W = -\frac{qq_0}{4\pi\varepsilon_0} \int_0^r x^{-2} dx$$

$$\Rightarrow \quad W = -\frac{qq_0}{4\pi\varepsilon_0} \left(\frac{x^{-2+1}}{-2+1} \Big|_\infty^r \right)$$

$$\Rightarrow \quad W = \frac{qq_0}{4\pi\varepsilon_0} \left(\frac{1}{r} - \frac{1}{\infty} \right)$$

$$\Rightarrow \quad W_{\infty \to P} = \frac{qq_0}{4\pi\varepsilon_0 r} = W$$

So, work done by the external force in bringing the test charge q_0 from ∞ to the point P under the influence of source charge q is

$$W = W_{\infty \to P} = \frac{qq_0}{4\pi\varepsilon_0 r}$$

This work done is stored in the form of electrostatic interaction energy (U) of the system. So,

$$U = \frac{qq_0}{4\pi\varepsilon_0 r}$$

Please note that if the two charges are of opposite nature, then

$$U = -\frac{qq_0}{4\pi\varepsilon_0 r}$$

ILLUSTRATION 60

Two charges, $+q$ and $-q$, each of mass m, are revolving in a circle of radius R, under mutual electrostatic force. Find (a) speed of each charge (b) kinetic energy of system (c) potential energy of the system (d) total energy of the system

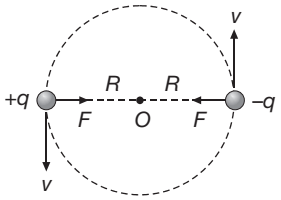
SOLUTION

In this situation, the centripetal force is provided by the electrostatic force of attraction.

(a) By Newton's second law we get,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(4R)m}}$$



(b) Kinectic energy of the system is $K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$$\Rightarrow \quad K = \frac{q^2}{4\pi\varepsilon_0(4R)} = \frac{q^2}{16\pi\varepsilon_0 R}$$

(c) Potential energy of the system is $U = \frac{1}{4\pi\varepsilon_0}\frac{q(-q)}{2R}$

$$\Rightarrow \quad U = \frac{-1}{4\pi\varepsilon_0}\frac{q^2}{(2R)} = -\frac{q^2}{8\pi\varepsilon_0 R}$$

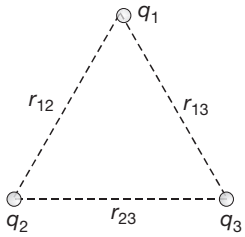
(d) Total energy of the system is $E = K.E. + P.E.$

$$\Rightarrow \quad E = \frac{-q^2}{4\pi\varepsilon_0(4R)} = -\frac{q^2}{16\pi\varepsilon_0 R}$$

So, we observe that $T.E. = -(K.E.) = \frac{1}{2}(P.E.)$

ELECTROSTATIC INTERACTION ENERGY FOR A SYSTEM OF PARTICLES

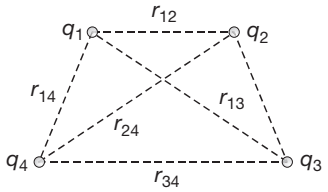
When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles with no pair repeated.



For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}\right] = U_{12} + U_{13} + U_{23}$$

Similarly, for an assembly of four charges q_1, q_2, q_3 and q_4 (having total number of six interaction $= {}^4C_2$), we have



$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_4}{r_{34}} + \frac{q_4q_1}{r_{14}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_4}{r_{24}}\right]$$

$$\Rightarrow \quad U = U_{12} + U_{23} + U_{34} + U_{41} + U_{13} + U_{24}$$

From above we see that the total potential energy is simply the sum of contributions due to distinct pairs. Generalising to N charges, we get

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ (j>i)}}^{N}\frac{q_iq_j}{r_{ij}}$$

where $j > i$ assures that the pairs are not repeated. Other way round, we can count every pair twice and divide the result by $\frac{1}{2}$. So,

$$U = \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{q_iq_j}{r_{ij}} = \frac{1}{2}\sum_{i=1}^{N}q_i\left(\frac{1}{4\pi\varepsilon_0}\sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_i}{r_{ij}}\right)$$

$$\Rightarrow \quad U = \frac{1}{2}\sum_{i=1}^{N}q_iV(r_i)$$

where $V(r_i)$ is the potential due to all other charges at the location of charge q_i (i.e., r_i)

ILLUSTRATION 61

Calculate the work required to be done to make an arrangement of three particles each having a charge $+q$ such that the particles lie at the vertices of an equilateral triangle of side a. What work will be done by electric field when the particles are shifted away so that the side of triangle becomes $2a$?

SOLUTION

The work required to make an arrangement of charges is equal to potential energy of the system

$$\Rightarrow \quad W = \frac{q^2}{4\pi\varepsilon_0 a} \times 3 = \frac{3q^2}{4\pi\varepsilon_0 a}$$

Work done by electric field equals the decrease in potential energy. So, we have

$$W_{E\text{-Field}} = -\Delta U = U_i - U_f$$

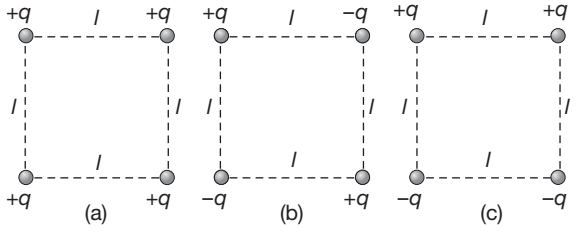
Since $U_i = \frac{q^2}{4\pi\varepsilon_0 (a)} \times 3$ and

$$U_f = \frac{q^2}{4\pi\varepsilon_0 (2a)} \times 3$$

$$\Rightarrow \quad W_{E\text{-Field}} = U_i - U_f = \frac{3q^2}{4\pi\varepsilon_0 a} - \frac{3q^2}{8\pi\varepsilon_0 a} = \frac{3q^2}{8\pi\varepsilon_0 a}$$

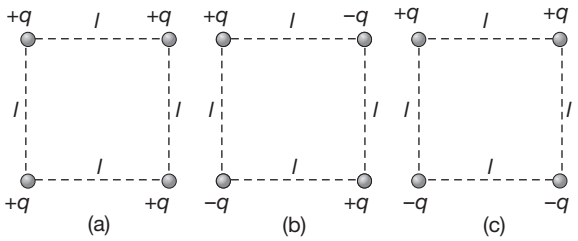
ILLUSTRATION 62

Determine the interaction energy of the point charges located at the corners of a square of side l in the figures shown.



SOLUTION

Interaction energy of any two point charges q_1 and q_2 at separation r is given by $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$



Let interaction energy of the system in Figure (a), (b) and (c) be U_a, U_b and U_c respectively, then

$$U_a = 4\frac{q^2}{4\pi\varepsilon_0 l} + 2\frac{q^2}{4\pi\varepsilon_0 (\sqrt{2}l)} = \frac{q^2}{4\pi\varepsilon_0 l}(4 + \sqrt{2})$$

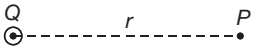
$$U_b = 4\frac{-q^2}{4\pi\varepsilon_0 l} + 2\frac{q^2}{4\pi\varepsilon_0 (\sqrt{2}l)} = \frac{q^2}{4\pi\varepsilon_0 l}(-4 + \sqrt{2})$$

and $U_c = 2\frac{q^2}{4\pi\varepsilon_0 l} - \frac{2q^2}{4\pi\varepsilon_0 l} - \frac{2q^2}{4\pi\varepsilon_0 (\sqrt{2}l)} = -\frac{\sqrt{2}q^2}{4\pi\varepsilon_0 l}$

ELECTROSTATIC POTENTIAL (V)

The electrostatic potential at a point P due to a source charge is the work done per unit test charge in bringing the test charge (q_0) from infinity to the point P under the electrostatic influence of source charge Q. So,

$$V_P = \text{Potential at point } P = \frac{W_{\infty \to P}}{q_0}$$



Since $W_{\infty \to P} = \frac{Qq_0}{4\pi\varepsilon_0 r}$

$$\Rightarrow \quad V_P = \frac{W_{\infty \to P}}{q_0} = \frac{Q}{4\pi\varepsilon_0 r}$$

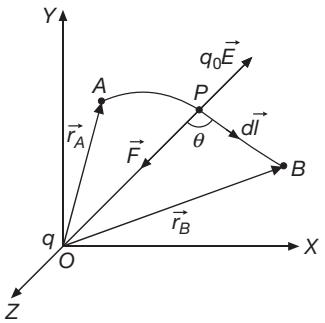
Electrostatic potential is a scalar physical quantity with SI unit joule per coulomb (JC^{-1}) and $1\ \text{JC}^{-1} = 1$ volt $= 1\ \text{V}$.

ELECTROSTATIC POTENTIAL DIFFERENCE (ΔV)

Consider a charge $+q$ to be located at the origin O. Let us arbitrarily choose two points A and B. Let $\vec{E}$ be the electric field strength at point P on this curve. The

test charge q_0 will experience an electric force $q_0\vec{E}$ directed radially away from charge $+q$. In order to prevent the test charge from accelerating in the direction of $q_0\vec{E}$, the external agent is required to apply a force $\vec{F}$ in the opposite direction. So,

$$\vec{F} = -q_0\vec{E}$$



If dW is the infinitesimally small amount of work done by external agent in giving small displacement $d\vec{l}$, then

$$dW = \vec{F}\cdot d\vec{l}$$

$$\Rightarrow \quad W_{AB} = \int_A^B \vec{F}\cdot d\vec{l}$$

$$\Rightarrow \quad W_{AB} = \int_A^B -q_0\vec{E}\cdot d\vec{l} = -q_0\int_A^B \vec{E}\cdot d\vec{l}$$

$$\Rightarrow \quad \frac{W_{AB}}{q_0} = -\int_A^B \vec{E}\cdot d\vec{l}$$

But $\quad V_B - V_A = \dfrac{W_{AB}}{q_0}$

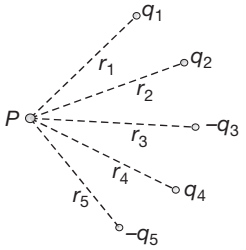
$$\Rightarrow \quad -\int_A^B \vec{E}\cdot d\vec{l} = V_B - V_A$$

$$\Rightarrow \quad V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = \frac{W_{A\to B}}{q_0}$$

So, electrostatic potential difference between two points is work done by external agent in moving unit positive charge from one point to another.

POTENTIAL DUE TO AN ASSEMBLY OF CHARGES

Before moving further with this discussion, let us know this and keep in mind that electrostatic potential is a scalar quantity and is positive due to positive charges and negative due to negative charges. Let us now calculate the electrostatic potential due to an assembly of charges at point P shown.



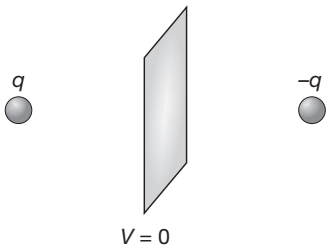
The electrostatic potential V at point P due to the assembly of charges shown is the algebraic sum of the potential due to each of the charge in the assembly. So,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} + \frac{q_4}{r_4} - \frac{q_5}{r_5}\right)$$

POINTS WITH ZERO POTENTIAL DUE TO TWO POINT CHARGES

CASE-1: If both charges are like, then there is no point at a finite distance where net potential zero.

CASE-2: If the charges are equal and unlike then the net potential is zero at all points on the equatorial plane as shown below.



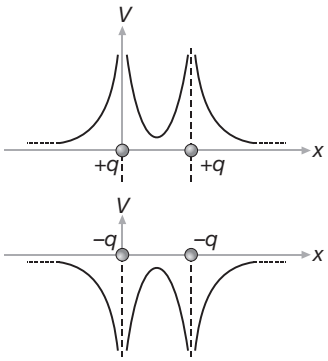
CASE-3: If the charges are unequal and unlike then all such points where resultant potential is zero lie on a closed surface.

However, on the line joining the two charges, only two such points exist, one between the charges and the other outside the charges. Both the above points lie nearer to charge of smaller magnitude.

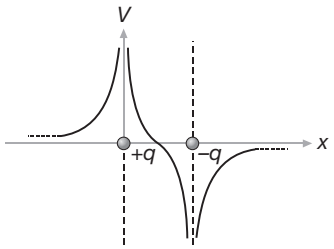
GRAPHICAL REPRESENTATION OF POTENTIAL OF A SYSTEM OF TWO POINT CHARGES

As we move on the line joining two charges from one charge towards the other, the variation of potential V with distance x is shown below.

CASE-1: For like charges of equal magnitude.



CASE-2: For unlike charges of equal magnitude.



CASE-3: For unlike charges of unequal magnitude.

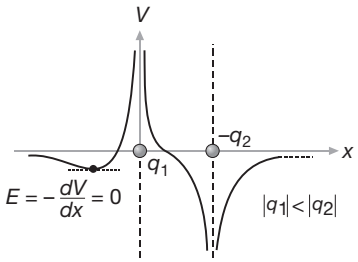


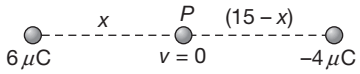
ILLUSTRATION 63

Two charges $6\ \mu C$ and $-4\ \mu C$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

SOLUTION

CASE-1: Between the charges, let $V = 0$ at distance x from 6×10^{-6} C charge. So, we have

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{6\times10^{-6}}{x}-\frac{4\times10^{-6}}{15-x}\right)=0$$



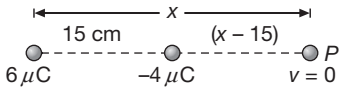
$$\Rightarrow \quad \frac{3}{x}=\frac{2}{15-x}$$

$$\Rightarrow \quad x = 9 \text{ cm}$$

CASE-2: Outside the charges, let $V = 0$ at distance x from 6×10^{-6} C charge. So, we have

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{6\times10^{-6}}{x}-\frac{4\times10^{-6}}{x-15}\right)=0$$

$$\Rightarrow \quad x = 45 \text{ cm}$$



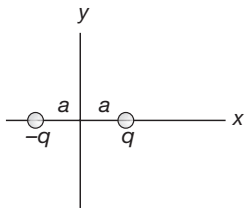
So, electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge. Note that the formula for potential used in the calculation required choosing potential to be zero at infinity.

Conceptual Note(s)

The electric field for such an arrangement of charges is zero only at one point where as the potential is zero for two different points (other than ∞).

ILLUSTRATION 64

Consider a system of two charges shown in figure. The arrangement is also called an Electric Dipole. Find the electric potential at an arbitrary point on the x-axis and make a plot of $\frac{V(x)}{V_0}$ Vs $\frac{x}{a}$ where $V_0 = \frac{q}{4\pi\varepsilon_0 a}$.



Electric dipole

SOLUTION

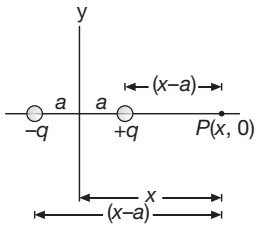
The electric potential can be found by the superposition principle. At a point $P(x, 0)$ on the x-axis, we have

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|x-a|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|x+a|}$$

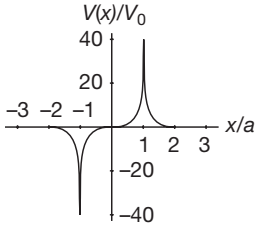
$$\Rightarrow \quad V(x) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{|x-a|} - \frac{1}{|x+a|}\right]$$

The above expression is rewritten as

$$\frac{V(x)}{V_0} = \frac{1}{\left|\frac{x}{a}-1\right|} - \frac{1}{\left|\frac{x}{a}+1\right|}$$



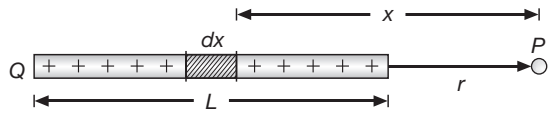
where $V_0 = \frac{q}{4\pi\varepsilon_0 a}$. The plot of the dimensionless electric potential as a function of $\frac{x}{a}$ is shown in figure.



As can be seen from the graph, $V(x)$ diverges at $\frac{x}{a} = \pm 1$, where the charges are located.

ELECTROSTATIC POTENTIAL AT THE AXIS OF A UNIFORMLY CHARGE ROD

Consider a non-conducting charged rod of length L, having a uniform charge Q. Let us find the electrostatic potential at a point P due to the rod at a distance r from one end of the rod.



For this we consider an infinitesimal element of length dx at a distance x from the point P. Charge on this element is

$$dq = \frac{Q}{L}dx$$

The potential dV due to this element at point P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{Lx}dx$$

Net electric potential at point P is obtained by integrating this expression

$$\Rightarrow \quad V = \int dV = \frac{1}{4\pi\varepsilon_0}\int_r^{r+L}\frac{Q}{Lx}dx$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\log_e x\Big|_r^{r+L}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\log_e\left(\frac{r+L}{r}\right)$$

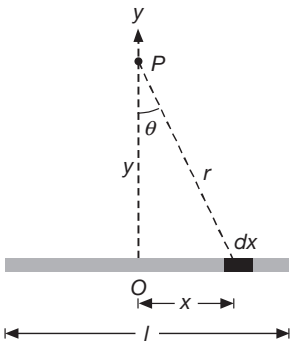
If charge density of the rod is λ, then $\lambda = \frac{Q}{L}$. So,

$$V = \frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{r+L}{r}\right)$$

ELECTROSTATIC POTENTIAL AT POINT P LYING ON PERPENDICULAR BISECTOR OF ROD

Consider a non-conducting rod of length l having a uniform charge density λ. The electric potential at P, a perpendicular distance y above the midpoint of the rod is calculated by considering a differential element of length dx which carries a charge $dq(=\lambda\,dx)$ as shown in figure. The source element is located at $(x,\ 0)$, while the observation point P is located on the y-axis at $(0,\ y)$. The distance of P from element is $r=\sqrt{x^2+y^2}$. Its contribution to the potential dV is given by

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{\left(x^2+y^2\right)^{1/2}}$$



A non-conducting rod of length l and uniform charge density λ.

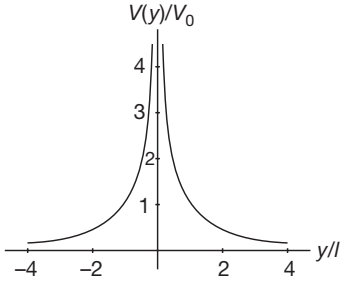
Taking V to be zero at infinity, the total potential due to the entire rod is

$$V=\frac{\lambda}{4\pi\varepsilon_0}\int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{dx}{\sqrt{x^2+y^2}}=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left[x+\sqrt{x^2+y^2}\right]\Bigg|_{-\frac{l}{2}}^{\frac{l}{2}}$$

Using $\int\frac{dx}{\sqrt{x^2+y^2}}=\log_e\left(x+\sqrt{x^2+y^2}\right)$

$$\Rightarrow\quad V=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left[\frac{\left(\frac{l}{2}\right)+\sqrt{\left(\frac{l}{2}\right)^2+y^2}}{-\left(\frac{l}{2}\right)+\sqrt{\left(\frac{l}{2}\right)^2+y^2}}\right]$$

A plot of $\frac{V(y)}{V_0}$, where $V_0=\frac{\lambda}{4\pi\varepsilon_0}$, as a function of $\frac{y}{l}$ is shown in figure.



In the limit $l\gg y$, the potential becomes

$$V=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left[\frac{\frac{l}{2}+\frac{l}{2}\sqrt{1+\left(\frac{2y}{l}\right)^2}}{-\left(\frac{l}{2}\right)+\frac{l}{2}\sqrt{1+\left(\frac{2y}{l}\right)^2}}\right]$$

$$\Rightarrow\quad V=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left[\frac{1+\sqrt{1+\left(\frac{2y}{l}\right)^2}}{-1+\sqrt{1+\left(\frac{2y}{l}\right)^2}}\right]$$

Since $\sqrt{1+\left(\frac{2y}{l}\right)^2}\simeq 1+\frac{2y^2}{l^2}$

$$\Rightarrow\quad V=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left[\frac{2+\frac{2y^2}{l^2}}{\frac{2y^2}{l^2}}\right]$$

Since, $\frac{y^2}{l^2}\ll 1$

$$\Rightarrow\quad 2+\frac{2y^2}{l^2}\simeq 2$$

$$\Rightarrow\quad V\approx\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{2}{\frac{2y^2}{l^2}}\right)=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{l^2}{y^2}\right)$$

$$\Rightarrow\quad V=\frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{l}{y}\right)$$

The corresponding electric field can be obtained by using

$$E_y = -\frac{\partial V}{\partial y} = \frac{\lambda}{2\pi\varepsilon_0 y}\frac{\frac{l}{2}}{\sqrt{\left(\frac{l}{2}\right)^2 + y^2}}$$

which is in complete agreement with the result.

ILLUSTRATION 65

On a thin rod of length $l = 1$ m, lying along the x-axis with one end at the origin $x = 0$, there is uniformly distributed charge per unit length $\lambda = Kx$, where $K = 10^{-9}$ Cm^{-2}. Find the work done in displacing a charge $q = 1000\ \mu$C from a point $A\left(0,\ \sqrt{0.44}\right)$ m to $B(0,\ l)$ m.

SOLUTION

Let us calculate the potential due to the rod at A and then at B.

Potential at $A\left(0,\ \sqrt{0.44}\right)$ m

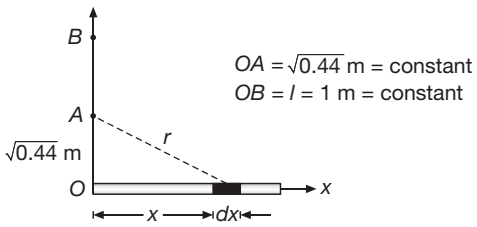
Potential at A due to rod is calculated by taking an infinitesimal element of length dx at a distance x from $O(0,\ 0)$ on the rod. Then

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda\, dx}{4\pi\varepsilon_0\sqrt{OA^2 + x^2}} \qquad \ldots(1)$$

$$\Rightarrow \quad dV = \frac{kx\, dx}{4\pi\varepsilon_0\sqrt{0.44 + x^2}}$$

$$\Rightarrow \quad \int_{V_0}^{V_A} dV = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{0.44 + x^2}\Big|_0^{l=1\text{ m}}\right]$$

$$\Rightarrow \quad V_A - V_0 = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{1.44} - \sqrt{0.44}\right] \qquad \ldots(2)$$



Potential at $B(0,\ 1)$ m

$$dV = \frac{dq}{4\pi\varepsilon_0 r'} = \frac{kx\, dx}{4\pi\varepsilon_0\sqrt{OB^2 + x^2}} = \frac{kx\, dx}{4\pi\varepsilon_0\sqrt{1 + x^2}}$$

$$\Rightarrow \quad V_B - V_0 = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{1 + x^2}\Big|_0^1\right]$$

$$\Rightarrow \quad V_B - V_0 = \frac{k}{4\pi\varepsilon_0}\left(\sqrt{2} - 1\right) \qquad \ldots(3)$$

Now, by definition, $W_{A\to B} = q\left(V_B - V_A\right)$

$$\Rightarrow \quad W_{A\to B} = W = \frac{qk}{4\pi\varepsilon_0}\left[\sqrt{2} - 1 - \sqrt{1.44} + \sqrt{0.44}\right]$$

(Please observe, that V_0 will cancel in the process)

$$\Rightarrow \quad W_{A\to B} = \frac{qk}{4\pi\varepsilon_0}[1.414 - 1 - 1.2 + 0.66]$$

$$\Rightarrow \quad W_{A\to B} = 9\times10^9\times1000\times10^{-6}\times10^{-9}[-0.126]$$

$$\Rightarrow \quad W_{A\to B} = -1.1\times10^{-3}\ \text{J}$$

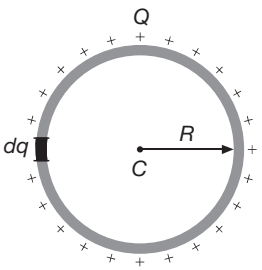
ELECTROSTATIC POTENTIAL DUE TO A CHARGED RING AT ITS CENTRE

Let us first find potential dV at centre C due to an infinitesimal charge dq on ring which is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R}$$

Total potential at C is $V = \int dV$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$



Since all the infinitesimal dq's of the ring are situated at same distance R from the ring centre C so, we can directly say that the total electric potential at centre of ring is $V_C = \frac{Q}{4\pi\varepsilon_0 R}$.

Here we must see that even if charge Q is non-uniformly distributed on ring, the electric potential C will remain same (we shall discuss this with an example below) because in that case too

$$V = \frac{1}{4\pi\varepsilon_0 R}\int dq = \frac{\int dq}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{Q_{\text{total on ring}}}{4\pi\varepsilon_0 R}$$

ILLUSTRATION 66

Consider a charged ring of radius R. Let the charge on this ring be varying according to the relation $\lambda = \lambda_0 \cos^2\phi$, where λ_0 is a positive constant and ϕ is the azimuthal angle. Find the electrostatic potential due to this ring at the centre.

SOLUTION

Let us first calculate the total charge possessed by the ring. Since,

$$dq = \int \lambda\, dx$$

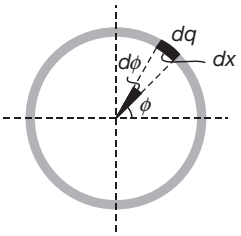
$$\Rightarrow \quad dq = \int \lambda R d\phi$$

$$\Rightarrow \quad dq = \lambda_0 R \int_0^{2\pi} \cos^2\phi\, d\phi$$

$$\Rightarrow \quad q = \int dq = \frac{\lambda_0 R}{2}\int_0^{2\pi}[1+\cos(2\phi)]d\phi$$

$$\Rightarrow \quad q = \frac{\lambda_0 R}{2}\left[\int_0^{2\pi} d\phi + \int_0^{2\pi}\cos 2\phi\, d\phi\right]$$

$$\Rightarrow \quad q = \frac{\lambda_0 R}{2}\left[2\pi + \frac{\sin(2\phi)}{2}\Big|_0^{2\pi}\right]$$



$$\Rightarrow \quad q = \lambda_0 R\pi + 0$$

$$\Rightarrow \quad q = \lambda_0 \pi R$$

Since, $dV = \frac{dq}{4\pi\varepsilon_0 R}$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0 R}\int dq$$

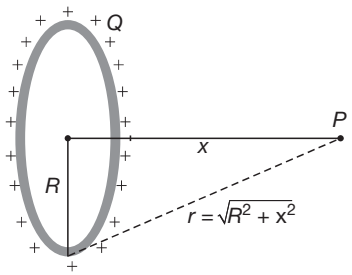
$$\Rightarrow \quad V = \frac{\lambda_0 \pi R}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad V = \frac{\lambda_0}{4\varepsilon_0}$$

ELECTROSTATIC POTENTIAL AT A POINT *P* ON THE AXIS OF THE RING AT DISTANCE *X* FROM ITS CENTRE

If we wish to find the electrostatic potential at a point P lying on the axis of ring, we can directly calculate the result because here too, all points of ring are at same distance $\sqrt{R^2+x^2}$ from the point P. So, potential at P is

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+x^2}}$$



VARIATION OF ELECTROSTATIC POTENTIAL ON THE AXIS OF A CHARGED RING

Since $V_{\text{centre}} = \frac{Q}{4\pi\varepsilon_0 R}$

and $V_{\text{axis}} = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}$

From the above two expressions, we conclude that electrostatic potential is maximum at the centre and

goes on decreasing when we move away from the centre on its axis.

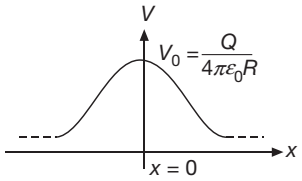


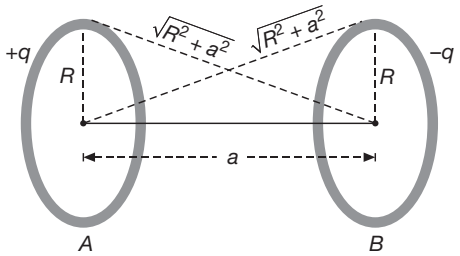
ILLUSTRATION 67

There are two thin wire rings, each of radius R, whose axes coincide. The charges of the rings are q and $-q$. Find the potential difference between the centres of the rings separated by a distance a. Also calculate the work done to move a charge q_0 from centre of first ring to the centre of other ring.

SOLUTION

Net potential at centre of ring A is

$$V_A = \begin{pmatrix}\text{Potential at } A\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Potential at } A\\ \text{due to } B\end{pmatrix}$$



$$\Rightarrow \quad V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R} + \frac{(-q)}{\sqrt{R^2+a^2}}\right)$$

$$\Rightarrow \quad V_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+a^2}}\right]$$

Similarly net potential at centre of ring B is

$$V_B = \begin{pmatrix}\text{Potential at } B\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Potential at } B\\ \text{due to } A\end{pmatrix}$$

$$\Rightarrow \quad V_B = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}}\right]$$

Thus potential difference,

$$\Delta V = V_B - V_A$$

$$\Rightarrow \quad \Delta V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}}\right]$$

$$\Rightarrow \quad \Delta V = \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

$$\Rightarrow \quad \Delta V = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

Since, $W_{A\to B} = q_0(V_B - V_A)$

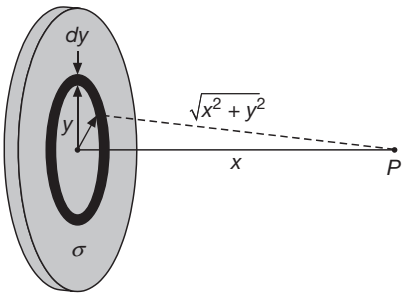
$$\Rightarrow \quad W_{A\to B} = \frac{qq_0}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

ELECTROSTATIC POTENTIAL DUE TO A UNIFORMLY CHARGED DISC AT A POINT P ON ITS AXIS

Consider a uniformly charged disc of radius R with surface charge density σ. We wish to find electric potential at point P lying on its axis at distance x from the centre. For this we consider an infinitesimal elemental ring of radius y and thickness dy concentric with the disc. Then charge on this infinitesimal element is

$$dq = \sigma\,(\text{Area of element})$$

$$\Rightarrow \quad dq = \sigma(2\pi y\,dy)$$



The electric potential at point P due to this infinitesimal element is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}}$$

(Please note here that once the point P is taken, then x becomes a fixed value)

$$\Rightarrow \quad dV = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi y\,dy)}{\sqrt{x^2+y^2}}$$

Net electric potential at point P due to entire disc is calculated by integrating this expression.

$$\Rightarrow \quad V = \int dV = \int_0^R \frac{\sigma}{2\varepsilon_0} \frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \quad V = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}}$$

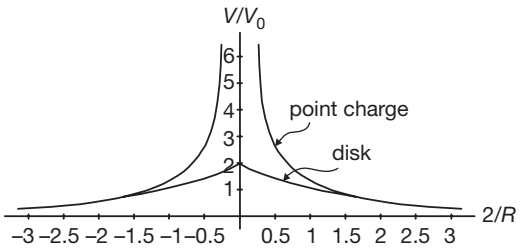
$$\Rightarrow \quad V = \frac{\sigma}{4\varepsilon_0} \int_0^R \left(x^2+y^2\right)^{-\frac{1}{2}} 2y\,dy$$

Using $\int \left[f(x)\right]^n f'(x)\,dx = \dfrac{\left[f(x)\right]^{n+1}}{n+1}, n \neq -1$

$$\Rightarrow \quad V = \frac{\sigma}{4\varepsilon_0} \frac{\left(x^2+y^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_0^R$$

$$\Rightarrow \quad V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+y^2}\Big|_0^R\right]$$

$$\Rightarrow \quad V_P = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+x^2} - x\right]$$



Comparison of the electric potentials of a non-conducting disk and a poin charge. The electric potential is measured in terms of $V_0 = Q/4\pi\varepsilon_0 R$

Also, at the centre of the disc, we have $x = 0$, so

$$V_{\text{centre}} = V_C = \frac{\sigma R}{2\varepsilon_0}$$

In the limit $x \gg R$, we have

$$\sqrt{R^2+x^2} = x\left(1+\frac{R^2}{x^2}\right)^{\frac{1}{2}} \simeq x\left(1+\frac{R^2}{2x^2}+.....\right)$$

$$\Rightarrow \quad V = \frac{\sigma}{2\varepsilon_0}\left[x+\frac{R^2}{2x}-x\right] = \frac{\sigma R^2}{4\varepsilon_0 x} = \frac{\sigma(\pi R^2)}{4\pi\varepsilon_0 x}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0 x} = V_{\substack{\text{due to a point charge} \\ \text{at distance } x \text{ from it}}}$$

ILLUSTRATION 68

A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The particle has $\dfrac{q}{m} = \dfrac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

SOLUTION

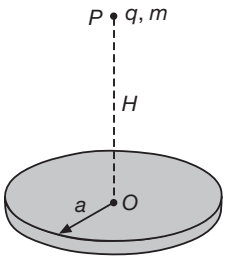
Since, $V_p = \dfrac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+H^2} - H\right]$

Potential at centre, (O) will be

$$V_O = \frac{\sigma a}{2\varepsilon_0} \qquad \{H = 0\}$$

(a) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy decrease in gravitational potential energy = increase in electrostatic potential energy

$\left(\Delta KE = 0 \text{ because } K_i = K_f = 0\right)$



$$\Rightarrow \quad mgH = q\left[V_O - V_P\right]$$

$$\Rightarrow \quad gH = \left(\frac{q}{m}\right)\left(\frac{\sigma}{2\varepsilon_0}\right)\left[a - \sqrt{a^2 + H^2} + H\right] \qquad \ldots(1)$$

Since, $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$

$$\Rightarrow \quad \frac{q\sigma}{2\varepsilon_0 m} = 2g$$

Substituting in equation (1), we get

$$gH = 2g\left[a + H - \sqrt{a^2 + H^2}\right]$$

$$\Rightarrow \quad \frac{H}{2} = (a + H) - \sqrt{a^2 + H^2}$$

$$\Rightarrow \quad \sqrt{a^2 + H^2} = a + \frac{H}{2} \text{ or } a^2 + H^2 = a^2 + \frac{H^2}{4} + aH$$

$$\Rightarrow \quad \frac{3}{4}H^2 = aH$$

$$\Rightarrow \quad H = \frac{4}{3}a \text{ and } H = 0$$

$$\Rightarrow \quad H = \left(\frac{4}{3}\right)a$$

(b) If U be the potential energy, U_e be the electrostatic energy and U_g be the gravitational potential energy of the particle at height h, then

$$U = U_e + U_g$$

where $U_e = qV = \frac{\sigma q}{2\varepsilon_0}\left[\sqrt{a^2 + H^2} - H\right]$

and $U_g = mgH$

$$\Rightarrow \quad U = \frac{\sigma q}{2\varepsilon_0}\left[\sqrt{a^2 + H^2} - H\right] + mgH \qquad \ldots(2)$$

At equilibrium position

$$F = \frac{-dU}{dH} = 0$$

Differentiating equation (2) w.r.t. H, we get

$$mg + \frac{\sigma q}{2\varepsilon_0}\left[\left(\frac{1}{2}\right)(2H)\frac{1}{\sqrt{a^2 + H^2}} - 1\right] = 0$$

$$\left\{\because \ \frac{\sigma q}{2\varepsilon_0} = 2mg\right\}$$

$$\Rightarrow \quad mg + 2mg\left[\frac{H}{\sqrt{a^2 + H^2}} - 1\right] = 0$$

$$\Rightarrow \quad 1 + \frac{2H}{\sqrt{a^2 + H^2}} - 2 = 0$$

$$\Rightarrow \quad \frac{2H}{\sqrt{a^2 + H^2}} = 1$$

$$\Rightarrow \quad \frac{H^2}{a^2 + H^2} = \frac{1}{4}$$

$$\Rightarrow \quad 3H^2 = a^2$$

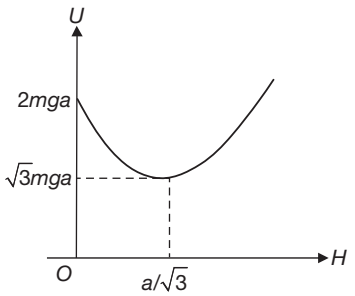
$$\Rightarrow \quad H = \frac{a}{\sqrt{3}}$$

From equation (2), we can see that,

$U = 2mga$ at $H = 0$ and

$U = U_{\min} = \sqrt{3}mga$ at $H = \frac{a}{\sqrt{3}}$

Therefore, U-H graph will be as shown.

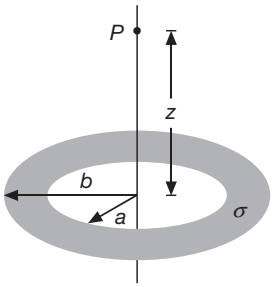


Also we note that at $H = \frac{a}{\sqrt{3}}$, U is minimum.

Therefore, $H = \frac{a}{\sqrt{3}}$ is the stable equilibrium position.

ELECTRIC POTENTIAL OF AN ANNULUS

Consider an annulus of uniform charge density σ, as shown in figure. The electric potential at a point P along the symmetric axis is calculated by taking an infinitesimal elemental ring of radius y and thickness dy concentric with the annulus. If dq be the charge on this infinitesimal element, then



An annulus of uniform charge density

$$dq = \sigma dA = \sigma(2\pi y\, dy)$$

Its contribution to the electric potential at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi y\, dy)}{\sqrt{z^2+y^2}}$$

Integrating over the entire annulus and using

$$\int \frac{y\, dy}{\sqrt{y^2+x^2}} = \sqrt{y^2+x^2}$$

We obtain

$$V = \frac{\sigma}{4\pi\varepsilon_0}\int_a^b \frac{\sigma(2\pi y\, dy)}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \quad V = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_a^b \frac{y\, dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right]$$

$$\Rightarrow \quad V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right)$$

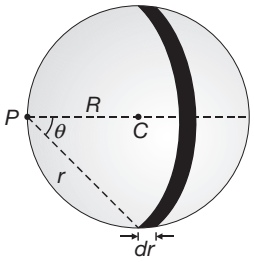
In the limit $a \to 0$ and $b \to R$, the potential becomes

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - |z|\right]$$

which coincides with the result of a non-conducting disk of radius R

ELECTROSTATIC POTENTIAL ON THE EDGE OF A UNIFORMLY CHARGED DISC

Consider a uniformly charged disc of radius R, having surface charge density σ. Let us take a point P on the edge of the disc.



For this, assume the disc to be made up of a large number of concentric rings with P as the centre. Let us consider one such infinitesimal ring of radius r, thickness dr and charge dq. Then potential due to this infinitesimal ring is

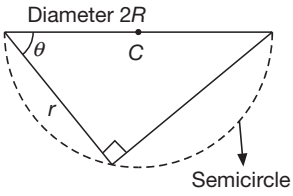
$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

where $dq = \sigma(\text{Area of infinitesimal element})$

$$\Rightarrow \quad dq = \sigma(2r\theta)dr \qquad \{\because \text{ length of element } = 2r\theta\}$$

$$\Rightarrow \quad dV = \frac{2\sigma r\theta\, dr}{4\pi\varepsilon_0 r} = \frac{\sigma}{2\pi\varepsilon_0}\theta\, dr$$

Also, we observe that (see figure)



$$r = 2R\cos\theta$$

$$\Rightarrow \quad dr = -2R\sin\theta\, d\theta$$

$$\Rightarrow \quad dV = -\frac{\sigma}{2\pi\varepsilon_0}2R\theta\sin\theta d\theta$$

$$\Rightarrow \quad V = \int_{\frac{\pi}{2}}^{0} dV = -\frac{\sigma R}{\pi\varepsilon_0}\int_{\frac{\pi}{2}}^{0}\theta\sin\theta\, d\theta$$

Since $-\int_b^a f(x)dx = \int_a^b f(x)dx$

$$\Rightarrow \quad V = \frac{\sigma R}{\pi\varepsilon_0}\int_0^{\frac{\pi}{2}}\theta\sin\theta\, d\theta$$

Using ILATE property to calculate the integral, we get

$$V=\frac{\sigma R}{\pi\varepsilon_0}\left[\theta\int_0^{\frac{\pi}{2}}\sin\theta\,d\theta-\int_0^{\frac{\pi}{2}}(-\cos\theta)\,d\theta\right]$$

$$\Rightarrow\quad V=\frac{\sigma R}{\pi\varepsilon_0}\left[-\theta\cos\theta\Big|_0^{\frac{\pi}{2}}+\sin\theta\Big|_0^{\frac{\pi}{2}}\right]$$

$$\Rightarrow\quad V=\frac{\sigma R}{\pi\varepsilon_0}\left[0+\sin\left(\frac{\pi}{2}\right)-\sin 0\right]$$

$$\Rightarrow\quad V=\frac{\sigma R}{\pi\varepsilon_0}$$

Also, we know that at the centre of the disc

$$V_C=\frac{\sigma R}{2\varepsilon_0}$$

Hence the potential at the centre is more than that at the edge

ILLUSTRATION 69

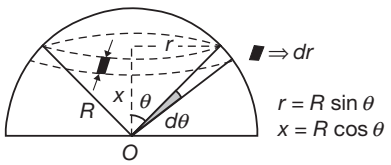
Find the electric field potential at the centre of a hemisphere of radius R having uniform surface charge density σ.

SOLUTION

Consider an infinitesimal ring element as shown. The charge on this element is given by

$$dq=(2\pi R\sin\theta)(R\,d\theta)\sigma$$

{Because radius of ring is $r=R\sin\theta$}



The potential due to this element at the centre O of the hemisphere

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{R}=\frac{1}{4\pi\varepsilon_0}\times(2\pi R\sigma\sin\theta)\,d\theta$$

$$\Rightarrow\quad dV=\frac{\sigma R}{2\varepsilon_0}\sin\theta\,d\theta$$

Potential due to the hemisphere is

$$V=\frac{\sigma R}{2\varepsilon_0}\int_0^{\frac{\pi}{2}}\sin\theta\,d\theta=\frac{\sigma R}{2\varepsilon_0}\left[-\cos\theta\Big|_0^{\frac{\pi}{2}}\right]$$

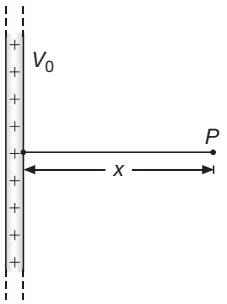
$$\Rightarrow\quad V=\frac{\sigma R}{2\varepsilon_0}$$

ILLUSTRATION 70

The electrostatic potential at surface of thin non-conducting sheet with charge density σ is V_0. Find the potential at a distance x from infinite sheet.

SOLUTION

Potential at point P at a distance x is



$$V_P=V_0-\int_0^x E\,dx$$

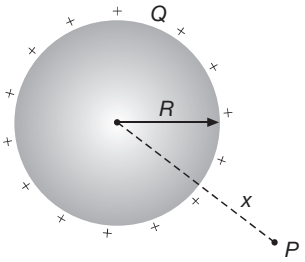
$$\Rightarrow\quad V_P=V_0-\int_0^x\frac{\sigma}{2\varepsilon_0}dx$$

$$\Rightarrow\quad V_P=V_0-\frac{\sigma}{2\varepsilon_0}x$$

ELECTROSTATIC POTENTIAL DUE TO A CHARGED CONDUCTING SPHERE/ CHARGED SHELL

For points located outside the charged sphere we can assume the entire charge to be concentrated at its centre. Thus electrostatic potential at a distance r from the centre of sphere, outside it will be

$$V_{\text{outside}}=V=\frac{Q}{4\pi\varepsilon_0 r}$$



At the points on surface of sphere, the potential is given by

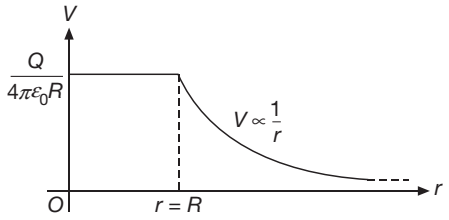
$$V_{\text{surface}} = V_S = \frac{Q}{4\pi\varepsilon_0 R}$$

At the interior points of sphere the electric field is zero at all points inside, so, we can say that the inner portion of the conducting sphere / charged shell is an equipotential region and hence at every interior point potential is same as that on its surface. Thus we have

$$V_{\text{inside}} = V_{\text{surface}} = \frac{Q}{4\pi\varepsilon_0 R}$$

The variation of V with distance r from the centre is shown in figure

Also, the same results are valid for a uniformly charged hollow sphere



Problem Solving Technique(s)

To find the electric potential due to a conducting sphere (or shell) we should keep in mind the following two points:

(a) Electric potential on the surface and at any point inside the sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

(R = radius of sphere)

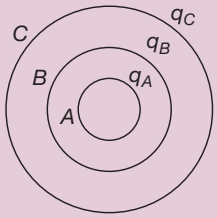
(b) Electric potential at any point outside the sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

$\{r$ = distance of the point from the centre$\}$

For example, in the figure shown, potential at A is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A}+\frac{q_B}{r_B}+\frac{q_C}{r_C}\right]$$



Similarly, potential at B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B}+\frac{q_B}{r_B}+\frac{q_C}{r_C}\right]$$

and potential at C is ,

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_C}+\frac{q_B}{r_C}+\frac{q_C}{r_C}\right]$$

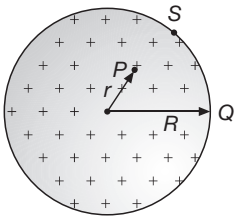
ELECTROSTATIC POTENTIAL DUE TO A NON-CONDUCTING UNIFORMLY CHARGED SPHERE

Here too, for outer and surface points the potential remains same as that of a conducting sphere (as if the entire charge is concentrated at the centre). So,

$$V_{\text{outside}} = V_o = \frac{Q}{4\pi\varepsilon_0 r} \qquad \{\text{for } r > R\}$$

$$V_{\text{surface}} = V_S = \frac{Q}{4\pi\varepsilon_0 R} \qquad \{\text{for } r = R\}$$

For an interior point, unlike to a conducting sphere, potential will not remain uniform as electric field exists inside this region. Inside a uniformly charged sphere electric field is in radially outward direction, so as we move away from centre, in the direction of electric field the potential decreases.



Consider a point P at a distance r from the centre of sphere. The potential difference between points P and S is

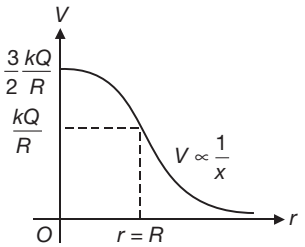
$$V_S - V_P = -\int_r^R \frac{Qrdr}{4\pi\varepsilon_0 R^3}$$

Since $V_S = \dfrac{Q}{4\pi\varepsilon_0 R}$

$$\Rightarrow \quad V_P - \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q}{8\pi\varepsilon_0 R^3}(R^2 - r^2)$$

$$\Rightarrow \quad V_P = \frac{Q}{8\pi\varepsilon_0 R^3}(R^2 - r^2) + \frac{Q}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad V_P = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2)$$



Also we observe that, potential at centre of sphere $(r = 0)$ is

$$V_C = \frac{3Q}{8\pi\varepsilon_0 R} = \frac{3}{2}V_S$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{3}{2}V_{\text{surface}}$$

Thus at centre, potential is maximum and is equal to $\dfrac{3}{2}$ times that on the surface

If we assume the charge density of the sphere to be ρ, then we get

$$V = \begin{cases} \dfrac{\rho R^3}{3\varepsilon_0 r} & r \geq R \\ & \text{(outside and at surface)} \\ \dfrac{\rho}{6\varepsilon_0}(3R^2 - r^2) & r < R \\ & \text{(inside)} \end{cases}$$

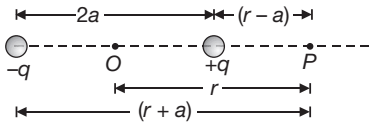
POTENTIAL DUE TO A DIPOLE

On Axial Line

Consider a point P lying on axial line of the dipole at a distance r from its centre. Then potential at this point is

$$V_P = -\frac{q}{4\pi\varepsilon_0(r+a)} + \frac{q}{4\pi\varepsilon_0(r-a)}$$

$$\Rightarrow \quad V_P = \frac{q}{4\pi\varepsilon_0}\frac{2a}{(r^2 - a^2)}$$



$$\Rightarrow \quad V = \frac{p}{4\pi\varepsilon_0(r^2 - a^2)}$$

For $a << r$ we have

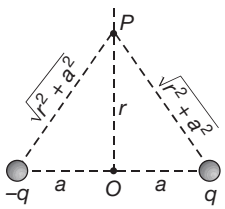
$$V \simeq \frac{p}{4\pi\varepsilon_0 r^2}$$

On Equitorial Line

Consider a point P lying on the equatorial line of the dipole at a distance r from its centre. Then potential at this point is

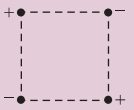
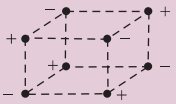
$$V_P = \frac{q}{4\pi\varepsilon_0\sqrt{r^2 + a^2}} + \frac{-q}{4\pi\varepsilon_0\sqrt{r^2 + a^2}}$$

$$\Rightarrow \quad V_P = 0$$



Remark(s)

As a matter of fact we do have four basic kinds of categories for charge distributions shown below.

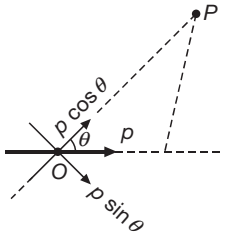
NAME	REPRESENTATION	VARIATION OF V AT FAR OFF DISTANCE
Monopole		$V-\frac{1}{r}$
Dipole		$V-\frac{1}{r^2}$
Quadrupole		$V-\frac{1}{r^3}$
Octopole		$V-\frac{1}{r^4}$

POTENTIAL AT POINT $P(r, \theta)$ DUE TO A SMALL DIPOLE

Consider a short dipole of dipole moment p. Let us calculate the electric potential due to this small dipole at the point P. For doing this we resolve the dipole moment p in two components

(i) $p\cos\theta$ along the line joining centre of dipole (O) to point P.

(ii) $p\sin\theta$ perpendicular to the line joining centre of dipole (O) to point P (as shown).



The point P lies on the axial line of dipole having dipole moment $p\cos\theta$ and equatorial line of dipole having dipole moment $p\sin\theta$.

$$V_{\text{axial}}=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \text{ and } V_{\text{equitorial}}=0$$

$$\Rightarrow \quad V=V_{\text{axial}}+V_{\text{equitorial}}=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

BINDING ENERGY OF A DIPOLE

The energy required to be supplied to the dipole so that the charges forming it become separated from each other's electrostatic influence.

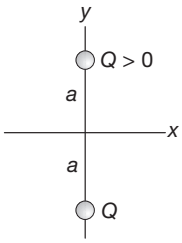
$$U=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

Test Your Concepts-VII

Based on Electrostatic Potential and Energy

(Solutions on page H.24)

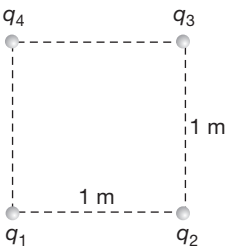
1. Two point charges of equal magnitude are located along the y-axis equal distances above and below the x-axis, as shown in figure.



(a) Plot a graph of the potential at points along the x-axis over the interval $-3a < x < 3a$. You should plot the potential in units of $\frac{Q}{4\pi\varepsilon_0 a}$.

(b) Let the charge located at $-a$ be negative and plot the potential along the y-axis over the interval $-4a < y < 4a$.

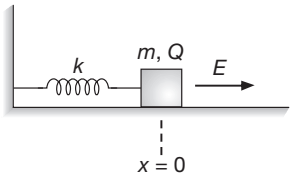
2. Four charges $q_1 = 1\,\mu C$, $q_2 = 2\,\mu C$, $q_3 = -3\,\mu C$ and $q_4 = 4\,\mu C$ are kept on the vertices of a square of side 1 m. Find the electric potential energy of this system of charges.



3. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9 V to a point where the potential is –5 V? (The potential in each case is measured relative to a common reference point).

4. Three point charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed at $(1\,m, 0, 0)$, $(0, 2\,m, 0)$ and $(0, 0, 3\,m)$ respectively. Find the electrostatic potential due to the charges at origin.

5. A block having mass m and charge $+Q$ is connected to a spring having constant k. The block lies on a frictionless horizontal surface, and the system is immersed in a uniform electric field of magnitude E, directed as shown in figure. If the block is released from rest when the spring is unstretched (at $x = 0$)



(a) By what maximum amount does the spring expand?

(b) What is the equilibrium position of the block?

(c) Show that the block's motion is simple harmonic, and determine its period.

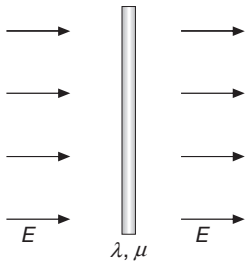
(d) By what maximum amount does the spring expand, if the coefficient of kinetic friction between block and surface is μ_k.

6. A charge $Q = 10\,\mu C$ is distributed uniformly over the circumference of a ring of radius 3 m placed on x-y plane with its centre at origin. Find the electric potential at a point $P(0, 0, 4\,m)$.

7. Consider a planet where free-fall acceleration is the same as that on Earth but there is also a strong downward electric field that is uniform close to the planet's surface. A 2 kg ball having a charge of $+5\,\mu C$ is thrown upward at a speed of $20.1\,ms^{-1}$, and it hits the ground after an interval of 4.1 s. What is the potential difference between the starting point and the top point of the trajectory?

8. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. Find the change in electrical energy of Q (approximately) when it is displaced by a small distance along the x-axis.

9. An insulating rod having linear charge density $\lambda = 40\,\mu Cm^{-1}$ and linear mass density $\mu = 0.1\,kgm^{-1}$ is released from rest in a uniform electric field $E = 100\,Vm^{-1}$ directed perpendicular to the rod. Determine the speed of the rod after it has travelled a distance of 2 m.



10. Two point charges are located on the x-axis, $q_1 = -1\,\mu C$ at $x = 0$ and $q_2 = +1\,\mu C$ at $x = 1\,m$.

(a) Find the work that must be done by an external force to bring a third point charge $q_3 = +1\,\mu C$ from infinity to $x = 2$ m.

(b) Find the total potential energy of the system of three charges.

11. Consider two concentric spherical metal shells of radii a and b, where $b > a$. The outer shell has charge Q but the inner shell is grounded. Find the charge on the inner shell.

12. At a certain distance from a point charge, the magnitude of the electric field is $500\,Vm^{-1}$ and the electric potential is -3 kV.

(a) What is the distance to the charge?

(b) What is the magnitude of the charge?

13. Find out the points on the line joining two charges $+q$ and $-3q$ (kept at a distance of 1 m) where electric potential is zero.

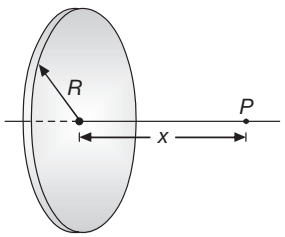
14. A light unstressed spring has length d. Two identical particles, each with charge q, are connected to the opposite ends of the spring. The particles are held stationary a distance d apart and then released at the same time. The system then oscillates on a horizontal frictionless table. The spring has a bit of internal kinetic friction, so the oscillation is damped. The particles eventually stop vibrating when the distance between them is $3d$. Find the increase in internal energy that appears in the spring during the oscillations. Assume that the system of the spring and two charges is isolated.

15. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?

16. A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in this field from $y_i = a$ to $y_f = 2a$ along the y-axis.

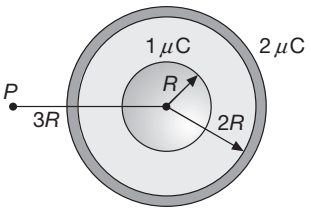
17. A wire having a uniform linear charge density λ is bent into the shape shown in figure. Find the electric potential at point O.

18. A disk of radius R has a non-uniform surface charge density $\sigma = \sigma_0 r$, where σ_0 is a positive constant and r is measured from the center of the disk. Find the potential at P.



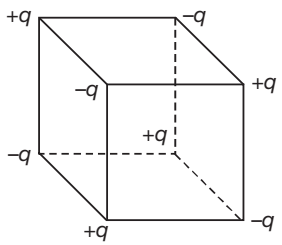
19. An infinite number of charges each equal to q are placed along the x axis at $x = 1, x = 4, x = 8, ...$ and so on. Find the potential and electric field at the point $x = 0$ due to this set of charges.
What will be potential and electric field if in the above set up the consecutive charge have opposite sign?

20. Two concentric spheres of radii R and $2R$ are charged. The inner sphere has a charge of $1\,\mu C$ and the outer sphere has a charge of $2\,\mu C$ of the same sign. The potential is 9000 V at a distance $3R$ from the common center. What is the value of R?



21. A point charge Q is held fixed at origin. A second point charge $-q$ having mass m is placed on the x-axis, at $(2\ell, 0)$ from the origin. The second point charge is released from rest. What is its speed when it is at $\left(\frac{\ell}{4}, 0\right)$ from the origin?

22. Eight point charges are placed at the corners of a cube of edge a as shown in figure. Find the work done in disassembling this system of charges.

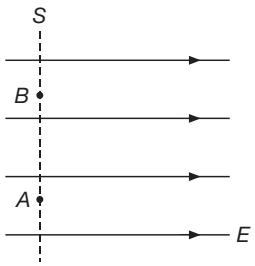


23. A conducting bubble of radius a, thickness $t\,(t \ll a)$ has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet.

24. Two point charges $2\,\mu C$ and $-4\,\mu C$ are kept 6 m apart. At what points on the line joining the charges, the electric potential will be zero.

EQUIPOTENTIAL SURFACES: INTRODUCTION

Consider a situation in which a charge is displaced from a point A to B on a surface S which is perpendicular to the direction of electric field. Then work done in displacing the charge will be zero, because electric force is normal to the direction of displacement.

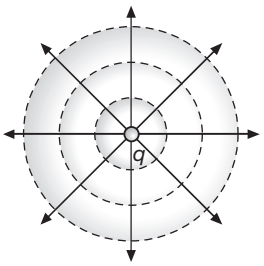


Since no work is done in moving the charge from A to B, hence we can say that A and B are at same potential or we can say that all the points lying on surface S are at same potential. So, we call the surface S to be an Equipotential Surface.

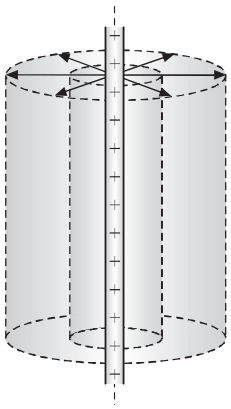
Following figures show equipotential surfaces in the surrounding of point charge, a long charged wire and at far off distance from both.

Mathematically equipotential surface is the locus of all points which have the same potential due to a charge distribution.

Any surface in an electric field at every point of which the direction of electric field is normal to the surface can be regarded as equipotential surface.

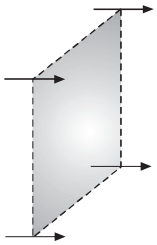


Spherical equipotential surfaces for point charge



Cylindrical equipotential surfaces for line charge

(Continued)



Planar equipotential surface

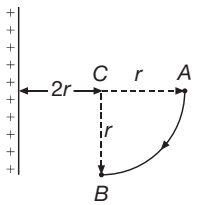
EQUIPOTENTIAL SURFACE: PROPERTIES

Since, we have already read that an equipotential surface is the locus of all points which have the same electrostatic potential due to a charge distribution. Equipotential surfaces possess the following properties.

(a) The work done in moving a charge along an equipotential surface is always zero.
(b) An electric field line must cut the equipotential surface at right angles.
(c) The tangential component of the electric field along the equipotential surface is zero, otherwise non-zero work would be done to move a charge from one point on the surface to another point on it.
(d) No two equipotential surfaces cross each other.
(e) An equipotential surface for a point charge must be a family of concentric spheres, for a line charge must be a family of coaxial cylinders.
(f) The equipotential surface for a point charge or a line charge at far off distance is a family of planes perpendicular to the field lines.

ILLUSTRATION 71

A charge q_0 is transported from point A to B along the arc AB with centre at C as shown in figure near a long charged wire with linear density λ lying in the same plane. Find the work done in doing so.



SOLUTION

Since, electric field is conservative in nature, so work done does not depend upon the path followed between the two points.

$$\Rightarrow \quad W_{A\to B} = W_{A\to C} + W_{C\to B}$$

But $W_{C\to B} = 0$ (because B and C are at same potentials or B and C lie on the same equipotential line)

$$\Rightarrow \quad W_{A\to B} = W_{A\to C}$$

and $$W_{A\to C} = -q_0 \int_{3r}^{2r} \vec{E}\cdot d\vec{r}$$

$$\Rightarrow \quad W_{A\to C} = \frac{q_0}{2\pi\varepsilon_0}\int_{2r}^{3r} \frac{\lambda}{r}dr \qquad \left\{\because E = \frac{\lambda}{2\pi\varepsilon_0 r}\right\}$$

$$\Rightarrow \quad W_{A\to C} = \frac{q_0\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{3}{2}\right)$$

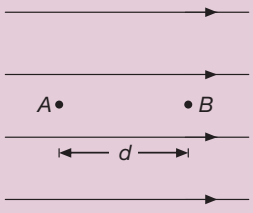
Remark(s)

(a) A region in which at every point electric field is zero, can be regarded as an equipotential region.

(b) As we have already discussed whenever charge is given to a metal body, it is distributed on its outer surface in such a way that net electric field at every interior point of body is zero. Thus if inside a metal body, a charge is displaced, no work is done in the process because electric field at every point is zero. Hence we can say that the whole metal body is equipotential at the inside.

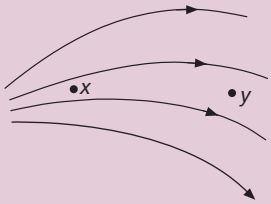
(c) For uniform electric field as shown in figure, we have

$$V_A - V_B = Ed \qquad \{\text{As } E = \text{constant}\}$$



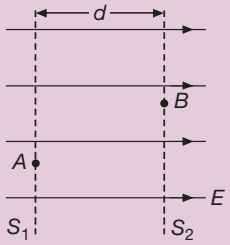
(d) For a non-uniform electric field like shown in figure, we have (between two points),

$$V_x - V_y = \int_x^y E\,dx$$



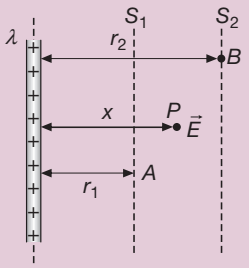
(e) Figure shows two equipotential surfaces in a uniform electric field E. If we have to calculate the potential difference between two points A and B as shown in figure, we simply find the potential difference between the two equipotential surfaces on which the points lie. So,

$$V_A - V_B = Ed$$



(f) Similarly, the figure shows a line charge with linear charge density λ. To find potential difference between points A and B which lie on equipotential surfaces S_1 and S_2, we find the potential difference between these surfaces and for that we consider a point P at a distance x from wire as shown. The electric field at point P is

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}$$



Now the potential difference between surface S_1 and S_2 is given by

$$V_B - V_A = -\int_{r_1}^{r_2} E\,dx$$

$$\Rightarrow \quad V_A - V_B = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 x}\,dx$$

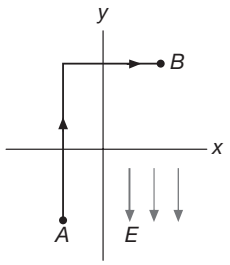
$$\Rightarrow \quad V_A - V_B = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_2}{r_1}\right)$$

Test Your Concepts-VIII

Based on Equipotential Surfaces

(Solutions on page H.30)

1. A uniform electric field of magnitude $325\ \text{Vm}^{-1}$ is directed in the negative y direction in figure. The coordinates of point A are $(-20,\ -30)$ cm, and those of point B are $(40,\ 50)$ cm. Calculate the potential difference $V_B - V_A$, along the path shown.

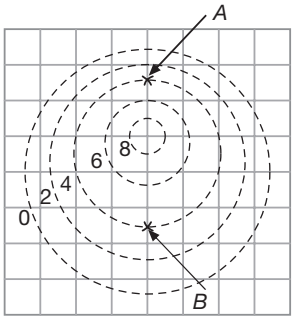


2. Figure shows several equipotential lines each labelled by its potential in volt. The distance between the lines of the square grid represents 2 cm.

(a) Where the magnitude of the field is larger, at A or at B? Explain.

(b) What is the electric field at B?

(c) Represent what the field looks like by drawing at least eight field lines in all directions of the page.



3. A point charge q is located at $x = -R$, and a point charge $-2q$ is located at the origin. Prove that the equipotential surface that has zero potential is a sphere centered at $\left(-\frac{4R}{3},\ 0,\ 0\right)$ and having a radius $r = \frac{2R}{3}$.

4. Three infinitely long linear charges of charge density λ, λ and -2λ are placed in space. A point in space is specified by its perpendicular distance r_1, r_2 and r_3 respectively from the linear charges. Prove that for the points which are equipotential, we have

$$\frac{r_1 r_2}{r_3^2} = \text{constant}$$

RELATION BETWEEN ELECTROSTATIC FIELD AND POTENTIAL

Since $dV = -\vec{E}\cdot d\vec{l}$

In cartesian coordinates, we have

$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ and $d\vec{l} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$, so we get

$$dV = -\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad dV = -\left(E_x dx + E_y dy + E_z dz\right)$$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y} \text{ and } E_z = -\frac{\partial V}{\partial z}$$

Now, by introducing a new differential quantity called the "del (gradient) operator $(\vec{\nabla})$", we get electrostatic field as the negative gradient of potential. So,

$$\vec{E} = -\vec{\nabla}V \qquad \ldots(1)$$

where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$$\Rightarrow \quad \vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

where

$$\frac{\partial V}{\partial x} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } x \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } x \text{ keeping } y \text{ and } z \text{ constant}\end{array}\right)$$

$$\frac{\partial V}{\partial y} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } y \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } y \text{ keeping } x \text{ and } z \text{ constant}\end{array}\right)$$

$$\frac{\partial V}{\partial z} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } z \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } z \text{ keeping } x \text{ and } y \text{ constant}\end{array}\right)$$

Just to make you understand the concept discussed here, I have a small problem discussed here for your comfort.

PROBLEM: Find the electrostatic field $\vec{E}$ for the potential $V = kxy$.

SOLUTION: Since

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right),$$

where $\frac{\partial V}{\partial x} = ky\left(\frac{\partial x}{\partial x}\right) = ky;\ \frac{\partial V}{\partial y} = kx\left(\frac{\partial y}{\partial y}\right) = kx$

and $\frac{\partial V}{\partial z} = 0$

$$\Rightarrow \quad \vec{E} = -k\left(y\hat{i} + x\hat{j}\right)$$

Notice that $\vec{\nabla}$ operates on a scalar quantity (electric potential) and results in a vector quantity (electric field). Mathematically, we can think of $\vec{E}$ as the negative of the gradient of the electric potential V. Physically, the negative sign implies that if V increases as a positive charge moves along some direction, say x, with $\frac{\partial V}{\partial x} > 0$, then there is a non vanishing component of $\vec{E}$ in the opposite direction $(-E_x \neq 0)$, because E always goes from higher V to lower V.

If the charge distribution possesses spherical symmetry, then the resulting electric field is a function of the radial distance r, i.e., $\vec{E} = E_r\hat{r}$. In this case, $dV = -E_r dr$. If $V(r)$ is known, then $\vec{E}$ may be obtained as

$$\vec{E} = E_r\hat{r} = -\left(\frac{dV}{dr}\right)\hat{r}$$

For example, the electric potential due to a point charge q is $V(r) = \frac{q}{4\pi\varepsilon_0 r}$. Using the above formula, the electric field is simply

$$\vec{E} = \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)\hat{r}$$

ILLUSTRATION 72

Suppose that the electric potential in some region of space is given by

$$V(x,\ y,\ z) = V_0\exp(-az)\cos ax,$$

where a is a positive constant. Find the electric field everywhere.

SOLUTION

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$E_x = -\frac{\partial V}{\partial x} = -V_0 \exp(-az)\frac{\partial}{\partial x}[\cos(ax)]$$

$$\Rightarrow \quad E_x = \frac{\partial V}{\partial x} = +aV_0 \sin(ax)\exp(-az)$$

$$E_y = \frac{\partial V}{\partial y} = 0$$

$\{\because$ the given expression has no dependence only$\}$

$$E_z = -\frac{\partial V}{\partial z} = -V_0 \cos(ax)\frac{\partial}{\partial z}\left[\exp(-az)\right]$$

$$\Rightarrow \quad E_z = -\frac{\partial V}{\partial z} = aV_0 \cos(ax)\exp(-az)$$

$$\Rightarrow \quad E = E_x\hat{i} + E_z\hat{k} \qquad \left\{\because E_y = 0\right\}$$

$$\Rightarrow \quad \vec{E} = aV_0 \exp(-az)\left[\sin(ax)\hat{i} + \cos(ax)\hat{k}\right]$$

ILLUSTRATION 73

Find the potential function $V(x, y)$ of an electrostatic field $\vec{E} = 2axy\,\hat{i} + a\left(x^2 - y^2\right)\hat{j}$ where a is a constant.

SOLUTION

Let V_0 be the potential at origin.

Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \int_{(0,0)}^{(x,y)} dV = -\int_{(0,0)}^{(x,y)}\left[2axy\,\hat{i} + a\left(x^2 - y^2\right)\hat{j}\right]\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\Rightarrow V(x,y) - V(0,0) = -\int_{(0,0)}^{(x,y)}\left(2axy\,dx + ax^2\,dy\right) - ay^2 dy$$

$$\Rightarrow \quad V(x, y) - V_0 = -\int_{(0,0)}^{(x,y)}\left\{d\left(ax^2y\right) - ay^2dy\right\}$$

$$\Rightarrow \quad V(x, y) - V_0 = \left[\left(-ax^2y + \frac{ay^3}{3}\right)\Bigg|_{(0,0)}^{(x,y)}\right]$$

$$\Rightarrow \quad V(x, y) = V_0 - ax^2y + \frac{ay^3}{3}$$

Remark(s)

(a) For an attractive system U is always **NEGATIVE**.
(b) For a repulsive system U is always **POSITIVE**.
(c) For a stable system U must be **MINIMUM**

i.e., $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

Since $F = -\frac{dU}{dx}$

$$\Rightarrow \quad F = -\frac{dU}{dx} = 0 \qquad \text{(FOR A STABLE SYSTEM)}$$

(d) If we are given $V(x, y, z)$, then

$$\vec{E} = -\vec{\nabla}V = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + k\frac{\partial V}{\partial z}\right)$$

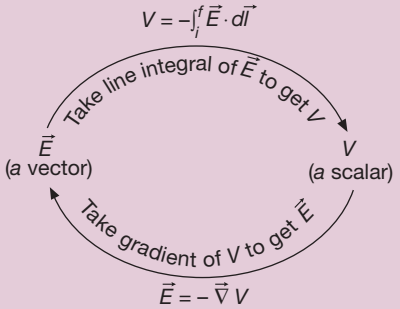
(e) If we are given $\vec{E}(x, y, z) = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$, then

$$V = -\int \vec{E}\cdot d\vec{\ell}$$

where, $d\vec{\ell} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad V = -\left[\int E_x dx + \int E_y dy + \int E_z dx\right]$$

The integrals are to be calculated within specified limits.



PROBLEM: An electrostatic field is given by $\vec{E} = -k\left(y\hat{i} + x\hat{j}\right)$. Find the electrostatic potential generating such a field.

SOLUTION: Since $V = -\int \vec{E}\cdot d\vec{r}$

Let $d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$

$$\Rightarrow \quad V = \int k\left(y\hat{i} + x\hat{j}\right)\cdot\left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$\Rightarrow \quad V = k\int y\,dx + x\,dy$$

$$\Rightarrow \quad V = k\int d(xy) \qquad \{\because d(xy) = x\,dy + y\,dx\}$$

$$\Rightarrow \quad V = k(xy) + \text{constant}$$

(f) Electrostatic potential at a point due to a positive charge is positive and due to a negative charge is negative.

(g) Electrostatic potential due an assembly of charges q_1, q_2, ..., q_n at a point P at distance r_1, r_2, ..., r_n respectively from the charges is given by

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

(h) If two points A and B are at potentials V_A and V_B, then work done in taking a test charge q_0 from A to B is $W_{A\to B} = q_0(V_B - V_A)$

(i) A positive charge always moves from higher potential to lower potential whereas a negative charge always moves from lower potential to higher potential.

(j) Commonly used unit of electrostatic energy is electronvolt (eV) and $1\,\text{eV} = 1.6\times10^{-19}$ J.

(k) If a charge q of mass m accelerates through a potential V, then velocity of q is calculated by using the Work-Energy Theorem, according to which work done equals the change in kinetic energy i.e.,

$$qV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

(l) Consider two points A and B situated in a uniform electric field at a distance d such that the line joining A and B is parallel to the field. If V be the potential difference between them, then

$$V = Ed(\text{in magnitude})$$

Test Your Concepts-IX

Based on Relation Between Electrostatic Field and Potential

(Solutions on page H.31)

1. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y and z components of the electric field over this region. Also calculate the magnitude of the field at the point P that has coordinates $(1,\ 0,\ -2)$ m?

2. When an uncharged conducting sphere of radius a is placed at the origin of an xyz cartesian coordinate system that lies in an initially uniform electric field $E = E_0\hat{k}$, the resulting electric potential is $V(x, y, z) = V_0$ for points inside the sphere and

$$V(x, y, z) = V_0 - E_o z + E_0 a^3 \frac{z}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y and z components of the resulting electric field.

3. In a region of space, the electric potential is represented by $V = 2x + 3y - z$. Obtain an expression for the electric field strength.

4. In a region of space the electric field is given by $\vec{E} = \left(x\hat{i} - 2y\hat{j} + z\hat{k}\right)\ \text{Vm}^{-1}$. Calculate the potential difference V_{AB} between A(2, 1, 0) m and B(0, 2, 4) m.

5. Find potential difference V_{AB} between $A(0, 0, 0)$ m and $B(1, 1, 1)$ m in an electric field given by

(a) $\vec{E} = y\hat{i} + x\hat{j}$

(b) $\vec{E} = 3x^2y\hat{i} + x^3\hat{j}$.

What can you say about the nature of the fields? Explain.

6. Determine the electric field strength vector if the potential of this field depends on x, y coordinates as

(a) $\varphi = a\left(x^2 - y^2\right)$

(b) $\varphi = axy$

where a is a positive constant. Draw the approximate shape of these fields using lines of force (in the x, y plane).

7. The potential of a certain electrostatic field has the form $V = a\left(x^2 + y^2\right) + bz^2$, where a and b are constants. Find the magnitude and direction of the electric field strength vector. What shape will the equipotential surface have for $a > 0,\ b > 0$?

8. Determine the potential $V(x, y, z)$ of an electrostatic field $E = ay\hat{i} + (ax + bz)\hat{j} + by\hat{k}$, where a and b are constants, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors of the axes x, y, z.

MOTION OF CHARGED PARTICLES AND CONSERVATION LAWS

For a variety of problems, we have to understand the Conservation Laws that can be applied to the electrostatic system of particles.

CASE-1: Law of Conservation of Linear Momentum

For an electrostatic system of charges, when no external force acts on the system, then we observe that no net internal forces will be acting on the system because electrostatic forces always form an action reaction pair. So, by a suitable selection of a system or subsystem we can apply Law of Conservation of Linear Momentum.

Total Initial Momentum = Total Final Momentum

CASE-2: Law of Conservation of Energy

Since electrostatic forces are conservative in nature, so, in the absence of an external force and a dissipative force (such as forces of friction) we can easily use this law as

$$\begin{pmatrix}\text{Total Initial Energy}\\ \text{of the system}\end{pmatrix} = \begin{pmatrix}\text{Total Final Energy}\\ \text{of the system}\end{pmatrix}$$

Here we need to read the problem carefully for the energies possessed by the system (as selected) initially and finally.

Another point which should be kept in mind is that electric field is a conservative field. Work done by electric field on a moving charge only depends on the positions of the charge. If a charge is freely moving in electric field, workdone by electric field on charge is equal to the change in kinetic energy of the charge i.e.,

$$W = \Delta K$$

In the presence of an external force and the dissipative non conservative force, the law will be suitably modified as below i.e.,

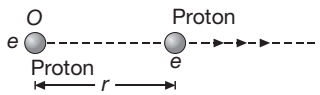
$$W_{\text{ext}} + W_{nc} = \Delta U + \Delta K$$

CASE-3: Law of Conservation of Angular Momentum

Since electrostatic forces are central in nature, so torque due to electrostatic force will be zero. So, we can easily use Law of Conservation of Angular Momentum too.

ILLUSTRATION 74

A proton is fixed at origin. Another proton is released from rest, from a point at a distance r from origin. Taking charge of proton as e and mass as m, find the speed of the proton (a) at a distance $2r$ from origin (b) at a large distance from origin.



SOLUTION

(a) The proton moves away under electrostatic repulsion. So, by Conservation of Energy, we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad \frac{e^2}{4\pi\varepsilon_0 r} + 0 = \frac{e^2}{4\pi\varepsilon_0 (2r)} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0 (2r)}$$

$$\Rightarrow \quad v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 rm}}$$

(b) Similarly, we have

$$U_i + K_i = U_\infty + K_\infty$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

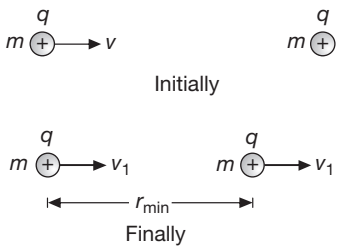
$$\Rightarrow \quad v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 rm}}$$

ILLUSTRATION 75

A charge q of mass m initially lying at infinity is projected head on with speed v toward another stationary particle of same mass and charge, initially at rest. Find the distance of closest approach of the two particles.

SOLUTION

As the moving charge comes closer to the charge initially at rest, due to electric repulsion it also starts moving and the first one starts retarding.



Since no external force is acting on system, total momentum remains conserved. The two charges get close to each other and their separation will be minimum when each will move with equal common speed say v_1. When their speeds are equal say v_1, then by Law of Conservation of Linear Momentum, we have

$$mv = mv_1 + mv_1$$

$$\Rightarrow \quad v_1 = \frac{v}{2}$$

To find the minimum separation between the two particles, using Law of Conservation of Energy,

Total Initial Energy = Total Final Energy

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 + \frac{q^2}{4\pi\varepsilon_0 r_{\min}}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = m\left(\frac{v}{2}\right)^2 + \frac{q^2}{4\pi\varepsilon_0 r_{\min}} \qquad \left\{\because\ v_1 = \frac{v}{2}\right\}$$

$$\Rightarrow \quad \frac{mv^2}{4} = \frac{q^2}{4\pi\varepsilon_0 r_{\min}}$$

$$\Rightarrow \quad r_{\min} = \frac{q^2}{\pi\varepsilon_0 mv^2}$$

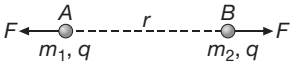
ILLUSTRATION 76

Two particles, each of charge q, and masses m_1 and m_2, are released from rest, from points A and B, r distance apart. Calculate the final relative velocity of separation of the particles at large separation from each other. (Neglect gravitational interaction of the particles)

SOLUTION

Both particles move under mutual electrostatic repulsion. Let v_1 and v_2 are final speed (when the separation becomes large). So, $v_{rel} = v_1 + v_2$

Since no external force is acting on particles, so by Law of Conservation of Linear Momentum, we have



$$-m_1v_1 + m_2v_2 = 0$$

$$\Rightarrow \quad m_1v_1 = m_2v_2 = p(\text{say})$$

By Law of Conservation of Mechanical Energy we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad \frac{q^2}{4\pi\varepsilon_0 r} + 0 + 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

$$\Rightarrow \quad \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{q^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad p^2 = \left(\frac{2m_1m_2}{m_1 + m_2}\right)\frac{q^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad p = \sqrt{\left(\frac{2m_1m_2}{m_1 + m_2}\right)\frac{q^2}{4\pi\varepsilon_0 r}}$$

$$\Rightarrow \quad v_1 = \frac{p}{m_1} = \frac{1}{m_1}\sqrt{\left(\frac{2m_1m_2}{m_1 + m_2}\right)\frac{q^2}{4\pi\varepsilon_0 r}}$$

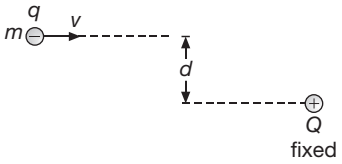
and $v_2 = \dfrac{p}{m_2} = \dfrac{1}{m_2}\sqrt{\left(\dfrac{2m_1m_2}{m_1 + m_2}\right)\dfrac{q^2}{4\pi\varepsilon_0 r}}$

$$\Rightarrow \quad v_r = v_1 + v_2$$

$$\Rightarrow \quad v_r = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 r}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

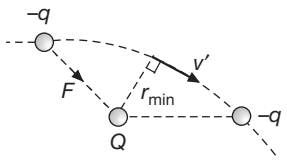
ILLUSTRATION 77

Figure shows a charge $+Q$ fixed at a position in space. From a large distance another charged particle of charge $-q$ and mass m is thrown with a speed v towards $+Q$ with an impact parameter d (it is the perpendicular distance between the velocities of the two particles) as shown. Find the distance of closest approach of the two particles.



SOLUTION

Here we can see that when $-q$ moves toward $+Q$, an attractive force acts on $-q$ toward $+Q$. Here as the line of action of force passes through the fix charge, no torque act on $-q$ relative to the fix point charge $+Q$, thus here we can say that with respect to $+Q$, the angular momentum of $-q$ must remain constant. The charge $-q$ will be closest to $+Q$ when it is moving perpendicularly to the line joining the two charges as shown.



If the closest separation in the two charges is r_{min} then, from conservation of angular momentum we have

$$mvd = mv'r_{min} \qquad \ldots(1)$$

Further, by Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r_{min}}$$

From (1), we get

$$v' = \frac{vd}{r_{min}}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}mv^2\left(\frac{d^2}{r_{min}^2}\right) - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r_{min}} \qquad \ldots(2)$$

$$\Rightarrow \quad r_{min}^2 + \left(\frac{2Qq}{4\pi\varepsilon_0 mv^2}\right)r_{min} - d^2 = 0$$

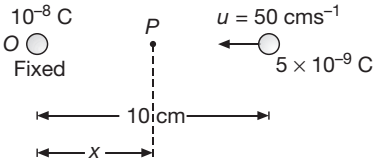
$$\Rightarrow \quad r_{min} = \frac{Qq}{2\pi\varepsilon_0 mv^2}\left[\sqrt{1+\frac{16\pi^2\varepsilon_0^2 m^2 v^4 d^2}{Q^2q^2}} - 1\right]$$

ILLUSTRATION 78

A particle of mass 40 mg having a charge of 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed positive point charge it has a velocity of 50 cms^{-1}. At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

SOLUTION

Let it come to momentary rest at point P at distance x from O. Then, by Law of Conservation of Energy.



$$(U+K)_{initial} = (U+K)_{final}$$

$$\Rightarrow \quad \frac{Qq}{4\pi\varepsilon_0 r_0} + \frac{1}{2}mu^2 = \frac{Qq}{4\pi\varepsilon_0 x} + \frac{1}{2}m(0)^2$$

where $Q = 10^{-8}$ C

$$q = 5\times10^{-9} \text{ C}$$

$$r_0 = 10 \text{ cm} = \frac{10}{100} \text{ m}$$

$$m = 40 \text{ mg} = \frac{40}{1000} \text{ g} = 40\times10^{-6} \text{ kg}$$

$$u = 50 \text{ cms}^{-1} = 0.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{9\times10^9\times5\times10^{-17}}{0.1} + \frac{1}{2}(40\times10^{-6})(0.5)^2 = \frac{9\times10^9\times5\times10^{-17}}{x}$$

$$\Rightarrow \quad 45\times10^{-7} + 50\times10^{-7} = \frac{45\times10^{-8}}{x}$$

$$\Rightarrow \quad x = \frac{45\times10^{-8}}{95\times10^{-8}} = 0.473 \text{ m}$$

$$\Rightarrow \quad x = 4.73 \text{ cm}$$

The force of repulsion increases as the particle approaches the fixed charge. So the acceleration is not constant during the motion.

ILLUSTRATION 79

Four point charges $+8\ \mu C$, $-1\ \mu C$, $-1\ \mu C$ and $+8\ \mu C$ are fixed at the points $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the Y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\ \mu C$ moves along the $-x$ direction. Its speed at $x\to\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free.

SOLUTION

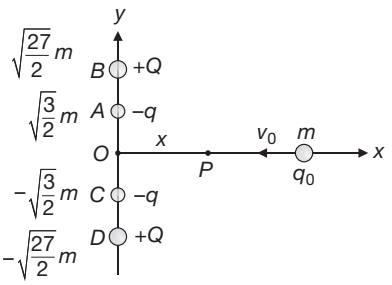
In the figure,

$$q = 1\ \mu C = 10^{-6}\ C,$$

$$q_0 = +0.1\ \mu C = 10^{-7}\ C,$$

$$m = 6\times10^{-4}\ \text{kg}$$

and $\quad Q = 8\ \mu C = 8\times10^{-6}\ C$



Let P be any point at a distance x from origin O. Then

$$AP = CP = \sqrt{\frac{3}{2}+x^2}$$

$$BP = DP = \sqrt{\frac{27}{2}+x^2}$$

Electric potential at point P will be

$$V = \frac{2Q}{4\pi\varepsilon_0(BP)} - \frac{2q}{4\pi\varepsilon_0(AP)}$$

$$\Rightarrow \quad V = 2\times9\times10^9\left[\frac{8\times10^{-6}}{\sqrt{\frac{27}{2}+x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2}+x^2}}\right]$$

$$\Rightarrow \quad V = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}+x^2}} - \frac{1}{\sqrt{\frac{3}{2}+x^2}}\right] \qquad \ldots(1)$$

Since, electric field at P is given by $E = -\frac{dV}{dx}$, so

$$E = -1.8\times10^4\left[(8)\left(-\frac{1}{2}\right)\left(\frac{27}{2}+x^2\right)^{-\frac{3}{2}} - (1)\left(-\frac{1}{2}\right)\left(\frac{3}{2}+x^2\right)^{-\frac{3}{2}}\right](2x)$$

So, $E = 0$ on x-axis either at $x = 0$ or when

$$\frac{8}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \frac{(4)^{3/2}}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \left(\frac{27}{2}+x^2\right) = 4\left(\frac{3}{2}+x^2\right)$$

This equation gives $x = \pm\sqrt{\frac{5}{2}}$ m

The least value of kinetic energy of the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m because at $x = +\sqrt{\frac{5}{2}}$ m, $E = 0$, so electrostatic force on charge q_0 is zero or $F_e = 0$.

For at $x > \sqrt{\frac{5}{2}}$ m, E is repulsive (towards positive x-axis)

and for $x < \sqrt{\frac{5}{2}}$ m, E is attractive (towards negative x-axis)

Now, from equation (1), potential at $x = \sqrt{\frac{5}{2}}$ m

$$V = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}+\frac{5}{2}}}-\frac{1}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]$$

$\Rightarrow \quad V = 2.7\times10^4$ V

Applying energy conservation at $x\to\infty$ and $x = \sqrt{\frac{5}{2}}$ m

$$\frac{1}{2}mv_0^2 = q_0V \qquad \ldots(2)$$

$\Rightarrow \quad v_0 = \sqrt{\frac{2q_0V}{m}}$

Substituting the values $v_0 = \sqrt{\frac{2\times10^{-7}\times2.7\times10^4}{6\times10^{-4}}}$

$v_0 = 3$ ms^{-1}

$\therefore$ Minimum value of v_0 is 3 ms^{-1}

From equation (1), potential at origin $(x = 0)$ is

$$V_0 = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}}}-\frac{1}{\sqrt{\frac{3}{2}}}\right] = 2.4\times10^4 \text{ V}$$

Let T be the kinetic energy of the particle at origin. Applying energy conservation at $x = 0$ and at $x = \infty$

$$T + q_0V_0 = \frac{1}{2}mv_0^2$$

But $\frac{1}{2}mv_0^2 = q_0V$ {from equation (2)}

$\Rightarrow \quad T = q_0(V - V_0)$

$\Rightarrow \quad T = (10^{-7})(2.7\times10^4 - 2.4\times10^4)$

$\Rightarrow \quad T = 3\times10^{-4}$ J

Please note here that, $E = 0$ or F on q_0 is zero at $x = 0$ and $x = \pm\sqrt{\frac{5}{2}}$ m. Of these $x = 0$ is stable equilibrium position and $x = \pm\sqrt{\frac{5}{2}}$ m is unstable equilibrium position.

Test Your Concepts-X

Based on Motion of Charged Particles in Electric Field

(Solutions on page H.32)

1. Consider an electron to be released from rest in a uniform electric field whose magnitude is 6 kVm^{-1}. Find,

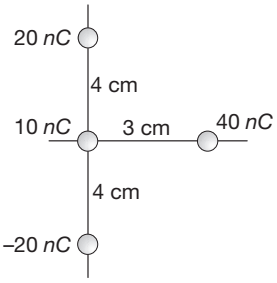
(a) through what potential difference will it have passed after moving 1 cm?

(b) how fast will the electron be moving after it has travelled 1 cm?

2. **(a)** Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V.

(b) Calculate the speed of an electron that is accelerated through the same potential difference.

3. Two particles, having charges 20 nC and –20 nC, are placed at the points with coordinates (0, 4) cm and (0, –4) cm, as shown. A particle with charge 10 nC is located at the origin.



(a) Find the electric potential energy of the configuration of the three fixed charges.

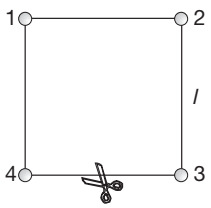
(b) A fourth particle, with a mass of 2×10^{-13} kg and a charge of 40 nC, is released from rest at the point (3, 0) cm. Find its speed after it has moved freely to a very large distance away.

4. In 1911 Ernest Rutherford and his assistants Geiger and Marsden conducted an experiment in which they scattered alpha particles from thin sheets of gold. An alpha particle, having charge $+2e$ and mass 6.67×10^{-27} kg, is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of the mass of an atom is in a very small nucleus, with electrons in orbit around it his planetary model of the atom. Assume an alpha particle, initially very far from a gold nucleus, is fired with a velocity of 2×10^{7} ms^{-1} directly toward the nucleus (charge $+79e$). How close does the alpha particle get to the nucleus before turning around? Assume the gold nucleus remains stationary.

5. Four identical particles each have charge q and mass m. They are released from rest at the vertices of a square of side L. How fast is each charge moving at the instant when their distance from the center of the square doubles?

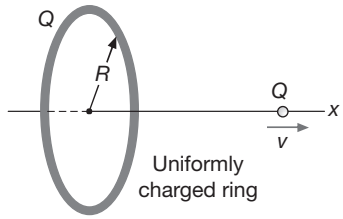
6. An electron is released from rest on the axis of a uniform positively charged ring, 0.1 m from the ring's center. If the linear charge density of the ring is $0.1\,\mu$Cm^{-1} and the radius of the ring is 0.2 m, how fast will the electron be moving when it reaches the center of the ring?

7. Four balls, each with mass m, are connected by four non-conducting strings to form a square with side ℓ, as shown in Figure. The assembly is placed on a horizontal nonconducting frictionless surface. Balls 3 and 4 each have charge q, and balls 1 and 2 are uncharged. Find the maximum speed of balls 3 and 4 after the string connecting them is cut.

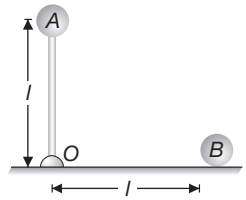


8. The x-axis is the symmetry axis of a stationary uniformly charged ring of radius R and charge Q.
A point charge Q of mass M is located initially at the center of the ring. When it is displaced slightly, the point charge accelerates along the x-axis to infinity. Show that the ultimate speed of the point charge is

$$v=\sqrt{\frac{Q^2}{2\pi\varepsilon_0 MR}}$$



9. Figure shows a small ball A of mass m and charge q connected at one end of light rod of length ℓ. Another identical ball B is placed on smooth ground at a distance ℓ from the point O, where the rod is clamped. A slight jerk on ball A makes the rod start rotating clockwise. At the instant when A strikes the ground, ball B is moving with a velocity u towards right and is at a position 2ℓ away from the point O. Calculate the kinetic energy of the ball A when it strikes the floor.



10. A charged particle of charge $+q$, mass m is thrown towards another fixed charge $+Q$ from a very large distance away with speed v_0 with an impact parameter ℓ. Find the distance of closest approach of the two charges.

11. The potential difference between two large parallel plates is varied as $V=at$, where a is a constant and t is time. An electron starts from rest at $t=0$ from the plate which is at negative potential. If the distance between the plates is ℓ, mass of electron is m and charge on electron is $-e$ then find the velocity of the electron when it reaches the other plate.

12. A point charge $-q$ revolves around a fixed charge $+Q$ in elliptical orbit. The minimum and maximum distance of q from Q are r_1 and r_2, respectively. The

mass of revolving particle is m. $Q > q$ and assume no gravitational effects. Find the velocity of q at positions when it is at a distance r_1 and r_2 from Q.

13. A proton of mass m approaches a free proton from a very large distance with a velocity v_0 along the straight line joining their centres. Calculate the distance of closest approach between the two protons.

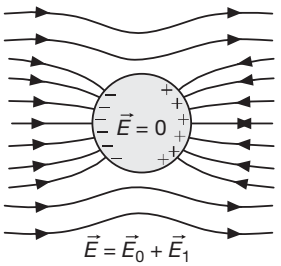
CONDUCTORS AND THEIR PROPERTIES

Insulators such as glass or paper are materials in which electrons are attached to some particular atoms and cannot move freely. On the other hand conductors (such as metals) have electrons capable of moving around freely. If a resultant electric field exists in the conductor then free charges will experience a force which will set a current flow. When no current flows, the resultant force and the electric field must be zero. Thus, under electrostatic conditions the value of $\vec{E}$ at all points within a conductor is zero. This idea, together with the Gauss's Law can be used to prove several interesting facts regarding a conductor.

The Electric Field is Zero Inside a Conductor

If we place a solid spherical conductor in a constant external field $\vec{E}_0$, the positive and negative charges will move toward the polar regions of the sphere (the regions on the left and right of the sphere in figure shown), thereby inducing an electric field $\vec{E}_i$. Inside the conductor, $\vec{E}_i$ points in the direction opposite to $\vec{E}_0$. Since charges are mobile, they will continue to move until $\vec{E}_i$ completely cancels $\vec{E}_0$ inside the conductor. At electrostatic equilibrium, $\vec{E}$ must vanish inside a conductor. Outside the conductor, the electric field $\vec{E}_i$ due to the induced charge distribution corresponds to a dipole field, and the total electric field is simply the resultant of both, so $\vec{E} = \vec{E}_0 + \vec{E}_i$. The field lines are depicted in figure.

Thus if a metal body is placed in an electric field, charge induction on the body surface starts and the continuous flow of electrons takes place inside the metal body (conductor) till the net electric field inside the body becomes zero.



Placing a conductor in a uniform electric field $\vec{E}$

Hence $E - E_i = 0$

Thus we can say that when all charges in a metal body are at rest, net electric field inside it is zero or the electric field inside the body due to the charges on its surface always balances the external electric field in the body. This is the condition of ELECTROSTATIC EQUILIBRIUM.

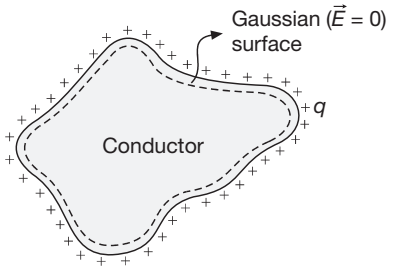
Net Excess Charge on a Conductor Resides on Its Outer Surface

Consider a charged conductor carrying a charge q and no currents are flowing in it i.e., in electrostatic equilibrium. Now consider a Gaussian surface inside the conductor everywhere on which $\vec{E} = 0$ (because it lies inside the conductor). Since, according to Gauss's Law,

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Since $\vec{E} = \vec{0}$, so $q_{enc} = 0$ (because it lies inside the conductor)

Thus, the sum of all charges inside the Gaussian surface is zero.



This Gaussian surface has been taken just inside the surface of the conductor. So, any charge on the conductor must reside on the surface of the conductor. In other words,

"Under electrostatic conditions (condition of electrostatic equilibrium) the excess charge on a conductor resides on its outer surface".

The Tangential Component of $\vec{E}$ is Zero on the Surface of a Conductor

We have already seen that for an isolated conductor, the electric field is zero in its interior. Any excess charge placed on the conductor must then distribute itself on the surface, as implied by Gauss's Law.

Consider the line integral $\oint \vec{E}\cdot d\vec{l}$ around a closed path $abcda$ shown in figure.

Since the electric field $\vec{E}$ is conservative, the line integral around the closed path $abcda$ must be zero.

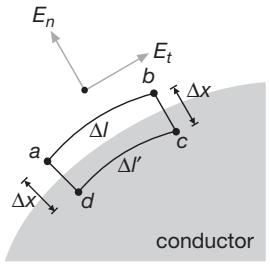
$$\Rightarrow \quad \oint_{abcda} \vec{E}\cdot d\vec{l} = E_t(\Delta l) + E_n(\Delta x)\cos(180) +$$

$$0(\Delta l) + E_n(\Delta x)\cos 0 = 0$$

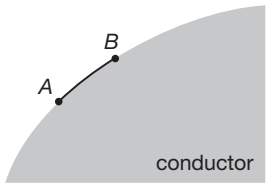
$$\Rightarrow \quad \oint_{abcda} \vec{E}\cdot d\vec{l} = E_t(\Delta l) = 0$$

where E_t and E_n are the tangential and the normal components of the electric field, respectively. We have oriented the infinitesimal segment ab so that it is parallel to E_t. Also here we take $ad = bc$. If we take $ad \neq bc$ then too since we have taken an infinitesimal loop $abcda$ so in the limit where both Δx and $\Delta x' \to 0$, we will have $E_t \Delta l = 0$. However, since the length element Δl is finite, we conclude that the tangential component of the electric field on the surface of a conductor vanishes.

$$E_t = 0 \text{ (on the surface of a conductor)}$$



This implies that the surface of a conductor in electrostatic equilibrium is an equipotential surface. Consider two points A and B on the surface of a conductor as shown,



Since the tangential component $E_t = 0$, the potential difference is

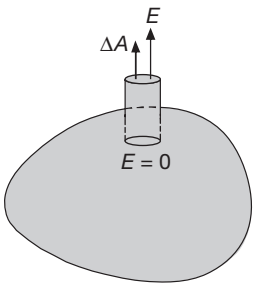
$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l} = 0$$

because $\vec{E}$ is perpendicular to $d\vec{l}$. Thus, points A and B are at the same potential with $V_A = V_B$.

Electric Field at any Point Close to the Charged Conductor is σ/ε_0

Consider a charged conductor of irregular shape. In general, surface charge density will vary from place to place. At a small surface ΔA, let us assume it to be a constant σ. Now construct a Gaussian surface in the form of a cylinder also called as Gaussian Pill Box of cross-section ΔA. One plane face of the cylinder is inside the conductor and other outside the conductor close to it. The surface inside the conductor does not contribute to the flux as $\vec{E}$ is zero everywhere inside the conductor. The curved surface outside the conductor also does not contribute to flux as $\vec{E}$ is always normal to the charged conductor and hence

parallel to the curved surface. Thus the only contribution to the flux is through the plane face outside the conductor.



So, from Gauss's Law,

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

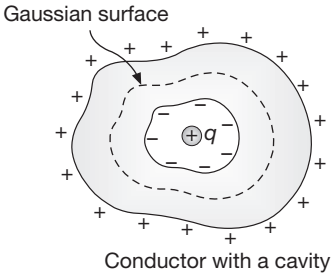
$$\Rightarrow \quad E\Delta A = \frac{(\sigma)(\Delta A)}{\varepsilon_0}$$

$$\Rightarrow \quad E = E_n = \frac{\sigma}{\varepsilon_0}$$

(This result holds good for a conductor of any arbitrary shape)

Conductor with Charge Inside a Cavity

Consider a hollow conductor shown in figure. Suppose the net charge carried by the conductor is $+Q$. In addition, there is a charge q inside the cavity. What is the charge on the outer surface of the conductor?



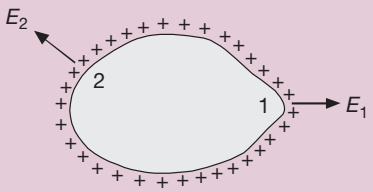
Since the electric field inside a conductor must be zero, the net charge enclosed by the Gaussian surface shown in figure must be zero. This implies that a charge $-q$ must have been induced on the cavity surface. Since the conductor itself has a charge $+Q$, the amount of charge on the outer surface of the conductor must be $Q+q$. (Think about the concept of charge conservation to get this as an answer).

The Potential of a Charged Conductor Throughout Its Volume is Same

In any region in which $\vec{E}=0$ at all points, such as the region vary far from all charges or the interior of a charged conductor, the line integral of $\vec{E}$ is zero along any path. It means that any two points in the conductor are at the same potential or have zero potential difference or the interior of a charged conductor is an equipotential region.

Remark(s)

(a) Electric field changes discontinuously at the surface of a conductor. Just inside the conductor it is zero and just outside the conductor it is $\frac{\sigma}{\varepsilon_0}$. In fact, the field gradually decreases from $\frac{\sigma}{\varepsilon_0}$ to zero in a small thickness of about 4 to 5 atomic layers at the surface.



(b) For a nonuniform conductor the surface charge density (σ) varies inversely as the radius of curvature (ρ) of that part of the conductor, i.e.,

$$\sigma \propto \frac{1}{\text{radius of curvature } (\rho)}$$

For case in the figure, $\rho_1 < \rho_2$

$$\Rightarrow \quad \sigma_1 > \sigma_2$$

$$\Rightarrow \quad E_1 > E_2 \qquad \left\{\because E = \frac{\sigma}{\varepsilon_0}\right\}$$

ILLUSTRATION 80

Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.

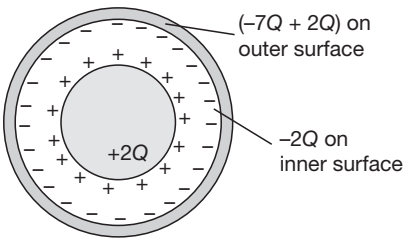
(a) Calculate the charge on the outer and inner surface of the outer sphere.

(b) If a wire is connected between the inner and outer spheres, after electrostatic equilibrium is established, how much total charge is on the outside sphere? How much charge is on the outer and inner surface of outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected?

(c) Now, if we return to the original condition as specified in (a) and connect the outer sphere to the ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner and outer surface of the outer sphere?

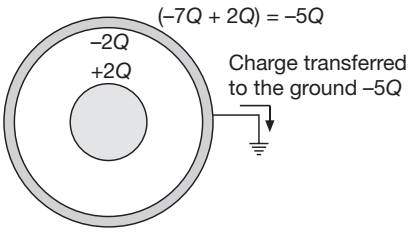
SOLUTION

(a) The charges on the inner sphere induce equal magnitude of charge, but opposite in sign, on the inner surface of outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must come on the outer surface. Thus outer and inner surface of outer sphere have charges $-5Q$ and $-2Q$ respectively.



(a)

(b) In electrostatic equilibrium, charge does not reside inside a conductor. So, when outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from inner sphere.



(b)

Total charge on outer surface of outer sphere is $-5Q$ and the total charge on inner surface is zero.

The electric field at the surface of the inner sphere goes to zero after connection. Consider a Gaussian surface just on the surface of inner sphere. So, according to Gauss's Law

$$\phi_E = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

$$(\text{as } Q_{\text{enclosed}} = 0)$$

$$\Rightarrow \quad E = 0$$

(c) When the outer sphere is grounded the charge on the surface is transferred to ground, thus charge is reduced to zero. The final charge distribution is shown in figure (b).

ILLUSTRATION 81

A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's Law to find

(a) the charge per unit length on the inner and outer surfaces of the cylinder and

(b) the electric field outside the cylinder, a distance r from the axis.

SOLUTION

(a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside $\frac{\text{charge}}{\text{length}} = -\lambda$

$$0 = \lambda l + q_{in}$$

So $\frac{q_{in}}{l} = -\lambda$

$\Rightarrow \quad 2\lambda l = q_{in} + q_{out}$

Outside surface: The total charge on the metal cylinder is $q_{out} = 2\lambda l + \lambda l$ so the outside $\frac{\text{charge}}{\text{length}}$ is 3λ

(b) $E = \frac{3\lambda}{2\pi \in_0 r}$ radially outward $\left\{\because E = \frac{\lambda_{enc}}{2\pi\varepsilon_0 r}\right\}$

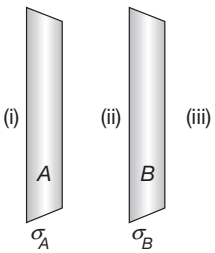
Test Your Concepts-XI

Based on Conductors

(Solutions on page H.35)

1. Two identical conducting spheres each having a radius of 0.5 cm are connected by a light 2 m long conducting wire. A charge of 60 μC is placed on one of the conductors. Assume that the surface distribution of charge on each sphere is uniform. Determine the tension in the wire.

2. Two infinite parallel planes carry equal but opposite uniform charge densities σ_A and σ_B (shown in figure). Find the field in each of the three regions:



(i) to the left of both
(ii) between them
(iii) to the right of both.
Also find the field in the three regions for $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$.

3. In a particular region of air at an altitude of 500 m above the ground the electric field is found to be 120 NC^{-1} directed downwards. At 600 m above the ground the electric field is 100 NC^{-1} downwards. Calculate the average volume charge density in the layer of air between these two elevations. Is it positive or negative?

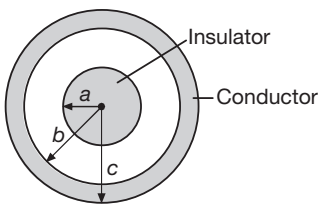
4. The electric field on the surface of an irregularly shaped conductor varies from 56 kNC^{-1} to 112 kNC^{-1}. Calculate the local surface charge density at the point on the surface where the radius of curvature of the surface is
(a) greatest and
(b) smallest

5. A conducting spherical shell of inner radius *a* and outer radius *b* carries a net charge *Q*. A point charge *q* is placed at the center of this shell. Determine the surface charge density on
(a) the inner surface of the shell and
(b) the outer surface of the shell.

6. A hollow conducting sphere is surrounded by a larger concentric spherical conducting shell. The inner sphere has charge –*Q*, and the outer shell has net charge +3*Q*. The charges are in electrostatic equilibrium. Using Gauss's Law, find the charges and the electric fields everywhere.

7. A positive point charge is at a distance $\frac{R}{2}$ from the center of an uncharged thin conducting spherical shell of radius *R*. Sketch the electric field lines set up by this arrangement both inside and outside the shell.

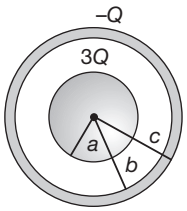
8. A solid, insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are *b* and *c*, as shown in figure.



(a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b$, $b < r < c$, and $r > c$.

(b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.

9. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Figure



(a) Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface.

(b) Find the magnitude and the direction of electric field at $r > c$.

(c) Find the electric field in the region with radius r where $b < r < c$.

(d) Construct a spherical gaussian surface of radius r, where $b < r < c$, and find the net charge enclosed by this surface.

(e) Construct a spherical gaussian surface of radius r, where $a < r < b$, and find the net charge enclosed by this surface.

(f) Find the electric field in the region $a < r < b$.

(g) Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r.

(h) Find the electric field in the region $r < a$.

(i) Determine the charge on the inner surface of the conducting shell.

(j) Determine the charge on the outer surface of the conducting shell.

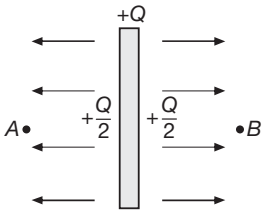
(k) Make a plot of the magnitude of the electric field versus r.

CHARGE DISTRIBUTION ON A SYSTEM OF PARALLEL PLATES

CASE-1:

Consider a large conducting plate that has been given a charge $+Q$. Due to symmetry, this charge is distributed equally on both sides of its surfaces as shown. If surface area of each face is A, charge density on the surface is given by

$$\sigma = \frac{Q}{2A}$$

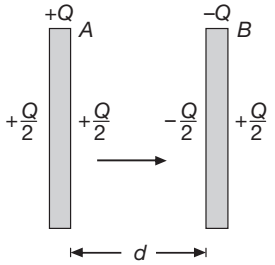


The electric field, on both sides of the plate, at points A and B is given by

$$E_A = E_B = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

CASE-2:

Now, if another uncharged plate is placed in front of this charged sheet as shown in figure. On the sides facing each other, plate of this plate an equal and opposite charge $-\frac{Q}{2}$ is induced and on the back surface $+\frac{Q}{2}$ is induced. The electric field, between the plates is $\frac{\sigma}{\varepsilon_0}$, where σ is the charge density of the surface. In figure the direction of electric field in different region is indicated by arrows.



Now we find the potential difference between the two plates A and B as

$$V_A - V_B = \frac{\sigma}{\varepsilon_0} d$$

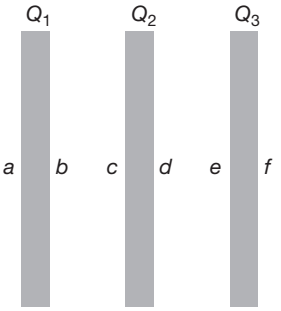
{As electric field is uniform between the plates}

$$\Rightarrow \quad V_A - V_B = \frac{Q}{2A\varepsilon_0} d \qquad \left\{\because \sigma = \frac{Q}{2A}\right\}$$

CASE-3:

Consider three parallel metallic plates each of area A which are kept as shown and charges Q_1, Q_2 and Q_3 are given to them. Let us find the resulting charge distribution on the six surfaces. Neglecting edge effects as usual, we observe that the plate separations do not affect the distribution of charge here. Let the plates 1, 2 and 3 have charges Q_1, Q_2 **and** Q_3**.** If we assume the charge on side a to be Q_a on the side c to be Q_c and that on the side e to be Q_e, then we have the charges on the left out surfaces given by

$$Q_b = Q_1 - Q_a\,,\ Q_d = Q_2 - Q_c \text{ and } Q_f = Q_3 - Q_e\,.$$



Electric field at point P is zero

$$E_P = 0$$

At P, charge Q_a will give an electric field towards right. All other charges Q_b, Q_c …, etc., will give the electric field towards left. So,

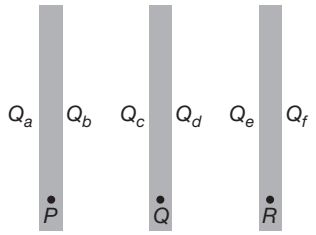
$$\frac{\left[Q_a - (Q_1 - Q_a) - Q_c - (Q_2 - Q_c) - Q_e - (Q_3 - Q_e)\right]}{2\varepsilon_0 A} = 0$$

$$\Rightarrow \quad 2Q_a - Q_1 - Q_2 - Q_3 = 0$$

$$\Rightarrow \quad Q_a = \frac{Q_1 + Q_2 + Q_3}{2}$$

Similarly the condition, $E_R = 0$ will give the result,

$$Q_f = \frac{Q_1 + Q_2 + Q_3}{2}$$



From here we may conclude that, half of the sum of all charges appears on each of the two outermost surfaces of the system of plates.

Further we have a condition,

$$E_Q = 0$$

$$\frac{\left[Q_a + (Q_1 - Q_a) + Q_c - (Q_2 - Q_c) - Q_e - (Q_3 - Q_e)\right]}{2\varepsilon_0 A} = 0$$

$$\Rightarrow \quad Q_1 + 2Q_c - Q_2 - Q_3 = 0$$

$$\Rightarrow \quad Q_c = \frac{Q_2 + Q_3 - Q_1}{2}$$

and $Q_b = Q_1 - Q_a = \dfrac{Q_1 - Q_2 - Q_3}{2} = -Q_c$

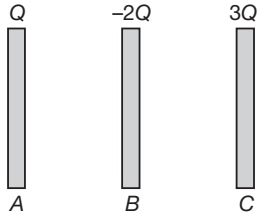
Similarly, we can show that, $Q_d = -Q_e$.

From here we can find another important result that the pairs of opposite surfaces like b, c and d, e carry equal and opposite charges and the outer surfaces carry equal charges.

The properties can often be applied to similar problems, provided additional conditions or constraints are not applied. These conditions (or constraints) can be in the form of some plates being connected together by conducting wires or some plates being held at fixed potentials through the use of batteries.

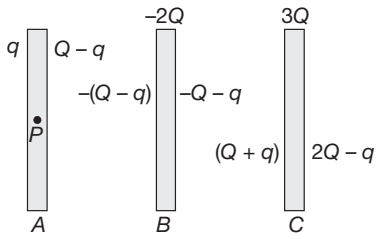
ILLUSTRATION 82

Consider a system of parallel plates A, B and C. The plates are given charges Q, $-2Q$ and $3Q$ respectively. Find the distribution of charge on the faces of the three plates.



SOLUTION

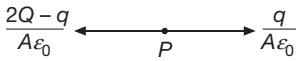
Let us assume that after final distribution of charge on the left face of plate A, charge is q then on right side of it remaining charge will be $(Q-q)$ as shown. Always remember that "In a system of large parallel conducting plates, the facing surfaces of plates always carry equal and opposite charges".



So, using this concept, we distribute the charges on the plates as shown in figure. Let us consider a point P inside the plate A. The net electric field at point P must be zero as it is inside a conducting body. This fact helps us to determine the charge q and hence the complete charge distribution.

Also, we observe that the electric field at point P is only due to the left face of A and right face of C, because all other charges produces electric field at point P in equal and opposite direction

Thus we have electric field at point P as



Since the point P lies inside a conductor, so net electric field at point P has to be zero.

$$\Rightarrow \quad \frac{2Q-q}{A\varepsilon_0}-\frac{q}{A\varepsilon_0}=0$$

$$\Rightarrow \quad q=Q$$

So, using this concept, we calculate the final distribution of charges on the system of parallel plates shown in figure.

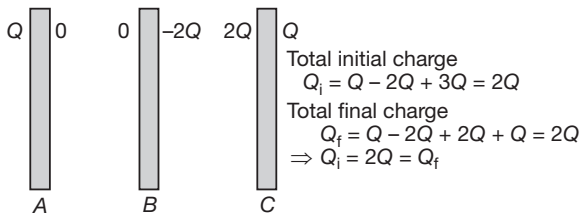
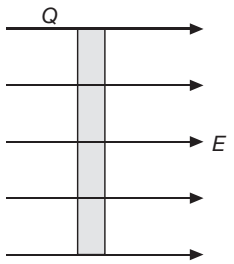


ILLUSTRATION 83

An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers the entire sheet. Find the charges appearing on the two surfaces of the sheet.

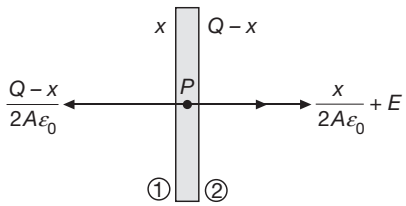


SOLUTION

Let charge on left side of plate be x, then $(Q-x)$ is charge on right side of plate.

Since inside the plate, at point P, we have

$$E_P = 0$$



$$\Rightarrow \quad \frac{x}{2A\varepsilon_0}+E=\frac{Q-x}{2A\varepsilon_0}$$

$$\Rightarrow \quad \frac{x}{A\varepsilon_0}=\frac{Q}{2A\varepsilon_0}-E$$

$$\Rightarrow \quad x=\frac{Q}{2}-\varepsilon_0 AE \text{ and } (Q-x)=\frac{Q}{2}+\varepsilon_0 AE$$

So charge on one side is $\left(\frac{Q}{2}-EA\varepsilon_0\right)$ and other side $\left(\frac{Q}{2}+EA\varepsilon_0\right)$

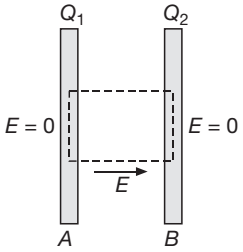
If $Q=0$ i.e. the sheet is neutral, then a charge $-\varepsilon_0 AE$ and $+\varepsilon_0 AE$ appear on the two surfaces of conducting sheet.

ILLUSTRATION 84

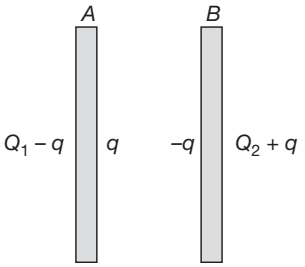
Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove that the charges on the inner facing surfaces are of equal magnitude and opposite sign. Also calculate the charges on the inner and outer surfaces of the plates.

SOLUTION

Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.



The distribution of charges is shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Since we have $E = \frac{q}{2\varepsilon_0 A}$, So at P



field due to charge $Q_1 - q$ is $\frac{Q_1 - q}{2A\varepsilon_0}$ (downwards)

field due to charge $+q$ is $\frac{q}{2A\varepsilon_0}$ (upwards)

field due to charge $-q$ is $\frac{q}{2A\varepsilon_0}$ (downwards)

and field due to charge $Q_2 + q$ is $\frac{Q_2 + q}{2A\varepsilon_0}$ (upwards)

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$E_p = \frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

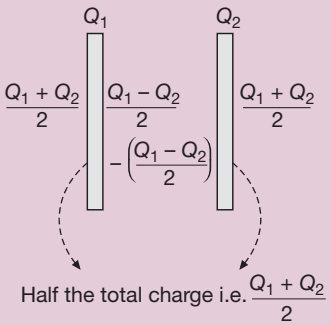
As the point P lies inside the conductor so, this field should be zero. Hence,

$$Q_1 - q - q + q - Q_2 - q = 0$$

$$\Rightarrow \quad q = \frac{Q_1 - Q_2}{2}$$

Problem Solving Technique(s)

So, when charged conducting plates are placed parallel to each other, then the outermost surfaces get equal charge (that is exactly half the total charge on the plates) and the plate surfaces facing each other have equal and opposite charge.



For Example:

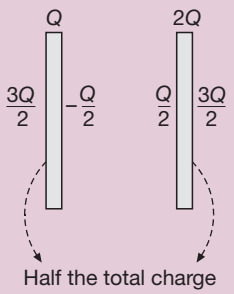
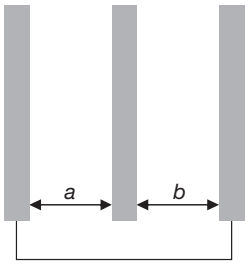


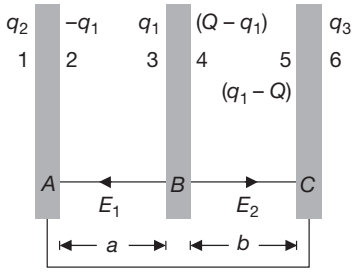
ILLUSTRATION 85

Three identical metallic plates are kept parallel to one another at a separation of a and b. The outer plates are connected by a thin conducting wire and a charge Q is placed on the central plate. Find the final charges on all the six faces of the plates shown.



SOLUTION

Let the charge distribution in all the six faces be as shown in figure. While distributing the charge on different faces, we have used the fact that two opposite faces have equal and opposite charges on them.



Net charge on plates A and C is zero. Hence,

$$q_2 - q_1 + q_3 + (q_1 - Q) = 0$$

$$\Rightarrow \quad q_2 + q_3 = Q \qquad \ldots(1)$$

Further A and C are at same potentials, if E_1 be the field between the plates A and B and that between B and C is E_2, then,

$$V_B - V_A = V_B - V_C$$

$$\Rightarrow \quad E_1 a = E_2 b$$

$$\Rightarrow \quad \left(\frac{q_1}{A\varepsilon_0}\right)a = \left(\frac{Q-q_1}{A\varepsilon_0}\right)b \qquad \{A = \text{Area of plates}\}$$

$$\Rightarrow \quad q_1 a = (Q - q_1)b$$

$$\Rightarrow \quad q_1 = \frac{Qb}{a+b} \qquad \ldots(2)$$

Electric field inside any conducting plate (say inside C) is zero. Therefore,

$$\frac{q_2}{2A\varepsilon_0} - \frac{q_1}{2A\varepsilon_0} + \frac{q_1}{2A\varepsilon_0} + \frac{Q-q_1}{2A\varepsilon_0} + \frac{q_1 - Q}{2A\varepsilon_0} - \frac{q_3}{2A\varepsilon_0} = 0$$

$$\Rightarrow \quad q_2 - q_3 = 0 \qquad \ldots(3)$$

Solving equations, (1), (2) and (3), we get

$$q_1 = \frac{Qb}{a+b},\ q_2 = q_3 = \frac{Q}{2}$$

Hence, charge on different faces are as follows :

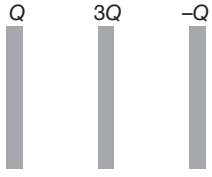
Face	Charge	Face	Charge
1	$q_2 = \frac{Q}{2}$	4	$(Q - q_1) = \frac{Qa}{a+b}$
2	$-q_1 = -\frac{Qb}{a+b}$	5	$(q_1 - Q) = -\frac{Qa}{a+b}$
3	$q_1 = \frac{Qb}{a+b}$	6	$q_3 = \frac{Q}{2}$

Test Your Concepts-XII

Based on Charge Distribution

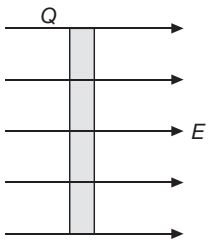
(Solutions on page H.37)

1. Figure shows three metallic plates with charges $-Q$, $+3Q$ and Q respectively. Determine the final charges on all the surfaces.

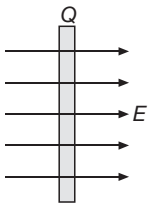


2. Prove that if an isolated large conducting sheet (isolated means no charges are near the sheet) is given a charge, then the charge distributes equally on its two surfaces.

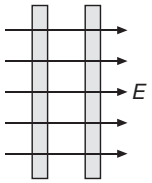
3. An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers the entire sheet. Find the charges appearing on the two surfaces of the sheet. Also calculate the charge appearing on each face of the sheet if the sheet is neutrals.



4. An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Calculate the resultant electric field on the left and right side of the plate.



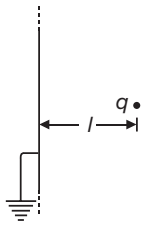
5. Two uncharged parallel conducting sheets each of area A are placed in a uniform electric field E at a finite distance from each other such that electric field is perpendicular to sheets and covers the entire sheets arrangement. Find out charges appearing on its two surfaces.



6. There are two large parallel metallic plates S_1 and S_2 carrying surface charge densities σ_1 and σ_2 respectively $(\sigma_1 > \sigma_2)$ placed at a distance d apart in vacuum. Find the work done by the electric field in moving a point charge q a distance $a(a < d)$ from S_1 towards S_2 along a line making an angle $\frac{\pi}{4}$ with the normal to the plates.

METHOD OF IMAGES

This method can be used for Boundary Value Problems in which we are asked to find electrostatic potential which results from a given charge configuration in the neighbourhood of an array (arrangement) of grounded conductors in the region outside the conductors. **Here, by Grounded Conductors we mean that conductors along the surface of which electrostatic potential V is set to zero.**



Such problems can be solved by reducing to an equivalent simpler problem. We begin by "removing" the conductor(s) and then introducing a new auxiliary system of charges. The **auxiliary** charges are also called **IMAGE** charges. The reason behind this nomenclature is that the auxiliary charges are considered as being practically the mirror images of the real charges (at least when the conductor is spherical or planar) in the sense that their positions and relative magnitudes are related in a manner exactly similar to the relations between the positions and relative linear magnification of the object and its corresponding image in Geometrical optics (also called Gaussian ray optics).

EXAMPLE

If we are asked to find the force between an infinite earthed conductor and a point charge q placed at

perpendicular distance ℓ from the earthed conductor (see figure), than we proceed as follows.
Firstly, the conductor being earthed implies $V = 0$.
So, we redraw the situation in which we replace the conductor and introduce an IMAGE charge $-q$ as shown.
The force between the two charges (object charge q and image charge $-q$) is the electrostatic force between the infinite grounded conductor and q. So,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2\ell)^2}$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4\ell^2} \text{ (attractive in nature)}$$

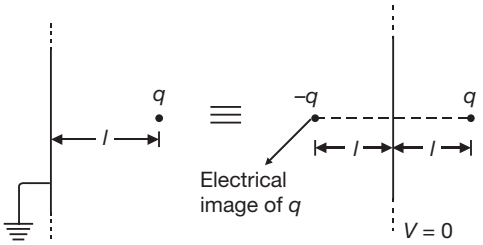


ILLUSTRATION 86

Calculate the work done in taking the charge q to infinity, when, initially it is at a distance l from an infinite conducting plane.

SOLUTION

While taking the charge to infinity let it (at any instant) be at a distance x from the grounded surface. The force with which it is attracted towards grounded surface is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2x)^2}$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0(4x^2)}$$

Let it be further moved through dx away from grounded surface (or towards infinity), then

$$dW = F dx\cos(180^\circ)$$

$$\Rightarrow \quad dW = -\frac{q^2}{4\pi\varepsilon_0(4x^2)}dx$$

$$\Rightarrow \quad W = \int dW = -\frac{q^2}{16\pi\varepsilon_0}\int_l^\infty x^{-2}dx$$

$$\Rightarrow \quad W = -\frac{q^2}{16\pi\varepsilon_0}\left[\frac{x^{-2+1}}{-2+1}\Big|_l^\infty\right]$$

$$\Rightarrow \quad W = \frac{q^2}{16\pi\varepsilon_0}\left(\frac{1}{\infty}-\frac{1}{l}\right)$$

$$\Rightarrow \quad W = -\frac{q^2}{16\pi\varepsilon_0 l}$$

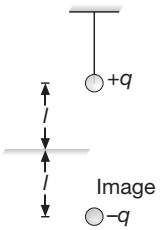
So, this much amount of energy i.e., $\frac{q^2}{16\pi\varepsilon_0 l}$ is to be supplied to the charge (system) to set it free from the electrostatic influence of the grounded conductor.

ILLUSTRATION 87

A small ball is suspended over an infinite horizontal conducting plane by means of an insulating elastic thread of stiffness k. As soon as the ball was charged, it descended by x and its new separation from the plane became equal to l. Find the charge of the ball.

SOLUTION

When the ball is charged, for the equilibrium of ball, electric force on it must counter balance the excess spring force, exerted, on the ball due to the extension in the string.



Thus $F_{el} = F_{spr}$

The force on the charge q is considered due to attraction by the electrical image

$$\Rightarrow \quad \frac{q^2}{4\pi\varepsilon_0(2l)^2} = kx$$

$$\Rightarrow \quad q = 4l\sqrt{\pi\varepsilon_0 kx}$$

ILLUSTRATION 88

Two point charges, q and $-q$, are separated by a distance L, both being located at a distance $\frac{L}{2}$ from the infinite conducting plane. Find

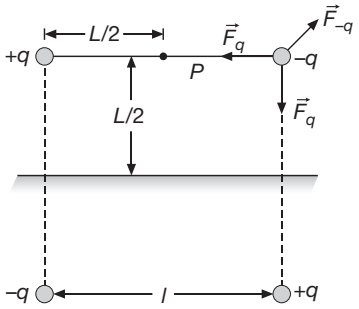
(a) the modulus of the vector of the electric force acting on each charge;
(b) the magnitude of the electric field strength vector at the mid-point between these charges.

SOLUTION

(a) Using the concept of electrical image, it is clear that the magnitude of the force acting on each charge,

$$|\vec{F}| = \sqrt{2}\frac{q^2}{4\pi\varepsilon_0 L^2} - \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}L)^2}$$

$$\Rightarrow \quad |\vec{F}| = \frac{q^2}{8\pi\varepsilon_0 L^2}(2\sqrt{2}-1)$$



(b) Also, from the figure, magnitude of electric field strength at P

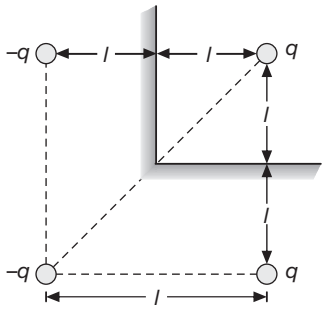
$$E = 2\left(1 - \frac{1}{5\sqrt{5}}\right)\frac{q}{\pi\varepsilon_0 L^2}$$

ILLUSTRATION 89

A point charge q is located between two mutually perpendicular conducting half-planes. Its distance from each half-plane is equal to l. Find the modulus of the vector of the force acting on the charge.

SOLUTION

Using the concept of electrical image, it is easily seen that the force on the charge q is,



$$F = \frac{\sqrt{2}q^2}{4\pi\varepsilon_0(2l)^2} + \frac{(-q)^2}{4\pi\varepsilon_0(2\sqrt{2}l)^2}$$

$$\Rightarrow \quad F = \frac{(2\sqrt{2}-1)q^2}{32\pi\varepsilon_0 l^2} \quad \text{(It is attractive)}$$

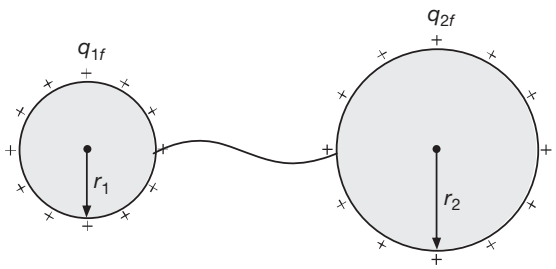
CHARGE DISTRIBUTION ON METAL OBJECT

Whenever a charge is given to a metal body then, due to mutual repulsion, it automatically spreads on the outer surface of the body. Let us consider an example for understanding the distribution of charges in detail.

Figure shows two conducting metal spheres of radii r_1 and r_2 at large separation having charges q_1 and q_2 initially.

If the two spheres are connected by a metal wire, charge flow takes place from the sphere at higher potential to the one at lower potential till final potential of the two spheres become equal.
After final distribution the charges on the two spheres be $(q_1)_f$ and $(q_2)_f$ respectively such that

$$\frac{(q_1)_f}{4\pi\varepsilon_0 r_1} = \frac{(q_2)_f}{4\pi\varepsilon_0 r_2} \qquad \ldots(1)$$



If after distribution the final surface charge densities on the two spheres are σ_1 and σ_2, then we have

$$(q_1)_f = \sigma_1\left(4\pi r_1^2\right)$$

and $$(q_2)_f = \sigma_2\left(4\pi r_2^2\right)$$

Now from equation (1) we get

$$\frac{\sigma_1\left(4\pi r_1^2\right)}{4\pi\varepsilon_0 r_1} = \frac{\sigma_2\left(4\pi r_2^2\right)}{4\pi\varepsilon_0 r_2}$$

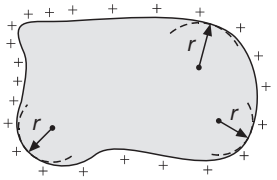
$$\Rightarrow \quad \sigma_1 r_1 = \sigma_2 r_2$$

$$\Rightarrow \quad \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$$

Thus we can say $\sigma \propto \frac{1}{r}$

$$\Rightarrow \quad \sigma r = \text{constant}$$

This shows that for charged metal bodies at same potential, the surface charge density is inversely proportional to the radius of curvature of the body.



Consider a randomly shaped metal body that has been given some charge as shown, then charge is distributed on the outer surface of this body in such a way that at sharp edges of body (where radius of curvature is less), charge density is more and on the broader parts of body (where radius of curvature is large) charge density is less because we already know that at every point of a metal body potential is same.

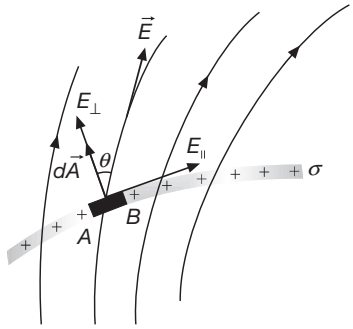
ELECTRIC PRESSURE ON A CHARGED SURFACE DUE TO EXTERNAL ELECTRIC FIELD

Consider a surface uniformly charged with charge density σ placed in electric field E. Let us consider a small portion AB having area dA, on the surface. Charge on AB is given by

$$dq = \sigma\, dA$$

If the external electric field on the surface AB is E in the direction shown, then it has two components, one parallel to the surface, other perpendicular to the surface, given by

$$E_\perp = E\cos\theta \text{ and } E_\parallel = E\sin\theta$$



Due to $E_\parallel$, the force on surface is tangential which only stretches the surface and due to $E_\perp$, the surface experiences an outward force and hence an outward pressure. The outward force on AB due to external electric field is

$$dF = E_\perp dq$$

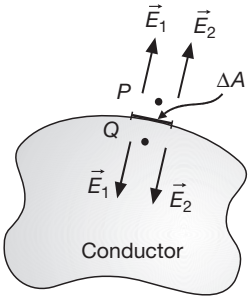
$$\Rightarrow \quad dF = (\sigma E_\perp)\, dA$$

Thus outward electrostatic pressure P_e is given by

$$P_e = \frac{dF}{dA} = \sigma E_\perp$$

MECHANICAL FORCE ON A CHARGED CONDUCTOR

Similar charges repel each other, hence the charge on any part of surface of the conductor is repelled by the charge on its remaining part. The surface of the conductor thus experiences a mechanical force.



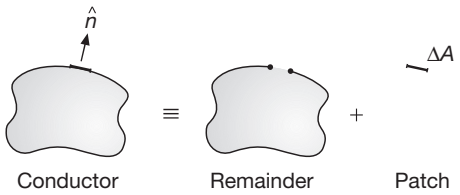
Also, we have seen that at the boundary of a conductor with uniform charge density σ, the tangential component of the electric field is zero, and hence is continuous. But the normal component of the electric field exhibits discontinuity with $E_n = \frac{\sigma}{\varepsilon_0}$ just outside and zero inside the conductor. Let us now consider a small patch of charge on the surface of conductor as shown.

A question comes to our mind "what is the force experienced by this patch"? Since by Newton's Third Law, the patch cannot exert the force on itself and hence the force on the patch must come only from the remaining part of the conductor **(called as remainder)**. Assume the patch to be a flat surface of area A on the surface of the conductor.

The electric field $(\vec{E})$ at any point P near the conductor surface is the sum of the electric field due to a small part of the surface (called as Patch) of area say A immediately in the neighbourhood of the point under consideration and due to the remaining part of the surface (called as Remainder). Let $\vec{E}_{\text{Patch}}$ and $\vec{E}_{\text{Remainder}}$ be the field intensities due to these parts respectively.

Then, total electric field,

$$\vec{E} = \vec{E}_{\text{Patch}} + \vec{E}_{\text{Remainder}}$$



Now, since $\vec{E}$ has a magnitude $\frac{\sigma}{\varepsilon_0}$ at any point P just outside the conductor and is zero at point Q just inside the conductor. Thus,

at P, $\vec{E}_{\text{Patch}} + \vec{E}_{\text{Remainder}} = \frac{\sigma}{\varepsilon_0}$ and

at Q, $E_{\text{Patch}} - \vec{E}_{\text{Remainder}} = \vec{0}$

$$\Rightarrow \quad \vec{E}_{\text{Patch}} = \vec{E}_{\text{Remainder}}$$

$$\Rightarrow \quad \left|\vec{E}_{\text{Patch}}\right| = \left|\vec{E}_{\text{Remainder}}\right| = \frac{\sigma}{2\varepsilon_0}$$

Hence, the force experienced by small surface of area A due to the charge on the rest of the surface is

$$\Rightarrow \quad F = qE_{\text{Remainder}} = (\sigma A)\left(E_{\text{Remainder}}\right) = \frac{\sigma^2 A}{2\varepsilon_0}$$

Thus, the force acting on the Patch is

$$\vec{F} = q\vec{E}_{\text{Remainder}} = \left(\frac{\sigma^2 A}{2\varepsilon_0}\right)\hat{n}$$

This is precisely the force needed to drive the charges on the surface of a conductor to an equilibrium state where the electric field just outside the conductor has a value $\frac{\sigma}{\varepsilon_0}$ and vanishes inside. The most interesting thing we must note here that irrespective of the sign of σ, the force always tends to pull the patch into the field.

Using the result obtained above, we may define the electrostatic pressure on the patch as

$$\frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \qquad \left\{\text{as } E = \frac{\sigma}{\varepsilon_0}\right\}$$

$$\Rightarrow \quad \text{Electrostatic Pressure} = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$$

Here, we also note that the pressure is being transmitted via the electric field.

Also, this is the value of the energy density u_e due to the field. So,

$$u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$$

(Please keep in mind that actually pressure and energy density possess the same dimensional formula i.e., $ML^{-1}T^{-2}$)

ELECTROSTATIC ENERGY DENSITY (u_e)

Electrostatic energy density is defined as the energy stored in unit volume in any electric field. Its mathematical formula is given by

$$\text{Energy density } u_e = \frac{1}{2}\varepsilon E^2$$

where $E =$ electric field intensity at that point and

$\varepsilon = \varepsilon_o \varepsilon_r$ electric permittivity of medium

ILLUSTRATION 90

Calculate the energy stored in an imaginary cubical volume of side a placed in front of an infinitely large nonconducting sheet of uniform charge density σ.

SOLUTION

Since electrostatic energy stored is

$$U = \int \frac{1}{2}\varepsilon_0 E^2 d\tau \text{ where } dr \text{ is small volume}$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0 E^2 \int d\tau \text{, where } E = \frac{\sigma}{2\varepsilon_0} = \text{constant}$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0 \left(\frac{\sigma^2}{4\varepsilon_0^2}\right) a^3 = \frac{\sigma^2 a^3}{8\varepsilon_0}$$

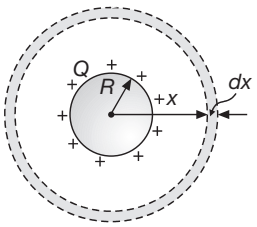
ILLUSTRATION 91

Using the concept of energy density, show that the total energy stored by a shell having charge Q and radius R is $\frac{Q^2}{8\pi\varepsilon_0 R}$.

SOLUTION

Since we have discussed that in space wherever electric field exists, there must be some field energy stored which has energy density, given as

$$u_e = \frac{1}{2}\varepsilon_0 E^2$$



Here we can see that when the sphere was uncharged, there was no electric field in its surroundings. But when the sphere becomes fully charged, electric field will exist in its surrounding from its surface to infinity. Let us calculate the field energy associated with this charged conducting sphere.

We know electric field due to a sphere at outer points varies with distance from centre as

$$E = \frac{Q}{4\pi\varepsilon_0 x^2}$$

To find the total field energy due to this sphere, we consider an elemental spherical shell of radius x and thickness dx as shown. The volume enclosed in this shell is

$$d\tau = (4\pi x^2)dx$$

Here we have taken symbol for volume as τ (and not V) as V is the symbol for potential.

Thus the field energy stored in the volume of this elemental shell is

$$u_e = \frac{dU}{d\tau} = \left(\frac{1}{2}\varepsilon_0 E^2\right)$$

$$\Rightarrow \quad dU = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2 (4\pi x^2)dx$$

$$\Rightarrow \quad dU = \frac{Q^2}{8\pi\varepsilon_0 x^2}dx$$

Thus total field energy associated with the sphere can be calculated by integrating this expression from surface of sphere to infinity (as electric field inside the sphere is zero).

Total field energy in the surrounding of sphere is

$$U = \int dU = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{x^2}dx$$

$$\Rightarrow \quad U = \frac{Q^2}{8\pi\varepsilon_0}\left[-\frac{1}{x}\Big|_R^\infty\right]$$

$$\Rightarrow \quad U_{R\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

ILLUSTRATION 92

A non-conducting sphere of radius R has a charge Q distributed uniformly on it. Using the concept of energy density, find the

(a) energy stored outside the sphere.
(b) energy stored inside the sphere and
(c) total energy stored inside and outside the sphere.

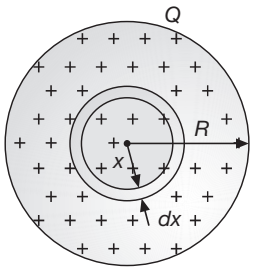
SOLUTION

(a) We know in outside region of a non-conducting uniformly charged sphere, every point is same as that of a conducting sphere of same radius.

Thus field energy in the surrounding of this sphere from surface to infinity is also

$$U_{R\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

(b) As in the case of conducting sphere (where $E_{\text{inside}} = 0$), in non-conducting sphere, at interior points $E \neq 0$.
Thus field energy also exists in the interior region. This can be calculated by considering an elemental shell of radius x and thickness dx inside the sphere as shown.



Here field energy in the volume of this elemental shell is

$$dU = \left(\frac{1}{2}\varepsilon_0 E^2\right) d\tau$$

where $d\tau = 4\pi x^2 dx$ and $E = E_{\text{inside}} = \dfrac{Qx}{4\pi\varepsilon_0 R^3}$

$$\Rightarrow \quad dU = \frac{1}{2}\varepsilon_0 \left(\frac{Qx}{4\pi\varepsilon_0 R^3}\right)^2 \left(4\pi x^2\, dx\right)$$

$$\Rightarrow \quad dU = \frac{Q^2}{8\pi\varepsilon_0 R^6} x^4\, dx$$

Total field energy inside the sphere can be given as

$$U = \int dU = \frac{Q^2}{8\pi\varepsilon_0 R^6}\int_0^R x^4 dx$$

$$\Rightarrow \quad U = \frac{Q^2}{8\pi\varepsilon_0 R^6}\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U_{0\to R} = \frac{Q^2}{40\pi\varepsilon_0 R}$$

(c) So, total self-energy of this sphere, also called ***Electrostatic Self Energy*** is

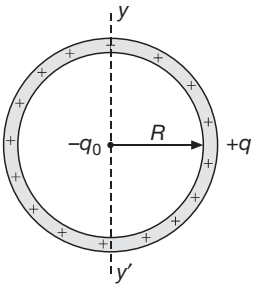
$$U_{\text{self}} = U_{0\to R} + U_{R\to\infty}$$

$$\Rightarrow \quad U_{\text{self}} = \frac{Q^2}{40\pi\varepsilon_0 R} + \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$\Rightarrow \quad U_{\text{self}} = U_{0\to R} + U_{R\to\infty} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

ILLUSTRATION 93

Consider a thin spherical shell of radius R having charge q distributed uniformly on it. At the centre of shell a negative point charge $-q_0$ is placed. The shell is cut in two identical hemispherical portions along a diametrical section yy' as shown. Due to mutual repulsion the two hemispherical parts tend to move away from each other but due to the attraction of $-q_0$, they may stay in equilibrium. Find the condition of equilibrium of the hemispherical shells. Calculate minimum value of q_0 to attain this equilibrium.



SOLUTION

Let the outward electric pressure at every point of the spherical shell, due to its own charge q be P_1. Then

$$P_1 = \frac{\sigma^2}{2\varepsilon_0}$$

$$\Rightarrow \quad P_1 = \frac{1}{2\varepsilon_0}\left(\frac{q}{4\pi R^2}\right)^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4} \quad \left\{\because\ \sigma = \frac{q}{4\pi R^2}\right\}$$

Due to charge $-q_0$, the electric field on the surface of shell is

$$E = \frac{q_0}{4\pi\varepsilon_0 R^2} \qquad \{\text{radially inwards}\}$$

This electric field pulls every point of the shell in an inward direction. Let the inward pressure on the surface of shell due to this negative point charge at centre be P_2. Then

$$P_2 = \sigma E \qquad \{\text{as done already}\}$$

$$\Rightarrow \quad P_2 = \left(\frac{q}{4\pi R^2}\right)\left(\frac{q_0}{4\pi\varepsilon_0 R^2}\right) = \frac{qq_0}{16\pi^2\varepsilon_0 R^4}$$

For equilibrium of hemispherical shell or for the shells not to separate,

$$P_2 \geq P_1$$

$$\Rightarrow \quad \frac{qq_0}{16\pi^2\varepsilon_0 R^4} \geq \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

$$\Rightarrow \quad q_0 \geq \frac{q}{2}$$

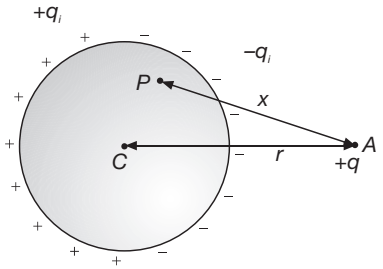
$$\Rightarrow \quad q_{\min} = \frac{q}{2}$$

ELECTROSTATIC FIELD AND POTENTIAL DUE TO INDUCED CHARGES

Electrostatic Field

Consider the situation in which a metal sphere of radius R is placed at a distance r from the point charge $+q$. Consider a point P in the sphere at a distance x from $+q$ where we wish to find the electric field and potential due to the induced charges on the surface of sphere.

The charge $+q$ at A will induce a negative and a positive charge $-q_i$ and $+q_i$ as shown at the nearer and farther end respectively.



Now since P lies inside the metal body,

so $\left(\vec{E}_P\right)_{\text{NET}} = \vec{0}$

i.e., the total field at E due to the point charge $+q$ (at A) and due to the charges induced on the sphere must be zero.

$$\Rightarrow \quad \vec{E}_{\substack{\text{at } P \text{ due to} \\ +q}} + \vec{E}_{\substack{\text{at } P \text{ due to} \\ \text{induced charges}}} = \vec{0}$$

$$\Rightarrow \quad \vec{E}_{\substack{\text{at } P \text{ due to} \\ \text{induced charges}}} = -\vec{E}_{\substack{\text{at } P \text{ due to} \\ +q}}$$

$$\Rightarrow \quad \left|E_{\substack{\text{at } P \text{ due to} \\ \text{induced charges}}}\right| = \left|E_{\substack{\text{at } P \text{ due} \\ \text{to } +q}}\right| = \frac{q}{4\pi\varepsilon_0 x^2}$$ (in magnitude) and $E_{\substack{\text{at } P \text{ due to} \\ \text{induced charges}}}$ will have a direction opposite to the direction of field at P due to $+q$. So, field due to induced charges is $\frac{q}{4\pi\varepsilon_0 x^2}$ directed from P to A i.e., towards $+q$.

Electrostatic Potential

Since, inside the sphere at every point $E = 0$ and due to this the entire region is equipotential. Hence the potential at the centre of sphere, will be only due to the charge $+q$, because potential due to induced charges at the centre will be zero (due to symmetrical placement of induced charges about C). So, net potential at the centre of sphere due to q at A is

$$V_C = \frac{q}{4\pi\varepsilon_0 r}$$

Since the entire sphere is equipotential, so potential at point P must be equal to the potential at point C. However, at P the potential due to induced charges will be non-zero because induced charges are not equidistant from point P. Thus net potential at point P will be

V_P = Potential at P due to $+q$ at A + Potential due to induced charges

$$\Rightarrow \quad V_P = \frac{q}{4\pi\varepsilon_0 x} + V_i$$

and since

$$V_P = V_C = \frac{q}{4\pi\varepsilon_0 r}$$

(because the sphere is equipotential inside) Hence

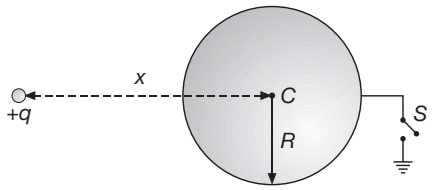
$$V_P = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 x} + V_i$$

$$\Rightarrow \quad V_i = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{x}\right)$$

EARTHING OF CHARGED OR UNCHARGED METAL BODIES

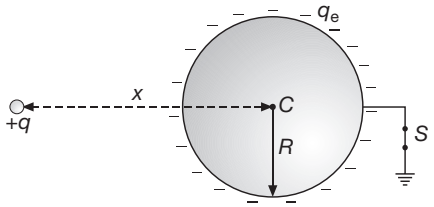
As far as electrical behaviour of the earth is concerned, the earth is assumed to be a very large conducting sphere of radius 6400 km. If some charge Q is given to earth, its potential V_E becomes

$$V_E = \frac{Q}{4\pi\varepsilon_0 R} \qquad \{\text{where } R = \text{Radius of Earth}\}$$



As R is very large V_E becomes a negligible value. Thus for very small bodies whose dimensions are negligible compared to earth we can assume the earth to be at zero potential.

If we connect a small body to earth then charge flow takes place between earth and the body till both acquire the same potential i.e., zero potential. The potential of earth will always remain zero, no matter if charge flows into earth or from earth. This implies that if a body at some positive potential is connected to earth, earth will supply some negative charge to this body so that the final potential of body becomes zero.



Consider a solid uncharged conducting sphere of radius R. A point charge q is placed in front of the sphere at a distance x as shown. Due to q, the potential at sphere is

$$V = \frac{q}{4\pi\varepsilon_0 x}$$

Here we have ignore the charges induced due to q because potential at C due to induced charges on the sphere is zero. If we close the switch S, earth supplies a charge q_E on to the sphere to make its final potential zero. Thus the final potential on sphere can be taken as

$$V = \frac{q}{4\pi\varepsilon_0 x} + \frac{q_E}{4\pi\varepsilon_0 R} = 0$$

$$\Rightarrow \quad q_E = -\frac{qR}{x}$$

Here it is obvious that earth has supplied a negative charge so as to develop a negative potential on sphere to nullify the initial positive potential on it due to q.

Remark(s)

Always remember whenever a metal body is connected to earth, we consider that earth supplies a charge to it (say q_E) to make its final potential zero due to all the charges including the charge on body and the charges in its surrounding.

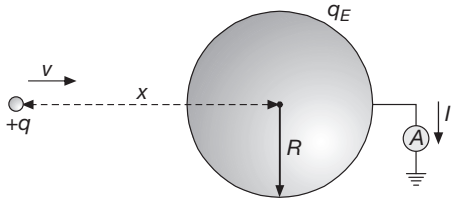
Problem Solving Technique(s)

Earthing a conductor: Potential of earth is often taken to be zero. If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e., zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

CURRENT DUE TO MOVEMENT OF CHARGES THROUGH EARTH

Consider a metal ball of radius R connected to earth through an ammeter. Let a charge $+q$ be moving towards the ball at speed v, due to which the potential of ball increases continuously. When the charge is at a distance x from ball, the potential of ball due to this charge is given by

$$V = \frac{q}{4\pi\varepsilon_0 x}$$



Since the metal ball is connected to the earth, so the earth supplies a charge q_E (say) to make the potential of the ball to be zero. This q_E is given by (as done earlier)

$$q_E = -\frac{qR}{x}$$

Since, x is continuously decreasing hence the charge on ball must increase continuously. As a result of this the ammeter shows a current I, which is given by

$$I = \frac{dq_E}{dt} = \frac{d}{dt}\left(-\frac{qR}{x}\right) = \frac{qR}{x^2}\left(\frac{dx}{dt}\right)$$

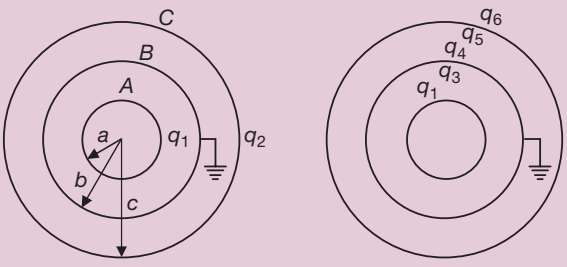
$$\Rightarrow \quad I = \left(\frac{qR}{x^2}\right)v$$

So, in the surrounding of an earthed conducting body whenever one or more charges are in motion, a continuous current flows in the wires connecting body to earth.

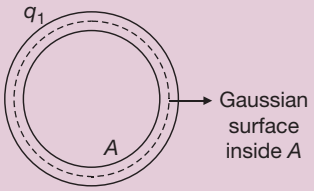
Problem Solving Technique(s)

Charges appearing on different surfaces of concentric spherical conducting shells:

Figure shows three concentric thin spherical conducting shells *A*, *B* and *C* of radii *a*, *b* and *c*. The shells *A* and *C* are given charges q_1 and q_2 and the shell *B* is earthed. We are interested in finding the charges on inner and outer surfaces of *A*, *B* and *C*. To solve such type of problems following points should keep in mind:



(a) The entire charge q_1 will come on the outer surface of *A* unless some charge is kept inside *A*.



To understand this, let us consider a Gaussian surface (a sphere) through the material of *A*. As the electric field in a conducting material is zero. The flux through this Gaussian surface is zero. Using Gauss's Law, the total charge enclosed must be zero.

(b) Similarly if we draw a Gaussian surface through the material of *B*, then we have

$$q_3 + q_1 = 0 \;\Rightarrow\; q_3 = -q_1$$

and if we draw a Gaussian surface through the material of *C*, then

$$q_5 + q_4 + q_3 + q_1 = 0 \;\Rightarrow\; q_5 = -q_4$$

(c) Since q_2 charge was given to shell *C*, so $q_5 + q_6 = q_2$

(d) Since B is earthed, so potential of B must be zero. Hence

$$V_B = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{b} + \frac{q_3 + q_4}{b} + \frac{q_5 + q_6}{c}\right) = 0$$

Using the above conditions we can find charges on different surfaces, we can summarise the above points as under

(i) Net charge inside a closed Gaussian surface drawn in any **conducting** shell is zero.

(ii) Potential of the conductor which is earthed is zero.

(iii) If two conductors are connected, they are at same potential.

(iv) Charge remains constant on all conductors except those which are earthed.

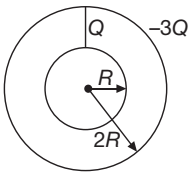
(v) Charge on the inner surface of the innermost shell is zero **provided no charge is kept inside it**. In all other shells charge resides on both the surfaces.

(vi) Equal and opposite charges appear on surfaces facing each other.

Based on this, try to understand the following ILLUSTRATIONS.

ILLUSTRATION 94

The two conducting spherical shells as shown in figure are joined by a conducting wire. After a long time, the wire is cut and the charge stops flowing. Calculate the final charge on each sphere after that.



SOLUTION

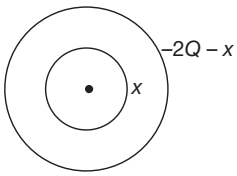
Since the shells had been connected by a conducting wire for a long time, so after the conducting wire is removed then both shells must be at the same potential. If x be the charge that has flown from inner shell to outer shell, then

Potential of inner shell is

$$V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$$

and potential of outer shell is

$$V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$$



Since $V_{out} = V_{in}$

$$\Rightarrow \quad \frac{-KQ}{R} = \frac{K(x-2Q)}{2R}$$

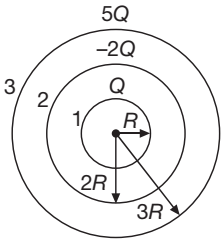
$$\Rightarrow \quad -2Q = x - 2Q$$

$$\Rightarrow \quad x = 0$$

So final charge on inner spherical shell is zero and on outer spherical shell is $-2Q$, i.e. entire charge on inner shell flows to the outer shell.

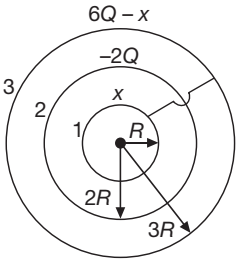
ILLUSTRATION 95

Calculate the final charge on each spherical shell after joining the inner most shell and outer most shell by a conducting wire.



SOLUTION

Let the charge on the innermost sphere be x. Then finally,



Potential of shell 1 = Potential of shell 3

$$\frac{Kx}{R} + \frac{K(-2Q)}{2R} + \frac{K(6Q-x)}{3R} = \frac{Kx}{3R} + \frac{K(-2Q)}{3R} + \frac{K(6Q-x)}{3R}$$

$$\Rightarrow \quad 3x - 3Q + 6Q - x = 4Q$$

$$\Rightarrow \quad 2x = Q$$

$$\Rightarrow \quad x = \frac{Q}{2}$$

Charge on innermost shell $= \frac{Q}{2}$

Charge on outermost shell $= \frac{5Q}{2}$

Charge on middle shell $= -2Q$

Final charge distribution is as shown in figure.

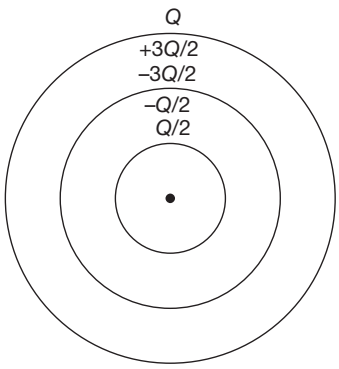
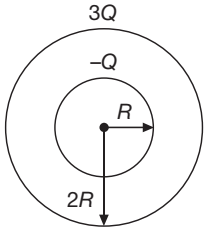


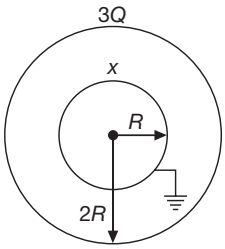
ILLUSTRATION 96

Two conducting hollow spherical shells of radii R and $2R$ carry charges $-Q$ and $3Q$ respectively. How much charge will flow into the earth if inner shell is grounded?



SOLUTION

When inner shell is grounded, then the potential of inner shell becomes zero.



$$\Rightarrow \quad \frac{Kx}{R}+\frac{K(3Q)}{2R}=0$$

$$\Rightarrow \quad x=\frac{-3Q}{2}$$

Since the charge has increased by

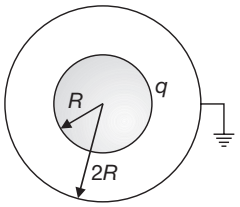
$$\Delta q=\frac{-3Q}{2}-(-Q)=\frac{-Q}{2}$$

Hence charge flown into the Earth is $\frac{Q}{2}$

ILLUSTRATION 97

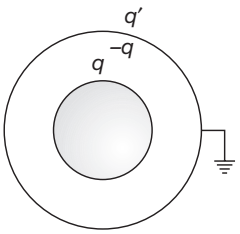
A charge q is distributed uniformly on the surface of a sphere of radius R. It is covered by an earthed concentric hollow conducting sphere of radius $2R$. Find the charges on inner and outer surfaces of hollow sphere when

(a) thickness of shell is negligible
(b) the shell has considerable thickness.



SOLUTION

(a) The charge on the inner surface should be $-q$, because if we draw a closed Gaussian surface through the material of the hollow sphere the total charge enclosed by this Gaussian surface should be zero. Let q' be the charge on the outer surface of the hollow sphere.



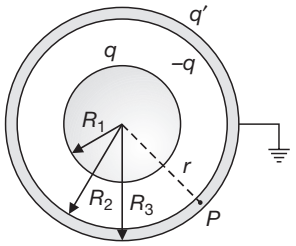
Since, the outer sphere is earthed, its potential should be zero. The potential on it is due to the charges q, $-q$ and q', Hence,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{2R}-\frac{q}{2R}+\frac{q'}{2R}\right)=0$$

$$\Rightarrow \quad q'=0$$

Therefore, there will be no charge on the outer surface of the hollow sphere.

(b)



In this case, we can set $V=0$ at any point on the hollow sphere. Let us select a point P a distance r from the centre. Where $R_2<r<R_3$. So,

$$V_P=0$$

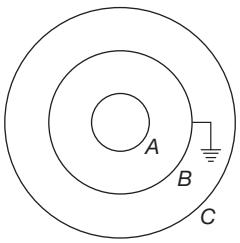
$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{q}{r}+\frac{q'}{R_3}\right)=0$$

$$\Rightarrow \quad q'=0$$

i.e., in this case also there will be no charge on the outer surface of the hollow sphere.

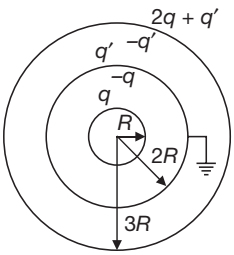
ILLUSTRATION 98

Figure shows three concentric thin spherical shells A, B and C of radii R, $2R$ and $3R$. The shell B is earthed and A and C are given charges q and $2q$ respectively. Find the charges appearing on all the surfaces of A, B and C.



SOLUTION

Since there is no charge inside A. The whole charge q given to the shell A will appear on its outer surface. Charge on its inner surface will be zero. Moreover if a Gaussian surface is drawn on the material of shell B net charge enclosed by it should be zero. Therefore, charge on its inner surface will be $-q$. Now let q' be the charge on its outer surface, then charge on the inner surface of C will be $-q'$ and on its outer surface will be, $2q-(-q')=2q+q'$ as total charge on C is $2q$.



Shell B is earthed. Hence, its potential should be zero.

$$V_B=0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{2R}-\frac{q}{2R}+\frac{q'}{2R}-\frac{q'}{3R}+\frac{2q+q'}{3R}\right)=0$$

Solving this equation, we get

$$q'=-\frac{4}{3}q$$

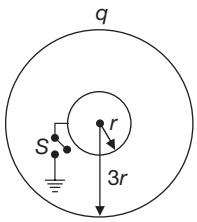
$$\Rightarrow \quad 2q+q'=2q-\frac{4}{3}q=\frac{2}{3}q$$

Therefore, charges on different surfaces in tabular form are given below:

A		B		C	
INNER	OUTER	INNER	OUTER	INNER	OUTER
0	q	$-q$	$-\frac{4}{3}q$	$\frac{4}{3}q$	$\frac{2}{3}q$

ILLUSTRATION 99

Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q. Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.



SOLUTION

Let q' be the charge on inner shell when it is earthed.

$$V_{\text{inner}}=0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{q'}{r}+\frac{q}{3r}\right)=0$$

$$\Rightarrow \quad q'=-\frac{q}{3}$$

i.e., $+\frac{q}{3}$ charge will flow from inner shell to earth.

FIELD ENERGY DENSITY OF ELECTRIC FIELD

An energy is associated with every region where electric field is present. This energy is called the field energy of the electric field. Using the concept of field

energy we can calculate the amount of energy stored in the space where electric field exists. To calculate the Field Energy Density (FED) we have two cases to be taken into account.

CASE-1:

Field Energy Density of a charged body, also called **Self Energy**.

"Self energy of a charged body is the total field energy, associated with the electric field due to the body itself in its surrounding".

CASE-2:

Field Energy Density of a system of charged bodies.

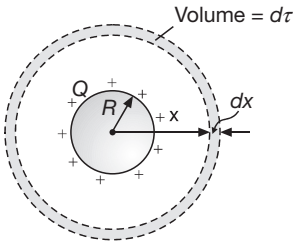
Here, we have to take Field Energy Density as the total of self energies associated with the bodies and the interaction energies of the pairs of charged bodies.

$$\Rightarrow \quad U = \left(\sum \begin{matrix} \text{self energy} \\ \text{of all charged} \\ \text{bodies} \end{matrix} \right) + \left(\sum \begin{matrix} \text{interaction} \\ \text{energy of all} \\ \text{pairs of} \\ \text{charged bodies} \end{matrix} \right)$$

ELECTROSTATIC SELF ENERGY OF A CHARGED CONDUCTING SPHERE

We have discussed that in space wherever electric field exist, there must be some field energy stored which has energy density, given as

$$U = \frac{1}{2}\varepsilon_0 E^2$$



Here we can see that when the sphere was uncharged, there was no electric field in its surroundings. But when the sphere becomes fully charged, electric field will exist in its surrounding from its surface to infinity. Let us calculate the field energy associated with this charged conducting sphere.

We know electric field due to a sphere at outer points varies with distance from centre as

$$E = \frac{Q}{4\pi\varepsilon_0 x^2}$$

To find the total field energy due to this sphere, we consider an elemental spherical shell of radius x and thickness dx as shown. The volume enclosed in this shell is

$$d\tau = (4\pi x^2)\,dx$$

Thus the field energy stored in the volume of this elemental shell is

$$dU = \left(\frac{1}{2}\varepsilon_0 E^2\right) d\tau$$

$$\Rightarrow \quad dU = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2 (4\pi x^2)\,dx$$

$$\Rightarrow \quad dU = \frac{Q^2}{8\pi\varepsilon_0 x^2}\,dx$$

Thus total field energy associated with the sphere can be calculated by integrating this expression from surface of sphere to infinity (as electric field inside the sphere is zero).

Total field energy in the surrounding of sphere is

$$U = \int dU = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{x^2}\,dx$$

$$\Rightarrow \quad U = \frac{Q^2}{8\pi\varepsilon_0}\left[-\frac{1}{x}\Big|_R^\infty\right]$$

$$\Rightarrow \quad U_{R\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \ldots(1)$$

Alternatively, whenever a system of charges is assembled, some work is done and this work is stored in the form of electrical potential energy of the system. Let consider an example of charging a conducting sphere of radius R.

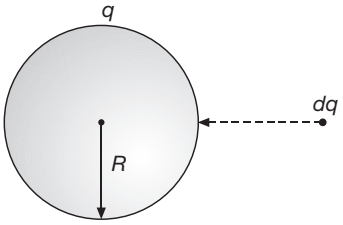
In the process of charging we bring charge to the sphere from infinity in steps of elemental charges dq. The charge on sphere opposes the elemental charge being brought to it. Let us assume that at an instant sphere has a charge q, due to which it has a potential given as

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

If now a charge dq is brought to its surface from infinity, work done in this process can be given as

$$dU = Vdq$$

$$\Rightarrow \quad dU = \left(\frac{q}{4\pi\varepsilon_0 R}\right)dq$$



Total work done in charging the sphere is

$$\Rightarrow \quad U = \int dU = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\,dq$$

$$\Rightarrow \quad U_{R\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \ldots(2)$$

Equation (1) gives the total work done in charging the sphere of radius R.

Here we can see that again result in equation (2) is same as that in equation (1)

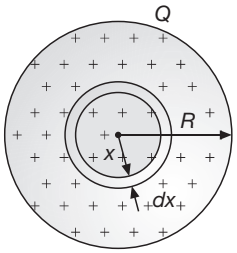
We can conclude by this that whatever work is done in charging a body is stored in its surrounding in the form of its field energy and can be regarded as self energy of that body. Once a body is charged in a given configuration, its self energy is fixed, if the body is now displaced or moved in any manner keeping its shape and charge distribution constant, its self energy does not charge.

ELECTROSTATIC SELF ENERGY OF A UNIFORMLY CHARGED NON-CONDUCTING SPHERE

METHOD 1:

We know in outside region of a non-conducting uniformly charged sphere, every point is same as that of a conducting sphere of same radius. Thus field energy in the surrounding of this sphere from surface to infinity is also

$$U_{R\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R}$$



As in the case of conducting sphere (where $E_{\text{inside}} = 0$), in non-conducting sphere, at interior points $E \neq 0$. Thus field energy also exists in the interior region. This can be calculated by considering an elemental shell of radius x and thickness dx inside the sphere as shown.

Here field energy in the volume of this elemental shell is

$$dU = \left(\frac{1}{2}\varepsilon_0 E^2\right)dV$$

where $E = E_{\text{inside}} = \dfrac{Qx}{4\pi\varepsilon_0 R^3}$

$$\Rightarrow \quad dU = \frac{1}{2}\varepsilon_0\left(\frac{Qx}{4\pi\varepsilon_0 R^3}\right)^2\left(4\pi x^2\,dx\right)$$

$$\Rightarrow \quad dU = \frac{Q^2}{8\pi\varepsilon_0 R^6}x^4\,dx$$

Total field energy inside the sphere can be given as

$$U = \int dU = \frac{Q^2}{8\pi\varepsilon_0 R^6}\int_0^R x^4 dx$$

$$\Rightarrow \quad U = \frac{Q^2}{8\pi\varepsilon_0 R^6}\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U_{0\to R} = \frac{Q^2}{40\pi\varepsilon_0 R}$$

So, total self energy of this sphere is

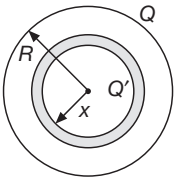
$$U_{\text{self}} = U_{0\to R} + U_{R\to\infty}$$

$$\Rightarrow \quad U_{\text{self}} = \frac{Q^2}{40\pi\varepsilon_0 R} + \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$\Rightarrow \quad U_{\text{self}} = U_{0\to R} + U_{R\to\infty} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

METHOD 2:

Let us assemble the sphere from infinitesimal concentric shells.



Now to add an infinitesimal spherical element of radius x, thickness dx having charge dq on a charged spherical core of radius x having charge q', the work that has to be done is given by

$$dU = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q'dq}{x}\right)$$

where $Q' = \left(\frac{4}{3}\pi x^3\right)\rho$, $dq = (4\pi x^2\,dx)\rho$ and $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$$\Rightarrow \quad dU = \frac{1}{4\pi\varepsilon_0}\frac{16\pi^2\rho^2}{3}x^4dx$$

$$\Rightarrow \quad U = \frac{1}{4\pi\varepsilon_0}\frac{16\pi^2\rho^2}{3}\int_0^R x^4dx$$

$$\Rightarrow \quad U = \frac{1}{4\pi\varepsilon_0}\left(\frac{16\pi^2}{3}\right)\left(\frac{Q^2}{\frac{16\pi^2}{9}R^6}\right)\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This work done in assembling the system is stored in the form of electrostatic self energy.

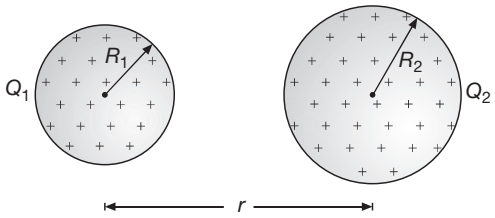
TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGED BODIES

Total electrostatic potential energy a system of charges will be given as

U = Total of self energy of all charged bodies + Total of interaction energy of all pairs of charged bodies

$$\Rightarrow \quad U = \begin{pmatrix}\text{Total self}\\ \text{energy of}\\ \text{all charged}\\ \text{bodies}\end{pmatrix} + \begin{pmatrix}\text{Total interaction}\\ \text{energy of all}\\ \text{pairs of charged}\\ \text{bodies}\end{pmatrix}$$

Let us understand this concept by taking this simple example. Figure shows uniformly charged non-conducting sphere of radius R_1 having charge Q_1 and a charged conducting sphere of radius R_2 having charge Q_2 respectively separated by a distance r.



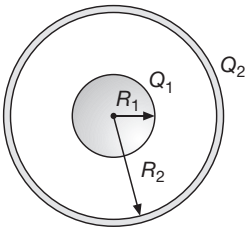
The total electrostatic energy of this system, will be

$$U = U_{\text{self}} + U_{\text{interaction}}$$

$$\Rightarrow \quad U = \frac{3Q_1^2}{20\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$$

ELECTROSTATIC ENERGY OF A SYSTEM OF CONCENTRIC SHELLS

Consider two concentric shells of radii R_1 and R_2 having uniform charges Q_1 and Q_2.



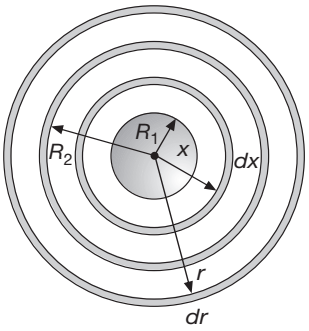
Here the total energy of this system will be

$$U_{\text{total}} = \begin{pmatrix}\text{self energy}\\ \text{of inner}\\ \text{shell}\end{pmatrix} + \begin{pmatrix}\text{self energy}\\ \text{of outer}\\ \text{shell}\end{pmatrix} + \begin{pmatrix}\text{interaction}\\ \text{energy of}\\ \text{the two}\\ \text{shells}\end{pmatrix}$$

$$\Rightarrow \quad U_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1^2}{2R_1} + \frac{Q_2^2}{2R_2} + \frac{Q_1Q_2}{R_2}\right)$$

The above result can also be obtained by following method.

Since, the total electrostatic energy of a system is stored in the form of field energy of the system hence here we can calculate the total electrostatic energy of the system by integrating the field energy density in the space surrounding the shells where the electric field exists.



Total field energy in the electric field associated with the system shown in figure can be given as

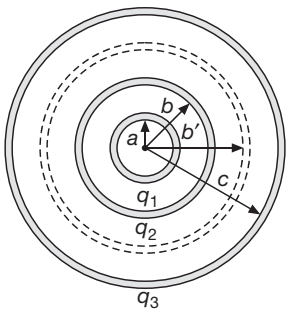
$$U=\int_{R_1}^{R_2}\frac{1}{2}\varepsilon_0\left(\frac{Q_1}{4\pi\varepsilon_0 x^2}\right)^2 4\pi x^2dx+\int_{R_2}^{\infty}\frac{1}{2}\varepsilon_0\left(\frac{(Q_1+Q_2)}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2dr$$

$$U=\frac{Q_1^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{(Q_1+Q_2)^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2}\right)$$

$$\Rightarrow\quad U=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1^2}{2R_1}+\frac{Q_2^2}{2R_2}+\frac{Q_1Q_2}{R_2}\right)$$

ILLUSTRATION 100

Three shells are shown carrying charge q_1, q_2 and q_3 and of radii a, b and c respectively. If the middle shell expands from radius b to $b'(b'<c)$. Find the work done by electric field in process.



SOLUTION

Since work done by a conservative field equals the decrease in potential energy, so we have

$$\Rightarrow\quad \begin{pmatrix}\text{Workdone}\\ \text{by}\\ \text{field}\end{pmatrix}=\begin{pmatrix}\text{Total initial}\\ \text{energy}\\ \text{of system}\end{pmatrix}-\begin{pmatrix}\text{Total final}\\ \text{energy of}\\ \text{system}\end{pmatrix}$$

Now, total initial energy of system is

$$U_i=SE_1+SE_2+SE_3+IE_{12}+IE_{13}+IE_{23}$$

(SE denotes self energy and IE denotes interaction energy)

$$\Rightarrow\quad U_i=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1^2}{2a}+\frac{q_2^2}{2b}+\frac{q_3^2}{2c}+\frac{q_1q_3}{c}+\frac{q_1q_2}{b}+\frac{q_2q_3}{c}\right)$$

Total final energy of system

$$U_f=SE_1+SE_2+SE_3+IE_{12}+IE_{13}+IE_{23}$$

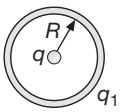
$$\Rightarrow\quad U_f=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1^2}{2a}+\frac{q_2^2}{2b'}+\frac{q_3^2}{2c}+\frac{q_1q_3}{c}+\frac{q_1q_2}{b'}+\frac{q_2q_3}{c}\right)$$

Work done by field $=U_i-U_f$

$$\Rightarrow\quad W=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_2^2}{2b}-\frac{q_2^2}{2b'}+\frac{q_1q_2}{b}-\frac{q_1q_2}{b'}\right)$$

ILLUSTRATION 101

Figure shows a shell of radius R having charge q_1 uniformly distributed over it.

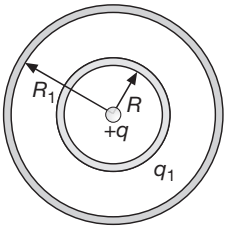


A point charge q is placed at the centre of shell. Find work required to increase radius of shell from R to $R_1(>R)$.

SOLUTION

Work done = Decrease in Potential Energy

$$\Rightarrow\quad \text{Work}=U_i-U_f$$



Now $U_i = (SE)_q + (SE)_{q_1} + (IE)$

$$\Rightarrow \quad U_i = (SE)_q + \frac{q_1^2}{8\pi\varepsilon_0 R} + \frac{q_1 q}{4\pi\varepsilon_0 R}$$

and $U_f = (SE)_q + \frac{q_1^2}{8\pi\varepsilon_0 R_1} + \frac{q_1 q}{4\pi\varepsilon_0 R_1}$

$$\Rightarrow \quad \text{Work done} = U_i - U_f$$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1^2}{2R} + \frac{q_1 q}{R} - \frac{q_1^2}{2R_1} - \frac{q_1 q}{R_1}\right)$$

ILLUSTRATION 102

Find the electrostatic energy stored in a cylindrical shell of length l, inner radius a and outer radius b, coaxial with a uniformly charged wire with linear charge density λ.

SOLUTION

For this we consider an elemental shell of radius x and width dx. The volume of this shell dV can be given as

$$dV = (2\pi x l)\,dx$$

The electric field due to the wire at the shell is

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}$$

The electrostatic field energy stored in the volume of this shell is

$$dU = \left(\frac{1}{2}\varepsilon_0 E^2\right)dV$$

$$\Rightarrow \quad dU = \frac{1}{2}\varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 x}\right)^2 (2\pi x l)\,dx$$

The total electrostatic energy stored in the above mentioned volume can be obtained by integrating the above expression within limits from a to b as

$$U = \int dU = \int_a^b \frac{1}{2}\varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 x}\right)^2 (2\pi x l)\,dx$$

$$\Rightarrow \quad U = \frac{\lambda^2 l}{4\pi\varepsilon_0}\int_a^b \frac{1}{x}\,dx$$

$$\Rightarrow \quad U = \frac{\lambda^2 l}{4\pi\varepsilon_0}\left(\log_e x\big|_a^b\right)$$

$$\Rightarrow \quad U = \frac{\lambda^2 l}{4\pi\varepsilon_0}\log_e\left(\frac{b}{a}\right)$$

Test Your Concepts-XIII

Based on Concept of Self and Interaction Energy

(Solutions on page H.39)

1. A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2. Calculate the work performed by the electric forces in this process.
2. A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2.
3. A spherical shell is uniformly charged with the surface density σ. Using the Energy Conservation Law, find the magnitude of the electric force acting on a unit area of the shell.
4. Calculate the work that must be done to charge a spherical shell of radius R to a total charge Q.

SOLVED PROBLEMS

PROBLEM 1

A rectangular tank of mass m_0 and charge Q over it is placed over a smooth horizontal floor. A horizontal electric field E exist in the region. Rain drops are falling vertically in the tank at the constant rate of n drops per second. Mass of each drop is m. Find the time taken by tank to reach to half the maximum speed.

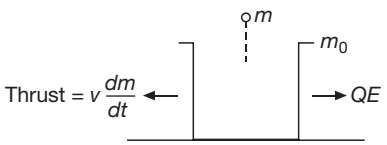
SOLUTION

Applying Newton's Second Law, we get

$$QE - v\frac{dm}{dt} = M\frac{dv}{dt}$$

$$\Rightarrow \quad QE - v(nm) = (m_0 + nmt)\frac{dv}{dt} \qquad \ldots(1)$$

When v becomes v_{max}, then $\frac{dv}{dt} = 0$



$$\Rightarrow \quad QE = v_{max}(nm)$$

$$\Rightarrow \quad v_{max} = v_0 = \frac{QE}{nm}$$

From (1), we get

$$\frac{QE - nmv}{m_0 + nmt} = \left(\frac{dv}{dt}\right)$$

$$\Rightarrow \quad \int_0^t \frac{dt}{m_0 + mnt} = \int_0^{\frac{v_0}{2}} \frac{dv}{QE - nmv}$$

$$\Rightarrow \quad \frac{1}{mn}\log\left(\frac{m_0 + nmt}{m_0}\right) = -\frac{1}{nm}\log\left(\frac{QE - \frac{mnv}{2}}{QE}\right)$$

$$\Rightarrow \quad \frac{1 + nmt}{m_0} = \frac{QE}{QE - mn\left(\frac{v}{2}\right)}$$

$$\Rightarrow \quad \frac{m_0 = nmt}{m_0} = \frac{2QE}{2QE - mn\left(\frac{v_0}{2}\right)}$$

$$\Rightarrow \quad \frac{m_0 + nmt}{m_0} = \frac{2QE}{2QE - \cancel{mn}\left(\frac{QE}{\cancel{mn}}\right)} \qquad \left\{\because\ v_0 = \frac{QE}{nm}\right\}$$

$$\Rightarrow \quad \frac{m_0 + nmt}{m_0} = \frac{2\cancel{QE}}{\cancel{QE}}$$

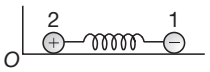
$$\Rightarrow \quad 1 + \frac{nmt}{m_0} = 2$$

$$\Rightarrow \quad \left(\frac{nm}{m_0}\right)t = 1$$

$$\Rightarrow \quad t = \frac{m_0}{nm}$$

PROBLEM 2

Two small identical balls lying on a horizontal plane are connected by a weightless spring. One ball is fixed at the origin and the other is free. The balls are charged identically as a result of which the spring length increases 'n' times. Determine the ratio of new frequency (when balls are charged) to old frequency (when balls were uncharged) when the free ball is displaced slightly from its mean position.



SOLUTION

When the balls are uncharged, $f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where k is force constant of the spring and m = mass of the oscillating ball (ball 1). When charged we have in the equilibrium position of ball 1,

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\eta l)^2} = k(\eta l - l) = kl(\eta - 1)$$

$$\Rightarrow \quad l^3 = \frac{q^2}{4\pi\varepsilon_0\eta^2(\eta - 1)k}$$

When the ball 1 is displaced by a small distance from the equilibrium position to the right, the unbalanced force to the right is given by

Resultant force to the right is

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\eta l+x)^2}-k(\eta l+x-l)$$

From Newton's Law, we have

$$m\frac{d^2x}{dt^2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}\left(1+\frac{x}{\eta l}\right)^{-2}-kl(\eta-1)-kx$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}\left(1-\frac{2x}{\eta l}\right)-kl(\eta-1)-kx$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2l^2}\frac{2x}{\eta l}-kl(\eta-1)-kx$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2}=-\left(\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{\eta^3l^3}+k\right)x$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2}=-\left(\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{\eta^3\dfrac{q^2}{4\pi\varepsilon_0\eta^2(\eta-1)k}}+k\right)x$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2}=-\left(\frac{2(\eta-1)}{\eta}k+k\right)x=-\frac{3\eta-2}{\eta}kx$$

$$\Rightarrow\quad \frac{d^2x}{dt^2}=-\frac{3\eta-2}{\eta}\frac{k}{m}x$$

$$\Rightarrow\quad \omega^2=\frac{3\eta-2}{\eta}\frac{k}{m}$$

$$\Rightarrow\quad f=\frac{1}{2\pi}\sqrt{\frac{3\eta-2}{\eta}\frac{k}{m}}$$

$$\Rightarrow\quad \frac{f}{f_0}=\sqrt{\frac{3\eta-2}{\eta}}$$

Thus the frequency is increased $\sqrt{\dfrac{3\eta-2}{\eta}}$ times

Here $\eta=2$ and so frequency increases $\sqrt{2}$ times.

PROBLEM 3

A point charge q is located at the centre of a thin ring of radius R with uniformly distributed charge $-q$. Find the magnitude of the electric field strength vector at the point lying on the axis of the ring at a distance x from its centre if $x \gg R$.

SOLUTION

Electric field at P due to ring

$$E_1=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$$

{charged toward centre as it is negatively}

Electric field at P due to $+q$

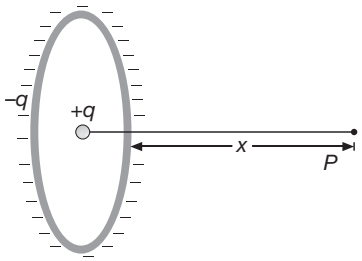
$$E_2=\frac{q}{4\pi\varepsilon_0x^2}$$ {away from centre}

Thus net field at P is

$$\vec{E}_{\text{net}}=\vec{E}_2+\vec{E}_1$$ {vectorially}

$$\Rightarrow\quad E_{\text{net}}=E_2-E_1$$

$$\Rightarrow\quad E_{\text{net}}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x^2}-\frac{x}{(x^2+R^2)^{3/2}}\right)$$



For $x \gg R$,

$$E_{\text{net}}=\frac{q}{4\pi\varepsilon_0x^2}\left[1-\left(1+\frac{R^2}{x^2}\right)^{-\frac{3}{2}}\right]$$

$$\Rightarrow\quad E_{\text{net}}=\frac{q}{4\pi\varepsilon_0x^2}\left[1-\left(1-\frac{3}{2}\frac{R^2}{x^2}\right)\right]$$

{using Binomial Approximation}

$$\Rightarrow\quad F_{\text{net}}=\frac{3qR^2}{8\pi\varepsilon_0x^4}$$

PROBLEM 4

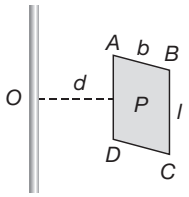
Find the electric flux crossing the wire frame $ABCD$ of length l width b and whose centre is at a distance $OP=d$ from an infinite line of charge with linear charge density λ. Consider that the plane of frame is perpendicular to the line OP.

SOLUTION

$$\phi_e = 2\int_0^{b/2} d\phi_e = 2\int_0^{b/2}\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)(l\,dx)\cos\theta$$

where, $r = \sqrt{d^2 + x^2}$

and $\cos\theta = \dfrac{d}{r} = \dfrac{d}{\sqrt{d^2 + x^2}}$



$$\Rightarrow \quad \phi = 2\int_0^{b/2}\left(\frac{\lambda l d}{2\pi\varepsilon_0}\right)\frac{dx}{d^2 + x^2}$$

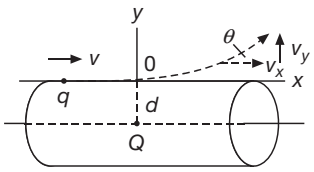
$$\Rightarrow \quad \phi = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\left(\frac{b}{2d}\right)$$

PROBLEM 5

A particle of mass m and charge q moves at high speed along the x axis. It is initially near $x \to -\infty$, and it ends up near $x \to +\infty$. A second charge Q is fixed at the point $x = 0$, $y = -d$. As the moving charge passes the stationary charge, its x component of velocity does not change appreciably, but it acquires a small velocity in the y direction. Determine the angle through which the moving charge is deflected.

SOLUTION

The vertical velocity component of the moving charge increases according to



$$m\frac{dv_y}{dt} = F_y$$

$$\Rightarrow \quad m\frac{dv_y}{dx}\frac{dx}{dt} = qE_y$$

Now $\dfrac{dx}{dt} = v_x$ has the nearly constant value v

$$\Rightarrow \quad dv_y = \frac{q}{mv}E_y dx$$

$$\Rightarrow \quad v_y = \int_0^{v_y} dv_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y dx \qquad \ldots(1)$$

The radially outward component of the electric field varies along the x-axis, and is described by using Gauss's Theorem

$$\int_{-\infty}^{\infty} E_y dA = \int_{-\infty}^{\infty} E_y (2\pi d)\,dx = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} E_y dx = \frac{Q}{2\pi d\varepsilon_0}$$

and hence from equation (1), we get

$$v_y = \frac{qQ}{mv2\pi d\varepsilon_0}$$

The angle of deflection is described by

$$\tan\theta = \frac{v_y}{v} = \frac{qQ}{2\pi\varepsilon_0 dmv^2}$$

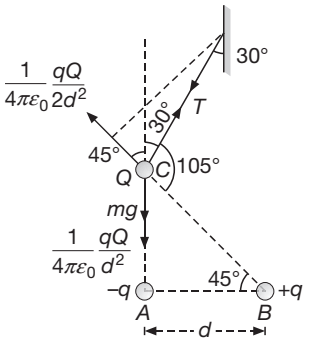
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{qQ}{2\pi\varepsilon_0 dmv^2}\right)$$

PROBLEM 6

Two small balls A and B with charges $-q$ and $+q$ respectively are fixed on a horizontal plane at a distance d from each other. A third ball C with charge $+Q$ is suspended from a string. The string makes an angle of $30°$ with the vertical when the ball C is in equilibrium at a height d vertically above the ball A. When the ball C is in an identical situation above ball B, find the angle which the string now makes with the vertical.

SOLUTION

Since ball C is in equilibrium, the sum of torques of all the forces about the point of suspension must be zero.



$$\frac{KqQ}{2d^2}l\sin 75° = \left(\frac{KqQ}{d^2} + mg\right)l\sin 30° \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{KqQ}{2d^2}\cos 15° - \frac{KqQ}{2d^2} = \frac{mg}{2}$$

$$\Rightarrow \quad Q = \frac{mg}{\frac{Kq}{2d^2}(\cos 15° - 1)\times 2}$$

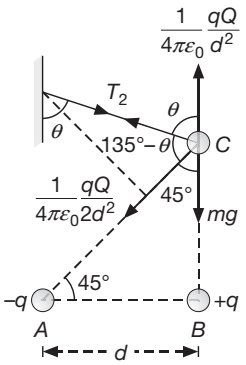
$$\Rightarrow \quad Q = \frac{mgd^2}{Kq(\cos 15° - 1)} \qquad \ldots(2)$$

Similarly, in second case,

$$\left(\frac{KqQ}{d^2} - mg\right)l\sin\theta = \frac{KqQ}{2d^2}l\sin(135° - \theta)$$

Since $\sin(135° - \theta) = \cos(45° - \theta)$

$$\Rightarrow \quad \left(\frac{KqQ}{d^2} - mg\right)l\sin\theta = \frac{KqQ}{2d^2}l\cos(45° - \theta) \qquad \ldots(3)$$



Substituting the value of charge Q from equation (2) in (3), we get

$$\left(\frac{mg}{(\cos 15^o - 1)} - mg\right)\sin\theta = \frac{mg\cos(45° - \theta)}{2(\cos 15° - 1)}$$

$$\Rightarrow \quad \left(\frac{1-(\cos 15° - 1)}{(\cos 15° - 1)}\right) = \frac{\cos 45°\cos\theta + \sin 45°\sin\theta}{2(\cos 15° - 1)\sin\theta}$$

$$\Rightarrow \quad (2 - \cos 15°) = \frac{\cos 45°\cot\theta + \sin 45°}{2}$$

$$\Rightarrow \quad (\cot\theta + 1) = 2\sqrt{2}(2 - \cos 15°)$$

$$\Rightarrow \quad \cot\theta = 2\sqrt{2}(2 - \cos 15°) - 1$$

Since $\cos(15°) = \cos(45° - 30°)$

$$\Rightarrow \quad \cos(15°) = \cos 45°\cos 30° + \sin 45°\sin 30°$$

$$\Rightarrow \quad \cos(15°) = \frac{1}{2\sqrt{2}}\left(\sqrt{3} + 1\right)$$

$$\Rightarrow \quad \cot\theta = 2\sqrt{2}\left(2 - \frac{1}{2\sqrt{2}}\left(\sqrt{3} + 1\right)\right) - 1$$

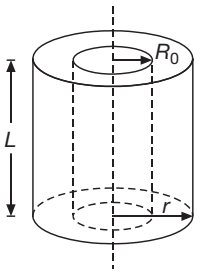
$$\Rightarrow \quad \cot\theta = \left(4\sqrt{2} - \sqrt{3} - 1\right) - 1$$

$$\Rightarrow \quad \cot\theta = 4\sqrt{2} - \sqrt{3} - 2$$

$$\Rightarrow \quad \theta = \cot^{-1}\left(4\sqrt{2} - \sqrt{3} - 2\right)$$

PROBLEM 7

A straight infinitely long cylinder of radius R_0 is uniformly charged with charge density σ. The cylinder serves as a source of electrons, with the velocity vector of emitted electrons perpendicular to its surface. What must be the electron velocity to ensure that the electron can move away from the axis of the cylinder to a distance greater than r.



SOLUTION

Let us, first of all determine electric field at a distance r from the cylinder. For that we consider a coaxial cylinder as a Gaussian surface, then according to Gauss's Law, we have

$$\varepsilon_0(2\pi rL)E = (2\pi R_0 L)\sigma$$

$$\Rightarrow \quad E = \frac{\sigma R_0}{\varepsilon_0 r} \qquad \ldots(1)$$

Applying Law of Conservation of Energy, we get

$$\frac{1}{2} m_e v_0^2 - eV_0 = -eV \qquad \ldots(2)$$

where V_0 is the potential of cylinder and V is the potential at a distance r from the cylinder's axis. Employing the relationship $E = -\frac{dV}{dr}$, we have

$$\frac{R_0 \sigma}{\varepsilon_0 r} = -\frac{dV}{dr} \qquad \ldots(3)$$

Integrating, we get

$$V = -\frac{R_0 \sigma}{\varepsilon_o} \log_e r + C \qquad \ldots(4)$$

where C is constant of integration. Also, from equation (4), we get

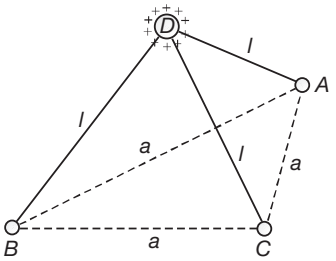
$$V_0 = -\frac{R_0 \sigma}{\varepsilon_0} \log_e (R_0) + C \qquad \ldots(5)$$

Substituting values of V and V_0 in (2), we get

$$v_0 = \sqrt{\frac{2eR_0\sigma}{\varepsilon_0 m_e} \log_e \left(\frac{r}{R_0}\right)}$$

PROBLEM 8

Three small identical neutral metal balls are at the vertices of an equilateral triangle. The balls are connected to a large charged sphere held above the plane of the triangle. The first and the second ball acquire charge q_1 and q_2 respectively. How much is q_3 ? The connecting wires are very thin.



SOLUTION

When a small ball is connected to a large charged sphere, the potential of the sphere will remain unchanged. Let l be the distance of each small ball from the large sphere D, r = radius of each small ball, a = each side of the equilateral triangle. Let Q be the charge on the sphere and V be its potential. First consider sharing of charge between the sphere and the first ball, i.e., between D and A. Then

$$\begin{pmatrix}\text{Potential}\\ \text{of small}\\ \text{sphere } A\end{pmatrix} = \begin{pmatrix}\text{Potential}\\ \text{due to}\\ \text{its own}\\ \text{charge}\end{pmatrix} + \begin{pmatrix}\text{Potential}\\ \text{due to}\\ \text{charge on the}\\ \text{sphere } D\end{pmatrix}$$

$$\Rightarrow \quad V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{l}$$

But potential of the ball is same as that of the sphere

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{Q}{l}\right) \qquad \ldots(1)$$

Next consider sharing of charge between $(A+D)$ and B.

The potential of B is $V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r} + \frac{q_1}{a} + \frac{Q}{l}\right)$

But potential of B is same as that of $(A+D)$, so

$$V_B = V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r} + \frac{q_1}{a} + \frac{Q}{l}\right) \qquad \ldots(2)$$

Consider sharing of charge between $(A+D+B)$ and C.

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3}{r} + \frac{q_1}{a} + \frac{q_2}{a} + \frac{Q}{l}\right) \qquad \ldots(3)$$

From equations (1) and (2), we get

$$\frac{q_1}{r} = \frac{q_2}{r} + \frac{q_1}{a}$$

$$\Rightarrow \quad \frac{r}{a} = \frac{q_1 - q_2}{q_1}$$

From equations (1) and (3), we get

$$\frac{q_1}{r} = \frac{q_3}{r} + \frac{q_1 + q_2}{a}$$

$$\Rightarrow \quad \frac{r}{a} = \frac{q_1 - q_3}{q_1 + q_2}$$

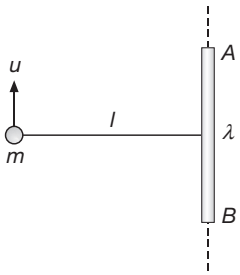
$$\Rightarrow \quad \frac{q_1 - q_2}{q_1} = \frac{q_1 - q_3}{q_1 + q_2}$$

$$\Rightarrow \quad q_1^2 - q_2^2 = q_1^2 - q_1 q_3$$

$$\Rightarrow \quad q_3 = \frac{q_2^2}{q_1}$$

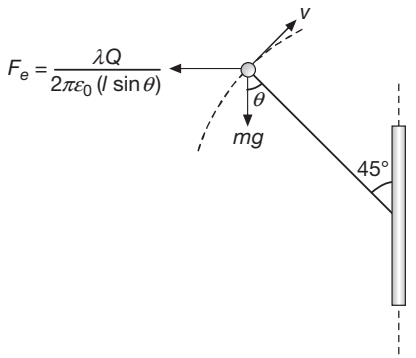
PROBLEM 9

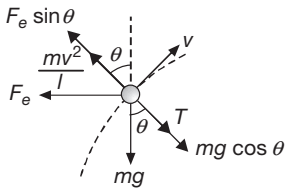
AB is a vertical rigid thin infinite wire carrying a linear charge of density $\lambda = 10\ \mu\text{Cm}^{-1}$. A particle having mass $m = 2$ g and charge is fixed to the wire by means of a light, insulating and inextensible string having length $l = 2\sqrt{2}$ m. Find the vertical velocity u with which it should be projected under gravity from the shown position so that the string slacks when its angle with vertical becomes 45°.



SOLUTION

Let us visualise the situation by drawing a diagram showing all the forces acting on the particle.





The electric field, when the particle is at a perpendicular distance $l\sin\theta$ from the wire, is

$$E = \frac{\lambda}{2\pi\varepsilon_0 (r_\perp)} = \frac{\lambda}{2\pi\varepsilon_0 (l\sin\theta)}$$

So, the electrostatic force F_e on the charge Q is

$$F_e = \frac{\lambda Q}{2\pi\varepsilon_0 (l\sin\theta)}$$

If the string slacks at $\theta = 45°$, then $T = 0$

$$\Rightarrow \quad mg\cos\theta = \frac{mv^2}{l} + \frac{\lambda Q\sin\theta}{2\pi\varepsilon_0 (l\sin\theta)} \qquad \ldots(1)$$

Using Work Energy principle, we have

$$\begin{pmatrix}\text{Work done}\\ \text{by gravity}\end{pmatrix} + \begin{pmatrix}\text{Work done}\\ \text{by electrostatic}\\ \text{forces}\end{pmatrix} = \begin{pmatrix}\text{Change}\\ \text{in } KE\end{pmatrix}$$

$$\Rightarrow \quad -mgl\cos\theta + \frac{1}{2\pi\varepsilon_0}\int_{l}^{l\sin\theta} \frac{Q(\lambda dx)}{x} = \frac{1}{2}m\left(v^2 - u^2\right)$$

$$\Rightarrow \quad -mgl\cos\theta + \left(\frac{\lambda Q}{2\pi\varepsilon_0}\right)\log_e(\sin\theta) = \frac{1}{2}m\left(v^2 - u^2\right)$$

Using (1), we get

$$-2mgl\cos\theta + \frac{\lambda Q}{\pi\varepsilon_0}\log_e(\sin\theta) = mgl\cos\theta - \frac{\lambda Q}{2\pi\varepsilon_0} - mu^2$$

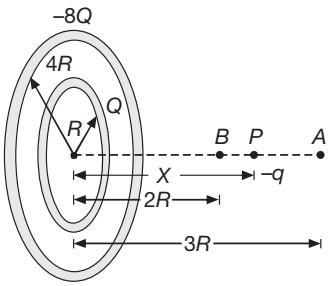
$$\Rightarrow \quad mu^2 = 3ml\cos\theta - 2\frac{1}{4\pi\varepsilon_0}\lambda Q\left[2\log_e(\sin\theta) + 1\right]$$

$$\Rightarrow \quad u = \sqrt{\frac{3gl}{\sqrt{2}} - \frac{\lambda Q}{2\pi\varepsilon_0 m}\left(-\log_e 2 + 1\right)}$$

$$\Rightarrow \quad u = 5.7\ \text{ms}^{-1}$$

PROBLEM 10

Two thin concentric rings are placed in a gravity free region in yz- plane, one of radius R carries a charge $+Q$ and the second of radius $4R$ and charge $-8Q$ distributed uniformly over it. Find the minimum velocity with which a point charge of mass m and charge $-q$ should be projected from a point at a distance $3R$ from the centre of the ring on its axis so that it will reach to the centre of the rings.



SOLUTION

Note that at a point distant $3R$ charge $-q$ will have repulsive force but on points close to the centre it will have an attraction force. First we have to find out a point where the electric field is zero because, beyond that, the charge will have attraction force and will be attracted to the centre of the rings. Electric field at a point P distant x from the centre.

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{\frac{3}{2}}} - \frac{1}{4\pi\varepsilon_0}\frac{8Qx}{(16R^2+x^2)^{\frac{3}{2}}} = 0$$

On solving for x, we get

$$x = 2R$$

We have to give the charge sufficient minimum KE so that it could reach this point, because for $x < 2R$ the field will be attractive.

Potential at A is

$$V_A = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{\sqrt{9R^2+R^2}} - \frac{8Q}{\sqrt{9R^2+16R^2}}\right]$$

$$\Rightarrow \quad V_A = -\frac{1}{4\pi\varepsilon_0}\left(\frac{16-\sqrt{10}}{10}\right)\frac{Q}{R}$$

Potential at point B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{4R^2+R^2}} - \frac{1}{4\pi\varepsilon_0}\frac{8Q}{\sqrt{4R^2+16R^2}} = -\frac{1}{4\pi\varepsilon_0}\frac{3}{\sqrt{5}}\frac{Q}{R}$$

From Conservation of Energy, we have

$$W_{A\to B} = q(V_B - V_A)$$

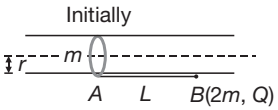
$$\text{Now } \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}\left[\frac{3}{\sqrt{5}} - \frac{16-\sqrt{10}}{10}\right]$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}\left[\frac{6\sqrt{5}+\sqrt{10}-16}{10}\right]$$

$$\Rightarrow \quad v_{\min} = \sqrt{\frac{Qq}{20\pi\varepsilon_0 mR}(6\sqrt{5}+\sqrt{10}-16)}$$

PROBLEM 11

In the figure shown R is a long smooth fixed non-conducting rod of uniform linear charge density λ and radius of cross-section r. A is non-conducting uncharged smooth ring of mass m which fits completely on the rod and can move horizontally on the rod. B is a non-conducting charged small particle of mass $2m$ and charge Q and connected to A by an inextensible light string of length l. B is released from rest from the position shown in figure. Find velocity of B and tension in the string when the string becomes vertical. Assume gravitational acceleration g and $\lambda Q > 0$.

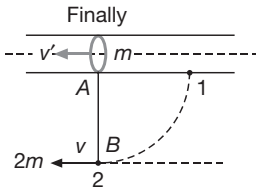


SOLUTION

The particle of charge q, mass $2m$ is released from rest (from 1) and it goes to 2, where it has a horizontal velocity v. Also, $2m$ is attached through string to m (the smooth ring) capable of moving along the rod. So, let the velocity of the ring, when the string becomes vertical be v'. Then by Law of Conservation of Linear Momentum (applied only along the horizontal)

$$mv' = (2m)v$$

$$\Rightarrow \quad v' = 2v$$



Further in going from 1 to 2, we have

$$W_{1\to 2} = q(V_2 - V_1) + \text{Work done by Gravitational Force}$$

$$\Rightarrow \quad \Delta K = Q\left[\frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r+L}{r}\right)\right] + (2m)gL$$

$$\left(\begin{matrix}\text{Change}\\ \text{in } KE\end{matrix}\right) = \left(\begin{matrix}\text{Work done}\\ \text{by electrostatic}\\ \text{forces}\end{matrix}\right) + \left(\begin{matrix}\text{Work done}\\ \text{by gravity}\end{matrix}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv'^2 + \frac{1}{2}(2m)v^2 = \frac{Q\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r+L}{r}\right) + (2m)gL$$

$$\Rightarrow \quad \frac{1}{2}m(4v^2) + \frac{1}{2}(2m)v^2 = \frac{Q\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r+L}{r}\right) + 2mgL$$

$$\Rightarrow \quad 3mv^2 = \frac{Q\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r+L}{r}\right) + 2mgL$$

$$\Rightarrow \quad v=\sqrt{\frac{2gL}{3}+\frac{Q\lambda}{6\pi\varepsilon_0 m}\log_e\left(\frac{r+L}{r}\right)}$$

Now, tension in the string, when it becomes vertical is

$$T=(2m)g+\frac{m\left(v_{\text{net}}^2\right)}{L}+QE$$

where $v_{\text{net}}=v+v'=3v$ and $E=\dfrac{\lambda}{2\pi\varepsilon_0(r+L)}$

$$\Rightarrow \quad T=2mg+\frac{9mv^2}{L}+\frac{\lambda Q}{2\pi\varepsilon_0(r+L)}$$

PROBLEM 12

A uniform electric field of strength $10^6\ \text{Vm}^{-1}$ is directed vertically downwards. A particle of mass 0.01 kg and charge 10^{-6} coulomb is suspended by an inextensible thread of length 1 m. The particle is displaced slightly from its mean position and released. Calculate the time period of its oscillation. What minimum velocity should be given to the particle at rest so that it completes a full circle in vertical plane without the thread getting slack? Calculate the maximum and minimum tension in the thread in this situation.

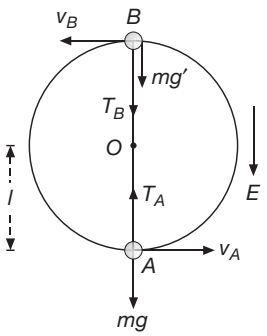
SOLUTION

The situation is shown in figure
Acceleration a due to electric field

$$a=\frac{qE}{m}=\frac{10^{-6}\times10^6}{0.01}=100\ \text{ms}^{-2}$$

Thus effective acceleration would be

$$g'=g+\frac{qE}{m}=9.8+100=109.8\ \text{ms}^{-2}$$



Now time period of particle would be given as

$$T=2\pi\sqrt{\left(\frac{l}{g'}\right)}=2\pi\sqrt{\left(\frac{1}{109.8}\right)}$$

$$T=0.6\ \text{s}$$

Now from figure, at point A we have

$$\frac{mv_A^2}{l}=T_A-mg'=T_A-(qE+mg) \qquad \ldots(1)$$

and $\dfrac{mv_B^2}{l}=T_B+(qE+mg)$

To complete the circle at point B tension T_B should be zero, thus we have

$$\frac{mv_{B_{\min}}^2}{l}=qE+mg \qquad \ldots(2)$$

Using Work Energy Theorem at points A and B, we have

$$\frac{1}{2}mv_A^2-2mgl-2gEl=\frac{1}{2}mv_B^2+U_B \qquad \ldots(3)$$

From equation (2) and (3), we get

$$v_A^2=\frac{5(qE+mg)l}{m}=5g'l=5\times109.8\times1$$

$$\left\{\text{where } g'=\frac{qE+mg}{m}\right\}$$

$$\Rightarrow \quad v_A=23.42\ \text{ms}^{-1}$$

Now, $T_A=(qE+mg)+\dfrac{mv_A^2}{l}$

$$T_A=6(qE+mg)=6(1+0.098)$$

$$T_A=6.588\ \text{N}$$

PROBLEM 13

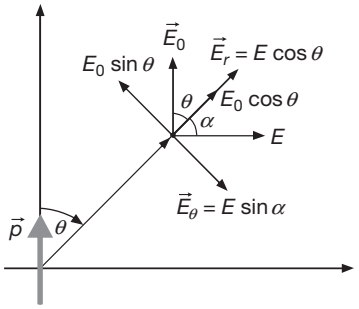
A point electric dipole with a moment p is placed in the external uniform electric field whose strength equals E_0 with p parallel to E_0. In this case one of the equipotential surfaces enclosing the dipole forms a sphere. Find radius of this sphere.

SOLUTION

The electric field due to dipole is given by

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p}{R^3}\sqrt{1+3\cos^2\theta}$$

and its direction with the radius vector $\vec{r}$ is given by $\tan\alpha = \frac{1}{2}\tan\theta$. Now given that external field $\vec{E}_0$ is parallel to $\vec{p}$. So that $\vec{E}_r$ and $\vec{E}_\theta$ are the radial and transverse component of electric field due to the dipole and $E_0 \sin\theta$ and $E_0 \cos\theta$ are two rectangular components of external field E_0. Since at equipotential surface, there is no tangential components of electric field. So $E_0 \sin\theta$ must be equal and opposite to $E_0 \sin\alpha$



Hence $E_0 \sin\theta = E\sin\alpha$

$$\Rightarrow \quad E_0 = \frac{E\sin\alpha}{\sin\theta}$$

$$\Rightarrow \quad E_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{p}{R^3}\sqrt{1+3\cos^2\theta}\right]\frac{\sin\theta}{\sqrt{1+3\cos^2\theta}} \times \frac{1}{\sin\theta}$$

$$\left\{\because \tan\alpha = \frac{1}{2}\tan\theta\right\}$$

$$\Rightarrow \quad E_0 4\pi\varepsilon_0 R^3 = p$$

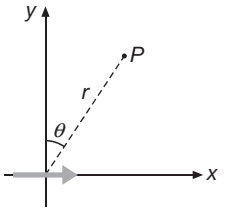
$$\Rightarrow \quad R = \sqrt[3]{\frac{p}{4\pi\varepsilon_0 E_0}}$$

PROBLEM 14

A short electric dipole is situated at the origin of coordinate axis with its axis along x-axis and equator along y-axis. It is found that the magnitudes of the electric field intensity and electric potential due to the dipole are equal at a point distant $r = \sqrt{5}$ m from origin. Find the position vector of this point.

SOLUTION

Consider a point P at distance r and angle θ from equation. Since



$$|E_P| = |V_P|$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\sin^2\theta} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^2}$$

$$\Rightarrow \quad \frac{\sqrt{1+3\sin^2\theta}}{\sqrt{5}} = \sin\theta$$

$$\Rightarrow \quad 1+3\sin^2\theta = 5\sin^2\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Positive vector $\vec{r}$ of point P is

$$\vec{r} = \sqrt{5}\left(\cos 45\hat{i} + \sin 45\hat{j}\right)6 = \sqrt{\frac{5}{2}}\left(\hat{i}+\hat{j}\right)$$

$$\Rightarrow \quad \vec{r} = \sqrt{\frac{5}{2}}\left(\hat{i}+\hat{j}\right)$$

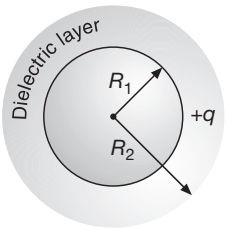
PROBLEM 15

Find the potential of an isolated ball shaped conductor of radius R_1 surrounded by an adjacent concentric layer of dielectric with dielectric constant K and outer radius R_2.

SOLUTION

Let a charge q be assumed to be given to the conductor

Since, $V = -\int E \cdot dr$



$$\Rightarrow \quad V = -\left[\int_{\infty}^{R_2}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr + \int_{R_2}^{R_1}\frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2}dr\right]$$

$$\Rightarrow \quad V = -\frac{q}{4\pi\varepsilon_0}\left[\int_{\infty}^{R_2}\frac{dr}{r^2} + \int_{R_2}^{R_1}\frac{dr}{Kr^2}\right]$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\left(-\frac{1}{r}\right)_{\infty}^{R_2}+\left(-\frac{1}{Kr}\right)_{R_2}^{R_1}\right]$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R_2}+\frac{1}{KR_1}-\frac{1}{KR_2}\right]$$

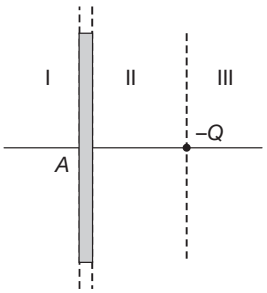
$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{KR_1+R_2-R_1}{KR_1R_2}\right]$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{R_1(K-1)+R_2}{KR_1R_2}\right]$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\left(\frac{K-1}{K}\right)\frac{1}{R_2}+\frac{1}{KR_1}\right]$$

PROBLEM 16

An infinitely long conducting wire of charge density $+\lambda$ and a point charge $-Q$ are at a distance from each other. In which of the three Regions (I, II or III) are there points that lie on the line passing through point charge perpendicular to the conductor and at which the field is zero?



SOLUTION

FOR REGION I

Now we check the region I, take a point to the left of wire at a distance x from it. The resultant field is

$$\vec{E}_R=\frac{\lambda}{2\pi\varepsilon_0 x}(-\hat{i})+\frac{Q}{4\pi\varepsilon_0(a+x)^2}\hat{i}$$

The two fields point in the opposite directions, so resultant field can be zero if,

$$\frac{\lambda}{2\pi\varepsilon_0 x}=\frac{Q}{4\pi\varepsilon_0(a+x)^2}$$

$$\Rightarrow \quad x^2+\left(2a-\frac{Q}{2\lambda}\right)x+a^2=0$$

$$\Rightarrow \quad x=\frac{1}{2}\left(\frac{Q}{2\lambda}-2a\right)\pm\sqrt{\frac{1}{4}\left(\frac{Q}{2\lambda}-2a\right)^2-a^2}$$

If the discriminant of the quadratic equation is real, we have two points where the field is zero. Discriminant is positive for $Q\geq 8a\lambda$.

FOR REGION II

In the region (II) the electric field of wire and point charge point in the same direction, positive x-axis. So no point can exist where the field is zero.

FOR REGION III

Now we take a point to the right of the point charge, at a distance x from it. Resultant field at this point is

$$\vec{E}_R=\frac{\lambda}{2\pi\varepsilon_0(x+a)}\hat{i}+\frac{Q}{4\pi\varepsilon_0 x^2}(-\hat{i})$$

Resultant field is zero if

$$\frac{\lambda}{(a+x)}=\frac{Q}{2x^2}$$

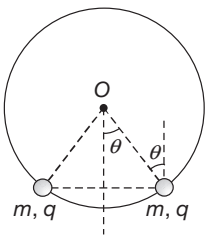
On solving the quadratic in x, we have

$$x=\frac{Q}{4\lambda}\pm\sqrt{\frac{Q^2}{16\lambda^2}+\frac{aQ}{2\lambda}}$$

The negative sign infront of the radical has no meaning because it would mean that the point is to the left of point charge, where field of wire and point charge are added, the magnitudes of the two fields are zero.

PROBLEM 17

Figure shows two identical beads of mass m and charge q. The beads can slide smoothly on a wire frame kept in a vertical frame.

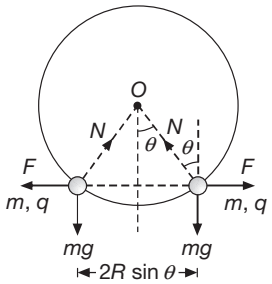


(a) Determine angular position θ w.r.t. vertical diameter.

(b) Now the beads are given a small angular displacement. Show that they perform simple harmonic motion.

SOLUTION

(a) Each bead is in equilibrium under the action of three forces
Coulomb force of repulsion, Weight mg and Normal reaction, N



From conditions of equilibrium, we have

$$N\sin\theta = F = \frac{q^2}{4\pi\varepsilon_0(2R\sin\theta)} \quad \ldots(1)$$

and $N\cos\theta = mg$ $\ldots(2)$

From equations (1) and (2), we have

$$\tan\theta = \frac{q^2}{4\pi\varepsilon_0 mg(2R\sin\theta)} \quad \ldots(3)$$

(b) After a small angular displacement, we may consider the Coulombic force F to be approximately constant.
In equilibrium, net torque about centre of circle is zero. So we have

$$-mgR\sin\theta + FR\cos\theta = 0 \quad \ldots(4)$$

After a small angular displacement ϕ of right bead, we have

$$-mgR\sin(\theta+\phi) + FR\cos(\theta+\phi) = I\alpha = mR^2\alpha \quad \ldots(5)$$

$$\Rightarrow \quad -mgR(\sin\theta\cos\phi + \cos\theta\sin\phi) + FR(\cos\theta\cos\phi - \sin\theta\sin\phi) = mR^2\alpha \quad \ldots(6)$$

For small ϕ, $\sin\phi \simeq \phi$ and $\cos\phi \simeq 1$, so equation (6) reduces to

$$-mgR(\sin\theta + \phi\cos\theta) + FR(\cos\theta - \phi\sin\theta) = mR^2\alpha \quad \ldots(7)$$

From equations (4) and (7), we get

$$-(mgR\cos\theta + FR\sin\theta)\phi = mR^2\alpha$$

$$\Rightarrow \quad \alpha = -\left(\frac{mgR\cos\theta + FR\sin\theta}{mR^2}\right)\phi$$

$$\Rightarrow \quad \ddot{\phi} + \left(\frac{mR\cos\theta + FR\sin\theta}{mR^2}\right)\phi = 0$$

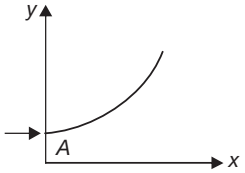
Comparing with standard equation of SHM i.e., $\ddot{\phi} + \omega^2\phi = 0$, we get

$$\omega = \sqrt{\frac{mgR\cos\theta + \dfrac{q^2}{4\pi\varepsilon_0(2R\sin\theta)^2}\sin\theta}{mR^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{8\pi\varepsilon_0 R^3 mg\sin^2\theta + q^2}{16\pi\varepsilon_0 mR^2\sin\theta}}$$

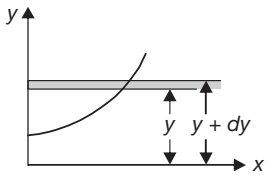
PROBLEM 18

In an insulating medium (di-electric constant $K = 1$) volumetric charge density varies with y-coordinate according to the law $\rho = ay$. A particle of mass m having positive charge q is at point $A(0, y_0)$ and projected with velocity $\vec{v} = v_0\hat{i}$ as shown in figure. Neglecting gravity and frictional resistance of the medium and assuming electric field strength to be zero at $y = 0$, calculate the slope of the trajectory of the particle as a function of y.



SOLUTION

As charge density varies with y-coordinate only, electric field must be directed along y-axis. Consider a thin layer of medium, having thickness dy at a distance y from x-axis as shown in figure.



Let strength of electric field at positions y and $(y+dy)$ be E and $(E + dE)$ respectively.

Then for area A of the layer, according to Gauss's Law,

$$(E+dE)A-EA=\frac{\rho A\,dy}{\varepsilon_0}$$

$$\Rightarrow \quad dE=\frac{\rho}{\varepsilon_0}dy=\frac{a}{\varepsilon_0}y\,dy$$

Integrating above equation with in appropriate limits, we get

$$\int_0^E dE=\frac{a}{\varepsilon_0}\int_0^y ydy$$

$$\Rightarrow \quad E=\frac{ay^2}{2\varepsilon_0} \qquad \ldots(1)$$

There is no force on the particle along x-axis, velocity v_0 of the particle along x-axis remains constant
Due to electric field E, the particle accelerates along positive y-direction with acceleration a_y, given by

$$a_y=\frac{qE}{m}=\frac{qay^2}{2m\varepsilon_0}$$

Let y-component of velocity of particle be v, then from Newton's Second Law we get

$$v\frac{dv}{dy}=\frac{qay^2}{2m\varepsilon_0}$$

$$\Rightarrow \quad v\,dv=\frac{qa}{2m\varepsilon_0}y^2\,dy$$

Since at point $A(y=y_0)$, y-component of velocity is $v=0$

$$\Rightarrow \quad \int_0^v v\,dv=\frac{qa}{2m\varepsilon_0}\int_{y_0}^y y^2dy$$

$$\Rightarrow \quad v=\sqrt{\frac{qa}{3m\varepsilon_0}\left(y^3-y_0^3\right)}$$

$$\text{Slope of trajectory}=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{v}{v_0}$$

$$\Rightarrow \quad \text{Slope}=\frac{dy}{dx}=\sqrt{\frac{qa}{3m\varepsilon_0 v_0^2}\left(y^3-y_0^3\right)}$$

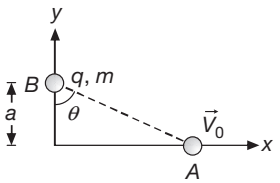
PROBLEM 19

A charged particle having a charge q moves along the x-axis with a constant velocity v_0. Another particle B with charge q and mass m is lying on the y-axis at $y=a$. The particle B is constrained to move along the y-axis, while the particle A moves along the x-axis.

Assuming that the velocity v_0 is very large, find the impulse imparted to B along the y-axis as the particle A moves from $-\infty$ to ∞ assuming that the motion of particle B is negligible.

SOLUTION

First of all, let us determine the angular speed of A relative to B at angular position θ shown in figure.



For particle A

$$\frac{dx}{dt}=\frac{d}{dt}(a\tan\theta)=v_0 \qquad \ldots(1)$$

$$\Rightarrow \quad a\sec^2\theta\frac{d\theta}{dt}=v_0$$

$$\Rightarrow \quad \left(\frac{d\theta}{dt}\right)=\frac{v_0}{a}\cos^2\theta \qquad \ldots(2)$$

Since particle A is moving very fast, we can assume that when it crosses along the x-axis, the y-component of the force on B due to A is

$$F_y=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2\sec^2\theta}\times\cos\theta \qquad \ldots(3)$$

$$\text{Impulse on } B=\int F_y dt=\int\frac{F_y d\theta}{\left(\frac{d\theta}{dt}\right)}$$

$$\Rightarrow \quad B=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\frac{a}{v_0}\int\frac{\cos^3\theta}{\cos^2\theta}d\theta$$

$$\Rightarrow \quad B=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{av_0}\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta$$

The impulse delivered to B, is given by

$$\Delta P_y = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{av_0} = \frac{q^2}{2\pi\varepsilon_0(av_0)} \qquad \ldots(4)$$

PROBLEM 20

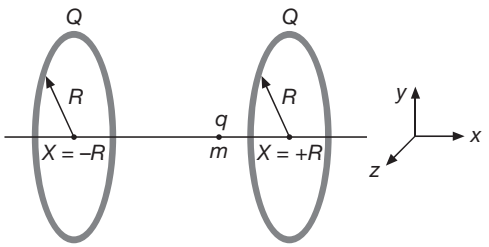
A particle of mass m and charge $+q$ is constrained to move along the x-axis. Two charged rings of radius R, charge Q are placed with their centres at $(-R,\ 0)$ and $(R,\ 0)$ as shown.

(a) Obtain an expression for the potential due to the rings as a function of x for $-R < x < R$.

(b) Show that in this region $V(x)$ is minimum at $x = 0$.

(c) Show that for $x \ll R$, the potential is of the form $V(x) = V(0) + \alpha x^2$.

(d) Derive an expression for the time period of oscillation of the mass m when it is displaced slightly from the origin and released.



SOLUTION

Electric potential due to a ring on its axis is given by

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R^2 + x^2)^{\frac{1}{2}}}$$

(a) At any point $(x,\ 0,\ 0)$ the potential due to both rings

$$V(x) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2 + (R-x)^2}} + \frac{1}{\sqrt{R^2 + (R+x)^2}}\right]$$

(b) For minimum $V(x)$,

$$\frac{d}{dx}[V(x)] = 0$$

$$\Rightarrow \quad \frac{dV(x)}{dx} = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{2}\frac{2(R-x)(-1)}{\left[R^2 + (R-x)^2\right]^{\frac{3}{2}}} - \frac{1}{2}\frac{2(R+x)}{\left[R^2 + (R+x)^2\right]^{\frac{3}{2}}}\right]$$

$$\Rightarrow \quad \frac{dV(x)}{dx} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R-x}{\left[R^2 + (R-x)^2\right]^{\frac{3}{2}}} - \frac{R+x}{\left[R^2 + (R+x)^2\right]^{\frac{3}{2}}}\right]$$

The expression at $x = 0$ is

$$\frac{dV}{dx} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R}{(2R^2)^{\frac{3}{2}}} - \frac{R}{(2R^2)^{\frac{3}{2}}}\right] = 0$$

Thus $V(x)$ is minimum at $x = 0$

$$V_{\min} = V(0) = \frac{Q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{R}$$

(c) For $x \ll R$,

$$V(x) = \frac{Q}{4\pi\varepsilon_0}\left\{\left[R^2 + (R-x)^2\right]^{-\frac{1}{2}} + \left[R^2 + (R+x)^2\right]^{-\frac{1}{2}}\right\}$$

$$\Rightarrow \quad V(x) = \frac{Q}{4\pi\varepsilon_0 R}\left\{\left[1 + \left(1 - \frac{x}{R}\right)^2\right]^{-\frac{1}{2}} + \left[1 + \left(1 + \frac{x}{R}\right)^2\right]^{-\frac{1}{2}}\right\}$$

Now we apply the binomial approximation, we have $(1+x)^n \simeq 1 + nx$ when $|x| \ll 1$

$$\left(1 - \frac{x}{R}\right)^2 \simeq 1 - \frac{2x}{R} \text{ and } \left(1 + \frac{x}{R}\right)^2 \simeq 1 + \frac{2x}{R}$$

$$\Rightarrow \quad V(x) = \frac{Q}{4\pi\varepsilon_0 R}\left[\left(2 - \frac{2x}{R}\right)^{-\frac{1}{2}} + \left(2 + \frac{2x}{R}\right)^{-\frac{1}{2}}\right]$$

Now we apply the binomial expansion,

$$\left(1-\frac{x}{R}\right)^{-\frac{1}{2}}=1-\frac{x}{2R}+\frac{3}{8}\frac{x^2}{R^2}-\cdots$$

$$\left(1+\frac{x}{R}\right)^{-\frac{1}{2}}=1+\frac{x}{2R}+\frac{3}{8}\frac{x^2}{R^2}+\cdots$$

where, we ignore the cubic and higher powers of x.

$$V(x)=\frac{Q}{4\pi\varepsilon_0\sqrt{2}R}\left[\left(1-\frac{x}{R}\right)^{-\frac{1}{2}}+\left(1+\frac{x}{R}\right)^{-\frac{1}{2}}\right]$$

$$\Rightarrow \quad V(x)=\frac{Q}{4\pi\varepsilon_0\sqrt{2}R}\left[2+\frac{3}{4}\frac{x^2}{R^2}\right]$$

$$\Rightarrow \quad V(x)=\frac{Q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{R}+\frac{Q}{4\pi\varepsilon_o}\frac{3}{4\sqrt{2}R^3}x^2$$

$$V(x)=V_0(0)+ax^2, \text{ where } a=\frac{3Q}{4\pi\varepsilon_0 4\sqrt{2}R^3}$$

(d) The charged particle is displaced from the origin by a distance x (say)

For $x \ll R$, the potential function is

$$V(x)=V(0)+ax^2$$

Since $E_x=-\frac{dV}{dx}$

$$\Rightarrow \quad E_x=-2ax$$

The force experienced by the charge is given by

$$F=qE_x=-Q(2ax)$$

So, acceleration of the charge is

$$a=-\frac{2aQ}{m}x$$

$$\Rightarrow \quad \ddot{x}+\left(\frac{2Qa}{m}\right)x=0$$

Now, we compare a with standard expression of SHM, $a=-\omega^2x$, or $(\ddot{x}+\omega^2x=0)$, we get

$$\omega=\sqrt{\frac{2aQ}{m}}=\frac{2\pi}{T}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{m}{2aQ}}$$

PROBLEM 21

A thin rod extends along the z-axis from $z=-d$ to $z=d$. The rod carries a positive charge Q uniformly distributed along its length $2d$ with charge density $\lambda=\frac{Q}{2d}$.

(a) Find the electric potential $V(z)$ at a point $z>d$ along the z-axis.

(b) Find the change in potential energy if an electron moves from $z=4d$ to $z=3d$.

(c) Assume that the electron started from rest at the point $z=4d$, then find its velocity at $z=3d$.

SOLUTION

(a) For simplicity, let's assume the potential to be zero at infinity, $V(\infty)=0$.

Consider an infinitesimal charge element $dq=\lambda dz$ located at a distance z along the z-axis. Its contribution to the electric potential at a point $z>d$ is

$$dV=\frac{\lambda}{4\pi\varepsilon_0}\frac{dz}{z}$$

Integrating over the entire length of the rod, we obtain

$$V(z)=\frac{\lambda}{4\pi\varepsilon_0}\int_{z+d}^{z-d}\frac{dz}{z}=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{z+d}{z-d}\right)$$

(b) Using the result derived in (a), the electrical potential at $z=4d$ is

$$V|_{z=4d}=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{4d+d}{4d-d}\right)=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{5}{3}\right)$$

Similarly, the electrical potential at $z=3d$ is

$$V|_{z=3d}=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{3d+d}{3d-d}\right)=\frac{\lambda}{4\pi\varepsilon_0}\log_e 2$$

The electric potential difference between the two points is given by

$$\Delta V=V|_{z=3d}-V|_{z=4d}=\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{6}{5}\right)>0$$

Using the fact that the electric potential difference ΔV is equal to the change in potential energy per unit charge, we have

$$\Delta U=q\Delta V=-\frac{e\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{5}{6}\right)$$

where $q=-e$ is the charge of the electron.

(c) If the electron starts out at rest from $z = 4d$, then the change in kinetic energy is

$$\Delta K = \frac{1}{2}mv_f^2$$

By conservation of energy

$$\Delta U + \Delta K = 0$$

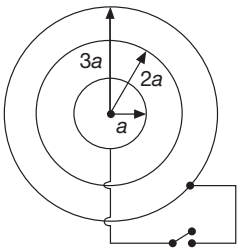
$$\Rightarrow \quad \Delta K = -\Delta U = \frac{e\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{6}{5}\right) > 0$$

Thus, the magnitude of the velocity at $z = 3d$ is

$$v_f = \sqrt{\frac{2e\lambda}{4\pi\varepsilon_0 m}\log_e\left(\frac{6}{5}\right)}$$

PROBLEM 22

Three concentric conducting shells of A, B and C radii a, $2a$ and $3a$ are shown in figure. The charge on the shell A, B and C is Q. When the key K is closed, find the charges on the innermost and outermost shells and ratio of charge densities of the shells.



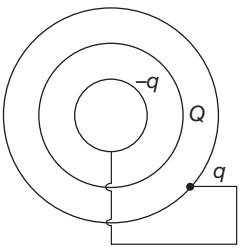
SOLUTION

When the key is closed, then the innermost and outermost shells will acquire the same potential. Let the charge on the outer shell be q and that on the inner shell be $-q$, the total charge on inner and outer shells is zero.

Potential on innermost shell,

$V_a =$ Sum of potentials due to $-q$, Q and q

$$\Rightarrow \quad V_a = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{a} + \frac{Q}{2a} + \frac{q}{3a}\right)$$



Similarly, potential on the outermost shell,

$$V_c = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{3a} + \frac{Q}{3a} + \frac{q}{3a}\right)$$

Since $V_a = V_c$, so we get

$$-\frac{q}{a} + \frac{Q}{2a} + \frac{q}{3a} = -\frac{q}{3a} + \frac{Q}{3a} + \frac{q}{3a} \qquad \ldots(1)$$

Equation (1) now becomes

$$-\frac{q}{a} + \frac{Q}{2a} = -\frac{q}{3a} + \frac{Q}{3a} \qquad \ldots(2)$$

$$\Rightarrow \quad q = \frac{Q}{4}$$

Thus charge on outermost shell is $q = \frac{Q}{4}$

Charge on innermost shell is $-q = -\frac{Q}{4}$, so

$$\sigma_A = \frac{1}{4\pi a^2}\left(-\frac{Q}{4}\right)$$

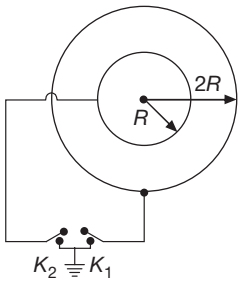
$$\sigma_B = \frac{Q}{4\pi(4a^2)}$$

$$\sigma_C = \frac{+Q}{4\pi(9a^2)}$$

$$\Rightarrow \quad \sigma_A : \sigma_B : \sigma_C = -9 : 9 : 1$$

PROBLEM 23

Two concentric shells of radii R and $2R$ are shown in figure. Initially a charge q is imparted to the inner shell. Now key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.



SOLUTION

When K_1 is closed first time, outer sphere is earthed and the potential on it becomes zero.
Let the charge on it be q_1'

V_1' = Potential due to charge on inner sphere and that due to charge on outer sphere

$$V_1' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R}+\frac{q_1'}{2R}\right]=0$$

$$\Rightarrow \quad q_1' = -q$$

When K_2 is closed first time, the potential V_2' on inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q_2'

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{R}+\frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(2R)}$$

$$\Rightarrow \quad q_2' = \frac{q}{2}$$

Now when K_1 will be closed second time, charge on outer sphere will be $-q_2'$, i.e., $-\frac{q}{2}$

After one event involving closure and opening of K_1 and K_2, charge is reduced to half its initial value.
Similarly, when K_1 will be closed nth time, charge on outer sphere will be $-\frac{q}{2^{n-1}}$ as each time charge will be reduced to half the previous value.
After closing K_2 nth time, charge on inner shell will be negative of half the charge on outer shell, i.e., $\left(+\frac{q}{2^n}\right)$ and potential on it will be zero.
For potential of outer shell,

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{\left(+\frac{q}{2^n}\right)}{2R}+\frac{1}{4\pi\varepsilon_0}\frac{\left(-\frac{q}{2^{n-1}}\right)}{2R}$$

$$\Rightarrow \quad V_0 = \frac{-q(-1+2)}{4\pi\varepsilon_0 2^{n+1}R}=\frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

$$\text{Potential difference} = V_0 - V_i = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}-0$$

$$\Rightarrow \quad V_0 - V_i = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

PROBLEM 24

Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere.

SOLUTION

Let q_1 and q_2 be the charges on the two spheres before connecting them.

Then, $q_1 = \sigma\left(4\pi R^2\right)$ and

$$q_2 = \sigma(4\pi)(2R)^2 = 16\sigma\pi R^2$$

Therefore, total charge (q) on both the spheres is

$$q = q_1 + q_2 = 20\sigma\pi R^2$$

Now, after connecting, the charge is distributed in the ratio of their capacities, which in turn depends on the ratio of their radii $(C = 4\pi\varepsilon_o R)$.

$$\therefore \quad \frac{q'_1}{q'_2}=\frac{R}{2R}=\frac{1}{2}$$

$$\Rightarrow \quad q'_1 = \frac{q}{3}=\frac{20}{3}\sigma\pi R^2$$

$$\text{and} \quad q'_2 = \frac{2q}{3}=\frac{40}{3}\sigma\pi R^2$$

Therefore, surface charge densities on the spheres are:

$$\sigma_1 = \frac{q'_1}{4\pi R^2}=\frac{(20/3)\sigma\pi R^2}{4\pi R^2}=\frac{5}{3}\sigma$$

$$\text{and} \quad \sigma_2 = \frac{q'_2}{4\pi(2R)^2}=\frac{(40/3)\sigma\pi R^2}{16\pi R^2}=\frac{5}{6}\sigma$$

Hence, surface charge density on the bigger sphere is σ_2 i.e., $\left(\frac{5}{6}\right)\sigma$.

PROBLEM 25

A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand S_2 is initially uncharged.

S_1 is given a charge Q brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

(b) What is the limiting value of this energy as $n \to \infty$?

SOLUTION

Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2, is q_1.

Therefore, charge on S_1 will be $Q - q_1$. Say it is q'_1.

$$\Rightarrow \quad \frac{q_1}{q'_1} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$$

It implies that Q charge is to be distributed in S_2 and S_1 in the ratio of R/r.

$$\Rightarrow \quad q_1 = Q\left(\frac{R}{R+r}\right) \qquad \ldots(1)$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R+r}\right)$$

This charge is again distributed in the same ratio.

Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1 + \frac{R}{R+r}\right)\left(\frac{R}{R+r}\right)$$

$$\Rightarrow \quad q_2 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

Similarly,

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

and $\quad q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots + \left(\frac{R}{R+r}\right)^n\right]$

$$\Rightarrow \quad q_n = Q\frac{R}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right] \qquad \ldots(2)$$

$$\left\{\because\ S_n = \frac{a\left(1-r^n\right)}{(1-r)}\right\}$$

Therefore, electrostatic energy of S_2 after n such contacts is

$$U_n = \frac{q_n^2}{2C} = \frac{q_n^2}{2\left(4\pi\varepsilon_o R\right)}$$

$$\Rightarrow \quad U_n = \frac{q_n^2}{8\pi\varepsilon_o R}$$

where q_n can be written from equation (2).

(b) $\quad q_n = \frac{QR}{R+r}\left[1 + \frac{R}{R+r} + \ldots\ldots + \ldots + \left(\frac{R}{R+r}\right)^{n-1}\right]$

Since, the sum of infinite GP is $S_\infty = \frac{a}{1-r}$

So, when $n \to \infty$, then

$$q_\infty = \frac{QR}{R+r}\left[\frac{1}{1 - \frac{R}{R+r}}\right]$$

$$\Rightarrow \quad q_\infty = \frac{QR}{R+r}\left(\frac{R+r}{R+r-R}\right)$$

$$\Rightarrow \quad q_\infty = \frac{QR}{R+r}\left(\frac{R+r}{r}\right) = Q\frac{R}{r}$$

$$\Rightarrow \quad U_\infty = \frac{q_\infty^2}{8\pi\varepsilon_0 R} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_o R}$$

$$\Rightarrow \quad U_\infty = \frac{Q^2R}{8\pi\varepsilon_o r^2}$$

PROBLEM 26

Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane.

(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.

(b) Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.

(c) If a particle of charge $+q$ starts form rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

SOLUTION

(a) Let $P(x,y)$ be a general point of x-y plane. Electric potential at point P would be,

$$V = (\text{potential due to } Q) + (\text{potential due to } -2Q)$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{\sqrt{(3a-x)^2+y^2}} + \frac{-2Q}{\sqrt{(3a+x)^2+y^2}}\right) \quad \ldots(1)$$

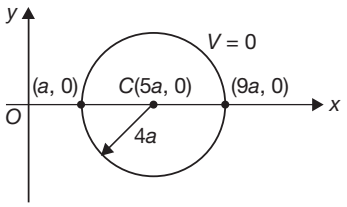
Given $V = 0$

$$\Rightarrow \quad 4\left[(3a-x)^2+y^2\right] = (3a+x)^2+y^2$$

Solving and rearranging, we get

$$(x-5a)^2+y^2 = (4a)^2$$

which is the equation of a circle of radius $4a$ and centre at $(5a, 0)$



(b) On x- axis, potential will be undefined (or say $\pm\infty$) at $x = 3a$ and $x = -3a$, because charge Q and $-2Q$ are placed at these two points. So, between $-3a < x < 3a$ we can find potential by putting $y = 0$ in equation (1). Therefore,

$$V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{3a-x} - \frac{2}{3a+x}\right), \text{ for } -3a < x < 3a$$

For $-3a < x < 3a$, we observe that

$V = 0$ at $x = a$

$V \to -\infty$ as $x \to -3a$

and $V \to +\infty$ as $x \to 3a$

For $x > 3a$, there is again a point where potential will become zero so for $x > 3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{x-3a} - \frac{2}{3a+x}\right) \text{ for } x > 3a$$

and observe that, $V = 0$ at $x = 9a$

For $x < -3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{3a-x} - \frac{2}{3a-x}\right) \text{ for } x < -3a$$

In this region potential will be zero only when $x \to -\infty$

So, we can summarise $V(x)$ as under

(i) At $x = 3a$, $V \to +\infty$

(ii) At $x = -3a$, $V \to -\infty$

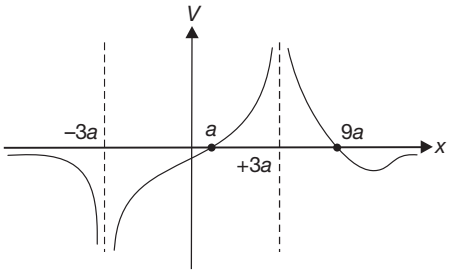
(iii) For $x < -3a$, $V(x) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{3a-x} - \frac{2}{3a+x}\right)$

(iv) For $-3a < x < 3a$, expression of $V(x)$ is same i.e.,

$$V(x) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{3a-x} - \frac{2}{3a+x}\right)$$

(v) For $x > 3a$, $V(x) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{x-3a} - \frac{2}{3a+x}\right)$

Potential on x-axis is zero at two places at $x = a$ and $x = 9a$. The V-x graph is shown here.



(c) Potential at centre i.e., at $x = 5a$ will be,

$$V_C = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{2a} - \frac{2}{8a}\right) = \frac{Q}{16\pi\varepsilon_o a} = \text{positive}$$

Potential on the circle will be zero.
Since, potential at centre > potential on circumference on it, the particle will cross the circle because positive charge moves from higher potential to lower potential. Speed of particle, while crossing the circle would be,

$$v = \sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{Qq}{8\pi\varepsilon_o ma}}$$

Here, ΔV is the potential difference between the centre and circumference of the circle.

PROBLEM 27

A positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity v from a point at distance $x(x > R)$ from the centre of the sphere. Calculate the minimum velocity v so that it penetrates a distance $\frac{R}{2}$ inside the sphere. Neglect any resistance other that electrostatic interaction. Assume that the charge on small mass remains constant through out the motion.

SOLUTION

By Law of Conservation of Energy

Total Energy at A = Total Energy at B

$$\Rightarrow \quad K_A + U_A = K_B + U_B$$

Energy of particle at A

$$E_A = \frac{1}{2}mv^2 + \frac{Qq}{4\pi\varepsilon_0 x}$$

At any point P inside the sphere at a distance r from its centre, the potential energy is given by

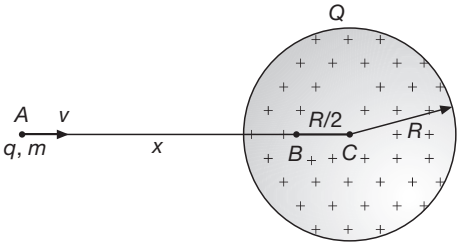
$$U_P = qV_P$$

where, $V_{\text{inside}} - V_P = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2)$

So, energy of particle at B is

$$E_B = 0 + \frac{Qq}{8\pi\varepsilon_0 R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$$

$$\Rightarrow \quad E_B = \frac{Q}{8\pi\varepsilon_0 R^3}\left(\frac{11R^2}{4}\right) = \frac{11}{32}\frac{Qq}{\pi\varepsilon_0 R}$$



$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{Qq}{4\pi\varepsilon_0 x} = \frac{11}{32}\frac{Qq}{\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{11}{8R} - \frac{1}{x}\right]$$

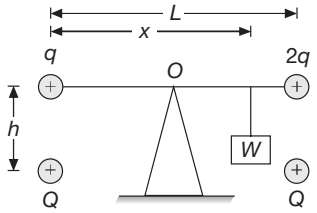
$$\Rightarrow \quad v = \sqrt{\frac{Qq}{2\pi\varepsilon_0 m}\left(\frac{11}{8R} - \frac{1}{x}\right)}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. An insulating long light rod of length L pivoted at its centre O and balanced with a weight W at a distance x from the left end as shown in figure. Charges q and $2q$ are fixed to the ends of the rod. Exactly below each of these charges at a distance h a positive charge Q is fixed. Then x is



(A) $\dfrac{QLq+\varepsilon_0^2h^2LW}{h^2W}$ (B) $\dfrac{QLq+\varepsilon_0h^2LW}{\varepsilon_0h^2W}$

(C) $\dfrac{4QLq+\varepsilon_0h^2LW}{8\pi h^2W}$ (D) $\dfrac{QLq+4\pi\varepsilon_0h^2LW}{8\pi\varepsilon_0h^2W}$

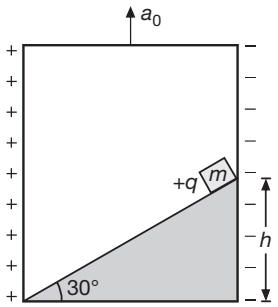
2. Three point charges q_1, q_2 and q_3 are taken such that when q_1 and q_2 are placed close together to form a single point charge, the force on q_3 at distance L from this combination is a repulsion of 2 unit in magnitude. When q_2 and q_3 are so combined the force on q_1 at distance L is an attractive force of magnitude 4 unit. Also q_3 and q_1 when combined exert an attractive force on q_2 of magnitude 18 unit at same distance L. The algebraic ratio of charges q_1, q_2 and q_3 is

(A) 1 : 2 : 3 (B) 2 : –3 : 4
(C) 4 : –3 : 1 (D) 4 : –3 : 2

3. In the process, when two bodies are charged by rubbing against the other, one becomes positively charged while the other becomes negatively charged. Then

(A) mass of each body remains unchanged
(B) mass of each body changes marginally
(C) mass of each body changes slightly and hence the total mass
(D) mass of each body changes slightly but the total mass remains the same

4. A small block of mass m, charge $+q$ is kept at the top of a smooth inclined plane of angle $30°$ placed in an elevator moving upward with an acceleration a_0. Electric field E exists between the vertical side walls of the elevator. The time taken by the block to come to the lowest point of inclined plane is (assuming the surface to be smooth)



(A) $t=\sqrt{\dfrac{2h}{g}}$

(B) $t=\sqrt{\dfrac{2h}{(g-a_0)+\dfrac{qE}{m}}}$

(C) $t=2\sqrt{\dfrac{2h}{(g+a_0)-\dfrac{\sqrt{3}qE}{m}}}$

(D) $t=\sqrt{\dfrac{2h}{(g+a_0)^2-\left(\dfrac{qE}{m}\right)h^2}}$

5. An electron of mass m, initially at rest, moves through a certain distance in a uniform electric field in time t. A proton of mass M, also initially at rest, takes time T to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio $\dfrac{T}{t}$ is nearly equal to

(A) $\dfrac{2M}{m}$ (B) $\sqrt{\dfrac{m}{M}}$

(C) $\sqrt{\dfrac{M}{m}}$ (D) $\dfrac{M}{m}$

6. A proton of mass m and accelerated by a potential difference V gets into a uniform electric field of a

parallel plate capacitor parallel to plates of length l at mid point of its separation between plates. The field strength in it varies with time as $E = at$, where a is a positive constant. Find the angle of deviation of the proton as it comes out of the capacitor. (Assume that it does not collide with any of the plates).

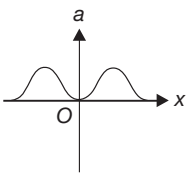
(A) $\theta = \tan^{-1}\left(\frac{al^2}{V}\sqrt{\frac{m}{8eV}}\right)$

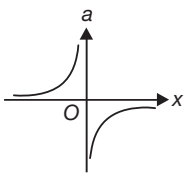
(B) $\theta = \tan^{-1}\left(\frac{al^2}{2V}\sqrt{\frac{m}{eV}}\right)$

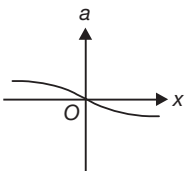
(C) $\theta = \tan^{-1}\left(\frac{al^2}{4V}\sqrt{\frac{m}{2eV}}\right)$

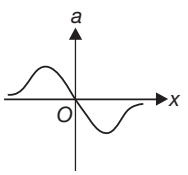
(D) $\theta = \tan^{-1}\left(\frac{al^2}{V}\sqrt{\frac{m}{2eV}}\right)$

7. Consider two identical positive charges which are fixed on the y-axis, at $(0, a)$ and $(0, -a)$. Let a particle having a negative charge start for the origin O from a point $P(x, 0)$ at a large distance for O, move along the x-axis, passes through O and moves far away from O. Let the acceleration a be taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-coordinate. Then a vs x curve is given by

(A) 

(B) 

(C) 

(D) 

8. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non–uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E} \cdot d\vec{l}$, ($l = 0$ being the centre of the ring) in volt is

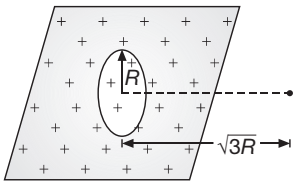
(A) +2 (B) –1
(C) –2 (D) 0

9. In a regular polygon of n sides, each corner is at a distance a from the centre. Identical charges of magnitude q are placed at $(n-1)$ corners. The field at the centre is

(A) $\frac{q}{4\pi\varepsilon_0 a^2}$ (B) $(n-1)\frac{q}{4\pi\varepsilon_0 a^2}$

(C) $\left(\frac{n}{n-1}\right)\frac{q}{4\pi\varepsilon_0 a^2}$ (D) $\left(\frac{n-1}{n}\right)\frac{q}{4\pi\varepsilon_0 a^2}$

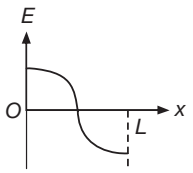
10. An infinite dielectric sheet having charge density σ has a hole of radius R in it. An electron is released on the axis of the hole at a distance $\sqrt{3}R$ from the centre. The speed with which it crosses the centre of the hole.

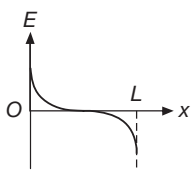


(A) $\sqrt{\frac{\sigma eR}{2m\varepsilon_0}}$ (B) $\sqrt{\frac{\sigma eR}{m\varepsilon_0}}$

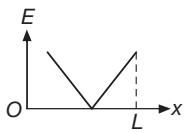
(C) $\sqrt{\frac{2\sigma eR}{m\varepsilon_0}}$ (D) None of these

11. Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge. The resultant field at P due to them is E. E is plotted against x for values of x from close to zero to slightly less than l. The curve that best represents the resulting plot of E for x values ranging from very close to zero to slightly less than l is

(A) 

(B) 

(C)

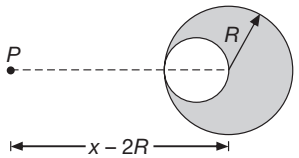
(D) 

12. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0, \ldots$ ad infinitum on the x-axis and a charge $-q$ is fixed at each of points $x = 2x_0, x = 4x_0, x = 6x_0 \ldots$ ad infinitum. Here x_0 is a positive constant. The potential at the origin due to the above system of charges is

(A) zero (B) $\frac{q}{8\pi\varepsilon_0 x_0 \log_e 2}$

(C) ∞ (D) $\frac{q\log_e 2}{4\pi\varepsilon_0 x_0}$

13. A solid spherical region has a spherical having a diameter R (equal to the radius of the spherical region), has a total charge Q. The electric field and potential at a point P at $x = 2R$ as shown is



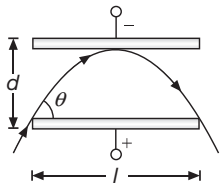
(A) $\frac{1}{27}\left(\frac{\rho R}{\varepsilon_0}\right), \frac{5}{12}\left(\frac{\rho R^2}{\varepsilon_0}\right)$

(B) $\frac{2}{27}\left(\frac{\rho R}{\varepsilon_0}\right), \frac{5}{36}\left(\frac{\rho R^2}{\varepsilon_0}\right)$

(C) $3\left(\frac{\rho R}{\varepsilon_0}\right), \frac{5}{6}\left(\frac{\rho R^2}{\varepsilon_0}\right)$

(D) $\frac{1}{9}\left(\frac{\rho R}{\varepsilon_0}\right), \frac{1}{9}\left(\frac{\rho R^2}{\varepsilon_0}\right)$

14. An electron enters the region between the plates of a parallel-plate capacitor with same initial velocity at an angle θ to the plates. The plate width is l and the plate separation is d. The electron follows the path shown, just missing the upper plate. Neglect gravity.



(A) $\tan\theta = \frac{2d}{l}$ (B) $\tan\theta = \frac{4d}{l}$

(C) $\tan\theta = \frac{6d}{l}$ (D) $\tan\theta = \frac{8d}{l}$

15. In a region of space, the electric field is in the x-direction, proportional to x and given by $\vec{E} = \alpha x\hat{i}$, where α is a positive constant. Consider an imaginary cubical volume of edge a, with its edges parallel to the axes of coordinates. The charge inside this cubical volume is

(A) $\varepsilon_0\alpha a^3$ (B) $\frac{\alpha a^3}{\varepsilon_0}$

(C) $\frac{1}{6}\left(\varepsilon_0\alpha a^2\right)$ (D) zero

16. An electrostatic potential is given by $V = kxy$, where k is a constant. A particle of charge q_0 is first taken from $(0, 0)$ to $(0, a)$ to (a, a), then directly from $(0, 0)$ to (a, a) and lastly from $(0, 0)$ to $(a, 0)$ to (a, a). If W_1, W_2 and W_3 be the work done for the individual paths respectively then

(A) $W_1 = W_2 = W_3 = -q_0 ka^2$

(B) $W_1 = W_3 = -\sqrt{2}q_0 ka^2$

(C) $W_1 = W_3 > W_2$

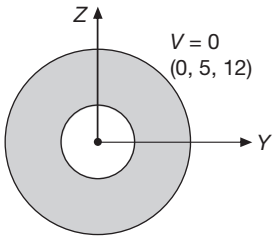
(D) $W_1 > W_2 > W_3$

17. A solid sphere of radius R is charged uniformly. At a point P, a distance r from the surface, the electrostatic potential is observed to be half of the potential at the centre. Then

(A) $r = 2R$ (B) $r = \frac{4R}{3}$

(C) $r = R$ (D) $r = \frac{R}{3}$

18. A line charge of charge density λ lies along the x-axis and let the surface of zero potential passes through $(0, 5, 12)$ m. The potential at point $(1, 3, -4)$ m is



(A) $\frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{13}{5}\right)$ (B) $\frac{2\lambda}{\pi\varepsilon_0}\log_e\left(\frac{13}{3}\right)$

(C) $\frac{\lambda}{4\pi\varepsilon_0}\log_e\left(\frac{13}{5}\right)$ (D) None of these

19. A point charge q is placed inside a conducting spherical shell of inner radius $2R$ and outer radius $3R$ at a distance of R from the centre of the shell. The electric potential at the centre of shell will be

(A) $\frac{q}{3\pi\varepsilon_0 R}$ (B) $\frac{q}{8\pi\varepsilon_0 R}$

(C) $\frac{q}{6\pi\varepsilon_0 R}$ (D) $\frac{5q}{24\pi\varepsilon_0 R}$

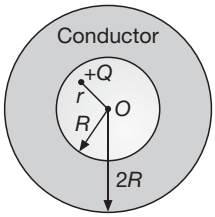
20. In a certain charge distribution, all points having zero potential can be joined by a circle C. Points inside C have positive potential, and points outside C have negative potential. A negative charge, which is free to move, is placed inside C. Then the negative charge
(A) may move, but will ultimately return to its starting point.
(B) will remain in equilibrium.
(C) can move inside C, but it cannot cross C.
(D) must cross C at some time.

21. Two identical thin rings, each of radius a metre are coaxial and placed a metre apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q coulomb from the centre of one ring to that of the other is

(A) zero (B) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{4\sqrt{2}\pi\varepsilon_0 a}$

(C) $\frac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 a}$ (D) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{4\sqrt{2}\pi\varepsilon_0 a}$

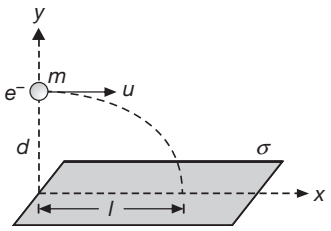
22. A point charge Q is placed at a distance $r(<R)$ from the centre O of a uncharged spherical shell of inner radius R and outer radius $2R$. The electric potential at the centre of the shell is given by



(A) $\frac{Q}{8\pi\varepsilon_0 R}$ (B) $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}+\frac{1}{2R}\right)$

(C) $\frac{Q}{4\pi\varepsilon_0 r}$ (D) $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{2R}\right)$

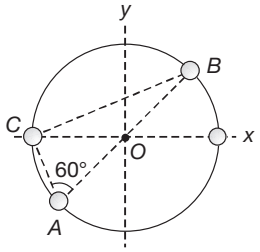
23. An electron is projected from a distance d and with initial velocity u parallel to a uniformly charged flat conducting plate as shown. It strikes the plate after travelling a distance l along the direction of projection. The surface charge density σ of the conducting plate is



(A) $\sigma=\frac{d\varepsilon_0 mu^2}{el}$ (B) $\sigma=\frac{d\varepsilon_0 mu}{el}$

(C) $\sigma=\frac{2d\varepsilon_0 mu^2}{el^2}$ (D) $\sigma=\frac{2d\varepsilon_0 mu}{el}$

24. Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB=60°$.



(A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis
(B) The potential energy of the system is zero
(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(D) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

25. Two particles A and B are suspended from the same support. The particles carry negative charges with A having more value of negative charge than B. They diverge and reach equilibrium with A and B making angles α and β with the vertical respectively.
(A) $\alpha>\beta$
(B) $\alpha<\beta$
(C) $\alpha=\beta$
(D) Data is insufficient to arrive at a conclusion.

26. 'Any excess charge on a conductor must reside only on its outer surface.' This statement is true
(A) for spherical conductors only (both solid and hollow).
(B) for hollow spherical conductors only.

(C) for conductors which do not have any sharp points or corners.
(D) for all types of conductors

27. A particle of mass m and charge q is set free a distance d from a perfectly conducting infinite plane. The time taken by the particle to hit the plane is

(A) $\frac{2}{3q}\sqrt{2md^3}$ (B) $\frac{1}{3q}\sqrt{2md^3}$

(C) $\frac{2}{q}\sqrt{2md^3}$ (D) $\frac{1}{q}\sqrt{2md^3}$

28. Two identical positive point charges each of value Q are fixed at the points $(a, 0)$ and $(-a, 0)$ on the x-axis. A particle of mass m and carrying charge $-q$ is released from rest at the point $P\left(0, \sqrt{3}a\right)$ on the y-axis. The velocity v_0 of the particle when it passes through the origin O is v_0. Then

(A) $v_0 = \sqrt{\frac{Qq}{\pi\varepsilon_0 ma}}$ (B) $v_0 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 ma}}$

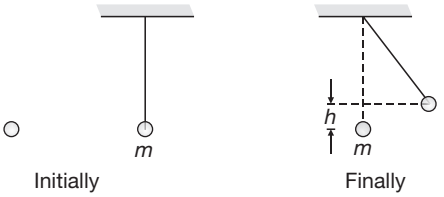
(C) $v_0 = \sqrt{\frac{Qq}{4\pi\varepsilon_0 ma}}$ (D) $v_0 = \sqrt{\frac{Qq}{8\pi\varepsilon_0 ma}}$

29. A charged particle passes through a uniform electric field existing between the parallel plates of a capacitor. The length of each plate is l. Its initial velocity is parallel to the plates. When it emerges from the field, its lateral deflection from its initial path is Δ and its angular deflection from its initial direction is θ. Then, $\tan\theta$ equals

(A) $\frac{\Delta}{l}$ (B) $\frac{2\Delta}{l}$

(C) $\frac{\Delta}{2l}$ (D) None of these

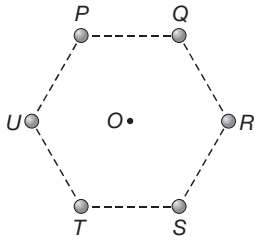
30. A small positively charged ball of mass m is suspended by an insulating thread of negligible mass. Another positively charged small ball is moved very slowly from a large distance until it is in the original position of the first ball. As a result, the first ball rises by h. How much work has been done?



(A) $2mgh$ (B) $3mgh$

(C) mgh (D) $4mgh$

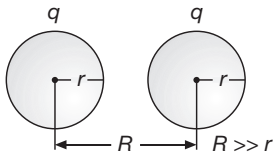
31. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for, P, Q, R, S, T and U respectively?



(A) +, −, +, −, −, + (B) +, −, +, −, +, −

(C) +, +, −, +, −, − (D) −, +, +, −, +, −

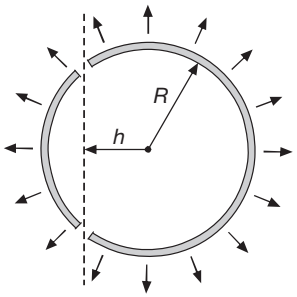
32. The potential energy of the system of two identically charged spheres as shown in the figure is equal to (Assume the charge distribution to be uniform)



(A) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{R}+\frac{1}{r}\right)$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r(R+r)}$ (D) None of these

33. Consider a metal sphere, of radius R that is cut in two along a plane whose minimum distance from the sphere's centre is h. Sphere is uniformly charged by a total electric charge Q. The force necessary to hold the two parts of the sphere together is



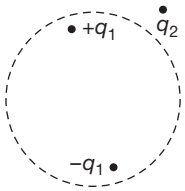
(A) $F=\dfrac{Q^2}{4\pi\varepsilon_0 R^4}(R^2-h^2)$

(B) $F=\dfrac{Q^2}{4\pi\varepsilon_0 R^2}$

(C) $F=\dfrac{Q^2}{32\pi\varepsilon_0 R^3}(R-h)$

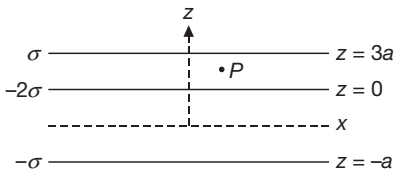
(D) $F=\dfrac{Q^2}{32\pi\varepsilon_0 R^4}(R^2-h^2)$

34. Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(A) q_2
(B) only the positive charges
(C) all the charges
(D) $+q_1$ and $-q_1$

35. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is



(A) $\dfrac{2\sigma}{\varepsilon_0}\hat{k}$ (B) $-\dfrac{2\sigma}{\varepsilon_0}\hat{k}$

(C) $\dfrac{4\sigma}{\varepsilon_0}\hat{k}$ (D) $-\dfrac{4\sigma}{\varepsilon_0}\hat{k}$

36. The potential on the Nth shell due to N concentric shells having charges Q, $2Q$, $3Q$, …….. NQ and radii a, $2a$, $3a$ …… Na respectively is

(A) $\dfrac{Q(N+1)}{8\pi\varepsilon_0 a}$ (B) $\dfrac{QN(N+1)}{8\pi\varepsilon_0 a}$

(C) $\dfrac{Q}{2\pi\varepsilon_0 a}$ (D) None of these

37. A hollow sphere of radius $2R$ is charged to a potential V and another smaller sphere of radius R is charged to a potential $\dfrac{V}{2}$. The smaller sphere is now placed inside the bigger sphere without changing the net charge on each sphere. The potential difference between the two spheres is

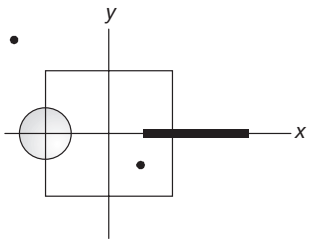
(A) $\dfrac{3V}{2}$ (B) $\dfrac{V}{4}$

(C) $\dfrac{V}{2}$ (D) V

38. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(A) V (B) $2V$
(C) $4V$ (D) $-2V$

39. A disk of radius $\dfrac{a}{4}$ having a uniformly distributed charge $6C$ is placed in the x-y plane with its centre at $\left(-\dfrac{a}{2}, 0, 0\right)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x=\dfrac{a}{4}$ to $x=\dfrac{5a}{4}$. Two point charges $-7C$ and $3C$ are placed at $\left(\dfrac{a}{4}, -\dfrac{a}{4}, 0\right)$ and $\left(-\dfrac{3a}{4}, \dfrac{3a}{4}, 0\right)$, respectively. Consider a cubical surface formed by six surfaces $x=\pm\dfrac{a}{2}$, $y=\pm\dfrac{a}{2}$, $z=\pm\dfrac{a}{2}$. The electric flux through this cubical surface is

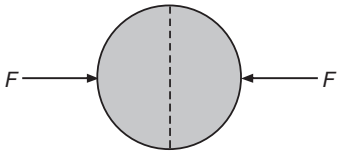


(A) $-\dfrac{2C}{\varepsilon_0}$ (B) $\dfrac{2C}{\varepsilon_0}$

(C) $\dfrac{10C}{\varepsilon_0}$ (D) $\dfrac{12C}{\varepsilon_0}$

40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held

together by pressing them with force F (see figure). F is proportional to



(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$

(C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

41. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air $= 1.8\times10^{-5}$ Nsm^{-2} and the density of oil $= 900$ kgm^{-3}, the magnitude of q is

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C

(C) 4.8×10^{-19} C (D) 8×10^{-19} C

42. A thin wire ring of radius r has an electric charge q_0. The increment of the force stretching the wire if a point charge $\frac{q_0}{2}$ is placed at the ring's centre is

(A) $\frac{q_0^2}{8\pi^2\varepsilon_0 r^2}$ (B) $\frac{q_0^2}{16\pi^2\varepsilon_0 r^2}$

(C) $\frac{q_0^2}{4\pi\varepsilon_0^2 r^2}$ (D) $\frac{2q_0^2}{\pi\varepsilon_0^2 r^2}$

43. Two point charges $q_1 = 4\ \mu$C and $q_2 = 9\ \mu$C are placed 20 cm apart. The electric field due to them will be zero on the line joining them at a distance of

(A) 8 cm from q_1 (B) 8 cm from q_2

(C) $\frac{80}{13}$ cm from q_1 (D) $\frac{80}{13}$ cm from q_2

44. A spherical region of space has a distribution of charge such that the volume charge density varies with the radial distance from the centre r as $\rho = \rho_0 r^3$, $0 \le r \le R$ where ρ_0 is a positive constant. The total charge Q on sphere is

(A) $Q = \frac{4}{5}\pi\rho_0 R^5$ (B) $Q = \frac{4}{5}\pi\rho_0 R^4$

(C) $Q = \frac{2}{3}\pi\rho_0 R^3$ (D) $Q = \frac{2}{3}\pi\rho_0 R^6$

45. An oil drop is found floating freely between the plates of a parallel plate condenser, the plates being horizontal and the lower plate carrying a charge $+Q$. The area of each plate is A and the distance of separation between them is D. The charge on the oil drop must be (g is the acceleration due to gravity)

(A) $\frac{Ag}{QM}$ (B) $\frac{\varepsilon_0 MgA}{Q}$

(C) $-\frac{AgQ}{D}$ (D) $-\frac{MgAQ}{\varepsilon_0}$

46. Two similar point charges q_1 and q_2 are placed at a distance r apart in air. Assume that a slab of thickness one third the separation between the charges is placed between the charges and it is observed that the Coulomb's repulsive force is reduced in the ratio $25:9$. Then the dielectric constant K of such a slab is

(A) 1 (B) 4

(C) 9 (D) 25

47. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by

(A) $2\pi R^2E$ (B) πR^2E

(C) $\left(\pi R^2 + \pi L^2\right)E$ (D) ZERO

48. Two concentric, thin metallic spheres of radii R_1 and $R_2\,(R_1 > R_2)$ bear charges Q_1 and Q_2 respectively. Then the potential at distance r between R_1 and R_2 will be $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

(A) $k\left(\frac{Q_1+Q_2}{r}\right)$ (B) $k\left(\frac{Q_1}{r}+\frac{Q_2}{R_2}\right)$

(C) $k\left(\frac{Q_2}{r}+\frac{Q_1}{R_1}\right)$ (D) $k\left(\frac{Q_1}{R_1}+\frac{Q_2}{R_2}\right)$

49. A wire of length L is placed along x-axis with one end at the origin. The linear charge density of the wire varies with distance x from the origin as $\lambda = \lambda_0\left(\frac{x^3}{L}\right)$, where λ_0 is a positive constant. The total charge Q on the rod is

(A) $Q=\frac{\lambda_0 L^2}{3}$ (B) $Q=\frac{\lambda_0 L^3}{3}$

(C) $Q=\frac{\lambda_0 L^3}{4}$ (D) $Q=\frac{\lambda_0 L^4}{5}$

50. A flat circular disc has a charge $+Q$ uniformly distributed on the disc. A charge $+q$ is thrown with kinetic energy E, towards the disc along its normal axis. The charge q will

(A) hit the disc at the centre
(B) return back along its path after touching the disc
(C) return back along its path without touching the disc
(D) any of the above three situations is possible depending on the magnitude of E

51. The electric intensity due to a uniformly charged infinite cylinder of radius R, at a distance $r(>R)$, from its axis is proportional to

(A) r^2 (B) r^3

(C) $\frac{1}{r}$ (D) $\frac{1}{r^2}$

52. Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result of immersion in the liquid the angle θ does not change. The density of the liquid is σ. The dielectric constant of the liquid is

(A) $\frac{\sigma}{\rho}$ (B) $\frac{\rho}{\sigma}$

(C) $\frac{\sigma}{\sigma-\rho}$ (D) $\frac{\rho}{\rho-\sigma}$

53. The radius of a hollow metallic sphere is R. If the potential difference between its surface and a point at a distance of $3R$ from its centre is V, then the electric field intensity at a distance of $3R$ from its centre is

(A) $\frac{V}{2R}$ (B) $\frac{V}{3R}$

(C) $\frac{V}{4R}$ (D) $\frac{V}{6R}$

54. The electric field due to a point charge at a distance R from it is E. If the same charge is placed on a metallic sphere of radius R, the electric field on the surface of the sphere will be

(A) ZERO (B) $\frac{E}{2}$

(C) E (D) $2E$

55. A positive point charge $50\ \mu C$ is located in the plane xy at the point with radius vector $\vec{r}_0=2\hat{i}+3\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. The electrostatic force $\vec{F}$ and its magnitude on a charge of $2\ \mu C$ placed at the point with radius vector $\vec{r}=8\hat{i}-5\hat{j}$ is (Here $\vec{r}_0$ and $\vec{r}$ are expressed in metre)

(A) 3 N (B) 9 mN

(C) 9 μN (D) 3 mN

56. Two free charges q and $4q$ are placed at a distance d apart. A third charge Q is placed between them at a distance x from charge q such that the system is in equilibrium. Then

(A) $Q=\frac{4q}{9},\ x=\frac{d}{3}$ (B) $Q=\frac{4q}{9},\ x=\frac{d}{4}$

(C) $Q=-\frac{4q}{9},\ x=\frac{d}{3}$ (D) $Q=-\frac{4q}{3},\ x=\frac{d}{4}$

57. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross-section 10^{-6} m^2. The ring has a uniform charge of π coulomb. The change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the centre of the ring is (Young's modulus of the metal is 2×10^{11} Nm^{-2})

(A) $\Delta R=2.25$ cm (B) $\Delta R=2.25$ mm

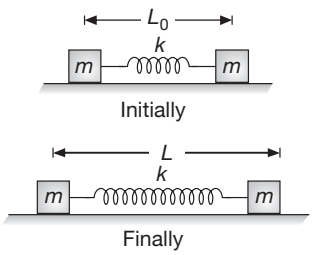
(C) $\Delta R=22.5$ cm (D) $\Delta R=22.5$ mm

58. Two isolated, charged conducting spheres of radii R_1 and R_2 produce the same electric field near their surfaces. The ratio of electric potentials on their surfaces is

(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$

(C) $\frac{R_1^2}{R_2^2}$ (D) $\frac{R_2^2}{R_1^2}$

59. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having a spring constant k and an unstretched length L_0. A total charge Q is slowly placed on the system, causing the spring to stretch to an equilibrium length $L=\frac{3L_0}{2}$, as shown. The value of Q, assuming that all the charge resides on the blocks and assuming the blocks as point charges is



(A) $Q = 3L_0\sqrt{2\pi\varepsilon_0 L_0}$ (B) $Q = 2L_0\sqrt{3\pi\varepsilon_0 L_0}$

(C) $Q = 3L_0\sqrt{3\pi\varepsilon_0 L_0}$ (D) $Q = 2L_0\sqrt{2\pi\varepsilon_0 L_0}$

60. A charged particle of mass m and charge q is released from rest in a uniform electric field E. The kinetic energy of the particle after time t is

(A) $\frac{2E^2t^2}{mq}$ (B) $\frac{Eq^2m}{2t^2}$

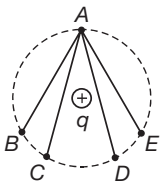
(C) $\frac{E^2q^2t^2}{2m}$ (D) $\frac{Eqm}{2t}$

61. Two identical oppositely charged metallic spheres placed 0.5 m apart, attract each other with a force of 0.108 N. When connected to each other by a copper wire for a short while, they begin to repel each other with a force of 0.036 N. The initial charge on each one of them is

(A) +1 μC, –3 μC (B) –1 μC, +3 μC

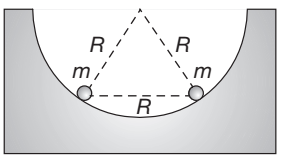
(C) –3 μC, +5 μC (D) 5 μC, $-\frac{3}{5}$ μC

62. In the electric field of a point charge q, a certain charge is carried from point A to B, C, D and E. Then the work done



(A) is least along the path AB
(B) is least along the path AD
(C) is zero along any one of the path AB, AC, AD and AE
(D) is least along AE

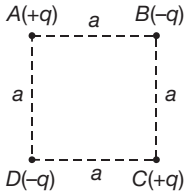
63. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, non-conducting walls, the beads move, and at equilibrium they are a distance R apart (shown in figure). The charge on each bead is



(A) $q = 2R\sqrt{\frac{4\pi\varepsilon_0 mg}{\sqrt{3}}}$ (B) $q = 2R\sqrt{\sqrt{3}\pi\varepsilon_0 mg}$

(C) $q = 2R\sqrt{\frac{\pi\varepsilon_0 mg}{\sqrt{3}}}$ (D) None of these

64. Charges $+q$, $-q$, $+q$ and $-q$ are placed at the corners A, B, C and D respectively of a square of side a. The potential energy of the system is $\frac{1}{4\pi\varepsilon_0}$ times



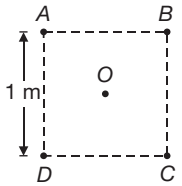
(A) $\frac{q^2}{a}(-4+\sqrt{2})$ (B) $\frac{q^2}{2a}(-4+\sqrt{2})$

(C) $\frac{4q^2}{a}$ (D) $-\frac{4\sqrt{2}q^2}{a}$

65. Two particles each of charge 10^{-7} C and mass 5 g, stay in limiting equilibrium on a horizontal surface. The particles have a separation of 10 cm between them. Assume the coefficient of friction between each particle and the table to be μ. Then μ is

(A) 1.8 (B) 0.018
(C) 0.18 (D) 1.08

66. Three charges, each of +4 μC, are placed at the corners B, C, D of a square ABCD of side 1 m. The electric field at the center O of the square is



(A) 7.2×10^4 N towards A

(B) 7.2×10^4 N towards C

(C) 3.6×10^4 N towards A

(D) 3.6×10^4 N towards C

67. The magnitude of the electric field E in the annular region of a charged cylindrical capacitor

(A) is same throughout
(B) is higher near the outer cylinder than near the inner cylinder

(C) varies as $\frac{1}{r}$, where r is the distance from the axis

(D) varies as $\frac{1}{r^2}$, where r is the distance from the axis

68. Four equal positive charges, each of charge Q are arranged at the corners of a square of side ℓ. A unit positive charge of mass m is placed at point P at height h above the centre of the square. Calculate Q, so that the unit charge is in equilibrium.

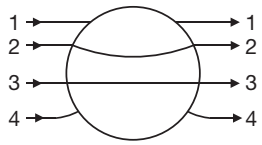
(A) $Q=\frac{\pi\varepsilon_0 mg}{l}\left(l^2+\frac{h^2}{2}\right)^{\frac{3}{2}}$

(B) $Q=\frac{\pi\varepsilon_0 mg}{h}\left(h^2+\frac{l^2}{2}\right)^{\frac{3}{2}}$

(C) $Q=\frac{\pi\varepsilon_0 mg}{4h}\left(h^2+\frac{l^2}{2}\right)^{\frac{3}{2}}$

(D) None of these

69. A metallic sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in the figure as



(A) 1 (B) 2
(C) 3 (D) 4

70. Three charge q, Q and $4q$ are placed in a straight line of length l at points distant 0, $\frac{l}{2}$ and l respectively from one end. In order to make the net force on q zero, the charge Q must be equal to

(A) $-q$ (B) $-2q$

(C) $\frac{-q}{2}$ (D) q

71. Three identical spheres each having a charge $2q$ and radius R are kept such that each touches the other two. Find the magnitude of the electric force on any sphere due to the other two.

(A) $\frac{3q^2}{4\pi\varepsilon_0 R^2}$ (B) $\frac{\sqrt{3}q^2}{2\pi\varepsilon_0 R^2}$

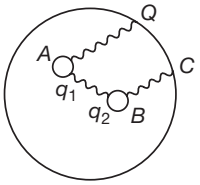
(C) $\frac{\sqrt{3}q^2}{8\pi\varepsilon_0 R^2}$ (D) $\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 R^2}$

72. A point charge q is placed at the midpoint of a cube of side L. The electric flux emerging from the cube is

(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{6L^2\varepsilon_0}$

(C) $\frac{6qL^2}{\varepsilon_0}$ (D) ZERO

73. Two small conductors A and B are given charges q_1 and q_2 respectively. Now they are placed inside a hollow metallic conductor C carrying a charge Q. If all the three conductors A, B and C are connected by a conducting wire as shown, the charges on A, B and C will be respectively



(A) $\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2}, Q$

(B) $\frac{Q+q_1+q_3}{3}, \frac{Q+q_1+q_2}{3}, \frac{Q+q_1+q_2}{3}$

(C) $\frac{q_1+q_2+Q}{2}, \frac{q_1+q_2+Q}{2}, 0$

(D) $0, 0, Q+q_1+q_2$

74. Two small equally charged spheres, each of mass m, are suspended from the same point by light silk threads of length l. The separation between the spheres is $x \ll l$. The rate $\frac{dq}{dt}$ with which the charge leaks off each sphere, if their velocity of approach varies as $v=\frac{6}{\sqrt{x}}$, (where α is a positive constant) is

(A) $\frac{3}{2}\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$ (B) $4\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$

(C) $9\sqrt{\frac{2\pi\varepsilon_0 mg}{3l}}$ (D) $9\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$

75. The electric field in a region of space is given by $\vec{E}=5\hat{i}+2\hat{j}\ \text{NC}^{-1}$. The electric flux due to this field through an area $2\ \text{m}^2$ lying in the YZ plane, in S.I. units, is

(A) 10 (B) 20

(C) $10\sqrt{2}$ (D) $2\sqrt{29}$

76. Consider an annular thin disc of inner radius a and outer radius $b(>a)$. The surface charge density σ varies with the distance r from the centre of the disc as $\sigma = \frac{\sigma_0}{r^2}$ where $a < r < b$ and σ_0 is a positive constant. The total charge on the disc is q, then Q equals

(A) $\sigma_0 \log_e\left(\frac{b}{a}\right)$ (B) $2\sigma_0 \log_e\left(\frac{b}{a}\right)$

(C) $2\pi\sigma_0 \log_e\left(\frac{a}{b}\right)$ (D) $2\pi\sigma_0 \log_e\left(\frac{b}{a}\right)$

77. If E_a be the electric field intensity due to a short dipole at a point on the axis and E_r be that on the right bisector at the same distance from the dipole, then

(A) $E_a = E_r$ (B) $E_a = 2E_r$

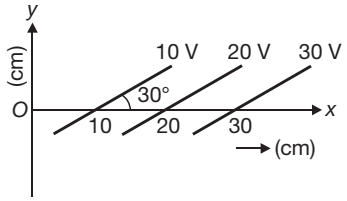
(C) $E_r = 2E_a$ (D) $E_a = \sqrt{2}E_r$

78. Two identical conducting spheres carrying different charges attract each other with a force F when placed in air medium at a distance d apart. The spheres are brought into contact and then taken to their original positions. Now the two spheres repel each other with a force whose magnitude is equal to that of the initial attractive force. The ratio between initial charges on the spheres is

(A) $-(3+\sqrt{8})$ only (B) $-3+\sqrt{8}$ only

(C) $-(3+\sqrt{8})$ or $(-3+\sqrt{8})$ (D) $+\sqrt{3}$

79. Equipotential surfaces are shown in figure. Then the electric field strength will be



(A) 100 Vm^{-1} along X-axis

(B) 100 Vm^{-1} along Y-axis

(C) 200 Vm^{-1} at an angle 120° with X-axis

(D) 50 Vm^{-1} at an angle 120° with X-axis

80. A solid sphere of radius R_1 and volume charge density $\rho = \frac{\rho_0}{r}$ is enclosed by a hollow sphere of radius R_2 with negative surface charge density σ, such that the total charge in the system is zero. ρ_0 is a positive constant and r is the distance from the centre of the spheres. The ratio $\frac{R_2}{R_1}$ is

(A) $\frac{\sigma}{\rho_0}$ (B) $\sqrt{\frac{2\sigma}{\rho_0}}$

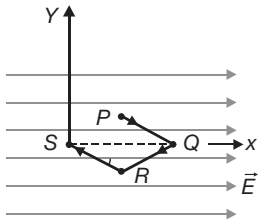
(C) $\sqrt{\frac{\rho_0}{2\sigma}}$ (D) $\frac{\rho_0}{\sigma}$

81. An electric charge is placed at the centre of a cube of side a. The electric flux through one of its faces will be

(A) $\frac{q}{6\varepsilon_0}$ (B) $\frac{q}{\varepsilon_0 a^2}$

(C) $\frac{q}{4\pi\varepsilon_0 a^2}$ (D) $\frac{q}{\varepsilon_0}$

82. Point charge q moves from point P to point S along the path PQRS (figure shown) in a uniform electric field E pointing coparallel to the positive direction of the X-axis. The coordinates of the points P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ respectively. The work done by the field in the above process is given by the expression



(A) qEa (B) $-qEa$

(C) $qEa\sqrt{2}$ (D) $qE\sqrt{(2a)^2 + b^2}$

83. A positively charged disc is placed on a horizontal plane. A charged particle is released from a certain height on its axis. The particle just reaches the center of the disc. Select the correct alternative.

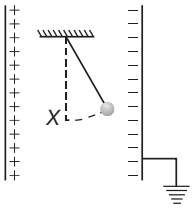
(A) Particle has negative charge on it.

(B) Total potential energy (gravitational + electrostatic) of the particle first increases then decreases.

(C) Total potential energy of the particle first decreases then increases.

(D) Total potential energy of the particle continuously decreases.

84. A simple pendulum has a length l, mass of bob m. The bob is given a charge q. The pendulum is suspended between the vertical plates of charged parallel plate capacitor. If E is the field strength between the plates, then time period T equals



(A) $2\pi\sqrt{\frac{l}{g}}$ (B) $2\pi\sqrt{\frac{l}{g+\frac{qE}{m}}}$

(C) $2\pi\sqrt{\frac{l}{g-\frac{qE}{m}}}$ (D) $2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

85. Two positive point charges are 3 m apart and their combined charge is 20 μC. If the force between them is 0.075 N, the charges are

(A) 10 μC, 10 μC (B) 15 μC, 5 μC

(C) 12 μC , 8 μC (D) 14 μC , 6 μC

86. Four charges of 6 μC, 2 μC, –12 μC and 4 μC are placed at the corners of a square of side 1 m. The square is in x-y plane and its center at its origin. Electric potential due to these charges is zero everywhere on the line

(A) $x = y,\ z = 0$ (B) $x = 0 = z$

(C) $x = 0 = y$ (D) $x = z,\ y = 0$

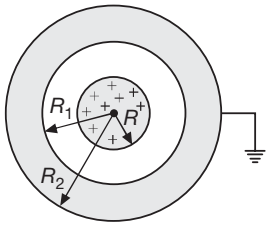
87. Two vertical metallic plates carrying equal and opposite charges are kept parallel to each other like a parallel plate capacitor. A small spherical metallic ball is suspended by a long insulated thread such that is hangs freely in the centre of the two metallic plates. The ball, which is uncharged, is taken slowly towards the positively charged plate and is made to touch that plate. Then the ball will

(A) stick to the positively charged plate
(B) come back to its original position and will remain there
(C) oscillate between the two plates touching each plate in turn
(D) oscillate between the two plates without touch them

88. As the electric charge on the surface of a hollow metal sphere increases, the electric field intensity inside the sphere

(A) decreases
(B) increases
(C) remains the same
(D) may increase or decrease depending on the radius of the sphere

89. A charge q is distributed uniformly over the volume of a insulating solid sphere of radius R. It is enclosed by an earthed conducting spherical shell of inner radius R_1 and outer radius R_2. The charge on the outer surface of the shell will be



(A) $+q$ (B) $\frac{R_2}{R_1}q$

(C) $\frac{R}{R_2}q$ (D) zero

90. A hollow metal sphere of radius 5 cm is charged so that the potential on its surface is 10 V. The potential at the centre of the sphere is

(A) ZERO
(B) 10 V
(C) same as at a point 5 cm away from the surface
(D) same as at a point 25 cm away from the surface

91. A rod of length L has a total charge q distributed uniformly along its length. It is bent in the shape of a semicircle. The electric potential at the center of the semicircle is

(A) $\frac{1}{4\pi\varepsilon_o}\frac{q}{L}$ (B) $\frac{q}{4\varepsilon_o L}$

(C) $\frac{q}{2\varepsilon_o L}$ (D) $\frac{1}{\pi\varepsilon_o}\frac{q}{L}$

92. Two charged conducting spheres of radii R_1 and R_2, separated by a large distance, are connected by a long wire. The ratio of the charges on them is

(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$

(C) $\frac{R_1^2}{R_2^2}$ (D) $\frac{R_2^2}{R_1^2}$

93. In PROBLEM 92, the ratio of the electric fields on the surfaces of the two spheres is

(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$

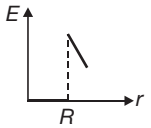
(C) $\frac{R_1^2}{R_2^2}$ (D) $\frac{R_2^2}{R_1^2}$

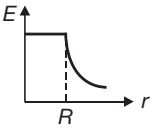
94. A spherical volume contains a uniformly distributed charge of density ρ. The electric field inside the sphere at a distance r from the center is

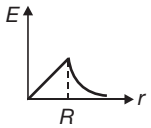
(A) $\frac{\rho r}{3\varepsilon_o}$ (B) $\frac{\rho r}{4\pi\varepsilon_o}$

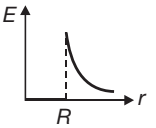
(C) $\frac{\rho}{\varepsilon_o r}$ (D) $\frac{\rho r}{4\pi\varepsilon_o}$

95. Which one of the following graphs represents the variation of electric field strength E with distance r from the centre of a uniformly charged non-conducting sphere

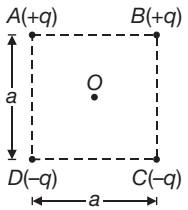
(A) 

(B) 

(C) 

(D) 

96. Four charges $+q$, $+q$, $-q$ and $-q$ are placed respectively at the corners A, B, C and D of a square of side a. The potential and field at the centre O of the square are respectively $\frac{1}{4\pi\varepsilon_0}$ times



(A) ZERO, $\frac{4q}{a^2}$ (B) ZERO, $\frac{4\sqrt{2}q}{a^2}$

(C) $\frac{4\sqrt{2}q}{a}$, $\frac{4q}{a^2}$ (D) $\frac{4\sqrt{2}q}{a}$, $\frac{4\sqrt{2}q}{a^2}$

97. Two fixed charges $-2Q$ and $+Q$ are located at points $(-3a, 0)$ and $(+3a, 0)$ respectively. Then

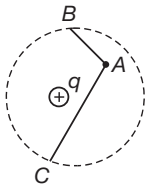
(A) points where the electric potential due to the two charges is zero, lie on a circle of radius $4a$ and center $(5a, 0)$

(B) potential is zero at $x = a$ and $x = 9a$

(C) If a particle of charge $+q$ is released from the center of the circle obtained in part (A) it will eventually cross the circle

(D) All of the above

98. In the electric field of a point charge q shown, a charge is carried from A to B and from A to C. Compare the work done



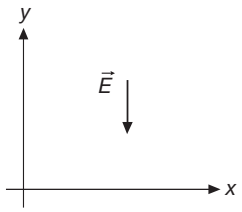
(A) work done is greater along the path AC than along AB

(B) work done is the same in both the cases

(C) work done is greater along the path AB than along AC

(D) work done is zero in both the cases

99. A particle of mass m and charge $-q$ is projected from the origin with a horizontal speed v into an electric field of intensity E directed downward. Select the incorrect statement.



(A) The kinetic energy after a displacement y is qEy

(B) The horizontal and vertical components of acceleration are $a_x = 0$, $a_y = \frac{qE}{m}$

(C) The equation of trajectory is $y = \frac{1}{2}\left(\frac{qEx^2}{mv^2}\right)$

(D) The horizontal and vertical displacements x and y after a time t are $x = vt$ and $y = \frac{1}{2}a_y t^2$

100. A positively charged thin metal ring of radius R is fixed in the X-Y plane with its centre at the origin O. A negatively charged particle P is released from rest at point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is

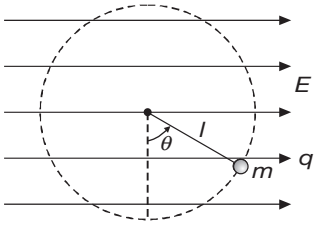
(A) simple harmonic for all values of z_0 satisfying $0 < z_0 < \infty$

(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \le R$

(C) approximately simple harmonic provided $z_0 << R$

(D) such that P crosses O and continues to move along the negative Z-axis towards $z = -\infty$

101. A small ball of mass m and charge $+q$ tied with a string of length l, is rotating in a vertical circle under gravity and a uniform horizontal electric field E as shown. The tension in the string will be minimum for



(A) $\theta = \tan^{-1}\left(\frac{qE}{mg}\right)$ (B) $\theta = \pi$

(C) $\theta = \pi - \tan^{-1}\left(\frac{qE}{mg}\right)$ (D) $\theta = \pi + \tan^{-1}\left(\frac{qE}{mg}\right)$

102. A solid metallic sphere has a charge $+3Q$. Concentric with this sphere is a conducting spherical shell having charge $-Q$. The radius of the sphere is a and that of the spherical shell is $b(b > a)$. The electric field at a distance R $(a < R < b)$ from the centre is

(A) $\frac{Q}{2\pi\varepsilon_0 R}$ (B) $\frac{3Q}{2\pi\varepsilon_0 R}$

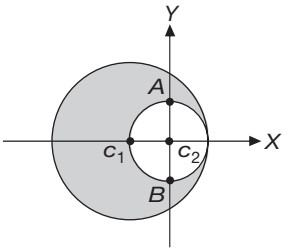
(C) $\frac{3Q}{4\pi\varepsilon_0 R^2}$ (D) $\frac{4Q}{4\pi\varepsilon_0 R^2}$

103. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2 respectively are placed at a distance R metre. The force per length on either wire will be $\left(K = \frac{1}{4\pi\varepsilon_0}\right)$

(A) $K\left(\frac{2\lambda_1\lambda_2}{R^2}\right)$ (B) $K\left(\frac{2\lambda_1\lambda_2}{R}\right)$

(C) $K\left(\frac{\lambda_1\lambda_2}{R^2}\right)$ (D) $K\left(\frac{\lambda_1\lambda_2}{R}\right)$

104. A uniformly charged sphere of radius $R = 2$ m and volume charge density ρ has a spherical cavity as shown. A point object of mass $m = 1$ g and charge $q = 2\times10^{-4}$ C is thrown, from point A. The minimum velocity required at A for it to reach point B is $\left(\rho = 3\varepsilon_0 \times 10^{-3}\ \text{Cm}^{-3}\right)$



(A) 0.02 ms^{-1} (B) 0.2 ms^{-1}

(C) 0.4 ms^{-1} (D) 0.04 ms^{-1}

105. The potential energy of an electric dipole in a uniform electric field is U. The magnitude of the torque acting on the dipole due to the field is N. Then

(A) U is minimum and N is zero when the dipole is parallel to the field.

(B) U is zero and N is zero when the dipole is perpendicular to the field.

(C) U is minimum and N is maximum when the dipole is perpendicular to the field

(D) U is minimum and N is zero when the dipole is anti-parallel to the field

106. Five point charges ($+q$ each) are placed at the five vertices of a regular hexagon of side $2a$. What is the magnitude of the net electric field at the centre of the hexagon?

(A) $\frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$ (B) $\frac{q}{16\pi\varepsilon_o a^2}$

(C) $\frac{\sqrt{2}q}{4\pi\varepsilon_o a^2}$ (D) $\frac{5q}{16\pi\varepsilon_o a^2}$

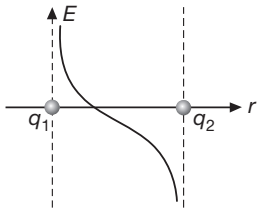
107. A charge Q is divided into two parts and the two parts are separated by a certain distance. The force between them will be maximum if one of the charges is

(A) $\frac{Q}{2}$ (B) $\frac{Q}{3}$

(C) $\frac{Q}{4}$ (D) None of these

108. The variation of electric field between the two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightward

direction of electric field as positive) as shown in the figure. Then the correct statement is



(A) q_1 and q_2 are positive and $q_1 < q_2$

(B) q_1 and q_2 are positive and $q_1 > q_2$

(C) q_1 is positive and q_2 is negative and $q_1 < q_2$

(D) q_1 and q_2 are negative and $q_1 < q_2$

109. A and B are two spherical conductors of same extent and size. A is solid and B is hollow. Both are charged to the same potential. If the charges on A and B are Q_A and Q_B respectively, then

(A) Q_A is less then Q_B

(B) Q_A is greater than Q_B but not double

(C) $Q_A = Q_B$

(D) $Q_A = 2Q_B$

110. Three point charges q, $2q$ and $8q$ are to be placed on a straight line 9 cm long. The system possesses minimum potential energy when

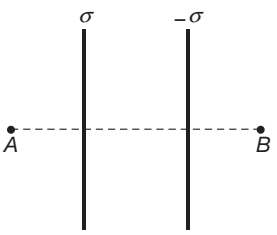
(A) $2q$ and q lie at ends with $8q$ at 3 cm from $2q$.

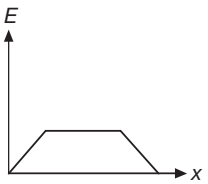
(B) $2q$ and $8q$ lie at ends with q at 6 cm from $8q$.

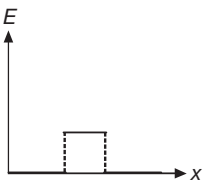
(C) q and $8q$ lie at ends with $2q$ at 6 cm from q.

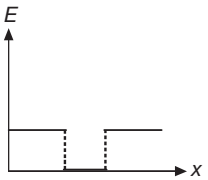
(D) $2q$ and $8q$ lie at ends with q at 3 cm from $8q$.

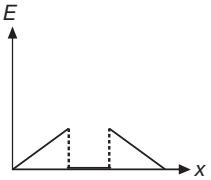
111. Two large parallel planes charged uniformly with surface charge density σ and $-\sigma$ are located as shown in the figure. Which one of the following graphs shows the variation of electric field along a line perpendicular to the planes as one moves from A to B?



(A) 

(B) 

(C) 

(D) 

112. An infinite number of charges, each equal to q coulomb, are placed along the x-axis at x (in metres) = 1, 2, 4, 8,..... and so on. The potential and field in SI units at $x = 0$ due to this set of charges are respectively $\frac{1}{4\pi\varepsilon_0}$ times

(A) $2q,\ 4q$ (B) $\frac{2q}{3},\ 4q$

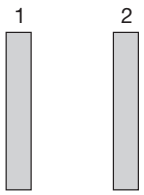
(C) $\frac{2q}{3},\ \frac{4q}{3}$ (D) $2q,\ \frac{4q}{3}$

113. In PROBLEM 112, if the consecutive charges have opposite signs, then the potential and field at $x = 0$ are respectively $\frac{1}{4\pi\varepsilon_0}$ times

(A) $\frac{2q}{3},\ \frac{4q}{7}$ (B) $\frac{2q}{5},\ \frac{4q}{5}$

(C) $\frac{2q}{3},\ \frac{4q}{5}$ (D) $\frac{2q}{5},\ \frac{4q}{7}$

114. Two conducting plates 1 and 2 each having large surface area A (on one side) are placed parallel to each other. The plate A is given a charge q, while the plate 2 is neutral. Then the electric field at a point in between the plates is



(A) $\frac{q}{2A\varepsilon_0}$ (B) $\frac{q}{A\varepsilon_0}$

(C) $\frac{2q}{A\varepsilon_0}$ (D) $\frac{3}{2}\frac{q}{A\varepsilon_0}$

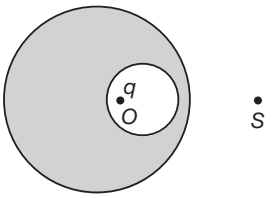
115. Two concentric metallic spherical shells are given positive charges of different value. Then
(A) the outer sphere is always at a higher potential
(B) the inner sphere is always at a higher potential
(C) both the spheres are at the same potential
(D) no prediction can be made about their potentials unless the actual values of charges and radii are known

116. Two thin infinite parallel plates have uniform charge densities $+\sigma$ and $-\sigma$. The electric field in the space between them is

(A) $\frac{\sigma}{2\varepsilon_0}$ (B) $\frac{\sigma}{\varepsilon_0}$

(C) $\frac{2\sigma}{\varepsilon_0}$ (D) ZERO

117. A charge q is placed at O in the cavity in a spherical uncharged conductor. Point S lies outside the conductor. If the charge is displaced from O towards S still remaining within the cavity, then



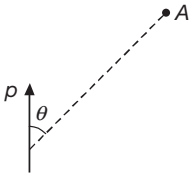
(A) electric field at S will increase
(B) electric field at S will decrease
(C) electric field at S will first increase and then decrease
(D) electric field at S will not change

118. The electric potential (in volt) in a region is given by

$$V = 6x - 8xy^2 - 8y + 6yz - 4x^2$$

Then electric force acting on a point charge of 2 C placed at the origin will be
(A) 2 N (B) 6 N
(C) 8 N (D) 20 N

119. The electric field at a point A due to dipole p is perpendicular to p. The angle θ is



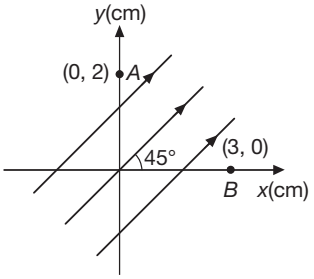
(A) 0° (B) 90°
(C) $\tan^{-1}(2)$ (D) $\tan^{-1}(\sqrt{2})$

120. Equal charges q are placed at the four corners A, B, C, D of a square of side a. The magnitude of the force on the charge at C will be

(A) $\frac{3q^2}{4\pi\varepsilon_0 a^2}$ (B) $\frac{4q^2}{4\pi\varepsilon_0 a^2}$

(C) $\left(\frac{1+2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$ (D) $\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$

121. A uniform electric field of $400\ \text{Vm}^{-1}$ is directed at $45°$ above the x-axis as shown in the figure. The potential difference $V_A - V_B$ is given by



(A) 0 (B) 4 V
(C) 6.4 V (D) 2.8 V

122. A charge Q is distributed over two concentric hollow spheres of radii r and $R(R>r)$ such that the surface densities are equal. The potential at the common centre is $\frac{1}{4\pi\varepsilon_0}$ times

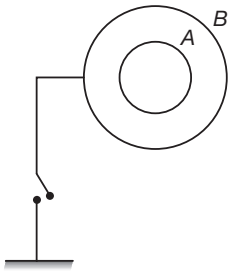
(A) $Q\left(\frac{r+R}{r^2+R^2}\right)$ (B) $\frac{Q}{2}\left(\frac{r+R}{r^2+R^2}\right)$

(C) $2Q\left(\frac{r+R}{r^2+R^2}\right)$ (D) zero

123. An electric dipole is placed at the origin along the x-axis. The electric field at any point, whose position vector makes an angle θ with the x-axis, makes an angle
(A) α (B) θ
(C) $\theta+\alpha$ (D) $\theta+2\alpha$

with the x-axis, where $\tan\alpha = \frac{1}{2}\tan\theta$.

124. Initially the spheres A and B are at potentials V_A and V_B respectively. Now sphere B is earthed by closing the switch. The potential of A becomes



(A) 0 (B) V_A

(C) $V_A - V_B$ (D) V_B

125. A charge Q is placed at each of the two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant force on Q is zero, then

(A) $Q = \sqrt{2}q$ (B) $Q = -\sqrt{2}q$

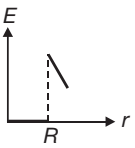
(C) $Q = 2\sqrt{2}q$ (D) $Q = -2\sqrt{2}q$

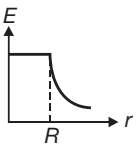
126. Electric charges q, q and $-2q$ are placed at the three corners of an equilateral triangle of side l. The magnitude of the electric dipole moment of the system is

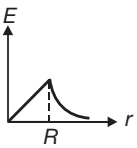
(A) ql (B) $2ql$

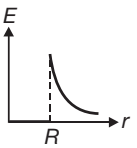
(C) $\sqrt{3}ql$ (D) $4ql$

127. Which one of the following graphs represents, variation of the electric field strength E with distance r from the centre of a charged conducting sphere?

(A) 

(B) 

(C) 

(D) 

128. A charged particle of mass m and charge q is released from rest from position $(x_0, 0)$ in a uniform electric field $E_0\hat{j}$. The angular momentum of the particle about origin

(A) is zero (B) is constant

(C) increases with time (D) decreases with time

129. An insulating solid sphere of radius R is charged in a non-uniform manner such that volume charge density $\rho = \frac{A}{r}$, where A is a positive constant and r the distance from centre. Electric field strength at any inside point at distance r_1 is

(A) $\frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi A}{r_1}\right)$ (B) $\frac{1}{4\pi\varepsilon_0}\left(\frac{A}{r_1}\right)$

(C) $\frac{A}{\pi\varepsilon_0}$ (D) $\frac{A}{2\varepsilon_0}$

130. An electric dipole is placed at the origin and is directed along the x-axis. At a point P, far away from the dipole, the electric field is parallel to the y-axis. OP makes an angle θ with the x-axis, then

(A) $\tan\theta = \sqrt{3}$ (B) $\tan\theta = \sqrt{2}$

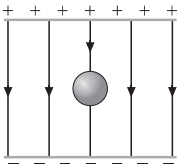
(C) $\theta = 45°$ (D) $\tan\theta = \left(\frac{1}{\sqrt{2}}\right)$

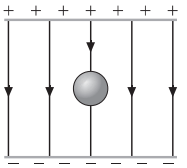
131. Two identical balls having like charges and placed at certain distance apart repel each other with a certain force. They are brought in contact and then moved apart to a distance equal to half their initial separation. The force of repulsion between them increases 4.5 times in comparison with the initial value. The ratio of the initial charges of the balls is

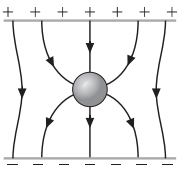
(A) 2 (B) 3

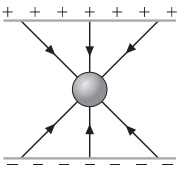
(C) 4 (D) 6

132. An uncharged sphere of metal is placed in uniform electric field produced by two oppositely charged plates. The lines of force will appear as

(A) 

(B) 

(C) 

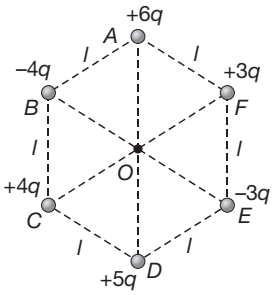
(D) 

133. Two small identical spheres having charges $+10\ \mu C$ and $-90\ \mu C$ attract each other with a force of F newton. If they are kept in contact and then separated by the same distance, the new force between them is

(A) $\frac{F}{6}$ (B) $16F$

(C) $\frac{16F}{9}$ (D) $9F$

134. The magnitude of electric field at the centre O of regular hexagon produced by the system of charges will be



(A) $\frac{q}{4\pi\varepsilon_0\ell^2}$ (B) $\frac{\sqrt{2}q}{4\pi\varepsilon_0\ell^2}$

(C) $\left(2+\sqrt{2}\right)\frac{q}{4\pi\varepsilon_0\ell^2}$ (D) 0

135. Two equal and opposite charges are placed at a certain distance, the force between them is F. If 25% of one charge is transferred to other, then the force between them is

(A) F (B) $\frac{9F}{16}$

(C) $\frac{15F}{16}$ (D) $\frac{4F}{5}$

136. The electric field inside a sphere which carries a charge density proportional to the distance from the origin $\rho=\alpha r$, where (α is a constant) is

(A) $\frac{\alpha r^3}{4\varepsilon_0}$ (B) $\frac{\alpha r^2}{4\varepsilon_0}$

(C) $\frac{\alpha r^2}{3\varepsilon_0}$ (D) None of these

137. A regular polygon has 20 sides. Equal charges, each Q, are placed at 19 vertices of the polygon and a charge q is placed at the centre of polygon. If the distance of each vertex from the centre O is a, net force experienced by q is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{20Qq}{a^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$

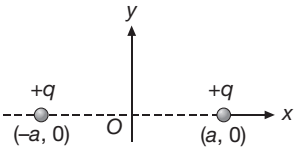
(C) $\frac{1}{4\pi\varepsilon_0}\frac{19Qq}{a^2}$ (D) zero

138. Two particles of masses m and 2 m and charges $2q$ and $2q$ are placed in a uniform electric field E and allowed to move for the same time. The ratio of kinetic energies will be

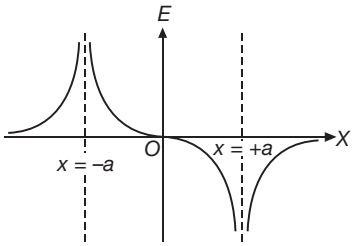
(A) 2 : 1 (B) 8 : 1

(C) 4 : 1 (D) 1 : 4

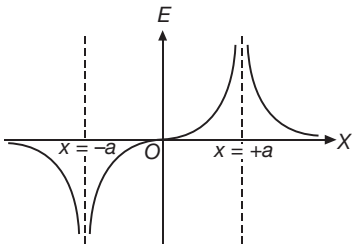
139. Two identical point charges q are placed at $x=-a$ and $x=a$, as shown in figure. The graph showing the variation of E along the x-axis is (assuming positive E to be taken along the $+x$-axis)



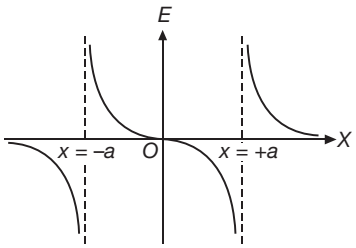
(A)



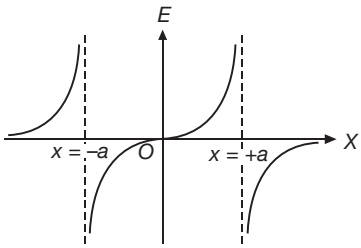
(B)



(C)



(D)



140. The number of electrons that should be removed from a coin of mass 1.6 g, so that it may float in electric field intensity 10^9 NC^{-1} directed upwards is

(A) 9.8×10^7 (B) 9.8×10^5

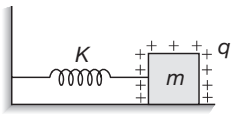
(C) 9.8×10^3 (D) 9.8×10^1

141. Two identical metallic spheres A and B carry charges $+Q$ and $-2Q$ respectively. The force between them is F Newton, when they are separated by a distance d in air. The spheres are allowed to touch each other and are moved back to their initial positions. The force between them now is

(A) $\frac{F}{2}$ (B) $\frac{F}{4}$

(C) $\frac{F}{8}$ (D) zero

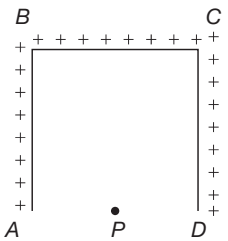
142. Initially the spring is in unstreched condition. An electric field (E) is switched on in right direction. The maximum extension in the spring will be



(A) $\frac{2qE}{K}$ (B) $\sqrt{\frac{2qE}{K}}$

(C) $\frac{qE}{K}$ (D) $\sqrt{\frac{qE}{K}}$

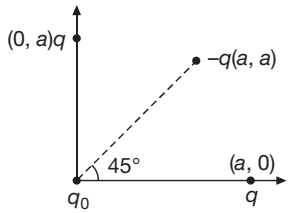
143. Three wires AB, BC, CD of equal length l are charged uniformly with linear charged density λ and are placed as shown. P is a point which lies at a distance l from the wire BC on its perpendicular bisector. The electric field at P is



(A) $\frac{\lambda}{2\sqrt{5}\varepsilon_0 l}(2\sqrt{5}-1)$ (B) $\frac{2\lambda}{2\sqrt{5}\varepsilon_0 l}(\sqrt{5}-1)$

(C) $\frac{\lambda}{4\sqrt{5}\varepsilon_0 l}(2\sqrt{5}-3)$ (D) $\frac{\lambda}{4\sqrt{5}\varepsilon_0 l}$

144. The net force acting on the charge q_0 present at the origin is



(A) $\frac{qq_0}{4\pi\varepsilon_0 a^2}\times\sqrt{2}$ (B) $\frac{qq_0}{4\pi\varepsilon_0\sqrt{2}a^2}+\frac{qq_0}{8\pi\varepsilon_0 a^2}$

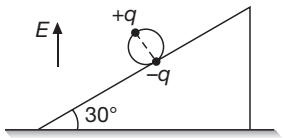
(C) $\frac{qq_0}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}-\frac{1}{2}\right)$ (D) $\frac{qq_0}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2}-\sqrt{2}\right)$

145. A neutral sphere of radius r and density ρ is intended to be suspended with an electric field E existing on the earth's surface directed vertically up. What fraction of the electrons should be removed from the sphere. Given that, the atomic number and atomic mass of the material of the sphere are Z & A respectively. Avogadro number is N_A. Assume that the sphere is able to hold the necessary charge without any leakage

(A) $\frac{2g}{eEZAN_A}$ (B) $\frac{3gA}{eEZ}$

(C) $\frac{gA}{eEZN_A}$ (D) $\frac{2gA}{eEZN_A}$

146. A wheel having mass m has charges $+q$ and $-q$ fixed at diametrically opposite ends. For the wheel to be in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E, the magnitude of E is



(A) $\frac{3mg}{2q}$ (B) $\frac{mg}{2q}$

(C) $\frac{\sqrt{3}mg}{2q}$ (D) $\frac{3mg}{4q}$

147. Seven identical charge particles each having charge q are placed on the vertices of a cube of edge ℓ, then the magnitude of electric field at centre of cube is

(A) $\frac{q}{4\pi\varepsilon_0\ell^2}$ (B) $\frac{q}{3\pi\varepsilon_0\ell^2}$

(C) $\frac{q}{8\pi\varepsilon_0\ell^2}$ (D) $\frac{q}{6\pi\varepsilon_0\ell^2}$

148. A simple pendulum of mass m and charge q is hanging from roof of a car moving on a circular track of radius r with constant speed v and string makes angle θ with vertical. Magnitude of horizontal electric field required so that string makes same angle θ on other side of vertical is

(A) $\frac{4mv^2}{qr}$ (B) $\frac{3mv^2}{qr}$

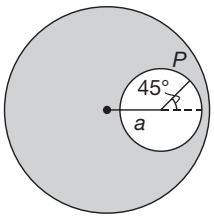
(C) $\frac{2mv^2}{qr}$ (D) $\frac{mv^2}{qr}$

149. Two identical charges repel each other with a force equal to 10 mg wt, when they are 0.6 m apart in air. The value of each charge is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 2 mC (B) 2×10^{-7} C

(C) 2 nC (D) 2 μC

150. A cavity of radius r is made inside a solid sphere. The volume charge density of the remaining sphere is ρ. An electron (charge e, mass m) is released from rest inside the cavity from point P as shown in figure. The centre of sphere and centre of cavity are separated by a distance a. The time after which the electron again touches the sphere is



(A) $\sqrt{\frac{6\sqrt{2}r\varepsilon_0 m}{e\rho a}}$ (B) $\sqrt{\frac{\sqrt{2r}\varepsilon_0 m}{6\rho a}}$

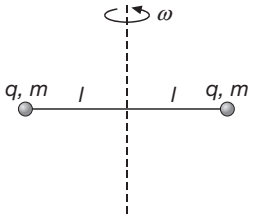
(C) $\sqrt{\frac{6r\varepsilon_0 m}{e\rho a}}$ (D) $\sqrt{\frac{r\varepsilon_0 m}{e\rho a}}$

151. A charged soap bubble having surface charge density σ and radius R. If pressure inside soap bubble and pressure outside it is same then the surface tension for soap bubble is

(A) $T = \frac{\sigma^2 R}{8\varepsilon_0}$ (B) $T = \frac{\sigma^2 R}{4\varepsilon_0}$

(C) $T = \frac{\sigma^2 R}{2\varepsilon_0}$ (D) $T = \frac{\sigma^2 R}{\varepsilon_0}$

152. Two identical point charges each q of mass m are connected by a insulating string of length 2ℓ and the system are rotated about an axis passing through mid point of string and perpendicular to its length as shown. Then neglecting gravity, tension in the string is (Assume only coulombic force between the charges)



(A) $\frac{q^2}{4\pi\varepsilon_0\ell^2} - m\ell\omega^2$ (B) $\frac{q^2}{4\pi\varepsilon_0\ell^2} + m\ell\omega^2$

(C) $\frac{q^2}{16\pi\varepsilon_0\ell^2} - m\ell\omega^2$ (D) $\frac{q^2}{16\pi\varepsilon_0\ell^2} + m\ell\omega^2$

153. A short dipole is placed along x-axis with centre at origin. The electric field at a point P, which is at a distance r from origin such that $\overrightarrow{OP} = \vec{r}$ makes an angle of 45° with x-axis, is directed along a direction making

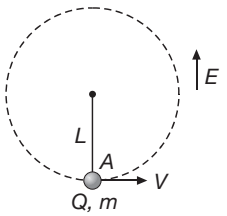
(A) $\tan^{-1}(0.5)$ with x-axis

(B) $\frac{\pi}{4} + \tan^{-1}(0.5)$ with x-axis

(C) $\frac{\pi}{4} + \tan^{-1}(0.5)$ with y-axis

(D) $\tan^{-1}(0.5)$ with y-axis

154. A particle of mass m and charge Q is attached to a string of length L. It is whirled into a vertical circle in field E. The speed of the particle at point A, so that the tension in the string at A is ten times the weight of particle, is (g = acceleration due to gravity)



(A) $\sqrt{5gL}$ (B) $\sqrt{\frac{QEL}{m}}$

(C) $\sqrt{L\left(9g + \frac{QE}{m}\right)}$ (D) None of these

155. In an experiment to demonstrate Coulomb's law in electrostatics, the force F between two small charged spheres is measured for various distances r between their centres. A graph is plotted of $\log_e F$ (y-axis) against $\log_e r$ (x-axis). The slope of this graph is

(A) -2 (B) $+\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) $+2$

156. Two point like charges Q_1 and Q_2 of whose strength are equal in absolute value of are placed at a certain distance from each other. Assuming the field strength to be positive in the positive direction of x-axis, the signs of the charges Q_1 and Q_2 for the graphs (field strength versus distance) shown in figure 1, 2, 3, and 4 are

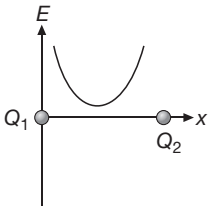


Figure 1

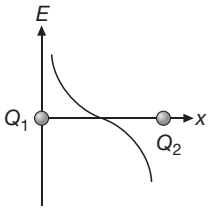


Figure 2

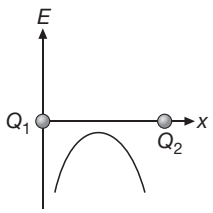


Figure 3

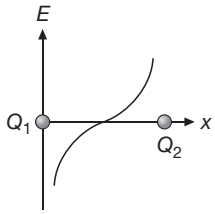


Figure 4

(A) Q_1 positive; Q_2 negative; both positive; Q_1 negative; Q_2 positive; both negative

(B) Q_1 negative; Q_2 positive; Q_1 positive; Q_2 negative; both positive; both negative

(C) Q_1 positive; Q_2 negative; both negative; Q_1 negative; Q_2 positive; both positive

(D) both positive; Q_1 positive; Q_2 negative; Q_1 negative; Q_2 positive; both negative

157. Two point charges q and $-q$ are at positions $(0, 0, d)$ and $(0, 0, -d)$ respectively. The electric field at $(a, 0, 0)$ is

(A) $\frac{2qd}{4\pi\varepsilon_0(d^2+a^2)^{\frac{3}{2}}}\hat{k}$ (B) $\frac{qd}{4\pi\varepsilon_0(d^2+a^2)^{\frac{3}{2}}}\hat{k}$

(C) $\frac{-2qd}{4\pi\varepsilon_0(d^2+a^2)^{\frac{3}{2}}}\hat{k}$ (D) $\frac{-qd}{4\pi\varepsilon_0(d^2+a^2)^{\frac{3}{2}}}\hat{k}$

158. A uniformly charged and infinitely long line having a linear charge density λ is placed at a normal distance y from a point O. Consider a sphere of radius R with O as centre and $R > y$. Electric flux through the surface of the sphere is

(A) zero (B) $\frac{2\lambda R}{\varepsilon_0}$

(C) $\frac{2\lambda\sqrt{R^2-y^2}}{\varepsilon_0}$ (D) $\frac{\lambda\sqrt{R^2+y^2}}{\varepsilon_0}$

159. Two charges of $-4\ \mu C$ and $+4\ \mu C$ are placed at points $A(1, 0, 4)$ and $B(2, -1, 5)$ located in an electric field $\vec{E} = 0.20\hat{i}\ \text{Vcm}^{-1}$. The torque acting on the dipole is

(A) 8×10^{-5} Nm (B) $\frac{8}{\sqrt{2}}\times10^{-5}$ Nm

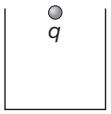
(C) $8\sqrt{2}\times10^{-5}$ Nm (D) $2\sqrt{2}\times10^{-5}$ Nm

160. The inward and outward electric flux from a closed surface are respectively 8×10^3 and 4×10^3 units. Then the net charge inside the closed surface is

(A) –4000 coulomb (B) 4000 coulomb

(C) $-\frac{4000}{\varepsilon_0}$ coulomb (D) $-4000\varepsilon_0$ coulomb

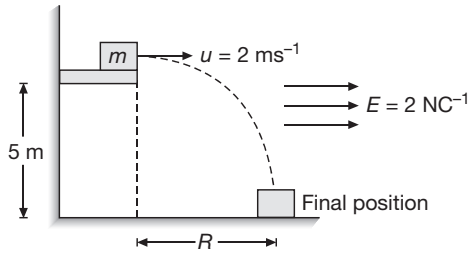
161. A charge q is placed at the centre of the open end of a cylindrical vessel. The flux of the electric field through the surface of the vessel is



(A) zero (B) $\frac{q}{\varepsilon_0}$

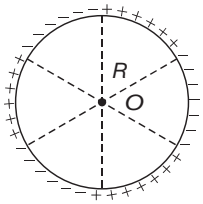
(C) $\frac{q}{2\varepsilon_0}$ (D) $\frac{2q}{\varepsilon_0}$

162. The block shown in the diagram has a mass of 2 microgram and a charge of 2×10^{-9} coulomb. If the block is given an initial velocity of $2\ \text{ms}^{-1}$ in x direction at $t = 0$ and an electric field of $2\ \text{NC}^{-1}$ in x direction is switch on at that moment, then R will be $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 4 m (B) 1 m
(C) 2 m (D) 3 m

163. A ring of radius R is marked in six equal parts and these parts are charged uniformly each with a charge of magnitude Q but positive and negative alternately as shown. The electric field at centre of ring will be



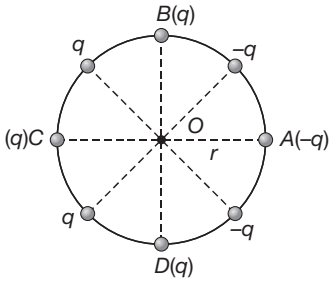
(A) $\frac{\lambda}{4\pi\varepsilon_0 r}$ where $\lambda = \frac{3Q}{\pi R}$

(B) $\frac{2\lambda}{4\pi\varepsilon_0 r}$ where $\lambda = \frac{3Q}{\pi R}$

(C) $\frac{3\lambda}{4\pi\varepsilon_0 r}$ where $\lambda = \frac{3Q}{\pi R}$

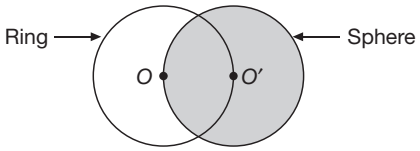
(D) $\frac{3\lambda}{2\pi\varepsilon_0 R}$, where $\lambda = \frac{3Q}{\pi R}$

164. Some point charges are placed on the circumference of circle at equal distance. (see figure) The direction of electric field at centre O will be along



(A) OA (B) OB
(C) OC (D) OD

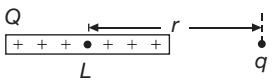
165. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring. Find the flux of the electric field through the surface of the sphere



(A) $\frac{Q}{4\varepsilon_0}$ (B) $\frac{Q}{6\varepsilon_0}$

(C) $\frac{Q}{3\varepsilon_0}$ (D) $\frac{Q}{2\varepsilon_0}$

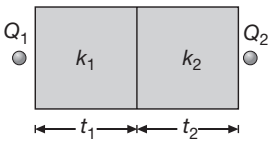
166. A charge Q is distributed over a line of length L. Another point charge q is placed at a distance r from the centre of the line distribution. Then the force experienced by q is



(A) $\frac{qQ}{4\pi\varepsilon_0\left(r^2 - L^2\right)}$ (B) $\frac{qQ}{\pi\varepsilon_0\left(4r^2 - L^2\right)}$

(C) $\frac{qQ}{4\pi\varepsilon_0 r^2}$ (D) $\frac{qQL}{4\pi\varepsilon_0 r^3}$

167. Two charges Q_1 and Q_2 are distance d apart. Two dielectrics of thickness t_1 and t_2 and dielectric constant k_1 and k_2 are introduced as shown. Find the force between the charges



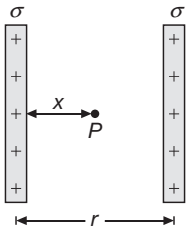
(A) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left[d - (t_1 + t_2) + k_1t_1 + k_2t_2\right]^2}$

(B) zero

(C) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left(d + \sqrt{k_1}\,t_1 + \sqrt{k_2}\,t_2\right)^2}$

(D) $\frac{Q_1Q_2}{4\pi\varepsilon_0\left[\left(\sqrt{k_1} - 1\right)t_1 + \left(\sqrt{k_2} - 1\right)t_2\right]^2}$

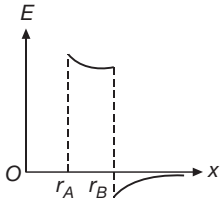
168. For two infinitely charged parallel sheets the electric field at P will be

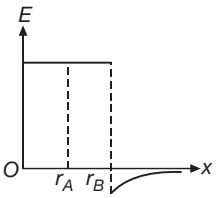


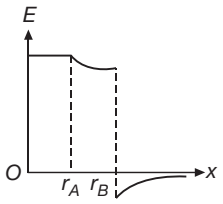
(A) $\frac{\sigma}{2x}-\frac{\sigma}{2(r-x)}$ (B) $\frac{\sigma}{2\varepsilon_0 x}+\frac{\sigma}{2\pi(r-x)\varepsilon_0}$

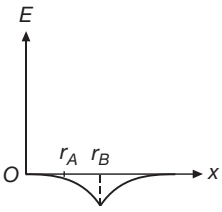
(C) $\frac{\sigma}{\varepsilon_0}$ (D) zero

169. Two concentric conducting thin spherical shells A, and B having radii r_A and $r_B\,(r_B>r_A)$ are charged to Q_A and $-Q_B\,(|Q_B|>|Q_A|)$. The variation of electric field (E) with distance r from centre is best represented by

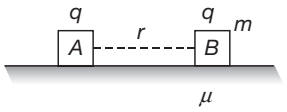
(A) 

(B) 

(C) 

(D) 

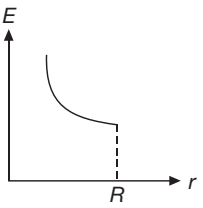
170. The minimum value of μ for which small block B of mass m remain at rest on horizontal is (Assume that block A is fixed and small block has identical charge q and separation is r)

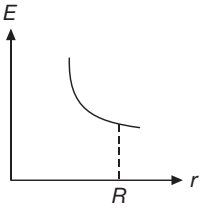


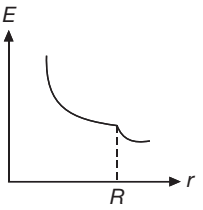
(A) $\frac{q^2}{8\pi\varepsilon_0 mgr^2}$ (B) $\frac{q^2}{4\pi\varepsilon_0 mgr^2}$

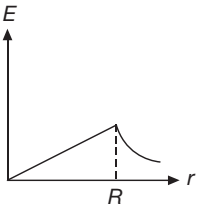
(C) $\frac{q^2}{2\pi\varepsilon_0 mgr^2}$ (D) $\frac{2q^2}{\pi\varepsilon_0 mgr^2}$

171. A conducting shell of radius R carries charge $-Q$. A point charge $+Q$ is placed at the centre of shell. The electric field E varies with distance r (from the centre of the shell) as

(A) 

(B) 

(C) 

(D) 

172. Two balls with equal charges are in a vessel with ice at $-10\ ^\circ C$ at a distance of $25\ cm$ from each other. On forming water at $0\ ^\circ C$, the balls are brought nearer to $5\ cm$ for the interaction between them to be same. If the dielectric constant of water at $0\ ^\circ C$ is 80, the dielectric constant of ice at $-10\ ^\circ C$ is

(A) 40 (B) 3.2

(C) 20 (D) 6.4

173. The electric field varies along the x-direction as $\vec{E}=(E_0x)\hat{i}$. Consider an imaginary cubical volume of edge l, with its edges parallel to the axes of coordinates. The charge inside this volume is

(A) zero (B) $\varepsilon_0 E_0 l^3$

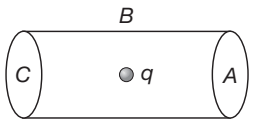
(C) $\frac{1}{\varepsilon_0}E_0 l^3$ (D) $\frac{1}{6}\varepsilon_0 E_0 l^2$

174. Four point charges, each $+q$, are fixed at the corners of a square of side a. Another point charge q_0 is placed at a height h vertically above the centre of square, assuming the square to be in a horizontal plane. Magnitude of force experienced by q_0 is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{\left(h^2+\frac{a^2}{2}\right)}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{\left(h^2+\frac{a^2}{2}\right)^{\frac{3}{2}}}$

(C) $\frac{1}{\pi\varepsilon_0}\frac{qq_0h}{\left(h^2+\frac{a^2}{2}\right)^{\frac{3}{2}}}$ (D) zero

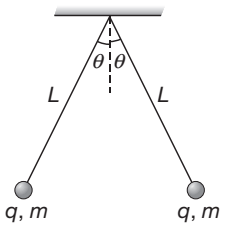
175. A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in units of voltmeter associated with the curved surface B, the flux linked with the plane surface A in units of voltmeter will be



(A) $\frac{q}{2\varepsilon_0}$ (B) $\frac{\phi}{3}$

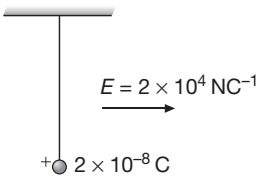
(C) $\frac{q}{\varepsilon_0}-\phi$ (D) $\frac{1}{2}\left(\frac{q}{\varepsilon_0}-\phi\right)$

176. Two small spheres each of mass m and charge q are tied from the same rigid support with the help of silk threads of length L. They make angle θ with the vertical as shown in the figure. If length L is decreased then angle θ with the vertical



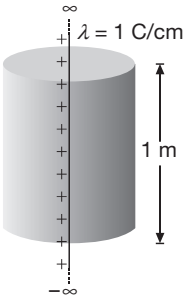
(A) increases (B) decreases
(C) unaffected (D) cannot say

177. A charge is suspended from a light thread in a uniform field as shown. The tension in the thread at equilibrium is



(A) 8.8 N
(B) 8.8×10^{2} N
(C) 8.8×10^{-4} N
(D) 8.8×10^{-3} N

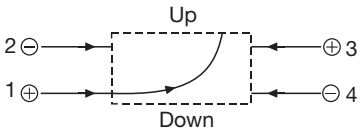
178. Total flux coming out of the cylindrical Gaussian surface is



(A) $\phi=100$ (B) $\phi=1$

(C) $\phi=\frac{1}{\varepsilon_0}$ (D) $\frac{100}{\varepsilon_0}$

179. The path of a positively charged particle 1 through a rectangular region of uniform electric field as shown in the figure. The direction of electric field and the direction of deflection of particles 2, 3 and 4 will be

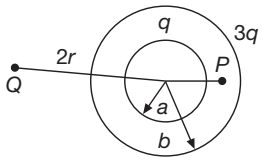


(A) Up, down, up, down
(B) Up, down, down, up
(C) Down, up, up, down
(D) Down, up, down, down

180. In a region of space the electric field is given by $\vec{E}=8\hat{i}+4\hat{j}+3\hat{k}$. The electric flux through a surface of area of 100 units in x-y plane is

(A) 800 units (B) 300 units
(C) 400 units (D) 1500 units

181. Ratio of electric field intensity at P & Q in the shown arrangement is



(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 4 : 3

182. An oil drop carrying a charge of two electrons has a mass of 3.2×10^{-17} kg. It is falling freely in air with terminal speed. The electric field required to make the drop move upwards with the same speed is (neglect buoyant force)

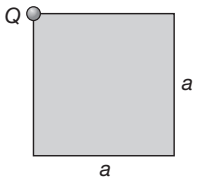
(A) 2×10^{3} Vm^{-1} (B) 4×10^{3} Vm^{-1}
(C) 3×10^{3} Vm^{-1} (D) 8×10^{3} Vm^{-1}

183. A charge q is located at the centre of a cube the electric flux through any face is

(A) $\dfrac{4\pi q}{6(4\pi\varepsilon_0)}$ (B) $\dfrac{\pi q}{6(4\pi\varepsilon_0)}$

(C) $\dfrac{q}{6(4\pi\varepsilon_0)}$ (D) $\dfrac{2\pi q}{6(4\pi\varepsilon_0)}$

184. A point charge Q is placed at the corner of a square plate of side a then the flux through the square plate is $90°$ and hence $\phi = 0$



(A) $\dfrac{Q}{8\varepsilon_0}$ (B) $\dfrac{Q}{4\varepsilon_0}$

(C) $\dfrac{Q}{\varepsilon_0}$ (D) zero

185. An isolated and charged spherical soap bubble has a radius r and the pressure inside is atmospheric. If T is the surface tension of soap solution, then charge on drop is

(A) $2\sqrt{\dfrac{2rT}{\varepsilon_0}}$ (B) $8\pi r\sqrt{2rT\varepsilon_0}$

(C) $8\pi r\sqrt{rT\varepsilon_0}$ (D) $8\pi r\sqrt{\dfrac{2rT}{\varepsilon_0}}$

186. If a small sphere of mass m and charge q is hanged from a silk thread at an angle θ with the surface of a vertical charged conducting plate, then for equilibrium of sphere the surface charge density of the plate is

(A) $\dfrac{\varepsilon_0 mg\tan\theta}{q}$ (B) $\dfrac{\varepsilon_0 2mg\tan\theta}{q}$

(C) $\varepsilon_0 mgq\tan\theta$ (D) $\dfrac{\varepsilon_0 mg\tan\theta}{3q}$

187. Two similar point charges q_1 and q_2 are placed at a distance r, apart in air. The force between them is F_1. A dielectric slab of thickness $t(<r)$ and dielectric constant K is placed between the charges. Then the force between the same charges is F_2. The ratio $\dfrac{F_1}{F_2}$ is

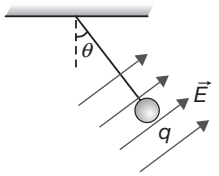
(A) 1 (B) K

(C) $\left(\dfrac{r-t+t\sqrt{K}}{r}\right)^2$ (D) $\left(\dfrac{r}{r-t+t\sqrt{K}}\right)^2$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A charged cork ball of mass 1 g is suspended on a light string in the presence of a uniform electric field as shown. When $\vec{E} = (3\hat{i} + 5\hat{j}) \times 10^5$ NC^{-1}, the ball is in equilibrium at $\theta = 37°$. T is the tension in the string and q is the charge on the ball (Take $\sin 37° = 0.60$ and $g = 10$ ms^{-2})



(A) $q = 11$ nC (B) $q = 12$ nC

(C) $T = 5.55 \times 10^{-3}$ N (D) $T = 4.55 \times 10^{-3}$ N

2. Consider a point charge Q placed at the origin O of the coordinate system. Let $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ be the electric fields at three points $A(1, 2, 3)$ m, $B(1, 1, -1)$ m and $C(2, 2, 2)$ m due to charge Q. Then

(A) $\vec{E}_A \perp \vec{E}_B$ (B) $\vec{E}_A \parallel \vec{E}_C$

(C) $|\vec{E}_B| = 4|\vec{E}_C|$ (D) $|\vec{E}_B| = 8|\vec{E}_C|$

3. Equipotential surfaces representing a uniform electric field must be

(A) plane surfaces.

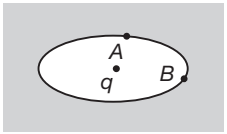
(B) normal to the direction of the field.

(C) spaced such that surfaces having equal differences in potential are separated by equal distances.

(D) having potentials decreasing in the direction of the field.

4. Consider two equipotential surfaces A and B having different potentials. Then
(A) A and B cannot intersect.
(B) A and B cannot both be plane surfaces.
(C) In the region between A and B, the field is maximum where equipotential surfaces are closest to each other.
(D) A line of force from A to B must be perpendicular to both.

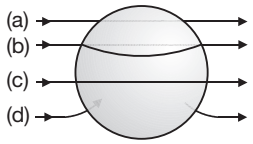
5. An ellipsoidal cavity is carved within a perfect conductor (see figure). A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in figure. Then,



(A) electric field near A in the cavity = electric field near B in the cavity.
(B) charge density at A = charge density at B.
(C) potential at A = potential at B.
(D) total electric flux through the surface of the cavity is $\frac{q}{\varepsilon_0}$.

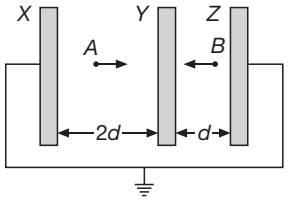
6. Consider two large, identical, parallel conducting plates have surfaces X and Y, facing each other. The charge per unit area on X is σ_1 and charge per unit surface area on Y is σ_2. Then,
(A) $\sigma_1 = -\sigma_2$ only if a charge is given to either X or Y
(B) $\sigma_1 = \sigma_2 = 0$ if equal charges are given to both X and Y
(C) $\sigma_1 > \sigma_2$ if X is given a charge more than that given to Y
(D) $\sigma_1 = -\sigma_2$ in all cases

7. A solid metallic sphere is place in a uniform electric field. Which of the curves shown in the figure represent the lines of force correctly?



(A) (a) (B) (b)
(C) (c) (D) (d)

8. X, Y and Z are parallel plates. Y is given some positive charge. Two electrons A and B start with zero initial velocity from X and Z respectively and reach Y in times t and T respectively.



(A) $t = T$ (B) $t = \sqrt{2}T$
(C) $2t = T$ (D) $t = 2T$

9. In the set up of PROBLEM 8, an electron E moves from X to Y and a proton P moves from Y to Z. Both particles start from rest.
(A) E reaches Y with greater energy than P.
(B) P reaches Y with greater energy than E.
(C) P and E reach Y with equal energies.
(D) P reaches Y with greater momentum than E.

10. A charged particle of mass m, charge q moves with a speed v in a circular path of radius r around a long uniformly charged conductor having charge density λ. Then
(A) $v \propto \sqrt{q}$ (B) $v \propto \sqrt{\lambda}$
(C) $v \propto \frac{1}{\sqrt{m}}$ (D) $v \propto \frac{1}{\sqrt{r}}$

11. Two charges Q_1 and Q_2 are placed at the points A and B having separation r lying inside and outside the uncharged conducting shell. The force on Q_1 is F and that on Q_2 is f. Then
(A) $F = 0$ (B) $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$
(C) $f = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ (D) $f = 0$

12. A particle of charge q and mass m is moving in an electric field
(A) must undergo change in velocity.
(B) must undergo change in speed.
(C) may not undergo change in velocity.
(D) may not undergo change in speed.

13. Two charges Q_1 and Q_2 have their respective placements at the inside and the outside of a closed surface S. If E be the field at any point on S and ϕ be the flux of E over the surface S, then if
(A) Q_1 changes, both E and ϕ will change.
(B) Q_2 changes, E will change but ϕ will not change.
(C) $Q_1 = 0$ and $Q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.
(D) $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

14. A particle having specific charge s starts from rest in a region where the electric field has constant direction but magnitude varying with time t as $E = E_0 t$. In time t, it is observed that the particle acquires a velocity v after covering a distance x, then

(A) $x = \frac{1}{2}(E_0 s)t^3$ (B) $x = \frac{1}{6}(E_0 s)t^3$

(C) $v = (E_0 s)t^2$ (D) $v = \frac{1}{2}(E_0 s)t^2$

15. At a point in space, let the field and potential be having values E and V respectively. Out of the following assertions which one(s) is/are correct?

(A) For $V = 0$, E must be zero

(B) For $V \neq 0$, E cannot be zero

(C) For $E \neq 0$, V cannot be zero

(D) None of these

16. In a parallel plate capacitor, the potential difference between the plates is V. A particle of mass m and charge $-Q$ leaves the negative plate and reaches the positive plate at distance d in time t with a momentum p. Then

(A) $p = \sqrt{mQV}$ (B) $p = \sqrt{2mQV}$

(C) $t = \sqrt{\frac{md^2}{QV}}$ (D) $t = \sqrt{\frac{2md^2}{QV}}$

17. A ring with a uniform charge Q and radius R, is placed in the yz plane with its centre at the origin O of the coordinates. Let V_O and E_O be the potential and the field at the origin, V and E be the potential and field at all points on the positive x axis, other than the origin, then

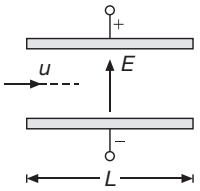
(A) $E_O = 0$

(B) $V_O = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

(C) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2 + R^2}}$

(D) $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$ and is maximum at $x = \frac{R}{\sqrt{2}}$

18. There is a uniform electric field E in the region between two parallel plates. A charged particle with a specific charge s enters the region with a speed u, in a direction parallel to the plates. The length of each plate is L. The time taken to come out of the plates is t and it suffers a vertical displacement Δ. Also as the particle crosses the plates, its direction of motion changes by an angle θ, then



(A) $\Delta = \frac{EsL^2}{2u^2}$ (B) $t = \frac{L}{u}$

(C) $\theta = \tan^{-1}\left(\frac{EsL}{u^2}\right)$ (D) $\theta = \tan^{-1}\left(\frac{Est}{u}\right)$

19. Three concentric metallic shells A, B and C are arranged such that shell A is the innermost and shell C is the outermost. Now, A is given some charge. Then the

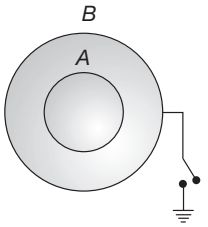
(A) inner surfaces of B and C have the same charge density.

(B) inner surfaces of B and C have the same charge.

(C) outer surfaces of A, B and C have the same charge density.

(D) outer surfaces of A, B and C have the same charge.

20. Two concentric shells A and B have radii R and $2R$, charges q_A and q_B and potentials $2V$ and $\frac{3V}{2}$ respectively. Now, the shell B is earthed and the new charges on them become q'_A and q'_B. Then



(A) $\frac{q_A}{q_B} = \frac{1}{2}$

(B) $\frac{q'_A}{q'_B} = 1$

(C) Potential difference between A and B after earthing becomes $\frac{3V}{2}$

(D) Potential difference between A and B after earthing becomes $\frac{V}{2}$

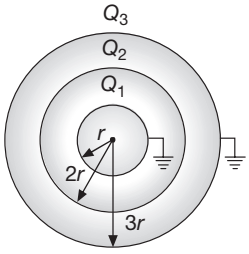
21. In a uniform electric field,

(A) all points are at the same potential.

(B) no two points can have the same potential.

(C) pairs of points separated by the same distance must have the same difference in potential.
(D) None of the above.

22. Three concentric conducting spherical shells A, B and C have radii r, $2r$ and $3r$ and possess charges Q_1, Q_2 and Q_3 respectively. The innermost and the outermost shells are earthed as shown in the figure. Select the mathematical relations between the charges that are correct.



(A) $Q_1 + Q_3 = -Q_2$ (B) $Q_1 = -\dfrac{Q_2}{4}$

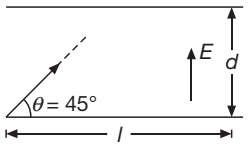
(C) $\dfrac{Q_3}{Q_1} = 3$ (D) $\dfrac{Q_3}{Q_2} = -\dfrac{1}{3}$

23. A spherical conductor A lies inside a hollow spherical conductor B. Charges Q_1 and Q_2 are given to A and B respectively.
(A) Charge Q_1 will appear on the outer surface of A.
(B) Charge Q_2 will appear on the outer surface of B.
(C) Charge $-Q_1$ will appear on the inner surface of B.
(D) Charge $Q_1 + Q_2$ will appear on the outer surface of B.

24. A particle of charge Q having a mass m moves rectilinearly under the action of an electric field that varies with distance x from the initial position of rest of the particle $E = \alpha - \beta x$, where α and β are positive constants. Then
(A) the motion of the particle is oscillatory
(B) the amplitude of the particle is $\dfrac{\alpha}{\beta}$
(C) the mean position of the particle is at $x = \dfrac{\alpha}{\beta}$
(D) the maximum acceleration of the particle is $\dfrac{q\alpha}{m}$

25. Two identical charges each having a charge $+Q$ are fixed at a separation $2a$. A small particle P having a charge of magnitude q is placed midway between them. Now, P is given a displacement $\Delta\,(\ll a)$, such that it undergoes simple harmonic motion. For this, we have the following (q, Δ) combinations.
(A) positive, along the line joining the two identical charges.
(B) positive, perpendicular to the line joining the two identical charges.
(C) negative, perpendicular to the line joining the two identical charges.
(D) negative, along the line joining the two identical charges.

26. An electron is projected between two plates at separation $d = 3$ cm each having length $l = 11$ cm as shown in figure, at a speed of 6×10^6 ms^{-1} at an angle of $45°$ in a uniform field $E = 2000$ Vm^{-1} directed upwards. The electron strikes the



(A) upper plate
(B) lower plate
(C) lower plate at the edge
(D) nowhere

27. Two identical particles of mass m carry a charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first particle from a large distance with a speed v. If d is the distance of closest approach then

(A) $d \propto Q^2$ (B) $d \propto \dfrac{1}{v^2}$

(C) $d \propto \dfrac{1}{v}$ (D) $d \propto \dfrac{1}{m}$

28. An electron moving with a speed of 5×10^6 ms^{-1} is shot parallel to an electric field of strength 1000 Vm^{-1}, arranged so as to retard its motion. The electron travels a distance s in the field before coming momentarily to rest in time t. It is observed that the electric field ends abruptly after 0.8 cm and due to this the electron loses a percentage fraction f of its initial energy, then

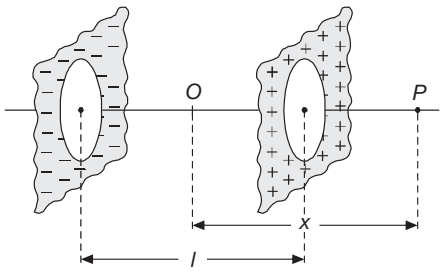
(A) $s = 0.14$ m (B) $s = 0.07$ m
(C) $t = 0.03\ \mu$s (D) $f = 11\%$

29. A positively charged thin metal ring of radius R is fixed in the xy plane, with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then the motion of P is
(A) periodic, for all value of z_0 satisfying $0 < z_0 < \infty$.
(B) simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$.

(C) approximately simple harmonic, provided $z_0 \ll R$.

(D) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$.

30. Two infinitely large plane sheets separated by a distance l carry a uniform surface charge densities $+\sigma$ and $-\sigma$. The planes have identical coaxial holes each of radius $R(l \gg R)$. The potential and the field at a point on the axis of the holes at a distance x from the midpoint O between the holes is V and E respectively. Then



(A) $V = \dfrac{\sigma l x}{2\varepsilon_0\sqrt{R^2 + x^2}}$ (B) $E = \dfrac{\sigma l R^2}{2\varepsilon_0\left(R^2 + x^2\right)^{3/2}}$

(C) $V = \dfrac{\sigma}{2\varepsilon_0 x^2}$ (D) $E = -\dfrac{\sigma}{2\varepsilon_0 R^5}$

31. At the corners of a square, four particles having charges of the same magnitude are placed. If the electrostatic potential and field at the centre of the square due to these four charges is V and E respectively, then by suitably selecting the signs of the four charges of identical magnitude, which of the following conditions can be achieved?

(A) $V = 0,\ E = 0$ (B) $V = 0,\ E \neq 0$

(C) $V \neq 0,\ E = 0$ (D) $V \neq 0,\ E \neq 0$

32. A pendulum bob of mass $m = 71$ mg, carrying a charge of $Q = 2 \times 10^{-8}$ C, is at rest in a horizontal, uniform electric field $E = 20000\ \text{Vm}^{-1}$. The tension T in the thread of the pendulum is T and the angle it makes with the vertical is θ. Then

(A) $\theta = 27°$ (B) $T = 880\ \mu\text{N}$

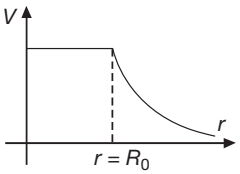
(C) $T = 803\ \mu\text{N}$ (D) $\theta = 30°$

33. Four identical charges each having a charge q are placed at the points $(a, 0, 0)$, $(0, a, 0)$, $(-a, 0, 0)$ and $(0, -a, 0)$. Let V_0 and E_0 be the potential and the field at the origin, V and E be the potential and field at all points on the z axis, other than the origin, then

(A) $V_0 \neq 0$ (B) $E_0 = 0$

(C) $V \neq 0$ (D) $E \neq 0$, along z-axis

34. For spherical symmetrical charge distribution, variation of electric potential with distance from centre is given in diagram. Given that $V = \dfrac{q}{4\pi\varepsilon_0 R_0}$ for $r \leq R_0$ and $V = \dfrac{q}{4\pi\varepsilon_0 r}$ for $r \geq R_0$



Then which option(s) is/are correct

(A) Total charge within $2R_0$ is q.

(B) Total electrostatic energy for $r \leq R_0$ is zero.

(C) At $r = R_0$ electric field is discontinuous.

(D) There will be no charge anywhere except at $r = R_0$.

35. In a region of space three charged particles are observed to be in equilibrium under their mutual electrostatic forces only. Then the particles

(A) must be collinear.

(B) cannot possess the same magnitude.

(C) cannot possess the same sign.

(D) will be in unstable equilibrium.

36. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(A) The angular momentum of the charge $-q$ is constant.

(B) The linear momentum of the charge $-q$ is constant.

(C) The angular velocity of the charge $-q$ is constant.

(D) The linear speed of the charge $-q$ is constant.

37. At the corners of an equilateral triangle of side l, three point charges are placed such that the only interaction force between them is the electrostatic force. Then this system of three charges will be

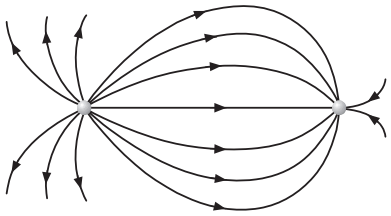
(A) in equilibrium if the charges have the same magnitude but all do not have the same sign.

(B) in equilibrium if the charges have different magnitudes but all do not have the same sign.

(C) in equilibrium if the charges rotate about the centroid of the triangle.

(D) never in equilibrium.

38. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$

(B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D) at a finite distance to the right of Q_2 the electric field is zero

39. Which of the following is/are incorrect statement?

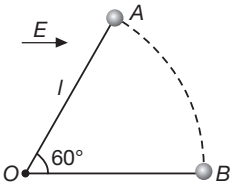
(A) Electric field is always conservative

(B) Electric field due to a varying magnetic field is non-conservative

(C) Electric field due to a stationary charge is conservative.

(D) Electric field lines are always closed loops.

40. A particle of mass m and charge q is fastened to one end of a string of length l. The other end of the string is fixed to the point O. The whole system lies on a frictionless horizontal plane. Initially, the mass is at rest at A. A uniform electric field in the direction shown is then switched on. Then



(A) the speed of the particle when it reaches B is $\sqrt{\frac{2qEl}{m}}$

(B) the speed of the particle when it reaches B is $\sqrt{\frac{qEl}{m}}$

(C) the tension in the string when particles reaches at B is qE

(D) the tension in the string when the particle reaches at B is $2qE$

41. A, B and C are three concentric metallic shells. Shell A is the inner most and shell C is the outermost. A is given some charge and shell C is earthed. Select the correct statement(s).

(A) The inner surfaces of B and C will have the same charge

(B) The inner surface of B and C will have same charge density

(C) The outer surface of A, B and C will have the same charge

(D) The outer surface of C will have no charge density

42. Two charge Q_1 & Q_2 placed at $(0, 0, 0)$ and $(a, 0, 0)$ are such that x component of field is zero at $\left(\frac{a}{2}, a, 0\right)$. Then

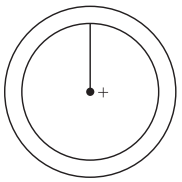
(A) y component of field is zero at $\left(\frac{a}{2}, 0, a\right)$

(B) $\frac{Q_1}{Q_2} = \frac{2}{1}$

(C) $\frac{Q_1}{Q_2} = 1$

(D) y component of field is zero at $\left(\frac{a}{2}, a, 0\right)$

43. An electrically isolated, hollow conducting sphere has a small positively charged ball suspended by an insulating rod from its inside surface, as shown. This causes the appearance of negative charge on the inner surface of the sphere. The electric field outside and far from the conducting sphere is approximately



(A) zero

(B) the same as if the sphere was not there

(C) half what it would be if the sphere was not there

(D) $\frac{Q}{4\pi\varepsilon_0 r^2}$, where r is the distance from centre of sphere

44. Two identical conducting balls have positive charges q_1 & q_2 respectively. The balls are brought together so that they touch and then put back in their original positions. The force between the balls may be

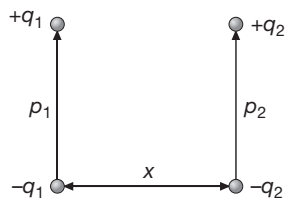
(A) The same as it was before the balls had been touched.

(B) greater than the force before the balls had been touched

(C) less than the force before the balls had been touched

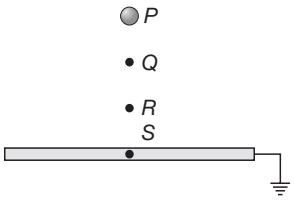
(D) zero

45. Figure shows two dipole moments parallel to each other and placed at a distance x apart (where x is very large compared to size of dipoles) then



(A) they will repel each other
(B) they will attract each other
(C) force of interaction is of magnitude of $\frac{3p_1p_2}{4\pi\varepsilon_0 x^4}$
(D) force of interaction is of magnitude of $\frac{6p_1p_2}{4\pi\varepsilon_0 x^4}$

46. A positive point charge is placed at P in front of an earthed metal sheet S. Two points Q and R lie between P and S as shown. If the electric field strength at Q and R are respectively E_Q and E_R and potential at Q and R are respectively V_Q and V_R, then



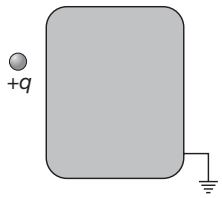
(A) $E_Q > E_R$ (B) $E_Q < E_R$
(C) $V_Q > V_R$ (D) $V_Q < V_R$

47. A particle of mass 2 kg and charge 1 mc is projected vertically with a velocity 10 ms^{-1}. There is a uniform horizontal electric field of 10^4 NC^{-1}. Select the correct statements.
(A) The horizontal range of the particle is 10 m.
(B) The time of flight of the particle is 2 s.
(C) The maximum height reached is 5 m.
(D) The horizontal range of the particle is zero.

48. A certain spherical region contains no charge, then
(A) Electric field at any point in the sphere must be zero.
(B) Electric field at any point in the sphere may not be zero.
(C) Electric potential at any point in the sphere must be zero.
(D) Electric potential at any point in the sphere may not be zero.

49. A dipole is placed in the electric field created by a uniformly charged long wire. It is possible that the dipole experiences
(A) No net force but a net torque
(B) No net force and no net torque
(C) A net force but no net torque
(D) A net force and a net torque

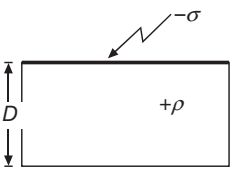
50. In front of an earthed conductor, a point charge $+q$ is placed as shown in figure. Select the correct statement(s).



(A) On the surface of conductor the net charge is always negative.
(B) On the surface of conductor at some points the charges are negative and at some points the charges may be positive distributed non uniformly.
(C) Inside the conductor electric field due to point charge is non-zero.
(D) None of these

51. A charged particle q is placed at a distance d from the centre of conducting sphere of radius $R(<d)$, then in static condition at the centre of sphere $\left(k=\frac{1}{4\pi\varepsilon_0}\right)$, we have
(A) magnitude of electric field due to induced charge is $\frac{kq}{d^2}$
(B) magnitude of electric field due to induced charge is $\frac{kq}{R^2}$
(C) magnitude of electric field due to induced charge is zero
(D) magnitude of electric field due to charge q is $\frac{kq}{d^2}$

52. We have an infinite non-conducting sheet of negligible thickness carrying a uniform surface charge density $-\sigma$ and next to it, an infinite parallel slab of thickness D with uniform volume charge density $+\rho$. All charges are fixed.



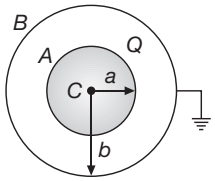
(A) Magnitude of electric field at a distance h above the negatively charged sheet is $\frac{\rho D-\sigma}{2\varepsilon_0}$

(B) Magnitude of electric field inside the slab at a distance h below the negatively charged sheet $(h<D)$ is $\frac{\sigma+\rho(D-2h)}{2\varepsilon_0}$

(C) Magnitude of electric field at a distance h below the bottom of the slab is $-\frac{\rho D-\sigma}{4\varepsilon_0}$

(D) Magnitude of electric field at a distance h below the bottom of the slab is $\frac{\rho D-\sigma}{2\varepsilon_0}$

53. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common centre of A and B, then the



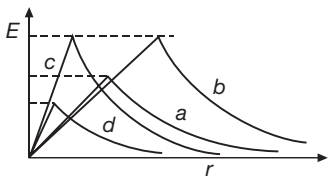
(A) field at a distance r from C, where $a\le r\le b$, is $\frac{Q}{4\pi\varepsilon_0 r^2}$

(B) potential at a distance r from C, where $a\le r\le b$, is $\frac{Q}{4\pi\varepsilon_0 r}$

(C) potential difference between A and B is $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

(D) potential at a distance r from C, where $a\le r\le b$, is $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$

54. Four spheres each of different radii named a, b, c, d are given. Each is solid, insulating and uniformly charged. The variation of electric field with distance r from the centre is given. Straight portion of curve c and d is overlapping and straight portion of curve a and b is overlapping. Select the correct statement(s).



(A) Radius of sphere $a>$ Radius of sphere b

(B) Radius of sphere $a<$ Radius of sphere b

(C) Volume charged density of $c>$ volume charged density of b

(D) Volume charged density of $c<$ volume charged density of b

55. A conductor A is given a charge $+Q$ and then placed inside a deep metal can B, without touching it. Then

(A) potential of A does not change when it is placed inside B

(B) if B is earthed, $+Q$ amount of charge flows from it into the earth

(C) if B is earthed, the potential of A is reduced

(D) if B is earthed, the potential of A is increased

56. A positive charge of q_1 is located 3 m to the left of a negative charge $-q_2$. Magnitude of charge q_1 greater than magnitude of negative charge q_2. The net electric field intensity is zero at a point on the line passing through charges and 1 m to the right of $-q_2$. On this line there are also points where the potential is zero. Location of these two points relative to the negative charge is/are

(A) at 0.2 m left of negative charge

(B) at 0.2 m right of negative charge

(C) at $\frac{3}{17}$ m left of negative charge

(D) at $\frac{3}{17}$ m right of negative charge

57. Select the correct statement(s).

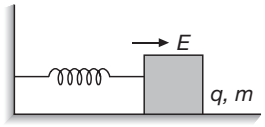
(A) When a positively charged sphere is brought near a metallic sphere, it is observed that a force of attraction exists between two, which means the metallic sphere become negative charged.

(B) A given conducting sphere cannot be charged to a potential greater than a certain value.

(C) Potential difference between two points lying in a uniform electric field may be equal to zero.

(D) Potential difference between two points lying in a non-uniform electric field may be equal to zero.

58. A block having mass m and charge q is connected by spring of force constant k. The block lies on a frictionless horizontal track and a uniform electric field E acts on system as shown. The block is released from rest when spring is unstretched (at $x=0$). Then



(A) Maximum stretch in the spring is $\frac{2qE}{k}$

(B) In equilibrium position, stretch in the spring is $\frac{qE}{k}$

(C) Amplitude of oscillation of block is $\frac{qE}{k}$

(D) Amplitude of oscillation is $\frac{2qE}{k}$

59. A uniform electric field of magnitude E_0 exists in a region at an angle $45°$ with x-axis. There are two point A $(a, 0)$ and B $(0, b)$ having potential V_A and V_B respectively, then

(A) $V_A > V_B$ if $a > b$ (B) $V_A = V_B$ if $a = b$

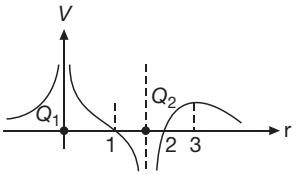
(C) $V_A > V_B$ if $a < b$ (D) $V_A < V_B$ if $a > b$

60. A simple pendulum of length ℓ has a bob of mass m, with a charge q on it. A vertical sheet of charge, with charge σ per unit area, passes through the point of suspension of the pendulum. At equilibrium, the string makes an angle θ with vertical. Its time period of oscillation is T in this position. Then

(A) $\tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$ (B) $\tan\theta = \frac{\sigma q}{\varepsilon_0 mg}$

(C) $T < 2\pi\sqrt{\frac{l}{g}}$ (D) $T > 2\pi\sqrt{\frac{l}{g}}$

61. Two point charges Q_1 and Q_2 lie along a line at a distance from each other. Figure show the potential variation along the line of charges



(A) Q_1 is positive and Q_2 is negative.
(B) Q_1 is negative and Q_2 is positive.
(C) Magnitude of Q_1 is more than magnitude of Q_2.
(D) Electric field is zero at 3.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If there exists coulombic attraction between two bodies both of them may not be charges.
Statement-2: In coulombic attraction two bodies are always oppositely charged.

2. **Statement-1:** Two charges q_1 and q_2 are placed at separation r. Then magnitude of force on each charge is $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$.
Statement-2: Now a third charge q_3 is placed near q_1 and q_2. Then force on q_1 due to q_2 remains F.

3. **Statement-1:** While going away from a point charge or a small electric dipole, electric field decreases at the same rate in both the cases.
Statement-2: Electric field is inversely proportional to square of distance from the point charge.

4. **Statement-1:** The potential of an uncharged conducting sphere of radius R, for a point charge q located at distance r from its centre $(r > R)$ is $\frac{q}{4\pi\varepsilon_0 r}$.
Statement-2: Electric field intensity inside the conductor is zero therefore potential at each point on conductor is zero.

5. **Statement-1:** When a neutral body is charged negatively, its mass increases slightly.
Statement-2: When a body is charged negatively, it gains some electrons and electron has finite mass : though quite small

6. **Statement-1:** Electric field at a point is always inversely proportional to $(\text{distance})^2$
Statement-2: Electric field due to a line charge at a point is inversely proportional to distance.

7. **Statement-1:** Any charge will move from electric potential V_1 to V_2 by its own only when $V_1 < V_2$.
Statement-2: Electron moves from $V_1 = 2V$ towards $V_2 = 4$ V.

8. **Statement-1:** Electric field intensity within an isolated conductor will be zero.
Statement-2: No net charge can exist within an isolated conductor.

9. **Statement-1:** Excess charge on a conductor resides entirely on the outer surface.
Statement-2: Like charges repel each another.

10. **Statement-1:** The surface of a charged conductor is always equipotential.
Statement-2: Electric field lines are always perpendicular to the equipotential surface.

11. **Statement-1:** When a charged body is brought near to an uncharged conducting body, equal and opposite charge is induced on the nearer surface of the conducting body.
Statement-2: Net electric field inside the conductor is zero.

12. A charge is launched with a velocity perpendicular to uniform electric field then
Statement-1: Initial power delivered by electric field is zero.
Statement-2: Path of charged particle is circular.

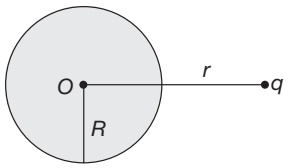
13. **Statement-1:** A charge conductor may have charged particle inside it.
Statement-2: There can't exist electric field line inside the conductor.

14. **Statement-1:** Electric field intensity at surface of uniformly charge spherical shell is E. If shell is punctured at a point then intensity at punctured point become $\frac{E}{2}$.
Statement-2: Electric field intensity due to spherical charge distribution can be found out by using Gauss Law.

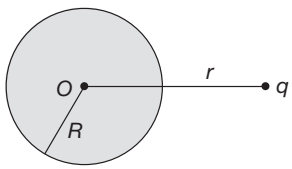
15. **Statement-1:** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
Statement-2: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

16. **Statement-1:** Induced charge does not contribute to electric field or potential at a given point.
Statement-2: A point charge q_0 is kept outside a solid metallic sphere, the electric field inside the sphere is zero.

17. **Statement-1:** A point charge q is placed in front of a solid conducting sphere. Electric field due to induced charges at the centre of sphere is zero.
Statement-2: Electric field at a point inside the solid body of conductor is zero.



18. **Statement-1:** Consider a conducting sphere of radius R. Now a charge q is placed in front of sphere. Electric potential at point O is $\frac{q}{4\pi\varepsilon_0 r}$.



Statement-2: Electric potential at the centre of sphere due to induced charges is zero.

19. **Statement-1:** A metallic shield in form of a hollow shell may be built to block an electric field.
Statement-2: In a hollow spherical shield, the electric field inside it is zero at every point.

20. **Statement-1:** If a point charge q is placed in front of an infinite grounded conducting plane surface, the point charge will experience a force.
Statement-2: This force is due to the induced charge on the conducting surface, which is at zero potential.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

In the classical model of a hydrogen atom, the electron revolves around the proton in a circle of radius $r = 0.53$ Å. The magnitude of the charge of the electron and proton is $e = 1.6 \times 10^{-19}$ C. Based on the above facts, answer the following questions.

1. The magnitude of the electric force between the proton and the electron is

(A) 8.2×10^{-6} N (B) 8.2×10^{-7} N

(C) 8.2×10^{-8} N (D) None of these

2. The magnitude of the electric field due to the proton at distance r is

(A) 5.76×10^{12} NC^{-1} (B) 5.76×10^{11} NC^{-1}

(C) 5.76×10^{10} NC^{-1} (D) None of these

3. The ratio of the magnitudes of the electrical and gravitational force between electron and proton is

(A) 2.2×10^{39}, dependent on r

(B) 2.2×10^{36}, dependent on r

(C) 2.2×10^{39}, independent of r

(D) 2.2×10^{36}, independent of r

Comprehension 2

An oil drop of radius $r = 1.64 \times 10^{-6}$ m having mass density $\rho_{oil} = 8.51 \times 10^2$ kgm^{-3} is allowed to fall from rest. It, then enters into a region of constant external field $\vec{E}$ applied in the downward direction. The oil drop has an unknown electric charge q. The magnitude of the electric field is adjusted till the gravitational force, $\vec{F}_g = m\vec{g} = -mg\hat{j}$ on the oil drop is exactly balanced by the electric force, $\vec{F}_e = q\vec{E}$. Suppose this balancing occurs when the value of electric field is given by $\vec{E} = -E_y\hat{j} = -(1.92 \times 10^5 \text{ NC}^{-1})\hat{j}$, with $E_y = 1.92 \times 10^5$ NC^{-1}. Based on the above facts, answer the following questions.

4. The mass of the oil drop is

(A) 1.57×10^{-14} kg (B) 1.57×10^{-13} kg

(C) 1.57×10^{-12} kg (D) 1.57×10^{-11} kg

5. The oil drop falls short of electrons or has the excess electrons amounting to N. Then

(A) $N(=4)$, short of electrons

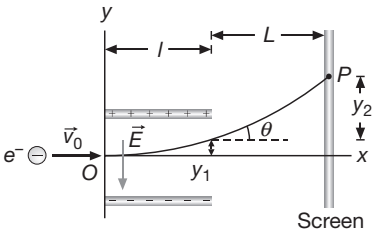
(B) $N(=5)$, short of electrons

(C) $N(=4)$, excess of electrons

(D) $N(=5)$, excess of electrons

Comprehension 3

An electron is injected (at $t = 0$) horizontally into a uniform field produced by two oppositely charged plates having length l and separation d, as shown in figure. The particle has an initial velocity $\vec{v}_0 = v_0\hat{i}$ perpendicular to $\vec{E} = E\hat{j}$.



Based on the above facts, and the figure provided, answer the following questions.

6. The path followed by the electron is a/an

(A) straight line (B) ellipse

(C) cycloid (D) parabola

7. The time t_1 taken by the electron to leave the plate is t_1. Then

(A) $t_1 = \sqrt{\dfrac{2lm}{eE}}$ (B) $t_1 = \dfrac{l}{v_0}$

(C) $t_1 = \sqrt{\dfrac{2md}{eE}}$ (D) $t_1 = \dfrac{d}{v_0}$

8. The velocity of the electron at time t_1 when it leaves the plates is

(A) $\left(\dfrac{eE}{m}\right)\dfrac{l}{v_0}$ (B) v_0

(C) $\dfrac{1}{mv_0}\sqrt{m^2v_0^4 + e^2l^2E^2}$ (D) None of these

9. The vertical displacement of the electron, y_1 after time t_1 when it leaves the plates is

(A) $y_1 = \dfrac{eEl^2}{mv_0}$ (B) $y_1 = \dfrac{eEl^2}{mv_0^2}$

(C) $y_1 = \dfrac{eEl^2}{2mv_0^2}$ (D) $y_1 = \dfrac{eEl^2}{m^2v^2}$

10. The electron makes an angle θ with the horizontal, when leaving the plates at time t_1, then

(A) $\sin\theta = \dfrac{eEl}{mv_0^2}$ (B) $\sin\theta = \dfrac{eEl}{2mv_0^2}$

(C) $\tan\theta = \dfrac{eEl}{mv_0^2}$ (D) $\tan\theta = \dfrac{eEl}{2mv_0^2}$

11. The electron hits the screen located a distance L from the end of the plates at a time t_2. The total vertical displacement of the electron from time $t = 0$ until it hits the screen at t_2 is y. Then

(A) $y = \dfrac{eEl}{mv_0^2}\left(\dfrac{L}{2} + l\right)$ (B) $y = \dfrac{eEl}{mv_0^2}\left(\dfrac{l}{2} + L\right)$

(C) $y = \dfrac{eEl}{2mv_0^2}(L + l)$ (D) $y = \dfrac{eEl}{2mv_0^2}\left(L + \dfrac{l}{2}\right)$

Comprehension 4

Two parallel plates having charges of equal magnitude but opposite sign are separated by 12 cm. Each plate has a surface charge density of 36 nCm^{-1}. A proton is released from rest at the positive plate. Based on the above facts, answer the following questions.

12. The potential difference between the plates is

(A) 288 V (B) 388 V

(C) 244 V (D) 488 V

13. The kinetic energy of the proton when it reaches the negative plate is

(A) 3.9×10^{-17} J (B) 7.8×10^{-17} J

(C) 4.6×10^{-17} J (D) 6.2×10^{-17} J

14. The speed of the proton just before it strikes the negative plate is

(A) 106 kms^{-1} (B) 235 kms^{-1}

(C) 306 kms^{-1} (D) 216 kms^{-1}

15. The acceleration of the proton is

(A) 3.9×10^{11} ms^{-2} (B) 4.68×10^{10} ms^{-2}

(C) 2.3×10^{11} ms^{-2} (D) 1.95×10^{11} ms^{-2}

16. The force acting on the proton

(A) 6.51×10^{-16} N (B) 7.81×10^{-17} N

(C) 3.84×10^{-16} N (D) 3.25×10^{-16} N

Comprehension 5

Two positive point charges each of magnitude q are placed on the y-axis at the points $(0, a)$ and $(0, -a)$. A positively charged particle of charge q_0 and mass m, when displaced slightly from the origin in the direction of the negative x-axis attains a speed v_0 at infinity. When the particle is projected towards the left along the x-axis from a point at a large distance towards the right of the origin with a velocity half the value of v_0, then it comes to rest at a point $P(x, 0)$. Again, if a negatively charged particle from the rest (assume the negatively charged particle to be at very large distance towards the left of origin), then this negatively charged particle crosses the origin with a speed v. Based on the above facts, answer the following questions.

17. The value of v_0 is

(A) $\sqrt{\dfrac{qq_0}{2\pi\varepsilon_0 ma}}$ (B) $\sqrt{\dfrac{qq_0}{\pi\varepsilon_0 ma}}$

(C) $\sqrt{\dfrac{qq_0}{3\pi\varepsilon_0 ma}}$ (D) None of these

18. The value of x is

(A) $\sqrt{3}a$ (B) $4a$

(C) $\sqrt{2}a$ (D) $\sqrt{15}a$

19. The value of v is

(A) $\dfrac{v_0}{3}$ (B) $\dfrac{v_0}{2}$

(C) v_0 (D) $\dfrac{v_0}{4}$

Comprehension 6

A particle of mass m_2 carrying a charge Q_2 is fixed on the surface of Earth. Another particle, mass m_1 and charge Q_1, is positioned right above the first one at an altitude $h \ll R$, with R the radius of the Earth. The charges Q_1 and Q_2 have the same sign. Based on the above facts, answer the following questions.

20. The velocity of m_1 at a point P very close to Q_2, a distance h_1 from the surface of earth, if the initial velocity of m_1 was zero and air drag and earth's electric field being ignored, is

(A) $\sqrt{2gh - \dfrac{Q_1Q_2}{2\pi\varepsilon_0 h_1 m_1}}$ (B) $\sqrt{2gh - \dfrac{Q_1Q_2}{4\pi\varepsilon_0 h_1 m_1}}$

(C) $\sqrt{2gh}$ (D) None of these

21. The magnitude of charge Q_2 at which the velocity of m_1, at altitude h_2 is zero is given by

(A) $Q_2 = \dfrac{\pi\varepsilon_0 m_1 g h h_2}{Q_1}$ (B) $Q_2 = \dfrac{4\pi\varepsilon_0 m_1 g h h_2}{Q_1}$

(C) $Q_2 = \dfrac{2\pi\varepsilon_0 m_1 g h h_2}{Q_1}$ (D) $Q_2 = \dfrac{\pi\varepsilon_0 m_1 g h h_2}{4Q_1}$

22. At what altitude h_3 will object m_1 be in equilibrium and what will be the nature of the object's oscillations if it is disturbed from equilibrium?

(A) $h_3 = \sqrt{\dfrac{Q_1 Q_2}{4\pi\varepsilon_0 m_1 g}}$, periodic

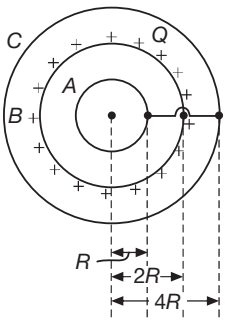
(B) $h_3 = \sqrt{\dfrac{Q_1 Q_2}{2\pi\varepsilon_0 m_1 g}}$, periodic and oscillatory

(C) $h_3 = \sqrt{\dfrac{Q_1 Q_2}{4\pi\varepsilon_0 m_1 g}}$, periodic and oscillatory

(D) None of these

Comprehension 7

Figure shows three concentric spherical conductors A, B and C with radii R, $2R$ and $4R$ respectively. A and C are connected by a conducting wire and B is having a uniform charge $+Q$.



Based on the above facts, answer the following questions.

23. Charge on the conductor A is q_A and that on C is q_C. Then

(A) $q_A = q_C = -\dfrac{Q}{3}$

(B) $q_A = \dfrac{Q}{3}$ and $q_C = \dfrac{2Q}{3}$

(C) $q_A = \dfrac{Q}{3}$ and $q_C = -\dfrac{Q}{3}$

(D) $q_A = -\dfrac{Q}{3}$ and $q_C = \dfrac{Q}{3}$

24. The potential of A is

(A) $-\dfrac{Q}{4\pi\varepsilon_0 R}$ (B) $-\dfrac{Q}{12\pi\varepsilon_0 R}$

(C) $\dfrac{Q}{16\pi\varepsilon_0 R}$ (D) None of these

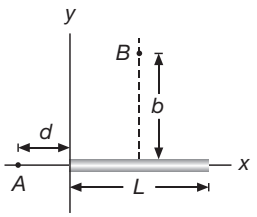
25. The potential of B is

(A) $\dfrac{Q}{4\pi\varepsilon_0 R}$ (B) $\dfrac{5Q}{12\pi\varepsilon_0 R}$

(C) $\dfrac{3Q}{6\pi\varepsilon_0 R}$ (D) $\dfrac{5Q}{48\pi\varepsilon_0 R}$

Comprehension 8

A rod of length L is placed along the x-axis with its left end at the origin and has a non-uniform charge density varying as $\lambda = \alpha x$, where α is a positive constant.



Based on the above facts and the figure provided, answer the following questions.

26. The dimensional formula for α is

(A) $M^{-2}T^{-1}A$ (B) $M^0L^{-2}TA$

(C) $M^0L^{-1}TA$ (D) $M^0L^{-2}T^{-1}A$

27. The electric potential at A, for $d = \dfrac{L}{4}$ is given by

(A) $\dfrac{\alpha}{4\pi\varepsilon_0}(L - \log_e 5)$ (B) $\dfrac{\alpha}{4\pi\varepsilon_0}(L + \log_e 5)$

(C) $\dfrac{\alpha}{4\pi\varepsilon_0}(1 - \log_e 5)$ (D) $\dfrac{\alpha}{4\pi\varepsilon_0}(1 + \log_e 5)$

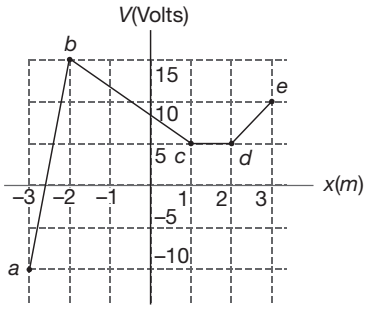
28. The electric potential at point B that lies along the perpendicular bisector of the rod a distance $b = \dfrac{\sqrt{3}L}{2}$ above the x-axis is given by

(A) $\dfrac{\alpha L}{4\pi\varepsilon_0}\log_e(3)$ (B) $\dfrac{\alpha L}{4\pi\varepsilon_0}\log_e(6)$

(C) $\dfrac{\alpha L}{8\pi\varepsilon_0}\log_e(3)$ (D) $\dfrac{\alpha L}{8\pi\varepsilon_0}\log_e(6)$

Comprehension 9

Suppose that the electric potential varies along the x-axis as shown in Figure below.



The potential does not vary in the y or z-direction. Of the intervals shown (ignore the behaviour at the end points of the intervals). The field E_x has a maximum absolute value of **FB1** Vm^{-1} in region **FB2**. Its value in the region CD is **FB3** Vm^{-1}. Based on the above facts, answer the following questions.

29. The value that fills **FB1** is

(A) $\frac{25}{2}$ (B) 25

(C) $\frac{15}{2}$ (D) –25

30. The region that fills **FB2** is

(A) ab (B) de

(C) bc (D) None of these

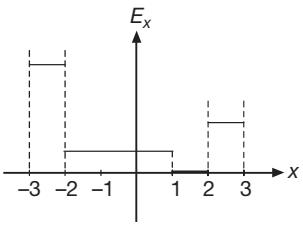
31. The value that fills **FB3** is

(A) 1 (B) 2

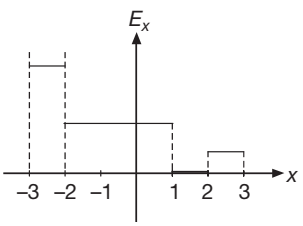
(C) 3 (D) ZERO

32. The plot of E_x vs x is

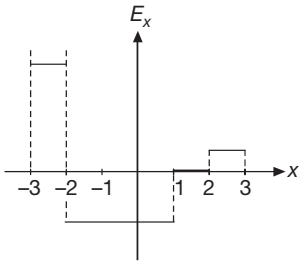
(A)



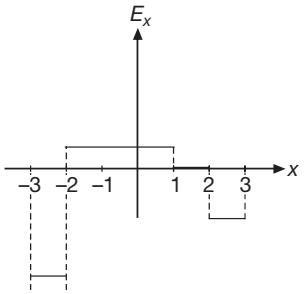
(B)



(C)



(D)



33. In the V vs x curve, the potential possesses a zero value at

(A) $x = -\frac{12}{5}$ m (B) $x = -\frac{5}{13}$ m

(C) $x = -\frac{13}{5}$ m (D) $x = -\frac{14}{5}$ m

Comprehension 10

A particle of charge $+3\times10^{-9}$ C is in a uniform field directed toward left. It is released from rest and moves a distance of 5 cm after which its kinetic energy is found to be 4.5×10^{-5} J. Based on the information supplied, answer the following questions.

34. What is the work that is being done by the electrical force?

(A) 90 μJ (B) 45 μJ

(C) 15 μJ (D) None of the above

35. The magnitude of the electric field is

(A) 0.3 MNC^{-1} (B) 3 MNC^{-1}

(C) 0.5 MNC^{-1} (D) 1.5 MNC^{-1}

36. The potential of starting point w.r.t. the end point is

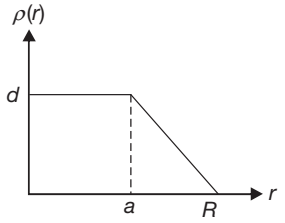
(A) 1 kV (B) 10 kV

(C) 1.5 kV (D) 15 kV

Comprehension 11

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is depended only on the radial

distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



37. The electric field at $r = R$ is
(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a

38. For $a = 0$, the value d (maximum value of ρ as shown in the figure) is

(A) $\dfrac{3Ze^2}{4\pi R^3}$ (B) $\dfrac{3Ze}{\pi R^3}$

(C) $\dfrac{4Ze}{3\pi R^3}$ (D) $\dfrac{Ze}{3\pi R^3}$

39. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

(A) $a = 0$ (B) $a = \dfrac{R}{2}$

(C) $a = R$ (D) $a = \dfrac{2R}{3}$

Comprehension 12

A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density is given by

$$\rho(r) = \begin{cases} \alpha \text{ for } r \le \dfrac{R}{2} \\ 2\alpha\left(1 - \dfrac{r}{R}\right) \text{ for } \dfrac{R}{2} \le r \le R \end{cases}$$

where α is a positive constant and $\rho(r) = 0$ for $r \ge R$.

Based on the above facts, answer the following questions.

40. The fraction of total charge contained in the region $r \le \dfrac{R}{2}$ is

(A) $\dfrac{4}{15}$ (B) $\dfrac{8}{15}$

(C) $\dfrac{7}{15}$ (D) $\dfrac{11}{15}$

41. If an electron is placed at the centre of this arrangement and slightly displaced then it will execute SHM. The time period of oscillation assuming $x < \dfrac{R}{2}$ is

(A) $2\pi\sqrt{\dfrac{\varepsilon_0 \pi m R^3}{8Qe}}$ (B) $2\pi\sqrt{\dfrac{15\pi\varepsilon_0 m R^3}{8Qe}}$

(C) $2\pi\sqrt{\dfrac{7\pi\varepsilon_0 m R^3}{8Qe}}$ (D) $2\pi\sqrt{\dfrac{13\pi\varepsilon_0 m R^2}{6Qe}}$

42. The electric field in a region $\dfrac{R}{2} < r < R$ is

(A) $\dfrac{\alpha R^3}{24 r^2 \varepsilon_0} + \dfrac{2\alpha r}{3\varepsilon_0}\left(1 - \dfrac{r}{R}\right)$

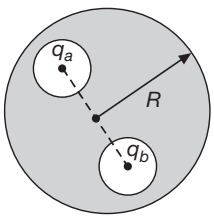
(B) $\dfrac{\alpha R^3}{8 r^2 \varepsilon_0} + \dfrac{2\alpha r}{6\varepsilon_0}\left(1 - \dfrac{r}{R}\right)$

(C) $-\dfrac{\alpha R^3}{16 r^2 \varepsilon_0} + \dfrac{2\alpha r}{3\varepsilon_0}\left(1 - \dfrac{r}{R}\right)$

(D) $-\dfrac{\alpha R^3}{96 \varepsilon_0 r^2} + \dfrac{2\alpha r}{3\varepsilon_0}\left(1 - \dfrac{3r}{4R}\right)$

Comprehension 13

Two spherical cavities, of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. At the centre of each cavity, a point charge is placed. Call these charges q_a and q_b, distance between q_a and q_b is r.



Based on the above facts, answer the following questions.

43. The force on charge q_a is.

(A) $\dfrac{q_a q_b}{4\pi\varepsilon_0 r^2}$ (B) $\dfrac{q_a(q_a + q_b)}{4\pi\varepsilon_0 R^2}$

(C) $\dfrac{(q_a)(q_a + q_b)}{4\pi\varepsilon_0 r^2}$ (D) zero

44. The field outside the conductor at any point P at a distance $r_0 (> R)$ from centre of conductor?

(A) $\frac{q_a}{4\pi\varepsilon_0 r_0^2}$ (B) $\frac{q_a+q_b}{4\pi\varepsilon_0 R^2}$

(C) $\frac{q_a+q_b}{4\pi\varepsilon_0 r_0^2}$ (D) $\frac{q_b}{4\pi\varepsilon_0 r_0^2}$

45. In each cavity, there is certain electric field say E. If another charge q_c were brought near the conductor then E
(A) increases.
(B) decreases.
(C) remains same.
(D) depends on the nature of charge q_c.

Comprehension 14

A small plastic ball of mass m covered with a thin zinc coating, is suspended by means of an insulating thread from a fixed point. There exists an electric field of magnitude E directed along horizontal. Now ultraviolet light is made to incident on the ball due to which it acquires a positive charge. Acceleration due to gravity is g. Based on the above facts, answer the following questions.

46. What is the origin of positive charge acquired by the ball?
(A) Excess of protons in plastic ball
(B) Deficiency of electrons in zinc coating
(C) Deficiency of electrons in plastic ball
(D) Some positively charged particles given by ultraviolet light

47. If the string makes an angle θ with the vertical, when the ball comes to equilibrium, the number of electrons lost by the ball is

(A) $\frac{mg}{eE}$ (B) $\frac{mg\sin\theta}{eE}$

(C) $\frac{mg\tan\theta}{eE}$ (D) $\frac{mg\cot\theta}{eE}$

48. If the charge that suddenly appears on the ball is q, what is the maximum angle by which the string gets deflected?

(A) $\tan^{-1}\left(\frac{mg}{qE}\right)$ (B) $\tan^{-1}\left(\frac{2mg}{qE}\right)$

(C) $2\tan^{-1}\left(\frac{qE}{mg}\right)$ (D) $\tan^{-1}\left(\frac{qE}{mg}\right)$

Comprehension 15

A ball of radius R carries a positive charge whose volume charge density depends only on the distance r form the ball's centre as.

$$\rho=\rho_0\left(1-\frac{r}{R}\right)$$

where ρ_0 is a constant. Assume ε as the permittivity of the ball, answer the following questions.

49. The magnitude of the electric field as a function of the distance r outside the ball is given by

(A) $E=\frac{\rho_0 R^3}{8\varepsilon r^2}$ (B) $E=\frac{\rho_0 R^3}{12\varepsilon r^2}$

(C) $E=\frac{\rho_0 R^2}{8\varepsilon r^3}$ (D) $E=\frac{\rho_0 R^2}{12\varepsilon r^3}$

50. The value of distance r_m, at which electric field intensity is maximum, is given by

(A) $r_m=\frac{R}{3}$ (B) $r_m=\frac{3R}{2}$

(C) $r_m=\frac{2R}{3}$ (D) $r_m=\frac{4R}{3}$

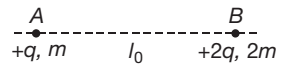
51. The maximum electric field intensity is

(A) $E_m=\frac{\rho_0 R}{9\varepsilon}$ (B) $E_m=\frac{\rho_0\varepsilon}{9R}$

(C) $E_m=\frac{\rho_0 R}{3\varepsilon}$ (D) $E_m=\frac{\rho_0 R}{6\varepsilon}$

Comprehension 16

Two positive point charges A and B have charge $+q$ and $2q$, mass m and $2m$ respectively as shown.



Both the charges are released from rest when they are at a distance l apart. Neglect gravity and also assume the only force acting on either charge is the electrostatic force due to each other. Based on the above facts, answer the following questions.

52. The speed of charge A at the instant separation between both charges is $2l_0$ is

(A) $\sqrt{\frac{q^2}{12\pi\varepsilon_0 ml}}$ (B) $\sqrt{\frac{q^2}{6\pi\varepsilon_0 ml}}$

(C) $\sqrt{\frac{q^2}{4\pi\varepsilon_0 ml}}$ (D) $\sqrt{\frac{q^2}{3\pi\varepsilon_0 ml}}$

53. The work done by electrostatic force on charge A while the separation between both charges changes from l to $2l$ is

(A) $\frac{q^2}{12\pi\varepsilon_0 l}$ (B) $\frac{q^2}{6\pi\varepsilon_0 l}$

(C) $\frac{q^2}{4\pi\varepsilon_0 l}$ (D) $\frac{q^2}{24\pi\varepsilon_0 l}$

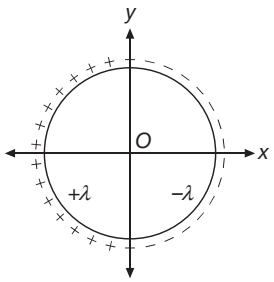
54. Total work done by electrostatic force on charge $A+$ charge B while the separation between both charges changes from l to $2l$ is

(A) $\frac{q^2}{12\pi\varepsilon_0 l}$ (B) $\frac{q^2}{6\pi\varepsilon_0 l}$

(C) $\frac{q^2}{4\pi\varepsilon_0 l}$ (D) $\frac{q^2}{24\pi\varepsilon_0 l}$

Comprehension 17

A thin ring of radius R metre is placed in x-y plane such that its centre lies at origin O. The half ring in region $x<0$ carries uniform linear charge density $+\lambda$ cm^{-1} and the remaining half ring in region $x>0$ carries uniform linear charge density $-\lambda$ cm^{-1}.



Based on the above facts, answer the following questions.

55. The electric potential (in volt) at point P whose coordinates are $\left(0, \frac{R}{2}\right)$ m is

(A) $\frac{\lambda}{2\pi\varepsilon_0}$ (B) 0

(C) $\frac{\lambda}{4\pi\varepsilon_0}$ (D) $\frac{\lambda}{6\pi\varepsilon_0}$

56. The direction of electric field at point P whose coordinates are $\left(0, \frac{R}{2}\right)$ m is

(A) along positive x-direction

(B) along negative x-direction

(C) along negative y-direction

(D) not possible to be determined.

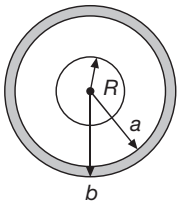
57. The dipole moment of the ring in Cm is

(A) $-(2\pi R^2\lambda)\hat{i}$ (B) $(2\pi R^2\lambda)\hat{i}$

(C) $-(4R^2\lambda)\hat{i}$ (D) $(4R^2\lambda)\hat{i}$

Comprehension 18

A metal sphere of radius R, carrying charge q_1 is surrounded by a thick concentric metal shell (inner radius a, outer radius b). The shell carries no net charge.



Based on the above facts, answer the following questions.

58. The surface charge density σ at $r=R$, $r=a$ and $r=b$ is

(A) $\sigma_R=\frac{q}{4\pi R^2}$, $\sigma_a=\frac{q}{4\pi a^2}$, $\sigma_b=\frac{q}{4\pi b^2}$

(B) $\sigma_R=\frac{q}{4\pi R^2}$, $\sigma_a=\frac{-q}{4\pi a^2}$, $\sigma_b=\frac{q}{4\pi b^2}$

(C) $\sigma_R=\frac{-q}{4\pi R^2}$, $\sigma_a=\frac{q}{4\pi a^2}$, $\sigma_b=\frac{q}{4\pi b^2}$

(D) $\sigma_R=\frac{q}{4\pi R^2}$, $\sigma_a=\frac{q}{4\pi a^2}$, $\sigma_b=\frac{-q}{4\pi b^2}$

59. The potential at the centre, assuming potential to be zero at infinity is

(A) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{a}\right)$ (B) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}+\frac{1}{a}+\frac{1}{b}\right)$

(C) $\frac{q}{4\pi\varepsilon_0 R}$ (D) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{R}\right)$

60. Now the outer surface is touched to a grounding wire, which lowers its potential to zero. Now the potential at the centre (Assume at infinity also potential is zero)

(A) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{a}\right]$ (B) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R}+\frac{1}{a}+\frac{1}{b}\right]$

(C) $\frac{q}{4\pi\varepsilon_0 R}$ (D) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{b}-\frac{1}{a}+\frac{1}{R}\right]$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

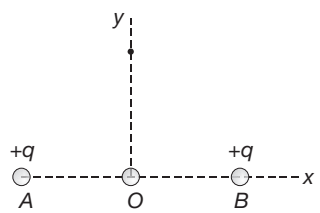
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Match the forces given in **COLUMN-I** with the properties given in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Electrostatic force of a nucleus on an electron	(p) Conservative
(B) The force attracting a man towards the centre of the earth	(q) Action-reaction force
(C) Force between earth and sun	(r) Depends on the nature of medium between the interacting objects
(D) Force between two protons	(s) Principle of Superposition is applicable
	(t) Repulsive

2. In the arrangement shown, identical charges, each $+q$ are fixed at A and B respectively. O is the midpoint of $AB = 2l$. A charge Q is placed at O and can be displaced slightly either along x-axis or along y-axis and then released. Assume Q to move only along the axis along which it had been displaced. Now match the options in **COLUMN-I** to the information in **COLUMN-II** (regarding Q at O)



COLUMN-I	COLUMN-II
(A) $Q = +q$, displaced along x-axis	(p) Force on Q before being displaced is zero
(B) $Q = -q$, displaced along x-axis	(q) Potential energy of the system is maximum
(C) $Q = q$, displaced along y-axis	(r) Potential energy of the system is minimum
(D) $Q = -q$, displaced along y-axis	(s) The charge Q is in stable equilibrium
	(t) Executes SHM about O

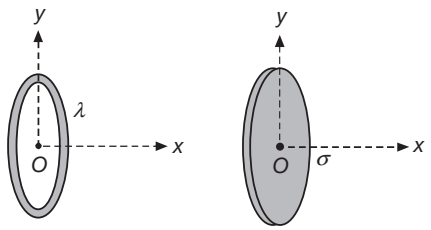
3. Match **COLUMN-I** with **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Force on an electron in an atom	(p) Gravitational force
(B) Force between a proton and a neutron inside nucleus	(q) Strong force
(C) Force between a proton and proton inside nucleus	(r) Coulomb force
(D) Conservative force	(s) Electric force
	(t) Spring force

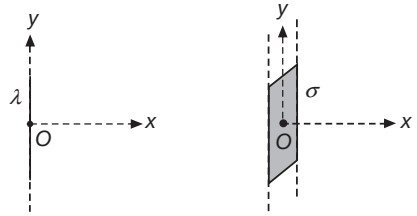
4. Match the facts given in **COLUMN-I** with the systems given in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Electric field strength is zero in the volume	(p) A non conducting solid sphere charged uniformly
(B) Electric field strength is non-zero in the volume but zero at the centre	(q) A conducting spherical shell charged uniformly
(C) Electric field strength is maximum on the surface	(r) A non conducting hollow sphere charged uniformly
(D) Electric field strength outside the system varies inversely as the square of the distance from the centre	(s) A conducting solid sphere charged uniformly

5. Figure (A) and (B) show a uniformly charged ring and a uniformly charged disc of large radius R, Figure (C) is a uniformly charged infinite thread and figure (D) shows a uniformly charged infinite sheet. In each figure let us consider the point O as the reference and then move away from O along the $+x$ axis. If E_1, E_2, E_3 and E_4 be the respective fields, then match the contents of **COLUMN-I** with those of **COLUMN-II**.



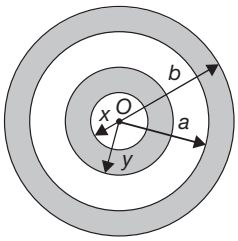
Ring of finite radius (A) — Disc of large radius (B)



Infinite line of charge (C) — Infinite sheet of charge (D)

COLUMN-I	COLUMN-II
(A) E_2	(p) decreases
(B) E_4	(q) is constant near it and then decreases
(C) E_1	(r) is discontinuous at O
(D) E_3	(s) is constant
	(t) first increases and then decreases

6. In figure a conducting spherical shell of inner radius x and outer radius y is concentric with a larger conducting spherical shell of inner radius a and outer radius b. The inner shell has a total charge $+3Q$ and the outer shell has a total charge $+5Q$. Consider a point P arbitrarily taken in the regions of the system of these two shells. Based on the facts provided, match the contents of **COLUMN-I** with the locations of the point P mentioned in **COLUMN-II**.

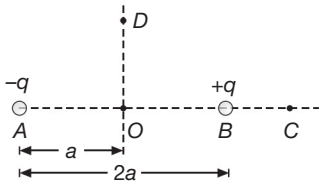


COLUMN-I	COLUMN-II
(A) Electric field strength is zero	(p) Outer surface of the larger spherical shell
(B) Electric field strength is non-zero	(q) Inner surface of the larger spherical shell
(C) Magnitude of charge on this surface is $3Q$	(r) Outer surface of the smaller spherical shell
(D) Charge on this surface is $+8Q$	(s) Region between inner and outer surface of bigger shell

(*Continued*)

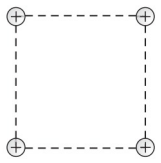
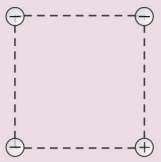
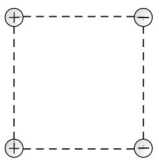
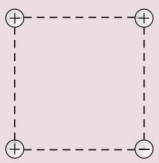
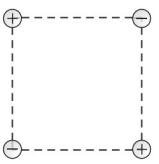
COLUMN-I	COLUMN-II
	(t) Region between inner and outer surface of smaller shell

7. For the electric dipole shown in figure, match the facts in **COLUMN-I** with the facts in **COLUMN-II**



COLUMN-I	COLUMN-II
(A) Electric potential is zero	(p) Point A
(B) Direction of electric field strength is opposite to the direction of electric dipole moment of the dipole	(q) Point C (on the axial line)
(C) Direction of electric field strength is the same as that of electric dipole moment	(r) Point O
(D) Electric field strength at C and D for $OC \gg a$ and $OD \gg a$ are E_C and E_D	(s) Any point on the equatorial line of the dipole.
	(t) $E_C > E_D$

8. Four charges, all having same magnitude are placed at the corners of a square of side a. At the centre of the square, the electrostatic field and potential due to the system of four charges is E and V respectively. Match the cases in **COLUMN-I** with the situations in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) $E \neq 0, V \neq 0$	(p) 
(B) $E = 0, V \neq 0$	(q) 
(C) $E = 0, V = 0$	(r) 
(D) $E \neq 0, V = 0$	(s) 
	(t) 

9. Match **COLUMN-I** with **COLUMN-II**

COLUMN-I	COLUMN-II
(A) $E \propto \frac{1}{r}$	(p) At large distance from the centre of the dipole
(B) $E \propto \frac{1}{r^2}$	(q) At large distance from the centre, at the axis of a uniformly charged ring.
(C) $E \propto \frac{1}{r^3}$	(r) Due to an infinite charged conducting plate.
(D) $E \propto \frac{1}{r^0}$	(s) At large distance from the axis of a thin conducting charged cylinder

(*Continued*)

COLUMN-I	COLUMN-II
	(t) At a distance r from the axis of a thin conducting cylinder

10. Match **COLUMN-I** with the standard quantities/symbols in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Jm^{-3}	(p) ε_0
(B) metre	(q) P_e
(C) $C^2J^{-1}m^{-1}$	(r) $\frac{U}{V^2}$
(D) farad	(s) $\frac{V}{E}$
	(t) $\frac{1}{2}\varepsilon_0 E^2$

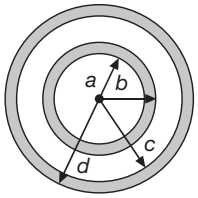
11. A particle of charge $-q$, mass m moves in a region of space between two plates from a plate at potential $-V$ to the plate at potential $+V$. The plate separation is d. If K, U, T and E be the respective kinetic energy, potential energy, total mechanical energy of the particle and E be the electric field between the plates, then match the facts in **COLUMN-I** with those in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) K	(p) constant
(B) U	(q) first increase and then decrease
(C) T	(r) increase
(D) E	(s) decrease
	(t) other than those in (p), (q), (r) or (s)

12. Match the dimensional formula in **COLUMN-I** with the quantities in **COLUMN-II**

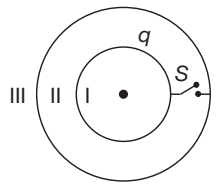
COLUMN-I	COLUMN-II
(A) $M^{-1}L^{-3}T^4A^2$	(p) V
(B) $ML^3T^{-3}A^{-1}$	(q) E
(C) $MLT^{-3}A^{-1}$	(r) ϕ_E
(D) $ML^2T^{-3}A^{-1}$	(s) ε_0
	(t) $\frac{\sigma}{2\varepsilon_0}$

13. A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d as shown in figure. The inner shell has total charge $+2q$ and the outer shell has charge $-4q$. Match the column for the electric field (magnitude and direction) in term of q and distance r from the common centre of two shells.



COLUMN-I	COLUMN-II
(A) $r < a$	(p) $\lvert E\rvert = 0$
(B) $b < r < c$	(q) $\lvert E\rvert = \frac{q}{2\pi\varepsilon_0 r^2}$
(C) $c < r < d$	(r) radially inward
(D) $r > d$	(s) radially outward

14. A charged sphere is enclosed by a concentric neutral conducting sphere as shown. The three region are marked as I, II and III. Region I is inside hollow charged sphere, region II is outside charged sphere and inside uncharged (outer) sphere, while region III is outside to both. Match the entries of **COLUMN-I** with the entries of **COLUMN-II**. All answers have to be given in steady state.

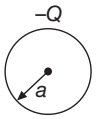
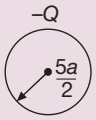
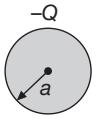
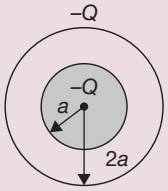


(*Continued*)

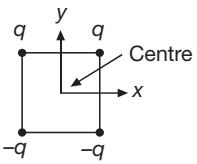
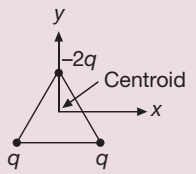
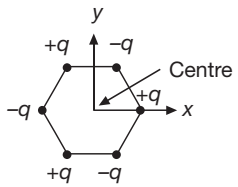
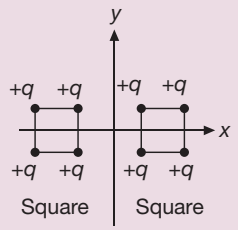
COLUMN-I	COLUMN-II
(A) When S is open, then in region II	(p) Electric field intensity is zero
(B) When S is open, then in region III	(q) Electric potential is zero

COLUMN-I	COLUMN-II
(C) When S is closed, then in region I	(r) Electric field intensity is non zero
(D) When S is closed, then in region II	(s) Electrical potential is non zero

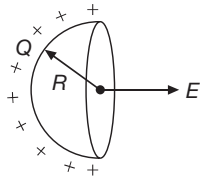
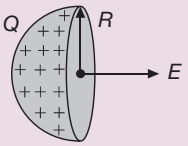
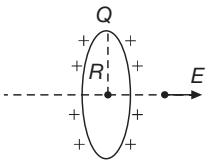
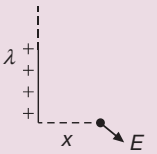
15. In each situation of **COLUMN-I**, some charge distributions are given with all details explained. The electrostatic potential energy and its nature is given situation in **COLUMN-II**. Then match situation in **COLUMN-I** with the corresponding results in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) A thin shell of radius a and having a charge $-Q$ uniformly distributed over its surface as shown 	(p) $\frac{Q^2}{8\pi\varepsilon_0 a}$ in magnitude
(B) A thin shell of radius $\frac{5a}{2}$ and having a charge $-Q$ uniformly distributed over its surface and a point charge $-Q$ placed at its centre as shown 	(q) $\frac{3Q^2}{20\pi\varepsilon_0 a}$ in magnitude
(C) A solid sphere of radius a and having a charge $-Q$ uniformly distributed throughout its volume as shown 	(r) $\frac{2Q^2}{5\pi\varepsilon_0 a}$ in magnitude
(D) A solid sphere of radius a and having a charge $-Q$ uniformly distributed throughout its volume. The solid sphere is surrounded by a concentric thin uniformly charged spherical shell of radius $2a$ and carrying charge $-Q$ as shown 	(s) Positive in sign
	(t) Negative in sign

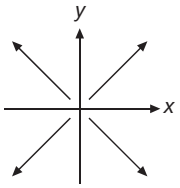
16. In **COLUMN-I** various configuration of charges are given and in **COLUMN-II** Information about electric field E and electric potential V is given. Match **COLUMN-I** with **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Square 	(p) Potential at any point on z-axis is zero.
(B) Equilateral triangle 	(q) y-component of electric field intensity at any point on z-axis is zero.
(C) Regular hexagon 	(r) Potential energy of system is negative.
(D) Both squares are equidistant from origin 	(s) z-component of electric field intensity at any point on z-axis is zero.
	(t) Potential is zero but electric field intensity is non-zero at z-axis.

17. Match the column for electric field given in **COLUMN-II** for the uniformly charged bodies in **COLUMN-I**.

COLUMN-I	COLUMN-II
(A) Hollow hemisphere, at its centre 	(p) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 x}$
(B) Solid hemisphere, at its centre 	(q) $\frac{Q}{8\pi\varepsilon_0 R^2}$
(C) On the axis of ring, near its centre 	(r) $\frac{3Q}{8\pi\varepsilon_0 R^2}$
(D) Semi-infinite wire 	(s) $\frac{\lambda x}{2\varepsilon_0 R^2}\left(1-\frac{3x^2}{2R^2}\right)$

18. In a certain region of x-y plane, potential function is given in **COLUMN-I** and corresponding electric lines of force are given in **COLUMN-II**. Match the potential function of **COLUMN-I** with their respective field line representation in **COLUMN-II**.

COLUMN-I (Potential Function)	COLUMN-II (Electric Lines of Force)
(A) $V = x^2 - y^2$	(p) 

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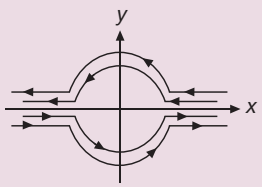
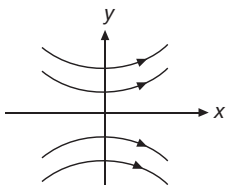
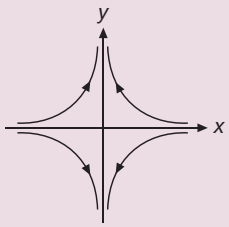
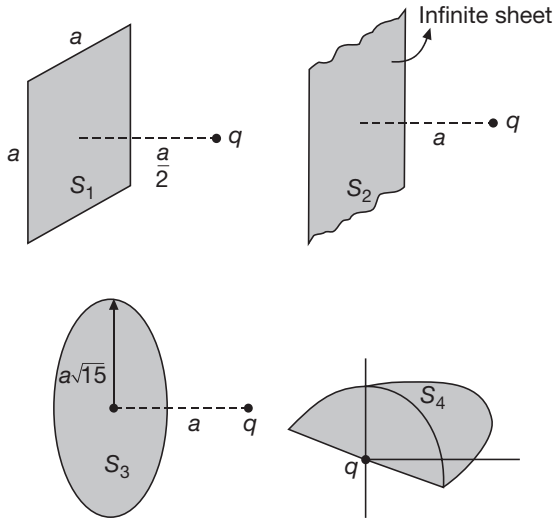
COLUMN-I (Potential Function)	COLUMN-II (Electric Lines of Force)
(B) $V = xy$	(q) 
(C) $V = x^2 + y^2$	(r) 
(D) $V = \frac{x}{y}$	(s) 

19. Consider the imaginary surfaces S_1, S_2, S_3 and S_4 drawn near a point charge as shown in figure.



COLUMN-I gives different surfaces and **COLUMN-II** corresponding electric flux. Match the entries of **COLUMN-I** to **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) S_1	(p) $\frac{3q}{8\varepsilon_0}$
(B) S_2	(q) $\frac{q}{2\varepsilon_0}$
(C) S_3	(r) $\frac{q}{6\varepsilon_0}$
(D) S_4	(s) $\frac{q}{4\varepsilon_0}$

20. Two spherical shells shown in figure have uniformly distributed charge q_1 and q_2. If r is the distance of a point from common centre and

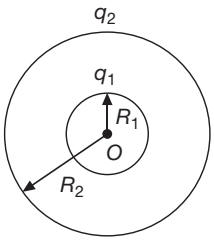
E_1 is electric field for $r < R_1$,

E_2 is electric field for $R_1 < r < R_2$,

V_1 is electric potential for $r < R_1$ and

V_2 is electric potential for $R_1 < r < R_2$,

then match the quantities in **COLUMN-I** with their respective property(ies) in **COLUMN-II**.

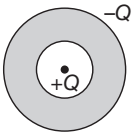
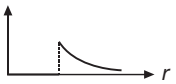
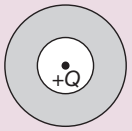
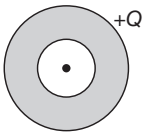
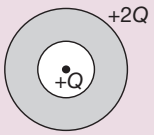
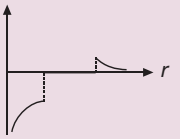


COLUMN-I	COLUMN-II
(A) E_1	(p) is constant for q_2 and varies for q_1.
(B) V_1	(q) is zero for q_2 and varies for q_1
(C) V_2	(r) is constant
(D) E_2	(s) is zero

21. n identical drops of a liquid each having charge q and radius r coalesce. Match the unfilled places in **COLUMN-I** with their respective fills given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $E_{big} = ...E_{small}$	(p) $n^{1/3}$
(B) $V_{big} = ...V_{small}$	(q) n^{3}
(C) $C_{big} = ...C_{small}$	(r) $n^{2/3}$
(D) $\sigma_{big} = ...\sigma_{small}$	(s) $n^{-1/3}$

22. Entries in **COLUMN-I** show four hollow metal spheres each with internal radius a and external radius b. Match each charge distribution in **COLUMN-I** with the corresponding graph of electric field versus radial distance r from centre as shown in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) 
(B) 	(q)
(C) 	(r)
(D) 	(s) 

(*Continued*)

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

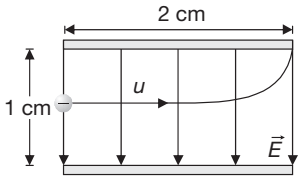
1. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross-section 10^{-6} m^2. The ring has a uniform charge of π coulomb. The change in the radius of the ring, when a charge of 10^{-8} coulomb is placed at the centre of the ring is $x \times 10^{-5}$ m. Find x, if the young's modulus of the metal is 2×10^{11} Nm^{-2}.

2. Three identical small balls, each of mass 0.1 g, are suspended from one point by silk threads each having a length of $l = 20$ cm. What charge, in nC, must be imparted to each ball so as to make each thread form an angle $\alpha = 30°$ with the vertical?

3. In classical experiments performed to measure the charge of an electron, a charged drop of oil is placed between the horizontal plates of a plane capacitor. Under the action of an electrostatic field, the drop moves uniformly upward, covering a certain distance during the time $t_1 = 6$ ms, or downward, when the sign of the charge on the plates changes, covering the same distance during the time $t_2 = 3$ ms. Assuming the force of friction between the drop and the air to be proportional to the velocity of the drop, find the time t, in millisecond during which the drop travels the same distance after the field is switched off.

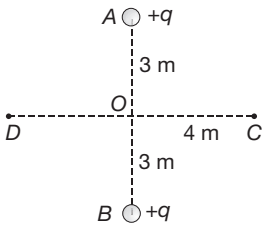
4. Four point charges each of magnitude 1 μC (each positive) are fixed at the vertices of a square $ABCD$. The diagonals of the square intersect at O. Length of each diagonal is 2 m. A particle of mass 2 g having a charge +2 μC is released from a point 1 cm from O and 99 cm from A. The motion of the particle is SHM with period $\frac{\pi\sqrt{x}}{3}$ s. Find the value of x. (Neglect gravity).

5. An electron is projected with an initial speed $u = 4 \times 10^6$ ms^{-1} into the uniform field between the parallel plates in figure. The direction of the field is vertically downward, and the field is zero except in the space between the plates. The electron enters the field at a point midway between the plates. If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field in NC^{-1}.



6. A uniform surface charge density 8σ exists over the entire x-y plane except for a circular hole of radius a centred at the origin. The electric field at a point $P(0, 0, \sqrt{3}a)$ on the z-axis is found to be $\sqrt{x}\dfrac{\sigma}{\varepsilon_0}$. Find the value of x.

7. Two fixed, equal, positive charges, each of magnitude 5×10^{-5} C are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. The distance of the farthest point D, in metre, where the negative charge will reach before returning towards C is $\sqrt{x}$. Find x.



8. Positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity v from a point A at distance $\dfrac{5R}{2}$ from the centre of the sphere. The minimum velocity v so that it can penetrate $\dfrac{R}{2}$ distance of the sphere is $\sqrt{\dfrac{Qqx}{2\pi\varepsilon_0 m}}$. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion. Find the integral value of x to the nearest approximation.

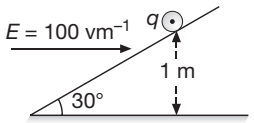
9. A particle A having a charge q is fixed on a vertical (insulated) wall. A second particle B of mass m, charge Q is suspended by a silk thread of length l from a point P on the wall that is at a distance l above the A. Calculate the angle in degree made by the thread with the vertical, when B stays in equilibrium for $Qq = 4\pi\varepsilon_0 mgl^2$.

10. Two positively charged particles each of mass 1.7×10^{-27} kg having charge of 1.6×10^{-19} C are placed at separation r. Calculate r, (in cm) to the nearest integer if each one experiences a repulsive force equal to its weight. $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$.

11. The bob of a simple pendulum has a mass of 40 g and a positive charge 4×10^{-6} C. It makes 20 oscillation in 45 s. A vertical electric field pointing upward and of magnitude 2.5×10^{4} NC^{-1} is switched on. How much time, in seconds will it now take to complete 20 oscillations.

12. A thin fixed ring of radius 1 meter has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. The time period of oscillations is $\dfrac{\pi}{k}$. Find k.

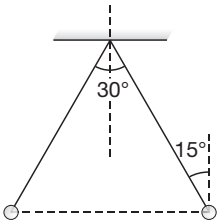
13. An inclined plane making an angle $30°$ with the horizontal is placed in a uniform horizontal electric field E of 100 Vm^{-1} as shown. A particle of mass 1 kg and charge 0.01 C is allowed to slide down from rest from a height of 1 m. If the coefficient of friction is 0.2, find the time, in millisecond it will take the particle to reach the bottom. (Take $g = 10$ ms^{-2})



14. A ball of mass m with a charge q can rotate in a vertical plane at the end of a string of length $l = 3.6$ m in a uniform electrostatic field whose lines of force are directed upwards. What horizontal velocity, in ms^{-1}, must be imparted to the ball in the upper position so that the tension in the string in the lower position of the ball is 15 times than the weight of the ball? Given $E = \dfrac{6mg}{5q}$ and $g = 10$ ms^{-2}.

15. Two equally charged identical metal spheres A and B repel each other with a force 2×10^{-5} N. Another identical uncharged sphere C is touched to B and then placed at the midpoint between A and B. What is the net electric force, in μN on C?

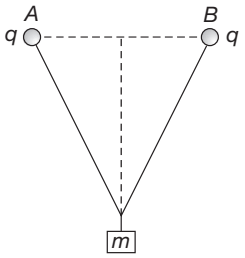
16. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of $30°$ with each other. When suspended in a liquid of density 0.8 gmcc^{-1} the angle remains same. What is the dielectric constant of liquid. Density of sphere $= 1.6$ gcc^{-1}.



17. Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each. The separation between the suspension points being 5 cm. In equilibrium the separation between the balls is 3 cm. Find the mass m in gram, nearest to an integer, of each ball and the tension in millinewton, nearest to two digit integer, in the strings. The charge on each ball has magnitude 2×10^{-8} C.

18. A charge $Q_1=+11\left(\frac{11}{3}\right)^{\frac{3}{2}}\times10^{-9}$ C is located at the origin in free space and another charge Q at $(2, 0, 0)$. If the x-component of the electric field at $(3, 1, 1)$ is zero, calculate the value of Q in nC.

19. Two similar helium filled spherical balloons tied to a 5 g weight with strings each carrying an electric charge q float in equilibrium as shown in figure.



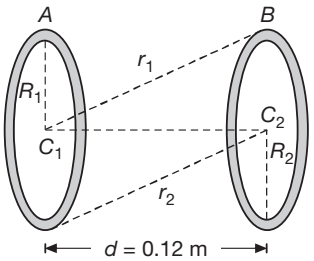
(a) Find the magnitude of q, in μC, assuming that the charge on each balloon as if it were concentrated at its centre.

(b) Find the volume of each balloon, in cc (neglect the weight of the unfilled balloons and assume that the density of air 1.29 kgm^{-3} and density of helium inside the balloon 0.2 kgm^{-3}).

20. A charge $Q=\frac{5}{100}$ nC is distributed over two concentric hollow spheres of radii $r=3$ cm and $R=6$ cm such that the surface densities are equal. Find the potential, in volt, at the common centre.

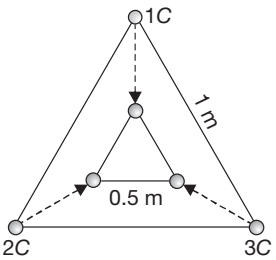
21. A charged dust particle of radius 5×10^{-7} m is located in a horizontal electric field having an intensity of 6.28×10^{5} Vm^{-1}. The surrounding medium is air with coefficient of voscosity $\eta=1.6\times10^{-5}$ Nsm^{-2}. If this particle moves with a uniform horizontal speed 0.02 ms^{-1}, find the number of electrons in it.

22. Two circular wire loops of radii 0.05 m and 0.09 m, respectively are put such that their axes coincide and their centres are 0.12 m apart. Charge of 10^{-6} C is spread uniformly on each loop. Find the potential difference, in volt between the centres of the loops.



23. The radii of internal and external spheres of concentric spherical air capacitor are 1 cm and 4 cm respectively. A potential difference of 3000 V is applied between the spheres. A velocity $x\times10^{5}$ ms^{-1} will be imparted to an electron when it approaches from a distance of $r_1=3$ cm to $r_2=3$ cm as measured from the centre of spheres. Then find x closest to three digit integer.

24. Three point charges of 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 1 m. The work required to move these charges to the corners of a smaller equilateral triangle of sides 0.5 m as shown in Figure is $x\times10^{9}$ J. Find x



25. Three point charges of 0.1 C are placed at the corners of an equilateral triangle with side $L=1$ m. If this system is supplied energy at the rate of 1 kW, how much time, in hours will be required to move one of the charges on to the mid point of the line joining the two charges.

26. An electron enters a region between the plates of a parallel plate capacitor at a point equidistant from either plate. The capacitor plates are 2×10^{-2} m apart and 10^{-1} m long. A potential difference of 300 V is

maintained across the plates. Assuming that the initial velocity of the electron is parallel to the capacitor plates, the largest value of the velocity of the electrons so that they do not fly out of the capacitor at the other end is $x \times 10^5$ ms^{-1}. Taking, $m_e = 9 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ coulomb, find the value of x closest to three digit integer.

27. Two plane parallel conducting plates 1.5×10^{-2} m apart are held horizontal one above the other in air. The upper plate is maintained at positive potential of 1.5 kV while the other plate is earthed.

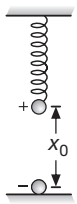
(a) Calculate the number of electrons which must be attached to a small oil drop of mass 4.9×10^{-15} kg between the plates to maintain it at rest, assuming that the density of air is negligible in comparison with that of oil.

(b) If the potential of above plate is suddenly changed to -1.5 kV and what is the initial acceleration, in ms^{-2}, of the charged drop?

(c) Also obtain the terminal velocity of the drop, in μms^{-1} if its radius is 5×10^{-6} m and the coefficient of viscosity of air is 1.8×10^{-5} Nsm^{-2}.

(Take $g = 10$ ms^{-2})

28. A positively charged sphere of mass $m = 5$ kg is attached by a spring of force constant $K = 10^4$ Nm^{-1}. The sphere is tied with a thread so that the spring is in its natural length. Another identical, negatively charged sphere is fixed to the floor, vertically below the positively charged sphere as shown in figure. Initial separation between sphere is $x_0 = 0.5$ m. Now thread is burnt and maximum elongation of the spring is 0.1 m, calculate the charge, in μC on each sphere. Take $g = 10$ ms^{-2}.



Initial position

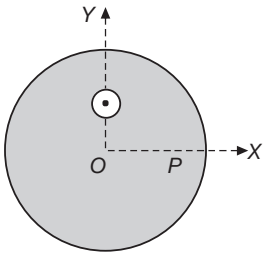
29. An infinite long straight conductor is uniformly charged, the charge per unit length being 6×10^4 Cm^{-1}.

(a) What is the force experienced by a proton, in μN, at a perpendicular distance 0.5 m from it?

(b) How much work in mJ will be required to bring the proton at a perpendicular distance from 0.5 m to 0.2 m from the conductor. Give your answer closest to the two digit integer, in this case.

30. Two concentric spherical shells have radii 0.15 m and 0.3 m. The inner sphere is given a charge -6×10^{-2} μC. An electron escapes from the inner sphere with negligible speed. Assuming that the region between the spheres is vacuum, the speed with which it will strike the outer sphere is $* \times 10^6$ ms^{-1}, where $*$ is not readable. Find the value of $*$. $\left(\frac{e}{m} \text{ for electron} = 1.76 \times 10^{11} \text{ Ckg}^{-1}\right)$.

31. A non-conducting sphere of radius $R = 5$ cm is centred at origin of coordinate system. It has a spherical cavity of radius 1 cm having centre at $(0, 3)$ cm. The material of sphere has a uniform charge density $\rho = \frac{10^{-6}}{\pi}$ Cm^{-3}. Calculate the electric potential at point $P(4 \text{ cm}, 0)$ to the nearest two digit integer.



32. Two small identical balls P and Q, each of mass 0.866 g, carry identical charges and are suspended by two threads of equal length of 50 cm each from the same point. At equilibrium, they are separated by a distance of 50 cm. Find the charge on each ball in nC. (Take $g = 10$ ms^{-2}).

33. A particle having a charge of $q = 8.85$ μC is placed on the axis of a circular ring of radius $R = 30$ cm at a point P at a distance of $a = 40$ cm from centre of ring. Calculate the electric flux passing through the ring in kNC^{-1}.

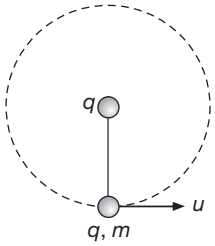
34. A particle of mass 2 μg and carrying a charge of 1×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has a velocity of 1 ms^{-1}. At what distance from the fixed point charge will the particle come momentarily to rest? (in mm)

35. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is $63.5\ \text{gmol}^{-1}$. Let us now take two pieces of copper each weighing 10 g. Let us transfer one electron from one piece to another for every 1000 atoms in that piece. The coulomb force between the two pieces after the transfer of electrons if they are 1 cm apart is $x\times10^{6}$ N. Find x, close to the nearest integer. (Avogadro number $N_A=6\times10^{23}\ \text{gmole}^{-1}$, charge on an electron $=-1.6\times10^{-19}$ coulomb)

36. It is required to hold four equal point charges each having a charge $Q=\frac{8}{7}\left(1-2\sqrt{2}\right)$ C in equilibrium at the corners of a square. Find the point charge, in coulomb, that will do this if placed at the centre of the square.

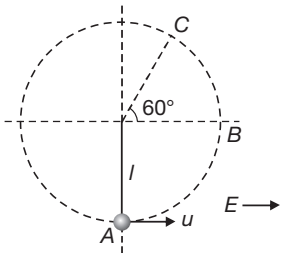
37. A radioactive source in the form of a metal sphere of radius 10^{-2} m, emits beta particles (electrons) at the rate of 5×10^{10} particles per second. The source is electrically insulated. How long, in μs will it take for its potential to be raised by 2 V assuming that 40% of the emitted beta particles escape the source.

38. A small ball of mass 2×10^{-3} kg having a charge of $1\ \mu$C is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity in cms^{-1} which should be imparted to the lower ball so that it can make complete revolution.



39. The centres of two metal spheres of radius 10 cm are 50 cm apart on the x-axis. The spheres are initially neutral, but a charge Q is transferred from one sphere to the other, creating a potential difference 100 V. A proton is released from rest from the surface of the positively charged sphere and travels to the negatively charged sphere. At what speed, in kms^{-1} does it strike the negatively charged sphere?

40. A simple pendulum with a bob of mass $m=1$ kg, charge $q=5\ \mu$C and string length $l=1$ m is given a horizontal velocity u in a uniform electric field $E=2\times10^{6}\ \text{Vm}^{-1}$ at its bottommost point A, as shown in figure. It is given that the speed u is such that the particle leaves the circle at point C. Find the speed u to the nearest integer. $\left(\text{Take } g=10\ \text{ms}^{-2}\right)$



41. Two insulating spheres have radii 0.3 cm and 0.5 cm, masses 0.1 kg and 0.7 kg, and uniformly distributed charges of $-2\ \mu$C and $3\ \mu$C. They are released from rest when their centers are separated by 1 m.

(a) How fast in cms^{-1} will each be moving when they collide?

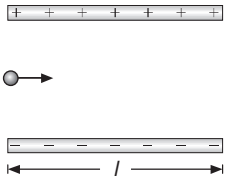
(b) If the spheres were conductors, would the speeds be greater or less than those as calculated in part (a)? Give reasonable explanation to your answer.

42. A particle having mass $m=1$ kg and charge $q=2\ \mu$C is thrown from a horizontal ground with a speed $u=20\ \text{ms}^{-1}$ that makes an angle $\alpha=45°$ with the horizontal. A horizontal electric field $E=2\times10^{7}\ \text{Vm}^{-1}$ exist in this space. Find the range, in metre on the horizontal ground of the projectile thrown.

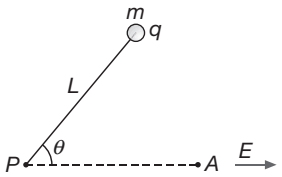
43. A ball of mass 2 kg, charge $1\ \mu$C is dropped from top of a high tower. In space electric field exist in horizontal direction away from tower which varies as $E=(5-2x)\times10^{6}\ \text{Vm}^{-1}$. Find maximum horizontal distance that the ball can go from the tower.

44. A particle having a charge of 1.6×10^{-19} C enters midway between the plates of a parallel plate condenser. The initial velocity of particle is parallel to the plates. A potential difference of 300 volts is applied to the capacitor plates. If the length of the capacitor plates is 10 cm and they are separated by 2 cm, calculate the greatest initial velocity, in kms^{-1} for which the particle will not be able to come out of the plates. The mass of the particle is 12×10^{-24} kg.

45. A uniform electric field E is created between two parallel charged plates as shown in figure. An electron enter the field symmetrically between the plates with a speed u. The length of each plate is l. Find the angle of deviation, in degree of the path of the electron as it comes out of the field for $u^2=\frac{\sqrt{3}Ele}{m}$.



46. A particle having charge $q = +2\ \mu C$ and mass $m = 0.01$ kg is connected to a string that is $L = 1.5$ m long and is tied to the pivot point P in figure. The particle, string and pivot point all lie on a frictionless horizontal table. The particle is released from rest when the string makes an angle $\theta = 60°$ with a uniform electric field of magnitude $E = 300\ Vm^{-1}$. Determine the speed of the particle in cms^{-1} when the string becomes parallel to the electric field (point A in figure).



47. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a.

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1. **[Online April 2019]**
The bob of a simple pendulum has mass 2 g and a charge of $5.0\ \mu C$. It is at rest in a uniform horizontal electric field of intensity $2000\ Vm^{-1}$. At equilibrium, the angle that the pendulum makes with the vertical is (take $g = 10\ ms^{-2}$)

(A) $\tan^{-1}(0.2)$ (B) $\tan^{-1}(5.0)$
(C) $\tan^{-1}(2.0)$ (D) $\tan^{-1}(0.5)$

2. **[Online April 2019]**
A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is

(A) $-2V$ (B) V
(C) $2V$ (D) $4V$

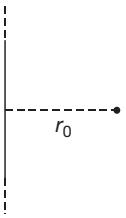
3. **[Online April 2019]**
The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is

(A) 180 V (B) –520 V
(C) 320 V (D) –48 V

4. **[Online April 2019]**
An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is

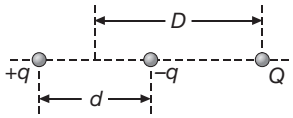
(A) $\sqrt{\frac{2qE}{md}}$ (B) $2\sqrt{\frac{qE}{md}}$
(C) $\sqrt{\frac{qE}{2md}}$ (D) $\sqrt{\frac{qE}{md}}$

5. **[Online April 2019]**
A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to



(A) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$ (B) $v \propto e^{+\frac{r}{r_0}}$
(C) $v \propto \ln\left(\frac{r}{r_0}\right)$ (D) $v \propto \left(\frac{r}{r_0}\right)$

6. **[Online April 2019]**
A system of three charges are placed as shown in the figure



If $D \gg d$, the potential energy of the system is best given by

(A) $\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}-\frac{qQd}{D^2}\right)$ (B) $\frac{1}{4\pi\varepsilon_0}\left(+\frac{q^2}{d}+\frac{qQd}{D^2}\right)$

(C) $\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}+\frac{2qQd}{D^2}\right)$ (D) $\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}-\frac{qQd}{2D^2}\right)$

7. [Online April 2019]

Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y=-2d$, $y=-d$, $y=+d$ and $y=+2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x=D$, with $D \gg d$, will behave as

(A) $E \propto \frac{1}{D^3}$ (B) $E \propto \frac{1}{D}$

(C) $E \propto \frac{1}{D^4}$ (D) $E \propto \frac{1}{D^2}$

8. [Online April 2019]

A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at $z=4a$. The minimum value of v such that it crosses the origin is

(A) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$ (B) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$

(C) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$ (D) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$

9. [Online April 2019]

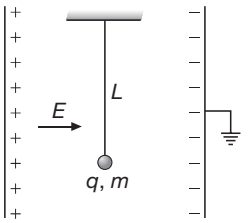
In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is [Take $\frac{1}{4\pi\varepsilon_0}=9\times10^9$ Nm^2C^{-2}]

(A) 2.0×10^3 ms^{-1} (B) 3.0×10^4 ms^{-1}

(C) 1.5×10^2 ms^{-1} (D) 6.3×10^4 ms^{-1}

10. [Online April 2019]

A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by



(A) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (B) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$

(C) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$ (D) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

11. [Online April 2019]

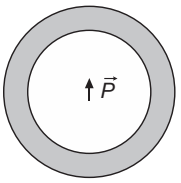
A point dipole $\vec{p}=-p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively (Take $V=0$ at infinity)

(A) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$ (B) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(C) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$ (D) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

12. [Online April 2019]

Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case



(A) Surface charge density on the outer surface depends on $|\vec{P}|$

(B) Surface charge density on the inner surface is uniform and equal to $\frac{\left(\frac{Q}{2}\right)}{4\pi a^2}$

(C) Electric field outside the shell is the same as that of point charge at the centre of the shell

(D) Surface charge density on the inner surface of the shell is zero everywhere

13. [Online April 2019]

Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then

(A) $a = \dfrac{3R}{2^{\frac{1}{4}}}$ (B) $a = \dfrac{R}{\sqrt{3}}$

(C) $a = 2^{-\frac{1}{4}}R$ (D) $a = 8^{-\frac{1}{4}}R$

14. [Online January 2019]

Three charges $+Q$, q, $+Q$ are placed respectively, at distance, 0, $\dfrac{d}{2}$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero then value of q is

(A) $+\dfrac{Q}{2}$ (B) $-\dfrac{Q}{2}$

(C) $-\dfrac{Q}{4}$ (D) $+\dfrac{Q}{4}$

15. [Online January 2019]

For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is

(A) $\dfrac{R}{\sqrt{2}}$ (B) $\dfrac{R}{\sqrt{5}}$

(C) R (D) $R\sqrt{2}$

16. [Online January 2019]

Two point charges $q_1(\sqrt{10}\ \mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at $x = 1$ m and $x = 4$ m respectively. The electric field (in Vm^{-1}) at a point $y = 3$ m on y-axis is, $\left[\text{Take } \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}\right]$

(A) $\left(63\hat{i} - 27\hat{j}\right)\times10^2$

(B) $\left(81\hat{i} - 81\hat{j}\right)\times10^2$

(C) $\left(-81\hat{i} + 81\hat{j}\right)\times10^2$

(D) $\left(-63\hat{i} + 27\hat{j}\right)\times10^2$

17. [Online January 2019]

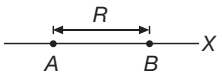
Charge is distributed within a sphere of radius R with volume charge density $\rho(r) = \dfrac{A}{r^2}e^{-\frac{2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

(A) $\dfrac{a}{2}\log\left(\dfrac{1}{1-\dfrac{Q}{2\pi aA}}\right)$ (B) $\dfrac{a}{2}\log\left(1-\dfrac{Q}{2\pi aA}\right)$

(C) $a\log\left(1-\dfrac{Q}{2\pi aA}\right)$ (D) $a\log\left(\dfrac{1}{1-\dfrac{Q}{2\pi aA}}\right)$

18. [Online January 2019]

Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure.



The distance from A at which both of them produce the same potential is

(A) $\dfrac{\sqrt{2}R}{\sqrt{2}-1}$ (B) $\dfrac{\sqrt{2}R}{\sqrt{2}+1}$

(C) $\dfrac{R}{\sqrt{2}-1}$ (D) $\dfrac{R}{\sqrt{2}+1}$

19. [Online January 2019]

A charge Q is distributed over three concentric spherical shells of radii a, b, c $(a < b < c)$ such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where $r < a$, would be

(A) $\dfrac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$ (B) $\dfrac{Q}{4\pi\varepsilon_0(a+b+c)}$

(C) $\dfrac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$ (D) $\dfrac{Q}{12\pi\varepsilon_0}\dfrac{ab+bc+ca}{abc}$

20. [Online January 2019]

Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), $(4, -2)$ and $(0, -2)$. The work required to put a fifth charge Q at the origin of the coordinate system will be

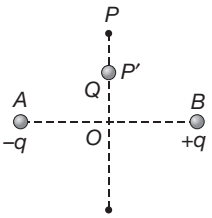
(A) $\dfrac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (B) $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{3}}\right)$

(C) $\dfrac{Q^2}{4\pi\varepsilon_0}$ (D) $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{5}}\right)$

21. [Online January 2019]

Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where $OP = y$ and

$y \gg 2a$. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} \gg 2a\right)$

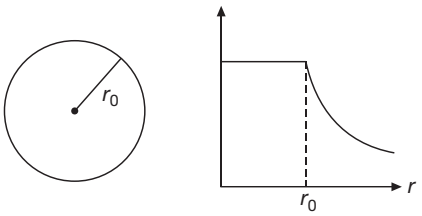


(A) $\frac{F}{3}$ (B) $9F$

(C) $27F$ (D) $3F$

22. [Online January 2019]

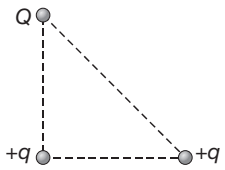
The given graph shows variation (with distance r from centre) of



(A) Potential of a uniformly charged spherical shell
(B) Electric field of a uniformly charged sphere
(C) Electric field of uniformly charged spherical shell
(D) Potential of a uniformly charged sphere

23. [Online January 2019]

Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angle isosceles triangles as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is



(A) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (B) $+q$

(C) $-2q$ (D) $\frac{-q}{1+\sqrt{2}}$

24. [Online January 2019]

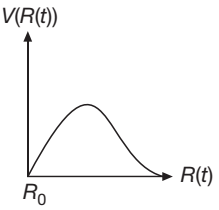
An electric field of $1000\ \text{Vm}^{-1}$ is applied to an electric dipole at angle of $45°$. The value of electric dipole moment is 10^{-29} Cm. What is the potential energy of the electric dipole?

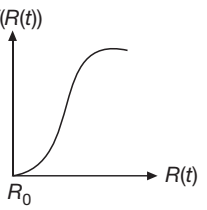
(A) -9×10^{-20} J (B) -10×10^{-29} J

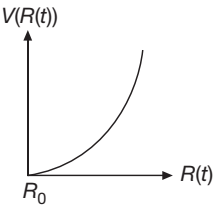
(C) -7×10^{-27} J (D) -20×10^{-18} J

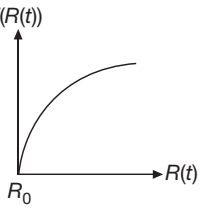
25. [Online January 2019]

There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $V(R(t))$ of the distribution as a function of its instantaneous radius $R(t)$ is

(A)


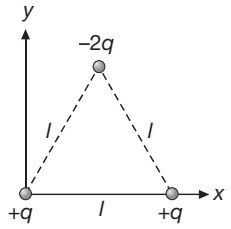
(B)


(C)


(D)


26. [Online January 2019]

Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure



(A) $2ql\hat{j}$ (B) $\sqrt{3}ql\left(\frac{\hat{j}-\hat{i}}{\sqrt{2}}\right)$

(C) $-\sqrt{3}ql\hat{j}$ (D) $ql\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

27. [Online January 2019]

A parallel plate capacitor with plates of area $1\ \text{m}^2$ each, are at a separation of 0.1 m. If the electric field between the plates is $100\ \text{NC}^{-1}$, the magnitude of charge on each plate is (Take $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$)

(A) 8.85×10^{-10} C (B) 9.85×10^{-10} C

(C) 6.85×10^{-10} C (D) 7.85×10^{-10} C

28. [2018]

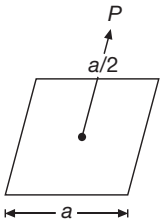
Three concentric metal shells A, B and C of respective radii, a, b and $c(a<b<c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(A) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{a}+c\right)$ (B) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$

(C) $\frac{\sigma}{\varepsilon_0}\left(\frac{b^2-c^2}{b}+a\right)$ (D) $\frac{\sigma}{\varepsilon_0}\left(\frac{b^2-c^2}{c}+a\right)$

29. [Online 2018]

A charge Q is placed at a distance $\frac{a}{2}$ above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is



(A) $\frac{Q}{3\varepsilon_0}$ (B) $\frac{Q}{6\varepsilon_0}$

(C) $\frac{Q}{\varepsilon_0}$ (D) $\frac{Q}{2\varepsilon_0}$

30. [Online 2018]

A solid ball of radius R has a charge density ρ given by $\rho=\rho_0\left(1-\frac{r}{R}\right)$ for $0\le r\le R$. The electric field outside the ball is

(A) $\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ (B) $\frac{4\rho_0 R^3}{3\varepsilon_0 r^2}$

(C) $\frac{3\rho_0 R^3}{4\varepsilon_0 r^2}$ (D) $\frac{\rho_0 R^3}{\varepsilon_0 r^2}$

31. [Online 2018]

Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameters and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B and then removed. As a result, the force between A and B would be equal to

(A) $\frac{3F}{8}$ (B) $\frac{F}{2}$

(C) $\frac{3F}{4}$ (D) F

32. [Online 2018]

A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at $x=0$, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct?

(A) The total energy of the system is $\frac{1}{2}m\omega^2A^2-\frac{1}{2}\frac{q^2E^2}{k}$.

(B) The new equilibrium position is at a distance $\frac{2qE}{k}$ from $x=0$.

(C) The new equilibrium position is at a distance $\frac{qE}{2k}$ from $x=0$.

(D) The total energy of the system is $\frac{1}{2}m\omega^2A^2+\frac{1}{2}\frac{q^2E^2}{k}$.

33. [2017]

An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1=E\hat{i}$, it experiences a torque $\vec{T}_1=\tau\hat{k}$. When subjected to another electric field $\vec{E}_2=\sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2=-\vec{T}_1$. The angle θ is

(A) 30° (B) 45°

(C) 60° (D) 90°

34. [Online 2017]

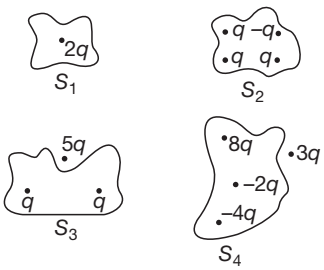
There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between the limits 589 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field?

(A) 589.2 V (B) 589.6 V

(C) 589.5 V (D) 589.4 V

35. [Online 2017]

Four closed surfaces and corresponding charge distributions are shown below.



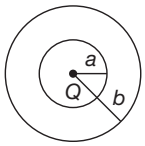
Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3 and Φ_4. Then

(A) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ (B) $\Phi_1 > \Phi_3$; $\Phi_2 < \Phi_4$

(C) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$ (D) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$

36. [2016]

The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre.



At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is

(A) $\frac{Q}{2\pi a^2}$ (B) $\frac{Q}{2\pi(b^2 - a^2)}$

(C) $\frac{2Q}{\pi(a^2 - b^2)}$ (D) $\frac{2Q}{\pi a^2}$

37. [Online 2016]

The potential (in volts) of a charge distribution is given by

$V(z) = 30 - 5z^2$ for $|z| \le 1$ m

$V(z) = 35 - 10|z|$ for $|z| \ge 1$ m

$V(z)$ does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ε_0) which is spread over a certain region, then choose the correct statement

(A) $\rho_0 = 20\varepsilon_0$ in the entire region.

(B) $\rho_0 = 10\varepsilon_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere

(C) $\rho_0 = 20\varepsilon_0$ for $|z| \le 1$ m and $\rho_0 = 0$ elsewhere

(D) $\rho_0 = 40\varepsilon_0$ in the entire region

38. [Online 2016]

Within a spherical charge distribution of charge density $\rho(r)$, N equipotential surfaces of potential V_0, $V_0 + \Delta V$, $V_0 + 2\Delta V$, ... $V_0 + N\Delta V (\Delta V > 0)$, are drawn and have increasing radii r_0, r_1, r_2, ..., r_N, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then

(A) $\rho(r) = \text{constant}$ (B) $\rho(r) \propto \frac{1}{r^2}$

(C) $\rho(r) \propto \frac{1}{r}$ (D) $\rho(r) \propto r$

39. [2015]

A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(A) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

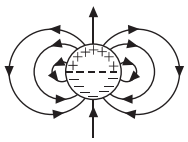
(B) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

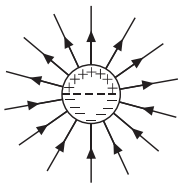
(C) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

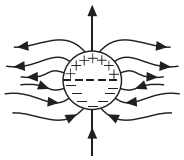
(D) $2R < R_4$

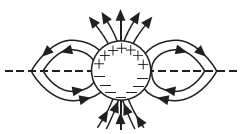
40. [2015]

A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in
(figures are schematic and not drawn to scale)

(A) 

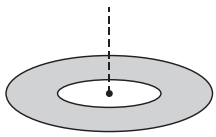
(B) 

(C) 

(D) 

41. [Online 2015]

A thin disc of radius $b = 2a$ has a concentric hole of radius a in it (shown in figure). It carries uniform surface charge σ on it. If the electric field on its axis at height $h (h \ll a)$ from its centre is given as Ch then value of C is

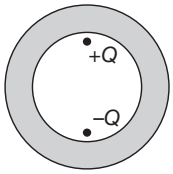


(A) $\frac{\sigma}{a\varepsilon_0}$ (B) $\frac{\sigma}{2a\varepsilon_0}$

(C) $\frac{\sigma}{4a\varepsilon_0}$ (D) $\frac{\sigma}{8a\varepsilon_0}$

42. [Online 2015]

Shown in the figure are two point charges $+Q$ and $-Q$ inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then



(A) $\sigma_1 \neq 0,\ Q_1 \neq 0;\ \sigma_2 \neq 0,\ Q_2 \neq 0$

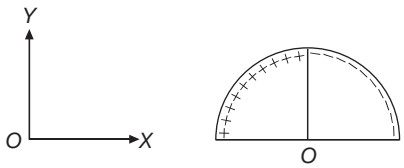
(B) $\sigma_1 \neq 0,\ Q_1 = 0;\ \sigma_2 \neq 0,\ Q_2 = 0$

(C) $\sigma_1 \neq 0,\ Q_1 = 0;\ \sigma_2 = 0,\ Q_2 = 0$

(D) $\sigma_1 = 0,\ Q_1 = 0;\ \sigma_2 = 0,\ Q_2 = 0$

43. [Online 2015]

A wire, of length $L(=20\text{ cm})$, is bent into a semi-circular arc. If the two equal halves of the arc, were each to be uniformly charged with charges $\pm Q$, ($|Q| = 10^3\varepsilon_0$ Coulomb where ε_0 is the permittivity (in SI units) of free space) the net electric field at the centre O of the semi-circular arc would be



(A) $(50\times10^3\ \text{NC}^{-1})\hat{j}$ (B) $(25\times10^3\ \text{NC}^{-1})\hat{i}$

(C) $(25\times10^3\ \text{NC}^{-1})\hat{j}$ (D) $(50\times10^3\ \text{NC}^{-1})\hat{i}$

44. [Online 2015]

An electric field $\vec{E} = (30\hat{i} + 30\hat{j})\ \text{NC}^{-1}$ exists in a region of space. If the potential at the origin is taken to be zero then the potential at $x = 2$ m, $y = 2$ m is

(A) $-130\ \text{JC}^{-1}$ (B) $-120\ \text{JC}^{-1}$

(C) $-140\ \text{JC}^{-1}$ (D) $-110\ \text{JC}^{-1}$

45. [2014]

Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(A) 120 V (B) –120 V

(C) –80 V (D) 80 V

46. [2013]

Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y \ll a)$ along the y-axis, the net force acting on the particle is proportional to

(A) y (B) $-y$

(C) $\frac{1}{y}$ (D) $-\frac{1}{y}$

47. [2013]

A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is

(A) $\frac{Q\ln 2}{8\pi\varepsilon_0 L}$ (B) $\frac{3Q}{4\pi\varepsilon_0 L}$

(C) $\frac{Q}{4\pi\varepsilon_0 L\ln 2}$ (D) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

48. [2012]

This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinity is zero.

Statement 1: When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement 2: The electric field at a distance $r(r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(A) Statement 1 is true, Statement 2 is false.

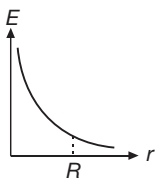
(B) Statement 1 is false, Statement 2 is true.

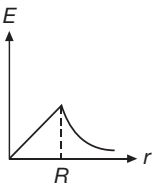
(C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

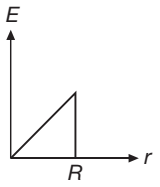
(D) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of Statement 1.

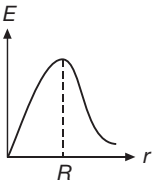
49. [2012]

In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be

(A) 

(B) 

(C) 

(D) 

50. [2011]

Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d \ll l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(A) $v \propto x^{-1/2}$ (B) $v \propto x^{-1}$

(C) $v \propto x^{1/2}$ (D) $v \propto x$

51. [2011]

The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(A) $-24\pi a\varepsilon_0 r$ (B) $-6a\varepsilon_0 r$

(C) $-24\pi a\varepsilon_0$ (D) $-6a\varepsilon_0$

52. [2010]

Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(A) 1 (B) 4

(C) 3 (D) 2

53. [2010]

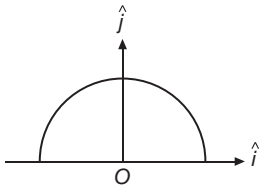
Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by

(A) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (B) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(C) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (D) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

54. [2010]

A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(A) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (B) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(C) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (D) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

55. [2009]

A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the $\frac{Q}{q}$ equals

(A) $-2\sqrt{2}$ (B) -1

(C) 1 (D) $-\frac{1}{\sqrt{2}}$

56. [2009]

Two points P and Q are maintained at the potentials of 10 V and -4 V respectively. The work done in moving 100 electrons from P to Q is

(A) -9.60×10^{-17} J

(B) 9.60×10^{-17} J

(C) -2.24×10^{-16} J

(D) 2.24×10^{-16} J

57. [2009]

Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(A) 0 (B) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$

(C) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (D) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

58. [2009]

This question contains Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q

Statement 2: The network done by a conservative force on an object moving along a closed loop is zero.

(A) Statement 1 is true, Statement 2 is false.

(B) Statement 1 is true, Statement 2 is true; Statement 2 is the correct explanation of Statement 1.

(C) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of Statement 1.

(D) Statement 1 is false, Statement 2 is true.

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2019]

A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2 (\alpha \ll 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

(A) The potential at the centre of the shell is reduced by $2\alpha V_0$.

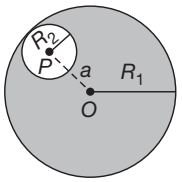
(B) The magnitude of electric field at the centre of the shell is reduced by $\frac{\alpha V_0}{2R}$.

(C) The magnitude of electric field at a point, located on a line passing through the hole and shell's centre, on a distance $2R$ from the centre of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$.

(D) The ratio of the potential at the centre of the shell to that of the point at $\frac{1}{2}R$ from centre towards the hole will be $\frac{1-\alpha}{1-2\alpha}$.

2. [JEE (Advanced) 2015]

Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statements is/are



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

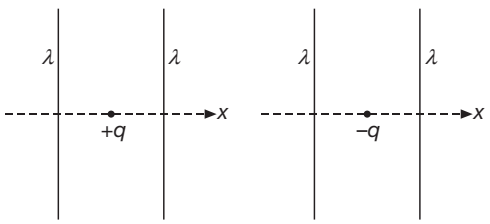
(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

3. [JEE (Advanced) 2015]

The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements is/are



(A) both charges execute simple harmonic motion

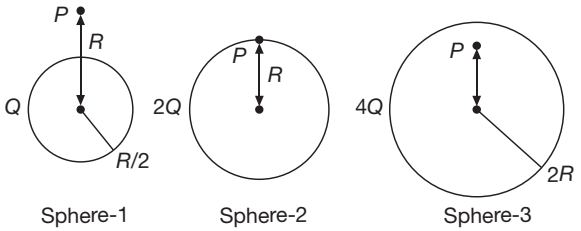
(B) both charges will continue moving in the direction of their displacement

(C) charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement

(D) charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement

4. [JEE (Advanced) 2014]

Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $\frac{R}{2}$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then

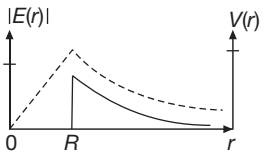


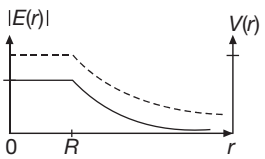
(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$

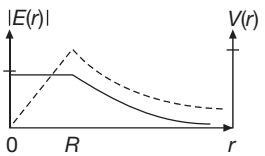
(C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

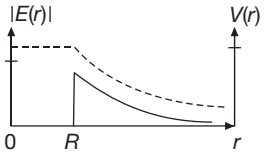
5. [IIT-JEE 2012]

Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|E(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph?

(A)


(B)


(C)


(D)


6. [IIT-JEE 2012]

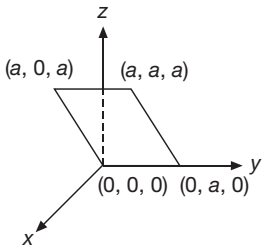
Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at $45°$ to the vertical just after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V

(C) 1×10^{-9} V (D) 1×10^{-10} V

7. [IIT-JEE 2011]

Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is

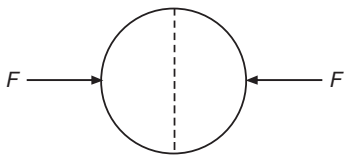


(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$

(C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

8. [IIT-JEE 2010]

A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$

(C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

9. [IIT-JEE 2010]

A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air $= 1.8\times10^{-5}$ Nsm^{-2} and the density of oil $= 900$ kgm^{-3}, the magnitude of q is

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C

(C) 4.8×10^{-19} C (D) 8×10^{-19} C

10. [IIT-JEE 2009]

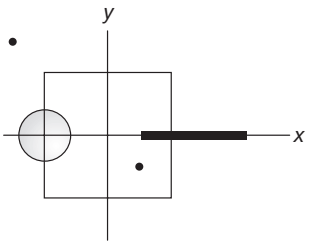
Three concentric metallic spherical shells of radii R, $2R$ and $3R$ are given charges Q_1, Q_2 and Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) 1:2:3 (B) 1:3:5

(C) 1:4:9 (D) 1:8:18

11. [IIT-JEE 2009]

A disk of radius $\frac{a}{4}$ having a uniformly distributed charge $6C$ is placed in the x-y plane with its centre at $\left(-\frac{a}{2}, 0, 0\right)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = \frac{a}{4}$ to $x = \frac{5a}{4}$. Two point charges $-7C$ and $3C$ are placed at $\left(\frac{a}{4}, -\frac{a}{4}, 0\right)$ and $\left(-\frac{3a}{4}, \frac{3a}{4}, 0\right)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm\frac{a}{2}$, $y = \pm\frac{a}{2}$, $z = \pm\frac{a}{2}$. The electric flux through this cubical surface is

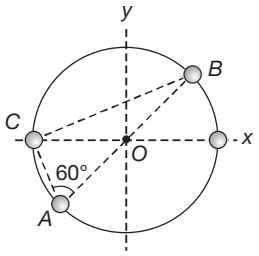


(A) $-\frac{2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$

(C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

12. [IIT-JEE 2008]

Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$.



(A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis

(B) The potential energy of the system is zero

(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$

(D) The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

13. [IIT-JEE 2007]

Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, -\frac{a}{2}\right)$, respectively, The work done by the electric field when another positive point charge in moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

(A) positive
(B) negative
(C) zero
(D) depends on the path connecting the initial and final positions

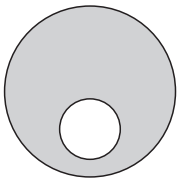
14. [IIT-JEE 2007]

Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

(A) negative and distributed uniformly over the surface of the sphere
(B) negative and appears only at the point on the sphere closest to the point charge
(C) negative and distributed non-uniformly over the entire surface of the sphere
(D) zero

15. [IIT-JEE 2007]

A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is



(A) zero everywhere
(B) non-zero and uniform
(C) non-uniform
(D) zero only at its centre

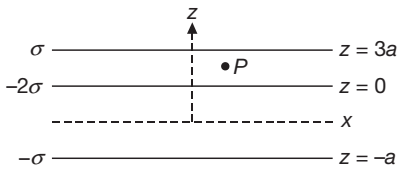
16. [IIT-JEE 2007]

A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.

(A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder
(B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder
(C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
(D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders

17. [IIT-JEE 2005]

Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is

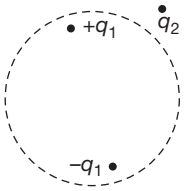


(A) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (B) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$

(C) $\frac{4\sigma}{\varepsilon_0}\hat{k}$ (D) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

18. [IIT-JEE 2004]

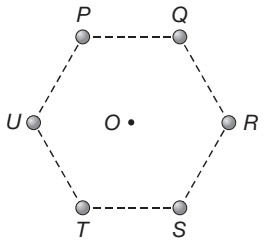
Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(A) q_2
(B) only the positive charges
(C) all the charges
(D) $+q_1$ and $-q_1$

19. [IIT-JEE 2004]

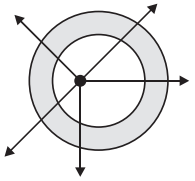
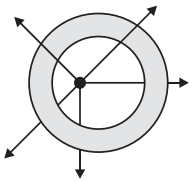
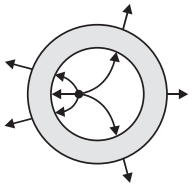
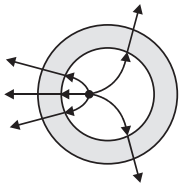
Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for, P, Q, R, S, T and U respectively?



(A) +, −, +, −, −, + (B) +, −, +, −, +, −
(C) +, +, −, +, −, − (D) −, +, +, −, +, −

20. [IIT-JEE 2003]

A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces?

(A) 
(B) 
(C) 
(D) 

21. [IIT-JEE 2002]

Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(A) x (B) x^2

(C) x^3 (D) $\frac{1}{x}$

22. [IIT-JEE 2002]

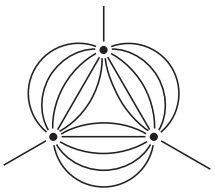
Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

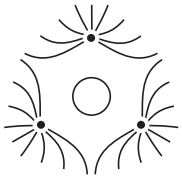
(A) x (B) x^2

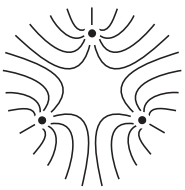
(C) x^3 (D) $\frac{1}{x}$

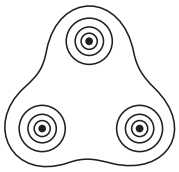
23. [IIT-JEE 2001]

Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(A) 

(B) 

(C) 

(D) 

24. [IIT-JEE 2001]

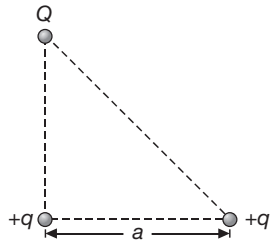
A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy

(A) $V_A < V_B$ (B) $V_A > V_B$

(C) $V_A < V_C$ (D) $V_A > V_C$

25. [IIT-JEE 2000]

Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle triangle (isosceles triangle) as shown. The net electrostatic energy of the configuration is zero, if Q is equal to



(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$

(C) $-2q$ (D) $+q$

26. [IIT-JEE 1998]

A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$,.... ad infinitum on the x-axis and a charge $-q$ is fixed at each of points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$.... ad infinitum. Here x_0 is a positive constant. The potential at the origin due to the above system of charges is

(A) zero (B) $\frac{q}{8\pi\varepsilon_0 x_0 \log_e 2}$

(C) ∞ (D) $\frac{q\log_e 2}{4\pi\varepsilon_0 x_0}$

27. [IIT-JEE 1997]

An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also, initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio $\frac{t_2}{t_1}$ is nearly equal to

(A) 1 (B) $\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}$

(C) $\left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}$ (D) 1836

28. [IIT-JEE 1997]

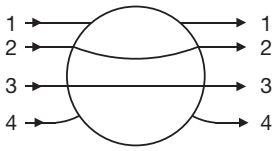
A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E}\cdot d\vec{l}$, ($l = 0$ being the centre of the ring) in volt is

(A) +2 (B) –1

(C) –2 (D) 0

29. [IIT-JEE 1996]

A metallic sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in the figure as



(A) 1 (B) 2
(C) 3 (D) 4

30. [IIT-JEE 1995]

Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$ respectively of a x-y co-ordinate system. Then:

(A) the electric field E at all points on the x-axis has the same direction
(B) work has to be done in bringing at a test charge from ∞ to the origin
(C) electric field at all point on y-axis is along x-axis
(D) the dipole moment is $2qd$ along the x-axis

31. [IIT-JEE 1992]

Two identical thin rings, each of radius a metre are coaxial and placed a metre apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q coulomb from the centre of one ring to that of the other is

(A) zero (B) $\dfrac{q(Q_1-Q_2)(\sqrt{2}-1)}{4\sqrt{2}\pi\varepsilon_0 a}$

(C) $\dfrac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 a}$ (D) $\dfrac{q(Q_1+Q_2)(\sqrt{2}+1)}{4\sqrt{2}\pi\varepsilon_0 a}$

32. [IIT-JEE 1989]

A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(A) V (B) $2V$
(C) $4V$ (D) $-2V$

33. [IIT-JEE 1987]

A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

(A) $-\dfrac{Q}{2}$ (B) $-\dfrac{Q}{4}$
(C) $+\dfrac{Q}{4}$ (D) $+\dfrac{Q}{2}$

34. [IIT-JEE 1984]

Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$ on the y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will

(A) Execute simple harmonic motion about the origin
(B) Move to the origin and remains at rest
(C) Move to infinity
(D) Execute oscillatory but not simple harmonic motion.

35. [IIT-JEE 1983]

A hollow metal sphere of radius 5 cm is charged so that the potential on its surface is 10 V. The potential at the centre of the sphere is

(A) ZERO
(B) 10 V
(C) same as at a point 5 cm away from the surface
(D) same as at a point 25 cm away from the surface

36. [IIT-JEE 1981]

An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of

(A) 1 Å (B) 10^{-10} cm
(C) 10^{-12} cm (D) 10^{-15} cm

Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2019]

A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its centre same as that of the shell. Here, centre of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?
(ε_0 is the permittivity of free space)

(A) If $h > 2R$ and $r = \dfrac{3R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

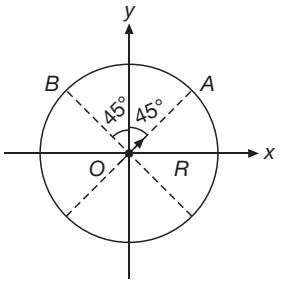
(B) If $h < \dfrac{8R}{5}$ and $r = \dfrac{3R}{5}$ then $\phi = 0$

(C) If $h > 2R$ and $r > R$ then $\phi = \dfrac{Q}{\varepsilon_0}$

(D) If $h > 2R$ and $r = \dfrac{4R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

2. [JEE (Advanced) 2019]

An electric dipole with dipole moment $\dfrac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are (ε_0 is permittivity of free space. $R \gg$ dipole size)



(A) Total electric field at point B is $\vec{E}_B = 0$

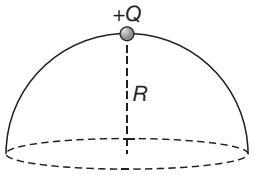
(B) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(C) $R = \left(\dfrac{p_0}{4\pi\varepsilon_0 E_0}\right)^{\frac{1}{3}}$

(D) The magnitude of total electric field on any two points of the circle will be same

3. [JEE (Advanced) 2017]

A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



(A) The electric flux passing through the curved surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$.

(B) The component of the electric field normal to the flat surface is constant over the surface

(C) Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

(D) The circumference of the flat surface is an equipotential

4. [JEE (Advanced) 2014]

Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, and infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

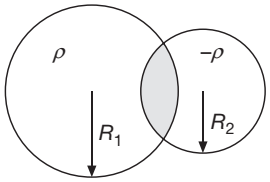
(B) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(C) $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$

(D) $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

5. [JEE (Advanced) 2013]

Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region



(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction

6. [JEE (Advanced) 2013]

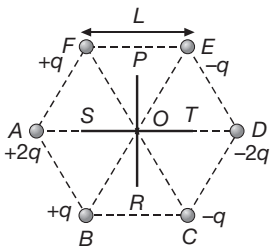
Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centre of the spheres, is zero. The ratio $\dfrac{\rho_1}{\rho_2}$ can be

(A) –4 (B) $-\dfrac{32}{25}$

(C) $\dfrac{32}{25}$ (D) 4

7. [IIT-JEE 2012]

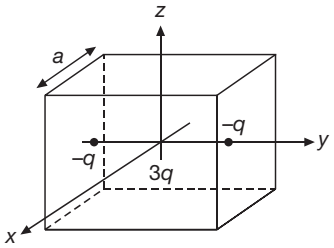
Six point charges are kept at the vertices of a regular hexagon of side L and centre O as shown in the figure. Given that $K = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{L^2}$, which of the following statements(s) is(are) correct.



(A) The electric field at O is $6K$ along OD
(B) The potential at O is zero
(C) The potential at all points on the line PR is same
(D) The potential at all points on the line ST is same

8. [IIT-JEE 2012]

A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $\left(0, -\frac{a}{4}, 0\right)$, $+3q$ at (0, 0, 0) and $-q$ at $\left(0, +\frac{a}{4}, 0\right)$. Choose the correct option(s).



(A) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

(B) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$

9. [IIT-JEE 2011]

Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss's law will still be valid

(B) The Gauss's law can be used to calculate the field distribution around an electric dipole

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

10. [IIT-JEE 2011]

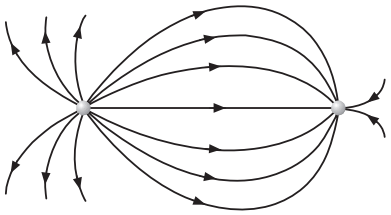
A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then

(A) $E_A^{\text{inside}} = 0$ (B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (D) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

11. [IIT-JEE 2010]

A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$

(B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D) at a finite distance to the right of Q_2 the electric field is zero

12. [IIT-JEE 2009]

Under the influence of the coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(A) The angular momentum of the charge $-q$ is constant

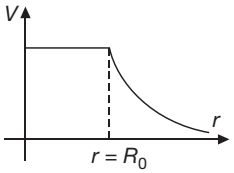
(B) The linear momentum of the charge $-q$ is constant

(C) The angular velocity of the charge $-q$ is constant

(D) The linear speed of the charge $-q$ is constant

13. [IIT-JEE 2006]

For spherical symmetrical charge distribution, variation of electric potential with distance from centre is given in diagram. Given that $V = \frac{q}{4\pi\varepsilon_0 R_0}$ for $r \le R_0$ and $V = \frac{q}{4\pi\varepsilon_0 r}$ for $r \ge R_0$



Then which option(s) is/are correct

(A) Total charge within $2R_0$ is q.

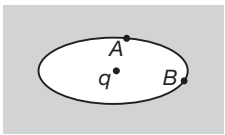
(B) Total electrostatic energy for $r \le R_0$ is zero.

(C) At $r = R_0$ electric field is discontinuous.

(D) There will be no charge anywhere except at $r = R_0$.

14. [IIT-JEE 1999]

An ellipsoidal cavity is carved within a perfect conductor (see figure). A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in figure. Then,



(A) electric field near A in the cavity = electric field near B in the cavity.

(B) charge density at A = charge density at B.

(C) potential at A = potential at B.

(D) total electric flux through the surface of the cavity is $\frac{q}{\varepsilon_0}$.

15. [IIT-JEE 1998]

A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre

(A) increases as r increases for $r < R$

(B) decreases as r increases for $0 < r < \infty$

(C) decreases as r increases for $R < r < \infty$

(D) is discontinuous at $r = R$

16. [IIT-JEE 1998]

A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is

(A) periodic for all values of z_0 satisfying $0 < z_0 < \infty$

(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \le R$

(C) approximately simple harmonic provided $z_0 \ll R$

(D) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$

Reasoning Based Problems

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2008]

Statement-1: For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

Statement-2: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

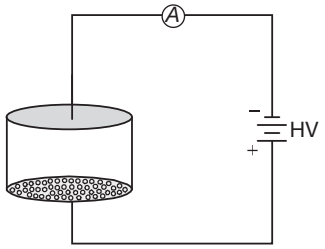
Linked Comprehension Type Problems

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$.

Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



1. [JEE (Advanced) 2016]

Which one of the following statements is correct?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) The balls will execute simple harmonic motion between the two plates

(C) The balls will bounce back to the bottom plate carrying the same charge they went up with

(D) The balls will stick to the top plate and remain there

2. [JEE (Advanced) 2016]

The average current in the steady state registered by the ammeter in the circuit will be

(A) proportional to $V_0^{1/2}$

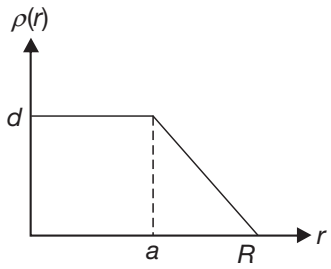
(B) proportional to V_0^2

(C) proportional to the potential V_0

(D) zero

Comprehension 2

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is depended only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



3. [IIT-JEE 2008]

The electric field at $r = R$ is

(A) independent of a

(B) directly proportional to a

(C) directly proportional to a^2

(D) inversely proportional to a

4. [IIT-JEE 2008]

For $a = 0$, the value d (maximum value of ρ as shown in the figure) is

(A) $\dfrac{3Ze^2}{4\pi R^3}$ (B) $\dfrac{3Ze}{\pi R^3}$

(C) $\dfrac{4Ze}{3\pi R^3}$ (D) $\dfrac{Ze}{3\pi R^3}$

5. [IIT-JEE 2008]

The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

(A) $a = 0$ (B) $a = \dfrac{R}{2}$

(C) $a = R$ (D) $a = \dfrac{2R}{3}$

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

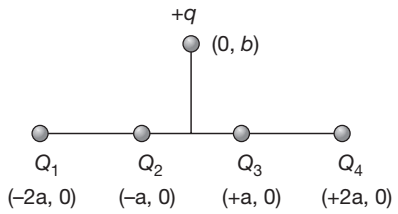
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

1. [JEE (Advanced) 2014]

Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a$, $-a$, $+a$ and $+2a$ respectively. A positive charge q is placed on the positive y-axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match

List I with List II and select the correct answer using the code given below the lists.



List I	List II
P. Q_1, Q_2, Q_3, Q_4 all positive	1. $+x$
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. $-x$
R. Q_1, Q_4 positive; Q_2, Q_3 negative	3. $+y$
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. $-y$

Codes

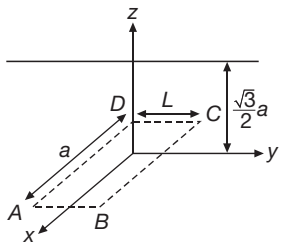
(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

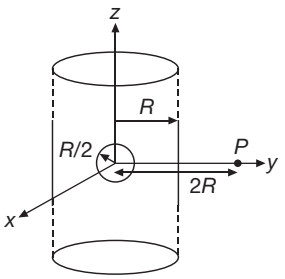
1. [JEE (Advanced) 2015]

An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



2. [IIT-JEE 2012]

An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $\frac{R}{2}$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



3. [IIT-JEE 2011]

Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left(\frac{q^2}{\gamma}\right)^{\frac{1}{N}}$, where k is a constant. Then N is.

4. [IIT-JEE 2009]

A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Coulomb's Law)

1. $K = 4$
2. 2×10^{16} N
3. $\frac{\lambda_0 L^2}{3}$
4. $\frac{\rho}{\rho-\sigma}$
5. 4×10^{42}, 10^{36}, 8.6×10^{-11} Ckg^{-1}
6. $9\times10^{-4}\left(6\hat{i}-8\hat{j}\right)$ N, 9×10^{-3} N
7. $\left(-21\hat{i}-28\hat{j}+84\hat{k}\right)\times10^{-5}$ N
8. 2.25 mm
9. 33×10^{-9} C
11. $2\pi\sqrt{\frac{4\pi\varepsilon_0 mL^3}{Qq}}$
12. $Q = 2L\sqrt{4\pi\varepsilon_0 k\left(L-L_0\right)}$
13. 3 μC and -1 μC
14. 38 μC or 12 μC
15. 4.1×10^{16} rads^{-1}, 1.5×10^{-16} s
17. $\left(\frac{4\pi\varepsilon_0 mgR^2}{\sqrt{3}}\right)^{\frac{1}{2}}$
18. $\mu = 0.18$
19. 1

Test Your Concepts-II (Based on Principle of Superposition)

1. $\frac{Q^2}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{i}+\hat{j}+\hat{k}\right)$

   $\frac{\sqrt{3}Q^2}{4\pi\varepsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)$ MAGNITUDE
2. $\frac{mg\pi\varepsilon_0}{h}\left(h^2+\frac{\ell^2}{2}\right)^{3/2}$
3. Radially outwards $\pm\frac{a}{\sqrt{2}}$, $\frac{Qq_0}{3\sqrt{3}\pi\varepsilon_0 a^2}$
4. $\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 R^2}$
5. $\frac{q^2}{4\pi\varepsilon_0 a^2}$
6. (a) zero

   (b) $F = \frac{qQ}{4\pi\varepsilon_0 a^2}$
7. $2\pi\sqrt{2\sqrt{2}\left(\frac{Qq}{\pi\varepsilon_0 ma^3}\right)}$
8. $\frac{qQ}{4\pi\varepsilon_0 a^2}$
9. 10 N (parallel to AB)

Test Your Concepts-III (Based on Electric Field)

1. $\frac{q}{4\pi\varepsilon_0 a^2}$
2. $\frac{Qx}{8\pi^2\varepsilon_0 R^3}$
3. 0.2 N (due North), 0.5 N (due South)
4. $\frac{\sqrt{6}q}{4\pi\varepsilon_0\ell^2}$
5. $E = \frac{mg}{q}$
7. (a) Q_2 is negative and Q_1 is positive.

   (b) $\left|\frac{Q_1}{Q_2}\right| = \left(\frac{\ell+a}{a}\right)^2$

   (c) $\frac{1}{\left(\frac{Q_1}{Q_2}\right)^{1/3}-1}$
8. $f = \lambda R E_0$ along positive x-axis.
9. (a) $\frac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}}$ {towards the centre}

   (b) The motion of the bead is periodic between $\pm x_0$

(c) $m\frac{d^2x}{dt^2} = -\frac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}}$ with $\frac{dx}{dt}=0$ at $x=\pm x_0$

and $x=x_0$ at $t=0$

(d) $x=x_0\cos(\omega t)$ and $v=-x_0\omega\sin(\omega t)$, where

$\omega=\sqrt{\frac{2Qq}{4\pi\varepsilon_0 mR^3}}$

(e) $\sqrt{\frac{4\pi^3\varepsilon_0 mR^3}{2Qq}}$

10. $2\pi\sqrt{\frac{mR\pi}{4qE}}$

11. $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$

12. $\frac{mgd}{q\sqrt{\ell^2-d^2}}$

13. $T=2\pi\sqrt{\frac{\ell}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2-2g\left(\frac{qE}{m}\right)\cos\beta}}}$

14. $2\sqrt{\frac{2md}{qE}}$ not a simple harmonic motion but periodic in nature.

15. $\frac{\lambda\sqrt{2}}{4\pi\varepsilon_0\ell}\left(-2\hat{i}+14\hat{j}\right)$

16. $-\left(\frac{3\lambda}{\pi\varepsilon_0\ell}\right)\hat{j}$

17. (a) $q=\frac{mg}{(A\cot\theta+B)}$

(b) $T=\frac{mgA}{(A\cos\theta+B\sin\theta)}$

18. (a) $\frac{mv^2}{qR}$

(b) $\frac{mv^2}{\sqrt{2qR}}$

19. 5.25 μC

20. $\frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{\frac{3}{2}}}\hat{i}$

21. $\frac{\lambda_0}{8\pi\varepsilon_0 x_0}$

22. $E_x=\frac{\lambda}{4\pi\varepsilon_0 b}$ and $E_y=\frac{\lambda}{4\pi\varepsilon_0 b}$ with $E_{\text{net}}=\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 b}$

Test Your Concepts-IV (Based on Dipoles and Dipole Moment)

1. $2p\cos\frac{\theta}{2}$

2. $\left|\vec{p}\right|=\sqrt{17}qa$

at an angle $\tan^{-1}\left(\frac{1}{4}\right)$ with the z axis.

3. $T=2\pi\sqrt{\frac{4\pi\varepsilon_0 \mathrm{I}}{\sigma}}$

4. (a) $2\pi\sqrt{\frac{md}{2\lambda LE}}$

(b) (i) λLdE

(ii) $2\lambda LdE$

5. (a) $\pi\sqrt{\frac{m}{\lambda\sin^2\theta_0 E}}$

(b) $8\lambda R^2 E\sin^2\theta_0$

6. (a) zero

(b) $-\frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2}$

(c) $\frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2}$

7. $\pi R^2\lambda_0$

8. (a) $\frac{qp}{4\pi\varepsilon_0 d^2}$

(b) $\frac{pq}{2\pi\varepsilon_0 d^3}\hat{i}$

Test Your Concepts-V (Based on Flux)

1. $\frac{q}{\varepsilon_0}\left(1-\frac{\ell}{\sqrt{R^2+\ell^2}}\right)$

2. $\frac{q}{\varepsilon_0}\left(\frac{L}{\sqrt{L^2+4R^2}}\right)$

3. $\frac{\lambda R}{2\varepsilon_0}$

4. $\left(1+\frac{\sqrt{3}}{2}\right)\frac{q}{2\varepsilon_0}$

5. $2HRE$

6. $E\left(2RH\cos\theta+\pi R^2\sin\theta\right)$

7. $\frac{ch\ell^2}{2}$

8. (a) 858 Nm^2C^{-1}
   (b) 0
   (c) 428.75 Nm^2C^{-1}

9. 180 kN m^2C^{-1}

10. 1 MNC^{-1}

11. (a) −2.34 kNm^2C^{-1}
    (b) +2.34 kNm^2C^{-1}
    (c) 0

12. +1.87 kNm^2C^{-1}

13. ERh

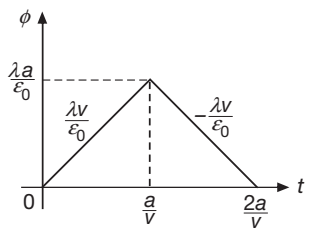
14. (a) $+\frac{Q}{2\varepsilon_0}$
    (b) $-\frac{Q}{2\varepsilon_0}$

Test Your Concepts-VI (Based on Gauss's Law)

1. zero

2. (a) $\frac{q}{\varepsilon_0}$
   (b) $\frac{q}{6\varepsilon_0}$

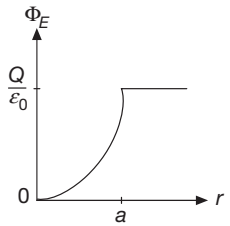
3. $4\pi\varepsilon_0\alpha R$

4. 

5. $\frac{1}{3\varepsilon_0}kx^2$

6. (a) $\frac{q}{2\varepsilon_0}$
   (b) $\frac{q}{2\varepsilon_0}$

7. $\frac{Q-6q}{6\varepsilon_0}$, $q=\frac{Q}{6}$

8. (a) $\frac{Qr^3}{\varepsilon_0 a^3}$
   (b) $\frac{Q}{\varepsilon_0}$
   (c) 

9. When $R\le d$, $\Phi_E=\frac{q_{enc}}{\varepsilon_0}=0$

   When $R>d$, $\Phi_E=\frac{2\lambda\sqrt{R^2-d^2}}{\varepsilon_0}$

10. (a) $\frac{\rho_0 R^5}{5\varepsilon_0 r^2}$
    (b) $\frac{\rho_0 r^3}{5\varepsilon_0}$

11. (a) $\begin{cases}\frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for } x>\frac{d}{2}\\ -\frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for } x<-\frac{d}{2}\end{cases}$

    (b) $\begin{cases}\frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for } x>0\\ -\frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for } x<0\end{cases}$

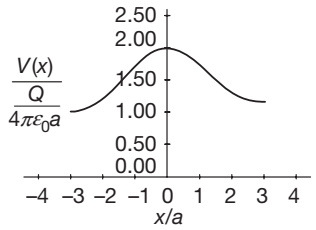
12. $\frac{\rho_0}{2\,\varepsilon_0}$

13. (b) $g=\left(\frac{GM_E}{R_E^3}\right)r$

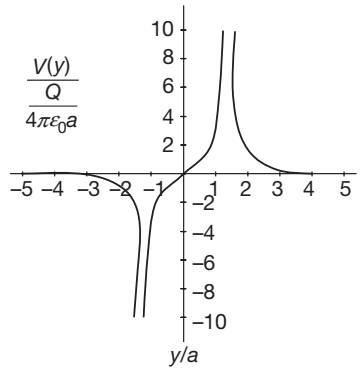
15. $\frac{\rho\vec{a}}{2\varepsilon_0}$

Test Your Concepts-VII (Based on Electrostatic Potential and Energy)

1. (a)



(b)



2. -7.62×10^{-2} J

3. 1.35 MJ

4. 9×10^{3} V

5. (a) $\dfrac{2QE}{k}$

(b) $\dfrac{QE}{k}$

(c) $2\pi\sqrt{\dfrac{m}{k}}$

(d) $\dfrac{2\left(QE-\mu_k mg\right)}{k}$

6. 1.8×10^{4} V

7. 40 kV

8. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2Qqx^2}{a^3}$

9. 0.4 ms^{-1}

10. (a) 4.5×10^{-3} J

(b) -4.5×10^{-3} J

11. $-Q\left(\dfrac{a}{b}\right)$

12. (a) 6 m

(b) $-2\ \mu$C

13. $r = 0.5$ m

$r = 0.25$ m

14. $\dfrac{5}{9}\left(\dfrac{q^2}{4\pi\varepsilon_0 d}\right)$

15. $-0.553\dfrac{Q}{4\pi\varepsilon_0 R}$

16. $-q_0E_0a$

17. $\dfrac{\lambda}{4\pi\varepsilon_0}\left(\pi+2\log_e 3\right)$

18. $\left(\dfrac{\pi\sigma_0}{4\pi\varepsilon_0}\right)\left[R\sqrt{R^2+x^2}+x^2\log_e\left(\dfrac{x}{R+\sqrt{R^2+x^2}}\right)\right]$

19. (i) $\dfrac{2q}{4\pi\varepsilon_0}, \dfrac{q}{3\pi\varepsilon_0}$

(ii) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{2q}{3}\right), \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{4q}{5}\right]$

20. 1 m

21. $\sqrt{\dfrac{7Qq}{4\pi\varepsilon_0 m\ell}}$

22. $5.824\left(\dfrac{1}{4\pi\varepsilon_o}\cdot\dfrac{q^2}{a}\right)$

23. $V\left(\dfrac{a}{3t}\right)^{1/3}$

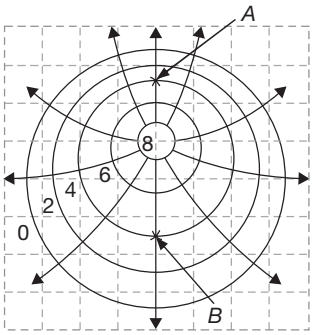
Test Your Concepts-VIII (Based on Equipotential Surfaces)

1. $+260V$

2. (a) $E_A > E_B$

(b) 200 NC^{-1} downwards

(c)



Test Your Concepts-IX (Based on Relation Between Electrostatic Field and Potential)

1. $5\sqrt{2}$ NC^{-1}
2. $E_x = 3E_0a^3xz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$

   $E_y = 3E_0a^3yz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$

   $E_z = E_0 + E_0a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$
3. $-2\hat{i}-3\hat{j}+\hat{k}$
4. 3 V
5. (a) 1 V (b) 1 V
6. (a) $-2a\left(x\hat{i}-y\hat{j}\right)$

   (b) $-ay\hat{i}-ax\hat{j}$
7. $\vec{E} = -\left(2ax\hat{i}+2ay\hat{j}+2bz\hat{k}\right)$

   $|\vec{E}| = 2\sqrt{a^2\left(x^2+y^2\right)+b^2z^2}$

   An ellipsoid of revolution with semi-axis

   $\sqrt{\frac{V}{a}}, \sqrt{\frac{V}{a}}, \sqrt{\frac{V}{b}}$.
8. $-(axy+byz)+C$

Test Your Concepts-X (Based on Motion of Charged Particles in Electric Field)

1. (a) 60 V

   (b) 4.6×10^6 ms^{-1}
2. (a) 1.52×10^5 ms^{-1}

   (b) 6.5×10^6 ms^{-1}
3. (a) -4.5×10^{-5} J

   (b) 3.46×10^4 ms^{-1}
4. 27.3 fm
5. $\sqrt{\left(1+\frac{1}{\sqrt{8}}\right)\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mL}}$
6. 1.45×10^7 ms^{-1}
7. $\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{3m\ell}}$
8. $\sqrt{\frac{Q^2}{2\pi\varepsilon_0MR}}$
9. $mg\ell - \frac{1}{2}mu^2 + \frac{q^2}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}\ell}\left(1-\sqrt{2}\right)$
10. $\frac{Qq}{4\pi\varepsilon_0mv_0^2} + \sqrt{\left(\frac{Qq}{4\pi\varepsilon_0mv_0^2}\right)^2+\ell^2}$
11. $\left(\frac{ae}{2m\ell}\right)\left(\frac{6m\ell^2}{ae}\right)^{\frac{2}{3}}$
12. $v_1 = \sqrt{\frac{Qqr_2}{2\pi\varepsilon_0mr_1(r_1+r_2)}}$, $v_2 = \sqrt{\frac{Qqr_1}{2\pi\varepsilon_0mr_2(r_1+r_2)}}$
13. $\frac{e^2}{\pi\varepsilon_0mv_0^2}$

Test Your Concepts-XI (Based on Conductors)

1. 2 N
2. $-\frac{1}{2\varepsilon_0}(\sigma_A+\sigma_B)$, $\frac{1}{2\varepsilon_0}(\sigma_A-\sigma_B)$, $\frac{1}{2\varepsilon_0}(\sigma_A+\sigma_B)$

   $E_I = E_{III} = 0$, $E_{II} = \frac{\sigma}{\varepsilon_0}$
3. 1.77 pCm^{-3}
4. (a) 496 nCm^{-2}

   (b) 992 nCm^{-2}

5. (a) $\frac{-q}{4\pi a^2}$

(b) $\frac{Q+q}{4\pi b^2}$

6. Charge on the outer surface of the sphere is $-Q$

Charge on the inner surface of the shell is $+Q$

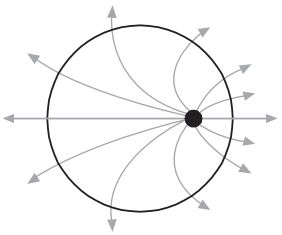
Charge on the outer surface of the shell is $+2Q$

Inside the sphere within the material of the shell, $E = 0$

Between the sphere and shell $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

Outside the shell $E = \frac{2Q}{4\pi\varepsilon_0 r^2}$

7.



8. (a) For $r < a$, $E = \frac{\rho r}{3\varepsilon_0}$

For $a < r < b$ and $c < r$, $E = \frac{Q}{4\pi r^2 \varepsilon_0}$

For $b \le r \le c$, $E = 0$

(b) $\frac{Q}{4\pi c^2}$

9. (a) $+2Q$

(b) $\frac{2Q}{4\pi\varepsilon_0 r^2}$

(c) $E = 0$

(d) 0

(e) $+3Q$

(f) $\frac{3Q}{4\pi\varepsilon_0 r^2}$

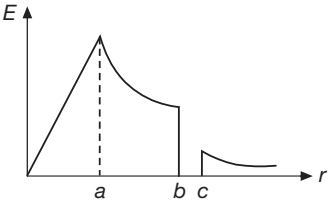
(g) $+3Q\left(\frac{r^3}{a^3}\right)$

(h) $\left(\frac{3Q}{4\pi\varepsilon_0 a^3}\right) r$

(i) $-3Q$

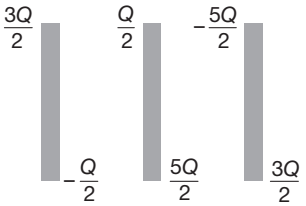
(j) $+2Q$

(k)



Test Your Concepts-XII (Based on Charge Distribution)

1.



3. $\left(\frac{Q}{2} - \varepsilon_0 AE\right), \left(\frac{Q}{2} + \varepsilon_0 AE\right)$

$-\varepsilon_0 AE, +\varepsilon_0 AE$

4. $E_{\text{left}} = E - \frac{Q}{2A\varepsilon_0}$, $E_{\text{right}} = E + \frac{Q}{2A\varepsilon_0}$

5. $AE\varepsilon_0, -AE\varepsilon_0$

6. $\frac{(\sigma_1 - \sigma_2)qa}{\sqrt{2}\,\varepsilon_o}$

Test Your Concepts-XIII (Based on Concept of Self and Interaction Energy)

1. $\frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

2. $\frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

3. $\frac{\sigma^2}{2\varepsilon_0}$

4. $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2R}$

Single Correct Choice Type Questions

1. D	**2.** D	**3.** D	**4.** C	**5.** C	**6.** C	**7.** D	**8.** A	**9.** A	**10.** B
11. B	**12.** D	**13.** B	**14.** B	**15.** A	**16.** A	**17.** D	**18.** A	**19.** D	**20.** D
21. B	**22.** D	**23.** C	**24.** C	**25.** C	**26.** D	**27.** A	**28.** B	**29.** B	**30.** B
31. D	**32.** A	**33.** D	**34.** C	**35.** B	**36.** A	**37.** B	**38.** A	**39.** A	**40.** A
41. D	**42.** B	**43.** A	**44.** D	**45.** B	**46.** C	**47.** D	**48.** C	**49.** C	**50.** D
51. C	**52.** D	**53.** D	**54.** C	**55.** B	**56.** C	**57.** B	**58.** A	**59.** A	**60.** C
61. B	**62.** C	**63.** C	**64.** A	**65.** C	**66.** A	**67.** C	**68.** B	**69.** D	**70.** A
71. D	**72.** A	**73.** D	**74.** D	**75.** A	**76.** D	**77.** B	**78.** C	**79.** C	**80.** C
81. A	**82.** B	**83.** C	**84.** D	**85.** B	**86.** C	**87.** C	**88.** C	**89.** D	**90.** B
91. B	**92.** A	**93.** B	**94.** A	**95.** C	**96.** B	**97.** D	**98.** B	**99.** A	**100.** C
101. D	**102.** C	**103.** B	**104.** A	**105.** A	**106.** B	**107.** A	**108.** A	**109.** C	**110.** B
111. B	**112.** D	**113.** C	**114.** A	**115.** B	**116.** B	**117.** D	**118.** D	**119.** D	**120.** C
121. D	**122.** A	**123.** C	**124.** C	**125.** D	**126.** C	**127.** D	**128.** C	**129.** D	**130.** B
131. A	**132.** C	**133.** C	**134.** D	**135.** B	**136.** B	**137.** B	**138.** A	**139.** C	**140.** A
141. C	**142.** A	**143.** A	**144.** C	**145.** C	**146.** B	**147.** B	**148.** C	**149.** D	**150.** A
151. A	**152.** D	**153.** B	**154.** C	**155.** A	**156.** B	**157.** C	**158.** C	**159.** C	**160.** D
161. A	**162.** D	**163.** D	**164.** A	**165.** C	**166.** B	**167.** D	**168.** D	**169.** A	**170.** B
171. A	**172.** B	**173.** B	**174.** C	**175.** D	**176.** A	**177.** C	**178.** D	**179.** A	**180.** B
181. C	**182.** A	**183.** A	**184.** D	**185.** B	**186.** B	**187.** C			

Multiple Correct Choice Type Questions

1. A, C	**2.** A, C	**3.** A, B, C, D	**4.** A, C, D	**5.** C, D
6. A, C	**7.** D	**8.** D	**9.** C, D	**10.** A, B, C
11. A, C	**12.** A, B	**13.** A, B, C	**14.** B, D	**15.** D
16. B, D	**17.** A, B, C, D	**18.** A, B, C, D	**19.** B, D	**20.** A, D
21. D	**22.** A, B, C	**23.** A, C, D	**24.** A, B, C, D	**25.** A, C
26. B, C	**27.** A, B, D	**28.** B, C, D	**29.** A, C, D	**30.** A, B
31. A, B, C, D	**32.** C, D	**33.** A, B, C, D	**34.** A, B, C, D	**35.** A, B, C, D
36. A	**37.** D	**38.** A, D	**39.** A, D	**40.** B, D
41. A, D	**42.** A, C	**43.** B, D	**44.** A, B	**45.** A, C
46. A, C	**47.** A, B, C	**48.** B, D	**49.** A, C, D	**50.** A, B, C
51. A, D	**52.** A, B, D	**53.** A, C, D	**54.** B, C	**55.** A, B, C
56. B, C	**57.** B, C, D	**58.** A, B, C	**59.** B, C, D	**60.** A, D
61. A, C, D				

Reasoning Based Questions

1. C	**2.** B	**3.** D	**4.** C	**5.** A	**6.** D	**7.** D	**8.** A	**9.** B	**10.** A
11. D	**12.** C	**13.** B	**14.** B	**15.** B	**16.** D	**17.** D	**18.** A	**19.** A	**20.** A

Linked Comprehension Type Questions

1. C	**2.** B	**3.** C	**4.** A	**5.** D	**6.** D	**7.** B	**8.** C	**9.** C	**10.** C
11. B	**12.** D	**13.** B	**14.** C	**15.** A	**16.** A	**17.** B	**18.** D	**19.** C	**20.** A
21. B	**22.** C	**23.** D	**24.** C	**25.** D	**26.** B	**27.** A	**28.** C	**29.** B	**30.** A
31. D	**32.** D	**33.** C	**34.** B	**35.** A	**36.** D	**37.** A	**38.** B	**39.** C	**40.** A
41. B	**42.** D	**43.** D	**44.** C	**45.** C	**46.** B	**47.** C	**48.** C	**49.** B	**50.** C
51. A	**52.** D	**53.** B	**54.** C	**55.** B	**56.** A	**57.** C	**58.** B	**59.** D	**60.** A

Matrix Match/Column Match Type Questions

1.	A → (p, q, r, s)	B → (p, q, s)	C → (p, q, s)	D → (p, q, r, s, t)
2.	A → (p, r, s, t)	B → (p, q)	C → (p, q)	D → (p, r, s, t)
3.	A → (p, r, s)	B → (p, q)	C → (p, q, r, s)	D → (p, r, s, t)
4.	A → (q, r, s)	B → (p)	C → (p, q, r, s)	D → (p, q, r, s)
5.	A → (q, r)	B → (s)	C → (t)	D → (p)
6.	A → (s, t)	B → (p, q, r)	C → (q, r)	D → (p)
7.	A → (r, s)	B → (p, r, s)	C → (q)	D → (t)
8.	A → (q, s)	B → (p)	C → (t)	D → (r)
9.	A → (s, t)	B → (q)	C → (p)	D → (r, s)
10.	A → (q, t)	B → (s)	C → (p)	D → (r)
11.	A → (r)	B → (s)	C → (p)	D → (p)
12.	A → (s)	B → (r)	C → (q, t)	D → (p)
13.	A → (p)	B → (q, s)	C → (p)	D → (q, r)
14.	A → (r, s)	B → (r, s)	C → (p, s)	D → (p, s)
15.	A → (p, s)	B → (q, s)	C → (q, s)	D → (s)
16.	A → (p, r, s, t)	B → (p, r, s, t)	C → (p, q, r, s)	D → (q)
17.	A → (q)	B → (r)	C → (s)	D → (p)
18.	A → (s)	B → (r)	C → (p)	D → (q)
19.	A → (r)	B → (q)	C → (p)	D → (s)
20.	A → (s)	B → (r)	C → (p)	D → (q)
21.	A → (p)	B → (r)	C → (p)	D → (p)
22.	A → (q)	B → (r)	C → (p)	D → (s)

Integer/Numerical Answer Type Questions

1. 225	**2.** 33	**3.** 12	**4.** 2	**5.** 2275
6. 12	**7.** 72	**8.** 1	**9.** 60	**10.** 12

11. 52	**12.** 5	**13.** 1320	**14.** 24	**15.** 20
16. 2	**17.** 8	**18.** 33	**19.** (a) 55, (b) 2293	**20.** 9
21. 30	**22.** 7080	**23.** 154	**24.** 99	**25.** 50
26. 365	**27.** (a) 3, (b) 20, (c) 58	**28.** 100	**29.** 345, 16	**30.** 8
31. 35	**32.** 373	**33.** 100	**34.** 47	**35.** 2
36. 2	**37.** 700	**38.** 586	**39.** 758	**40.** 6
41. 1080, 155	**42.** 200	**43.** 5	**44.** 10	**45.** 30
46. 30	**47.** 2			

ARCHIVE: JEE MAIN

1. D	**2.** B	**3.** A	**4.** A	**5.** A	**6.** A	**7.** C	**8.** D	**9.** D	**10.** C
11. C	**12.** C	**13.** D	**14.** C	**15.** A	**16.** A	**17.** A	**18.** B	**19.** A	**20.** D
21. C	**22.** A	**23.** A	**24.** C	**25.** D	**26.** C	**27.** A	**28.** B	**29.** B	**30.** A
31. A	**32.** D	**33.** C	**34.** A	**35.** A	**36.** B	**37.** B	**38.** C	**39.** C, D	**40.** A
41. C	**42.** C	**43.** B	**44.** D	**45.** C	**46.** A	**47.** D	**48.** B	**49.** B	**50.** A
51. D	**52.** D	**53.** C	**54.** D	**55.** A	**56.** D	**57.** C	**58.** D		

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Single Correct Choice Type Problems

1. D	**2.** D	**3.** C	**4.** C	**5.** D	**6.** C	**7.** C	**8.** A	**9.** D	**10.** B
11. A	**12.** C	**13.** C	**14.** D	**15.** B	**16.** A	**17.** B	**18.** C	**19.** D	**20.** C
21. B	**22.** B	**23.** C	**24.** B	**25.** B	**26.** D	**27.** B	**28.** A	**29.** D	**30.** C
31. B	**32.** A	**33.** B	**34.** D	**35.** B	**36.** C				

Multiple Correct Choice Type Problems

1. A, B, C	**2.** A, C	**3.** A, D	**4.** C	**5.** C, D
6. B, D	**7.** A, B, C	**8.** A, C	**9.** C, D	**10.** A, B, C, D
11. A, D	**12.** A	**13.** A, B, C, D	**14.** C, D	**15.** A, C
16. A, C				

Reasoning Based Problems

1. B

Linked Comprehension Type Problems

1. A	**2.** B	**3.** A	**4.** B	**5.** C

Matrix Match/Column Match Type Questions

1. A

Integer/Numerical Answer Type Questions

1. 6	**2.** 6	**3.** 3	**4.** 2

CHAPTER

2 Capacitance and Applications

Learning Objectives

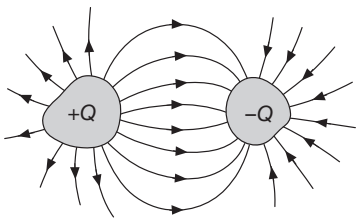
After reading this chapter, you will be able to understand concepts and problems based on:

(a) Capacitors
(b) Capacitance
(c) Energy Concepts
(d) Effect of Dielectric Insertion
(e) Capacitor Circuits
(f) Spherical Capacitor and Cylindrical Capacitor

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

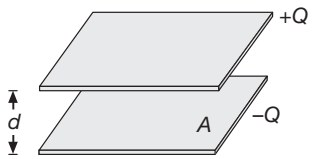
CAPACITORS: INTRODUCTION

A capacitor is a device which stores electric charge. Capacitors vary in shape and size, but the basic configuration involves two conductors carrying equal but opposite charges (shown in figure). Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency dependent and independent voltage dividers when combined with resistors.



In the uncharged state, the charge on either one of the conductors in the capacitor is zero. During the charging process, a charge Q is moved from one conductor to the other one, giving one conductor a charge $+Q$, and the other one a charge $-Q$ as a result of which a potential difference ΔV is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

The simplest example of a capacitor consists of two conducting plates of area A, which are parallel to each other, and separated by a distance $d (\ll A)$, as shown in figure (called as a Parallel Plate Capacitor).



Experimentally it has been verified that the amount of charge Q stored in a capacitor is linearly proportional to ΔV, the electric potential difference between the plates. Thus, we may write

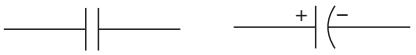
$$Q = C|\Delta V|$$

$$\Rightarrow \quad C = \frac{Q}{|\Delta V|}$$

where C is a positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference ΔV. The SI unit of capacitance is the farad (F).

$$1\ \text{F} = 1\ \text{farad} = 1\ \text{coulomb volt}^{-1} = 1\ \text{CV}^{-1}$$

Figure (a) shows the symbol which is used to represent capacitors in circuits. For a polarized fixed capacitor which has a definite polarity. Figure (b) is rarely used.



(a) (b)

CALCULATION OF CAPACITANCE

As discussed, the method for the calculation of capacitance involves integration of the electric field between two conductors or the plates which are just equipotential surfaces to obtain the potential difference ΔV. Thus,

$$\Delta V = -\int_b^a \vec{E} \cdot d\vec{r}$$

Since, by definition, we have $C = \frac{Q}{|\Delta V|}$

$$\Rightarrow \quad C = \frac{Q}{|\Delta V|} = \frac{Q}{\left| -\int_b^a \vec{E} \cdot d\vec{r} \right|}$$

DEFINITION OF C

$$C = \frac{Q}{\Delta V} \qquad \ldots(1)$$

If $\Delta V = 1$ volt, then

$$C = Q \quad \text{(numerically)}$$

i.e., capacitance of a conductor is numerically equal to the charge that raises its potential by 1 volt.

Practically farad is a very big unit for capacitance and generally smaller units such as μF (microfarad), nF (nanofarad), pF (picofarad) are used.

$$1\ \mu\text{F} = 10^{-6}\ \text{F}$$

$$1\ \text{nF} = 10^{-9}\ \text{F}$$

$$1\ \text{pF} = 10^{-12}\ \text{F} = 1\ \mu\mu\text{F}$$

CGS unit of capacitance is statfarad (SF) and

$$1\ \text{farad} = 9 \times 10^{11}\ \text{statfarad}$$

Dimensional formula for capacitance is,

$$[C] = M^{-1}L^{-2}T^4A^2$$

ENERGY STORED IN A CHARGED CAPACITOR

Whenever any conductor is charged, the charge produces an electric field outside it. Now if, further similar charge is to be given to the conductor some work has to be done by the external agency against the electrostatic forces of repulsion. This work done is stored as the electrostatic potential energy which is stored in the electrostatic field of the conductor.

Let us consider that charge is given in a stepwise manner to the conductor. If V be the potential of the conductor at the instant when it has a charge q, then

$$V = \frac{q}{C}$$

If dq is the additional infinitesimal charge that is to be given to the conductor and dW is the corresponding work done, then

$$dW = V dq$$

$$\Rightarrow \quad dW = \frac{q}{C} dq$$

Total work done to charge the conductor to Q is

$$W = \int dW = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{C}\frac{q^2}{2}\Big|_0^Q$$

$$\Rightarrow \quad W = \frac{Q^2}{2C}$$

If V_0 is the potential that develops on the conductor finally on account of charge Q then

$$V_0 = \frac{Q}{C}$$

$$\Rightarrow \quad W = \frac{1}{2}CV_0^2 = \frac{Q^2}{2C} = \frac{1}{2}QV_0$$

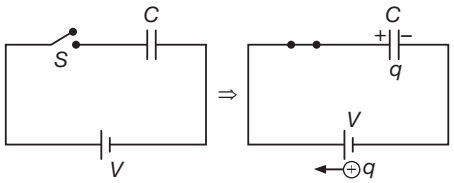
ILLUSTRATION 1

Prove that while charging a capacitor half of the energy supplied by the battery dissipates in the form of heat.

SOLUTION

When the switch S is closed, the charge stored in the capacitor is

$$q = CV$$



Charge transferred from the battery is also q. So, energy supplied by the battery $= qV = (CV)(V) = CV^2$. However the capacitor stores only $\frac{1}{2}CV^2$ which is exactly half this energy. So, the remaining half or 50% or $\frac{1}{2}CV^2$ must have been dissipated as heat.

CAPACITANCE OF AN ISOLATED SPHERE

Let a conducting sphere of radius R acquire a potential V when a charge Q is given to it. The potential acquired by the sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

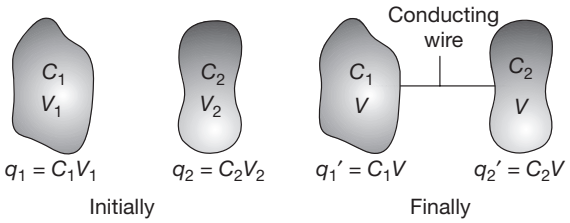
In CGS system $\frac{1}{4\pi\varepsilon_0} = 1$

So, $C = R$ (numerically).

Hence, in cgs system the capacitance of an isolated sphere equals its radius.

CHARGE SHARING BETWEEN TWO CHARGED CONDUCTORS

Consider two charged conductors having capacitance C_1 and C_2 at potential V_1 and V_2 respectively. Let the two be connected by a conducting wire such that the charge now starts flowing from the conductor at higher potential to the conductor at lower potential till both the conductors acquire the same potential.



Total initial charge $= q_1 + q_2 = C_1V_1 + C_2V_2$

If V is the common potential after charge sharing takes place then

Total final charge $= q_1' + q_2' = C_1V + C_2V$

By Law of Conservation of Charge

Total Initial Charge = Total Final Charge

$$\Rightarrow \quad C_1V_1 + C_2V_2 = (C_1 + C_2)V$$

$$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \qquad \ldots(1)$$

Further after sharing the new potential is the same. So,

$$\frac{q_1'}{q_2'} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2}$$

i.e., charges are shared in the ratio of the capacitances after common potential (V) is acquired.

Further, there is always a loss in energy during the sharing process as some energy gets converted to heat.

Loss $= -\Delta U =$ Total Initial Energy $-$ Total Final Energy

$$\Rightarrow \quad \text{Loss} = -\Delta U = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right)$$

$$\Rightarrow \quad \text{Loss} = -\Delta U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

From (1) put value of V above we get

$$\text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

ILLUSTRATION 2

A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and then charged again.

If the process is being repeated n times, the potential of the large capacitor has now fallen to V. Calculate C.

SOLUTION

Let V_1 be the potential of C_0 after first charging, then

$$(C+C_0)V_1 = C_0V_0$$

$$\Rightarrow \quad V_1 = \frac{C_0V_0}{C+C_0}$$

Let V_2 be the potential of C_0 after second charging then

$$C_0V_1 = (C+C_0)V_2$$

$$\Rightarrow \quad V_2 = \left(\frac{C_0}{C+C_0}\right)\left(\frac{C_0}{C+C_0}\right)V_0 = \left(\frac{C_0}{C+C_0}\right)^2 V_0 \text{ and so on}$$

$$\Rightarrow \quad V_n = V = \left(\frac{C_0}{C+C_0}\right)^n V_0$$

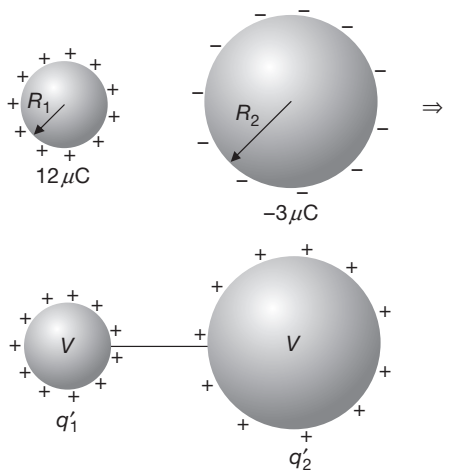
$$\Rightarrow \quad C = \left[\left(\frac{V_0}{V}\right)^{\frac{1}{n}} - 1\right]C_0$$

ILLUSTRATION 3

Two isolated spherical conductors have radii 6 cm and 12 cm respectively. They have charges of 12 μC and -3 μC. Find the charges after they are connected by a conducting wire. Also find the common potential after redistribution.

SOLUTION

Net charge $= (12-3)\ \mu\text{C} = 9\ \mu\text{C}$



Since the charge is distributed in the ratio of their capacitance (or radii in case of spherical conductors), i.e.,

$$\frac{q_1'}{q_2'} = \frac{R_1}{R_2} = \frac{6}{12} = \frac{1}{2}$$

$$\Rightarrow \quad q_1' = \left(\frac{1}{1+2}\right)(9) = 3\ \mu\text{C}$$

and $\quad q_2' = \left(\frac{2}{1+2}\right)(9) = 6\ \mu\text{C}$

The common potential is given by

$$V = \frac{q_1+q_2}{C_1+C_2} = \frac{(9\times10^{-6})}{4\pi\varepsilon_0(R_1+R_2)}$$

$$\Rightarrow \quad V = \frac{(9\times10^{-6})(9\times10^{9})}{(18\times10^{-2})}$$

$$\Rightarrow \quad V = 4.5\times10^5\ \text{V}$$

ILLUSTRATION 4

A capacitor of 20 μF charged to 500 V, is connected in parallel to another capacitor of 10 μF charged to 200 V, find the common potential.

SOLUTION

The common potential,

$$V_{\text{common}} = \frac{C_1V_1+C_2V_2}{C_1+C_2}$$

Here $C_1 = 20\ \mu\text{F} = 20\times10^{-6}$ F, $V_1 = 500$ V

$C_2 = 10\ \mu\text{F} = 10\times10^{-6}$ F, $V_2 = 200$ V

$$\Rightarrow \quad V_{\text{common}} = \frac{20\times10^{-6}\times500+10\times10^{6}\times200}{20\times10^{-6}+10\times10^{-6}}$$

$$\Rightarrow \quad V_{\text{common}} = 400\ \text{V}$$

Remark(s)

CAPACITOR OR CONDENSER

An arrangement which has capability of collecting (and storing) charge and whose capacitance can be varied is called **a capacitor (or condenser)**.

In a capacitor two charged conductors are placed near each other so that the potential of each conductor is determined not only by its own charge but also by the magnitude and sign of the charge on the neighbouring conductor. The capacitance of a capacitor depends

(a) directly on the size of the conductors of the capacitor.

(b) directly on the dielectric constant K of the medium between the conductors.

(c) inversely on the distance of separation between the conductors.

The capacitance of a capacitor also depends on the presence of conductors present in neighbourhood and is independent of the material of metallic conductor.

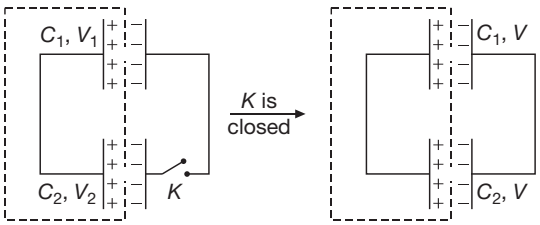
SHARING OF CHARGES BETWEEN CAPACITORS AND COMMON POTENTIAL

Just like conductors, when two isolated capacitors (either both of them are charged or one of them is charged) are connected to each other, then redistribution of charge takes place due to potential difference between them. The charge flows from higher potential to lower potential till both of them acquire the same potential called common potential.

Consider two capacitors, C_1 and C_2 charged to potentials, V_1 and V_2. Let the capacitors be connected such that

CASE-1:

Their similar plates are connected to each other (i.e. positive plate of one capacitor is connected to positive plate of other capacitor and so on).



Consider the part of circuit inside dotted loop. This part is isolated from other part. Therefore, total charge of this part remains constant. Hence

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

During this redistribution of charge, there is a loss of energy in the form of heat and electromagnetic radiations, given by

Loss of energy is $-\Delta U = U_i - U_f$.

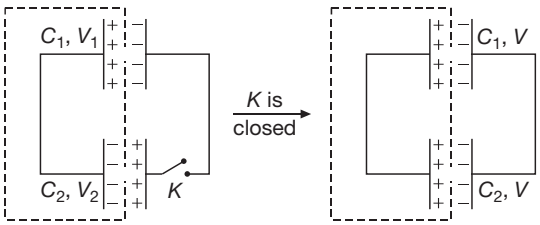
$$\Rightarrow \quad -\Delta U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

Substituting $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$ in above equation, we get

$$\text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

CASE-2:

Their dissimilar plates are connected together (i.e. positive plate of one capacitor is connected to the negative plate of other capacitor and so on).



Again applying the Law of Conservation of Charge, we get

$$C_1V_1 - C_2V_2 = C_1V + C_2V$$

$$\Rightarrow \quad V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

So, loss in energy is given by

$$\text{Loss} = -\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$$

ILLUSTRATION 5

A parallel plate capacitor of capacitance C has been charged, so that the potential difference between its plates is V. Now, the plates of this capacitor are

connected to another uncharged capacitor of capacitance 2C. Find the common potential acquired by the system and loss of energy.

SOLUTION

In this problem, we have

$$C_1 = C,\ C_2 = 2C,\ V_1 = V \text{ and } V_2 = 0$$

Common potential is given by

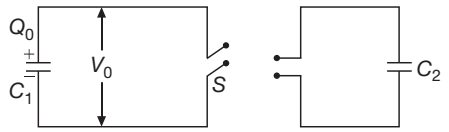
$$V_{\text{Common}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V_{\text{Common}} = \frac{CV + 0}{3C} = \frac{V}{3}$$

and $$\text{Loss} = -\Delta U = \frac{1}{2}\frac{C \times 2C}{3C} \times (V - 0)^2 = \frac{1}{3}CV^2$$

ILLUSTRATION 6

In the circuit shown, $C_1 = 8\ \mu\text{F}$, $C_2 = 4\ \mu\text{F}$ and $V_0 = 120$ V. When the switch S is closed, the charged capacitor C_1 gets connected to an uncharged capacitor C_2. Calculate the



(a) charge on each capacitor after switching S.
(b) total energy after switching S.

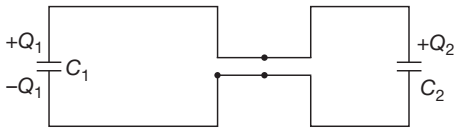
SOLUTION

(a) Before switching on S, $Q_0 = C_1V_0 = 960\ \mu\text{C}$ and

$$\text{Energy stored } U = \frac{1}{2}Q_0V_0$$

$$U = \frac{1}{2}(960 \times 10^{-6}\ \text{C})(120\ \text{V})$$

$$U = 0.058\ \text{J}$$



(b) After we close switch S, we have

$$Q_0 = Q_1 + Q_2$$

In the steady state, we have

$$Q_1 = C_1V \quad \ldots(1)$$

$$Q_2 = C_2V \quad \ldots(2)$$

Adding equation (1) and (2), we get

$$(C_1 + C_2)V = Q_1 + Q_2 = Q_0$$

$$\Rightarrow \quad V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu\text{C}}{12\ \mu\text{F}} = 80\ \text{V}$$

$$\Rightarrow \quad Q_1 = 640\ \mu\text{C},\ Q_2 = 320\ \mu\text{C}$$

The total energy stored is given by

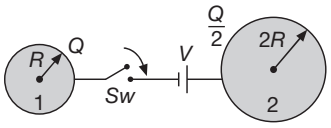
$$U = \frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}Q_0V$$

$$\Rightarrow \quad U = \frac{1}{2}(960 \times 10^{-6})(80\ \text{V}) = 0.038\ \text{J}$$

The energy after closing the switch is less than the original energy of 0.058 J. This difference has been converted to energy of some other form (e.g. heat energy or energy radiated as electromagnetic waves).

ILLUSTRATION 7

Two spheres 1 and 2 of radii R and $2R$ having charges Q and $\frac{Q}{2}$, respectively are connected with a cell of potential difference V as shown in Figure. When the switch Sw is closed, calculate the final charge on each sphere.

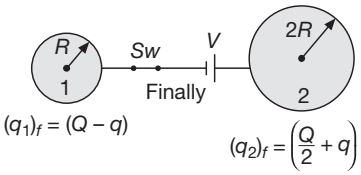


SOLUTION

When the switch is closed, the potential difference between the spheres should be V. Let charge q flow from the sphere of radius R to sphere of radius $2R$. Then

$$\frac{(q_2)_{\text{final}}}{C_2} - \frac{(q_1)_{\text{final}}}{C_1} = V$$

$$C_2 = 4\pi\varepsilon_0(2R) \text{ and } C_1 = 4\pi\varepsilon_0R$$



$$\Rightarrow \quad \frac{\left(\frac{Q}{2}+q\right)}{4\pi\varepsilon_0(2R)} - \frac{(Q-q)}{4\pi\varepsilon_0(R)} = V$$

$$\Rightarrow \quad \frac{Q}{2} + q - 2Q + 2q = 4\pi\varepsilon_0(2R)V$$

$$\Rightarrow \quad 3q = 8\pi\varepsilon_0 RV + \frac{3Q}{2}$$

$$\Rightarrow \quad q = \frac{8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$$

So the final charges on each of the spheres are

$$(q_1)_f = Q - q = \frac{Q}{2} - \frac{8\pi\varepsilon_0 RV}{3}$$

and $$(q_2)_f = \frac{Q}{2} + q = Q + \frac{8\pi\varepsilon_0 RV}{3}$$

Test Your Concepts-I

Based on General Capacitance

(Solutions on page H.126)

1. A capacitor of 20 μF charged to 500 V, is connected in parallel to another capacitor of 10 μF charged to 200 V, find the common potential.
2. Calculate the heat generated when a condenser of 100 μF capacity and charged to 200 V is discharged through a 2 Ω resistance.
3. The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0° to 180°. With the dial set at 180°, the capacitor is connected to a 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is turned at 0°.
   (a) What is the potential difference across the capacitor when the dial reads 0°?
   (b) How much work is required to turn the dial, if friction is neglected?
4. A battery of 10 V is connected to a capacitor of capacitance 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further compare this energy with the initial energy stored in the capacitors.
5. Consider two conducting spheres with radii R_1 and R_2 separated by a distance much greater than radius of either. A total charge Q is shared between the spheres, subject to the condition that the electric potential energy of the system has the smallest possible value. The spheres being very far apart, you can assume that the charge of each is uniformly distributed over the surface of each.
   (a) Determine the values of charges on the spheres in terms of Q, R_1 and R_2.
   (b) Find the potential difference between the surfaces of the spheres.
6. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and charged again, the process being repeated 10 times. The potential of the capacitor C_0 has now fallen to V. Calculate C.

PRINCIPLE OF A PARALLEL PLATE CAPACITOR

Consider a conducting plate A which is given a charge Q (as shown in Figure 1(a)) such that its potential rises to V. Then

$$C = \frac{Q}{V}$$

Let us place another identical conducting plate B parallel to it such that charge is induced on plate B (as shown in Figure 1(b)).

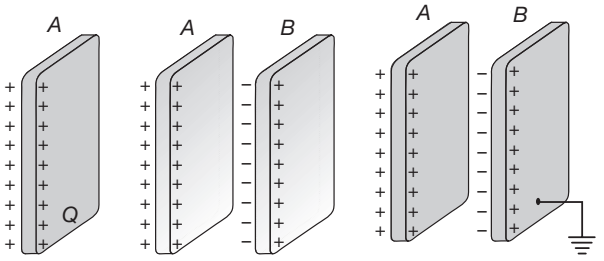


Figure 1(a) Figure 1(b) Figure 1(c)

If V_- is the potential at A due to induced negative charge on B and V_+ is the potential at A due to induced positive charge on B, then

$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since $V' < V$ (as the induced negative charge lies closer to the plate A in comparison to induced positive charge).

$\Rightarrow \quad C' > C$

Further, if B is earthed from the outer side (as shown in Figure 1(c)) then $V'' = V - V_-$ as the entire positive charge flows to the earth. So

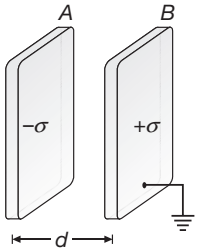
$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-}$$

$\Rightarrow \quad C'' \gg C$

So, if an identical earthed conductor is placed in the viscinity of a charged conductor then the capacitance of the charged conductor increases appreciably. This is the principle of a parallel plate capacitor.

PARALLEL PLATE CAPACITOR

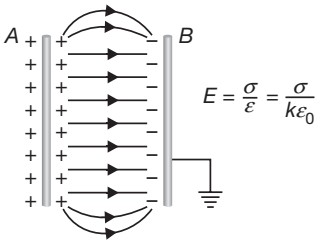
It consists of two metallic plates A and B each of area A at separation d. Plate A is positively charged and plate B is earthed. If K is the dielectric constant of the material medium and E is the field that exists between the two plates, then



A = Area of plate
d = Separation between the plates.

$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{K\varepsilon_0}$$

$$\Rightarrow \quad \frac{V}{d} = \frac{q}{K\varepsilon_0 A} \qquad \left\{\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right\}$$



$$\Rightarrow \quad C = \frac{q}{V} = \frac{K\varepsilon_0 A}{d}$$

If medium between the plates is air or vacuum, then

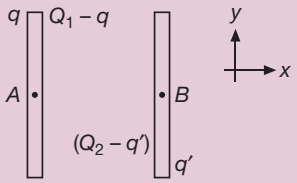
$$K = 1$$

$$\Rightarrow \quad C_0 = \frac{\varepsilon_0 A}{d}$$

Remark(s)

Suppose charges Q_1 and Q_2 are given to the two plates. The resulting charge distribution on the various surfaces is shown in the figure and is derived using the concept that no field exists in the thickness of the conductor.

$$E_A = 0$$



$$\Rightarrow \quad \frac{1}{2\varepsilon_0 A}\left[q - (Q_1 - q) - (Q_2 - q') - q'\right] = 0$$

$$\Rightarrow \quad q - Q_1 + q - Q_2 + q' - q' = 0$$

$$\Rightarrow \quad 2q - Q_1 - Q_2 = 0$$

$$\Rightarrow \quad q = \frac{Q_1 + Q_2}{2}$$

Similarly $E_B = 0$

$$\Rightarrow \frac{1}{2\varepsilon_0 A}\left[q + (Q_1 - q) + (Q_2 - q') - q'\right] = 0$$

$$\Rightarrow Q_1 + Q_2 - 2q' = 0$$

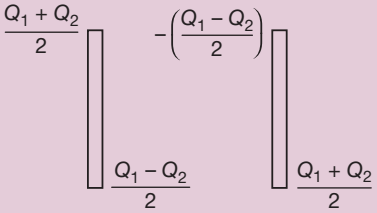
$$\Rightarrow q' = \frac{Q_1 + Q_2}{2}$$

$$\Rightarrow Q_1 - q = \frac{2Q_1 - Q_1 - Q_2}{2} = \frac{Q_1 - Q_2}{2}$$

and $Q_2 - q' = Q_2 - \left(\frac{Q_1 + Q_2}{2}\right)$

$$\Rightarrow Q_2 - q' = \frac{2Q_2 - Q_1 - Q_2}{2} = \frac{Q_2 - Q_1}{2} = -\left(\frac{Q_1 - Q_2}{2}\right)$$

So, the distribution of charges is as shown below



The charges on the inner surfaces are equal and opposite and the field between the plates is,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{(q_1 - q_2)}{2A\varepsilon_0}$$

Thus, the potential difference between the plates is,

$$V = Ed = \frac{(q_1 - q_2)d}{2A\varepsilon_0}$$

In order to find $C = \frac{q}{V}$ we must now use the value $\frac{(q_1 - q_2)}{2}$ for q because this is the charge responsible for creating the field between the two plates. The outer charges $\frac{(q_1 + q_2)}{2}$ do not come in the picture.

Thus, we get $C = \frac{\varepsilon_0 A}{d}$ as before.

ELECTROSTATIC FORCE BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

The plates of the capacitor each carry equal and opposite charges, hence they must attract each other with a force, say F.

At any instant let the plate separation be x, then

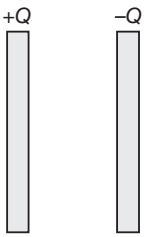
$$C = \frac{\varepsilon_0 A}{x}$$

Also $U = \frac{Q^2}{2C}$

$$\Rightarrow \quad U = \left(\frac{Q^2}{2\varepsilon_0 A}\right)x$$

Let the plates be moved towards each other through dx, such that the new separation between the plates is $(x - dx)$. If U_f is the final potential energy, then

$$U_f = \frac{Q^2}{2C'} = \frac{Q^2}{2\varepsilon_0 A}(x - dx)$$



If dU is the change in potential energy, then

$$dU = U_f - U_i$$

$$\Rightarrow \quad dU = \frac{Q^2}{2\varepsilon_0 A}(x - dx) - \frac{Q^2}{2\varepsilon_0 A}x$$

$$\Rightarrow \quad dU = -\frac{Q^2}{2\varepsilon_0 A}dx$$

Further since

$$F = -\frac{dU}{dx}$$

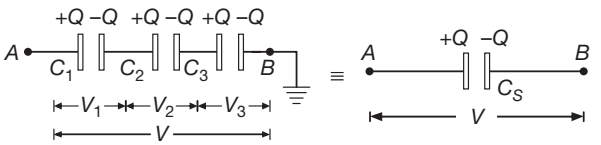
$$\Rightarrow \quad F = \frac{Q^2}{2\varepsilon_0 A} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)A = \left(\frac{1}{2}\varepsilon_0 E^2\right)A$$

$$\left\{\because Q = \sigma A,\ E = \frac{\sigma}{\varepsilon_0}\right\}$$

From here too, we observe that electrostatic pressure

$$P_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$$

CAPACITORS IN SERIES



In this arrangement of capacitors the charge has no alternative path(s) to flow.

(a) The charges on each capacitor are equal

i.e., $Q = C_1V_1 = C_2V_2 = C_3V_3$ …(1)

(b) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances.

$$V = V_1 + V_2 + V_3 \quad \ldots(2)$$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Further $V_1 = \dfrac{Q}{C_1}$ and $V = \dfrac{Q}{C_S}$

$$\Rightarrow \frac{V_1}{V} = \frac{\frac{1}{C_1}}{\frac{1}{C_S}}$$

$$\Rightarrow V_1 = \left(\frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right)V = \left(\frac{C_S}{C_1}\right)V$$

Similarly, $V_2 = \left(\dfrac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right)V = \left(\dfrac{C_S}{C_2}\right)V$

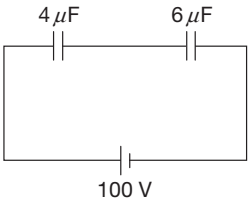
and $V_3 = \left(\dfrac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right)V = \left(\dfrac{C_S}{C_3}\right)V$

(c) C_S has a value smaller than the least capacitance of the circuit, provided all capacitors are connected in series.

ILLUSTRATION 8

In the circuit shown in figure, find

(a) the equivalent capacitance
(b) the charge stored in each capacitor and
(c) the potential difference across each capacitor.

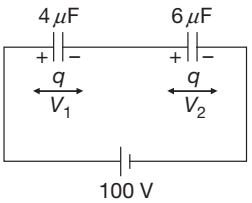


SOLUTION

(a) The equivalent capacitance

$$C = \frac{C_1C_2}{C_1 + C_2}$$

$$\Rightarrow \quad C = \frac{(4)(6)}{4+6} = 2.4\ \mu\text{F}$$



(b) The charge q, stored in each capacitor is,

$$q = CV = (2.4\times10^{-6})(100)\ \text{C}$$

$$\Rightarrow \quad q = 240\ \mu\text{C}$$

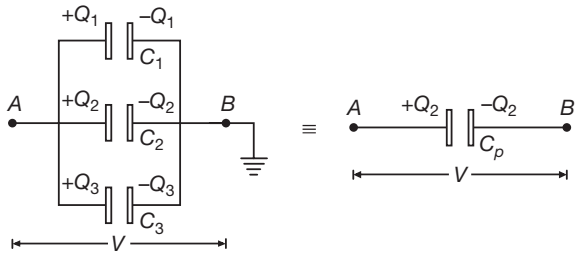
(c) In series combination, $V \propto \dfrac{1}{C}$ $\{\because\ q = \text{constant}\}$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\Rightarrow \quad V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V = \left(\frac{6}{4+6}\right)(100) = 60\ \text{V}$$

So, $V_2 = V - V_1 = 100 - 60 = 40\ \text{V}$

CAPACITORS IN PARALLEL



In such an arrangement of capacitors the charge has an alternative path(s) to flow.

(a) The potential difference across each capacitor is same and equals the total potential applied.

i.e., $V = V_1 = V_2 = V_3$ …(1)

$$\Rightarrow \quad V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} \qquad \ldots(2)$$

(b) The total charge Q is shared by each capacitor in the direct ratio of the capacitances.

$$Q = Q_1 + Q_2 + Q_3 \qquad \ldots(3)$$

If C_p is the net capacitance for the parallel combination of capacitors then

$$C_P V = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow \quad C_P = C_1 + C_2 + C_3$$

Further $Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$, $Q = C_P V$

$$\Rightarrow \quad \frac{Q_1}{Q} = \frac{C_1}{C_P}$$

$$\Rightarrow \quad Q_1 = \left(\frac{C_1}{C_P}\right) Q$$

Similarly $Q_2 = \left(\frac{C_2}{C_P}\right) Q$

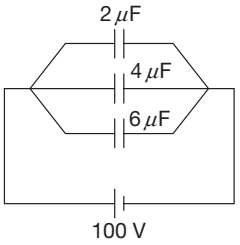
$$Q_3 = \left(\frac{C_3}{C_P}\right) Q$$

(c) C_P has a value greater than the greatest capacitance of the circuit.

ILLUSTRATION 9

In the circuit shown in figure, find

(a) the equivalent capacitance and
(b) the charge stored in each capacitor.



SOLUTION

(a) The capacitors are in parallel. Hence the equivalent capacitance is,

$$C = C_1 + C_2 + C_3$$

$$\Rightarrow \quad C = (2 + 4 + 6) = 12\ \mu\text{F}$$

(b) Total charge drawn from the battery

$$q = CV = 12 \times 100\ \mu\text{C} = 1200\ \mu\text{C}$$

This charge will be distributed in the ratio of their capacitances. Hence,

$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 2 : 4 : 6$$

$$\Rightarrow \quad q_1 = \left(\frac{2}{2+4+6}\right) \times 1200 = 200\ \mu\text{C}$$

$$\Rightarrow \quad q_2 = \left(\frac{4}{2+4+6}\right) \times 1200 = 400\ \mu\text{C}$$

and $q_3 = \left(\frac{6}{2+4+6}\right) \times 1200 = 600\ \mu\text{C}$

Problem Solving Technique(s)

(a) If C_1, C_2, C_3,... are capacitors connected in series and if total potential across all is V, then potential across each capacitor is

$$V_1 = \left(\frac{\frac{1}{C_1}}{\frac{1}{C_S}}\right) V;\ V_2 = \left(\frac{\frac{1}{C_2}}{\frac{1}{C_S}}\right) V;\ V_3 = \left(\frac{\frac{1}{C_3}}{\frac{1}{C_S}}\right) V$$

and so on, where $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$

(b) If C_1, C_2, C_3,... are capacitors connected in parallel and if Q is total charge on the combination, then charge on each capacitor is

$$Q_1 = \left(\frac{C_1}{C_P}\right)Q;\ Q_2 = \left(\frac{C_2}{C_P}\right)Q;\ Q_3 = \left(\frac{C_3}{C_P}\right)Q$$

and so on, where $C_P = C_1 + C_2 + C_3 + ... + C_n$

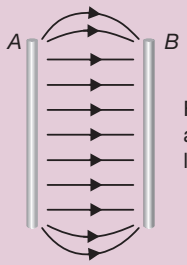
(c) The current through a capacitor is zero as long as the voltage across it does not change with time (or as long as capacitor is in the steady state i.e., charge on the capacitor has build up to attain a constant maximum value).

(d) It is not possible to change the voltage across a capacitor by a finite amount in zero time as this process requires an infinite amount of current to flow through the capacitor.

(e) A capacitor can store a finite amount of energy in it even if the current flowing through it is zero when voltage across the capacitor is constant.

(f) A true mathematical (theoretical) model of a capacitor should never dissipate energy but can only store it whereas a physical (practical) model never does so.

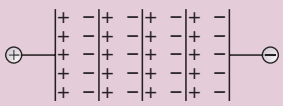
(g) A parallel plate capacitor must have plates of large area (A) in comparison to the distance of separation (d) between the plates to avoid **Fringing Effect**. Fringing Effect is the bending of field lines at the corners of a capacitors with large d (see figure).



Fringing effect for a capacitor with large d.

(h) n plates arranged as shown in figure constitute $(n-1)$ capacitors in parallel, each of value $\left(\frac{\varepsilon_0 A}{d}\right)$, so that

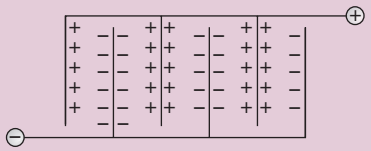
$$C_P = (n-1)\frac{\varepsilon_0 A}{d}$$



In this situation except two extreme plates each plate is common to two adjacent capacitors.

(i) n plates arranged as shown in figure constitute $(n-1)$ capacitors in series each of value $\left(\frac{\varepsilon_0 A}{d}\right)$, so that

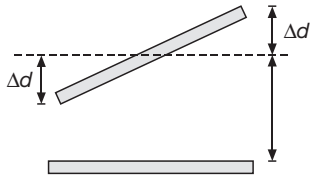
$$C_S = \frac{\varepsilon_0 A}{d(n-1)}$$



In this situation except two extreme plates each plate is common to two adjacent capacitors.

ILLUSTRATION 10

An air-dielectric capacitor is formed by two non-parallel plates, each of area A. An edge view of the arrangement is shown in figure. Note that the top plate is tilted relative to the bottom plate so that on one edge the plate separation is $d + \Delta d$, while on the other edge it is $d - \Delta d$. Assuming that $\Delta d \ll d$ and that d is small compared with the length of the plate, show that $C = \frac{\varepsilon_0 A}{d}\left[1 + \frac{1}{3}\left(\frac{\Delta d}{d}\right)^2\right]$.



SOLUTION

Consider an infinitesimal element of thickness dx at a distance x from O.

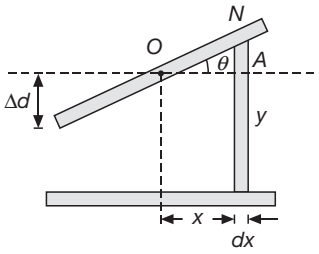
Then the capacitor is thought of being made from such infinitesimal elements having their capacitances in parallel. If dC be the capacitance of such an element, then

$$dC = \frac{\varepsilon_0 (dA)}{y}$$

Since, $\Delta d \ll d$, so

$$AN \cong x\theta$$

$$\Rightarrow \quad dC = \frac{\varepsilon_0 (b\,dx)}{d + x\theta}$$



where b = breadth of the plate and l = length of the plate

So, total capacitance C is given by

C = sum of capacitances of infinitesimal elements

$$\Rightarrow \quad C = \int dC = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{\varepsilon_0 b\,dx}{d + x\theta}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \log_e (d + x\theta) \Big|_{-\frac{l}{2}}^{\frac{l}{2}}$$

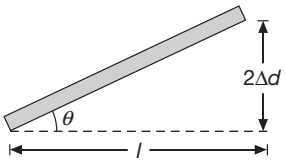
$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \left[\log_e \left(d + \frac{l\theta}{2} \right) - \log_e \left(d - \frac{l\theta}{2} \right) \right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \left[\log \left(1 + \frac{l\theta}{2d} \right) - \log \left(1 - \frac{l\theta}{2d} \right) \right]$$

Using $\log_e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\ldots$

and $\log_e (1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\ldots \right)$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \left\{ \left[\frac{l\theta}{2d} - \frac{1}{2}\left(\frac{l\theta}{2d} \right)^2 + \frac{1}{3}\left(\frac{l\theta}{2d} \right)^3 + \ldots. \right] + \left[\frac{l\theta}{2d} + \frac{1}{2}\left(\frac{l\theta}{2d} \right)^2 + \frac{1}{3}\left(\frac{l\theta}{2d} \right)^3 + \ldots.. \right] \right\}$$



$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \left[2\left(\frac{l\theta}{2d} \right) + 2\frac{1}{3}\left(\frac{l\theta}{2d} \right)^3 \right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{\theta} \left[\frac{l\theta}{d} + \frac{1}{12} \frac{l^3\theta^3}{d^3} \right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 lb}{d} \left[1 + \frac{1}{12} \frac{l^2}{d^2} \theta^2 \right]$$

But $\frac{\varepsilon_0 lb}{d} = \frac{\varepsilon_0 A}{d}$

and $\theta \simeq \frac{2\Delta d}{l}$

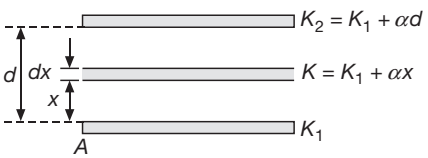
$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d} \left[1 + \frac{1}{12} \frac{l^2}{d^2} \left(\frac{4(\Delta d)^2}{l^2} \right) \right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d} \left[1 + \frac{1}{3}\left(\frac{\Delta d}{d} \right)^2 \right]$$

ILLUSTRATION 11

A parallel plate square capacitor has the space between the plates filled with a medium whose dielectric constant increases uniformly with distance x from one of its plate as $K = K_1 + \alpha x$. If d is distance between the plates and K_1 and K_2 are dielectric constants of the medium at the two square plates, find the capacity of the capacitor.

SOLUTION



Since $K = K_1 + \alpha x$

and at $x = d$, $K = K_2$

$$\Rightarrow \quad \alpha = \frac{K_2 - K_1}{d}$$

The capacitance of the elementary capacitor is

$$dC = \frac{A\varepsilon_0 (K_1 + \alpha x)}{dx}$$

Since these elementary capacitors are joined in series, so

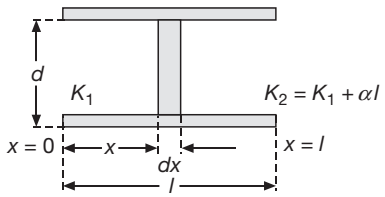
$$\frac{1}{C} = \int \frac{1}{dC} = \frac{1}{A\varepsilon_0}\int_0^d \frac{dx}{(K_1 + \alpha x)} = \frac{1}{A\varepsilon_0}\left(\frac{\ln(K_1 + \alpha x)}{\alpha}\right)\Bigg|_0^d$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{A\varepsilon_0 \alpha}\ln\left(\frac{K_1 + \alpha d}{K_1}\right) = \frac{d}{A\varepsilon_0 (K_2 - K_1)}\ln\left(\frac{K_2}{K_1}\right)$$

$$\Rightarrow \quad C = \frac{A\varepsilon_0 (K_2 - K_1)}{d\ln\left(\frac{K_2}{K_1}\right)}$$

ILLUSTRATION 12

A parallel plate square capacitor has the space between the plates filled with a dielectric whose dielectric constant increases linearly with distance x from one edge to the other as $K = K_1 + \alpha x$, where K_1 and K_2 are dielectric constants of the medium at the two edges of the square plates, find the capacity of the capacitor.



SOLUTION

Since $K = K_1 + \alpha x$ and at $x = l$, we have

$$K = K_2$$

$$\Rightarrow \quad \alpha = \frac{K_2 - K_1}{l}$$

The capacitance of the elementary capacitor is

$$dC = \frac{\varepsilon_0 (ldx)(K_1 + \alpha x)}{d}$$

Since, these elementary capacitors are joined in parallel, so

$$C = \int dC = \frac{\varepsilon_0 l}{d}\int_0^l (K_l + \alpha x)\,dx = \frac{\varepsilon_0 l^2}{d}\left(K_1 + \frac{\alpha l}{2}\right)$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 l^2}{d}\left[K_1 + \frac{1}{2}\left(\frac{K_2 - K_1}{l}\right)l\right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

ELECTROSTATIC ENERGY DENSITY (u_E)

For a parallel plate capacitor, we have

$$C = \frac{\varepsilon_0 A}{d} \text{ and } V = Ed, \quad \text{where } E = \frac{\sigma}{\varepsilon_0}$$

Since, $U = \frac{1}{2}CV^2$

$$\Rightarrow \quad U = \frac{1}{2}\frac{\varepsilon_0 A}{d}E^2 d^2$$

$$\Rightarrow \quad U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0 E^2 \tau$$

where $\tau (= Ad)$ is volume of the capacitor

$$\Rightarrow \quad \frac{U}{\tau} = u_E = \frac{\text{Electrostatic Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$$

This energy is stored in the capacitor in the form of electrostatic field. Since,

$$\frac{\text{Electrostatic Energy}}{\text{Volume}} = \text{Electrostatic Pressure}$$

$$\Rightarrow \quad \text{Electrostatic Pressure} = \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0} = u_E$$

Also we note that if we take $d\tau$ as the volume element, then

$$u_E = \int u_E d\tau$$

I, have taken a new symbol (τ) for the volume, because we may confuse V for potential.

Remark(s)

The potential energy U of a charged conductor or a capacitor is stored in the electric field. The energy per unit volume is called the energy density (u_E). Energy density (u_E) in a dielectric medium is given by,

$$u_E = \frac{1}{2}\varepsilon_0 K E^2$$

This relation shows that the energy stored per unit volume depends on E^2. If E is the electric field in a space of volume $d\tau$, then the total stored energy in an electrostatic field is given by,

$$U = \frac{1}{2}\varepsilon_0 K \int E^2 \, d\tau$$

and if E is uniform throughout the volume (electric field between the plates of a capacitor is uniform), then the total stored energy can be given by,

$$U = \frac{1}{2} K \varepsilon_0 E^2 \tau$$

ENERGY FOR SERIES AND PARALLEL COMBINATIONS

Series Combination

For a series combination of capacitors Q = constant and

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$\Rightarrow \quad \frac{Q^2}{2C_S} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \ldots$$

$$\Rightarrow \quad U_S = U_1 + U_2 + U_3 + \ldots$$

i.e., if U_1 is energy stored in capacitor C_1, if U_2 is energy stored in capacitor C_2 and so on, and U_S is total energy for series combination of C_1, C_2, C_3,… then,

$$\begin{pmatrix}\text{Total Energy}\\ \text{for the series}\\ \text{combination}\end{pmatrix} = \begin{pmatrix}\text{Sum of individual}\\ \text{energies stored in}\\ \text{each capacitor}\end{pmatrix}$$

Parallel Combination

For a parallel combination of capacitors V =constant and

$$C_P = C_1 + C_2 + C_3 + \ldots$$

$$\Rightarrow \quad \frac{1}{2}C_P V^2 = \frac{1}{2}C_1 V^2 + \frac{1}{2}C_2 V^2 + \frac{1}{2}C_3 V^2 + \ldots$$

$$\Rightarrow \quad U_P = U_1 + U_2 + U_3 + \ldots$$

i.e., if U_1 is energy stored in capacitor C_1, if U_2 is energy stored in capacitor C_2 and so on, and U_P is total energy for parallel combination of C_1, C_2, C_3,… then too

$$\begin{pmatrix}\text{Total Energy}\\ \text{for the parallel}\\ \text{combination}\end{pmatrix} = \begin{pmatrix}\text{Sum of individual}\\ \text{energy stored in}\\ \text{each capacitor}\end{pmatrix}$$

Problem Solving Technique(s)

So, to conclude, if capacitors are connected in series or in parallel the total energy across the combination (any one) is equal to the sum of the individual energies stored in each capacitor.

CALCULATING THE NET CAPACITANCE OF CIRCUITS

To calculate the net capacitance of a circuit/network, the steps given below should be followed.

STEP-1: Identify the two points across which the equivalent capacitance is to be calculated.

STEP-2: Imagine a battery to be connected between these points.

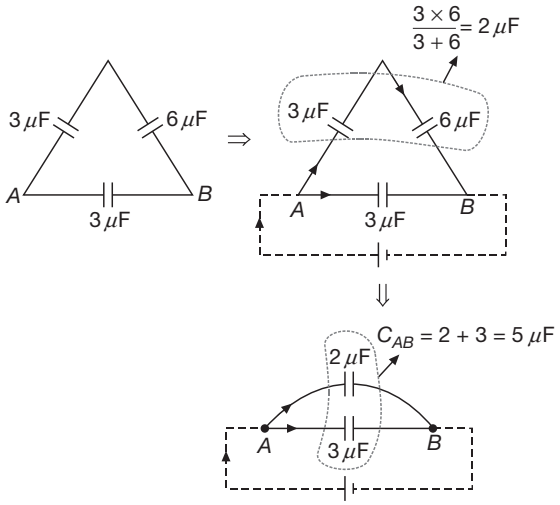
STEP-3: Start solving the circuit from the reference point which is farthest from the points between which the equivalent capacitance has to be calculated. This point is likely to be not a node.

SIMPLE CIRCUITS

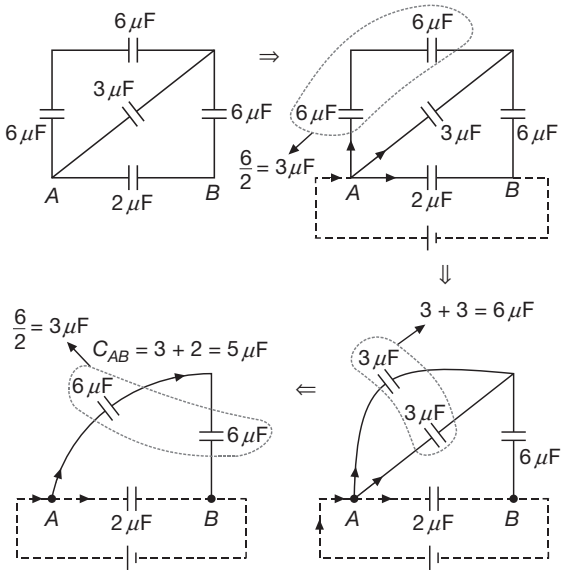
Analyse the circuit carefully to conclude which pair of capacitors are in series and which are in parallel (This all should be done keeping in mind the points across which net capacitance has to be calculated).

Find their net capacitance and again draw an equivalent diagram to apply the above specified technique repeatedly so as to get the total capacitance between the points A and B as Illustrated below.

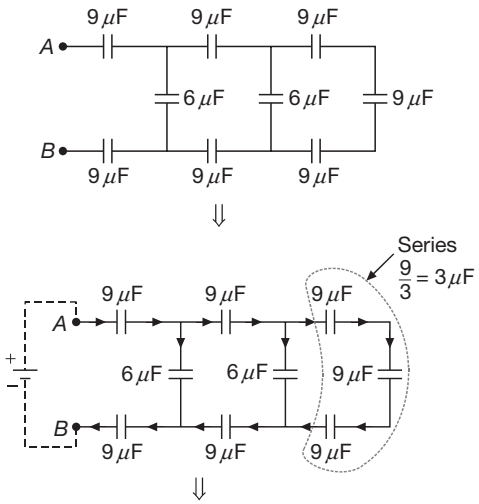
(i)



(ii)



(iii)



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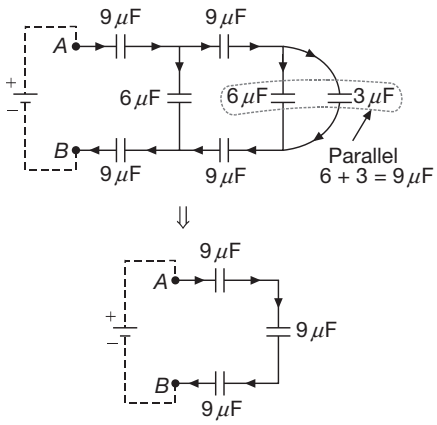
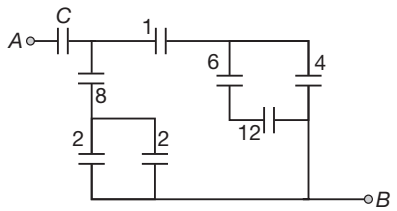


By similar process $C_{AB} = 3\ \mu F$

ILLUSTRATION 13

Find C if the equivalent capacitance between the points A and B in the circuit shown is 1 μF. All the capacitances are in μF.



SOLUTION

After a brief simple analysis, we conclude the following information

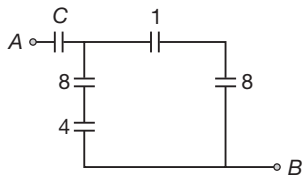
(a) 2 and 2 are in parallel to give $4\,\mu F$

(b) 6 and 12 are in series to give $\frac{6\times12}{6+12} = 4\ \mu F$

(c) This 4 μF (net capacitance of 6 and 12) is in parallel combination to another 4, so as to give us 8 μF

So, using the above information, we redraw the above circuit

Further, we conclude



(d) 1 and 8 are in series, so we get $\frac{1\times 8}{1+8}=\frac{8}{9}\ \mu F$

(e) 8 and 4 are in series, so we get $\frac{8\times 4}{8+4}=\frac{32}{12}\ \mu F$

(f) $\frac{8}{9}\ \mu F$ and $\frac{32}{12}\ \mu F$ are in a parallel combination to give net capacitance $\frac{8}{9}+\frac{32}{12}=\frac{32}{9}\ \mu F$

Finally $\frac{32}{9}\ \mu F$ and C are in series between A and B, so as to give $1\ \mu F$. So, the final equivalent circuit is shown in figure.

$$\Rightarrow \quad \frac{\frac{32}{9}C}{\frac{32}{9}+C}=1$$

$$\Rightarrow \quad 32C=32+9C$$

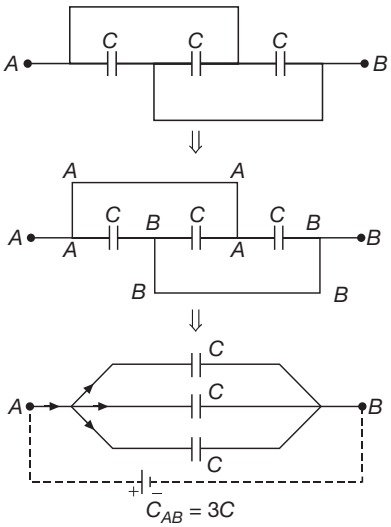
$$\Rightarrow \quad 23C=32$$

$$\Rightarrow \quad C=\frac{32}{23}\ \mu F$$

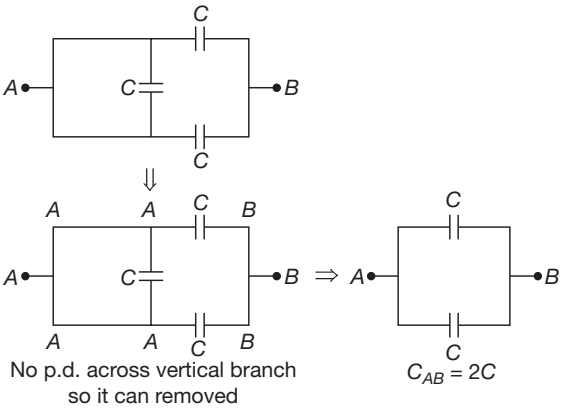
CIRCUITS WITH EXTRA WIRES

If there is no capacitor or resistor in any branch of a circuit, then every point of this branch will be at same potential. Suppose equivalent capacitance is to be determined in following cases.

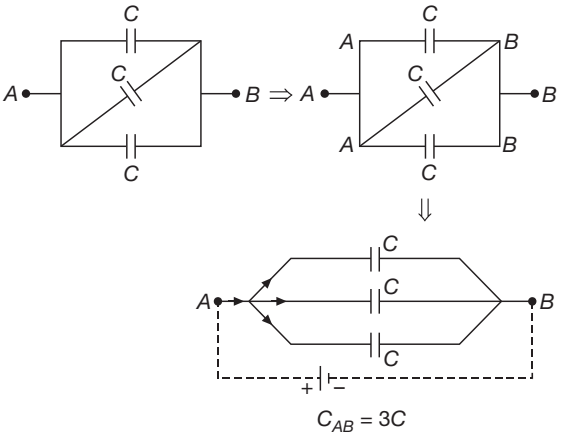
(i)



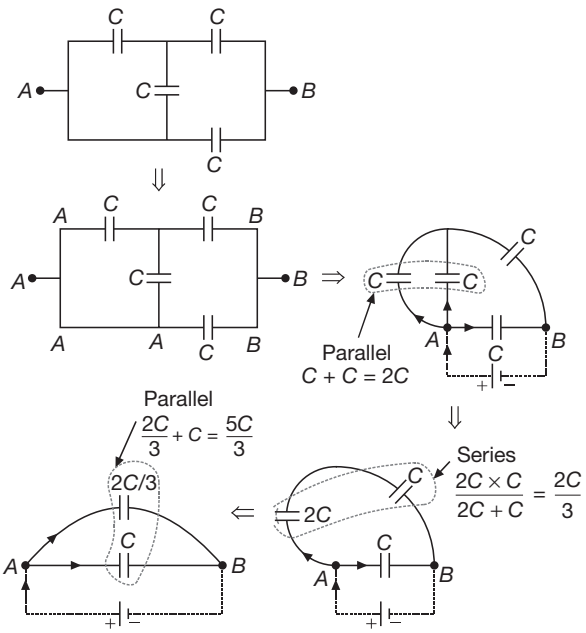
(ii)



(iii)

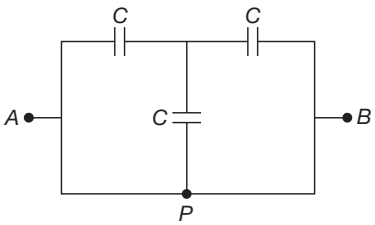


(iv)



Hence equivalent capacitance between A and B is $\frac{5C}{3}$.

(v) Since there is no capacitor in the path APB, the points A, P and B are electrically same i.e., the input and output points are directly connected (short circuited).



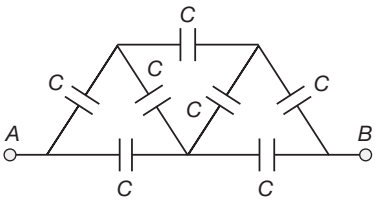
Thus, entire charge will prefer to flow along path APB. It means that the capacitors connected in the circuit will not receive any charge for storing. Thus equivalent capacitance of this circuit is zero.

CONCEPT OF LINE OF SYMMETRY

Line of symmetry (L.O.S.) is an imagination of our mind to divide a highly symmetric circuit into two equal halves such that the points of the circuit through which LOS passes are at equal potential.

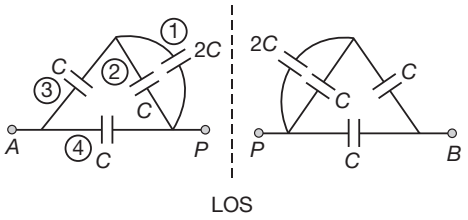
ILLUSTRATION 14

Find the net capacitance of the circuit shown between the points A and B.



SOLUTION

This circuit is highly symmetric and so we can consider the line of symmetry to pass through the circuit to divide it into two equal (identical) halves. If line of symmetry passes through a branch possessing a capacitor, then on each side of Line of Symmetry (LOS) the capacitance will become $2C$ ($2C$ and $2C$ in series will gives C), as shown.



Now, the concept of Line of Symmetry makes our job easy to calculate capacitance across AP. (1) and (2) are in parallel further in series with (3), whose resultant capacitance is in parallel with (4).

Resultant of (1) and (2) is $3C$

Resultant of $3C$ and (3) is $\frac{3C}{4}$.

Resultant of $\frac{3C}{4}$ and (4) is $\frac{7C}{4}$.

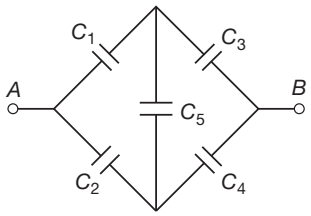
So total capacitance across AB is

$$C_{AB} = \frac{C_{AP}}{2}$$

$$\Rightarrow \quad C_{AB} = \frac{7C}{8}$$

BALANCED WHEATSTONE BRIDGE

If in a circuit, five capacitors are arranged as shown in following figure, the circuit is called Wheatstone Bridge type circuit.



It is called a Balanced Wheatstone Bridge (BWSB), when

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

In this situation we observe that no charge flows through C_5 and hence C_5 can be removed from the circuit to get the equivalent capacitance between A and B.

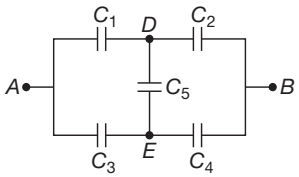
Conceptual Note(s)

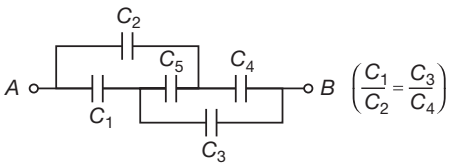
If $\frac{C_1}{C_2}=\frac{C_3}{C_4}$, then no charge exists in the branch containing C_5 (also called fifth branch) and hence it can be omitted (just neglect it as if it was never present).

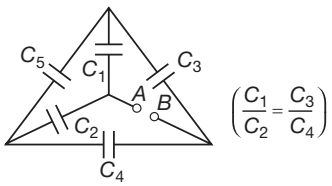
As a consequence of this we can say that C_1 and C_3 are in series to give $\frac{C_1C_3}{C_1+C_3}$, C_2 and C_4 are in series to give $\frac{C_2C_4}{C_2+C_4}$ and both in parallel. So, net capacitance

$$C_{net}=\frac{C_1C_3}{C_1+C_3}+\frac{C_2C_4}{C_2+C_4}$$

Other shapes of Balanced Wheatstone Bridge are indicated below.

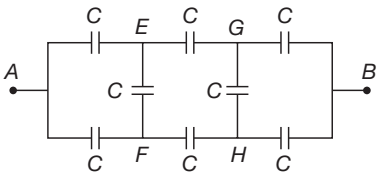






EXTENDED WHEATSTONE BRIDGE

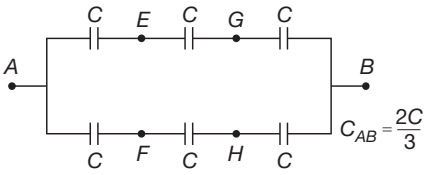
The given figure consists of two Wheatstone Bridges connected together. One bridge is connected between points $AEGHFA$ and the other is connected between points $EGBHFE$.



This circuit is known as Extended Wheatstone Bridge and it has two branches EF and GH to the left and right of which symmetry in the ratio of capacitances can be seen.

$$\frac{C}{C}=\frac{C}{C}=\frac{C}{C}=1$$

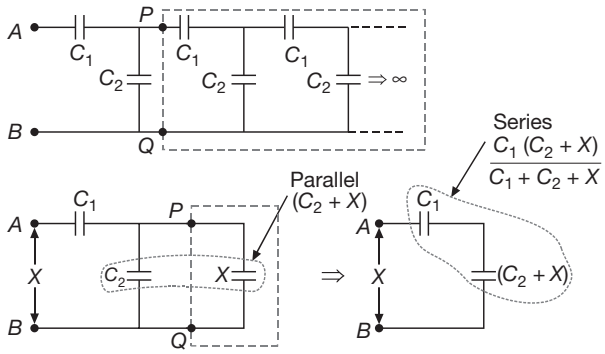
It can be seen that ratio of capacitances in branches AE and EG is same as that between the capacitances of the branches AF and FH. Thus, in the bridge $AEGHFA$; the branch EF can be removed. Similarly in the bridge $EGBHFE$ branch GH can be removed.



INFINITE CHAIN OF CAPACITORS

In the following infinite circuits, the equivalent capacitance between A and B is to be calculated.

Suppose the effective capacitance between A and B is X. Since the network is infinite, so even if we remove one repetitive unit of capacitors from the chain, then too the remaining network would still have infinite pair of capacitors, i.e., effective capacitance between P and Q will still be X



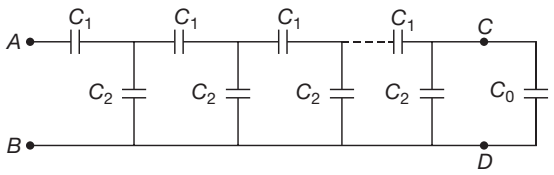
Hence equivalent capacitance between A and B

$$C_{AB} = \frac{C_1(C_2 + X)}{C_1 + C_2 + X} = X$$

$$\Rightarrow \quad C_{AB} = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

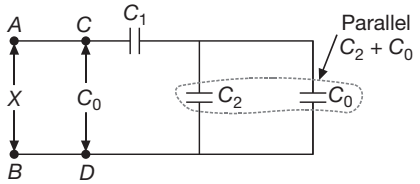
ILLUSTRATION 15

For what value of C_0 in the circuit shown below will the net effective capacitance between A and B be independent of the number of sections in the chain?



SOLUTION

Suppose there are n sections between A and B and the network is terminated by C_0 with equivalent capacitance X.



Now if we add one more sections to the network between D and C (as shown in figure), the equivalent capacitance of the network X will be independent of number of sections if the capacitance between D and C still remains C_0 i.e.,

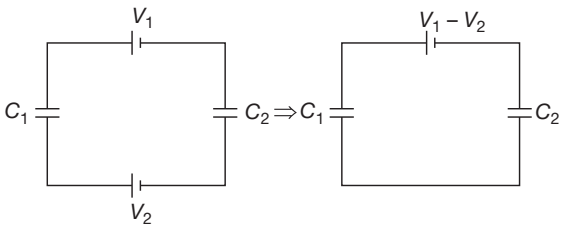
$$C_0 = \frac{C_1 \times (C_2 + C_0)}{C_1 + C_2 + C_0}$$

$$\Rightarrow \quad C_0^2 + C_2C_0 - C_1C_2 = 0$$

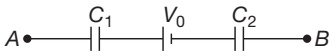
On simplification $C_0 = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$

NETWORK WITH MORE THAN ONE CELL

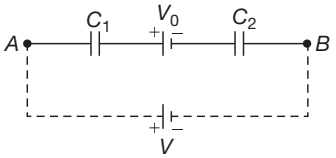
(a) Potential difference across C_1 is $\left(\frac{C_2}{C_1 + C_2}\right)$ $(V_1 - V_2)$ and potential difference across C_2 is $\left(\frac{C_1}{C_1 + C_2}\right)(V_1 - V_2)$



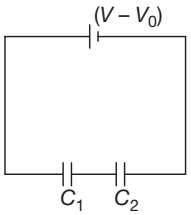
(b) Consider the arrangement shown.



This arrangement is connected across a battery of voltage $V(> V_0)$ as shown.



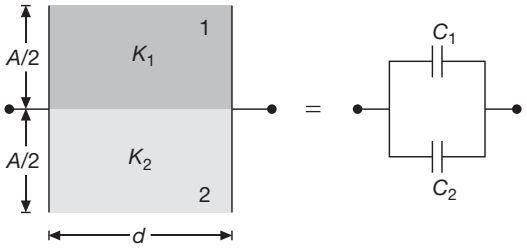
Then, potential difference between the ends of this arrangement is $(V - V_0)$.



CASE OF COMPOUND DIELECTRICS

If several dielectric medium filled between the plates of a parallel plate capacitor in different ways as shown.

(a) In the arrangement shown, these two capacitors are in parallel and



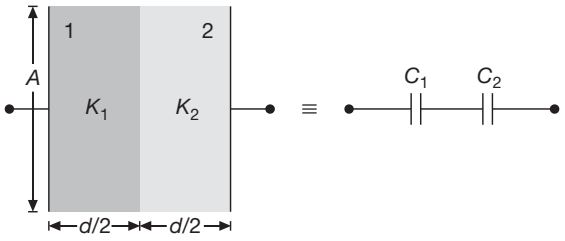
$$C_1 = \frac{K_1\varepsilon_0 A}{2d} \text{ and } C_2 = \frac{K_2\varepsilon_0 A}{2d}$$

Since, $C_{eq} = C_1 + C_2$

$$\Rightarrow \quad C_{eq} = \left(\frac{K_1 + K_2}{2}\right).\frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad K_{eq} = \frac{K_1 + K_2}{2}$$

(b) The system can be assumed to be made up of two capacitors C_1 and C_2 which may be said to connected in series



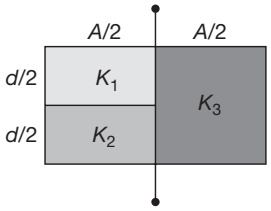
$$C_1 = \frac{K_1\varepsilon_0 A}{d/2},\ C_2 = \frac{K_2\varepsilon_0 A}{d/2}$$

Since, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

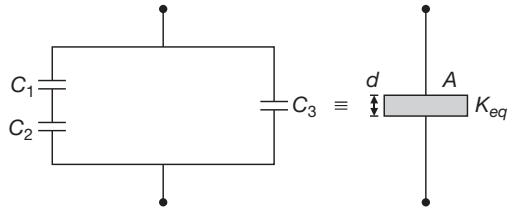
$$\Rightarrow \quad C_{eq} = \left(\frac{2K_1K_2}{K_1 + K_2}\right)\frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

(c)



The equivalent capacitor circuit diagram for this arrangement is shown in figure.



where $C_1 = K_1\frac{\varepsilon_0(A/2)}{(d/2)} = K_1\left(\frac{\varepsilon_0 A}{d}\right)$

$$C_2 = K_2\frac{\varepsilon_0(A/2)}{d/2} = K_2\left(\frac{\varepsilon_0 A}{d}\right)$$

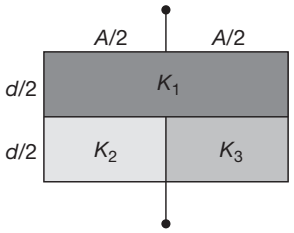
$$C_3 = K_3\frac{\varepsilon_0(A/2)}{d} = K_3\left(\frac{\varepsilon_0 A}{2d}\right)$$

So, $C_{eq} = \frac{C_1C_2}{C_1 + C_2} + C_3 = \frac{\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1 + K_2} + \frac{K_3}{2}\right)$

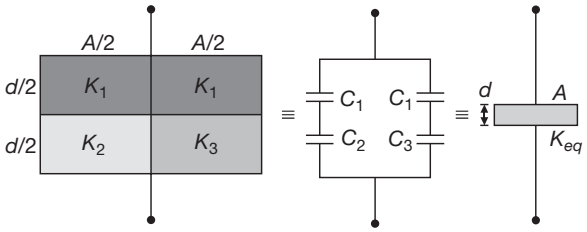
$$C_{eq} = K_{eq}\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow \quad K_{eq} = \frac{K_1K_2}{K_1 + K_2} + \frac{K_3}{2}$$

(d) To calculate equivalent capacitance in this case, we have



to proceed as follows by redrawing the arrangement



where $C_1 = K_1 \dfrac{\varepsilon_0 (A/2)}{(d/2)} = K_1 \left(\dfrac{\varepsilon_0 A}{d} \right)$

$$C_2 = K_2 \frac{\varepsilon_0 (A/2)}{(d/2)} = K_2 \left(\frac{\varepsilon_0 A}{d} \right)$$

$$C_3 = K_3 \frac{\varepsilon_0 (A/2)}{(d/2)} = K_3 \left(\frac{\varepsilon_0 A}{d} \right)$$

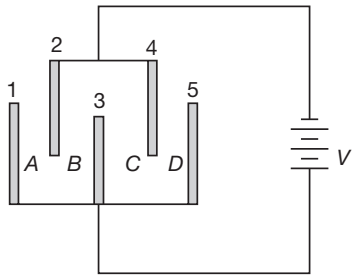
So, $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} + \dfrac{C_1 C_3}{C_1 + C_3}$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{d} \left[\frac{K_1 K_2}{K_1 + K_2} + \frac{K_1 K_3}{K_1 + K_3} \right] = K_{eq} \left(\frac{\varepsilon_0 A}{d} \right)$$

$$\Rightarrow \quad K_{eq} = \frac{K_1 K_2}{K_1 + K_2} + \frac{K_1 K_3}{K_1 + K_3}$$

ILLUSTRATION 16

Five identical capacitor plates each of area A are arranged in such a way that adjacent plates are at a distance d apart. The plates are connected to source of emf V. Calculate the magnitude and nature of charge on plate 1 and 4 respectively.



SOLUTION

These five plates constitute four identical capacitors in parallel each of capacitance C given by

$$C = \left(\frac{\varepsilon_0 A}{d} \right)$$

Now plate-1 connected to positive terminal of battery and part of one capacitor

$$q_1 = +\left(\frac{\varepsilon_0 AV}{d} \right)$$

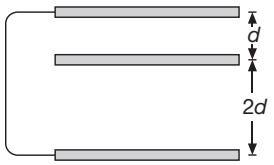
Since plate 4 is connected to negative terminal and is common between two capacitors in parallel, so

$$q_4 = q_C + q_D = -2q_1 = \left(-2\left[\frac{\varepsilon_0 AV}{d} \right] \right)$$

$$q_4 = -\left(\frac{2\varepsilon_0 AV}{d} \right)$$

ILLUSTRATION 17

Two large parallel metal plates are oriented horizontally and separated by a distance $3d$. A grounded conducting wire joins them, and initially each plate carries no charge. Now a third identical plate carrying charge Q is inserted between the two plates, parallel to them and located a distance d from the upper plate, as in Figure.



(a) What induced charge appears on each of the two original plates?

(b) What potential difference appears between the middle plate and each of the other plates? Each plate has area A.

SOLUTION

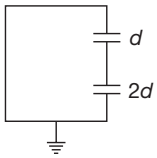
Imagine the centre plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same ΔV and carrying total charge Q.

The upper has a capacitance $C_1 = \dfrac{\varepsilon_0 A}{d}$ and the lower has a capacitance $C_2 = \dfrac{\varepsilon_0 A}{2d}$.

Charge flows from ground onto each of the outside plates of capacitors, so that

$$Q_1 + Q_2 = Q \qquad \ldots(1)$$

and $\Delta V_1 = \Delta V_2 = \Delta V$



$$\Rightarrow \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\Rightarrow \quad \frac{Q_1 d}{\varepsilon_0 A} = \frac{Q_2 2d}{\varepsilon_0 A}$$

$$\Rightarrow \quad Q_1 = 2Q_2$$

From (1), we get

$$2Q_2 + Q_2 = Q$$

$$\Rightarrow \quad Q_2 = \frac{Q}{3}$$

(a) $Q_2 = \frac{Q}{3}$

So, on the lower plate the charge is $-\frac{Q}{3}$

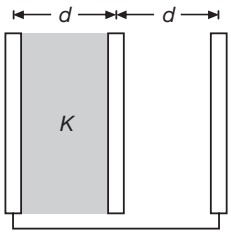
$$Q_1 = \frac{2Q}{3}$$

So, on the upper plate the charge is $-\frac{2Q}{3}$

(b) $\Delta V = \frac{Q_1}{C_1} = \frac{2Qd}{3\varepsilon_0 A}$

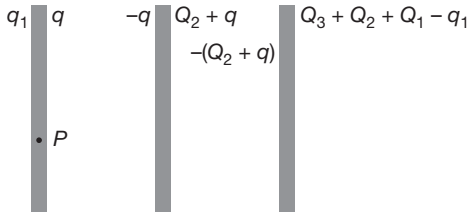
ILLUSTRATION 18

Three large conducting plates are placed a distance d apart in air. The space between the first two plates is completely filled with a dielectric slab of dielectric constant $K = 2$ as shown in Figure. The plates are given charges $Q_1 = 7Q$, $Q_2 = 3Q$ and $Q_3 = 2Q$ respectively. The outer two plates are now connected with a conducting wire. Find the charges on all the six surfaces.



SOLUTION

Charges on the surfaces facing each other will be equal and opposite (from Gauss's Law). Let the distribution be as shown in the following diagram and the charges on the six surfaces are as shown.



As electric field inside the conductor (let at point P) is zero.

$$q_1 = Q_3 + Q_2 + Q_1 - q_1$$

$$\Rightarrow \quad q_1 = \frac{Q_3 + Q_2 + Q_1}{2}$$

Since, the outer plates are connected hence, their potential will be same.

$$\Rightarrow \quad \frac{Q_2 + q}{\varepsilon_0 A} d + \frac{q}{K\varepsilon_0 A} d = 0$$

$$\Rightarrow \quad Q_2 + q + \frac{q}{K} = 0$$

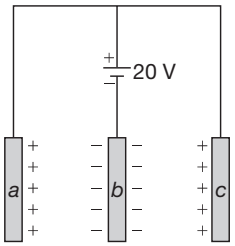
$$\Rightarrow \quad KQ_2 + (K+1)q = 0$$

$$\Rightarrow \quad q = \frac{-KQ_2}{K+1} = -2Q \qquad \{\because \ Q_2 = 3Q\}$$

So, the final charge configuration on the six surfaces will be as shown here.

ILLUSTRATION 19

Each of the three plates shown in Figure has $200\ \text{cm}^2$ area on one side, and the gap between the adjacent plates is 0.2 mm. The emf of the battery is 20 V. Calculate the distribution of charge on various surfaces of the plates and the equivalent capacitance of the system between the terminal points?



SOLUTION

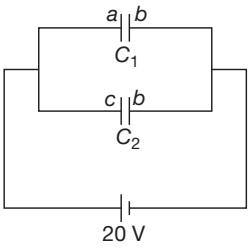
As the potentials of a and c are equal, the capacitors C_{ab} and C_{bc} are in parallel. Therefore, equivalent capacitance is

$$C = C_{ab} + C_{bc} = \frac{2\varepsilon_0 A}{d}$$

where, $C_{ab} = C_{bc} = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 2\times 10^{-2}}{0.2\times 10^{-3}} = 100\varepsilon_0$

$$\Rightarrow \quad C = \frac{2\times \varepsilon_0 \times 2\times 10^{-2}}{0.2\times 10^{-3}} = 200\varepsilon_0$$

The charge on plate b is negative on both faces. Thus, the charge on faces a and c is



$$q_a = q_c = \frac{\varepsilon_0 A}{d} V$$

$$\Rightarrow \quad q_a = q_c = 100\varepsilon_0 \times 20$$

$$\Rightarrow \quad q_a = q_c = 2000\varepsilon_0$$

$$\Rightarrow \quad Q = 2\times 2000\varepsilon_0$$

$$\Rightarrow \quad Q = 4000\varepsilon_0$$

Test Your Concepts-II

Based on Series and Parallel Combination of Capacitors

(Solutions on page H.127)

1. Two identical parallel-plate capacitors, each with capacitance C, are charged to potential difference ΔV and connected in parallel. Then the plate separation in one of the capacitors is doubled.

(a) Find the total energy of the system of two capacitors before the plate separation is doubled.

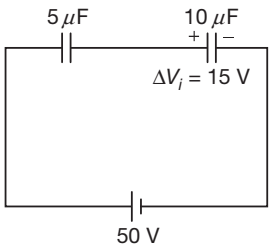
(b) Find the potential difference across each capacitor after the plate separation is doubled.

(c) Find the total energy of the system after the plate separation is doubled.

(d) Reconcile the difference in the answers to parts (a) and (c) with the Law of Conservation of Energy.

2. Two identical parallel plate capacitors are first connected in series and then in parallel. In each case, the plates of one capacitor are brought closer by a distance Δd and the plates of the other capacitor are moved away by same distance Δd. How does the total capacitance of the system change in both cases by the above displacement of plates?

3. A 10 μF capacitor is charged to 15 V. It is next connected in series with an uncharged 5 μF capacitor. The series combination is finally connected across a 50 V battery, as shown in Figure. Find the new potential differences across the 5 μF and 10 μF capacitors.



4. Each plate of a parallel plate capacitor has an area A. What amount of work has to be performed

to slowly increase the distance between the plates from x_1 to x_2, if

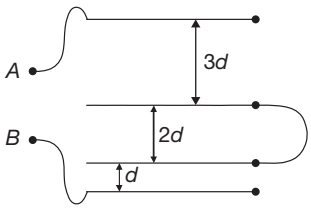
(a) the charge of the capacitor, q is kept constant in the process.

(b) the voltage across the capacitor, V, is kept constant in the process.

5. Two capacitors A and B are connected in series across a 100 V supply and it is observed that the potential difference across them are 60 V and 40 V. A capacitor of 2 μF capacitance is now connected in parallel with A and the potential difference across B rises to 90 V. Determine the capacitance of A and B.

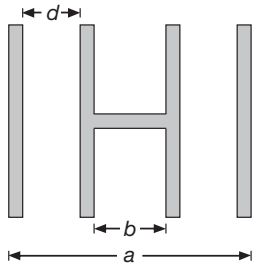
6. Two capacitors are in parallel and the energy of the combination is 0.1 J, when the differences of potential between terminals is 2 V. With the same two condensers now connected in series, the energy is 1.6×10^{-2} J for the same difference of potential across the series combination. Calculate the capacitances of the capacitors.

7. If the area of each plate is A and the successive separation are $3d$, $2d$ and d. Then find the equivalent capacitance across A and B.



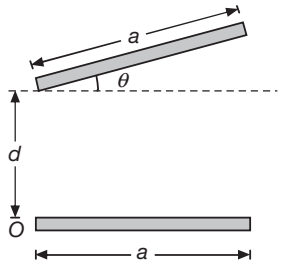
8. Two capacitors A and B each having a dielectric of dielectric constant 2 are connected in series. When they are connected across a 230 V D.C. supply it is found that the potential difference across A is 130 V and that across B is 100 V. If the dielectric in the smaller capacitor is replaced by another dielectric of dielectric constant 5, what will be the new values of the potential difference across each?

9. Two capacitors are joined in series shown in figure. The central rigid H-shaped part is movable. Find the equivalent capacitance of the combination and hence show that it is independent of position of the central H-shaped part. The area of each plate is A.

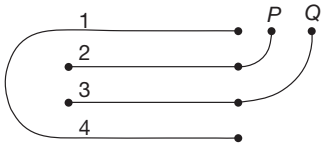


10. A capacitor has square plates, each of side a making an angle θ with each other as shown in figure. Show that for small θ, the capacitance C is given by

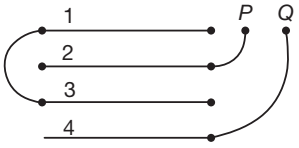
$$C=\frac{\varepsilon_0 a^2}{d}\left(1-\frac{a\theta}{2d}\right)$$



11. Find the effective capacitance of the following arrangement of four identical metallic plates between P and Q. The area of each plate is A and separation between successive plates is d.



ARRANGEMENT 1

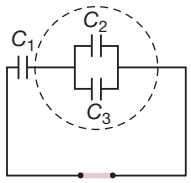


ARRANGEMENT 2

12. A radio capacitor of variable capacitance is made of n plates each of area A and separated from each other by a distance d. The alternate plates are connected together. One group of alternate plates is

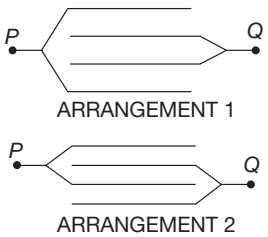
fixed while the other is movable. Find the maximum capacitance of the capacitor.

13. (a) Find the equivalent capacitance of the combination shown in figure, where $C_1 = 6\ \mu F$, $C_2 = 4\ \mu F$ and $C_3 = 8\ \mu F$.

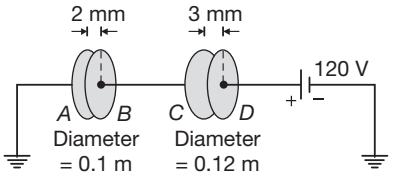


(b) If a battery is connected between a and b, determine the charge on each capacitor and the potential difference across each, if $V_{ab} = 12$ V.

14. Find the capacitance of the following combinations between P and Q. Area of the each plate is A and separation of successive plates is d.



15. The circular plates A and B of a parallel plate air capacitor have a diameter of 0.1 m and are 2 mm apart. The plates C and D, of a similar capacitor have a diameter 0.12 m and are 3 mm apart. Plate A is earthed and the plates B and D are connected together. Plate C is connected to the positive pole of a 120 V battery whose negative terminal is earthed. Calculate



(a) the combined capacitance of the arrangement and

(b) the energy stored in it.

Take $\frac{1}{\varepsilon_0} = 36\pi \times 10^9\ \text{Nm}^2\text{C}^{-2}$

16. Using the concept of energy density, find the total energy stored in a

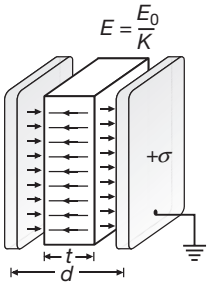
(a) parallel plate capacitor

(b) charged spherical conductor

DIELECTRIC SLAB INSERTED IN A PARALLEL PLATE CAPACITOR

When the space between the parallel plate capacitor is partly filled with a dielectric of thickness $t (< d)$

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d.



On inserting the slab of thickness t, a field $E = \frac{E_0}{K}$ exists inside the slab of thickness t and a field E_0 exists in remaining space $(d - t)$. If V is total potential then

$$V = E_0 (d - t) + Et$$

$$\Rightarrow \quad V = E_0 \left[d - t + \left(\frac{E}{E_0} \right) t \right]$$

But $\frac{E_0}{E} = K$ = Dielectric Constant

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0} \left[d - t + \frac{t}{K} \right]$$

$$\Rightarrow \quad V = \frac{q}{A\varepsilon_0} \left[d - t + \frac{t}{K} \right]$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

So, on introducing a dielectric slab of thickness t and dielectric constant K the capacitance increases by the same amount as the effective air spacing between the plates is made $t\left(1 - \frac{1}{K}\right)$

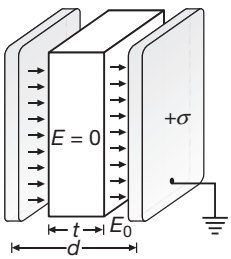
CONDUCTING SLAB INSERTED IN A PARALLEL PLATE CAPACITOR

When the space between the parallel plate capacitor is partly filled by a conducting slab of thickness $t (< d)$.

If no conducting slab is introduced between the plates, then a field $E_0 = \frac{\sigma}{\varepsilon_0}$ exists in a space d. If C_0 be the capacitance (without the introduction of conducting slab), then

$$C_0 = \frac{\varepsilon_0 A}{d}$$

On inserting the slab, field inside it is zero and so a field $E_0 = \frac{\sigma}{\varepsilon_0}$ now exists in a space $(d - t)$



$$\Rightarrow \quad V = E_0 (d - t)$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}(d - t)$$

$$\Rightarrow \quad V = \frac{q}{A\varepsilon_0}(d - t)$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d\left(1 - \frac{t}{d}\right)}$$

$$\Rightarrow \quad C = \frac{C_0}{\left(1 - \frac{t}{d}\right)}$$

Since $d - t < d$

$$\Rightarrow \quad C > C_0$$

i.e., Capacitance increases on insertion of conducting slab between the plates of capacitor.

If $t = d$, then $C \to \infty$ i.e., if a conducting slab occupies the complete space between the plates of the capacitor, then $C \to \infty$.

CHARGE INDUCED ON A DIELECTRIC AND GAUSS'S LAW FOR DIELECTRICS

Consider a parallel plate capacitor which has a surface charge density σ and $-\sigma$ on each plate. If E_0 is the electric field between the plates of air capacitor, then

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

Let a dielectric of dielectric constant K be now introduced between the plates of the capacitor. If σ_p is induced surface charge density (see figure) due to the already existing field, then electric field due to induced charge is

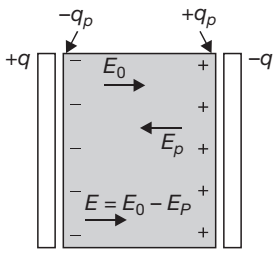
$$E_p = \frac{\sigma_p}{\varepsilon_0}$$

So, resultant dielectric field within the plates is

$$E = E_0 - E_p$$

$$\Rightarrow \quad E = \frac{1}{\varepsilon_0}\left(\sigma - \sigma_p\right) \qquad \ldots(1)$$

Also $E = \frac{\sigma}{K\varepsilon_0}$ $\qquad \ldots(2)$



Compare (1) and (2), we get

$$\frac{1}{\varepsilon_0}(\sigma-\sigma_p)=\frac{\sigma}{K\varepsilon_0}$$

$$\Rightarrow \quad \sigma_p=\sigma\left(1-\frac{1}{K}\right)$$

$$\Rightarrow \quad \frac{q_p}{A}=\frac{q}{A}\left(1-\frac{1}{K}\right)$$

$$\Rightarrow \quad q_p=q\left(1-\frac{1}{K}\right)$$

Since according to Gauss's Law, for dielectric

$$\oint \vec{E}\cdot d\vec{A}=\frac{1}{\varepsilon_0}(\Sigma q_{\text{enc}})=\frac{1}{\varepsilon_0}(q-q_p)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{A}=\frac{1}{\varepsilon_0}\left[q-q\left(1-\frac{1}{K}\right)\right]=\frac{q}{K\varepsilon_0}$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{A}=\frac{q}{K\varepsilon_0}=\frac{q}{\varepsilon} \qquad \{\because\ \varepsilon=\varepsilon_r\varepsilon_0=K\varepsilon_0\}$$

$$\Rightarrow \quad \oint K\varepsilon_0\vec{E}\cdot d\vec{A}=q$$

$$\Rightarrow \quad \oint \vec{D}\cdot d\vec{A}=q$$

where $\vec{D}=K\varepsilon_0\vec{E}$ is called the **Electric Displacement Vector**.

Problem Solving Technique(s)

Since $q_p=q\left(1-\frac{1}{K}\right)$

For a conductor $K\to\infty$. Hence,

$q_p=q$, $\sigma_p=\sigma$ and $E=0$

Hence, we may conclude the above discussion as under

(a) $E_{\text{vacuum}}=E_0=\frac{\sigma}{\varepsilon_0}=\frac{q}{A\varepsilon_0}$

(b) $E_{\text{dielectric}}=\frac{E_0}{K}$ {K = dielectric constant}

(c) $E_{\text{conductor}}=0$ {as $K\to\infty$}

Regarding dielectrics, it is worth noting that:

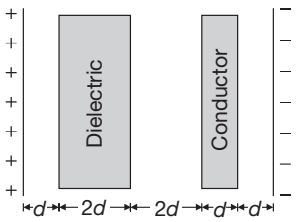
(a) These are non-conductors upto a certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength,** dielectric loses its insulating property and begins to conduct.

(b) These have either permanent dipole moment (polar-dielectrics, e.g., water) or acquire induced dipole moment **(non-polar dielectrics)** when placed in an electric field.

(c) The dielectric constant of polar dielectric depends on its temperature and due to thermal agitation with rise in temperature decreases.

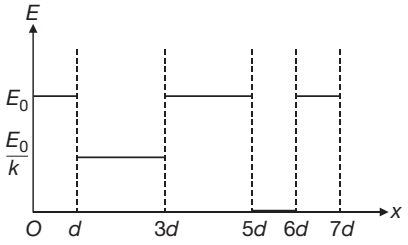
ILLUSTRATION 20

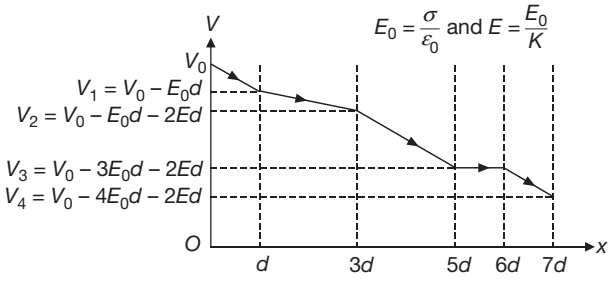
Two parallel conducting plates of area A charge $+q$ and $-q$ are as shown. A dielectric slab of dielectric constant K, thickness $2d$ and a conducting plate of thickness d is inserted between them. Taking $x=0$ at positive plate and $x=7d$ at negative plate, plot E vs x and V vs x graphs. Here E is the electric field and V is the potential and consider the potential at the positive plate to be V_0.



SOLUTION

The electric field in air/vacuum is $E_0=\frac{\sigma}{\varepsilon_0}=\frac{q}{A\varepsilon_0}$. In dielectric, field is $E=\frac{E_0}{K}$ and in conductor field is zero. Hence, the E-x graph is as shown in figure.





Using $V = Ed$ (in uniform field) the V-x graph is shown (slope of this graph gives electric field in that region).

ILLUSTRATION 21

A parallel plate capacitor with air as dielectric has a plate area of 200 cm^2 and plate separation of 4 mm. Calculate the percentage change in the capacitance if a layer of varnish $(k = 3)$ of thickness 0.1 mm is given on the inside of both plates.

SOLUTION

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{C_0}{1 - \frac{t}{d} + \frac{t}{kd}} \qquad \left\{\because\ C_0 = \frac{\varepsilon_0 A}{d}\right\}$$

where $t = 2(0.1\text{ mm}) = 0.2$ mm, $d = 4$ mm and $K = 3$

Putting values, we get $C_0 = 44.25$ pF

Now, $C = \dfrac{44.25}{1 - \frac{0.2}{4} + \frac{0.2}{(3)(4)}}$

$$\Rightarrow \quad C = \frac{44.25}{1 - \frac{1}{20} + \frac{1}{60}}\ \text{pF}$$

$$\Rightarrow \quad C = \frac{44.25 \times 60}{60 - 3 + 1} = \frac{2655}{58}$$

$$\Rightarrow \quad C = 45.77\%$$

So, %age change is

$$\frac{\Delta C}{C_0} \times 100 = \left(\frac{45.77 - 44.25}{44.25}\right) \times 100 = 3.4\%$$

So, percentage increase in the value of capacitance is 3.4%

EFFECT OF INSERTION OF DIELECTRIC IN A PARALLEL PLATE CAPACITOR

The effect of insertion of a dielectric on other physical quantities such as charge, potential difference, electric field and energy associated with a capacitor depends on the fact that whether the charged capacitor is isolated (i.e., charge is kept constant) or is attached to the battery (i.e. potential is kept constant).

If q_0, C_0, V_0, E_0 and U_0 represent the charge, capacitance, potential difference, electric field and energy associated with charged air capacitor respectively. On introduction of a dielectric slab of dielectric constant K between the plates let the respective quantities becomes q, C, V, E and U.

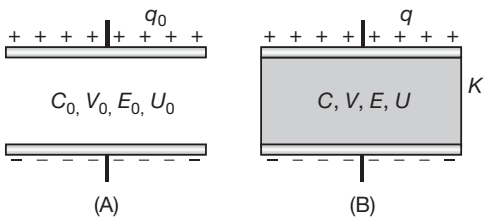
CASE-1: When charge is kept constant OR Capacitor is Isolated OR Battery is Disconnected

(a) Charge remains unchanged, i.e., $q = q_0$, as in an isolated system charge is conserved.

(b) Capacitance increases and becomes $C = KC_0$, because due to the presence of a dielectric, the capacitance becomes K times.

(c) Potential difference between the plates decreases and becomes $V = \left(\dfrac{V_0}{K}\right)$

$$\left(\because\ V = \frac{q}{C} = \frac{q_0}{KC_0},\ \text{as } q = q_0 \text{ and } C = KC_0\right)$$



(d) Field between the plates decreases and becomes

$$E = \left(\frac{E_0}{K}\right)$$

$$\Rightarrow \quad E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \qquad \left\{\because V = \frac{V_0}{K} \text{ and } E_0 = \frac{V_0}{d}\right\}$$

(e) Energy stored in the capacitor decreases and becomes $U = \left(\dfrac{U_0}{K}\right)$

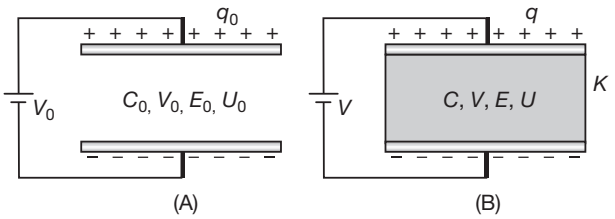
Since $U = \dfrac{q^2}{2C}$

$$\Rightarrow \quad U = \frac{q_0^2}{2KC_0} = \frac{U_0}{K} \qquad \{\because\ q = q_0 \text{ and } C = KC_0\}$$

CASE-2: When potential is kept constant OR battery remains attached

(a) Potential difference remains constant, i.e., $V = V_0$, because battery is a source of constant potential difference.

(b) Capacitance increases and becomes $C = KC_0$, because by presence of a dielectric capacitance becomes K times.



(c) Charge on capacitor increases, i.e., $q = Kq_0$, because

$$q = CV$$

$$\Rightarrow \quad q = (KC_0)V = Kq_0 \qquad \{\because\ q_0 = C_0V\}$$

(d) Electric field remains unchanged, so $E = E_0$

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0 \quad \left\{\because\ V = V_0 \text{ and } \frac{V_0}{d} = E_0\right\}$$

(e) Energy stored in the capacitor increases and becomes $U = KU_0$

$$\text{Since, } U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2$$

$$\Rightarrow \quad U = \frac{1}{2}KU_0 \qquad \left\{\because C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right\}$$

Problem Solving Technique(s)

While solving problems of this type always keep in mind that, whenever a battery is disconnected then,

q = constant

and if battery remains attached, then V = constant

ILLUSTRATION 22

An air capacitor of capacity $C = 10\ \mu\text{F}$ is connected to a constant voltage battery of 12 V. Now the space between the plates is filled with a liquid of dielectric constant $k = 5$. Calculate the charge that flows from battery to the capacitor.

SOLUTION

Initially charge on the capacitor $Q_i = 10 \times 12 = 120\ \mu\text{C}$

When dielectric medium is filled, so capacitance becomes k times, i.e., new capacitance is

$$C' = kC = 5 \times 10 = 50\ \mu\text{C}$$

Final charge on the capacitor $Q_f = 50 \times 12 = 600\ \mu\text{C}$

Hence additional charge supplied by the battery is

$$\Delta Q = Q_f - Q_i = 480\ \mu\text{C}$$

ILLUSTRATION 23

The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0° to 180°. With the dial set at 180°, the capacitor is connected to a 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is turned at 0°.

(a) What is the potential difference across the capacitor when the dial reads 0°?

(b) How much work is required to turn the dial, if friction is neglected?

SOLUTION

When the dial is at 0°, the capacitance of the capacitor is given by

$$C_1 = 50\ \text{pF} = 50 \times 10^{-12}\ \text{F}$$

When dial is at 180°, the capacitance is given by

$$C_2 = 950\ \text{pF} = 950 \times 10^{-12}\ \text{F}$$

The potential difference across capacitor C_2, is given by

$$V_2 = 400\ \text{V}$$

Charge on capacitor C_2 is

$$q = C_2V_2 = 950 \times 10^{-12} \times 400$$

$$\Rightarrow \quad q = 380 \times 10^{-9}\ \text{C}$$

(a) When battery is disconnected the charge remains the same and so, $q = \text{constant}$. Let V_1 be the potential difference across capacitor when dial reads 0°. Then

$$q = C_1 V_1$$

$$\Rightarrow \quad 380 \times 10^{-9} = 50 \times 10^{-12} \times V_1$$

$$\Rightarrow \quad V_1 = \frac{380 \times 10^{-9}}{50 \times 10^{-12}} = 7600 \text{ V}$$

(b) Work required to turn the dial from 180° to 0° is $W =$ Gain in energy of capacitor

$$\Rightarrow \quad W = \frac{q^2}{2C_1} - \frac{q^2}{2C_2} = \frac{q^2}{2}\left(\frac{1}{C_1} - \frac{1}{C_2}\right)$$

$$\Rightarrow \quad W = \frac{q^2 (C_2 - C_1)}{2C_1 C_2}$$

$$\Rightarrow \quad W = \frac{(380 \times 10^{-9})^2 (950 - 50) \times 10^{-12}}{2 \times 50 \times 10^{-12} \times 950 \times 10^{-12}}$$

$$\Rightarrow \quad W = 1.368 \times 10^{-3} \text{ J}$$

ILLUSTRATION 24

Two parallel capacitors each of capacitance C are connected in series to a battery of emf V. Now, one of the capacitors is filled completely with a dielectric of dielectric constant K.

(a) Calculate the ratio of the electric field strength in the capacitor to the electric field when the dielectric is introduced. Does the field increase or decrease after insertion of dielectric?

(b) Find the amount of charge that flows through the battery.

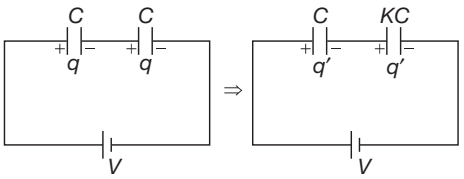
SOLUTION

(a) Net capacitance without inserting the slab is, $\frac{C}{2}$

$$\Rightarrow \quad q = \left(\frac{C}{2}\right)V$$

Initial electric field E is given by

$$E = \frac{\text{Potential Difference}}{\text{separation between the plates}} = \frac{\left(\frac{V}{2}\right)}{d} = \frac{V}{2d}$$



Net capacitance after inserting the slab is

$$C' = \left(\frac{K}{K+1}\right)C$$

$$\Rightarrow \quad q' = CV\left(\frac{K}{K+1}\right)$$

Final electric field E' is given by

$$E' = \frac{\text{Potential Difference}}{\text{Separation between the plates}}$$

$$\Rightarrow \quad E' = \frac{V\left(\frac{1}{K+1}\right)}{d} \quad \frac{V}{(K+1)d}$$

So, electric field decreases by a factor

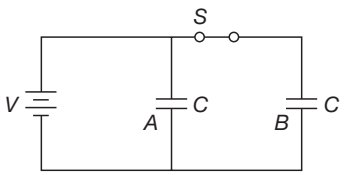
$$\frac{E}{E'} = \frac{\left(\frac{V}{2d}\right)}{\left(\frac{V}{(K+1)d}\right)} = \frac{K+1}{2}$$

(b) Charge that flows through the battery is $q' - q$

$$\text{Charge Flowing} = \frac{CV(K-1)}{2(K+1)}$$

ILLUSTRATION 25

Figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



SOLUTION

With the switch S closed, the potential difference across capacitors A and B is same. So,

$$V = \frac{Q_A}{C} = \frac{Q_B}{C}$$

The initial charges on the capacitors are given by

$$Q_A = Q_B = CV$$

When dielectric is introduced, the new capacitance of either capacitor.

$$C' = KC = 3C$$

Now, when the switch S is opened, let the potential difference across capacitor A be V volt and that across the capacitor B be V' volt.

When dielectric is introduced with the switch open (i.e., battery disconnected) then, the charge on capacitor B remains unchanged, so

$$Q_B = CV = C'V'$$

$$\Rightarrow \quad V' = \frac{C}{C'}V = \frac{V}{3} \text{ volt}$$

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

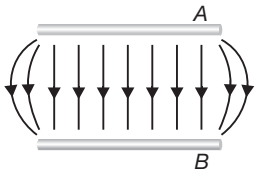
$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{5}{3}CV^2$$

$$\Rightarrow \quad \frac{U_i}{U_f} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

FORCE ON A DIELECTRIC SLAB BEING INSERTED BETWEEN CAPACITOR PLATES

When a dielectric slab is being inserted between the plates of an initially charged capacitor, then due to the electric field between the plates of the capacitor, bound charges of opposite nature are induced inside the dielectric. Also it is observed that while inserting the dielectric slab, fringing of electric field lines at the edges of the plates takes place. (*Fringing Effect is the bending of field lines at the corners of a capacitor as shown*).



Fringing effect for a capacitor with large d.

A component of the electric field along the surface of the dielectric pulls the dieletric slab inside the capacitor. Thus force on the dielectric slab is given by

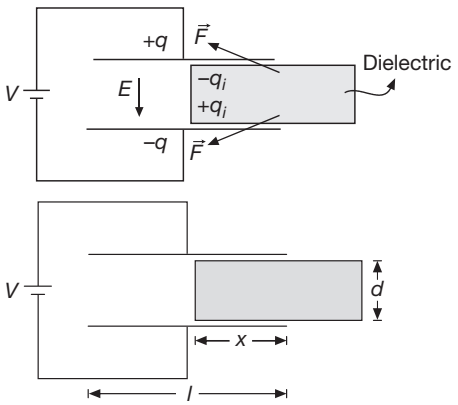
$$F = \left|\frac{dU}{dx}\right|$$

Let us discuss the two cases under which the dielectric slab can be introduced between the plates of the capacitor.

CASE-1: FOR CAPACITOR CONNECTED TO BATTERY (i.e. V = Constant)

Consider a dielectric slab inserted partially between the plates of a capacitor. Due to positive charge on the upper plate, some negative charge is induced on the top surface of the dielectric. Similarly, some positive charge is induced at the bottom surface of the slab. The negative induced charge is attracted towards positive charge on upper plate and the positive induced charge is attracted towards negative charge on lower plate as shown. As a result, the net force on the slab acts inwards and the slab is attracted into the capacitor.

To calculate this force, let us consider a parallel plate capacitor having plates of length l, width b and separation d. Let the capacitor be connected to a battery of voltage V. Let us find the force on a slab of thickness d and dielectric constant K when a portion of length $x(<l)$ lies inside the plates.



The capacitance of the capacitor so formed is calculated by assuming the capacitor in this position to be made up of two capacitors in parallel, one without a dielectric (of length $(l-x)$, separation d and width b) and the other with a dielectric (of length x, separation d and width b).

The capacitance of the portion of capacitor without dielectric is,

$$C_1 = \frac{\varepsilon_0 b(l-x)}{d}$$

and that of capacitor with dielectric is

$$C_2 = \frac{K\varepsilon_0 bx}{d}$$

So, the net capacitance of the capacitor in this position is,

$$C = C_1 + C_2$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[l + x(K-1)] \qquad \ldots(1)$$

The electric field inside the capacitor attracts the dielectric slab with a force F (say). For the dielectric slab to slowly enter the capacitor, an external force $F_{ext} = F$ must be applied opposite to F. Due to this external force, the dielectric slab doesnot gain any kinetic energy while entering the capacitor plates.

Let the slab move further inside the capacitor by an infinitesimal distance dx. Due to this the capacitance of the capacitor also increases from C to $C+dC$. Since the capacitor is connected to the battery, so potential difference V remains constant and hence the battery has to supply a charge (dQ) to the capacitor given by

$$dQ = V(dC)$$

During this process work done by the battery is

$$dW_{battery} = VdQ = (dC)V^2$$

Also work done by the external force F during this displacement dx is

$$dW_{external} = Fdx\cos(180°) = -Fdx$$

Total work done on the capacitor is

$$dW = dW_{battery} + dW_{external}$$

$$\Rightarrow \quad dW = (dC)V^2 - Fdx$$

This work done dW is equal to the increase in the energy $dU = \frac{1}{2}(dC)V^2$ stored in the capacitor (*assuming that no heat losses are produced while inserting the dielectric inside the capacitor*). So, we have

$$\frac{1}{2}(dC)V^2 = (dC)V^2 - Fdx$$

$$\Rightarrow \quad Fdx = \frac{1}{2}(dC)V^2$$

$$\Rightarrow \quad F = \frac{1}{2}V^2\left(\frac{dC}{dx}\right)$$

From (1), we get $dC = \frac{\varepsilon_0 b}{d}(K-1)dx$

$$\Rightarrow \quad \frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K-1)$$

$$\Rightarrow \quad F = \frac{\varepsilon_0 bV^2(K-1)}{2d}$$

Thus, the electric field attracts the dielectric into the capacitor with the force F given by

$$F = \frac{\varepsilon_0 bV^2(K-1)}{2d}$$

Also, we note that this force is constant and is independent of the distance x.

CASE-2: FOR AN ISOLATED CAPACITOR (i.e., q = Constant)

When the battery is removed after charging the capacitor then q remains constant. In that case,

$$U = \frac{q^2}{2C}$$

$$\Rightarrow \quad dU = d\left[\frac{q^2 d}{2\varepsilon_0 b\{l + x(K-1)\}}\right]$$

$$\Rightarrow \quad dU = -\frac{q^2 d(K-1)}{2\varepsilon_0 b}\left(\frac{1}{l + x(K-1)}\right)^2 dx$$

Here, since work done by battery is zero and so only the electrostatic force does the work.

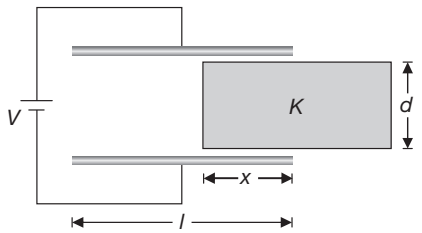
Hence, $F = -\frac{dU}{dx}$

$$\Rightarrow \quad F = \frac{q^2 d(K-1)}{2\varepsilon_0 b}\left(\frac{1}{l + x(K-1)}\right)^2$$

This force also acts inwards, but in this case it is not constant and is a function of x.

ILLUSTRATION 26

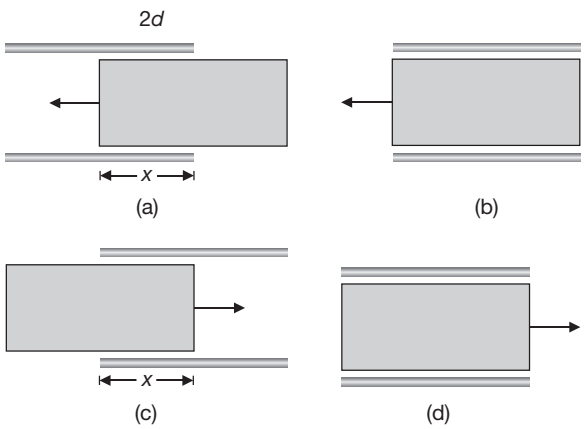
In the situation shown in figure the length of the plates is l, area of the plates is A and separation between then is d. A dielectric slab of dielectric constant K, mass m and thickness d is released from rest. Prove that the slab will execute periodic motion and find its time period. Assume friction to be absent between inner portion of the plates and the outer surface of the slab. Also see that initially the slab has its length x inserted in the plates of the capacitor.



SOLUTION

As derived and seen earlier that the electric forces pull the slab inwards with a constant force, given by

$$F = \frac{\varepsilon_0 b V^2 (K-1)}{2d}$$



where, b = width of plate $= \frac{A}{l}$

So, acceleration of slab, $a = \frac{F}{m}$

$$\Rightarrow \quad a = \frac{\varepsilon_0 \left(\frac{A}{l}\right) V^2 (K-1)}{2md}$$

$$\Rightarrow \quad a = \frac{\varepsilon_0 A V^2 (K-1)}{2mld}$$

At the instant when the slab is fully inside the plates, we get the state of equilibrium for the slab. The slab will execute periodic motion in the phases as shown in figure.

So, time taken to reach from position (a) to position (b) equals t (say).

Then, $t = \frac{T}{4}$

Since, $s = \frac{1}{2} a t^2$

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} \qquad \{\because \quad s = l - x\}$$

$$\Rightarrow \quad t = \sqrt{\frac{2(l-x)}{\frac{\varepsilon_0 A V^2 (K-1)}{2mld}}} = \sqrt{\frac{4(l-x)mld}{\varepsilon_0 A V^2 (K-1)}}$$

The desired time period is

$$T = 4t = 8\sqrt{\frac{(l-x)mld}{\varepsilon_0 A V^2 (K-1)}}$$

DIELECTRIC BREAKDOWN

When a dielectric material is subjected to a strong electric field then there comes a breakdown field strength at which it loses its insulating or dielectric ability and the dielectric becomes a conductor. This occurs when the electric field is so strong that electrons are ripped loose from their molecules. This maximum electric field magnitude that a material can withstand without the occurrence of breakdown is called its **Dielectric Strength**. The dielectric strength of dry air is about 3×10^6 Vm^{-1}.

ELECTRIC ENERGY DENSITY OF DRY AIR

The breakdown field strength at which dry air loses its insulating ability and allows a discharge to pass through is $E_b = 3\times10^6\ \text{Vm}^{-1}$. At this field strength, the electric energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.85\times10^{-12})(3\times10^6)^2 = 40\ \text{Jm}^{-3}$$

ILLUSTRATION 27

Four identical plane capacitors with an air dielectric are connected in series. The intensity of the field at which the air is punctured is $E_a = 3\times10^4\ \text{Vcm}^{-1}$. The distance between the plates $d = 0.7$ cm.

(a) What maximum voltage can be fed to this battery of capacitors?

(b) What will this maximum voltage be if one of the capacitors is replaced by a similar one in which glass is used as a dielectric?

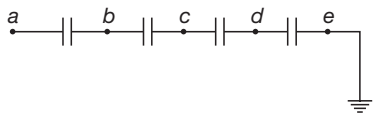
The permitivity of glass $K = 7$ and the puncturing field intensity of glass $E_g = 9\times10^4\ \text{Vcm}^{-1}$.

SOLUTION

(a) Potential across each capacitor $= V_0 = Ed$

$$\Rightarrow\quad V_0 = (3\times10^4\times10^2\ \text{Vm}^{-1})(0.7\times10^{-2}\ \text{m})$$

$$\Rightarrow\quad V_0 = 2.1\times10^4\ \text{V}$$



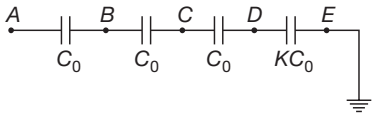
Now $\Delta V = \Delta V_{ab} + \Delta V_{bc} + \Delta V_{cd} + \Delta V_{de}$

$$\Rightarrow\quad \Delta V = 2.1\times10^4 + 2.1\times10^4 + 2.1\times10^4 + 2.1\times10^4$$

$$\Rightarrow\quad (\Delta V)_{\text{Max}} = 4(2.1\times10^4) = 8.4\times10^4\ \text{V}$$

(b) For glass, breakdown voltage is given by

$$V_g = 6.3\times10^4\ \text{V}$$



If the glass capacitor is connected between the points D and E, then

$$q = C_0(\Delta V_{AB}) = C_0(\Delta V_B C) = C_0(\Delta V_{CD}) = 7C_0(\Delta V_{DE})$$

$$\Rightarrow\quad \Delta V_{AB} = \Delta V_{BC} = \Delta V_{CD} = K\Delta V_{DE}$$

$$\Rightarrow\quad \Delta V_{AB} = \Delta V_{BC} = \Delta V_{CD} = 7\Delta V_{DE}$$

OPTION 1

Take $\Delta V_{AB} = 2.1\times10^4$ V and find ΔV_{DE}

From the above equality, we get

$$\Delta V_{DE} = \frac{\Delta V_{AB}}{7} = 0.3\times10^4\ \text{V}$$

OPTION 2

Take $\Delta V_{DE} = 6.3\times10^4$ V (The breakdown voltage of glass), then

$$\Delta V_{AB} = \Delta V_{BC} = \Delta V_{CD} = 7(6.3\times10^4\ \text{V})$$

$$\Rightarrow\quad \Delta V_{AB} = \Delta V_{BC} = \Delta V_{CD}$$

$$\Delta V_{AB} = 4.41\times10^5\ \text{V} > 2.1\times10^4\ \text{V}$$

So, as per OPTION 2, all the first three capacitors are going to breakdown, so, we have to accept

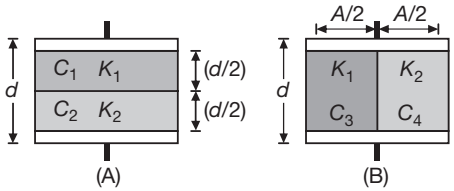
$$\Delta V_{DE} = 0.3\times10^4\ \text{V}\ \text{(for glass capacitor)}$$

Test Your Concepts-III

Based on Dielectrics and Breakdown

(Solutions on page H.132)

1. A capacitor is filled with two dielectrics of same dimensions but of dielectric constant 2 and 3 respectively. Find the ratio of capacities in the two arrangements.



2. A parallel-plate capacitor of plate separation d is charged to a potential difference ΔV_0. A dielectric slab of thickness d and dielectric constant κ is introduced between the plates while the battery remains connected to the plates.

(a) Show that the ratio of energy stored after the dielectric is introduced to the energy stored in the empty capacitor is $\frac{U}{U_0} = \kappa$. Give a physical explanation for this increase in stored energy.

(b) What happens to the charge on the capacitor?

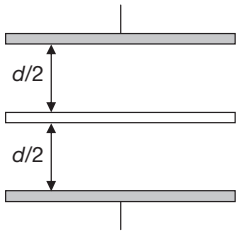
3. Two parallel capacitors each of capacitance C are connected in series to a battery of emf V. Now, one of the capacitors is filled completely with a dielectric of dielectric constant K.

(a) Calculate the ratio of the electric field strength in the capacitor to the electric field when the dielectric is introduced. Does the field increase or decrease after insertion of dielectric?

(b) Find the amount of charge that flows through the battery.

4. A parallel plate capacitor with air as dielectric has a plate area of 200 cm^3 and plate separation of 4 mm. Calculate the percentage change in the capacitance if a layer of varnish $(k = 3)$ of thickness 0.1 mm is given on the inside of both plates.

5. A thin conducting plate is inserted in halfway between the plates of a parallel plate capacitor.

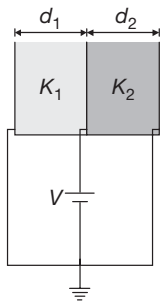


(a) Compute the capacitance before the plate was inserted.

(b) What is the capacitance after the plate was inserted?

(c) What does the value of capacitance becomes when the thin plate and the upper plate are shortened (i.e., connected by a conducting wire)? Area of each plate is A.

6. A capacitor is composed of three parallel conducting plates. All three plates are of the same area A. The first pair of plates are kept a distance d_1 apart, and the space between them is filled with a medium of a dielectric K_1. The corresponding data for the second pair are d_2 and K_2, respectively.



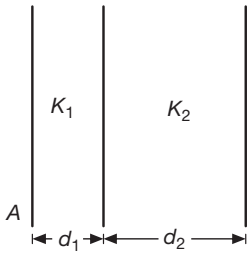
The middle plate is connected to the positive terminal of a constant voltage source V, and the external plates are connected the other terminal of V.

(a) Find the capacitance of the system.

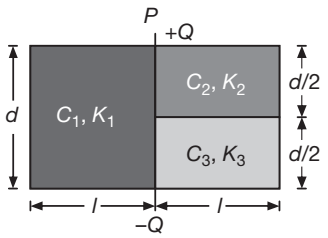
(b) What is the surface charge density on the middle plate?

(c) Compute the energy density in the medium K_1.

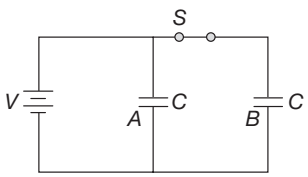
7. Find the capacitance of a system of three parallel plates, each of area A, separated by d_1 and d_2. The space between them is filled with dielectrics of relative dielectric constants K_1 and K_2.



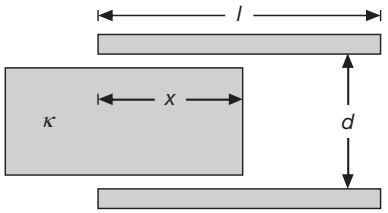
8. A parallel plate capacitor is constructed using three different dielectric materials as shown in figure. The parallel plates across which a potential difference is applied, are of area $A = 1\,\text{cm}^2$ and are separated by a distance $d = 2$ mm. If $K_1 = 4$, $K_2 = 6$ and $K_3 = 2$, find capacitance across P and Q. (Take $\varepsilon_0 = 8.8 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$)



9. Figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



10. A capacitor is constructed from two square plates of sides ℓ and separation d. A material of dielectric constant κ is inserted a distance x into the capacitor, as shown in Figure. Assume that d is much smaller than x.



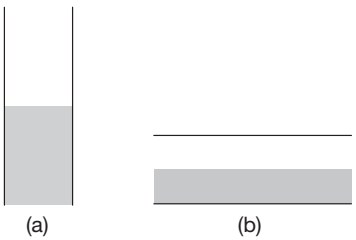
(a) Find the equivalent capacitance of the device.

(b) Calculate the energy stored in the capacitor, letting ΔV represent the potential difference.

(c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference ΔV. Ignore friction.

(d) Obtain a numerical value for the force assuming that $\ell = 5$ cm, $\Delta V = 2000$ V, $d = 2$ mm and the dielectric is glass ($\kappa = 4.5$).

11. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is 2. When this capacitor is positioned horizontally, what fraction of it should be filled with the same dielectric in order for the two capacitors to have equal capacitance?



KIRCHHOFF'S LAWS FOR CAPACITOR CIRCUITS

Kirchhoff's Junction Law (KJL)

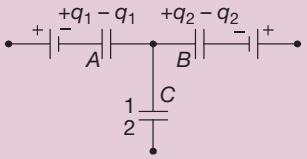
According to KJL, charge can never accumulate at a junction i.e., at the junction

$$\sum q_{\text{junction}} = 0$$

KJL is based on Law of Conservation of Charge.

Problem Solving Technique(s)

This law is helpful in determining the nature of charge on an unknown capacitor plate.



Charge on capacitor C can be determined by using this rule. As no charge must accumulate at the junction O, so if x is charge on plate 1 of C, then

$$-q_1 + q_2 + x = 0$$

$$\Rightarrow \quad x = q_1 - q_2$$

i.e., plate 1 has a charge $(q_1 - q_2)$ and plate 2 has a charge $-(q_1 - q_2)$.

Kirchhoff's Loop Law (KLL)

In a closed loop/circuit (a closed loop is the one which starts and ends at the same point), the algebraic sum of potential differences across each element of a closed loop/circuit is zero.

$$\Rightarrow \quad \sum V = 0$$

Conventions Followed to Apply Loop Law

(a) In a loop, across a battery, if we travel from negative terminal of battery to the positive terminal then there is a potential rise and a +ve sign is applied with voltage of the battery.

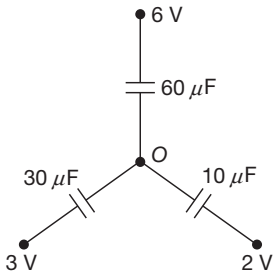
(b) In a loop, across a battery, if we travel from positive terminal of the battery to the negative terminal then there is a potential fall and a –ve sign is applied with voltage of the battery.

(c) In a loop, across a capacitor, if we go from negative plate to the positive plate of the capacitor then there is a potential rise and a +ve sign is to be taken with potential difference across the capacitor i.e., $\Delta V = +\frac{q}{C}$.

(d) In a loop, across a capacitor, if we go from positive plate to the negative plate of the capacitor then there is a potential fall and a –ve sign is to be taken with the potential difference across the capacitor i.e., $\Delta V = -\frac{q}{C}$.

ILLUSTRATION 28

Three uncharged capacitors having capacitances $60\ \mu\text{F}$, $30\ \mu\text{F}$ and $10\ \mu\text{F}$ are connected to each other as shown. Calculate the potential at the point O.



SOLUTION

According to Kirchhoff's Junction Law (KJL), charge cannot accumulate at the junction (O), so

$$\Sigma q_0 = 0\text{, where } q = C\Delta V$$

Let V_0 be the potential of junction O, then

$$60(6 - V_0) + 10(2 - V_0) + 30(3 - V_0) = 0$$

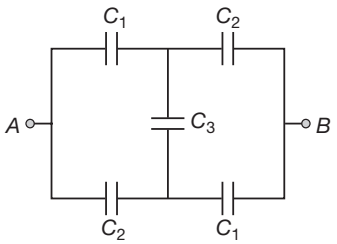
$$\Rightarrow \quad 360 + 20 + 90 = (60 + 10 + 30)V_0$$

$$\Rightarrow \quad V_0 = \frac{470}{100}$$

$$\Rightarrow \quad V_0 = 4.7\ \text{V}$$

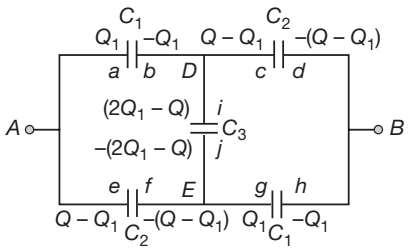
ILLUSTRATION 29

Find the equivalent capacitance between the point A and B in figure.



SOLUTION

Let us connect a battery between the points A and B. The charge distribution is shown in figure. Suppose the positive terminal of the battery supplies a charge $+Q$ and the negative terminal a charge $-Q$. The charge Q is divided between plates a and e.



Let a charge Q_1 goes to the plate a and the rest $Q-Q_1$ goes to the plate e. The charge $-Q$ supplied by the negative terminal is divided between plates d and h. Using the symmetry of the figure, charge $-Q_1$ goes to the plate h (as it has a capacitance C_1) and $-(Q-Q_1)$ to the plate d (as it has a capacitance C_2). This is because if we look into the circuit from A or from B, the circuit looks identical. The division of charge at A and at B should, therefore, be similar. The charges on the other plates may be written easily. The charge on the plate i is $2Q_1-Q$ which ensures that the total charge on plates b, c and i remains zero as these three plates form an isolated system.

We have $V_A-V_B=(V_A-V_D)+(V_D-V_B)$

$$\Rightarrow \quad V_A-V_B=\frac{Q_1}{C_1}+\frac{Q-Q_1}{C_2} \qquad \ldots(1)$$

Also, $V_A-V_B=(V_A-V_D)+(V_D-V_E)+(V_E-V_B)$

$$\Rightarrow \quad V_A-V_B=\frac{Q_1}{C_1}+\frac{2Q_1-Q}{C_3}+\frac{Q_1}{C_1} \qquad \ldots(2)$$

We have to eliminate Q_1 from these equations to get the equivalent capacitance $\dfrac{Q}{(V_A-V_B)}$.

The first equation may be written as

$$V_A-V_B=Q_1\left(\frac{1}{C_1}-\frac{1}{C_2}\right)+\frac{Q}{C_2}$$

$$\Rightarrow \quad \frac{C_1C_2}{C_2-C_1}(V_A-V_B)=Q_1+\frac{C_1}{C_2-C_1}Q \qquad \ldots(3)$$

The second equation may be written as

$$V_A-V_B=2Q_1\left(\frac{1}{C_1}+\frac{1}{C_3}\right)-\frac{Q}{C_3}$$

$$\Rightarrow \quad \frac{C_1C_2}{2(C_1+C_3)}(V_A-V_B)=Q_1-\frac{C_1}{2(C_1+C_3)}Q \qquad \ldots(4)$$

Subtracting (4) from (3)

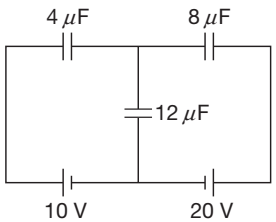
$$(V_A-V_B)\left[\frac{C_1C_2}{C_2-C_1}-\frac{C_1C_3}{2(C_1+C_3)}\right]=\left[\frac{C_1}{C_2-C_1}+\frac{C_1}{2(C_1+C_3)}\right]Q$$

$$\Rightarrow \quad (V_A-V_B)\left[2C_1C_2(C_1+C_3)-C_1C_3(C_2-C_1)\right]=C_1\left[2(C_1+C_3)+(C_2-C_1)\right]Q$$

$$\Rightarrow \quad C=\frac{Q}{V_A-V_B}=\frac{2C_1C_2+C_2C_3+C_3C_1}{C_1+C_2+2C_3}$$

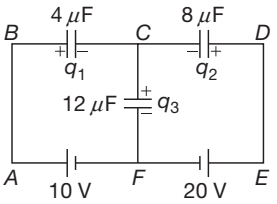
ILLUSTRATION 30

Find the charges on the three capacitors shown in figure.



SOLUTION

Let the charges in three capacitors be as shown in figure



Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2. Then,

$$q_1 + q_2 = q_3 \quad \ldots(1)$$

This relation can also be obtained in a different manner. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries. Thus,

$$q_3 - q_1 - q_2 = 0$$

$$\Rightarrow \quad q_3 = q_1 + q_2$$

Applying Second Law in Loops *BCFAB* and *CDEFC*, we have

$$-\frac{q_1}{4} - \frac{q_3}{12} + 10 = 0$$

$$\Rightarrow \quad q_3 + 3q_1 = 120 \quad \ldots(2)$$

and $$\frac{q_2}{8} - 20 + \frac{q_3}{12} = 0$$

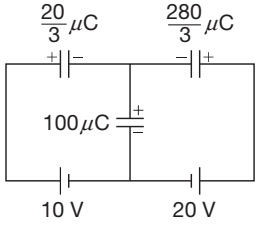
$$\Rightarrow \quad 3q_2 + 2q_3 = 480 \quad \ldots(3)$$

Solving the above three equations, we have

$$q_1 = \frac{20}{3}\ \mu\text{C},$$

$$q_2 = \frac{280}{3}\ \mu\text{C} \text{ and } q_3 = 100\ \mu\text{C}$$

Thus, charges on different capacitors in μC, are shown in figure.



FLOW OF CHARGE

Consider a circuit having a switch, say S. Whenever the switch S in the circuit is opened or closed (or transferred from one mode to the other), then a charge flow takes place through certain points/branches of the circuit. To solve such like problems, we may follow the following steps.

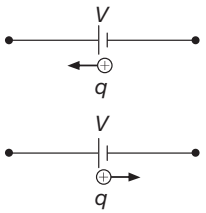
STEP-1: Find the charges on different capacitors for the initial and final positions/modes of the switch.
STEP-2: The charge flowing through a point/branch is calculated by the difference between the charges in the initial and the final positions/modes of the switch.

$$q_{\substack{\text{flowing through}\\ \text{a point/branch}}} = \left| \sum q_i - \sum q_f \right|$$

ENERGY SUPPLIED/CONSUMED BY THE BATTERY

We must take a note that whenever a charge $+q$ leaves the positive terminal of the battery of emf V, then work is done by the battery or the battery supplies an energy given by

$$E_{\text{supplied}} = W_{\text{by the battery}} = qV$$



However, if a charge $+q$ enters the positive terminal of the battery then work is done on the battery or the battery consumes an energy given by

$$E_{\text{consumed}} = W_{\text{on the battery}} = qV$$

While solving numerical problems we must keep in mind that the energy supplied by the battery will be taken as positive and the energy consumed by the battery will be taken as negative.

GENERATION OF HEAT

Again, let us consider a circuit having a switch S (as discussed earlier). Now, whenever the switch S in the circuit is opened or closed (or transferred from

open mode to the close mode), then some amount of heat will be generated in the circuit. If W_{supplied} be the energy supplied by the battery/batteries, W_{consumed} be the energy consumed by the battery/batteries, ΣU_i be the initial energy stored in all the capacitors and ΣU_f be the final energy stored in all the capacitors, then heat generated is given by

$$\Delta H = \underbrace{\left(W_{\text{supplied}} - W_{\text{consumed}}\right)}_{\text{by the battery/batteries}} + \underbrace{\left(\sum U_i - \sum U_f\right)}_{\text{for the capacitors}}$$

because, we have

$$W_{\text{battery}} = \Delta U_C + \Delta H$$

ILLUSTRATION 31

A capacitor of capacitance C which is initially charged upto a potential difference V is connected with a battery of emf $\frac{V}{2}$ such that the positive terminal of battery is connected with positive plate of capacitor. After a long time, calculate the

(a) charge flow through the battery
(b) work done by battery
(c) heat dissipated in the circuit during the process of charging

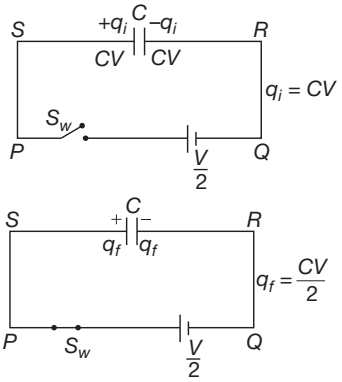
SOLUTION

(a) Initial charge on the capacitor is $q_i = CV$

Final charge on the capacitor is $q_f = \frac{CV}{2}$

Charge flown $= \Delta q = q_f - q_i = -\frac{CV}{2}$

i.e. charge must have flown into the battery.



(b) Since a charge $\frac{CV}{2}$ flows into the battery i.e., a charge $\frac{CV}{2}$ enters the positive terminal of the battery so energy is consumed by the battery and is given by $W_{\text{battery}} = (\Delta q)V_{\text{battery}} = -\frac{CV^2}{4}$.

(c) Finally, change in energy of capacitor is

$$\Delta U_c = U_{\text{final}} - U_{\text{initial}}$$

$$\Rightarrow \quad \Delta U_c = \frac{1}{2}C\left(\frac{V}{2}\right)^2 - \frac{1}{2}CV^2$$

$$\Rightarrow \quad \Delta U_c = \frac{1}{8}CV^2 - \frac{1}{2}CV^2 = -\frac{3}{8}CV^2$$

Since $W_{\text{battery}} = \Delta U_c + \Delta H$

$$\Rightarrow \quad \Delta H = W_{\text{battery}} - \Delta U_c$$

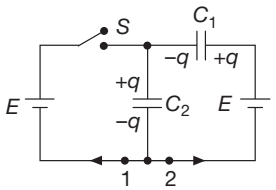
$$\Rightarrow \quad \Delta H = -\frac{CV^2}{4} - \left(-\frac{3}{8}CV^2\right)$$

$$\Rightarrow \quad \Delta H = \frac{3}{8}CV^2 - \frac{CV^2}{4}$$

$$\Rightarrow \quad \Delta H = \frac{CV^2}{8}$$

ILLUSTRATION 32

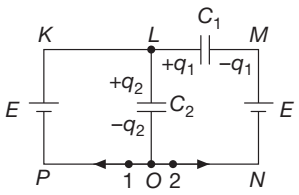
What charges will flow after shorting of the switch S in the circuit illustrated in figure through sections 1 and 2 in the directions indicated by the arrows?



SOLUTION

The charge distribution, when the switch is closed, is shown here.
In figure, the two capacitors are connected in series and hence their equivalent capacitance is given by

$$C_S = \frac{C_1C_2}{(C_1 + C_2)}$$



If q is the charge on either of the capacitors, then

$$q = \left(\frac{C_1C_2}{C_1 + C_2}\right)E$$

For the closed loop $LKPOL$, applying Kirchhoff's Loop Law, we get

$$E - \frac{q_2}{C_2} = 0 \qquad \{\because -\Delta V = 0\}$$

$$\Rightarrow \quad q_2 = C_2E \qquad \ldots(1)$$

For closed loop $MLONM$, applying Kirchhoff's Loop Law, we get

$$-\frac{q_1}{C_1} + \frac{q_2}{C_2} - E = 0 \qquad \ldots(2)$$

Substituting the value of q_2 from equation (1) in equation (2), we get

$$-\frac{q_1}{C_1} + \frac{C_2E}{C_2} - E = 0$$

$$\Rightarrow \quad q_1 = 0$$

So, charge flowing through 1 is given by

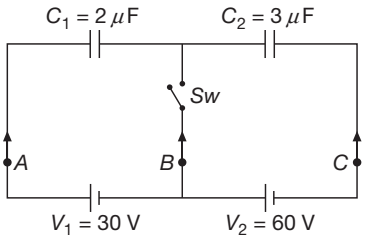
$$\Delta q_1 = C_2E$$

and the charge that flows through section 2 is given by

$$\Delta q_2 = -q_1 - q = -\left(\frac{C_1C_2}{C_1 + C_2}\right)E$$

ILLUSTRATION 33

In the circuit shown, calculate the charge that flows through A, B and C (in the directions shown), when the switch Sw is closed. Also calculate the loss in energy.

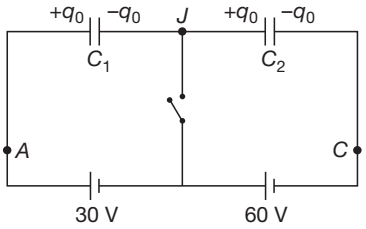


SOLUTION

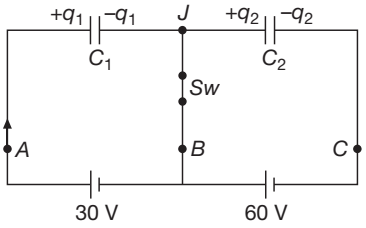
When the switch Sw is open, then both capacitors $2\,\mu F$ and $3\,\mu F$ are in series. So, their equivalent capacitance is $C_{eq} = \frac{(2)(3)}{2+3} = \frac{6}{5}\,\mu F$

Since both in series, so charge on both will be the same, say q_0. So,

$$q_0 = (C_{eq})V_{total} = \left(\frac{6}{5}\right)(30 + 60) = 108\ \mu C$$



When the switch Sw is closed then let q_1 and q_2 be the charges on capacitors C_1 and C_2 (as shown).



Now, $q_1 = C_1V_1 = 60\ \mu C$ and

$$q_2 = C_2V_2 = 180\ \mu C$$

Now initial charge on left plate of C_1 is $q_0 = 108\ \mu C$ and final charge on left plate of C_1 is $q_1 = 60\ \mu C$, which makes us conclude that a charge of $-48\ \mu C$ is

being sent (via A) to left plate of capacitor C_1. Hence charge flowing from A in the direction shown is

$$\Delta q_1 = (q_1)_{\text{final}} - (q_1)_{\text{initial}}$$

$$\Rightarrow \quad \Delta q_1 = 60 - 108 = -48\ \mu\text{C}$$

Similarly, final charge on right plate of C_2 is $-q_2 = -180\ \mu$C whereas initial charge on right plate of C_2 was $-q_0 = -108\ \mu$C. This simply means that a charge $-72\ \mu$C flows (via C) to right plate of capacitor C_2. Hence charge flowing through A in the diection shown is

$$\Delta q_2 = (q_2)_{\text{final}} - (q_2)_{\text{initial}}$$

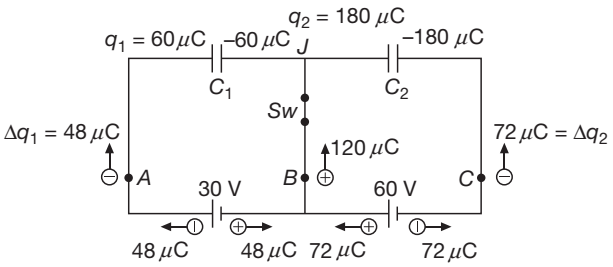
$$\Rightarrow \quad \Delta q_2 = -180 - (-108) = -72\ \mu\text{C}$$

Initial charge at the junction J is $+q_0 - q_0 = \text{zero}$

However, final charge at the junction J is $(q_2 - q_1)$

$$\Rightarrow \quad q_2 - q_1 = 180 - 60 = 120\ \mu\text{C}$$

So, finally charges flowing in the circuit are shown in figure.



Now, we observe that energy is supplied by $V_1 = 60$ V battery and consumed by $V_2 = 30$ V battery. So,

$$\text{Energy supplied} = |\Delta q_2| V_2 = (72 \times 10^{-6})(60)$$

$$\Rightarrow \quad \text{Energy supplied} = 4.32\ \text{mJ}$$

$$\text{Energy consumed} = |\Delta q_1| V_1 = (48 \times 10^{-6})(30)$$

$$\Rightarrow \quad \text{Energy consumed} = 1.44\ \text{mJ}$$

Initial energy stored by capacitors is

$$\Sigma U_{\text{initial}} = \frac{1}{2} C_{\text{eq}} V_{\text{total}}^2 = \frac{1}{2}\left(\frac{6}{5} \times 10^{-6}\right)(90)^2$$

$$\Rightarrow \quad \Sigma U_{\text{initial}} = 4.86\ \text{mJ}$$

Final energy stored by the capacitors is

$$\Sigma U_{\text{final}} = \frac{1}{2}(2 \times 10^{-6})(30)^2 + \frac{1}{2}(3 \times 10^{-6})(60)^2$$

$$\Rightarrow \quad \Sigma U_{\text{final}} = 6.3\ \text{mJ}$$

So, loss in energy is given by

$$\text{Loss} = (E_{\text{supplied}} - E_{\text{consumed}}) + (\Sigma U_i - \Sigma U_f)$$

$$\Rightarrow \quad \text{Loss} = (4.32 - 1.44) + (4.86 - 6.3)$$

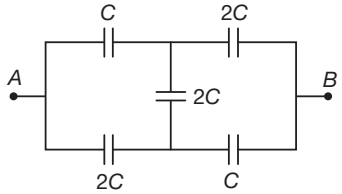
$$\Rightarrow \quad \text{Loss} = 1.44\ \text{mJ}$$

Test Your Concepts-IV

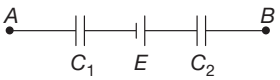
Based on Capacitor Circuits, Kirchhoff's Laws, Charge Flown and Generation of Heat

(Solutions on page H.135)

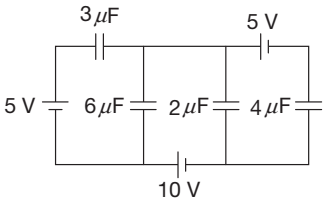
1. Find the equivalent capacitance between *A* and *B*.



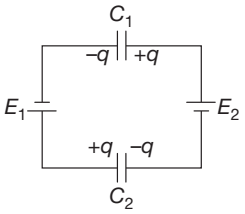
2. A circuit has section *AB* shown in figure. The emf of the source equals $E = 10$ V, the capacitor capacitances are equal to $C_1 = 1\ \mu$F and $C_2 = 2\ \mu$F, and the potential difference $V_A - V_B = 5$ V. Find the voltage across each capacitor.



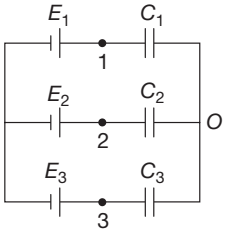
3. In the circuit shown, find the charges on 6 μF and 4 μF capacitors.



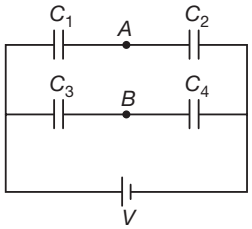
4. Find the charges on capacitor C_1 and C_2 in the circuit shown in figure.



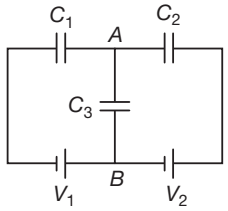
5. Determine the potential at point 1 of the circuit shown in figure, assuming the potential at the point *O* to be equal to zero. Using the symmetry of the formula obtained, write the expressions for the potential at points 2 and 3.



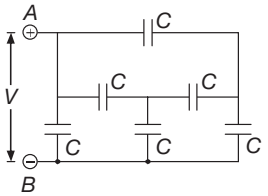
6. Calculate the potential difference between points *A* and *B* in the circuit shown in Figure. Under what condition is it equal to zero?



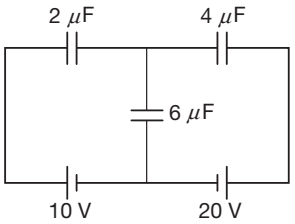
7. Calculate the potential difference between points *A* and *B* in the circuit shown in figure.



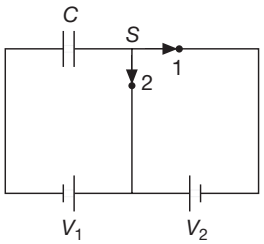
8. Determine the capacitance C_{AB} of the battery of identical capacitors shown in Figure.



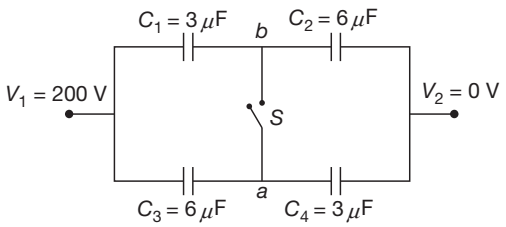
9. Find the charges on the three capacitors shown in figure.



10. What amount of heat will be generated in the circuit shown in Figure after the switch *S* is shifted from position 1 to position 2?



11. The arrangement shows a capacitor circuit with a switch *S*.

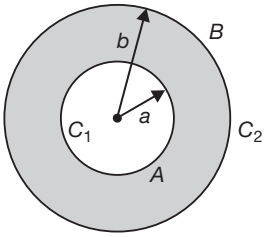


(i) Calculate the potential difference between a and b when the switch S is open and when S is closed.

(ii) Also calculate the charge that flows through the switch when it is closed.

SPHERICAL CAPACITOR

It consists of two concentric spherical conductors of radii a and b $(a < b)$ respectively. The space between the two conductors is filled with dielectric material of dielectric constant K.



The inner conductor is given a charge q. Depending upon which conductor (inner or outer) is earthed, we get the net capacitance as discussed in the following cases.

Let C_1 be the capacitance in between the two conductors and C_2 be capacitance outside both.

To Find C_1

Imagine the outer surface of B to be earthed. Then $-q$ is the charge induced on the inner surface of B.

If V is the potential difference between the two surfaces, then

$$V = \frac{q}{4\pi\varepsilon_0 Ka} + \frac{-q}{4\pi\varepsilon_0 Kb}$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0 K}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow \quad C_1 = \frac{q}{V} = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right) \qquad \ldots(1)$$

To Find C_2

Imagine A to be made open circuited (i.e. made non conducting), then

$$C_2 = 4\pi\varepsilon_0 Kb \qquad \ldots(2)$$

CASE-1: When battery is connected to B and A is earthed. Then C_1 and C_2 are in parallel

$$\Rightarrow \quad C = C_1 + C_2$$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right) + 4\pi\varepsilon_0 Kb$$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 K\left(\frac{b^2}{b-a}\right)$$

CASE-2: When battery is connected to A, then C_1 and C_2 are in series.

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{b-a}{ab}\frac{1}{4\pi\varepsilon_0 K} + \frac{1}{4\pi\varepsilon_0 Kb}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{4\pi\varepsilon_0 Kb}\left(\frac{b-a}{a} + 1\right)$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{4\pi\varepsilon_0 Kb}\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 Ka$$

CASE-3: When battery connected to A and B is earthed. Then C_2 can be omitted as it will not receive any charge.

So, $\quad C = C_1$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right)$$

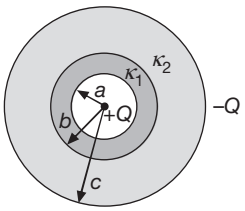
CASE-4: When battery connected to B and A is open circuited (or made non conducted) then C_1 can be omitted (as it is open circuited). So,

$$C = C_2$$

$$\Rightarrow \quad C = 4\pi\varepsilon_0 Kb$$

ILLUSTRATION 34

Consider a conducting spherical shell with an inner radius a and outer radius c. Let the space between two surfaces be filled with two different dielectric materials so that the dielectric constant is κ_1 between a and b and κ_2 between b and c, as shown in figure. Determine the capacitance of this system.



SOLUTION

The system can be treated as two capacitors connected in series, since the total potential difference across the capacitors is the sum of potential differences across individual capacitors. The equivalent capacitance for a spherical capacitor of inner radius r_1 and outer radius r_2 filled with dielectric with dielectric constant κ is given by

$$C = 4\pi\varepsilon_0\kappa\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

Thus, the equivalent capacitance of this system is

$$\frac{1}{C} = \frac{1}{\frac{4\pi\varepsilon_0\kappa_1 ab}{(b-a)}} + \frac{1}{\frac{4\pi\varepsilon_0\kappa_2 bc}{(c-b)}} = \frac{\kappa_2 c(b-a)+\kappa_1 a(c-b)}{4\pi\varepsilon_0\kappa_1\kappa_2 abc}$$

$$\Rightarrow \quad C = \frac{4\pi\varepsilon_0\kappa_1\kappa_2 abc}{\kappa_2 c(b-a)+\kappa_1 a(c-b)}$$

Check Point

Let us check the limiting value of C when $\kappa_1, \kappa_2 \to 1$. In this case, the above expression reduces to

$$C = \frac{4\pi\varepsilon_0 abc}{c(b-a)+a(c-b)} = \frac{4\pi\varepsilon_0 abc}{b(c-a)} = \frac{4\pi\varepsilon_0 ac}{(c-a)}$$

which agrees with equation for a spherical capacitor of inner radius a and outer radius c.

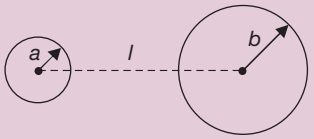
Remark(s)

(a) Capacitance of a two sphere capacitor of radii a and b when spheres are placed at large distance is

$$C = 4\pi\varepsilon_0\left(\frac{ab}{a+b}\right), \text{ for } \ell \gg a,\ \ell \gg b$$

(b) For a spherical capacitor of inner radius a and outer radius b, we have

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$



If both a and b are made extremely large but $b-a=d$ is kept fixed, then

$$ab \simeq a^2 \,(\text{or } b^2)$$

$$\Rightarrow \quad C = \varepsilon_0\left(\frac{4\pi a^2}{d}\right) = \frac{\varepsilon_0 A}{d},$$

where A is area of sphere $(\simeq 4\pi a^2 \simeq 4\pi b^2)$

The above result is the capacitance of a parallel plate capacitor of plate area A and plate separation d.

ILLUSTRATION 35

In case of two conducting spherical shells having radii a and $b(>a)$ calculate the capacity of the system if

(a) shells are concentric and inner is given a charge while outer earthed

(b) shells are concentric and the outer is given a charge while the inner is earthed

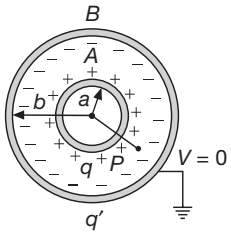
(c) shells carry equal and opposite charges and are separated by a distance d. Can you try to solve this part, using the concept of self energy and interaction energy?

SOLUTION

(a) Since the outer sphere is earthed hence, its potential will be zero, i.e., $V_B = 0$. If q' is the charge induced on shell B, then

$$\frac{(q+q')}{4\pi\varepsilon_0 b} = 0$$

$$\Rightarrow \quad q' = -q$$



So electric field at a point P between the shells is

$$E = E_A + E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \{\text{as } E_B = E_{\text{in}} = 0\}$$

$$\Rightarrow \quad \frac{-dV}{dr} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \left\{\text{as } E = -\frac{dV}{dr}\right\}$$

$$\Rightarrow \quad -\int_V^0 dV = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Since, $C = \dfrac{q}{V}$

$$\Rightarrow \quad C = \frac{4\pi\varepsilon_0 ab}{b-a} \qquad \ldots(1)$$

From this it is clear that

(i) As $b \to \infty$ equation (1) reduces to $C \to 4\pi\varepsilon_0 a$. So, we can say that, a spherical conductor is a spherical capacitor with its other plate of infinite radius.

(ii) As a and b both become very large such that $b-a$ has a finite and small value, say d, then $4\pi ab \simeq 4\pi a^2 \simeq 4\pi b^2 = A$ and so equation (1) reduces to $C = \left(\dfrac{\varepsilon_0 A}{d}\right)$ and hence we observe that a spherical capacitor behaves as a parallel plate capacitor when its spherical surfaces have large radii and are close to each other:

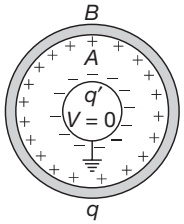
(b) In this situation $V_A = 0$ so if q' is the charge induced on shell A, then

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q'}{a} + \frac{q}{b}\right) = 0$$

$$\Rightarrow \quad q' = -\frac{a}{b}q$$

Since no field will exist at P due to B as P lies inside the conductor, so the field at a point P, between the shells is

$$E = E_A + E_B = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}$$



$$\Rightarrow \quad -\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{a}{b}q\right)\frac{1}{r^2}$$

$$\left\{\because \ E = -\frac{dV}{dr} \text{ and } q' = -\frac{a}{b}q\right\}$$

$$\Rightarrow \quad -\int_0^V dV = -\frac{q}{4\pi\varepsilon_0}\frac{a}{b}\int_a^b \frac{dr}{r^2}$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\frac{(b-a)}{b^2}$$

So, $C_1 = \dfrac{q}{V} = \dfrac{4\pi\varepsilon_0 b^2}{(b-a)}$

$$\Rightarrow \quad C_1 = \frac{b}{a}C(>C) \qquad \{\text{as } b > a\} \qquad \ldots(2)$$

Also, we note that in this situation $q_B \neq -q_A$ and hence the system is not a capacitor. However, we observe that

$$C_1 = \frac{4\pi\varepsilon_0 b^2}{(b-a)} = \frac{4\pi\varepsilon_0 ab}{(b-a)} + 4\pi\varepsilon_0 b = C_{AB} + C_B$$

This system is equivalent to a spherical capacitor (of inner radius a and outer radius b and a spherical conductor of radius b connected in parallel. This is because the charge q given to the outer shell distributes in such a way that $\left(\frac{a}{b}\right)q$ remains on its inner side while the remaining $\left[q - \left(\frac{a}{b}\right)q\right]$ lies on its outer side. Hence,

$$\left[\frac{q_{\text{in}}}{q_{\text{out}}}\right]_B = \frac{\left(\frac{a}{b}\right)q}{q - \left(\frac{a}{b}\right)q} = \frac{a}{(b-a)}$$

and the system becomes equivalent to two capacitors in parallel having a common potential V.

(c) To find the capacitance in this situation, let us consider a point P at a distance r from centre of A on the line joining the centres of two spheres. If E be the net field at this point P, then

$$E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{q}{4\pi\varepsilon_0 (d-r)^2}$$

$$\Rightarrow \quad -\frac{dV}{dr} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{1}{(d-r)^2}\right] \quad \left\{\text{as } E = -\frac{dV}{dr}\right\}$$

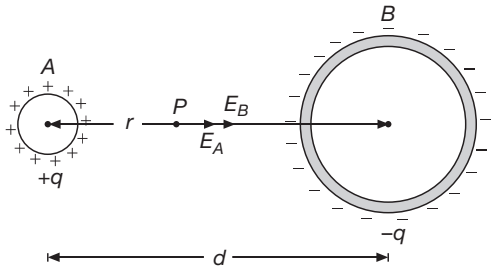
$$\Rightarrow \quad -\int_{V_A}^{V_B} dV = \frac{q}{4\pi\varepsilon_0}\int_a^{(d-b)} \left[\frac{1}{r^2} + \frac{1}{(d-r)^2}\right] dr$$

$$\Rightarrow \quad V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r} + \frac{1}{(d-r)}\right]\Bigg|_a^{(d-b)}$$

$$\Rightarrow \quad V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{1}{(d-a)} - \frac{1}{(d-b)}\right]$$

For $d \gg a$ and $d \gg b$, we get

$$\Rightarrow \quad V_A - V_B = V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$$



Since $C = \frac{q}{|\Delta V|}$

$$\Rightarrow \quad C_2 = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)} \qquad \ldots(3)$$

Using Concept of Self Energy and Interaction Energy

The self energy of a spherical conductor is

$$U_S = \frac{1}{2}qV = \frac{1}{2}\left[\frac{q^2}{4\pi\varepsilon_0 R}\right] \qquad \left\{\text{as } V = \frac{q}{4\pi\varepsilon_0 R}\right\}$$

while interaction energy of two point charges separated by a distance r

$$U_I = qV = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

So total potential energy of the system is

$$U = \begin{pmatrix}\text{Self}\\ \text{Energy}\\ \text{of } A\end{pmatrix} + \begin{pmatrix}\text{Self}\\ \text{Energy}\\ \text{of } B\end{pmatrix} + \begin{pmatrix}\text{Interaction}\\ \text{Energy}\\ \text{of } AB\end{pmatrix}$$

$$\Rightarrow \quad U = U_A + U_B + U_{AB}$$

$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0 a} + \frac{(-q)^2}{8\pi\varepsilon_0 b} + \frac{q(-q)}{4\pi\varepsilon_0 d}$$

$$\Rightarrow \quad U = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

Since, we have $U = \left(\frac{q^2}{2C}\right)$

$$\Rightarrow \quad C_2 = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)}$$

This result is identical to the result obtained earlier, using the method of electric field.

Now, if $d \to \infty$ then

$$C_2' = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}+\frac{1}{b}\right)}$$

$$\Rightarrow \quad C_2' < C_2$$

Also, $\frac{1}{C_2'} = \frac{1}{4\pi\varepsilon_0 a} + \frac{1}{4\pi\varepsilon_0 b} = \frac{1}{C_A} + \frac{1}{C_B}$

and hence the given system becomes equivalent to two capacitors $C_A\left(=4\pi\varepsilon_0 a\right)$ and $C_B\left(=4\pi\varepsilon_0 b\right)$ in series.

Also, we observe that in this ILLUSTRATION, (a) and (c) represent capacitors (as here the two conductors have equal and opposite charges) while arrangement given in (b) is not a capacitor (because q_B is not equal to $-q_A$).

ILLUSTRATION 36

An isolated conductor initially free from charge is charged by repeated contacts with a plate which after each contact is replenished to a charge Q. If q is the charge on the conductor after first operation prove that the maximum charge which can be given to the conductor in this way is $\frac{Qq}{Q-q}$.

SOLUTION

Let C_1, be the capacitance of the plate and C_2 that of the conductor. After first contact charge on conductor is q. Therefore, charge on plate will remain $Q-q$. As the charge redistributes in the ratio of capacitances, so

$$\frac{Q-q}{q} = \frac{C_1}{C_2} \qquad \ldots(1)$$

Let $q_{\max}$ be the maximum charge which can be given to the conductor. Then flow of charge from the plate to the conductor will stop when, both get equal potential. So,

$$V_{\text{conductor}} = V_{\text{plate}}$$

$$\Rightarrow \quad \frac{q_{\max}}{C_2} = \frac{Q}{C_1}$$

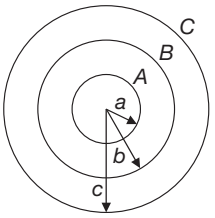
$$\Rightarrow \quad q_{\max} = \left(\frac{C_2}{C_1}\right)Q$$

Substituting $\frac{C_2}{C_1}$ from equation (1), we get

$$q_{\max} = \frac{Qq}{Q-q}$$

ILLUSTRATION 37

Three concentric conducting shells A, B and C of radii a, b and c are as shown in figure. Find the capacitance of the assembly between A and C. Now if, the space between A and B is filled with a dielectric material of dielectric constant K, then find the new capacitance between A and C.



SOLUTION

Consider a positive charge q on A and a negative charge $-q$ on C. Find the potential difference V between A and C. The desired capacitance will be

$$C = \frac{q}{V}$$

While calculating V, notice that net charge on B is zero. Moreover, we can apply the generator principle while calculating V.

$$V = V_A - V_C = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{c}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{c-a}{ac}\right)$$

So, the desired capacitance is,

$$C = \frac{q}{V} = 4\pi\varepsilon_0\left(\frac{ac}{c-a}\right)$$

Now when the dielectric is filled between A and B, the electric field will change in this region. Therefore the potential difference and hence the capacitance of the system will change. So, first find the electric field $E(r)$ in the region $a \le r \le c$. Then find the potential difference (V) between A and C and finally the capacitance of the system will be,

$$C = \frac{q}{V}$$

where, q = charge on A

$$E(r) = \begin{cases} \dfrac{q}{4\pi\varepsilon_0 K r^2} & \text{for} \quad a \le r \le b \\ \dfrac{q}{4\pi\varepsilon_0 r^2} & \text{for} \quad b \le r \le c \end{cases}$$

Since $\Delta V = \int dV = -\int \vec{E}\cdot d\vec{r}$

So, the potential difference between A and C is

$$V = V_A - V_C = -\int_a^b \frac{q}{4\pi\varepsilon_0 K r^2}dr - \int_b^c \frac{q}{4\pi\varepsilon_0 r^2}dr$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{K}\left(\frac{1}{a}-\frac{1}{b}\right)+\left(\frac{1}{b}-\frac{1}{c}\right)\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(b-a)}{Kab}+\frac{(c-b)}{bc}\right]$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0 Kabc}\left[c(b-a)+Ka(c-b)\right]$$

The desired capacitance is,

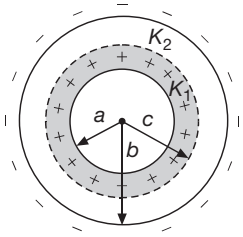
$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0 Kabc}{Ka(c-b)+c(b-a)}$$

ILLUSTRATION 38

A capacitor is formed of two concentric spherical conducting shells of radii a and b with $b > a$. If the medium between the spherical shells has a dielectric constant K_1, from a to c and K_2 from c to b, find the capacitance of the spherical capacitor.

SOLUTION

Let the charge on inner sphere be $+Q$ and that on the outer sphere be $-Q$.



$$V = -\int E dr = \frac{-Q}{4\pi\varepsilon_0}\left[\int_r^a \frac{dr}{K_1 r^2} + \int_b^r \frac{dr}{K_2 r^2}\right]$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{K_1}\left(\frac{1}{a}-\frac{1}{r}\right)+\frac{1}{K_2}\left(\frac{1}{r}-\frac{1}{b}\right)\right]$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(\frac{1}{K_2}-\frac{1}{K_1}\right)+\left(\frac{1}{K_1 a}-\frac{1}{K_2 b}\right)\right]$$

$$\Rightarrow \quad C = \frac{Q}{V} = 4\pi\varepsilon_0\left[\frac{1}{r}\left(\frac{1}{K_2}-\frac{1}{K_1}\right)+\left(\frac{1}{K_1 a}-\frac{1}{K_2 b}\right)\right]^{-1}$$

CYLINDRICAL CAPACITOR

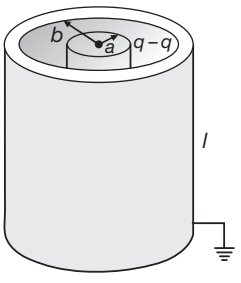
It consists of two coaxial metallic cylinders of inner radius a and outer radius b. The outer surface of the cylinder of radius b (outer one) is earthed. The space between the two cylinders is filled with material of dielectric constant K. Let inner cylinder be given a charge per unit length of $\lambda\left(=\frac{q}{l}\right)$. A charge $-q$ is induced on length l at inner surface of outer cylinder

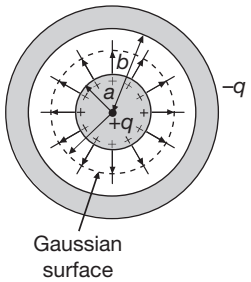
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } a < r < b$$

$$\Rightarrow \quad -\frac{dV}{dr} = \frac{\lambda}{2\pi\varepsilon_0 K r}$$

$$\Rightarrow \quad \int_{\text{inner surface}}^{\text{outer surface}} dV = -\frac{\lambda}{2\pi\varepsilon_0 K}\int_{r=a}^{r=b}\frac{dr}{r}$$

$$\Rightarrow \quad V_{\substack{\text{outer}\\\text{surface}}} - V_{\substack{\text{inner}\\\text{surface}}} = -\frac{\lambda}{2\pi\varepsilon_0 K}\log_e\left(\frac{b}{a}\right)$$





Since, inner surface is at higher potential and outer at lower potential, so

$$V_{\substack{\text{inner}\\\text{surface}}} - V_{\substack{\text{outer}\\\text{surface}}} = \frac{\lambda}{2\pi\varepsilon_0 K}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad V_{\substack{\text{inner}\\\text{surface}}} - V_{\substack{\text{outer}\\\text{surface}}} = \frac{q}{2\pi\varepsilon_0 lK}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad C = \frac{q}{V_{\substack{\text{inner}\\\text{surface}}} - V_{\substack{\text{outer}\\\text{surface}}}} = \frac{2\pi\varepsilon_0 lK}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C = \frac{2\pi\varepsilon_0 lK}{\log_e\left(\frac{b}{a}\right)}$$

ENERGY STORED IN A SPHERICAL SHELL

The electric field associated with a spherical shell of radius a is given by

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & r > a \\ \vec{0} & r < a \end{cases}$$

The corresponding energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

outside the sphere, and zero inside it. Since the electric field is non-vanishing outside the spherical shell, we must integrate over the entire region of space from $r = a$ to $r \to \infty$. In spherical coordinates the infinitesimal volume element $d\tau = 4\pi r^2 dr$, we have

$$U = \int u_E\, d\tau = \int_a^\infty \left(\frac{Q^2}{32\pi^2\varepsilon_0 r^4}\right)4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0}\int_a^\infty \frac{dr}{r^2}$$

$$\Rightarrow \quad U = \frac{Q^2}{8\pi\varepsilon_0 a} = \frac{1}{2}QV$$

where $V = \dfrac{Q}{4\pi\varepsilon_0 a}$ is the electric potential on the surface of the shell, with $V(\infty) = 0$.

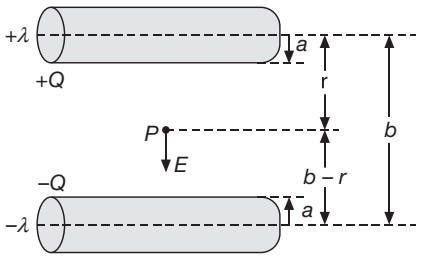
We can readily verify that the energy of the system is equal to the work done in charging the sphere. To show this, suppose at some instant the sphere has charge q and is at a potential $V = \dfrac{q}{4\pi\varepsilon_0 a}$. The work required to add an additional charge dq to the system is $dW = Vdq$. Thus, the total work is

$$W = \int dW = \int Vdq = \int_0^Q dq\left(\frac{q}{4\pi\varepsilon_0 a}\right) = \frac{Q^2}{8\pi\varepsilon_0 a}$$

ILLUSTRATION 39

Two long straight wires with equal cross-sectional radii a are located parallel to each other in air. The distance between their axes equals b. Find the mutual capacitance of the wires per unit length under the condition $b \gg a$.

SOLUTION



Let the charges on the two wires of length l be $+Q$ and $-Q$. The charge per unit length is $+\lambda$ and $-\lambda$ where $\lambda = \dfrac{Q}{l}$. The electric field at point P due to both wires is from positively charged wire towards the negatively charged wire and is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0 (b-r)}$$

The potential difference between the two wires is

$$V = -\int E dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{b-a}^{a}\left(\frac{1}{r} + \frac{1}{b-r}\right)dr$$

$$\Rightarrow \quad V = \frac{\lambda}{2\pi\varepsilon_0}(\ln r - \ln(b-r))\Big|_a^{b-a} = \frac{Q}{\pi\varepsilon_0 l}\ln\left(\frac{b-a}{a}\right)$$

$$\Rightarrow \quad C = \frac{Q}{V} = \frac{\pi\varepsilon_0 l}{\ln\left(\frac{b-a}{a}\right)}$$

Therefore, capacitance per unit length is

$$\frac{C}{l} = \frac{\pi\varepsilon_0}{\ln\left(\frac{b-a}{a}\right)}$$

Test Your Concepts-V

Based on Spherical and Cylindrical Capacitors

(Solutions on page H.139)

1. Two conducting spheres A and B each of radius $R = 9$ mm are separated by a very large distance $d = 9$ m. Calculate the capacitance of the system.

2. A spherical capacitor has the inner sphere of radius 2 cm and the outer one of 4 cm. If the inner sphere is earthed and the outer one is charged with a charge of 2 μC and isolated, calculate,

(a) the potential to which the outer sphere is raised.

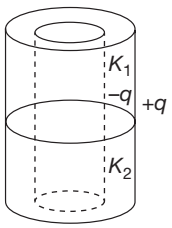
(b) the charge retained on the outer surface of the outer sphere. Give an account for the remaining charge.

3. N drops of mercury of equal radii and possessing equal charges combine to form a big drop. Compare the charge, capacitance, potential of bigger drop and energy stored by bigger with the corresponding quantities of individual drop.

4. Find the capacitance of a spherical capacitor made of two concentric spherical conductors of inner radius a and outer radius b, when the outer conductor is charged and the inner conductor is earthed.

5. Two conducting spheres are placed concentrically. The inner sphere is earthed and the outer given a charge 30 nC. The radii of sphere are 8 cm and 10 cm. What is the charge on the inner sphere? Will there be any charge on the inner surface of the outer sphere? Explain.

6. A cylindrical capacitor of inner and outer radii a and b respectively is filled with two dielectrics of constants K_1 and K_2. Each dielectric occupies half the length of the cylinder. Find the capacity of the system between the inner and outer cylinders.



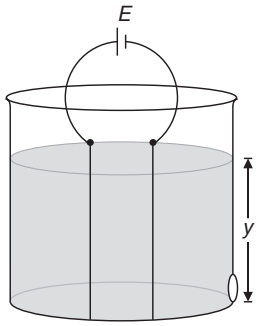
SOLVED PROBLEMS

PROBLEM 1

A parallel plate capacitor is placed in a cylindrical tank filled with a liquid of dielectric constant K. The area of cross-section of the tank is A and height of the liquid is equal to the length of the square plate of plate area l^2. The separation between the plates is d. A small orifice of area a is opened at the bottom of the tank at $t = 0$. If the capacitor in the process remains connected with a battery of emf E and assuming the level of liquid in the capacitor remains same as outside, find the current in the circuit as a function of time

SOLUTION

According to Equation of Continuity, Rate of Change of volume is av, where, v is the velocity of efflux given by



$$v = \sqrt{2gy}$$

$$\Rightarrow \quad A\left(-\frac{dy}{dt}\right) = av = a\sqrt{2gy}$$

$$\Rightarrow \quad -y^{-\frac{1}{2}}dy = \left(\frac{a\sqrt{2g}}{A}\right)dt$$

$$\Rightarrow \quad -\int_{l}^{y} y^{-\frac{1}{2}}dy = \left(\frac{a\sqrt{2g}}{A}\right)\int_{0}^{t} dt$$

$$\Rightarrow \quad 2\left(\sqrt{l} - \sqrt{y}\right) = \frac{a\sqrt{2g}}{A}t$$

$$\Rightarrow \quad \sqrt{y} = \sqrt{l} - \frac{a\sqrt{2g}}{2A}t$$

$$\Rightarrow \quad y = \left(\sqrt{l} - \frac{a\sqrt{2g}}{2A}t\right)^2 \qquad \ldots(1)$$

Capacitance of the capacitor at time t is,

$$C = \frac{\varepsilon_0 l(l-y)}{d} + \frac{K\varepsilon_0 yl}{d}$$

$\{\because$ the plate is a square plate of side $l\}$

Since, $q = EC$ and by definition of current,

$$I = \frac{dq}{dt}$$

Since here, q is decreasing with the passage of time, so

$$I = -\frac{dq}{dt}$$

$$\Rightarrow \quad I = \left(-\frac{dq}{dt}\right) = -E\left(\frac{dC}{dt}\right) = -E\left(\frac{K\varepsilon_0 l}{d} - \frac{\varepsilon_0 l}{d}\right)\frac{dy}{dt} \qquad \ldots(2)$$

From equation (1),

$$\frac{dy}{dt} = -\frac{a\sqrt{2g}}{A}\left(\sqrt{l} - \frac{a\sqrt{2g}}{2A}t\right)$$

Substituting in equation (2), we get

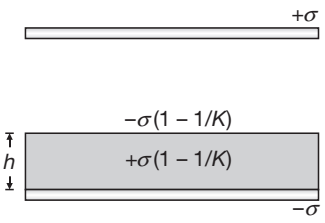
$$I = \frac{\varepsilon_0 El}{d}\left(\frac{a}{A}\sqrt{2gl} - \frac{a^2 gt}{A^2}\right)(K-1)$$

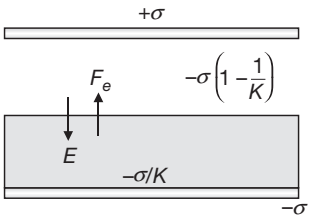
PROBLEM 2

A parallel plate capacitor is located horizontally so that one of its plate is just submerged into liquid while the other is over the surface. The permitivity of the liquid is equal to K and its density is ρ. To what height will the level of liquid in the capacitor rise after its plates get a charge of surface density σ.

SOLUTION

Let us consider the equilibrium of liquid inside the parallel plate capacitor. For that





Weight of the liquid (acting downwards) = electrostatic force on the liquid (acting upwards)

$$\Rightarrow \quad Ah\rho g = \sigma\left(1-\frac{1}{K}\right)AE \qquad \ldots(1)$$

where, $E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2K\varepsilon_0}$ …(2)

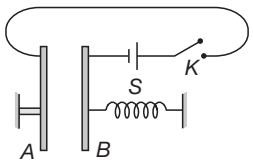
Substituting equation (2) in equation (1), we get

$$h = \frac{\sigma\left(1-\frac{1}{K}\right)\left(\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2K\varepsilon_0}\right)}{\rho g}$$

$$\Rightarrow \quad h = \frac{\sigma^2\left(K^2-1\right)}{2K^2\varepsilon_0\rho g}$$

PROBLEM 3

One plate A of a parallel plate capacitor AB is fixed, while the other is attached to the wall by a spring and can move parallel to the plate A. The two plates are joined to a cell through a key K which is initially open. When the key is closed permanently, the plate separation d decreases by $\eta = 10\%$. What will be the percentage fractional decrease between the plates if the key is closed for such a short interval that the separation between A and plate B does not change noticeably?



SOLUTION

When key is closed permanently, voltage between A and B remains constant. Now charge on capacitor changes because the capacitance changes as plate B moves towards plate A due to force of attraction between them.

The force of attraction between two capacitor plates is $F = \frac{1}{2}\varepsilon_0 E^2 A$

where A = Area of each plate and E = Electric field between plates.

Since plates move towards each other such that separation decreases by η. Hence, new separation d' becomes $d' = d - \eta d$.

If E' be the new electric field between plates, then

$$\Rightarrow \quad E' = \frac{V}{d-\eta d}$$

In equilibrium, force of attraction between plates is balanced by outward elastic force produced in the spring.

$$\Rightarrow \quad \text{Force of attraction} = \text{Elastic force}$$

$$\left(\frac{1}{2}\varepsilon_0 E'^2\right)A = K(d-d')$$

$$\Rightarrow \quad \frac{1}{2}\varepsilon_0\frac{V^2}{d^2(1-\eta)^2}A = K\eta d \qquad \ldots(1)$$

where ηd is the displacement from original position and K is the force constant of spring.

When key is closed for very short time, the capacitor collects some charge during that short interval at constant voltage and then B will move towards A, with constant voltage. If Q be the charge collected, then

$$Q = CV = \left(\frac{\varepsilon_0 A}{d}\right)V$$

Let new separation in equilibrium position be $(d - x_2)$

$$\text{Force of attraction} = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0\frac{V^2A}{d^2}$$

$$\text{where } E'' = \frac{V}{d-x_2} \cong \frac{V}{d} \qquad \{\because\ x_2 \ll d\}$$

$$\text{In equilibrium } \frac{1}{2}\varepsilon_0\frac{V^2A}{d^2} = Kx_2 \qquad \ldots(2)$$

Dividing (2) by (1)

$$\frac{Kx_2}{K\eta d} = \frac{\left(\frac{1}{2}\varepsilon_0\frac{V^2A}{d^2}\right)}{\left(\frac{\frac{1}{2}\varepsilon_0 V^2A}{d^2(1-\eta)^2}\right)}$$

$$\Rightarrow \quad \frac{x_2}{\eta d} = (1-\eta)^2$$

$$\Rightarrow \quad \frac{x_2}{d} = \eta(1-\eta)^2$$

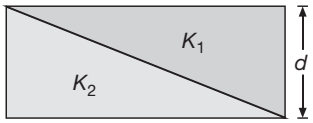
So, fractional change is

$$\frac{x_2}{d} = \eta(1-\eta)^2 = 0.1(1-0.1)^2 = 0.081$$

$\Rightarrow$ Percentage Fractional Change $= 8.1\%$

PROBLEM 4

The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively. Find capacitance of the resultant capacitor.

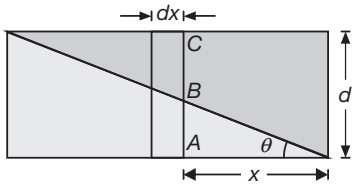


SOLUTION

Let the length of capacitor is l and breadth be b and d is distance between two plates and area of capacitor is A.

So $A = lb$

In this problem, first we will find the capacitance of small area considering a strip of width dx then integrate over. $x = 0$ to $x = l$ to get total capacitance of capacitor where l is length of each capacitor plate.



Hence consider a strip at distance x from right of width dx.

Now AB has K_1 and BC has K_2 dielectric constant

$$AB = x\tan\theta$$

$$BC = d - x\tan\theta$$

capacitance of strip

$$dC = \frac{\varepsilon_0 A_S}{\frac{AB}{K_1} + \frac{BC}{K_2}} \qquad \{\because\ A_S = \text{area of strip} = bdx\}$$

$$\Rightarrow \quad dC = \frac{\varepsilon_0 bdx}{\frac{x\tan\theta}{K_1} + \frac{d - x\tan\theta}{K_2}}$$

$$\Rightarrow \quad C = \int_0^C dC = \int_0^l \frac{\varepsilon_0 bdx}{\left(\frac{x\tan\theta}{K_1} + \frac{d - x\tan\theta}{K_2}\right)}$$

$$\Rightarrow \quad C = \varepsilon_0 bK_1K_2 \int_0^l \frac{dx}{K_1 d + (K_2 - K_1)x\tan\theta}$$

$$\Rightarrow C = \frac{\varepsilon_0 bK_1K_2}{(K_2 - K_1)\tan\theta}\left[\log_e\left(K_1 d + (K_2 - K_1)x\tan\theta\right)\right]\Big|_0^l$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 bK_1K_2}{(K_2 - K_1)\tan\theta}\log_e\left[\frac{K_1 d + (K_2 - K_1)l\tan\theta}{K_1 d}\right]$$

But $\tan\theta = \frac{d}{l}$ and $A = lb$

$$\Rightarrow \quad C = \frac{\varepsilon_0 bK_1K_2}{(K_2 - K_1)\frac{d}{l}}\log_e\left[\frac{K_1 d + (K_2 - K_1)d}{K_1 d}\right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 AK_1K_2}{(K_2 - K_1)d}\log_e\left(\frac{K_2}{K_1}\right)$$

PROBLEM 5

Consider a parallel plate capacitor whose plate separation is maintained by a dielectric of dielectric constant K and thickness d when the potential across the capacitor is zero. The dielectric strength of dielectric is E_0 and it is compressible having Young's modulus Y. The capacitance of the capacitor as the limit $V \to 0$ is C_0.

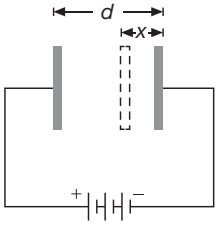
(a) Derive an expression for the capacitance as a function of voltage across the capacitor.

(b) Calculate the maximum voltage that can be applied to the capacitor?

Assume that dielectric constant of capacitor does not change under compression.

SOLUTION

(a) Let the voltage applied and compression in the dielectric be V and x respectively. Let us now consider an infinitesimal compression dx, during which the charge supplied by battery is dq.



Variable voltage source

From Conservation of Energy, we have

$$\begin{pmatrix}\text{Work done by battery}\end{pmatrix}=\begin{pmatrix}\text{Electrostatic energy in dielectric}\end{pmatrix}+\begin{pmatrix}\text{Elastic potential energy in dielectric}\end{pmatrix}$$

$$\Rightarrow \quad Vdq=\frac{1}{2}Vdq+\frac{YA}{d}xdx$$

$$\Rightarrow \quad \frac{1}{2}Vdq=\frac{YA}{d}xdx \qquad \ldots(1)$$

Since, $q=KCV$

$$\Rightarrow \quad dq=KV(dC) \qquad \ldots(2)$$

Substitute equation (2) in (1), we get

$$\frac{1}{2}KV^2dC=\frac{YA}{d}xdx \qquad \ldots(3)$$

Capacitance of the capacitor is given by

$$C=\frac{K\varepsilon_0 A}{(d-x)} \qquad \ldots(4)$$

$$\Rightarrow \quad \frac{dC}{dx}=\frac{K\varepsilon_0 A}{(d-x)^2} \qquad \ldots(5)$$

Now substituting equation (4) in (3) , we get

$$\frac{1}{2}KV^2\frac{K\varepsilon_0 A}{(d-x)^2}=\frac{YA}{d}x$$

$$\Rightarrow \quad V^2=\frac{2Y(d-x)^2}{K^2\varepsilon_0 d} \qquad \ldots(6)$$

For a small compression x, we have, $x \ll d$,

$$\Rightarrow \quad d-x\approx d$$

$$\Rightarrow \quad V^2\approx\frac{2Yd^2}{K^2\varepsilon_0 d}x$$

$$\Rightarrow \quad x=\left(\frac{K^2\varepsilon_0}{2Yd}\right)V^2 \qquad \ldots(7)$$

Substitute equation (7) in (4), we get

$$C=\frac{K\varepsilon_0 A}{d-\frac{V^2K^2\varepsilon_0}{2Yd}}=\frac{K\varepsilon_0 d}{d\left(1-\frac{V^2K^2\varepsilon_0}{2Yd^2}\right)}$$

For $x \ll d$, we have

$$C=\frac{K\varepsilon_0 A}{d}\left(1-\frac{V^2K^2\varepsilon_0}{2Yd^2}\right)^{-1}\approx\frac{K\varepsilon_0 A}{d}\left(1+\frac{V^2K^2\varepsilon_0}{2Yd^2}\right)$$

Since at $V=0$, we have $C\to C_0=\frac{K\varepsilon_0 A}{d}$

$$\Rightarrow \quad C=C_0\left(1+\frac{V^2K^2\varepsilon_0}{2Yd^2}\right)=C_0\left(1+\frac{x}{d}\right)$$

(b) Let x_0 be the compression when the dielectric breakdown occurs. The potential difference at this position is given by

$$V_0=E_0\left(d-x_0\right)$$

Substitute this in equation (6) , we get

$$E_0^2=\frac{2Y}{K^2\varepsilon_0 d}x_0$$

$$\Rightarrow \quad x_0=\frac{E_0^2K^2\varepsilon_0 d}{2Y}$$

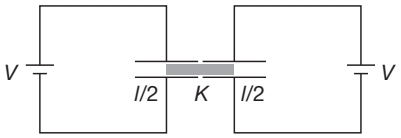
Substitute x_0 in expression for V_0, we get

$$V_0=E_0\left(d-\frac{E_0^2K^2\varepsilon_0 d}{2Y}\right)$$

$$\Rightarrow \quad V_0=E_0 d\left(1-\frac{E_0^2K^2\varepsilon_0}{2Y}\right)$$

PROBLEM 6

In figure, the capacitors have plate area $A(=l\times b)$ and separation 'd'. Now, if the slab of mass m is displaced slightly, calculate the time period of the oscillation. Is the motion simple harmonic?



SOLUTION

Let the slab be displaced to the left by a distance x, then the work done W is given by

W = change in stored energy + work done by battery

Let C_0 be the initial capacitance of capacitor, then

$$C_0 = \frac{\varepsilon_0 lb}{2d} + \frac{\varepsilon_0 kbl}{2d}$$

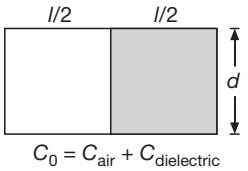
Let C_1 and C_2 be the final capacitances, then

$$C_1 = \frac{\varepsilon_0 b}{d}\left(\frac{l}{2} - x\right) + \frac{\varepsilon_0 bk}{d}\left(\frac{l}{2} + x\right) \text{ and}$$

$$C_2 = \frac{\varepsilon_0 b}{d}\left(\frac{l}{2} + x\right) + \frac{\varepsilon_0 bk}{d}\left(\frac{l}{2} - x\right)$$

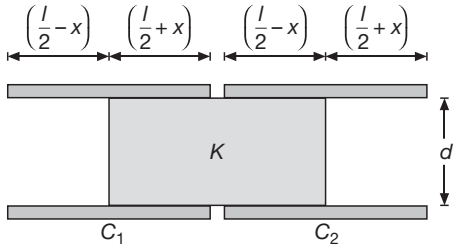
Since $F = \dfrac{2\varepsilon_0 b(K-1)V^2}{2d}$ {As done earlier}

$$\Rightarrow \quad a = \frac{F}{m} = \frac{\varepsilon_0 b}{md}(k-1)V^2$$



Hence, motion is oscillatory but not SHM because acceleration is constant. For constant acceleration motion,

$$x = \frac{1}{2}at^2$$

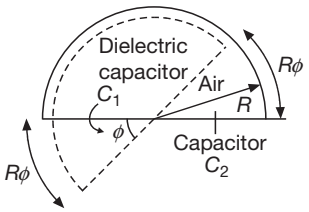


$$\Rightarrow \quad t = \sqrt{\frac{2x}{a}}$$

Time period $T = 4t = \sqrt{\dfrac{32mxd}{\varepsilon_0 b(k-1)V^2}}$

PROBLEM 7

A capacitor consists of two stationary plates shaped as a semi-circle of radius R and a movable plate made of dielectric with permittivity K and capable of rotating about an axis O between the stationary plates. The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in figure.



SOLUTION

Let C_0 be the initial capacitance of the condenser.

$$C_0 = \frac{K\varepsilon_0\left(\dfrac{\pi R^2}{2}\right)}{d} = \frac{K\varepsilon_0 \pi R^2}{2d} \qquad \ldots(1)$$

When the dielectric is turned through an angle ϕ it divides the capacitor into an air capacitor and a dielectric capacitor. Let C_1 be the new capacity of dielectric capacitor. Then

$$C_1 = \frac{K\varepsilon_0\left(\dfrac{\pi R^2}{2} - \dfrac{R^2\phi}{2}\right)}{d} \qquad \ldots(2)$$

If C_2 be the capacity of air, then

$$C_2 = \frac{\varepsilon_0\left(\dfrac{R^2\phi}{2}\right)}{d} = \frac{\varepsilon_0 R^2\phi}{2d} \qquad \ldots(3)$$

The arrangement is equivalent to two capacitors (of capacitance C_1 and C_2) connected in parallel because they have common plates. So,

$$C = C_1 + C_2$$

$$\Rightarrow \quad C = \frac{K\varepsilon_0(\pi R^2)}{2d} - \frac{K\varepsilon_0 R^2\phi}{2d} + \frac{\varepsilon_0 R^2\phi}{2d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 R^2}{2d}[K\pi + (1-K)\phi] \qquad \ldots(4)$$

Energy of capacitor $= \frac{1}{2}CV^2$

$$\Rightarrow \quad U = \frac{1}{4}\frac{\varepsilon_0 R^2}{d}[K\pi + (1-K)\phi]V^2$$

For a conservative force,

$$\tau = -\frac{\partial U}{\partial \phi}$$

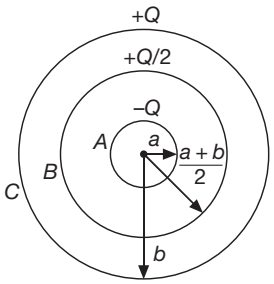
$$\Rightarrow \quad \tau = -\frac{\partial}{\partial \phi}\left[\frac{1}{4}\frac{\varepsilon_0 R^2}{d}\{K\pi + (1-K)\phi\}\right]V^2$$

$$\Rightarrow \quad \tau = -\frac{\varepsilon_0 R^2}{4d}(K-1)V^2$$

The negative sign shows that the torque acts opposite to angular displacement.

PROBLEM 8

Figure shows three conducting spherical shells A, B and C with charges $-Q$, $+\frac{Q}{2}$ and $+Q$ respectively. Calculate the capacitance of the system between points A and C.



SOLUTION

Potential at A, V_A is given by

$$(V_A) = \begin{pmatrix}\text{Potential}\\ \text{at } A \text{ due}\\ \text{to charge}\\ \text{on } A\end{pmatrix} + \begin{pmatrix}\text{Potential}\\ \text{at } A \text{ due}\\ \text{to charge}\\ \text{on } B\end{pmatrix} + \begin{pmatrix}\text{Potential}\\ \text{at } A \text{ due}\\ \text{to charge}\\ \text{on } C\end{pmatrix}$$

$$\Rightarrow \quad V_A = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{a} + \frac{\frac{Q}{2}}{4\pi\varepsilon_0\left(\frac{a+b}{2}\right)} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$\Rightarrow \quad V_A = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right)$$

Potential at C, V_C is given by

$$(V_C) = \begin{pmatrix}\text{Potential}\\ \text{at } C \text{ due}\\ \text{to charge}\\ \text{on } A\end{pmatrix} + \begin{pmatrix}\text{Potential}\\ \text{at } C \text{ due}\\ \text{to charge}\\ \text{on } B\end{pmatrix} + \begin{pmatrix}\text{Potential}\\ \text{at } C \text{ due}\\ \text{to charge}\\ \text{on } C\end{pmatrix}$$

$$\Rightarrow \quad V_C = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q}{8\pi\varepsilon_0 b}$$

Potential difference, $\Delta V = V_A - V_C$

$$\Rightarrow \quad \Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right) - \frac{Q}{8\pi\varepsilon_0 b}$$

$$\Rightarrow \quad \Delta V = \frac{Q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab(a+b)}\right)$$

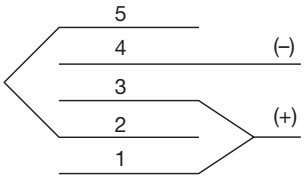
Capacitance of the arrangement is

$$C_{AC} = \frac{Q}{\Delta V} = \frac{Q}{V_A - V_C}$$

$$\Rightarrow \quad C_{AC} = \frac{Q}{\left\{\frac{Q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab(a+b)}\right)\right\}} = \frac{8\pi\varepsilon_0 ab(a+b)}{(a^2 - 2b^2 + ab)}$$

PROBLEM 9

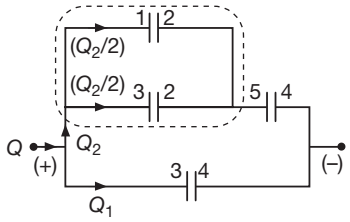
Five identical conducting plates 1, 2, 3, 4 and 5 are fixed parallel to and equidistant from each other as shown in Figure. Plates 2 and 5 are connected by a conductor while 1 and 3 are joined by another conductor. The junction of 1 and 3 and the plate 4 are connected to a source of constant emf V_0. If d is the distance between any two successive plates and A is the area of either face of each plate. Find



(a) the effective capacitance of the system between the terminals of the source.
(b) the charges on plates 3 and 5.

SOLUTION

(a) The equivalent circuit is shown here. The system consists of four capacitors, C_{12}, C_{32}, C_{34} and C_{54}. The capacitance of each capacitor is $C_0 = \dfrac{\varepsilon_0 A}{d}$.



The capacitors C_{12} and C_{32} are in parallel, and hence their net capacitance is $C_0 + C_0 = 2C_0$. The capacitor C_{54} is in series with the parallel combination of C_{12} and C_{32}. Hence the resultant capacitance will be $\dfrac{C_0 \times 2C_0}{C_0 + 2C_0}$

Since, C_{34} is again in parallel with the combination of C_{12}, C_{32}, C_{54}. Hence the effective capacitance C is

$$C = C_0 + \frac{C_0 \times 2C_0}{C_0 + 2C_0} = \frac{5}{3}C_0 = \frac{5}{3}\left(\frac{\varepsilon_0 A}{d}\right)$$

(b) Charge on the plate 5 = Charge on the upper half of parallel combination

$$\Rightarrow \quad Q_5 = V_0\left(\frac{2}{3}C_0\right) = \frac{2}{3}\left(\frac{\varepsilon_0 A V_0}{d}\right)$$

Charge on plate 3 on the surface facing 4 is

$$Q_{34} = V_0 C_0 = \frac{\varepsilon_0 A V_0}{d}$$

Charge on plate 3 on the surface facing 2 = (potential difference across 3 – 2 combination) C_0

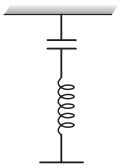
$$Q_{32} = V_0 \frac{C_0}{C_0 + 2C_0} C_0 = \varepsilon_0 \frac{A V_0}{3d}$$

$$\Rightarrow \quad Q_{32} = \frac{\varepsilon_0 A V_0}{d} + \varepsilon_0 \frac{A V_0}{3d}$$

$$\Rightarrow \quad Q_{32} = \frac{\varepsilon_0 A V_0}{d}\left(1 + \frac{1}{3}\right) = \frac{4}{3}\left(\frac{\varepsilon_0 A V_0}{d}\right)$$

PROBLEM 10

A parallel plate capacitor with air as a dielectric is arranged horizontally, such that its one plate is fixed and the other plate is connected with a perpendicular spring. The area of each plate is A. In the steady position, the separation between the plates is d_0. When the capacitor is connected with an electric source with the voltage V, a new equilibrium appears, such that the new separation between the plates becomes d_1. Assuming mass of the upper plate to be m, find the



(a) spring constant K.
(b) maximum voltage for a given K in which an equilibrium is possible.
(c) angular frequency of the oscillating system around the equilibrium value d_1 (amplitude of the oscillation << d_1).

SOLUTION

(a) Let the initial extension in the spring, in equilibrium situation be x_0. Then, for equilibrium of lower plate 2, we must have kx_0 (acting upwards) balancing mg (acting downwards)

$$Kx_0 = mg$$

When the voltage source is connected, then the plate separation changes from d_0 to d_1, as a result of which the extension of the spring becomes $(d_0 - d_1)$, so that net extension in the spring becomes $x_0 + (d_0 - d_1)$

When equilibrium is attained again, we have

$$K\left[x_0 + (d_0 - d_1)\right] = mg + \frac{1}{2}\varepsilon_0 E^2 A$$

Since, $Kx_0 = mg$

$$\Rightarrow \quad K(d_0-d_1)=\frac{1}{2}\varepsilon_0\left(\frac{V}{d_1}\right)^2 A \qquad \ldots(1)$$

$$\Rightarrow \quad K=\frac{\varepsilon_0 AV^2}{2d_1^2(d_0-d_1)}$$

(b) From equation (1), we have

$$V^2=\frac{2Kd_1^2(d_0-d_1)}{\varepsilon_0 A}=\frac{2K}{\varepsilon_0 A}\left(d_0d_1^2-d_1^3\right)\ldots(2)$$

Differentiating w.r.t. d_1, we get

$$2V\left(\frac{dV}{dd_1}\right)=\frac{2K}{A\varepsilon_0}\left(2d_1d_0-3d_1^2\right)$$

For V to be maximum, we have

$$\left(\frac{dV}{dd_1}\right)=0$$

$$\Rightarrow \quad 2d_1d_0-3d_1^2=0$$

$$\Rightarrow \quad d_1=\frac{2}{3}d_0$$

Substituting $d_1=\frac{2d_0}{3}$ in (2), we get

$$V_{\max}^2=\frac{2K\left(\frac{2}{3}d_0\right)^2\left(d_0-\frac{2}{3}d_0\right)}{\varepsilon_0 A}$$

$$\Rightarrow \quad V_{\max}=\frac{2d_0}{3}\sqrt{\frac{2Kd_0}{3A\varepsilon_0}} \qquad \ldots(3)$$

(c) Let a small displacement x be given to the upper plate in the downward direction from the equilibrium position. Then the net force on the plate is

$$F=-K\left[x_0+(d_0-d)+x\right]+mg+\frac{1}{2}\varepsilon_0 A\left[\frac{V^2}{(d_1-x)^2}\right]$$

$$\left\{\because\ E=\frac{V}{d_1-x}\right\}$$

$$\Rightarrow \quad F=-K(d_0-d)-Kx+\frac{1}{2}\frac{\varepsilon_0 AV^2}{d_1^2}\left(1-\frac{x}{d_1}\right)^{-2}$$

$$\Rightarrow \quad F=-K(d_0-d)-Kx+\frac{1}{2}\frac{\varepsilon_0 AV^2}{d_1^2}\left(1+\frac{2x}{d_1}\right)$$

$$\left\{\because\ \text{For } x\ll d_1,\ \left(\frac{1-x}{d_1}\right)^{-2}\simeq 1+\frac{2x}{d_1}\right\}$$

$$\Rightarrow \quad F=-K(d_0-d)-Kx+K(d_0-d_1)\left(1+\frac{2x}{d_1}\right)$$

$$\Rightarrow \quad F=-Kx\left(\frac{3d_1-2d_0}{d_1}\right)$$

$$\Rightarrow \quad m\ddot{x}=-Kx\left(\frac{3d_1-2d_0}{d_1}\right)$$

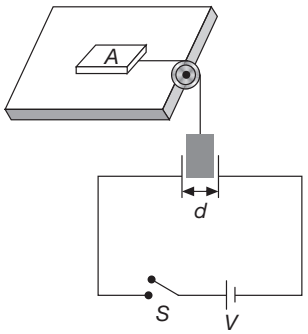
$$\Rightarrow \quad \ddot{x}+\frac{K}{m}\left(\frac{3d_1-2d_0}{d_1}\right)x=0$$

Compare with the standard equation of SHM, $\ddot{x}+\omega^2x=0$, we get

$$\omega=\sqrt{\frac{K}{m}\left(\frac{3d_1-2d_0}{d_1}\right)}$$

PROBLEM 11

A block A of mass m kept on a rough horizontal surface is connected to a dielectric slab of mass $\frac{m}{6}$ and dielectric constant k by means of a light and inextensible string passing over a fixed pulley as shown in figure. The dielectric can completely fill the space between the parallel plate capacitor of plate area $l\times l$ and separation between the plates d kept in vertical position. Initially switch S is open and length of the dielectric inside the capacitor is b. The coefficient of friction between the block A and the surface is $\mu=\frac{1}{4}$. Ignore any other friction.



(a) Find the minimum value of the emf V of the battery so that after closing the switch the block A will move.

(b) If $V = 2V_{\min}$ find the speed of the block A when the dielectric completely fills the space between the plates of the capacitor.

SOLUTION

(a) The forces acting on the dielectric are electrostatic attractive force of field of capacitor and its weight. The block will slip when

$$F_E + mg \geq \mu Mg$$

$$\Rightarrow \quad F_E \geq \frac{M}{4}g - \frac{M}{6}g$$

Since, $F_E = \frac{\varepsilon_0 l}{2d}(K-1)V^2$

$$\Rightarrow \quad \frac{1}{2}\frac{\varepsilon_0 l}{d}(K-1)V^2 \geq \frac{Mg}{12}$$

$$\Rightarrow \quad V_{\min} = \sqrt{\frac{Mg}{12}\times\frac{2d}{\varepsilon_0 l(K-1)}} = \sqrt{\frac{Mgd}{6\varepsilon_0 l(K-1)}}$$

(b) Now $V = 2V_{\min}$. In this case the block will accelerate. So, writing equations of motion for the dielectric and the block, we get

For dielectric

$$F_E' + mg - T = ma \qquad \ldots(1)$$

For block

$$T - \mu Mg = Ma \qquad \ldots(2)$$

Equation (1) and (2) give

$$a = \frac{F_E' + (m - \mu M)g}{m + M}$$

Since, $F_E = \frac{\varepsilon_0 l}{2d}(K-1)V^2$

So, at $V = 2V_{\text{MIN}} = 2\sqrt{\frac{Mgd}{6\varepsilon_0(K-1)l}}$, we get

$$F_E = \frac{\varepsilon_0 l}{2d}(K-1)\times 4\times\frac{Mgd}{6\varepsilon_0 l(K-1)} = \frac{Mg}{3}$$

$$\left\{\because\ V = 2V_{\text{MIN}} = 2\sqrt{\frac{Mgd}{6\varepsilon_0 l(K-1)}}\right\}$$

$$\text{Thus } a = \frac{\frac{Mg}{3} + \left(\frac{M}{6} - \frac{M}{4}\right)g}{\frac{M}{6} + M} = \frac{\frac{Mg}{3} - \frac{Mg}{12}}{\frac{7M}{6}} = \frac{3g}{14}$$

$$\Rightarrow \quad a = \frac{3g}{14}$$

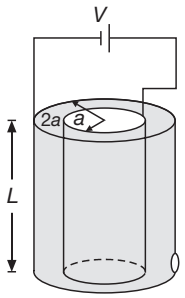
Using equation of motion, $v^2 = 2as$, we get

$$v^2 = 2\left(\frac{3g}{14}\right)(l-b)$$

$$\Rightarrow \quad v = \sqrt{\frac{3}{7}g(l-b)}$$

PROBLEM 12

A cylindrical capacitor of inner radius a, outer radius $2a$ and length L is kept with its axis vertical. Lower cross-section of the capacitor is sealed with very thin dielectric material and the curved space is filled completely with the oil of dielectric constant K. The plates are connected to a battery of emf V. Suddenly an orifice of cross-sectional area A is pierced at the bottom of the cylinder. Find the expression for current in the connecting wires (of negligible resistance) as a function of time.



SOLUTION

Let the liquid column inside the cylinder occupies a height x after time t. So, the air column gets a height $(L-x)$. The arrangement shown is now a parallel combination of two capacitors of capacitances C_1 and C_2 where

$$C_1 = \frac{2\pi\varepsilon_0 xK}{\log_e(2)} \qquad \ldots(1)$$

$$C_2 = \frac{2\pi\varepsilon_0(L-x)}{\log_e(2)} \qquad \ldots(2)$$

Hence, equivalent capacitance C is given by

$$C = C_1 + C_2 = \frac{2\pi\varepsilon_0}{\log_e(2)}[L + x(K-1)] \qquad \ldots(3)$$

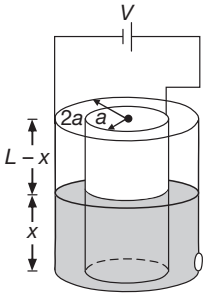
The cross-sectional area of cylinder in which the liquid is present is

$$A' = \pi(4a^2 - a^2) = 3\pi a^2$$

Using equation of continuity, we get

$$A\sqrt{2gx} = -A'\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad dt = -3\frac{dx}{\sqrt{2gx}} \qquad \{\because A' = 3\pi a^2 \text{ and } A = \pi a^2\}$$



Integrating, we get

$$\int_0^t dt = -\frac{3}{\sqrt{2g}}\int_L^x \frac{dx}{\sqrt{x}}$$

$$\Rightarrow \quad t = -\frac{3}{\sqrt{2g}}\left[2(\sqrt{x})\Big|_L^x\right] = 3\sqrt{\frac{2}{g}}(\sqrt{L} - \sqrt{x})$$

$$\Rightarrow \quad t = 3\sqrt{\frac{2}{g}}(\sqrt{L} - \sqrt{x})$$

$$\Rightarrow \quad x = \left(\sqrt{L} - \frac{t}{3}\sqrt{\frac{g}{2}}\right)^2$$

$$\Rightarrow \quad x = L - \frac{2}{3}\sqrt{\frac{gL}{2}}t + \frac{g}{18}t^2 \qquad \ldots(4)$$

This variation in x will change the value of C and hence q changes with t so as to produce a current, say I. To calculate C as a function of t, substitute x from equation (4) in equation (3), we get

$$C = \frac{2\pi\varepsilon_0}{\log_e(2)}\left[L + \left(L - \left(\frac{2}{3}\right)\sqrt{\frac{gL}{2}}t + \frac{g}{18}t^2\right)(K-1)\right]$$

$$\Rightarrow \quad C = \frac{2\pi\varepsilon_0}{\log_e(2)}\left[KL - \frac{1}{3}\sqrt{2gL}(K-1)t + \frac{g}{18}(K-1)t^2\right]$$

Now, the charge on capacitor is $Q = CV$

So, current in the connecting wires is given by

$$I = \frac{dQ}{dt} = V\left(\frac{dC}{dt}\right) \qquad \{\text{Since, } V \text{ is a constant}\}$$

where $\frac{dC}{dt} = \frac{2\pi\varepsilon_0}{\log_e(2)}(K-1)\left(\frac{gt}{18} - \frac{\sqrt{2gL}}{3}\right)V$

$$\Rightarrow \quad I = \frac{2\pi\varepsilon_0(K-1)V}{\log_e(2)}\left(\frac{gt}{18} - \frac{\sqrt{2gL}}{3}\right)$$

PROBLEM 13

A charge Q is imparted to two identical plane capacitors connected in parallel. At the moment of time $t = 0$ the distance between the plates of the first capacitor begins to increase uniformly according to the law $d_1 = d_0 + vt$, and the distance between the plates of the second capacitor to decrease uniformly according to the law $d_2 = d_0 - vt$.

(a) Neglecting the resistance of the connecting wires, find the current in the circuit when the plates of the capacitors move.

(b) Find the work performed by an electrostatic field when the distance between the plates of the first capacitor increases and that between the plates of the second capacitor simultaneously decreases by a.

SOLUTION

(a) Let us denote the charges on the first and second capacitors at time t by q_1 and q_2. So, we must have

$$q_1 + q_2 = Q \text{ and} \qquad \ldots(1)$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \ldots(2)$$

Since, at time t

$$C_1 = \frac{\varepsilon_0 A}{(d_0 + vt)} \text{ and } C_2 = \frac{\varepsilon_0 A}{(d_0 - vt)}$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt} \qquad \ldots(3)$$

$$\Rightarrow \quad q_1 = Q\left(\frac{d_0 - vt}{2d_0}\right) \text{ and } q_2 = Q\left(\frac{d_0 + vt}{2d_0}\right)$$

The reduction in the charge on the first capacitor is equal to the increase of the charge on the second capacitor. The current is

$$I = -\frac{\Delta q_1}{\Delta t} = \frac{\Delta q_2}{\Delta t} = \frac{Qv}{2d_0}$$

The current will flow from the positively charged plate of the first capacitor to the positively charged plate of the second capacitor.

(b) Since the resistance of the connecting wires is zero, the quantity of heat liberated is also zero. Hence the change in the electrostatic energies of the two capacitors will be equal to the work performed by the electrostatic field.

At time t, the energies of the first and the second capacitors will be

$$U_1 = \frac{q_1^2}{2C_1} = \frac{1}{8\varepsilon_0}\frac{Q^2}{Ad_0^2}(d_0 - vt)^2(d_0 + vt)$$

$$U_1 = \frac{Q^2}{8\varepsilon_0 Ad_0^2}(d_0 - a)^2(d_0 + a)$$

and $$U_2 = \frac{q_2^2}{2C_2} = \frac{1}{8\varepsilon_0}\frac{Q^2}{Ad_0^2}(d_0 + vt)^2(d_0 - vt)$$

$$U_2 = \frac{Q^2}{8\varepsilon_0 Ad_0^2}(d_0 + a)^2(d_0 - a)$$

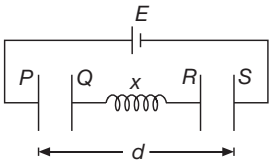
The total energy is given by

$$U = U_1 + U_2 = \frac{1}{4\varepsilon_0}\frac{Q^2}{Ad_0}(d_0^2 - a^2)$$

Therefore, the energy will drop by $\Delta U = \frac{Q^2}{4\varepsilon_0 Ad_0}a^2$ during the time t. This change will be equal to the work W performed by the electrostatic field.

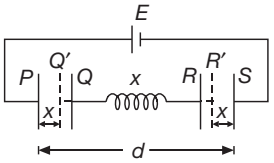
PROBLEM 14

Two parallel plate capacitors with area A are connected through a conducting spring of natural length l in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



SOLUTION

Let charge on capacitors be q and separation between plates P and Q and R and S be x at any time. Distance between plates P and Q and R and S is same because the force acting on them is same.



Capacitance of capacitor PQ is given by

$$C_1 = \frac{\varepsilon_0 A}{x}$$

Capacitance of capacitor RS is given by

$$C_2 = \frac{\varepsilon_0 A}{x}$$

From Kirchhoff's Voltage Law (KVL), we have

$$\frac{q}{C_1} + \frac{q}{C_2} = E$$

$$\Rightarrow \quad q = \frac{\varepsilon_0 AE}{2x}$$

At this moment extension in spring, $y = d - 2x - l$ and force on plate Q towards P is given by

$$F_1 = \frac{q^2}{2A\varepsilon_0} = \frac{\varepsilon_0^2 A^2 E^2}{8Ax^2\varepsilon_0} = \frac{A\varepsilon_0 E^2}{8x^2} \qquad \ldots(1)$$

Spring force on plate Q due to extension in spring is given by

$$F_2 = ky$$

At equilibrium,

Separation between the plates = Extension in the spring

Thus $x = y = d - 2x - l$

$$\Rightarrow \quad x = \frac{d-l}{3} \qquad \ldots(2)$$

$$\Rightarrow \quad F_1 = F_2 = kx = ky \qquad \ldots(3)$$

From equations (1) and (3), we get

$$\frac{A\varepsilon_0 E^2}{8x^2} = ky = kx$$

$$\Rightarrow \quad x = \left(\frac{A\varepsilon_0 E^2}{8k}\right)^{\frac{1}{3}} \qquad \ldots(4)$$

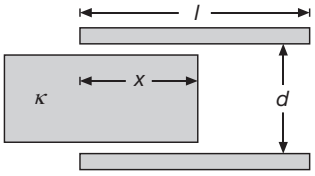
From equations (2) and (4), we get

$$\left(\frac{d-l}{3}\right)^3 = \frac{A\varepsilon_0 E^2}{8k}$$

$$\Rightarrow \quad k = \frac{27\varepsilon_0 AE^2}{8(d-l)^3}$$

PROBLEM 15

A capacitor is constructed from two square plates of sides l and separation $d(<<l)$, as suggested in figure. The plates carry charges $+Q_0$ and $-Q_0$. A block of metal has a width l, a length l and a thickness slightly less than d. It is inserted a distance x into the capacitor. The charges on the plates are not disturbed as the block slides in. In a static situation, a metal prevents an electric field from penetrating inside it.



(a) Calculate the stored energy as a function of x.

(b) Find the direction and magnitude of the force that acts on the metallic block.

(c) The area of the advancing front face of the block is essentially equal to ld. Considering the force on the block as acting on this face, find the stress (force per area) on it.

(d) For comparison, express the energy density in the electric field between the capacitor plates in terms of Q_0, l, d and ε_0.

SOLUTION

Since, the metal can be thought of as a dielectric having $\kappa \to \infty$, so the portion of the capacitor nearly filled by metal has capacitance $\frac{\kappa\varepsilon_0(lx)}{d} \to \infty$ and stored energy $\frac{Q^2}{2C} \to 0$

The unfilled portion has capacitance $\frac{\varepsilon_0 l(l-x)}{d}$

The charge on this portion is $Q = \frac{(l-x)Q_0}{l}$

(a) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{\left[(l-x)\frac{Q_0}{l}\right]^2}{2\varepsilon_0 \frac{l(l-x)}{d}} = \frac{Q_0^2(l-x)d}{2\varepsilon_0 l^3}$$

(b) $$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2(l-x)d}{2\varepsilon_0 l^3}\right) = +\frac{Q_0^2 d}{2\varepsilon_0 l^3}$$

$$\Rightarrow \quad F = \frac{Q_0^2 d}{2\varepsilon_0 l^3} \text{ to the right (into the capacitor)}$$

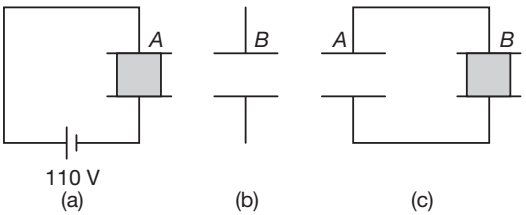
(c) $$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{ld} = \frac{Q_0^2}{2\varepsilon_0 l^4}$$

(d) Energy density

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0\left(\frac{Q_0}{\varepsilon_0 l^2}\right)^2 = \frac{Q_0^2}{2\varepsilon_0 l^4}$$

PROBLEM 16

Two parallel plate capacitors A and B have the same separation $d = 8.85\times10^{-4}$ m between the plates. The plate areas of A and B are $0.04\ \text{m}^2$ and $0.02\ \text{m}^2$ respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.



(a) The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

(b) The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

(c) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system.

SOLUTION

(a) Capacitor A is a combination of two capacitors C_K and C_O in parallel. Hence,

$$C_A = C_K + C_O = \frac{K\varepsilon_o A}{d} + \frac{\varepsilon_o A}{d} = (K+1)\frac{\varepsilon_o A}{d}$$

Here, $A = 0.02\ \text{m}^2$. Substituting the values, we have

$$C_A = (9+1)\frac{8.85\times10^{-12}(0.02)}{(8.85\times10^{-4})}\ \text{F}$$

$$\Rightarrow \quad C_A = 2\times10^{-9}\ \text{F}$$

Energy stored in capacitor A, when connected with a 110 V battery is

$$U_A = \frac{1}{2}C_A V^2 = \frac{1}{2}(2\times10^{-9})(110)^2$$

$$\Rightarrow \quad U_A = 1.21\times10^{-5}\ \text{J}$$

(b) Charge stored in the capacitor

$$q_A = C_A V = (2\times10^{-9})(110)$$

$$\Rightarrow \quad q_A = 2.2\times10^{-7}\ \text{C}$$

Now, this charge remains constant even after battery is disconnected. But when the slab is removed, capacitance of A will get reduced. Let it be C'_A

$$C'_A = \frac{\varepsilon_o(2A)}{d} = \frac{(8.85\times10^{-12})(0.04)}{8.85\times10^{-4}}\ \text{F}$$

$$\Rightarrow \quad C'_A = 0.4\times10^{-9}\ \text{F}$$

Energy stored in this case would be

$$U'_A = \frac{1}{2}\frac{(q_A)^2}{C'_A} = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(0.4\times10^{-9})}\ \text{J}$$

$$\Rightarrow \quad U'_A = 6.05\times10^{-5}\ \text{J} > U_A$$

Therefore, work done to remove the slab would be

$$W = U'_A - U_A = (6.05-1.21)\times10^{-5}\ \text{J}$$

$$\Rightarrow \quad W = 4.84\times10^{-5}\ \text{J}$$

(c) Capacity of B when filled with dielectric is given by

$$C_B = \frac{K\varepsilon_o A}{d} = \frac{(9)(8.85\times10^{-12})(0.02)}{(8.85\times10^{-4})}\ \text{F}$$

$$\Rightarrow \quad C_B = 1.8\times10^{-9}\ \text{F}$$

These two capacitors are in parallel. Therefore, net capacitance of the system is

$$C = C'_A + C_B = (0.4+1.8)\times10^{-9}\ \text{F}$$

$$\Rightarrow \quad C = 2.2\times10^{-9}\ \text{F}$$

Charge stored in the system is $q = q_A = 2.2\times10^{-7}\ \text{C}$

Therefore, energy stored, $U = \frac{1}{2}\frac{q^2}{C}$

$$\Rightarrow \quad U = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(2.2\times10^{-9})}$$

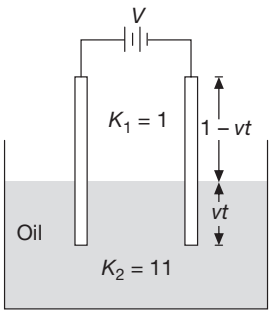
$$\Rightarrow \quad U = 1.1\times10^{-5}\ \text{J}$$

PROBLEM 17

Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of $0.001\ \text{ms}^{-1}$. Calculate the current drawn from the battery during the process. (Dielectric constant of oil $= 11$, $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-1}$).

SOLUTION

Let a be the side of the square plate.



As shown in figure, C_1 and C_2 are in parallel. Therefore, total capacity of capacitors in the position shown is

$$C = C_1 + C_2$$

$$\Rightarrow \quad C = \frac{\varepsilon_o a(a-x)}{d} + \frac{K\varepsilon_o a x}{d}$$

$$\Rightarrow \quad q = CV = \frac{\varepsilon_o a V}{d}(a - x + Kx)$$

As plates are lowered in the oil, C increases or charge stored will increase.

Therefore, $I = \frac{dq}{dt} = \frac{\varepsilon_o a V}{d}(K-1)\frac{dx}{dt}$

Substituting the values

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2\text{Nm}^{-2}$$

$a = 1$ m, $V = 500$ V, $d = 0.01$ m, $K = 1$ and

$\frac{dx}{dt}$ = speed of plate $= 0.001$ ms^{-1}

We get current

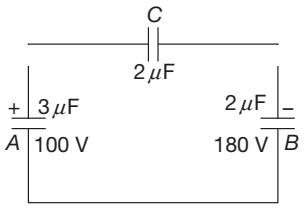
$$I = \frac{\left(8.85 \times 10^{-12}\right)(500)1(11-1)(0.001)\text{ A}}{(0.01)}$$

$$\Rightarrow \quad I = 4.43 \times 10^{-9}\text{ A}$$

PROBLEM 18

Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate:

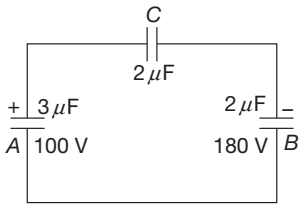
(a) the final charge on the three capacitors and
(b) the amount of electrostatic energy stored in the system before and after completion of the circuit.



SOLUTION

(a) Charge on capacitor A, before joining with an uncharged capacitor

$$q_A = CV = (100)(3)\ \mu\text{C} = 300\ \mu\text{C}$$



Similarly, charge on capacitor B

$$q_B = (180)(2)\ \mu\text{C} = 360\ \mu\text{C}$$

Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in figure alongside (q_1, q_2 and q_3 are in microcoulombs).

From conservation of charge

Net charge on plates 2 and 3 before joining = net charge after joining

$$\Rightarrow \quad 300 = q_1 + q_2 \qquad \ldots(1)$$

Similarly, net charge on plates 4 and 5 before joining = net charge after joining.

$$-360 = -q_2 - q_3$$

$$\Rightarrow \quad 360 = q_2 + q_3 \qquad \ldots(2)$$

Applying Kirchhoff's Second Law in closed loop ABCDA

$$\frac{q_1}{3} - \frac{q_2}{2} + \frac{q_3}{2} = 0$$

$$\Rightarrow \quad 2q_1 - 3q_2 + 3q_3 = 0 \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$q_1 = 90\ \mu\text{C},\ q_2 = 210\ \mu\text{C} \text{ and } q_3 = 150\ \mu\text{C}$$

Therefore, final charges on the three capacitors are as shown.

(b) (i) Electrostatic energy stored before, completing the circuit

$$U_i = \frac{1}{2}\left(3\times10^{-6}\right)(100)^2 + \frac{1}{2}\left(2\times10^{-6}\right)(180)^2$$

$$\left\{\because\ U = \frac{1}{2}CV^2\right\}$$

$$\Rightarrow \quad U_i = 4.74\times10^{-2}\ \text{J}$$

$$\Rightarrow \quad U_i = 47.4\ \text{mJ}$$

(ii) Electrostatic energy stored after, completing the circuit

$$U_f = \frac{1}{2}\frac{\left(90\times10^{-6}\right)^2}{\left(3\times10^{-6}\right)} + \frac{1}{2}\frac{\left(210\times10^{-6}\right)^2}{\left(2\times10^{-6}\right)} + \frac{1}{2}\frac{\left(150\times10^{-6}\right)^2}{\left(2\times10^{-6}\right)}$$

$$\Rightarrow \quad U_f = 1.8\times10^{-2}\ \text{J}$$

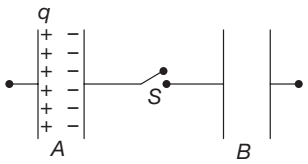
$$\Rightarrow \quad U_f = 18\ \text{mJ}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(A) zero (B) $\frac{q}{2}$

(C) q (D) $2q$

2. Two uncharged metal balls of radius 0.09 mm each collide such that an electron is transferred between them. The potential difference between the balls is

(A) 16 μV (B) 16 pV (C) 32 μV (D) 32 pV

3. A parallel plate capacitor of value 1.77 μF is to be designed using a dielectric material of dielectric constant 200, break through strength 3×10^6 Vm^{-1}. In order to make such a capacitor, which can withstand a potential difference of 20 V across the plates, the separation between the plates d and the area of the plates A (each) can be

(A) $d=10^{-6}$ m, $A=10^{-3}$ m^2

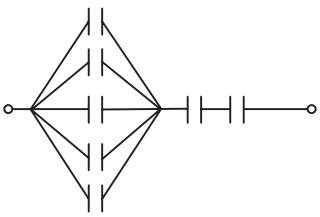
(B) $d=10^{-5}$ m, $A=10^{-2}$ m^2

(C) $d=10^{-4}$ m, $A=10^{-4}$ m^2

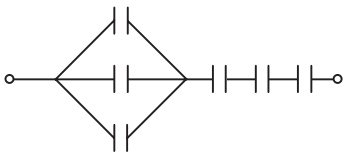
(D) $d=10^{-4}$ m, $A=10^{-5}$ m^2

4. Seven capacitors each of capacitance 2 μF are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{10}{11}\right)\mu$F. Which of combination(s) shown in figure, will achieve the desired results?

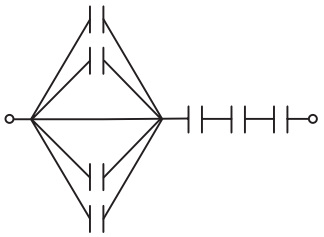
(A)



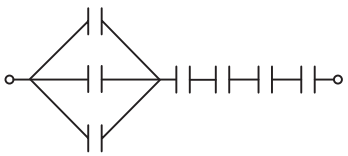
(B)



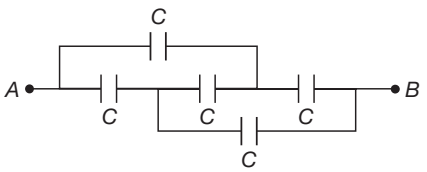
(C)



(D)



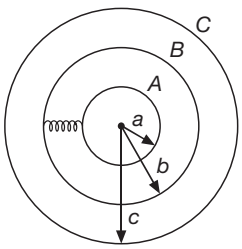
5. In the circuit shown, the equivalent capacitance between the points A and B is



(A) $\frac{C}{5}$ (B) $\frac{C}{3}$

(C) $\frac{C}{2}$ (D) C

6. Three conducting spheres A, B and C of radii a, b and c respectively are as shown in figure. The capacitance of the system, when A and B are connected by a conducting wire is



(A) $4\pi\varepsilon_0\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (B) $4\pi\varepsilon_0\left(\frac{abc}{ab+bc+ca}\right)$

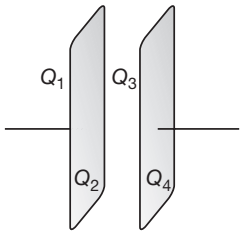
(C) $4\pi\varepsilon_0(a+b+c)$ (D) $4\pi\varepsilon_0\left(\frac{bc}{c-b}\right)$

7. If we treat the earth as a conducting sphere of radius 6400 km, its capacitance would be of the order of

(A) 1 F (B) 1 mF

(C) 1 μF (D) 1000 F

8. An isolated parallel plate capacitor of capacitance C has four surfaces with charges Q_1, Q_2, Q_3 and Q_4 as shown in figure. The potential difference between the plates is



(A) $\frac{Q_1+Q_2+Q_3+Q_4}{2C}$ (B) $\frac{Q_2+Q_3}{2C}$

(C) $\frac{Q_2-Q_3}{2C}$ (D) $\frac{Q_1+Q_4}{2C}$

9. A conducting sphere of radius R, carrying a charge Q is joined to an uncharged conducting sphere of radius $2R$. The charge flowing between them is q. Then

(A) $q=\frac{Q}{4}$ (B) $q=\frac{Q}{3}$

(C) $q=\frac{Q}{2}$ (D) $q=\frac{2Q}{3}$

10. The capacitances of two condensers are C_1 and C_2 and their respective potentials are V_1 and V_2. If they are connected by a thin wire, the loss in energy is

(A) $\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1+V_2)^2$ (B) $\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)$

(C) $\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$ (D) None of these

11. In a parallel plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is connected to a battery and the slab is taken out. It is observed that,

(A) some charge is drawn from the battery

(B) some charge is returned to the battery

(C) the potential difference across the capacitor is reduced

(D) the external agent does no work in taking the slab out

12. In a parallel plate capacitor of capacitance C, a metal sheet is inserted between the plates, parallel to them. The thickness of the sheet is half of the separation between the plates. The capacitance now becomes

(A) $4C$ (B) $2C$

(C) $\frac{C}{2}$ (D) $\frac{C}{4}$

13. In a parallel plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is charged from a battery and then disconnected from it. The slab is now taken out. It is observed that,

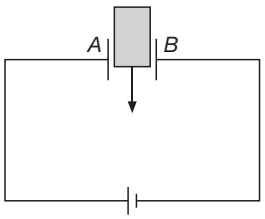
(A) the potential difference across the capacitor is reduced

(B) the potential difference across the capacitor is increased

(C) the energy stored in the capacitor is reduced

(D) the external agent does no work in taking the slab out

14. A dielectric slab is introduced between the plates of a capacitor in the manner as shown. Then current in outer circuit



(A) always flows from B to A

(B) always flows from A to B

(C) first flows from B to A and then from A to B

(D) first flows from A to B and then from B to A

15. Two capacitors of 3 pF and 6 pF are connected in series and a potential difference of 5 kV is applied across the combination. They are then disconnected and reconnected in parallel. The potential difference across the combination is

(A) 1.1 MV (B) 2.25 MV

(C) 2250 V (D) 1111 V

16. It is observed that the capacitance of an isolated sphere of radius a is increased n times when it is enclosed by an earthed concentric sphere of radius b. Then $\frac{b}{a}$ equals

(A) $\frac{n^2}{n-1}$ (B) $\frac{n}{n-1}$

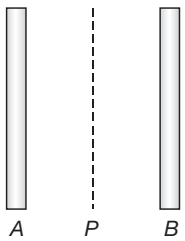
(C) $\frac{2n}{n+1}$ (D) $\frac{2n+1}{n+1}$

17. An infinite number of capacitances $2\ \mu F$, $4\ \mu F$, $8\ \mu F$ ……., are connected in series. The equivalent capacitance of the system is

(A) infinite (B) 0.25 μF

(C) 0.5 μF (D) 1 μF

18. An air capacitor consists of two parallel plates A and B having respective charges $2Q$ and $3Q$ as shown in the figure. If C_0 is the capacitance of the capacitor and P be the median plane of the capacitor, then



(A) $V_P - V_A = \frac{Q}{4C_0}$ (B) $V_P - V_A = \frac{Q}{2C_0}$

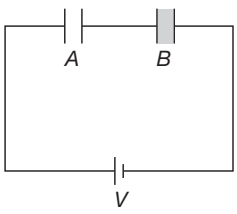
(C) $V_P - V_A = -\frac{Q}{C_0}$ (D) $V_P - V_B = -\frac{Q}{4C_0}$

19. A capacitor of capacitance 1 μF can withstand a maximum voltage of 6 kV and the other capacitor of capacitance 2 μF can withstand a maximum voltage of 4 kV. The series combination of the capacitor can withstand a maximum voltage of

(A) 4 kV (B) 6 kV

(C) 9 kV (D) 10 kV

20. Two identical capacitors A and B are connected in series to a battery as shown in figure. Capacitor B contains a dielectric slab of dielectric constant K as shown. Q_1 and Q_2 are the initial charges stored in the capacitors. Now the dielectric slab is removed and the final corresponding charges stored are q_1 and q_2. Then



(A) $\frac{q_2}{Q_2} = \frac{K+1}{2K}$ (B) $\frac{q_1}{Q_1} = \frac{K+1}{K}$

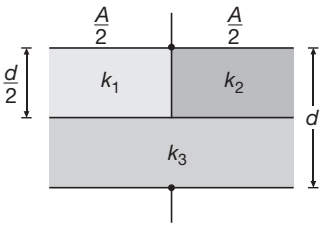
(C) $\frac{q_1}{Q_1} = \frac{K}{2}$ (D) $\frac{q_2}{Q_2} = \frac{K+1}{2}$

21. A parallel plate capacitor having capacitance C_0 is connected to a battery of emf E. It is then disconnected from the battery and a dielectric slab of dielectric constant k completely filling the air gap of the capacitor is inserted in it. If ΔU indicates the change in energy, then

(A) $\Delta U = 0$ (B) $\Delta U = \frac{1}{2}C_0E^2(k-1)$

(C) $\Delta U = \frac{1}{2}C_0E^2\left(1-\frac{1}{k}\right)$ (D) $\Delta U = \frac{1}{2}C_0E^2\left(\frac{1}{k}-1\right)$

22. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor then its dielectric constant k is given by



(A) $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{2k_3}$

(B) $\frac{1}{k} = \frac{1}{k_1 + k_2} + \frac{1}{2k_3}$

(C) $\frac{1}{k} = \frac{k_1 k_2}{k_1 + k_2} + 2k_3$

(D) $k = \frac{k_1 k_3}{k_1 + k_3} + \frac{k_2 k_3}{k_2 + k_3}$

23. Two identical capacitors are joined in parallel and charged to a potential V. Then these are separated and connected in series with the positive plate of one connected to the negative plate of the other. Then

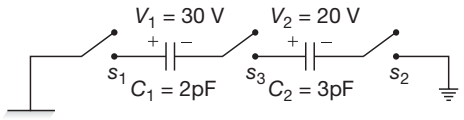
(A) the charges on the plates connected together are destroyed.

(B) the charges on the free plates are enhanced.

(C) the energy stored in the system increases.

(D) the potential difference between the plates becomes $2V$.

24. For the circuit shown, which of the following statements is true?



(A) With S_1 closed, $V_1 = 15$ V , $V_2 = 20$ V

(B) With S_3 closed, $V_1 = V_2 = 25$ V

(C) With S_1 and S_2 closed, $V_1 = V_2 = 0$

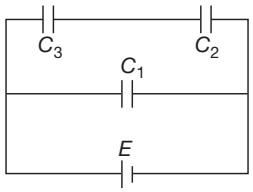
(D) With S_1 and S_3 closed, $V_1 = 30$ V , $V_2 = 20$ V

25. For the same kind of field in air and dielectric (of dielectric constant K), let U_a and U_d represent the energy density (energy per unit volume) in air and in a dielectric respectively. Then,

(A) $U_a = U_d$ (B) $U_a = KU_d$

(C) $U_d = KU_a$ (D) $U_a = (K-1)U_d$

26. The charge acquired by C_1 as shown in the figure is Q. The charge acquired by C_2 is $(C_1 = C_2 = C_3)$



(A) Q (B) $\frac{Q}{2}$

(C) $\frac{Q}{4}$ (D) $2Q$

27. The work done in increasing the voltage across the plates of the capacitor from 5 V to 10 V is W. The work done in increasing the voltage from 10 V to 15 V will be

(A) W (B) $\frac{4}{3}W$

(C) $\frac{5}{3}W$ (D) $2W$

28. The plates of a parallel plate capacitor are separated by d cm. A plate of thickness t cm with dielectric constant k_1 is inserted and the remaining space is filled with a plate of dielectric constant k_2. If Q is the charge on the capacitor and area of plates is A cm^2 each, then potential difference between the plates is

(A) $\frac{Q}{\varepsilon_0 A}\left(\frac{t}{k_1}+\frac{d-t}{k_2}\right)$ (B) $\frac{4\pi Q}{A}\left(\frac{t}{k_1}+\frac{d-t}{k_2}\right)$

(C) $\frac{4\pi Q}{A}\left(\frac{k_1}{t}+\frac{k_2}{d-t}\right)$ (D) $\frac{Q}{\varepsilon_0 A}\left(\frac{k_1}{t}+\frac{d-t}{k_2}\right)$

29. Two capacitors of 1 pF and 2 pF are charged to 200 V and 100 V and then connected by a wire. The potential of the connected system is

(A) 150 V (B) 106 V

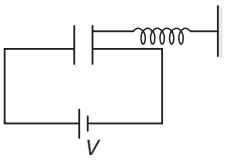
(C) 133 V (D) 187 V

30. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$

(C) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (D) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

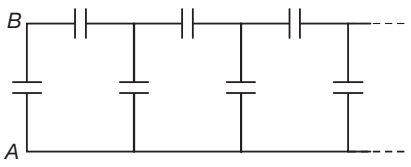
31. One plate of a capacitor is connected to a spring as shown in figure. Area of both the plates is A. In steady state separation between the plates is $\frac{4d}{5}$ (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately



(A) $\frac{\varepsilon_0 AV^2}{2d^3}$ (B) $\frac{4\varepsilon_0 AV}{d^2}$

(C) $\frac{4\varepsilon_0 AV^2}{d^3}$ (D) $\frac{6\varepsilon_0 V^2}{Ad^3}$

32. An infinite ladder of capacitors each 1 μF is made as shown in figure. The capacitance between A and B (in μF) is

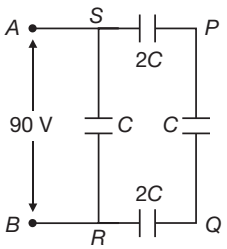


(A) 1 (B) 1.3
(C) 1.6 (D) 0

33. When a dielectric slab of thickness half that of separation is inserted between the plates the capacitance of a capacitor becomes $\frac{4}{3}$ times the original value. The dielectric constant, K of the slab is

(A) $K = 2$ (B) $K = 4$
(C) $K = 6$ (D) $K = 8$

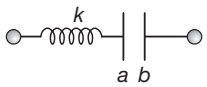
34. In the circuit shown, a potential difference of 90 V is applied across AB. The potential difference between the points P and Q is



(A) 15 V (B) 30 V
(C) 45 V (D) 60 V

35. Consider an air filled parallel plate capacitor with one plate connected to a spring having a force constant k, and another plate held fixed. The system rests on a frictionless table top.

If the charges placed on plates a and b are $+Q$ and $-Q$, respectively, the spring expand will have an expansion x, given by



Capacitor connected to a spring

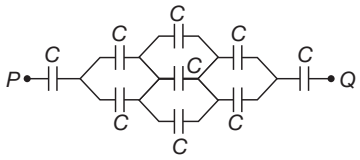
(A) $\frac{Q^2}{kA\varepsilon_0}$ (B) $\frac{2Q^2}{kA\varepsilon_0}$

(C) $\frac{Q^2}{2kA\varepsilon_0}$ (D) None of these

36. Three concentric metallic spherical shells of radii R, $2R$, $3R$ are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is

(A) 1:2:3 (B) 1:3:5
(C) 1:4:9 (D) 1:8:18

37. The equivalent capacitance between P and Q if $C = 3\ \mu F$ is



(A) 3 μF (B) 1 μF
(C) 0.9 μF (D) ∞ μF

38. A parallel plate capacitor of capacitance 100 μF is charged to 500 V. The plate separation is then reduced to half its original value. Then

(A) the potential on the capacitor becomes 1000 V
(B) the potential on the capacitor becomes 250 V
(C) the change in stored energy is 3.75×10^{-5} J
(D) the change in stored energy is 6.25×10^{-6} J

39. A parallel plate capacitor has circular plates of 0.08 m radius and 1.0×10^{-3} m separation. If a potential difference of 100 V is applied, the charge on the capacitor will be

(A) 1.8×10^{-10} C (B) 1.8×10^{-8} C
(C) 1.8×10^{-20} C (D) None of the above

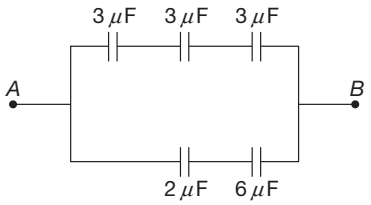
40. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration

(A) ZERO (B) $\frac{3}{2}CV^2$
(C) $\frac{25}{6}CV^2$ (D) $\frac{9}{2}CV^2$

41. The radii of two spheres forming a spherical condenser are 0.5 m and 0.6 m. If a medium of dielectric constant 6 is completely filled in between, the capacity of the condenser will be

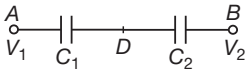
(A) 3.3×10^{-10} F (B) 2×10^{-9} F
(C) 2 F (D) 18 F

42. In the given arrangement of the capacitors, one $3\ \mu F$ capacitor has got $600\ \mu J$ of energy. Then the potential difference across $2\ \mu F$ capacitor is



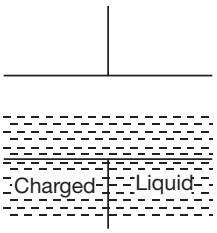
(A) 15 V
(B) 30 V
(C) 45 V
(D) 60 V

43. Two capacitors C_1 and C_2 in a circuit are joined as shown in figure. The potentials of points A and B are V_1 and V_2 respectively; then the potential of point D will be



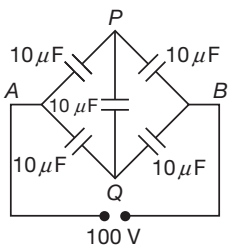
(A) $\frac{(V_1+V_2)}{2}$
(B) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$
(C) $\frac{C_1V_1+C_2V_2}{C_1+C_2}$
(D) $\frac{C_2V_1-C_1V_2}{C_1+C_2}$

44. A parallel plate capacitor is located horizontally such that one of the plates is submerged in a liquid while the other is above the liquid surface. When plates are charged the level of liquid



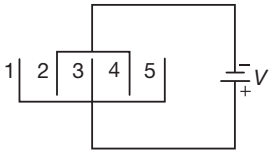
(A) rises
(B) falls
(C) remains unchanged
(D) may rise or fall depending on the amount of charge

45. Five capacitors, each of capacity 10 μF, are connected to a 100 V d.c. supply as shown. The equivalent capacitance between A and B is



(A) 40 μF
(B) 30 μF
(C) 20 μF
(D) 10 μF

46. Five identical plates, each of area A are joined as shown in figure. The distance between the plates is d. The plates are connected to a potential difference of V volt. The charge on plates 1 and 4 will be

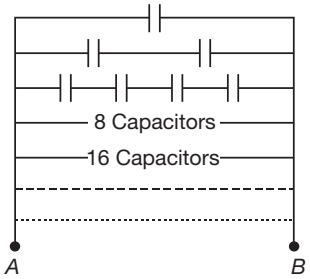


(A) $\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$
(B) $-\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$
(C) $\frac{\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}$
(D) $-\frac{\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}$

47. The capacities of two conductors are C_1 and C_2 and their respective potentials are V_1 and V_2. If they are connected by a thin wire, then the loss of energy will be

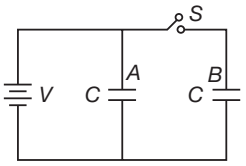
(A) $\frac{C_1C_2(V_1+V_2)}{2(C_1+C_2)}$
(B) $\frac{C_1C_2(V_1-V_2)}{2(C_1+C_2)}$
(C) $\frac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$
(D) $\frac{(C_1+C_2)(V_1-V_2)}{C_1C_2}$

48. An infinite number of identical capacitors, each of capacitance 1 μF, are connected as in the figure. The equivalent capacitance between A and B is



(A) $\frac{1}{2}$ μF
(B) 1 μF
(C) 2 μF
(D) ∞

49. The figure shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of constant 3. The ratio of the total energy stored in both capacitors before and after the introduction of the dielectric is



(A) $\frac{5}{3}$ (B) $\frac{3}{5}$

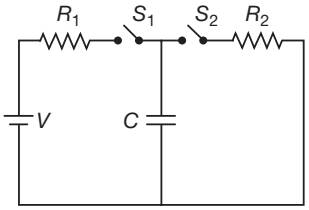
(C) $\frac{5}{2}$ (D) $\frac{2}{5}$

50. Two capacitances, 4 μF and 6 μF are connected in series. A potential difference of 500 V is applied between the outer plates of the system. The charge on each capacitor will be

(A) 1200 C (B) 6000 C

(C) 1200 μC (D) 6000 μC

51. A battery of e.m.f. V volt, resistors R_1 and R_2, a condenser C and switches S_1 and S_2 are connected in a circuit shown. The condenser will get fully charged to V volt when



(A) S_1 and S_2 are both closed

(B) S_1 and S_2 are both open

(C) S_1 is open and S_2 is closed

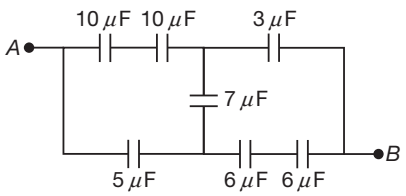
(D) S_1 is closed and S_2 is open

52. A capacitor connected to a 10 V battery collects a charge of 40 μC with air as dielectric and 100 μC with oil as dielectric. The dielectric constant of the oil is

(A) 4 (B) 2.5

(C) 0.4 (D) 1.0

53. In the figure, the equivalent capacitance between A and B is



(A) 3.75 μF (B) 5.25 μF

(C) 6.5 μF (D) 10.5 μF

54. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes

(A) $\frac{3V}{K+2}$ (B) KV

(C) $\frac{V}{K}$ (D) $\frac{3V}{K}$

55. A parallel plate capacitor with a slab of dielectric constant 3 filling the whole space between the plates is charged to a certain potential and isolated. Then the slab is drawn out and another slab of equal thickness but dielectric constant 2 is introduced between the plates. The ratio of the energy stored in the capacitor later to that stored initially is

(A) 2 : 3 (B) 3 : 2

(C) 4 : 9 (D) 9 : 4

56. A capacitor C_1 is charged to a potential difference V. The charging battery is then removed and the capacitor is connected to an uncharged capacitor C_2. The final potential difference across the combination is

(A) $V\frac{C_1}{C_1+C_2}$ (B) $V\frac{C_2}{C_1+C_2}$

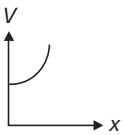
(C) $V\frac{C_1C_2}{C_1+C_2}$ (D) $\frac{V}{C_1+C_2}$

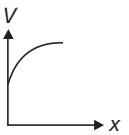
57. In PROBLEM 56, if U is the initial stored energy in capacitor C_1, the final stored energy in the combination is

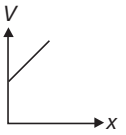
(A) $U\frac{C_1}{C_1+C_2}$ (B) $U\frac{C_2}{C_1+C_2}$

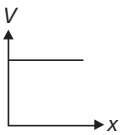
(C) $U\frac{C_1C_2}{C_1+C_2}$ (D) $\frac{U}{C_1+C_2}$

58. Between the plates of a parallel plate dielectric plate is introduced just to fill the space between the plates. The capacitor is charged and later disconnected from the battery. The dielectric plate is slowly drawn out of the capacitor parallel to plates. The plot of the potential difference V across the plates and the length of the dielectric plate drawn out is

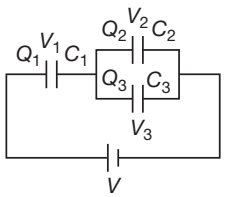
(A) 

(B) 

(C) 

(D) 

59. Three capacitors C_1, C_2 and C_3 are connected to a battery as shown. With symbols having their usual meanings, the correct conditions are



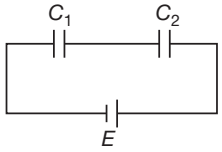
(A) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V$

(B) $V_1 = V_2 = V_3 = V$

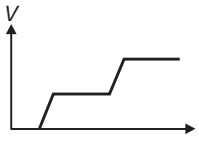
(C) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$

(D) $Q_2 = Q_3$ and $V_2 = V_3$

60. Figure (A) shows two capacitors connected in series and connected by a battery. The graph (B) shows the variation of potential as one moves from left to right on the branch containing the capacitors. Then



(A)



(B)

(A) $C_1 = C_2$

(B) $C_1 < C_2$

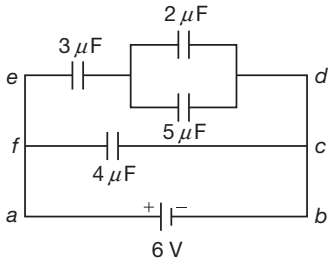
(C) $C_1 > C_2$

(D) C_1 and C_2 cannot be compared

61. A 5 μF capacitor is fully charged across a 12 V battery. It is then disconnected from the battery and connected to an uncharged capacitor. If the voltage across the capacitor becomes 3 V, the capacitance of the uncharged capacitor is

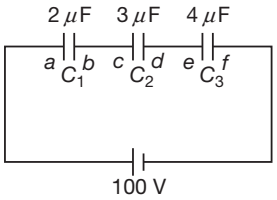
(A) 3 μF (B) 15 μF
(C) 25 μF (D) 60 μF

62. In the circuit given below, the charge in μC, on the capacitor having capacity 5 μF is



(A) 4.5 (B) 9
(C) 7 (D) 15

63. Three capacitors C_1, C_2 and C_3 are connected as shown. If the charges accumulated on the plates a, b, c, d, e and f are q_a, q_b, q_c, q_d, q_e and q_f respectively then



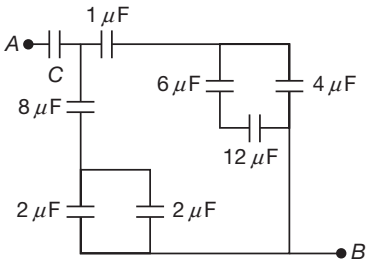
(A) $q_b + q_d + q_f = \left(\frac{100}{9}\right)$ C

(B) $q_b + q_d + q_f = 0$

(C) $q_d + q_c + q_e = 50$ C

(D) $q_b = q_d = q_f$

64. In the given network, the value of C so that the equivalent capacitance between A and B is 1 μF should be



(A) $\frac{32}{23}\mu$F (B) $\frac{23}{32}\mu$F

(C) $\frac{3}{2}\mu$F (D) $\frac{2}{3}\mu$F

65. An infinite number of capacitors, having capacitances 1 μF, 2 μF, 4 μF, 8 μF, ... are connected in series. The equivalent capacitance of the system is

(A) infinite (B) 0.25 μF
(C) 0.5 μF (D) 2 μF

66. Separation between the plates of a parallel plate capacitor is d and the area of each plate is A. When a slab of material of dielectric constant K and thickness $t(<d)$ is introduced between the plates, its capacitance becomes

(A) $\dfrac{\varepsilon_0 A}{d+t\left(1-\dfrac{1}{K}\right)}$ (B) $\dfrac{\varepsilon_0 A}{d+t\left(1+\dfrac{1}{K}\right)}$

(C) $\dfrac{\varepsilon_0 A}{d-t\left(1-\dfrac{1}{K}\right)}$ (D) $\dfrac{\varepsilon_0 A}{d-t\left(1+\dfrac{1}{K}\right)}$

67. Two thin infinite parallel plates have uniform charge densities $+\sigma$ and $-\sigma$. The electric field in the space between them is

(A) $\dfrac{\sigma}{2\varepsilon_0}$ (B) $\dfrac{\sigma}{\varepsilon_0}$

(C) $\dfrac{2\sigma}{\varepsilon_0}$ (D) ZERO

68. A parallel plate capacitor with air between the plates is charged to a potential difference of 500 V and then insulated. A plastic plate is inserted between the plates filling the whole gap. The potential difference between the plates now becomes 75 V. The dielectric constant of plastic is

(A) $\dfrac{10}{3}$ (B) 5

(C) $\dfrac{20}{3}$ (D) 10

69. A parallel plate capacitor is charged to a certain potential difference. A slab of thickness 3 mm is inserted between the plates and it becomes necessary to increase the distance between the plates by 2.4 mm to maintain the same potential difference. The dielectric constant of the slab is

(A) 3 (B) 5

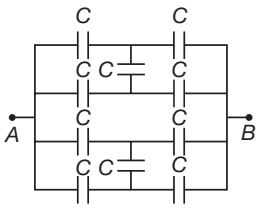
(C) 2.5 (D) 2.4

70. A system of three parallel plates, each of area A, are separated by distances d_1 and d_2. The space between them is filled with dielectrics of permittivities ε_1 and ε_2. The permittivity of free space is ε_0. The equivalent capacitance of the system is

(A) $\dfrac{\varepsilon_1\varepsilon_2 A}{\varepsilon_2 d_1+\varepsilon_1 d_2}$ (B) $\dfrac{\varepsilon_1\varepsilon_2\varepsilon_0 A}{(\varepsilon_1 d_1+\varepsilon_2 d_2)}$

(C) $\dfrac{\varepsilon_0 A}{\varepsilon_1 d_1+\varepsilon_2 d_2}$ (D) $\dfrac{\varepsilon_0 A}{\varepsilon_1 d_2+\varepsilon_2 d_1}$

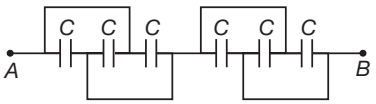
71. The effective capacitance of combination of equal capacitors between points A and B shown in figure is



(A) C (B) $2C$

(C) $3C$ (D) $\dfrac{C}{2}$

72. Six equal capacitors each of capacitance C are connected as shown. The equivalent capacitance between A and B is

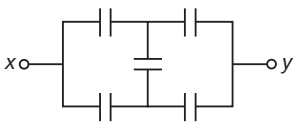


(A) $\dfrac{2}{3}C$ (B) $\dfrac{3}{2}C$

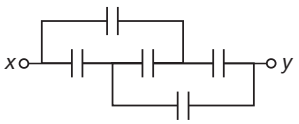
(C) $2C$ (D) $6C$

73. Four ways of making a network of five capacitors of the same value are shown here. Three out of four are identical. The one which is different is

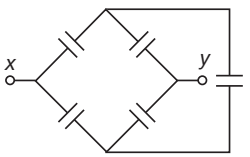
(A)



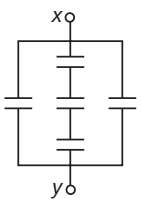
(B)



(C)



(D)

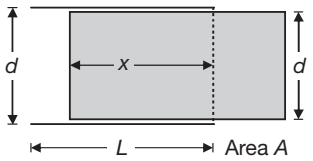


74. An isolated metallic object is charged in vacuum to a potential V, its electrostatic energy being E. It is then disconnected from the source of potential, its charge being left unchanged, and is immersed in a large volume of dielectric with dielectric constant K. Its electrostatic energy is

(A) $\frac{E}{K}$ (B) KE

(C) $\left(\frac{E}{V}\right)K$ (D) $\frac{V^2}{E}K$

75. In a capacitor with $C_0 = \frac{\varepsilon_0 A}{d}$ (in vacuum) and a charge q, a dielectric slab of dielectric constant K is inserted. If the inserted length is x and the edge effects are ignored, then the force on the slab is



(A) attractive and equal to $\frac{1}{2}\frac{q}{C_0 L}(K-1)$

(B) repulsive and equal to $\frac{1}{2}\frac{q}{C_0 L}(K-1)$

(C) attractive and equal to $\frac{1}{2}\frac{q^2}{C_0 L}\times\frac{K-1}{\left(1+\frac{x}{L}(K-1)\right)^2}$

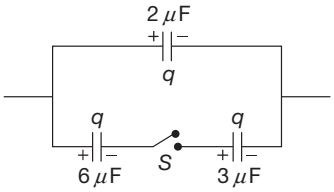
(D) repulsive and equal to $\frac{1}{2}\frac{q^2}{C_0 L}\times\frac{K-1}{\left(1+\frac{x}{L}(K-1)\right)^2}$

76. A capacitor is charged to potential V by a battery of emf V. If an identical capacitor is inserted in the circuit in series with the previous capacitor, the potential difference across the new capacitor is

(A) $2V$ (B) V

(C) $\frac{V}{2}$ (D) zero

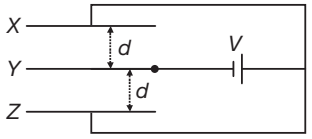
77. The flow of charge through switch S when it is closed is



(A) zero (B) $\frac{q}{4}$

(C) $\frac{2q}{3}$ (D) $\frac{q}{3}$

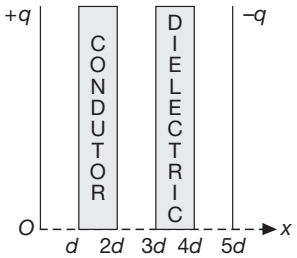
78. Consider the arrangement of three plates X, Y, and Z each of the area A and separation d. The energy stored when the plates are fully charged is

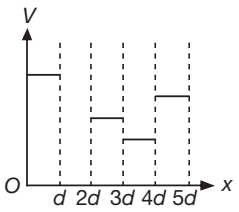


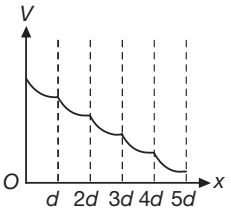
(A) $\frac{\varepsilon_0 AV^2}{2d}$ (B) $\frac{\varepsilon_0 AV^2}{d}$

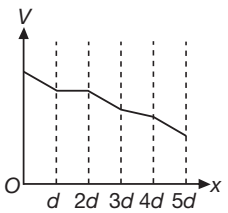
(C) $\frac{2\varepsilon_0 AV^2}{d}$ (D) $\frac{3\varepsilon_0 AV^2}{2d}$

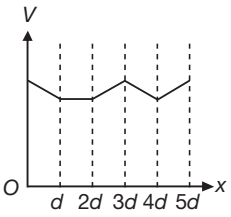
79. The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x=0$ and negatively charged plate is at $x=5d$. Two slabs one of conductor and the other of a dielectric of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be



(A)


(B)


(C)


(D)


80. Capacitance of a capacitor, having plate separation d, becomes $\frac{4}{3}$ times its original value if a dielectric slab of thickness half the separation is inserted between the plates. The dielectric constant of the slab is

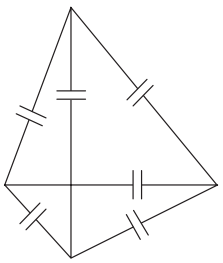
(A) 2 (B) 4

(C) 6 (D) 8

81. A capacitor is connected to a battery. The force of attraction between the plates when the separation between them is halved

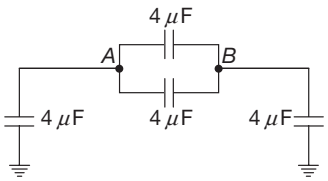
(A) remains the same (B) becomes eight times
(C) becomes four times (D) becomes two times

82. If the capacitance of each capacitor is C, then effective capacitance of the shown network across any two junctions is



(A) C (B) $5C$
(C) $\frac{C}{2}$ (D) $2C$

83. Find equivalent capacitance between A and B

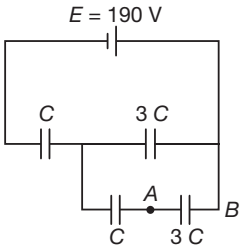


(A) 4 μF (B) 6 μF
(C) 8 μF (D) 10 μF

84. A capacitor of capacitance 10 μF is charged to a potential 50 V with a battery. The battery is now disconnected and an additional charge 200 μC is given to the positive plate of the capacitor. The potential difference across the capacitor will be

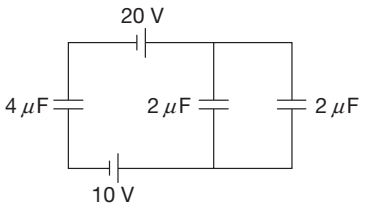
(A) 50 V (B) 60 V
(C) 80 V (D) 100 V

85. In the circuit shown in figure potential difference between A and B is



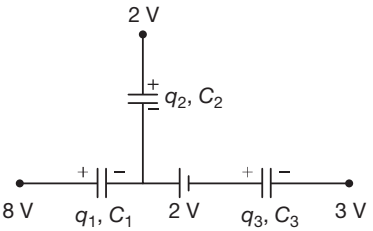
(A) 10 V (B) 30 V
(C) 60 V (D) 90 V

86. In the circuit shown in figure, charge stored in 4 μF capacitor is



(A) 10 μC (B) 20 μC
(C) 40 μC (D) 120 μC

87. A part of the circuit is shown in the figure. All the capacitors have capacitance of 2 μF, then charge on

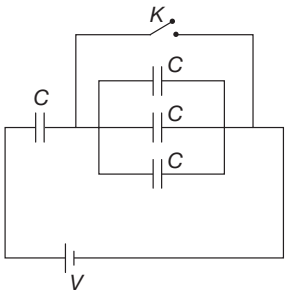


(A) capacitor C_1 is zero
(B) capacitor C_2 is zero
(C) capacitor C_3 is zero
(D) any capacitor cannot be determined

88. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now

(A) V (B) $V = \frac{Q}{C}$
(C) $V + \frac{Q}{2C}$ (D) $V + \frac{Q}{C}$

89. The charge flowing through the cell on closing the key K is equal to



(A) $\frac{CV}{4}$ (B) $4CV$

(C) $\frac{3CV}{4}$ (D) $\frac{4CV}{3}$

90. A capacitor is charged until its stored energy is 3 J and the charging battery is removed. Now another uncharged capacitor is connected across it and it is found that charge distributes equally. The final value of total energy stored in the electric fields is

(A) 1.5 J (B) 3 J

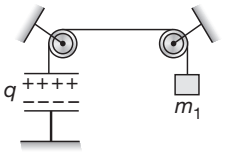
(C) 2.5 J (D) 2 J

91. The effective capacitance of two capacitors of capacitances C_1 and $C_2(C_2 > C_1)$ connected in parallel is $\frac{25}{6}$ times the effective capacitance when they are connected in series. The ratio $\frac{C_2}{C_1}$ is

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{25}{6}$

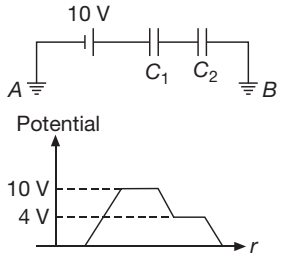
92. In the given system a capacitor of plate area A is charged upto charge q. The mass of each plate is m_2. The lower plate is rigidly fixed. Find the value of m_1 so that the system is in equilibrium



(A) $m_2 + \frac{q^2}{\varepsilon_0 Ag}$ (B) m_2

(C) $\frac{q^2}{2A\varepsilon_0 g} + m_2$ (D) None of these

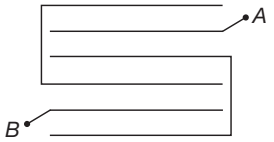
93. Figure shows two capacitors C_1 and C_2 connected with 10 V battery and terminal A and B are earthed. The graph shows the variation of potential as one moves from left to right. Then the ratio of $\frac{C_1}{C_2}$ is



(A) $\frac{5}{2}$ (B) $\frac{2}{3}$

(C) $\frac{2}{5}$ (D) $\frac{4}{3}$

94. If each plate has area A and separation between successive plates is d, then equivalent capacitance between A and B is



(A) $\frac{A\varepsilon_0}{d}$ (B) $\frac{A\varepsilon_0}{2d}$

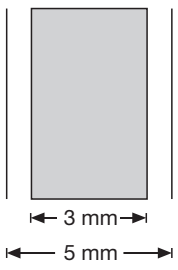
(C) $\frac{4A\varepsilon_0}{d}$ (D) $\frac{3A\varepsilon_0}{d}$

95. Four metallic plates each with a surface area of one side A, are placed at a distance d from each other. The two outer plates are connected to one point A and the two other inner plates to another point B as shown in the figure. Then the capacitance of the system is

(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$

(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{4\varepsilon_0 A}{d}$

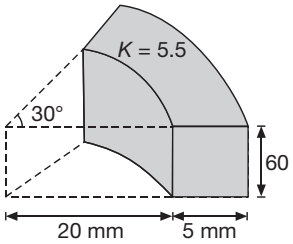
96. Separation between the plates of a parallel plate capacitor is 5 mm. This capacitor, having air as the dielectric medium between the plates, is charged to a potential difference 25 V using a battery. The battery is then disconnected and a dielectric slab of thickness 3 mm and dielectric constant $K = 10$ is placed between the plates, as shown. Potential difference between the plates after the dielectric slab has been introduced is



(A) 18.5 V (B) 13.5 V

(C) 11.5 V (D) 6.5 V

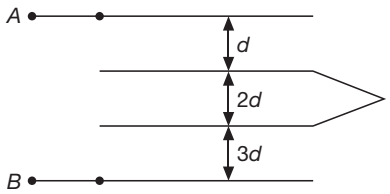
97. The capacitance between the inner and outer curved cylindrical conductor surface as shown in figure is (Space between conductor surface is filled with dielectric of $K = 5.5$)



(A) 6.86 pF (B) 1.86 pF

(C) 3.26 pF (D) 12.63 pF

98. If area of each plate is A and the successive separations are d, $2d$ and $3d$, then equivalent capacitances across A and B is



(A) $\frac{\varepsilon_0 A}{6d}$ (B) $\frac{\varepsilon_0 A}{4d}$

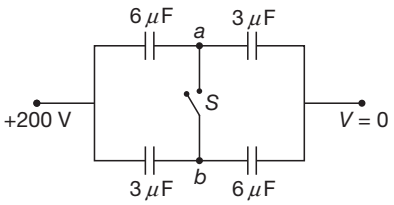
(C) $\frac{3\varepsilon_0 A}{4d}$ (D) $\frac{\varepsilon_0 A}{3d}$

99. An air capacitor is charged up to potential V_1. It is connected in parallel to an identical uncharged capacitor filled with a dielectric medium. After redistribution of charge is final potential difference of this combination is V, then the dielectric constant of the substance will be (Assume that both the capacitors with air medium has same capacitance)

(A) $V^2(V_1 - V)$ (B) $V(V_1 - V)$

(C) $\frac{V}{(V_1 - V)}$ (D) $\frac{(V_1 - V)}{V}$

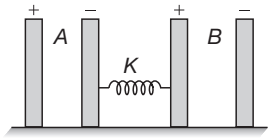
100. Find the amount of charge that will flow through the switch when it is closed



(A) 300 μC (B) 400 μC

(C) 500 μC (D) 100 μC

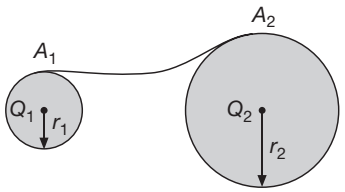
101. Two charged capacitors A and B have their outer plates fixed and inner plates connected by a spring of force constant k. The charge on each capacitor is q. The extension in the spring at equilibrium is



(A) $\frac{q^2}{2A\varepsilon_0 k}$ (B) $\frac{q^2}{4A\varepsilon_0 k}$

(C) $\frac{q^2}{A\varepsilon_0 k}$ (D) zero

102. Two spherical conductors A_1 and A_2 of radii r_1 and r_2 and carrying charges q_1 and q_2 are connected in air by a copper wire as shown in the figure. Then the equivalent capacitance of the system is



(A) $\frac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}$ (B) $4\pi\varepsilon_0(r_1 + r_2)$

(C) $4\pi\varepsilon_0 r_2$ (D) $4\pi\varepsilon_0 r_1$

103. A parallel plate capacitor is charged with a battery. Charging is completed, battery is disconnected. Now the separation between plates is decreased then which of following is correct?

(A) Electric field will not be same

(B) Potential differences between plates increases

(C) Capacitance will decrease

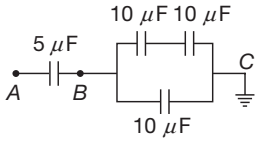
(D) Stored energy will decrease

104. Two identical capacitors are connected in parallel across a potential difference V. After they are fully charged, the positive of first capacitor is connected to negative of second and negative of first is connected to positive of other. The loss in energy will be

(A) $\frac{1}{2}CV^2$ (B) CV^2

(C) $\frac{CV^2}{4}$ (D) zero

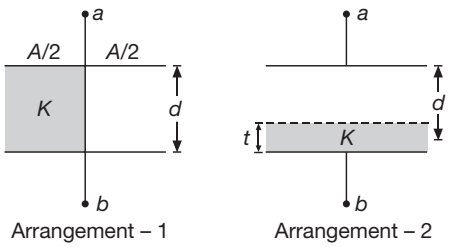
105. In the given circuit, if point C is connected to the earth and a potential of 12000 V is given to point A. The potential at B is



(A) 1500 V (B) 1000 V

(C) 400 V (D) 500 V

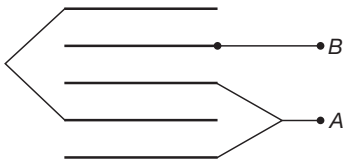
106. A capacitor is half filled with dielectric $(K = 2)$ as shown in Arrangement-1. If the same dielectric is to be filled in the same capacitor as shown in Arrangement-2, then the thickness of dielectric so that capacitor still has same capacitance is



(A) $\frac{2d}{3}$ (B) $\frac{3d}{2}$

(C) $\frac{3d}{4}$ (D) $\frac{4d}{3}$

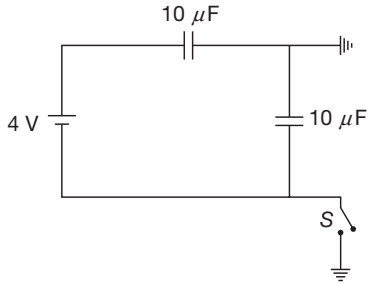
107. Five identical plates of equal area A are placed parallel to and at equal distance d from each other as shown in figure. The effective capacity of the system between the terminals A and B is



(A) $\frac{3}{5}\frac{\varepsilon_0 A}{d}$ (B) $\frac{5}{4}\frac{\varepsilon_0 A}{d}$

(C) $\frac{5}{3}\frac{\varepsilon_0 A}{d}$ (D) $\frac{4}{5}\frac{\varepsilon_0 A}{d}$

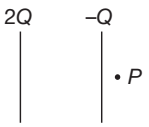
108. In the given figure, the work done by battery after the switch S is closed will be



(A) 100 μJ (B) –100 μJ

(C) 80 μJ (D) –80 μJ

109. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is C. P is a point outside the capacitor and close to the plate of charge $-Q$. The distance between the plates is d select incorrect alternative



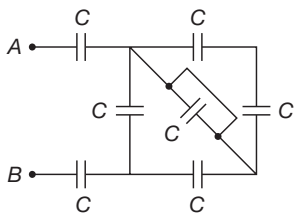
(A) A point charge at point P will experience electric force due to capacitor

(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

(D) The force on one plate due to the other plate is $\frac{Q^2}{2Cd}$

110. The net capacitance between A and B is



(A) $6C$ (B) $\frac{2C}{5}$

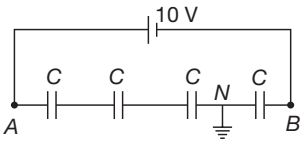
(C) $\frac{2C}{3}$ (D) None of these

111. Two identical parallel plate capacitors are placed in series and connected to a constant voltage source of V_0 volt. If one of the capacitors is completely immersed in a liquid with dielectric constant K, the potential difference between the plates of the other capacitor will change to

(A) $\frac{K+1}{K}V_0$ (B) $\frac{K}{K+1}V_0$

(C) $\frac{K+1}{2K}V_0$ (D) $\frac{2K}{K+1}V_0$

112. The potential at point A in the circuit is (N point is grounded. Grounding means that potential of that point is zero.)



(A) 10 V (B) 7.5 V

(C) 5 V (D) 2.5 V

113. A parallel plate capacitor has plate area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their final separation is $2d$. The work done in this process is

(A) $\frac{\varepsilon_0 AV^2}{2d}$ (B) $\frac{\varepsilon_0 AV^2}{d}$

(C) $\frac{2\varepsilon_0 AV^2}{d}$ (D) $W=0$

114. A parallel plate capacitor having a plate separation of 2 mm is charged by connecting it to a 300 V supply. The energy density is

(A) $10\ \text{Jm}^{-3}$ (B) $1.0\ \text{Jm}^{-3}$

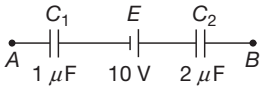
(C) $0.1\ \text{Jm}^{-3}$ (D) $0.01\ \text{Jm}^{-3}$

115. An air capacitor of capacity $C=10\ \mu\text{F}$ is connected to a constant voltage battery of 12 V. Now the space between the plates is filled with a liquid of dielectric constant 5. The (additional) charge that flows now from battery to the capacitor is

(A) $120\ \mu\text{C}$ (B) $600\ \mu\text{C}$

(C) $480\ \mu\text{C}$ (D) $24\ \mu\text{C}$

116. Potential difference across C_1 and C_2, when $V_A - V_B$ equals 5 V is



(A) 5 V and 10 V (B) 15 V and 15 V

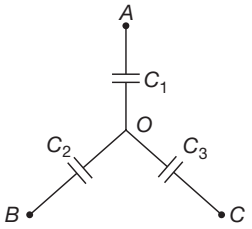
(C) 10 V and 5 V (D) 7.5 V and 7.5 V

117. Two identical capacitors are connected in parallel across a potential difference V. After they are fully charged the battery is removed and the positive of first capacitor is connected to negative of second and negative of first is connected to positive of other. The loss in energy will be

(A) $\frac{1}{2}CV^2$ (B) CV^2

(C) $\frac{CV^2}{4}$ (D) zero

118. Three capacitors of capacitors C_1, C_2, C_3 are connected as shown in the figure. The points A, B and C are at potential V_1, V_2 and V_3 respectively. Then the potential at O will be



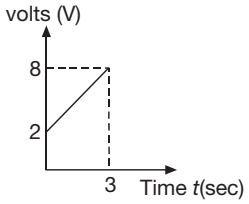
(A) $\frac{V_1+V_2+V_3}{2}$

(B) $\frac{V_1V_2+V_2V_3+V_3V_1}{V_1+V_2+V_3}$

(C) $\frac{V_1C_1+V_2C_2+V_3C_3}{C_1+C_2+C_3}$

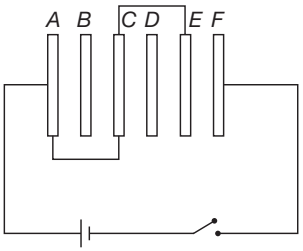
(D) zero

119. A circuit element is placed in a closed box. At time $t=0$, a constant current generator supplying a current of I amp is connected across the box. Potential difference across the box varies according to graph as shown in the figure. The element in the box is



(A) a resistance of $2\ \Omega$
(B) a battery of emf 6 V
(C) an inductance of 2 H
(D) a capacitance

120. A, B, C, D, E and F are conducting plates each of area A and any two consecutive plates separated by a distance d. The net energy stored in the system after the switch S is closed is



(A) $\frac{3\varepsilon_0 A}{2d}V^2$ (B) $\frac{5\varepsilon_0 A}{12d}V^2$

(C) $\frac{\varepsilon_0 A}{2d}V^2$ (D) $\frac{\varepsilon_0 A}{d}V^2$

121. Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (of mass density ρ, dielectric constant K). The inner one is maintained at potential V and the outer one is grounded. The equilibrium height (h) to which the oil rise in the space between the tubes is [Assume this height (h) as a equilibrium height]

(A) $\frac{2\varepsilon_0 V^2(K-1)}{\rho g(b^2-a^2)\log_e\left(\frac{b}{a}\right)}$

(B) $\frac{\varepsilon_0 V^2(K-1)}{\rho g(b^2-a^2)\log_e\left(\frac{b}{a}\right)}$

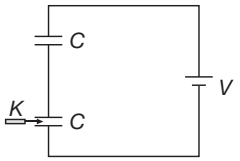
(C) $\frac{4\varepsilon_0 V^2(K-1)}{g\rho(b^2-a^2)\log_e\left(\frac{b}{a}\right)}$

(D) $\frac{6\varepsilon_0 V^2(K-1)}{\rho g(b^2-a^2)\log_e\left(\frac{b}{a}\right)}$

122. What fraction of the energy drawn from the charging battery is stored in a capacitor?

(A) 75% (B) 100%
(C) 25% (D) 50%

123. The work done in inserting the dielectric slab inside one of the capacitors as shown in diagram.



(A) $\frac{CV^2}{2}\left(\frac{K-1}{K+1}\right)$ (B) $\frac{CV^2}{4}\left(\frac{K-1}{K+1}\right)$

(C) $\frac{CV^2}{4}\left(\frac{K+1}{K-1}\right)$ (D) $\frac{CV^2}{2}\left(\frac{K+1}{K-1}\right)$

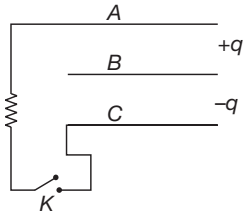
124. The two spherical shells are at large separation one of them has radius 10 cm and has $1.25\ \mu$C charge. The other is of 20 cm radius and has $0.75\ \mu$C charge. If they are connected by a conducting wire of negligible capacity, the charge on the shells are

(A) $1\ \mu$C, $1\ \mu$C (B) $\frac{2}{3}\ \mu$C, $\frac{4}{3}\ \mu$C

(C) $\frac{4}{3}\ \mu$C, $\frac{2}{3}\ \mu$C (D) $0.25\ \mu$C, $0.25\ \mu$C

125. Three identical metal plates of area A are at distance d_1 and d_2 from each other. Metal plate A is uncharged, while plate B and C have respective charges $+q$ and $-q$. If metal plates A and C are connected by switch K through a consumer of unknown resistance, the energy given by the consumer to its surrounding

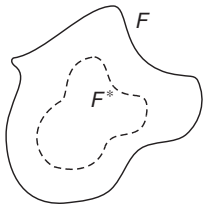
Assume $d_1 = d_2 = d$



(A) $\frac{q^2 d}{4\varepsilon_0 A}$ (B) $\frac{q^2 d}{\varepsilon_0 A}$

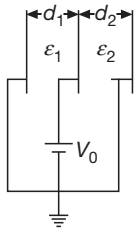
(C) $\frac{q^2 d}{2\varepsilon_0 A}$ (D) $\frac{2q^2 d}{\varepsilon_0 A}$

126. A closed body, whose surface F is made of metal foil, has an electrical capacitance C with respect to an uniformly distant point. The foil is now dented in such a way that the new surface F^* is entirely inside or an the original surface as shown in the figure. Then



(A) Capacitance of F^* > capacitance of F
(B) Capacitance of F^* < capacitance of F
(C) Capacitance of F^* = capacitance of F
(D) Nothing can be concluded from given

127. A capacitor is composed of three parallel conducting plates. All three plates are of same area A. The first pair of plates are kept a distance d_1 apart and the space between them is filled with a medium of a dielectric ε_1. The corresponding data for the second pair are d_2 and ε_2 respectively. The surface charge density on the middle plate is



(A) $\varepsilon_0 V\left(\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_2}{d_2}\right)$ (B) $-\varepsilon_0 V\left(\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_2}{d_2}\right)$

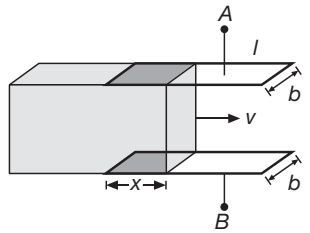
(C) $2\varepsilon_0 V\left(\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_2}{d_2}\right)$ (D) $-2\varepsilon_0 V\left(\frac{\varepsilon_1}{d_1}+\frac{\varepsilon_2}{d_2}\right)$

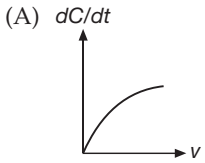
128. A capacitor of capacity C_1 is charged upto potential V volt and then connected in parallel to an uncharged capacitor of capacity C_2. The final potential difference across each capacitor will be

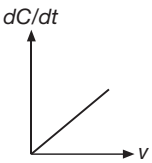
(A) $\frac{C_2 V}{C_1+C_2}$ (B) $\frac{C_1 V}{C_1+C_2}$

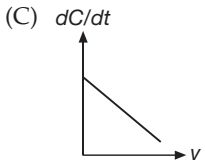
(C) $\left(1+\frac{C_2}{C_1}\right)V$ (D) $\left(1-\frac{C_2}{C_1}\right)V$

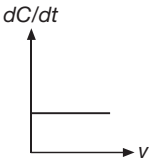
129. Let a dielectric slab be inserted inside the parallel plate capacitor with a speed v. Then the variation of rate of change of capacitance $\frac{dC}{dt}$ with respect to v is



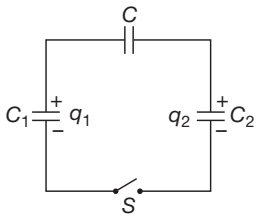
(A) 

(B) 

(C) 

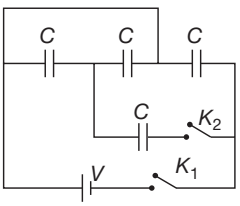
(D) 

130. Two capacitor C_1 and C_2, charged with q_1 and q_2 are connected in series with an uncharged capacitor C, as shown in figure. As the switch S is closed



(A) C gets charged in any condition
(B) C gets charged only when $q_1C_2 > q_2C_1$
(C) C gets charged only when $q_1C_2 < q_2C_1$
(D) C gets charged when $q_1C_2 \neq q_2C_1$

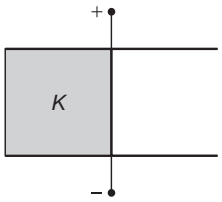
131. Initially K_1 is closed, now if K_2 is also closed, the heat dissipated in the connecting wires is



(A) $\frac{1}{2}CV^2$ (B) $\frac{2}{3}CV^2$

(C) $\frac{1}{3}CV^2$ (D) $\frac{1}{4}CV^2$

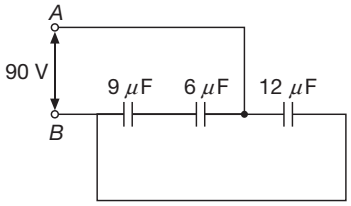
132. Half of the space of a parallel plate capacitor is filled with dielectric as shown. Potential difference V_0 is applied across the plates. The electric field between the plates in the space with dielectric is E_1 and in the space without dielectric is E_2 then



(A) $E_1 = E_2$ (B) $E_1 > E_2$

(C) $E_1 < E_2$ (D) None of these

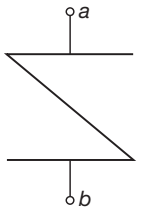
133. The charge on $6\ \mu F$ capacitor is



(A) 540 μC (B) 180 μC

(C) 324 μC (D) 810 μC

134. A thin metallic wire is joined between both the plates of parallel plate capacitor as shown in diagram. The capacitance of the system will be



(A) $\frac{C}{2}$ (B) $2C$

(C) 0 (D) Infinite

135. A capacitor consists of two parallel metal plate of area A separated by a distance d. A dielectric slab of area A, thickness b and dielectric constant k is placed inside the capacitor. If C_K is the capacitance of capacitor with dielectric, how must K and b be selected so that $C_K = 2C$, where C is capacitance without dielectric?

(A) $K = \frac{4b}{2b-d}$ and $\frac{d}{3} < b \le d$

(B) $K = \frac{2b}{2b-d}$ and $\frac{d}{2} < b \le d$

(C) $K = \frac{2b}{2b-d}$ and $\frac{d}{2} \le b \le 2d$

(D) $K = \frac{2b}{2b-d}$ and $\frac{d}{4} \le b \le d$

136. Capacitance of an isolated sphere is increased n times when it is enclosed by an earthed concentric sphere. The ratio of their radii is

(A) $\frac{n^2}{n-1}$ (B) $\frac{n}{n-1}$

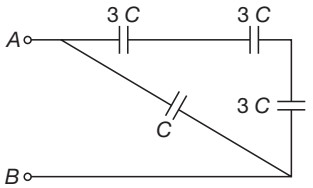
(C) $\frac{2n}{n+1}$ (D) $\frac{2n+1}{n+1}$

137. A capacitor of capacity $C_1 = 1\ \mu F$ can with stand the maximum voltage $V_1 = 16$ KV while another capacitor $C_2 = 2\ \mu F$ can with stand the maximum voltage $V_2 = 4$ KV. The maximum voltage of capacitance connected in series will be

(A) 10 KV (B) 12 KV

(C) 6 KV (D) 4 KV

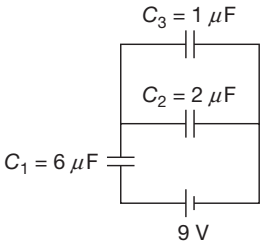
138. The equivalent capacitance between points A and B



(A) C (B) $2C$

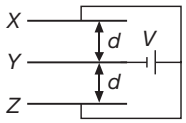
(C) $3C$ (D) $4C$

139. An arrangement of three capacitors is connected to the battery. C_1 and C_2 has fixed capacitances. If C_3 is increased and is made $10\ \mu F$ then



(A) charge on C_1 increases and that on C_2 also increases
(B) charge on C_1 decreases and that on C_2 decreases
(C) charge on C_1 increases but that on C_2 decreases
(D) charge on C_1 decreases but that on C_2 increases

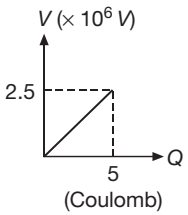
140. Consider an arrangement of three plates X, Y and Z each of area A and separation d. The energy stored, when the plates are fully charged is

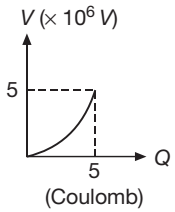


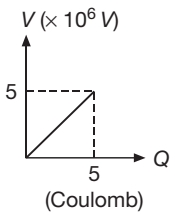
(A) $\frac{\varepsilon_0 AV^2}{2d}$ (B) $\frac{\varepsilon_0 AV^2}{d}$
(C) $\frac{2\varepsilon_0 AV^2}{d}$ (D) $\frac{3\varepsilon_0 AV^2}{d}$

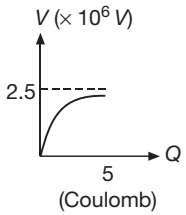
141. From a supply of identical capacitors rated 8 mF, 250 V the minimum number of capacitors required to form a composite 16 mF, 1000 V is
(A) 2 (B) 4
(C) 16 (D) 32

142. A capacitor of 2 μF is charged at a steady rate from zero to 5 coulomb. The graph correctly representing the variation of potential difference across its plates with respect to charge on the condenser is

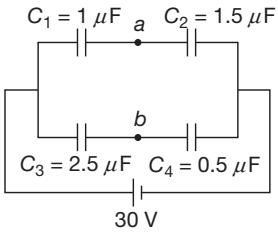
(A)


(B)


(C)


(D)


143. The potential difference between points a and b



(A) 5 V (B) 9 V
(C) –13 V (D) 13 V

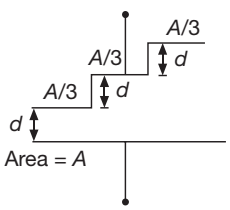
144. Two metallic charged sphere whose radii are 20 cm and 10 cm respectively, each having 150 μC positive charge. The common potential after they are connected by a conducting wire is
(A) 9×10^6 V (B) 4.5×10^6 V
(C) 1.8×10^7 V (D) 13.6×10^6 V

145. A capacitor when filled with a dielectric $K=3$ has charge Q_0, voltage V_0 and Electric field E_0. If the dielectric is replaced with another one having $K=9$, the new value of charge, voltage and field will be respectively
(A) $3Q_0$, $3V_0$, $3E_0$ (B) Q_0, $3V_0$, $3E_0$
(C) Q_0, $\frac{V_0}{3}$, $3E_0$ (D) Q_0, $\frac{V_0}{3}$, $\frac{E_0}{3}$

146. A capacitor with plate separation d is charged to V volts. The battery is disconnected and a dielectric slab of thickness $\frac{d}{2}$ and dielectric constant 2 is inserted between the plates. The potential difference across the terminals becomes
(A) V (B) $2V$
(C) $\frac{4V}{3}$ (D) $\frac{3V}{4}$

147. A parallel plate capacitor C_0 is charged to a potential V_0. The energy stored in the capacitor when the charging battery is kept connected and the plate separation is doubled is U_1. The energy stored in the capacitor when the charging battery is disconnected and the separation between plates is doubled is U_2, then $\frac{U_1}{U_2}$ is
(A) 4 (B) $\frac{1}{4}$
(C) 2 (D) $\frac{1}{2}$

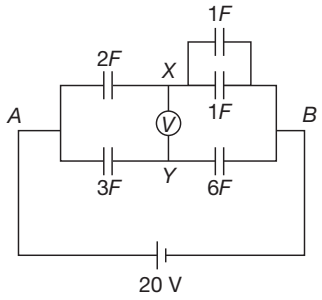
148. A capacitor is made of a flat plate of area A and a second plate having a stair like structure as shown in diagram. The capacitance of the arrangement is



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{18}{11}\frac{\varepsilon_0 A}{d}$

(C) $\frac{11}{18}\frac{\varepsilon_0 A}{d}$ (D) $\frac{\varepsilon_0 A}{3d}$

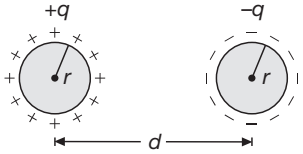
149. Calculate the reading of voltmeter between X and Y then $(V_x - V_y)$ is equal to



(A) 10 V (B) 13.33 V

(C) 3.33 V (D) 10.33 V

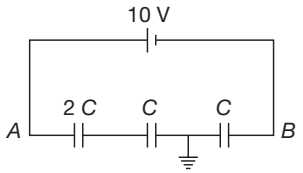
150. Two similar conducting spheres having charge $+q$ and $-q$ are placed at d separation from each other in air. The radius of each ball is r and the separation between their centre is $d(d \gg r)$. The capacitance of the two ball system is



(A) $4\pi\varepsilon_0 r$ (B) $2\pi\varepsilon_0 r$

(C) $4\pi \log_e\left(\frac{\varepsilon_0 r}{d}\right)$ (D) $4\pi \log_e\left(\frac{r}{d}\right)$

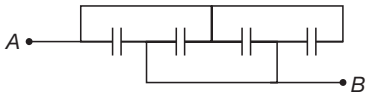
151. Potentials of A and B are respectively



(A) $+6V$, $-4V$ (B) $-6V$, $+4V$

(C) $+10V$, $0V$ (D) $+4V$, $+2V$

152. Four condensers are joined as shown in the adjoining figure. The capacity of each is $8\ \mu$F. The equivalent capacity between the points A and B will be



(A) 32 μF (B) 2 μF

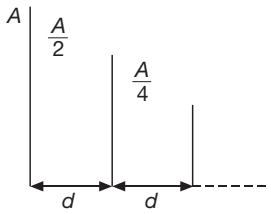
(C) 8 μF (D) 16 μF

153. A capacitor stores $50\ \mu$C charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of $100\ \mu$C flows through the battery. The dielectric constant of the material is

(A) 2.5 (B) 2

(C) 4 (D) 3

154. n conducting plates are placed face to face as shown. Distance between any two plates is d. Area of the plates is A , $\frac{A}{2}$, $\frac{A}{4}$ ... $\frac{A}{2^{(n-1)}}$

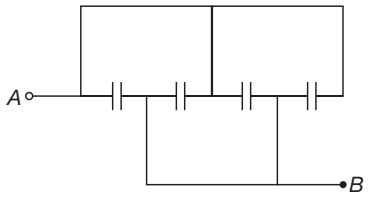


The equivalent capacitance of system is

(A) $\frac{\varepsilon_0 A}{2^n d}$ (B) $\frac{\varepsilon_0 A}{(2^n - 1)d}$

(C) $\frac{\varepsilon_0 A}{(2^n - 2)d}$ (D) $\frac{\varepsilon_0 A}{(2^{n-1} - 1)d}$

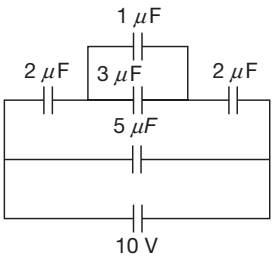
155. Four condensers are joined as shown in fig. The capacity of each is $8\ \mu$F. The equivalent capacity between points A and B will be



(A) 32 μF (B) 2 μF

(C) 8 μF (D) 16 μF

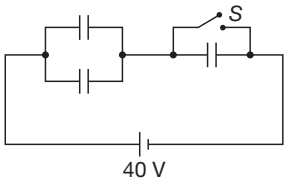
156. The ratio of potential difference between $1\ \mu$F and $5\ \mu$F capacitors is



(A) 1 : 2 (B) 3 : 1

(C) 1 : 5 (D) 10 : 1

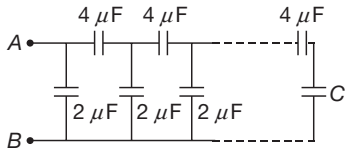
157. Each capacitor in the circuit has capacitance $4\ \mu F$. When the switch S is closed, the charge that flows through AB is



(A) $320\ \mu C$ (B) $213\ \mu C$

(C) $107\ \mu C$ (D) None of these

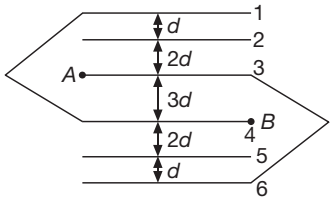
158. A finite ladder is constructed by connecting several sections of $2\ \mu F$, $4\ \mu F$ capacitor combinations as shown in the figure. It is terminated by a capacitor of capacitance C. The value of C, such that equivalent capacitance of the ladder between the point A and B becomes independent of the number of sections in between is



(A) $4\ \mu F$ (B) $2\ \mu F$

(C) $18\ \mu F$ (D) $6\ \mu F$

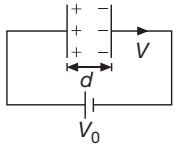
159. Six plates of equal area A and plate separation as shown are arranged equivalent capacitance between A and B is



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$

(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{\varepsilon_0 A}{4d}$

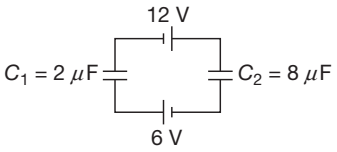
160. A parallel plate capacitor is connected to a battery. The plates are pulled apart with a uniform speed. If x is the separation between the plates, the time rate of change of electrostatic energy of capacitor is proportional to



(A) x^{-2} (B) x

(C) x^{-1} (D) x^2

161. In the circuit shown, the



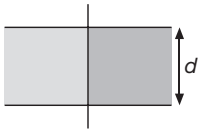
(A) charge on C_2 is greater than C_1

(B) charge on C_1 is greater than C_2

(C) potential difference across C_1 and C_2 are same

(D) potential difference across C_1 is greater than C_2

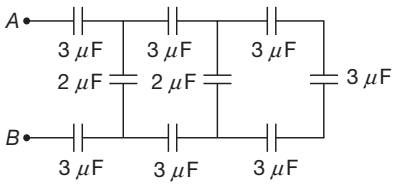
162. A capacitor of capacitance $1\ \mu F$ is filled with two dielectrics of dielectric constants 4 and 6. What is the new capacitance?



(A) $10\ \mu F$ (B) $5\ \mu F$

(C) $4\ \mu F$ (D) None of these

163. The resultant capacitance between (A) and (B) in the following figure is



(A) $1\ \mu F$ (B) $3\ \mu F$

(C) $2\ \mu F$ (D) $1.5\ \mu F$

164. A condenser of capacitance $10\ \mu F$ has been charged to $100\ V$. It is now connected to another uncharged condenser in parallel. The common potential becomes $40\ V$. The capacitance of another condenser is

(A) $15\ \mu F$ (B) $5\ \mu F$

(C) $10\ \mu F$ (D) $16\ \mu F$

165. Two parallel plate capacitors of capacitance C and $3C$ are connected in parallel and charged to a potential difference V with the help of a battery. The battery is disconnected and the space between these two capacitors are filled with dielectric of constant K and $\frac{K}{2}$ respectively. The voltage across the combination will be

(A) $\frac{4V}{3K}$ (B) $\frac{6V}{3K}$

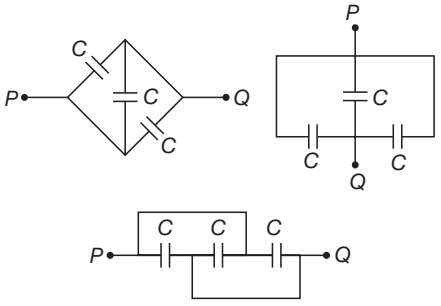
(C) $\frac{8V}{5K}$ (D) $\frac{3V}{5K}$

166. Two insulated metallic spheres of $3\ \mu F$ and $5\ \mu F$ capacitances are charged to $300\ V$ and $50\ V$ respectively. The energy loss, when they are connected by a wire, is

(A) 0.012 J (B) 0.0218 J

(C) 0.0585 J (D) 3.75 J

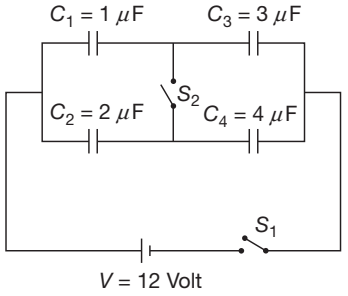
167. The ratio of equivalent capacitances between points P and Q in the following cases is



(A) 1:2:3 (B) 3:2:1

(C) 1:1:1 (D) 1:1:2

168. Initially only S_1 is closed. After a long time S_2 is also closed. The ratio of charge on C_1 when only S_1 is closed to the charge on C_1 when both S_1 and S_2 is closed is



(A) $\frac{1}{2}$ (B) $\frac{14}{15}$

(C) $\frac{15}{14}$ (D) $\frac{2}{1}$

169. Capacitor C_1 of capacitance $1\ \mu F$ and capacitor C_2 of capacitance $2\ \mu F$ are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at $t = 0$. Then

(A) at $t = 0$, the value of current in the circuit containing $1\ \mu F$ is more than current in the circuit containing $2\ \mu F$

(B) at $t = 0$, the current in $2\ \mu F$ capacitor circuit is more than current in $1\ \mu F$ capacitor circuit

(C) $1\ \mu F$ capacitor losses 50% charge sooner than $2\ \mu F$ capacitor

(D) $2\ \mu F$ capacitor losses 50% charge sooner than $1\ \mu F$ capacitor

170. A number of capacitors each of capacitance $1\ \mu F$ and each one of which get punctured if a potential difference just exceeding 500 volt is applied, are provided. Then an arrangement suitable for giving a capacitor of $2\ \mu F$ across which 3000 volt may be applied requires at least

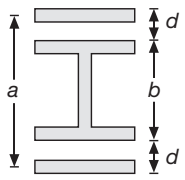
(A) 18 component capacitors

(B) 36 component capacitors

(C) 72 component capacitors

(D) 144 component capacitors

171. Two capacitors are joined in series as shown in figure. The area of each plate is A. The equivalent of the combination is

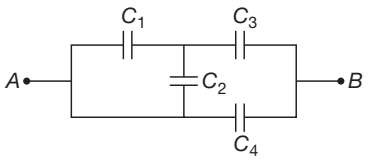


(A) $\frac{\varepsilon_0 A}{d_1 - d_2}$ (B) $\frac{\varepsilon_0 A}{a - b}$

(C) $\varepsilon_0 A\left(\frac{1}{a} - \frac{1}{b}\right)$ (D) $\varepsilon_0 A\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$

172. The charge on capacitor C_2 if a potential difference V is applied between A and B is

(assume $C_1 = C_2 = C_3 = C_4 = C$)



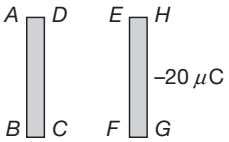
(A) CV (B) $\frac{CV}{3}$

(C) $\frac{CV}{2}$ (D) $\frac{CV}{4}$

173. A parallel plate capacitor is charged to a definite potential and the charging battery is disconnected. Now if the plates of the capacitor are moved apart, then

(A) the stored energy of the capacitor increases
(B) Charge on the capacitor increases
(C) Voltage of the capacitor decreases
(D) The capacitance increases

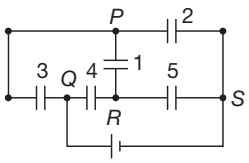
174. A charge $-20\ \mu C$ is given to side GH of the negative plate of the capacitor. The charge on positive plate at CD side is



(A) $+10\ \mu C$ (B) $+20\ \mu C$

(C) $-10\ \mu C$ (D) None of these

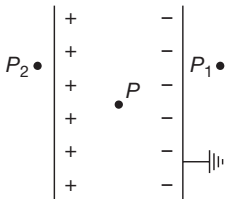
175. The capacitor that will get minimum amount of charge if a battery is connected between Q and S (all capacitors are identical) is



(A) 1 (B) 2

(C) 3 (D) 6

176. There are two metallic plates of a parallel plate capacitor. One plate is given a charge $+q$ while the other is earthed as shown . Points P, P_1 and P_2 are taken as shown in adjoining figure. Then the electric intensity is not zero at



(A) P only (B) P_1 only

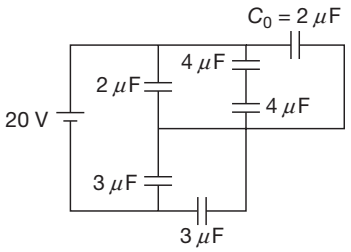
(C) P_2 only (D) P, P_1 and P_2

177. A capacitor of capacitance C_1 is charged to a potential V_0. The electrostatic energy stored in it is U_0. It is connected to another uncharged capacitor of capacitance C_2 in parallel. The energy dissipated in the process is

(A) $\frac{C_2}{C_1 + C_2} U_0$ (B) $\frac{C_1}{C_1 + C_2} U_0$

(C) $\left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$ (D) $\frac{C_1 C_2}{2(C_1 + C_2)} U_0$

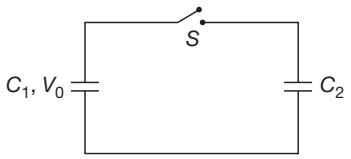
178. Charge on capacitor C_0 is



(A) $10\ \mu C$ (B) $20\ \mu C$

(C) $30\ \mu C$ (D) $0\ \mu C$

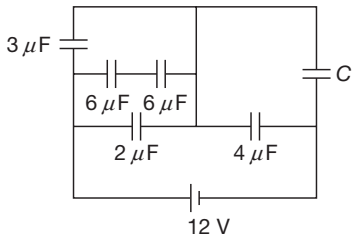
179. A capacitor of capacity C_1 is charged to the potential of V_0. On disconnecting with the battery, it is connected with a capacitor of capacity C_2 as shown in the figure. The ratio of energies before and after the connection of switch S will be



(A) $\frac{(C_1+C_2)}{C_1}$ (B) $\frac{C_1}{(C_1+C_2)}$

(C) C_1C_2 (D) $\frac{C_1}{C_2}$

180. In the given circuit the charge passing through the battery is $48\ \mu C$. The potential difference of capacitor C is



(A) 6 V (B) 8 V

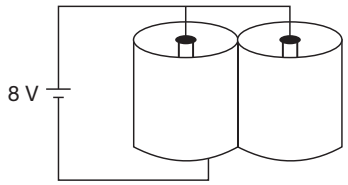
(C) 3 V (D) 4.5 V

181. A parallel plate capacitor is connected to a battery. The plates are pulled apart with a uniform speed. If x is the separation between the plates at any time, then the time rate of change of electrostatic energy of capacitor is proportional to

(A) x^{-2} (B) x

(C) x^{-1} (D) x^2

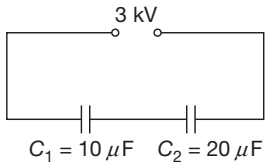
182. The outer cylinders of two cylindrical capacitors of capacitance $2\ \mu F$ each are kept in contact and the inner cylinders are connected through a wire. A battery of emf $8\ V$ is connected. The total charge supplied by battery



(A) $q = 2\ \mu C$ (B) $q = 8\ \mu C$

(C) $q = 16\ \mu C$ (D) $q = 32\ \mu C$

183. Capacitor $C_1(10\ \mu F)$ and $C_2(20\ \mu F)$ are connected in series across a $3\ kV$ supply as shown. The charge on capacitor C_1 is



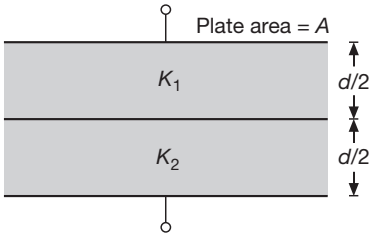
(A) 45000 μC (B) 20000 μC

(C) 15000 μC (D) 10000 μC

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A parallel plate capacitor is filled with two dielectrics having dielectric constants K_1 and K_2 as shown in the figure. The capacitance of the system is C and that of the two capacitors forming the system is C_1 and C_2. Then we have



(A) $C_1 = \frac{2K_1\varepsilon_0 A}{d}$

(B) $C_2 = \frac{2K_2\varepsilon_0 A}{d}$

(C) C_1 and C_2 are in series

(D) $C = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$

2. An air filled parallel plate capacitor having capacitance C_0 is connected to a battery of emf E and then disconnected from it. A dielectric slab having a dielectric constant K, which can just fill the air column in the capacitor, is now inserted in it. Then it is observed that the

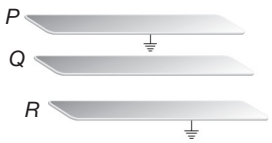
(A) change in energy is $\frac{1}{2}C_0E^2\left(1-\frac{1}{K}\right)$

(B) change in energy is $\frac{1}{2}C_0E^2(K-1)$

(C) energy stored in the capacitor decreases by a factor of K

(D) potential difference between the plates decreases by a factor of K

3. Three large, parallel conducting plates P, Q and R are placed horizontally such that the plates P and R are rigidly fixed and earthed and the plate Q is given some charge. Under the influence of the electrostatic and gravitational forces, the plate Q may stay in equilibrium



(A) midway between P and R
(B) if it is closer to P than to R
(C) if it is closer to R than to P
(D) which is always unstable

4. Consider a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 4d$. Now a dielectric slab of thickness $2d$ is inserted in the capacitor such that the slab is equidistant from both the plates. The capacitor is given some charge. As x increases from 0 to $4d$, we observe that the
(A) magnitude of the electric field does not change
(B) direction of the electric field does not change
(C) electrostatic potential increases continuously
(D) electrostatic potential first increases, then decreases and then again increases

5. A parallel plate capacitor having capacitance C has two plates X and Y each having a charge Q. The plate X is now connected to the positive terminal of a battery and the plate Y to the negative terminal of the same battery of emf $E = \frac{Q}{C}$. Then it is observed that

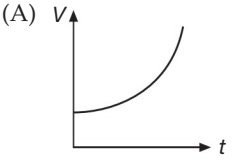
(A) an energy CE^2 is supplied by the battery
(B) Q amount of charge flows, through the capacitor, from the positive terminal to the negative terminal of the battery
(C) the total charge on the plate X is $2Q$
(D) the total charge on the plate Y is zero

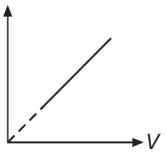
6. Consider a parallel plate capacitor that has been charged from a battery and then disconnected from it. Now the separation between the plates is doubled, then the
(A) external agent has to do some work on the plates
(B) energy stored in the capacitor doubles
(C) potential difference between the plates doubles
(D) field between the plates does not alter

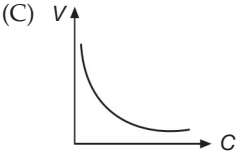
7. A parallel plate capacitor is charged from a cell and then isolated from it. If F be the force of attraction between the plates of the capacitor, E be the electric field in the region between the plates, U be the electrostatic energy stored in the capacitor and ΔV be the potential difference between the plates of the capacitor, then on increasing the separation between the plates we observe that

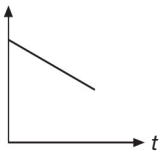
(A) U increases (B) F decreases
(C) ΔV decreases (D) E remains constant

8. A parallel plate capacitor has a dielectric slab that completely fills the empty space of the capacitor. The capacitor is charged by a battery and then battery is disconnected. Now the slab is pull out slowly at $t = 0$. If at time t, capacitance of the capacitor is C, potential difference across is V, and energy stored in it is U, then which of the following graphs are correct?

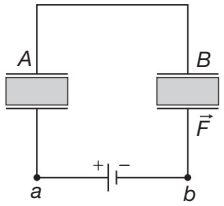
(A) 

(B) 

(C) 

(D) 

9. Consider two identical dielectric slabs that are inserted into two identical capacitors A and B connected to the battery as shown in figure. Now the slab of capacitor B is pulled out from it with the battery still remaining connected. Then during the process of removing the dielectric, we observe that the following statement(s) to be correct.



(A) Charge flows from a to b.
(B) Final charge on capacitor B will be less than that on capacitor A.
(C) Work is done by the external force F always appears a significant heat in the circuit.
(D) Internal energy of the battery increases.

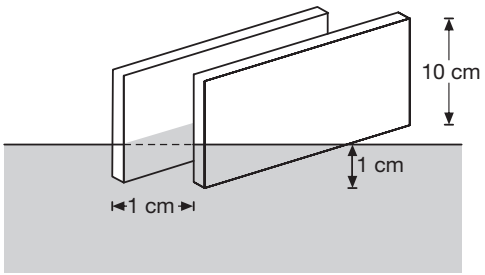
10. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of

the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in questions, in the process of inserting the slab, then

(A) $Q = \dfrac{\varepsilon_0 AV}{d}$ (B) $Q = \dfrac{\varepsilon_0 KAV}{d}$

(C) $E = \dfrac{V}{Kd}$ (D) $W = \dfrac{\varepsilon_0 AV^2}{2d}\left(1 - \dfrac{1}{K}\right)$

11. A large vessel is filled with water having dielectric constant $K = 81$. Two square plates each of length 10 cm and separated by 1 cm are held parallel to each other with their lower ends just touching the surface of water. The plates are charged by applying a voltage V and then disconnected from the voltage source. The liquid rises by 1 cm between the plates. The capacitor has capacitances C_0 and C without and with water respectively. Then



(A) $C_0 = 8.85\ \mu\text{F}$ (B) $C = 0.689\ \text{nF}$

(C) $V = 1110\ \text{V}$ (D) $C = 80\ \mu\text{F}$

12. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius $2R$. When joined by a metal wire,

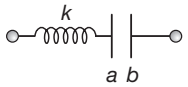
(A) $\dfrac{Q}{3}$ amount of charge will flow from the sphere to the shell

(B) $\dfrac{2Q}{3}$ amount of charge will flow from the sphere to the shell

(C) Q amount of charge will flow from the sphere to the shell

(D) $\dfrac{Q^2}{16\pi\varepsilon_0 R}$ amount of heat will be produced

13. Consider an air filled parallel plate capacitor with one plate connected to a spring having a force constant k and the other plate is held fixed. The system rests on a frictionless table top. The plates have charge density σ and $-\sigma$, area A, charges Q and $-Q$. The electrostatic force due to the electric field created by the plate b is F_e and the spring gets an extension x. Then

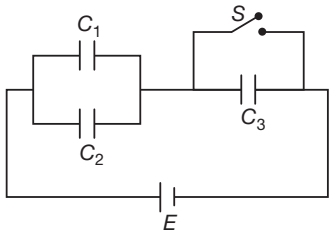


Capacitor connected to a spring

(A) $F_e = \dfrac{\sigma Q}{2\varepsilon_0}$ (B) $x = \dfrac{QE}{k}$

(C) $x = \dfrac{Q^2}{2\varepsilon_0 kA}$ (D) $F_e = \dfrac{Q^2}{2kA\varepsilon_0}$

14. In an arrangement of capacitors shown $C_1 = C_2 = C_3 = C$. The emf of the battery connected in the circuit is E. When the switch S is closed, then



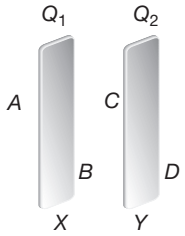
(A) some charge enters the positive terminal of the battery

(B) some charge flows out of the positive terminal of the battery

(C) a charge $\dfrac{4}{3}CE$ flows through the battery

(D) a charge CE flows through the battery

15. Two large, parallel conducting plates are placed close to each other. The inner surfaces of the two plates have surface charge densities $+\sigma$ and $-\sigma$. The outer surfaces are without charge. The electric field has a magnitude of



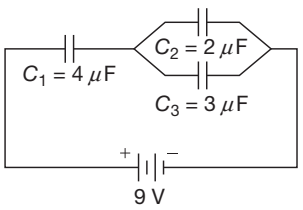
(A) $\dfrac{\sigma}{\varepsilon_0}$ in the region between the plates.

(B) $\dfrac{\sigma}{\varepsilon_0}$ in the region outside the plates.

(C) $\frac{2\sigma}{\varepsilon_0}$ in the region between the plates.

(D) zero in the region outside the plates.

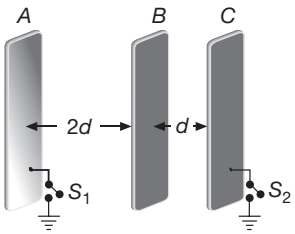
16. In the figure shown, the charges on capacitors C_1, C_2 and C_3 are Q_1, Q_2 and Q_3 respectively. Then



(A) $Q_1 = 20\ \mu C$ (B) $Q_2 = 12\ \mu C$

(C) $Q_2 = 8\ \mu C$ (D) $Q_3 = 12\ \mu C$

17. In the diagram shown, we have three large, identical, parallel conducting plates A, B and C placed such that the switches S_1 and S_2 are open initially and they can be used to earth the plates A and C by just closing them. A charge $+Q$ is given to the plate B. It is observed that a charge of amount



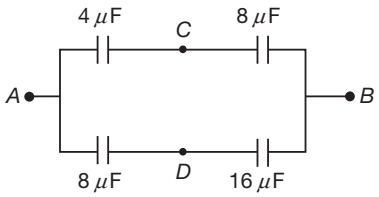
(A) Q will pass through S_1, when S_1 is closed and S_2 is open.

(B) Q will pass through S_2, when S_2 is closed and S_1 is open.

(C) $\frac{Q}{3}$ will pass through S_1, $\frac{2Q}{3}$ will pass through S_2, when S_1 and S_2 are closed together.

(D) $\frac{4Q}{3}$ will pass through S_1, $-\frac{Q}{3}$ will pass through S_2, when S_1 and S_2 are closed together.

18. In the circuit shown, some potential difference is applied between A and B. If C is joined to D



(A) no charge will flow between C and D

(B) some charge will flow between C and D

(C) equivalent capacitance between C and D will not change

(D) the equivalent capacitance between C and D will change

19. In a parallel plate capacitor of plate area A, plate separation d and charge q, the force of attraction between the plates is F

(A) $F \propto q^2$ (B) $F \propto d$

(C) $F \propto \frac{1}{d}$ (D) $F \propto \frac{1}{A}$

20. An air filled parallel plate capacitor has capacitance C_0. Now the air column in the capacitor is completely replaced by a dielectric slab having a dielectric constant K and the capacitor is connected to a battery of emf E and then the slab is taken out. In the process of taking out the dielectric, it is observed that

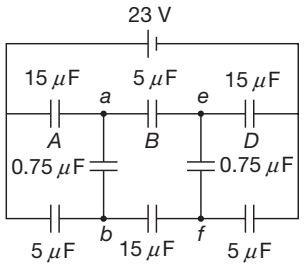
(A) the energy stored in the capacitor reduces by $C_0E^2(K-1)$

(B) a work $\frac{1}{2}C_0E^2(K-1)$ has to be done by the external agent to take the slab out

(C) an energy $C_0E^2(K-1)$ is absorbed by the cell

(D) a charge $C_0E(K-1)$ flows through the cell

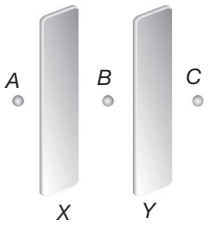
21. In the circuit shown the potential difference between the points ab and ef are V_{ab} and V_{ef} respectively. Then



(A) $V_{ab} = 5$ V (B) $V_{ef} = -5$ V

(C) $V_{ab} = 0$ (D) $V_{ef} = 0$

22. Two large, parallel conducting plates X and Y each having an area A are placed close to each other. The plate X is given a charge Q whereas the plate Y has no charge on it. Three points A, B and C are taken in the different regions of the plates as shown. If $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ be the respective fields at these points, then



(A) $|\vec{E}_B| = \dfrac{Q}{\varepsilon_0 A}$ (B) $|\vec{E}_B| = \dfrac{1}{2}\left(\dfrac{Q}{\varepsilon_0 A}\right)$

(C) $|\vec{E}_A| = |\vec{E}_B| = |\vec{E}_C|$ (D) $\vec{E}_A = -\vec{E}_C$

23. In an attempt to double the separation between the plates of a parallel plate capacitor while it still being connected to a battery, we observe that

(A) the electric field in the region between the plates becomes half

(B) some energy is absorbed by the battery

(C) the external agent has to do some work on the plates

(D) the charge on the capacitor becomes half

24. Consider a situation where a charge of amount Q is given to an isolated metal plate X of surface area A such that its surface charge density becomes σ_1. Now an isolated identical plate Y is brought close to X such that the surface charge density on X becomes σ_2. When the plate Y is earthed, the new surface charge density becomes σ_3, then which of the following relations seem to be the most appropriate and correct?

(A) $\sigma_1 = \dfrac{Q}{A}$ (B) $\sigma_2 = \sigma_1$

(C) $\sigma_1 = \dfrac{1}{2}\left(\dfrac{Q}{A}\right)$ (D) $\sigma_3 = \dfrac{Q}{A}$

25. Consider a capacitor C that is charged by a battery of emf V to a potential V. The capacitor is then disconnected from the battery and then connected again with it but now with its polarity reversed. Then the

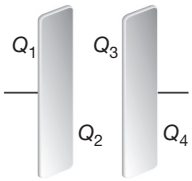
(A) work done by the battery is CV^2

(B) total charge that passes through battery is $2CV$

(C) initial and final energy of the capacitor is same

(D) work done is by the battery is $2CV^2$

26. Consider an isolated parallel plate capacitor having capacitance C. The four surfaces of the capacitor have charges Q_1, Q_2, Q_3 and Q_4, as shown. If ΔV is the potential difference between the plates then



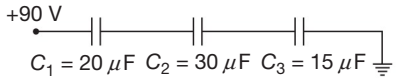
(A) $|Q_2| = C\Delta V$

(B) $Q_1 + Q_2 = C\Delta V$

(C) $(Q_1 + Q_4) + (Q_2 - Q_3) = C\Delta V$

(D) $|Q_3| = C\Delta V$

27. In the arrangement of the capacitors shown, select the correct options



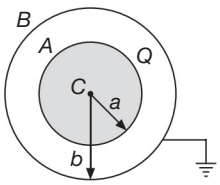
(A) Total charge in this series combination is $600\ \mu C$

(B) The potential difference between the plates of C_1 is 30 V

(C) The potential difference between the plates of C_2 is 20 V

(D) The potential difference between the plates of C_3 is 40 V

28. Consider a conducting sphere A having radius a, charge Q placed concentrically inside a conducting shell B having radius $b(>a)$ and earthed. Let C is the common centre of the sphere A and the shell B. If E and V be the respective electrostatic field and potential at a distance r $(a \le r \le b)$ from C and ΔV be the electrostatic potential difference between the two shells, then



(A) $E = 0$ (B) $E = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q}{r^2}\right)$

(C) $\Delta V = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$ (D) $V = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{b}\right)$

29. Consider two large parallel conducting plates placed close to each other. It is observed that the inner surfaces of the two plates have surface charge densities $+\sigma$ and $-\sigma$ whereas the outer surfaces possess no

charge. If $\vec{E}$ be the electric field between the plates of the capacitor then in the region

(A) between the plates, $|\vec{E}| = \frac{\sigma}{\varepsilon_0}$

(B) between the plates, $|\vec{E}| = \frac{2\sigma}{\varepsilon_0}$

(C) outside the plates, $|\vec{E}| = 0$

(D) outside the plates, $|\vec{E}| = \frac{\sigma}{\varepsilon_0}$

30. Two identical sheets of a metallic foil are separated by d and capacitance of the system is C_0 and charged to a potential difference V_0. Keeping the charge constant, the separation is increased by l. Then the new capacitance and potential difference are C and V respectively. Then

(A) $C = \frac{C_0}{\left(1+\frac{l}{d}\right)}$ (B) $C = C_0\left(1+\frac{l}{d}\right)$

(C) $V = V_0\left(1+\frac{l}{d}\right)$ (D) $V = \frac{V_0}{\left(1+\frac{l}{d}\right)}$

31. Two capacitors, $C_1 = 2\ \mu F$ and $C_2 = 8\ \mu F$ are connected in series across a 300 V source. Then

(A) the charge on each capacitor is 4.8×10^{-4} C

(B) the potential difference across C_1 is 60 V

(C) the potential difference across C_2 is 240 V

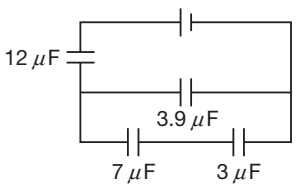
(D) the energy stored in the system is 7.2×10^{-2} J

32. A parallel plate capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous ones as

(A) $Q > Q_0$ (B) $V > V_0$

(C) $E > E_0$ (D) $U > U_0$

33. In the given circuit, the potential difference across the 7 μF capacitor is 6 V. Then



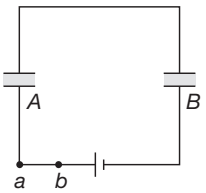
(A) the potential difference across the 12 μF capacitor is 10 V

(B) the potential difference across the 3.9 μF capacitor is 10 V

(C) the charge on the 3 μF capacitor is 42 μC

(D) the emf of the battery is 30 V

34. Identical dielectric slabs are inserted into two identical capacitors A and B. These capacitors and are connected as shown in figure. Now the slab of capacitor B is pulled out with battery remaining connected. Select the correct statement(s).



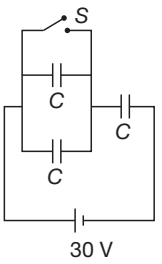
(A) During this process the charge flows from a to b.

(B) Final charge on capacitor B will be less than on capacitor A.

(C) During this process, work is done by external force F which appears as heat in the circuit.

(D) During this process, the battery receives energy.

35. Three capacitors each having capacitance $C = 2\ \mu F$ are connected with a battery of emf 30 V as shown in figure. Select the correct statement(s), when the switch S is closed.



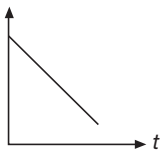
(A) The amount of charge flown through the battery is 20 μC

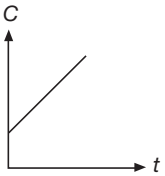
(B) The heat generated in the circuit is 0.6 mJ

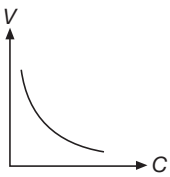
(C) The energy supplied by the battery is 0.6 mJ

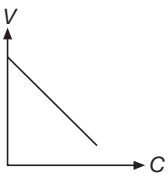
(D) The amount of charge flown through the switch S is 60 μC

36. A parallel plate capacitor has a dielectric slab in it. The slab just fills the space inside the capacitor. The capacitor is charged by a battery and then battery is disconnected. Now the slab is started to pull out slowly at $t = 0$. If at time t, the capacitance of the capacitor is C and potential difference between the plates of a capacitor is V then which of the following graphs is/are correct

(A) 

(B) 

(C) 

(D) 

37. A parallel plate air capacitor is connected to a battery. If plates of the capacitor are pulled further apart, then which of the following statement(s) is/are correct?

(A) Strength of electric field inside the capacitor remain unchanged, if battery is disconnected before pulling the plate.

(B) During the process, work is done by an external force applied to pull the plates whether battery is disconnected or it remain connected.

(C) Potential energy in the capacitor decreases if the battery remains connected during pulling plates apart.

(D) None of the above

38. A $2\ \mu F$ capacitor is charged to a potential of $15\ V$ and a $3\ \mu F$ is charged to a potential of $10\ V$ and the capacitors are connected such that positive plate of one is connected to the negative plate of the other capacitor and negative plate of one is connected to the positive plate of the other capacitor. Select the correct statement(s) about the final circuit.

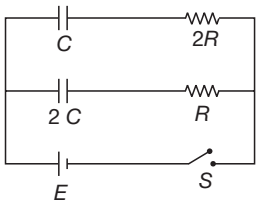
(A) Final charge on each capacitor is zero

(B) Final total electrical energy of the capacitor will be non zero

(C) Total charge flown from A to D is $30\ \mu C$

(D) Total charge flown from A to D is $-30\ \mu C$

39. In the circuit shown in the figure, switch S is closed at time $t = 0$. Select the correct statements



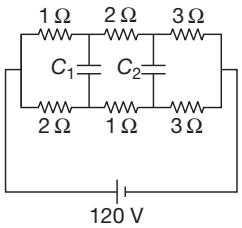
(A) Rate of increase of charge is same in both the capacitors

(B) Ratio of charge stored in capacitors C and $2C$ at any time t would be $1:2$

(C) Time constants of both the capacitors are equal

(D) Steady state charge in capacitors C and $2C$ are in the ratio of $1:2$

40. In the circuit shown in figure, $C_1 = C_2 = 2\ \mu F$. In steady state, the charge stored in



(A) capacitor C_1 is zero

(B) capacitor C_2 is zero

(C) both capacitors is zero

(D) capacitor C_1 is $40\ \mu C$

41. The two plates x and y of a parallel plate capacitor of capacitance C are given a charge of amount Q each. x is now joined to the positive terminal and y to the negative terminal of a cell of emf $V = \frac{Q}{C}$, then select the correct statements.

(A) A charge of amount Q will flow from the positive terminal to the negative terminal of the cell through the capacitor

(B) The total charge on the plate x will be $2Q$

(C) The total charge on the plate y will be zero

(D) The energy supplied by the cell is CV^2

42. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor. Select the correct statement(s).

(A) The energy stored in the capacitor will become K-times the initial energy.

(B) The electric field inside the capacitor will decrease K-times the initial field.

(C) The force of attraction between the plates will become K^2-times the initial force.

(D) The charge on the capacitor will become K-times the initial charge.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** A spherical conductor charged upto $50\ V$ is placed at the centre of a conducting shell which is charged upto $100\ V$ and connected by a wire. All the charge of the shell flows to the sphere.

   Statement-2: The positive charge always flows from higher to lower potential.

2. **Statement-1:** A single isolated conductor is always equivalent to a capacitor.

   Statement-2: The second plate of an isolated conductor can be assumed at infinity.

3. **Statement-1:** The two adjacent conductors carrying same charge can be at different potentials.

   Statement-2: The potential of a conductor depends on the charge on it, shape and size of its the surrounding charged bodies and the relative separation between them.

4. **Statement-1:** A parallel plate capacitor is charged using a battery and then a dielectric slab is inserted completely filling space between plates without disconnecting battery. Electric field between plates of capacitor will decrease.

   Statement-2: When the battery remains connected, then charge on plates of capacitor increases.

5. **Statement-1:** If two concentric conducting sphere which are connected by a conducting wire. No charge can exist on inner sphere.

   Statement-2: When charge on outer sphere will exist then potential of inner shell and outer shell will be same

6. **Statement-1:** Two concentric spherical shell of different radius are at potential V_A and V_B. If outer shell is earthed then potential difference will not be changed.

   Statement-2: Potential difference between the surfaces of two concentric spherical shells does not depends on the charge on the outer shell.

7. **Statement-1:** If three capacitors of capacitance $C_1 < C_2 < C_3$ are connected in parallel then their equivalent capacitance $C_{\text{parallel}} > C_{\text{series}}$

   Statement-2: $\dfrac{1}{C_{\text{parallel}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

8. **Statement-1:** A charged capacitor is disconnected from a battery. Now if its plates are separated further, the potential energy will fall.

   Statement-2: Energy stored in a capacitor is equal to the work done in charging it.

9. **Statement-1:** There cannot be a potential difference between two adjacent conductors that carry the same amount of positive charge.

   Statement-2: Potential of a conductor can be found by $\left(\dfrac{Q}{C}\right)$ ratio and capacitance C depends on the geometrical parameters like size, area etc.

10. **Statement-1:** When a capacitor is charged by a battery, both the plates receive charges of equal magnitude, irrespective of their sizes.

    Statement-2: The charge distribution on the plates of capacitor is in accordance with charge conservation principle.

11. **Statement-1:** The capacitance of any capacitor is always constant for any charge.

    Statement-2: If the charge on a capacitor increases, its capacitance increases as $C = \dfrac{Q}{V}$.

12. **Statement-1:** A parallel plate capacitor is connected across battery through a key. A dielectric slab of constant K is introduced between the plates. The energy which is stored becomes K times.

    Statement-2: The surface density of charge on the plate remains constant.

13. **Statement-1:** When one plate of a charge parallel plate capacitor is connected to the earth, its capacitance increases.
    Statement-2: Electric potential difference between the plates always decreases.

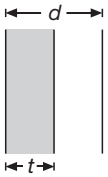
14. **Statement-1:** If the distance between parallel plates of a capacitor is halved and dielectric constant is made three times, then the capacitance becomes 6 times.
    Statement-2: Capacitance does not depend upon the nature of material of the capacitor plates.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A parallel plate capacitor having capacitance C_0 consists of two metal plates of area A and separation d. A slab of thickness $t(<d)$ is inserted between the plates with its faces parallel to the plates and having the same surface area as that of plates as shown.



Based on the above facts, answer the following questions.

1. The capacitance of the system for the slab to be dielectric having dielectric constant 2 and $d=\frac{t}{3}$ is

(A) $\frac{3}{2}C_0$ (B) $\frac{3}{4}C_0$

(C) $\frac{5}{6}C_0$ (D) $\frac{6}{5}C_0$

2. The ratio $\frac{t}{d}$ for which the capacitance of system will be $\left(\frac{3}{2}\right)$ times that of the conductor with air filling the full space is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) 1 (D) $\frac{1}{3}$

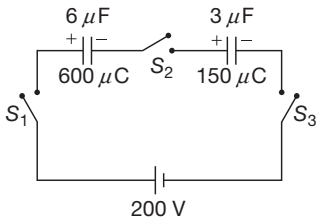
3. The ratio of energy in two cases mentioned in PROBLEM 2, is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Comprehension 2

Two capacitors of capacity $6\ \mu F$ and $3\ \mu F$ are charged to 100 V and 50 V separately and are then connected as shown. Now all the three switches S_1, S_2 and S_3 are closed.



Based on the above facts, answer the following questions.

4. It is observed that an isolated system is formed by plate(s)

(A) a and d separately (B) b and c separately

(C) a and d jointly (D) b and c jointly

5. In steady state, the charge on $6\ \mu F$ and $3\ \mu F$ capacitor respectively is

(A) $400\ \mu C,\ 400\ \mu C$ (B) $700\ \mu C,\ 250\ \mu C$

(C) $800\ \mu C,\ 350\ \mu C$ (D) $300\ \mu C,\ 450\ \mu C$

6. Suppose Q_1, Q_2 and Q_3 be the magnitudes of charges that flows from switches S_1, S_2 and S_3 after they are closed. Then

(A) $Q_1=Q_3$ and $Q_2=0$

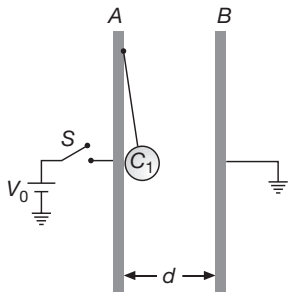
(B) $Q_1=Q_3=\frac{Q_2}{2}$

(C) $Q_1=Q_3=3Q_2$

(D) $Q_1=Q_2=Q_3$

Comprehension 3

In the given set up the parallel plate capacitor AB has vertical plates with plate separation 50 mm and capacitance C_0. From the plate A a small conducting ball of mass m, capacitance C hangs from a non-conducting silk thread of length 100 mm. The ball initially touches the plate A which is connected to a power supply of voltage V_0 for a short time by closing the switch S and then opening it again. The plate B is grounded. The motion of conducting ball is now observed. It is observed that due to the charge deposited on plates and ball, the ball swings across, touches the plate B, returns, touches A and finally swings out again such that it almost touches plate B. Taking $g=10\ ms^{-1}$ and based on the above facts, answer the following questions.



7. The angle made by ball and string with the plate A when the ball is in its final position is

(A) $30°$ (B) $45°$

(C) $60°$ (D) $0°$

8. The ball initially touches plate B, it loses its charge to B, swings back and touches A and then finally again swings out and almost touches plate B. The new common potential V is given by

(A) $\dfrac{CV_0}{C+C_0}$ (B) $\dfrac{C_0V_0}{C+C_0}$

(C) V_0 (D) $\left(1+\dfrac{C}{C_0}\right)V_0$

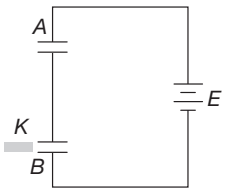
9. The required power supply voltage V_0 is given by

(A) $\left(1+\dfrac{C}{C_0}\right)\sqrt{\dfrac{m}{\sqrt{3}C}}$ (B) $\left(1+\dfrac{C_0}{C}\right)\sqrt{\dfrac{m}{2\sqrt{3}C}}$

(C) $\left(1+\dfrac{C}{C_0}\right)\sqrt{\dfrac{m}{2\sqrt{3}C}}$ (D) $\left(1+\dfrac{C_0}{C}\right)\sqrt{\dfrac{5m}{\sqrt{3}C}}$

Comprehension 4

Two identical capacitors A and B, each of capacitance C are connected in series. The combination is connected to a battery of emf E. A dielectric slab of dielectric constant K is slipped between the plates of capacitor B to cover entire space between the plates.



Based on these facts, answer the following questions.

10. After introduction of dielectric slab in B, the ratio of capacitance of A and B is

(A) $1:1$ (B) $1:K$

(C) $K:1$ (D) $1:\sqrt{K}$

11. After introduction of dielectric slab in B, the ratio of potential differences across A and B will be

(A) $1:1$ (B) $1:K$

(C) $K:1$ (D) $1:\sqrt{K}$

12. The ratio of potential differences across A before and after the introduction of dielectric slab in B will be

(A) $1:1$ (B) $1:K$

(C) $(K+1):2$ (D) $(K+1):2K$

13. The ratio of potential differences across B before and after the introduction of dielectric slab in B will be

(A) $1:1$ (B) $K:1$

(C) $(K+1):2$ (D) $(K+1):2K$

14. The ratio of energy stored in capacitors A and B after the introduction of dielectric slab in B is

(A) $1:1$ (B) $1:K$

(C) $K:1$ (D) $(K+1)^2:K^2$

Comprehension 5

Two spherical conductors A and B have capacitance $2\ \mu F$ and $3\ \mu F$. A is given a charge Q and allowed to share with B by means of a conducting wire. Based on these facts, answer the following questions.

15. The ratio of radii of spheres A and B is

(A) $2:3$ (B) $3:2$

(C) $4:9$ (D) $9:4$

16. The charge on A after sharing is

(A) $\dfrac{Q}{5}$ (B) $\dfrac{4Q}{5}$

(C) $\dfrac{2Q}{5}$ (D) $\dfrac{3Q}{5}$

17. The ratio of electrostatic energies of A before and after sharing is

(A) $\dfrac{2}{5}$ (B) $\dfrac{3}{5}$

(C) $\dfrac{4}{25}$ (D) $\dfrac{9}{25}$

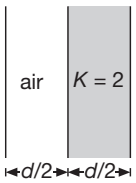
18. The ratio of final electrostatic energy of system to initial electrostatic energy of system is

(A) 1 : 1 (B) 2 : 5

(C) 3 : 5 (D) 2 : 3

Comprehension 6

Two metal plates form a parallel plate capacitor. The separation between the plates is d. Now half of the separation between the plates is filled with a dielectric of relative permittivity 2 and of same area.



The new arrangement gets punctured when a potential difference just exceeding 1200 V is applied. Based on these facts, answer the following questions.

19. The ratio of capacitance before and after insertion of dielectric is

(A) 1 : 2 (B) 2 : 1

(C) 3 : 4 (D) 4 : 3

20. The maximum potential difference that can exist across the dielectric is

(A) 300 V (B) 400 V

(C) 600 V (D) 800 V

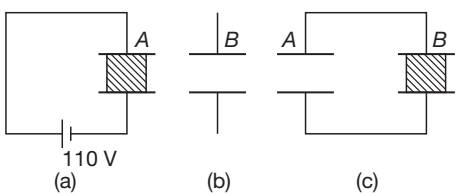
21. The ratio of electrostatic energy stored by air and dielectric parts of capacitor is

(A) 1 : 2 (B) 2 : 1

(C) 3 : 4 (D) 4 : 3

Comprehension 7

Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.



The dielectric slab is placed inside A as shown in Figure (a). A is then charged to a potential difference of 110 V. The capacitance of A is C_A and the energy stored in it is U_A.

The battery is disconnected and then the dielectric slab is removed from A. The work done by the external agency in removing the slab from A is W.

The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in Figure (c). The energy stored in the system is U. Based on the above facts, answer the following questions.

22. The value of C_A is

(A) 0.18 nF (B) 2 nF

(C) 3 nF (D) 1 nF

23. The value of U_A approximately is

(A) 10 μJ (B) 11 μJ

(C) 12 μJ (D) 13 μJ

24. The value of W is

(A) 48.4 μJ (B) 38.4 μJ

(C) 28.4 μJ (D) 18.4 μJ

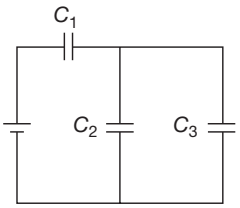
25. The value of U is

(A) 10 μJ (B) 11 μJ

(C) 12 μJ (D) 13 μJ

Comprehension 8

Three capacitors are connected to a battery of voltage V as shown in Figure. Their capacitances are $C_1 = 3C$, $C_2 = C$ and $C_3 = 5C$. Based on the information and the diagram provided, answer the following questions.



26. The equivalent capacitance of this set of capacitors is

(A) 4C (B) $\frac{23C}{6}$

(C) 2C (D) None of these

27. The ordering of the capacitors according to the charge they store, from largest to smallest is

(A) $C_1 > C_2 > C_3$ (B) $C_3 > C_2 > C_1$

(C) $C_3 > C_1 > C_2$ (D) $C_2 > C_1 = C_3$

28. Rank the capacitors according to the potential difference across them, from largest to smallest.

(A) $C_1 > C_2 > C_3$ (B) $C_1 < C_2 = C_3$

(C) $C_1 > C_2 = C_3$ (D) $C_1 < C_2 < C_3$

29. Now if, C_3 is increased then the charge on

(A) C_1 and C_2 increase, C_3 decreases

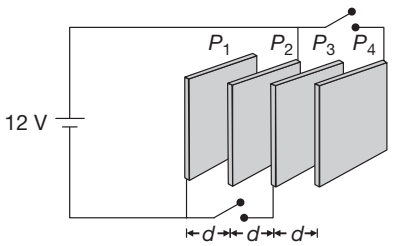
(B) C_1 and C_3 increase, C_2 decreases

(C) C_1 and C_2 decrease, C_3 increases

(D) C_1 and C_3 decrease, C_2 increases

Comprehension 9

Four parallel metal plates P_1, P_2, P_3 and P_4, each of area $7.5\ \text{cm}^2$ are separated successively by a distance $d = 1.19$ mm as shown in Figure. P_1 is connected to the negative terminal of a battery and P_2 to the positive terminal. The battery maintains a potential difference of 12 V. Based on the information and the diagram provided, answer the following questions.



30. If P_3 is connected to the negative terminal, the capacitance of the three-plate system $P_1P_2P_3$ is

(A) 12.1 pF (B) 11.2 pF

(C) 5.58 pF (D) 16.7 pF

31. The charge on P_2, in pC is

(A) 34 (B) 67

(C) 134 (D) None of these

32. P_4 is now connected to the positive terminal of the battery. The capacitance of the four plate system $P_1P_2P_3P_4$ is

(A) 12.1 pF (B) 11.2 pF

(C) 5.58 pF (D) 16.7 pF

33. The charge on P_4, in pC is

(A) 34 (B) 67

(C) 134 (D) 268

Comprehension 10

Given here are four SITUATIONS. Read them carefully to answer the questions that follow.

Situation 1: A $1\ \mu$F capacitor and a $2\ \mu$F capacitor are connected in series across a 300 V supply line. The charge on each capacitor is found to be Q_1 and Q_2 and potential across each is V_1 and V_2.

Situation 2: The charged capacitors are now disconnected from the line and reconnected with their positive plates together and the negative plates together, with no external voltage being applied. The respective charges and potentials across capacitors are found to be Q_1', Q_2' and V_1', V_2'.

Situation 3: Again the system is reset to the initial conditions and the charged capacitors are disconnected from the line and reconnected with plates of opposite polarity connected together, with no external voltage being applied. The respective charges and potentials across capacitors are found to be Q_1'', Q_2'' and V_1'' and V_2''.

Situation 4: The charged capacitors in situation 2 are disconnected and reconnected with the plates of opposite polarity together. The respective charges and potentials across the capacitors are found to be Q_1''', Q_2''' and V_1''', V_2'''.

Based on the Situations discussed above, answer the following questions.

34. The (Q, V) values in μC and volt respectively for SITUATION 1 are

(A) $Q_1 = Q_2 = 200\ ;\ V_1 = V_2 = 100$

(B) $Q_1 = Q_2 = 100\ ;\ V_1 = 100\ ;\ V_2 = 200$

(C) $Q_1 = 100\ ;\ Q_2 = 200\ ;\ V_1 = V_2 = 100$

(D) $Q_1 = Q_2 = 200\ ;\ V_1 = 200\ ;\ V_2 = 100$

35. The (Q, V) values in μC and volt respectively for SITUATION 2 are

(A) $V_1' = V_2' = 200\ ;\ Q_1 = Q_2 = 200$

(B) $V_1' = V_2' = \frac{200}{3}\ ;\ Q_1' = Q_2' = \frac{400}{3}$

(C) $V_1' = V_2' = \frac{400}{3}\ ;\ Q_1' = \frac{400}{3}\ ;\ Q_2' = \frac{800}{3}$

(D) $V_1' = \frac{200}{3}\ ;\ V_2' = \frac{400}{3}\ ;\ Q_1' = \frac{400}{3}\ .\ Q_2' = \frac{800}{3}$

36. The (Q, V) values in μC and volt respectively for SITUATION 3 are

(A) $V_1''=V_2''=0\,;\ Q_1''=Q_2''=0$

(B) $V_1''\neq V_2''=0\,;\ Q_1''=Q_2''\neq 0$

(C) $V_1''=V_2''\neq 0\,;\ Q_1''\neq Q_2''=0$

(D) $(V_1''=V_2'')\neq 0\,;\ (Q_1''=Q_2'')\neq 0$

37. The (Q, V) values in μC and volt respectively for SITUATION 4 are

(A) $V_1''=V_2''=\frac{200}{9}\,;\ Q_1'''=Q_2'''=\frac{800}{9}$

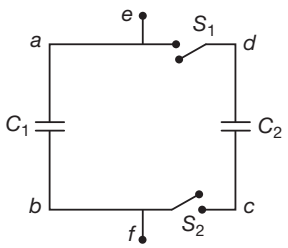
(B) $V_1'''=V_2'''=\frac{400}{9}\,;\ Q_1'''=\frac{400}{9}\,;\ Q_2'''=\frac{800}{9}$

(C) $V_1'''=\frac{400}{9}\,;\ V_2'''=\frac{800}{9}\,;\ Q_1'''=Q_2'''=\frac{400}{9}$

(D) $V_1'''=\frac{200}{9}\,;\ V_2'''=\frac{400}{9}\,;\ Q_1'''=\frac{200}{9}\,;\ Q_2'''=\frac{400}{9}$

Comprehension 11

Two capacitors $C_1=1\ \mu$F and $C_2=3\ \mu$F are each charged to a potential $V_0=100$ V but with opposite polarity, as shown. Switches S_1 and S_2 are now closed. Based on the above facts, answer the following questions.



38. What is the potential difference between points 'e' and 'f'?

(A) 25 V (B) 50 V

(C) 75 V (D) 100 V

39. What is the final charge on C_1?

(A) 50 μC (B) 75 μC

(C) 150 μC (D) 300 μC

40. What is the charge on C_2?

(A) 50 μC (B) 75 μC

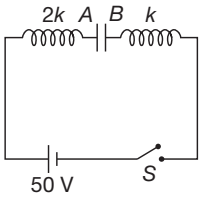
(C) 150 μC (D) 300 μC

41. Energy lost by the system of capacitors, after the switches are closed is

(A) 20 mJ (B) 5 mJ

(C) 15 mJ (D) 10 mJ

Comprehension 12



In the given circuit, two identical parallel conducting plates A and B are connected to a 50 V battery by metal springs of spring constants $2k$ and k respectively. Initially, the switch S is open and the plates are uncharged. In fact, the two plates A and B form a capacitor. When the switch S is closed, the separation between the plates becomes 2 mm and this is one fourth of the initial separation between plates. Also the electrostatic energy stored in the capacitor is found to be 10 mJ. Based on the information provided, answer the following questions.

42. Initially, when the plates were uncharged capacitance of the capacitor was

(A) 0.5 μF (B) 1 μF

(C) 2 μF (D) 4 μF

43. Extension in the spring connected to plate B is

(A) 3 mm (B) 4 mm

(C) 5 mm (D) 6 mm

44. Spring constant of the spring connected to plate A is

(A) 625 Nm^{-1} (B) 1250 Nm^{-1}

(C) 2500 Nm^{-1} (D) 5000 Nm^{-1}

Comprehension 13

Inside a parallel plate capacitor, there is a dielectric slab parallel to the outer plates whose thickness is equal to 70% of the gap width. The dielectric constant of material of the slab is 7. When the slab is absent, the capacitance of the capacitor equals 10 μF. The capacitor was connected to a 10 V battery. Now the capacitor is disconnected from the battery and the slab is removed from the gap. After removal of the slab, the positively charged plate of the capacitor is reconnected with negative terminal of the same battery and

negatively charged plate with positive terminal of the battery. Based on the information provided, answer the following questions.

45. Energy stored in capacitor before reconnection is

(A) 500 μJ (B) 1250 μJ

(C) 3215 μJ (D) 4250 μJ

46. Energy stored in the capacitor after reconnection is

(A) 500 μJ (B) 1250 μJ

(C) 3215 μJ (D) 4250 μJ

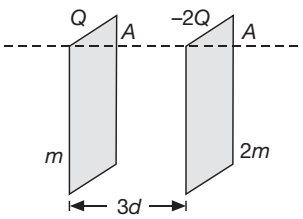
47. Heat generated in connecting wires after reconnection with the battery is

(A) 500 μJ (B) 1250 μJ

(C) 3215 μJ (D) 4250 μJ

Comprehension 14

A parallel plate capacitor has two plates of area A each. The mass of the left plate is m and its electric charge is Q. The mass of the right plate is $2m$ and its electric charge is $-2Q$. Initially the plates are fixed and the separation between the plates is $3d$. If C is the capacitance of the capacitor when plates would have been lying at the separation d, then based on the information provided, answer the following questions.



48. Potential difference across plates of capacitor is

(A) $\frac{3Q}{2C}$ (B) $\frac{9Q}{2C}$

(C) $\frac{6Q}{C}$ (D $\frac{3Q}{C}$

49. Energy possessed by the electric field between the plates is

(A) $\frac{27Q^2}{4C}$ (B) $\frac{9Q^2}{8C}$

(C) $\frac{27Q^2}{8C}$ (D) $\frac{81Q^2}{8C}$

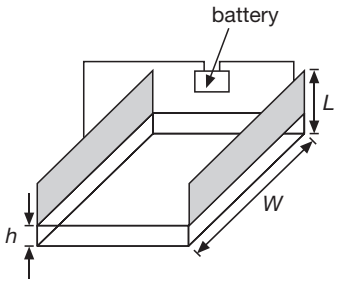
50. At a given moment plates are unfixed. What is relative velocity of plates when their distance is reduced to d

(A) $9Q\sqrt{\frac{3}{mC}}$ (B) $\frac{Q}{2}\sqrt{\frac{1}{mC}}$

(C) $\frac{2Q}{3}\sqrt{\frac{1}{3mC}}$ (D) $\frac{9Q}{2}\sqrt{\frac{1}{3mC}}$

Comprehension 15

A fuel gauge uses a capacitor to determine the height of the fuel in the tank. The effective dielectric constant K_{eff} changes from a value of 1 (when the tank is empty) to a value of K, where K is dielectric constant of the fuel (when the tank is full). The appropriate electronic circuitry can determine the effective dielectric constant of the combined air and fuel between the capacitor plates.



Each of the two rectangular plates has a width W and a length L. The height of the fuel between the plates is h. (Ignore fringing effect). Based on the information provided, answer the following questions.

51. The electronic gadget which senses the fuel height is called

(A) level tester (B) strain gauge

(C) transducer (D) capacitor

52. The value of K_{eff} as a function of h is

(A) $\frac{(K-1)h}{L}+1$ (B) $\frac{(K-1)h}{L}$

(C) $\frac{(K-1)h}{L}-1$ (D) $\frac{(K+1)h}{L}-1$

53. When the tank is $\frac{1}{4}$th empty, then the value of K_{eff} is

(A) $\frac{3K}{4}$ (B) $\frac{K-1}{4}$

(C) $(K-1)\frac{3}{4}$ (D) $\frac{K+1}{4}$

Comprehension 16

Consider a series combination of capacitor C_1, C_2 and C_3. Capacitor C_1 and C_3 have air as the dielectric medium between the plates while C_2 has a medium of dielectric constant $K = 3$ filled uniformly between the plates. A potential difference V_0 is applied across the combination. It is observed that potential difference across C_3 is 12 V. The dielectric material $(K = 3)$ between the plates of C_2 is now removed so that its capacity now becomes C_2'. In this situation, the potential difference across C_3 is found to be 9 V. It is also observed that the potential differences across C_1 and C_2' are equal. If the capacitance of C_2 is 6 μF, then based on the information provided, answer the following questions.

54. Applied potential difference V_0 is

(A) 10 V (B) 20 V

(C) 30 V (D) 36 V

55. Capacitance C_1 is

(A) 8 μF (B) 6 μF

(C) 3 μF (D) 2 μF

56. Capacitance C_3 is

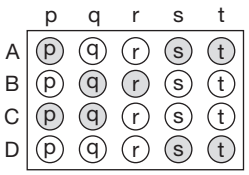
(A) 8 μF (B) 6 μF

(C) 3 μF (D) 2 μF

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

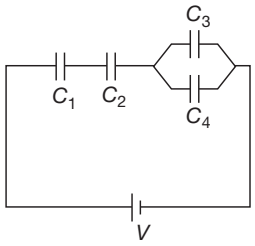


1. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Energy stored in a capacitor carrying charge Q. (potential difference $= V$ between the plates)	(p) CV^2
(B) Energy supplied to a capacitor by a battery of potential difference V.	(q) $\frac{Q^2}{2C}$
(C) Energy loss when plates of a charged capacitor are connected together.	(r) $\frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$
(D) Energy loss when charges of two parallel plates capacitors of capacitances C_1 and C_2 at potential differences V_1 and V_2 respectively are shared.	(s) $\frac{1}{2}CV^2$

(*Continued*)

2. In the circuit shown $C_1=C$, $C_2=2C$, $C_3=3C$, $C_4=4C$. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Charge across the capacitor C_3 (multiple of CV)	(p) $\frac{2}{23}$
(B) Potential difference across the capacitor C_4 (multiple of V)	(q) $\frac{6}{23}$
(C) Energy stored across the capacitor C_3 (multiple of CV^2)	(r) $\frac{1}{66}$
(D) Energy stored across the capacitor C_4 (multiple of CV^2)	(s) $\frac{1}{88}$

3. A parallel plate capacitor with air between its plates is charged using a battery and then disconnected from the battery. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.

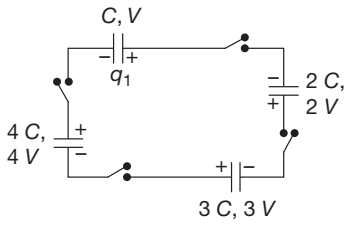
COLUMN-I	COLUMN-II
(A) Potential difference between the plates will decrease if	(p) Separation between the plates is increased to $\frac{K}{2}$ times the initial value and space between the plates after the separation has increased, is completely filled with a dielectric (here K is the dielectric constant)
(B) Electric field strength between the plates will reduce if	(q) Separation between the plates is increased
(C) Electric energy stored in the capacitor will decrease if	(r) A dielectric with $K>1$ is filled between the plates of capacitor
(D) Electric energy density will decrease if	(s) Separation between the plates is reduced

(*Continued*)

4. Three capacitors of capacitances 2 μF, 3 μF and 6 μF are connected in series with a 12 V battery. All the connecting wires are disconnected. The three positive plates are connected together and three negative plates are connected together. The charges on three capacitors after reconnection are Q_1, Q_2 and Q_3 respectively. The charge supplied by battery is Q. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.

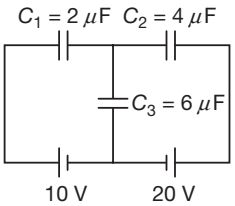
COLUMN-I	COLUMN-II
(A) Q_1	(p) 12 μC
(B) Q_2	(q) $\frac{216}{11}$ μC
(C) Q_3	(r) $\frac{72}{11}$ μC
(D) Q	(s) $\frac{108}{11}$ μC

5. Capacitors with capacitances C, $2C$, $3C$ and $4C$ are charged to the voltage V, $2V$, $3V$, $4V$ respectively. Circuit is closed. Assume voltages across capacitors in equilibrium are V_1, V_2, V_3 and V_4 respectively. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.



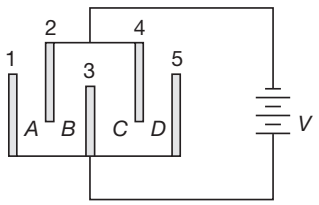
COLUMN-I	COLUMN-II
(A) V_1	(p) $\frac{2V}{5}$
(B) V_2	(q) $\frac{7V}{5}$
(C) V_3	(r) $\frac{19V}{5}$
(D) V_4	(s) $\frac{14V}{5}$

6. In the diagram shown, q_1, q_2 and q_3 are the charges on the capacitors C_1, C_2 and C_3. The potential difference across C_3 is V, where q_1, q_2 and q_3 are in μC and V is in volt. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) q_1	(p) 50
(B) q_2	(q) $\frac{10}{3}$
(C) q_3	(r) $\frac{140}{3}$
(D) V	(s) $\frac{25}{3}$

7. Five identical capacitor plates each of area A are arranged such that adjacent plates are at d distance apart. Plates are connected to a source of emf V as shown. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Charge on plate 1 $\left(\text{multiple of } \frac{\varepsilon_0 AV}{d}\right)$	(p) –2
(B) Charge on plate 4 $\left(\text{multiple of } \frac{\varepsilon_0 AV}{d}\right)$	(q) 1
(C) Potential difference between the plates 2 and 3 (multiple of 2 V)	(r) 0
(D) Potential difference between the plates 1 and 5 (multiple of 2 V)	(s) $\frac{1}{2}$

8. In case of two conducting spherical shells having radii a and $b(>a)$. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Shells are concentric and inner is given a charge while outer is earthed	(p) $C = 4\pi\varepsilon_0(a+b)$
(B) Shells are concentric and the outer is given a charge while inner is earthed	(q) $C = \frac{4\pi\varepsilon_0 ab}{b-a}$
(C) Shells carry equal and opposite charges and are separated by a distance d	(r) $C = \frac{4\pi\varepsilon_0 b^2}{b-a}$
(D) Shells are connected by a conducting wire	(s) $C = \frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$

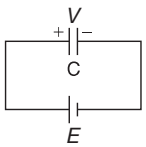
9. A parallel plate capacitor with air between its plates is charged using a battery and remains connected to the battery. Match the quantities in **COLUMN-I** with their respective answer(s) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Electric field strength between the plates will reduce if	(p) Separation between the plates is increased
(B) Electric energy stored in the capacitor will increase if	(q) Separation between the plates is reduced
(C) Electric energy density will increase if	(r) A dielectric with $K > 1$ is filled between the plates
(D) Charge in the capacitor will decrease if	(s) Separation between the plates is increased to $\frac{K}{2}$ times the initial value and space between the plates, after the separation has increased, is completely filled with a dielectric (here K is the dielectric constant and $K = 3$)

10.

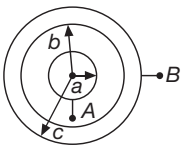
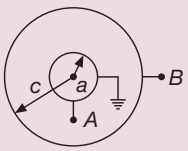
COLUMN-I	COLUMN-II
(A) If distance between plates of an isolated capacitor decreases	(p) Potential difference across plate is decreased
(B) If dielectric is inserted between plates of capacitor whose plates are connected with battery	(q) Capacitance of the capacitor will increase
(C) If area of plates of an isolated capacitor is increased	(r) Energy of capacitor will increase
(D) If distance between plates of capacitor is decreased when the capacitor is connected with battery	(s) Force between the plates will decrease

11. A capacitor of capacitance C is charged to a potential V. Now it is connected to a battery of e.m.f E as shown in the figure. Match the conditions in **COLUMN-I** with the arguments in **COLUMN-II.**

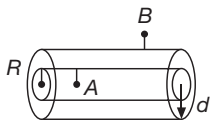
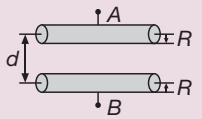


COLUMN-I	COLUMN-II
(A) If $V = E$, then	(p) non-zero charge is supplied by the +ve terminal of battery to +ve plate of capacitor.
(B) If $V > E$	(q) Zero charge is supplied by +ve terminal of battery to +ve plate of capacitor.
(C) If $V < E$, then	(r) non-zero thermal energy will be dissipated in the circuit.
(D) If $V \neq E$, then	(s) outer surfaces of the plates of capacitor have zero charge.

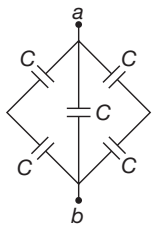
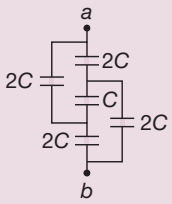
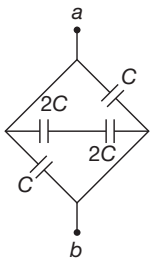
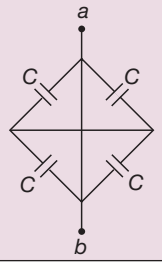
12. Match the columns for the capacitance of systems in **COLUMN-I** to their respective values of capacitance in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A)  (Concentric shells) capacitance C_{AB}	(p) $\frac{4\pi\varepsilon_0 c^2}{c-a}$
(B)  (Concentric shells) Capacitance C_{AB}	(q) $\frac{4\pi\varepsilon_0 ac}{c-a}$

(*Continued*)

COLUMN-I	COLUMN-II
(C)  (Concentric cylinders) Capacitance C_{AB} per unit length	(r) $\frac{2\pi\varepsilon_0}{\log_e\left(\frac{d}{R}\right)}$
(D)  (Parallel wires of radius R at separation d) capacitance C_{AB} per unit length)	(s) $\frac{\pi\varepsilon_0}{\log_e\left(\frac{d}{R}\right)}$

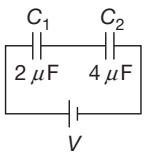
13. Match the Column

COLUMN-I	COLUMN-II
(A) 	(p) C_{eq} between a and b is $2C$
(B) 	(q) C_{eq} between a and b is C
(C) 	(r) C_{eq} between a and b is $3C$
(D) 	(s) C_{eq} between a and b is $4C$

14. Match the entries of **COLUMN-I** with the entries of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) When a dielectric slab is gradually inserted between the plates of an isolated parallel plate capacitor	(p) The electric potential energy of the system decreases
(B) When a dielectric gradually inserted between the plates of a parallel plate capacitor connected to battery	(q) Work done by external agent is positive
(C) When the plates of a parallel plate capacitor are pulled apart for a capacitor connected to battery	(r) Work done by battery is positive
(D) When the plates of a parallel plate isolated capacitor are pulled apart	(s) Work done by external agent is negative

15. In the given figure, the separation between the plates of C_1 is slowly increased to double of its initial value, then match the following

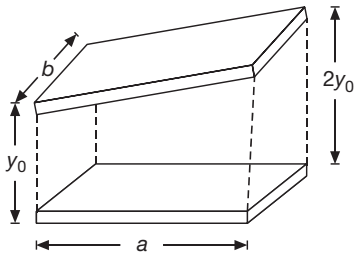


COLUMN-I	COLUMN-II
(A) The potential difference across C_1	(p) increases
(B) The potential difference across C_2	(q) decreases
(C) The energy stored in C_1	(r) increases by a factor of $\frac{6}{5}$
(D) The energy stored in C_2	(s) decreases by a factor of $\frac{18}{25}$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

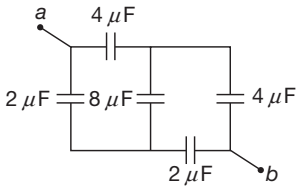
1. A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown in figure. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right where $y_0 \ll a$ and $y_0 \ll b$. The capacitance of the system is found to be $C = \frac{\varepsilon_0 ab}{y_0}\log_e \alpha$. Then find value of α.



2. A $10\ \mu F$ capacitor has plates with vacuum between them. Each plate carries a charge of magnitude $1000\ \mu C$. A particle with charge $-3\ \mu C$ and mass 2×10^{-16} kg is fired from the positive plate toward the negative plate with an initial speed of 2×10^{6} ms^{-1}. Does it reach the negative plate? If so, find its impact speed in kms^{-1}. If not, what fraction of the way across the capacitor does it travel?

3. Capacitors $C_1 = 6\ \mu F$ and $C_2 = 2\ \mu F$ are charged as a parallel combination across a 250 V battery. The capacitors are disconnected from the battery and from each other. They are then connected positive plate to negative plate and negative plate to positive plate. Calculate the resulting charge, in μC, on each capacitor.

4. Calculate the equivalent capacitance, in μF, between the points a and b in figure.



5. A detector of radiation called a Geiger tube consists of a closed, hollow, conducting cylinder with a fine wire along its axis. Suppose that the internal diameter of the cylinder is 2.5 cm and that the wire along the axis has a diameter of 0.2 mm. The dielectric strength of the gas between the central wire and the cylinder is 1.2×10^{6} Vm^{-1}. Calculate the maximum potential difference, in volt that can be applied between the wire and the cylinder before breakdown occurs in the gas.

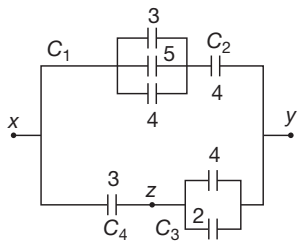
6. A radioactive source in the form of a metal sphere of diameter 10^{-3} m emits β particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long (in ns) will it take for its potential to rise by 1 V, assuming that 80% of emitted β particles escape from the surface.

7. A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm thick slab is introduced between the plates; in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.

8. The capacitance of all the capacitors shown in figure are in μF

(a) What is the equivalent capacitance, in μF, between x and y?

(b) If the charge on $5\ \mu F$ capacitor is $120\ \mu C$, what is the potential difference, in volt, between x and z?

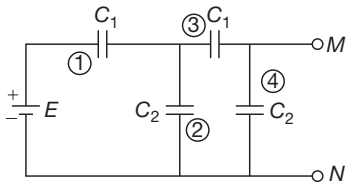


9. A capacitor of capacitance $C_1 = 1\ \mu F$ withstands the maximum voltage $V_1 = 0.6$ kV while another capacitor of capacitance $C_2 = 2\ \mu F$ withstands the maximum voltage $V_2 = 4$ kV. What maximum kilovolt will the system of these two capacitors withstands if they are connected in series?

10. A parallel capacitor of plate area $0.2\ m^2$ and spacing 10^{-2} m is charged to 10^{3} V and is then disconnected from the battery. How much work is required if the

plates are pulled apart to double the plate spacing? Calculate the final voltage, in kilo volt, of the capacitor if $\varepsilon_0 = \dfrac{1}{36\pi \times 10^7}$ Fm^{-1}.

11. Find the potential difference in volt, between points M and N of the system shown, if the e.m.f. is equal to $E = 110$ V and the capacitance ratio $\dfrac{C_2}{C_1} = \eta = 2$.



12. A condenser has capacitance $10\ \mu$F and is charged to a potential 150 V.

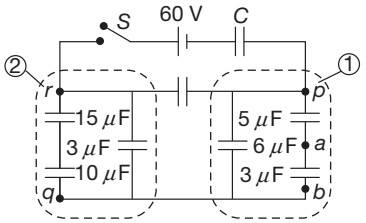
(a) Calculate the charge, in μC.

(b) A second condenser has a capacitance of $20\ \mu$F and is charged to a potential of 300 V. If after charging, the two condensers are connected in parallel by wires of negligible capacitance, how much energy, in mJ, is dissipated?

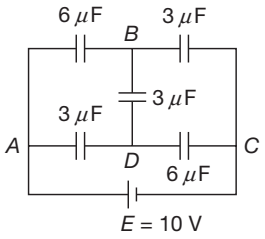
13. A parallel plate capacitor of plate area $A = 10^{-3}$ metre^2 and plate separation $d = 10^{-2}$ m is charged to $V_0 = 100$ V. Then after removing the charging battery, a slab of insulating material of thickness $b = 0.5 \times 10^{-2}$ m and dielectric constant $K = 7$ is inserted between the plates. Calculate the free charge, in pC, on the plates of the capacitor, electric field intensity, in kVm^{-1}, in air, electric field intensity, in Vm^{-1}, in the dielectric, potential difference, in volt, between the plates and capacitance, in pF, with dielectric present.

14. The distance between the parallel plates of a charged condenser is $d = 5$ cm and the intensity of the field $E = 300\ \text{V cm}^{-1}$. A slab of dielectric constant $K = 5$ and 1 cm wide is inserted parallel to the plates. Determine the potential difference, in volt, between the plates, before and after the slab is inserted. If the slab is replaced by a metal plate so that the final potential difference remains unchanged, what be the thickness, in mm, of the plate?

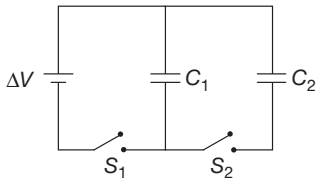
15. The circuit shown in Figure is in steady state. The potential difference between points a and b is 10 V. Find the capacitance, in μF, of the capacitor C.



16. In the circuit shown in Figure. Find the charge, in μC, on each capacitor.



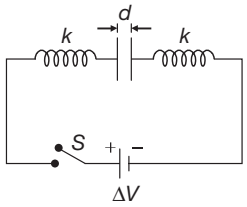
17. Consider the circuit shown in Figure, where $C_1 = 6\ \mu$F, $C_2 = 3\ \mu$F and $\Delta V = 20$ V. Capacitor C_1 is first charged by closing the switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by closing the switch S_2. Calculate



(a) the initial charge acquired by C_1, in μC.

(b) the final charge on each capacitor, in μC.

18. The circuit in figure consists of two identical parallel metal plates connected by identical metal springs to a 100 V battery. With the switch open, the plates are uncharged, are separated by a distance $d = 8$ mm and have a capacitance $C = 2\ \mu$F. When the switch is closed, the distance between the plates decreases by a factor of half.



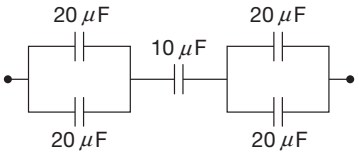
(a) How much charge, in μC, collects on each plate?

(b) What is the spring constant, in Nm^{-1}, for each spring?

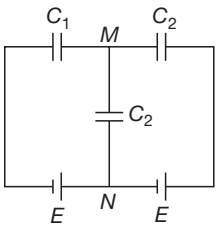
19. A certain storm cloud has a potential of 1×10^8 V relative to a tree. If, during a lightning storm, 50 C of charge is transferred through this potential difference and 1% of the energy is absorbed by the tree, how much sap, in kg (nearest to two digit integer) in the tree can be boiled away?

Assume that the "sap" is "water initially at 30 °C". Water has a specific heat of 4186 Jkg^{-1} °C, a boiling point of 100 °C, and a latent heat of vaporisation of 2.26×10^6 Jkg^{-1}

20. Each capacitor in the combination shown in figure has a breakdown voltage of 16 V. What is the breakdown voltage, in V, of the combination?



21. Find the potential difference, in volt, between points M and N of the system shown, if the emf is equal to $E = 110$ V and the capacitance ratio $\frac{C_2}{C_1}$ is 2.



22. A 20 pF parallel plate capacitor with air as medium is charged to 200 V and then disconnected from the battery. What is the energy U_i of the capacitor? The plates are then slowly pulled apart (in a direction normal to the plate area) so that the plate separation is doubled. What is the mechanical work done, in nJ, in the process?

23. A 3 μF parallel plate capacitor is connected to a battery of 400 V. The plates are then pulled apart as in above problem, so that the capacitance value becomes 1 μF. This operation is carried out while the capacitor is still connected to the battery of 400 V. Calculate the mechanical work done, in mJ. Account for the loss of energy of the capacitor.

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1. **[Online April 2019]**

Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 Vm^{-1}. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF? (given $\varepsilon_0 = 8.86\times10^{-12}$ C^2Nm^{-2})

(A) 3.8 (B) 4.5

(C) 8.5 (D) 6.2

2. **[Online April 2019]**

A parallel plate capacitor has 1 μF capacitance. One of its two plates is given +2 μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is

(A) 5 V (B) 1 V

(C) 3 V (D) 2 V

3. **[Online April 2019]**

A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(A) 2.55×10^{-6} J (B) 6.25×10^{-6} J

(C) 3.75×10^{-6} J (D) 2.16×10^{-6} J

4. **[Online April 2019]**

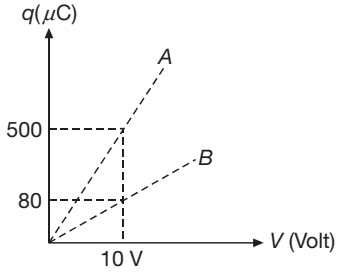
The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is L

(A) $\frac{(n+1)V}{(K+n)}$ (B) $\frac{V}{K+n}$

(C) V (D) $\frac{nV}{K+n}$

5. [Online April 2019]

Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are

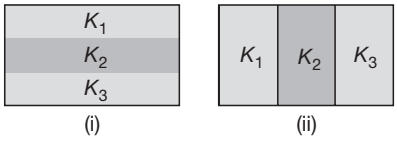


(A) 40 μF and 10 μF (B) 20 μF and 30 μF

(C) 60 μF and 40 μF (D) 50 μF and 30 μF

6. [Online April 2019]

Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in figure i, and the second one is filled as shown in figure ii. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II))



(A) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

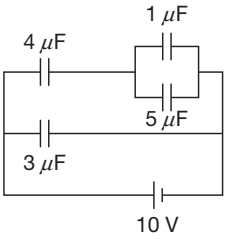
(B) $\frac{E_1}{E_2}=\frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

(C) $\frac{E_1}{E_2}=\frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(D) $\frac{E_1}{E_2}=\frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

7. [Online April 2019]

In the given circuit, the charge on 4 μF capacitor will be

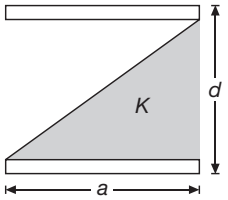


(A) 5.4 μC (B) 9.6 μC

(C) 13.4 μC (D) 24 μC

8. [Online January 2019]

A parallel plate capacitor is made of two square plates of side a, separated by a distance $d(d \ll a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is

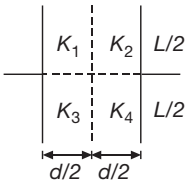


(A) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$ (B) $\frac{K\varepsilon_0 a^2}{d}\ln K$

(C) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (D) $\frac{1}{2}\frac{K\varepsilon_0 a^2}{d}$

9. [Online January 2019]

A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be



(A) $K=\frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

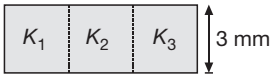
(B) $K=\frac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(C) $K=\frac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

(D) $K=\frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

10. [Online January 2019]

A parallel plate capacitor is of area $6\ cm^2$ and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be



(A) 36 (B) 14

(C) 12 (D) 4

11. [Online January 2019]

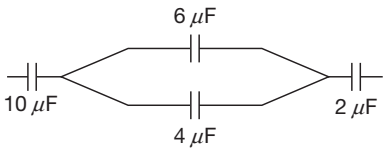
A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is

(A) 560 pJ (B) 692 pJ

(C) 508 pJ (D) 600 pJ

12. [Online January 2019]

In the figure shown below, the charge on the left plate of the $10\ \mu F$ capacitor is ... The charge on the right plate of the $6\ \mu F$ capacitor is



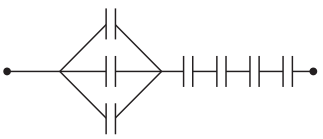
(A) $+18\ \mu C$ (B) $-12\ \mu C$

(C) $+12\ \mu C$ (D) $-18\ \mu C$

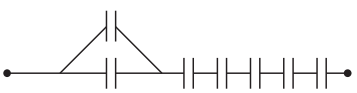
13. [Online January 2019]

Seven capacitors, each of capacitance $2\ \mu F$, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value?

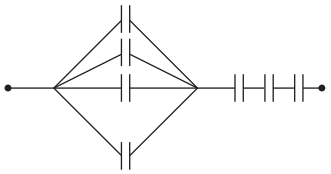
(A)



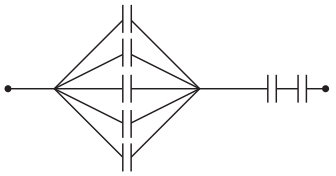
(B)



(C)

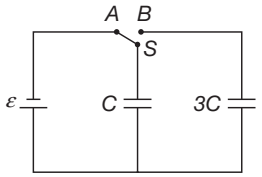


(D)



14. [Online January 2019]

In the figure shown, after the switch S is turned from position A to position B, the energy dissipated in the circuit in terms of capacitance C and total charge Q is

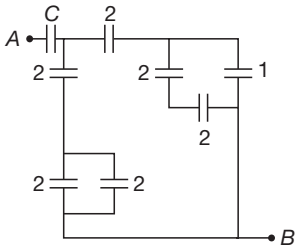


(A) $\frac{3}{8}\frac{Q^2}{C}$ (B) $\frac{1}{8}\frac{Q^2}{C}$

(C) $\frac{5}{8}\frac{Q^2}{C}$ (D) $\frac{3}{4}\frac{Q^2}{C}$

15. [Online January 2019]

In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\ \mu F$. All values in the circuit are in μF.

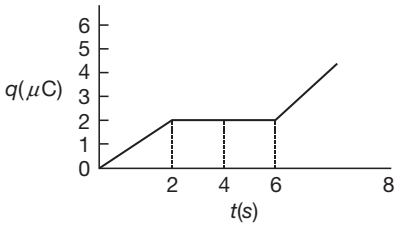


(A) $\frac{7}{11}\ \mu F$ (B) $4\ \mu F$

(C) $\frac{6}{5}\ \mu F$ (D) $\frac{7}{10}\ \mu F$

16. [Online January 2019]

The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure



What is the value of current at $t = 4$ s ?

(A) 2 μA (B) zero

(C) 3 μA (D) 1.5 μA

17. [2018]

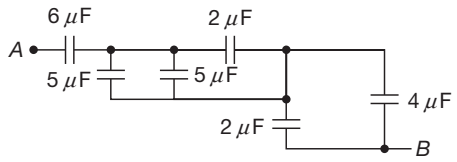
A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(A) 1.2 nC (B) 0.3 nC

(C) 2.4 nC (D) 0.9 nC

18. [Online 2018]

The equivalent capacitance between A and B in the circuit given below, is



(A) 5.4 μF (B) 4.9 μF

(C) 3.6 μF (D) 2.4 μF

19. [Online 2018]

A parallel plate capacitor with area 200 cm^2 and separation between the plates 1.5 cm, is connected across a battery of emf V. If the force of attraction between the plates is 25×10^{-6} N, the value of V is approximately

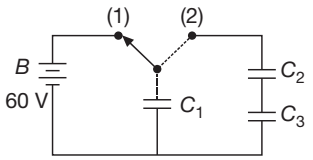
$\left(\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \right)$

(A) 150 V (B) 100 V

(C) 250 V (D) 300 V

20. [Online 2018]

A capacitor $C_1 = 1\ \mu$F is charged up to a voltage $V = 60$ V by connecting it to battery B through switch (1). Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors $C_2 = 3$ F and $C_3 = 6$ F through switch (2), as shown in the figure. The sum of final charges on C_2 and C_3 is



(A) 20 C (B) 40 C

(C) 36 C (D) 54 C

21. [2017]

A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(A) 2 (B) 16

(C) 24 (D) 32

22. [Online 2017]

The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be

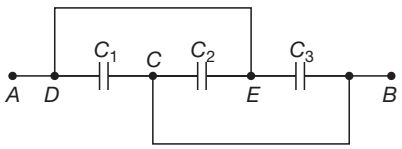
$\left[\text{Take } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2} \right]$

(A) 32 mm (B) 20 mm

(C) 16 mm (D) 28 mm

23. [Online 2017]

A combination of parallel plate capacitors is maintained at a certain potential difference.



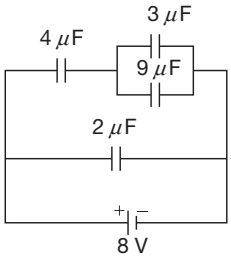
When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab

(A) 6 (B) 5

(C) 4 (D) 3

24. [2016]

A combination of capacitors is set up as shown in the figure. The magnitude of the electric field due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal



(A) 240 NC^{-1} (B) 360 NC^{-1}

(C) 420 NC^{-1} (D) 480 NC^{-1}

25. [Online 2016]

Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them

(A) all in series

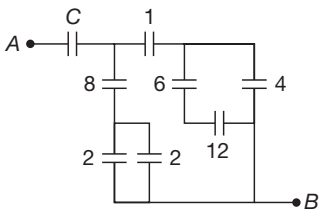
(B) all in parallel

(C) two in parallel and one in series

(D) two in series and one in parallel

26. [Online 2016]

Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is

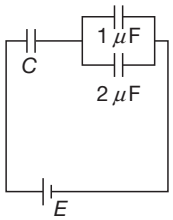


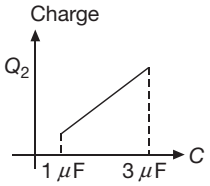
(A) $\frac{32}{23}$ μF (B) $\frac{31}{23}$ μF

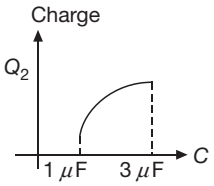
(C) $\frac{33}{23}$ μF (D) $\frac{34}{23}$ μF

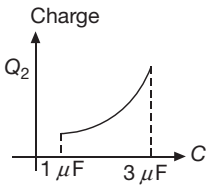
27. [2015]

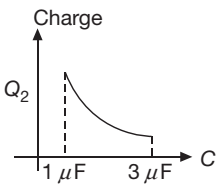
In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale)



(A)


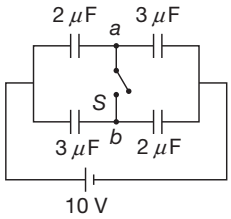
(B)


(C)


(D)


28. [Online 2015]

In figure is shown a system of four capacitors connected across a 10 V battery. Charge that will flow from switch S when it is closed is



(A) 5 μC from b to a

(B) 20 μC from a to b

(C) 5 μC from a to b

(D) zero

29. [2014]

A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

(A) 6×10^{-7} C/m^2

(B) 3×10^{-7} C/m^2

(C) 3×10^4 C/m^2

(D) 6×10^4 C/m^2

30. [2013]

Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(A) $5C_1 = 3C_2$ (B) $3C_1 = 5C_2$

(C) $3C_1 + 5C_2 = 0$ (D) $9C_1 = 4C_2$

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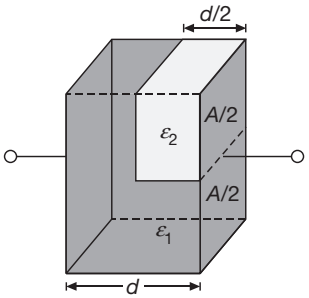
Single Correct Choice Type Problems

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2015]

A parallel plate capacitor having plates of area A and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is

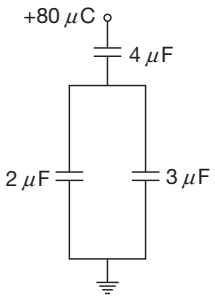


(A) $\frac{6}{5}$ (B) $\frac{5}{3}$

(C) $\frac{7}{5}$ (D) $\frac{7}{3}$

2. [IIT-JEE 2012]

In the given circuit, a charge of $+80\ \mu C$ is given to the upper plate of the $4\ \mu F$ capacitor. Then in the steady state, the charge on the upper plate of the $3\ \mu F$ capacitor is

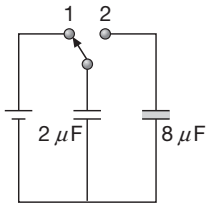


(A) $+32\ \mu C$ (B) $+40\ \mu C$

(C) $+48\ \mu C$ (D) $+80\ \mu C$

3. [IIT-JEE 2011]

A $2\ \mu F$ capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0% (B) 20%

(C) 75% (D) 80%

4. [IIT-JEE 2002]

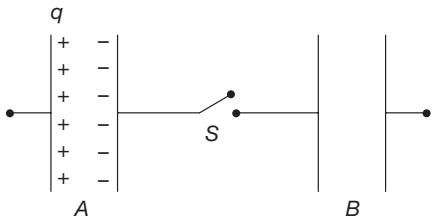
Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$

(C) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (D) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

5. [IIT-JEE 2001]

Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



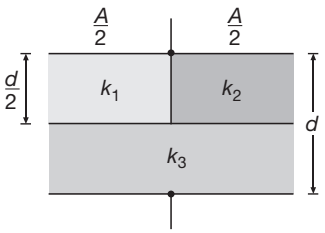
(A) zero (B) $\frac{q}{2}$

(C) q (D) $2q$

6. [IIT-JEE 2000]

A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be

used to have the same capacitance C in this capacitor then its dielectric constant k is given by



(A) $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{2k_3}$ (B) $\frac{1}{k}=\frac{1}{k_1+k_2}+\frac{1}{2k_3}$

(C) $\frac{1}{k}=\frac{k_1k_2}{k_1+k_2}+2k_3$ (D) $k=\frac{k_1k_3}{k_1+k_3}+\frac{k_2k_3}{k_2+k_3}$

7. [IIT-JEE 1999]

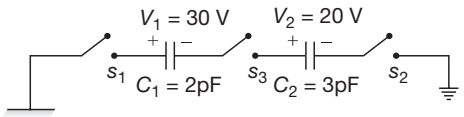
Two identical metal plates are given positive charges Q_1 and $Q_2(<Q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

(A) $\frac{(Q_1+Q_2)}{2C}$ (B) $\frac{(Q_1+Q_2)}{C}$

(C) $\frac{(Q_1-Q_2)}{C}$ (D) $\frac{(Q_1-Q_2)}{2C}$

8. [IIT-JEE 1999]

For the circuit shown, which of the following statements is true?



(A) With S_1 closed, $V_1=15$ V, $V_2=20$ V

(B) With S_3 closed, $V_1=V_2=25$ V

(C) With S_1 and S_2 closed, $V_1=V_2=0$

(D) With S_1 and S_3 closed, $V_1=30$ V, $V_2=20$ V

9. [IIT-JEE 1996]

The magnitude of the electric field E in the annular region of a charged cylindrical capacitor

(A) is same throughout

(B) is higher near the outer cylinder than near the inner cylinder

(C) varies as $\frac{1}{r}$, where r is the distance from the axis

(D) varies as $\frac{1}{r^2}$, where r is the distance from the axis

10. [IIT-JEE 1995]

A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration

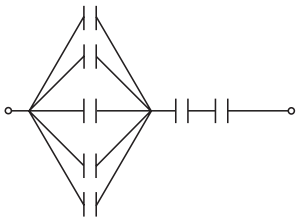
(A) ZERO (B) $\frac{3}{2}CV^2$

(C) $\frac{25}{6}CV^2$ (D) $\frac{9}{2}CV^2$

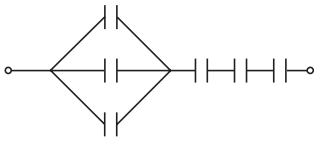
11. [IIT-JEE 1990]

Seven capacitors each of capacitance $2\,\mu$F are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{10}{11}\right)\mu$F. Which of combination(s) shown in figure, will achieve the desired results?

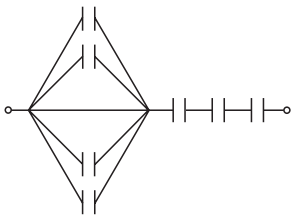
(A)



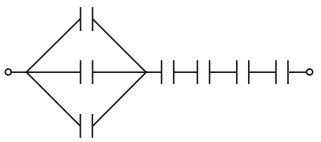
(B)



(C)



(D)

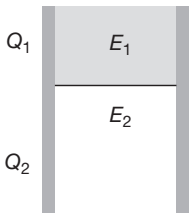


Multiple Correct Choice Type Problems

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [IIT-JEE 2014]

A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers $\frac{1}{3}$ of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.

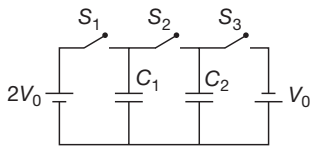


(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$

(C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

2. [JEE (Advanced) 2013]

In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time



(A) the charge on the upper plate of C_1 is $2CV_0$
(B) the charge on the upper plate of C_1 is CV_0
(C) the charge on the upper plate of C_2 is 0
(D) the charge on the upper plate of C_2 is $-CV_0$

3. [IIT-JEE 1998]

A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x=0$ and positive plate is at $x=3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$

(A) the magnitude of the electric field remains the same
(B) the direction of the electric field remains the same
(C) the electric potential increases continuously
(D) the electric potential increases at first, then decreases and again increases

4. [IIT-JEE 1991]

A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in questions, in the process of inserting the slab, then

(A) $Q=\frac{\varepsilon_0 AV}{d}$

(B) $Q=\frac{\varepsilon_0 KAV}{d}$

(C) $E=\frac{V}{Kd}$

(D) $W=\frac{\varepsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right)$

5. [IIT-JEE 1987]

A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles

(A) the charge on the capacitor increases
(B) the voltage across the plates increases
(C) the capacitance increases
(D) the electrostatic energy stored in the capacitor increases

6. [IIT-JEE 1985]

A parallel plate capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous ones as

(A) $Q>Q_0$ (B) $V>V_0$

(C) $E>E_0$ (D) $U>U_0$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. [JEE (Advanced) 2019]

A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1 + \frac{m}{N}\right)$. For a very large $N(>10^3)$, the capacitance C is $\alpha\left(\frac{K\varepsilon_0 A}{d\ln 2}\right)$. The value of α will be _____.

(ε_0 is the permittivity of free space)

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on General Capacitance)

1. 400 V

2. 2 J

3. (a) 7600 V

(b) 1.368×10^{-3} J

4. 5 J, $\frac{1}{2}$

5. (a) $\left(\frac{R_1}{R_1+R_2}\right)Q$, $\left(\frac{R_2}{R_1+R_2}\right)Q$

(b) 0

6. $\left[\left(\frac{V_0}{V}\right)^{\frac{1}{10}}-1\right]C_0$

Test Your Concepts-II (Based on Series and Parallel Combination of Capacitors)

1. (a) $C(\Delta V)^2$

(b) $\frac{4\Delta V}{3}$

(c) $4C\frac{(\Delta V)^2}{3}$

2. (a) Net capacitance remains same

(b) Net capacitance increases.

3. Across the 5 μF capacitor 23.3 V

Across the 10 μF capacitor 26.7 V

4. (a) $\frac{q^2}{2\varepsilon_0 A}(x_2-x_1)$

(b) $\frac{1}{2}\varepsilon_0 AV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)$

5. 0.24 μF, 0.16 μF

6. 40 mF and 10 mF

7. $\frac{\varepsilon_0 A}{4d}$

8. 78.7 V, 151.3 V.

9. $\frac{\varepsilon_0 A}{a-b}$

10. $\frac{\varepsilon_0 a^2}{d}\left[1-\frac{a\theta}{2d}\right]$

11. (a) Arrangement 1, $\frac{3}{2}\left(\frac{\varepsilon_0 A}{d}\right)$

(b) Arrangement 2, $\frac{2}{3}\left(\frac{\varepsilon_0 A}{d}\right)$

12. $(n-1)\frac{\varepsilon_0 A}{d}$

13. (a) 4 μF

(b) $V_1=8$ V, $Q_1=48$ μC

$V_2=4$ V, $Q_2=16$ μC

$V_3=4$ V, $Q_3=32$ μC

14. Arrangement 1, $\frac{2\varepsilon_0 A}{d}$

Arrangement 2, $\frac{3\varepsilon_0 A}{d}$

15. (a) 17 pF

(b) 1.224×10^{-7} J

16. (a) $\frac{q^2}{2C}$

(b) $\frac{q^2}{8\pi\varepsilon_0 R}$

Test Your Concepts-III (Based on Dielectrics and Breakdown)

1. $\frac{24}{25}$

2. (b) It becomes κ times the initial charge

3. (a) Electric field decreases by a factor $\left(\frac{K+1}{2}\right)$

(b) $\frac{CV(K-1)}{2(K+1)}$

4. 3.4%

5. (a) $\frac{\varepsilon_0 A}{d}$

(b) $\frac{\varepsilon_0 A}{d-t}$

(c) $\frac{2\varepsilon_0 A}{d}$

6. (a) $\varepsilon_0 A\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$ (b) $\varepsilon_0 V\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$

(c) $\frac{1}{2}\left(\frac{\varepsilon_0 K_1 V^2}{d_1^2}\right)$

7. $\frac{\varepsilon_0 K_1 K_2 A}{d_1 K_2 + d_2 K_1}$

8. 1.54 pF

9. $\frac{3}{5}$

10. (a) $\frac{\varepsilon_0}{d}\left[\ell^2 + \ell x(\kappa-1)\right]$

(b) $\frac{1}{2}\left(\frac{\varepsilon_0 (\Delta V)^2}{d}\right)\left[\ell^2 + \ell x(\kappa-1)\right]$

(c) $\frac{\varepsilon_0 (\Delta V)^2}{2d}\ell(\kappa-1)$

(d) 1.55×10^{-3} N

11. $\frac{2}{3}$

Test Your Concepts-IV (Based on Capacitor Circuits, Kirchhoff's Laws, Charge Flown and Generation of Heat)

1. $\frac{10}{7}C$

2. $V_1 = 10$ V, $V_2 = 5$ V

3. 10 μC and $\frac{40}{3}$ μC

4. $\frac{|E_2 - E_1| C_1 C_2}{C_1 + C_2}$

5. $V_1 = \frac{E_1(C_2+C_3) - E_2C_2 - E_3C_3}{C_1+C_2+C_3}$

$V_2 = \frac{E_2(C_1+C_3) - E_1C_1 - E_3C_3}{C_1+C_2+C_3}$

$V_3 = \frac{E_3(C_1+C_2) - E_1C_1 - E_2C_2}{C_1+C_2+C_3}$

6. $\frac{C_1C_4 - C_2C_3}{(C_1+C_2)(C_3+C_4)}V$, $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

7. $\frac{V_2C_2 - V_1C_1}{C_1+C_2+C_3}$

8. $2C$

9. Charge on 2 μF capacitor is $\frac{10}{3}$ μC

Charge on 4 μF capacitor is $\frac{140}{3}$ μC

Charge on 6 μF capacitor is 50 μC

10. $\frac{1}{2}V_2^2C$

11. (i) $\frac{200}{3}$ V, 0 V; (ii) 300 μC

Test Your Concepts-V (Based on Spherical and Cylindrical Capacitors)

1. 2 pF

2. (a) 2.5×10^4 V

(b) +1 μC

3. $\frac{Q}{q} = N$, $\frac{C}{c} = \frac{R}{r}$, $\frac{V}{v} = N^{\frac{2}{3}}$, $\frac{U}{u} = N^{\frac{5}{3}}$

4. $\frac{4\pi\varepsilon_0 b^2}{b-a}$

5. +24 nC

6. $\frac{\pi L}{\log_e\left(\frac{b}{a}\right)}(K_1+K_2)$

Single Correct Choice Type Questions

1. A	2. B	3. B	4. A	5. D	6. D	7. B	8. C	9. B	10. C
11. B	12. B	13. B	14. C	15. D	16. B	17. D	18. B	19. C	20. A
21. D	22. D	23. D	24. D	25. B	26. B	27. C	28. B	29. C	30. C
31. C	32. C	33. A	34. C	35. C	36. B	37. C	38. B	39. B	40. B
41. B	42. C	43. C	44. A	45. D	46. C	47. C	48. C	49. B	50. C
51. D	52. B	53. A	54. A	55. B	56. A	57. A	58. B	59. C	60. C
61. B	62. B	63. D	64. A	65. C	66. C	67. B	68. C	69. B	70. A
71. B	72. B	73. D	74. A	75. C	76. D	77. A	78. B	79. C	80. A
81. C	82. D	83. D	84. B	85. A	86. B	87. C	88. C	89. C	90. A
91. A	92. C	93. B	94. A	95. B	96. C	97. A	98. B	99. D	100. A
101. A	102. B	103. D	104. B	105. D	106. A	107. C	108. C	109. D	110. B
111. B	112. B	113. A	114. C	115. C	116. C	117. B	118. C	119. D	120. C
121. B	122. D	123. B	124. A	125. A	126. B	127. A	128. B	129. B	130. D
131. C	132. A	133. C	134. D	135. B	136. B	137. B	138. B	139. C	140. B
141. D	142. A	143. C	144. A	145. D	146. D	147. A	148. C	149. C	150. B
151. A	152. A	153. D	154. C	155. A	156. C	157. B	158. A	159. A	160. A
161. D	162. A	163. A	164. A	165. C	166. C	167. C	168. C	169. C	170. C
171. B	172. B	173. A	174. A	175. A	176. A	177. A	178. B	179. A	180. A
181. A	182. D	183. B							

Multiple Correct Choice Type Questions

1. A, B, C, D	2. A, C, D	3. B, D	4. B, C,	5. A, B, C, D
6. A, B, C, D	7. A, D	8. A, B, C, D	9. A, D	10. A, C, D
11. A, C, D	12. C, D	13. A, B, C,	14. B, C,	15. A, B, C, D
16. A, C, D	17. A, B, C,	18. A, C,	19. A, D	20. B, C, D
21. A, B,	22. B, C, D	23. A, B, C, D	24. B, C, D	25. B, C, D
26. A, D	27. A, B, C, D	28. B, C, D	29. A, C,	30. B, C,
31. A, D	32. A, D	33. A, C, D	34. A, D	35. A, C, D
36. A, C,	37. A, B, C,	38. A, C,	39. B, C, D	40. B, D
41. A, B, C, D	42. A, C, D			

Reasoning Based Questions

1. D	2. A	3. A	4. D	5. A	6. A	7. C	8. D	9. D	10. A
11. C	12. C	13. C	14. B						

Linked Comprehension Type Questions

1. D	2. A	3. B	4. D	5. B	6. D	7. A	8. B	9. C	10. B
11. C	12. D	13. C	14. C	15. A	16. C	17. C	18. B	19. C	20. B
21. B	22. B	23. C	24. A	25. B	26. C	27. A	28. C	29. B	30. B
31. C	32. D	33. B	34. D	35. C	36. A	37. B	38. B	39. A	40. C
41. C	42. C	43. B	44. C	45. B	46. A	47. D	48. B	49. C	50. D
51. C	52. A	53. D	54. D	55. D	56. C				

Matrix Match/Column Match Type Questions

1. A → (q, s)	B → (p)	C → (q, s)	D → (r)
2. A → (q)	B → (p)	C → (s)	D → (r)
3. A → (p, r, s)	B → (p, r)	C → (p, r, s)	D → (p, r)
4. A → (s)	B → (q)	C → (r)	D → (p)
5. A → (r)	B → (p)	C → (q)	D → (s)
6. A → (q)	B → (r)	C → (p)	D → (s)
7. A → (q)	B → (p)	C → (s)	D → (r)
8. A → (q)	B → (r)	C → (s)	D → (p)
9. A → (p, s)	B → (q, r, s)	C → (q, r, s)	D → (p)
10. A → (p, q)	B → (r, q)	C → (p, q, s)	D → (r, q)
11. A → (q, s)	B → (r, s)	C → (p, r, s)	D → (p, r, s)
12. A → (q)	B → (p)	C → (r)	D → (s)
13. A → (p)	B → (p)	C → (r)	D → (q)
14. A → (p, s)	B → (r, s)	C → (p, q)	D → (q)
15. A → (p, r)	B → (q)	C → (s, q)	D → (q)

Integer/Numerical Answer Type Questions

1. $\alpha = 2$	**2.** Yes, 1000	**3.** $q_{6\,\mu F} = 750\ \mu C$, $q_{2\,\mu F} = 250\ \mu C$		**4.** 3
5. 579	**6.** 6950	**7.** 5	**8.** (a) 5, (b) 64	**9.** 9
10. 2	**11.** 10	**12.** (a) 7500, (b) 75	**13.** 890, 10, 1430, 57, 16	**14.** 1500, 1260, 8
15. 9	**16.** Charge on 6 μF capacitor is 24 μC, Charge on 3 μF capacitor is 18 μC			**18.** (a) 400, (b) 2500
19. 10	**20.** 24	**21.** 550	**22.** 400	**23.** 160

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1. C	**2.** B	**3.** C	**4.** A	**5.** A	**6.** C	**7.** D	**8.** A	**9.** (*)	**10.** C
11. C	**12.** A	**13.** A	**14.** A	**15.** A	**16.** B	**17.** A	**18.** D	**19.** C	**20.** B
21. D	**22.** C	**23.** B	**24.** C	**25.** D	**26.** A	**27.** B	**28.** A	**29.** A	**30.** B

* No given option is correct

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Single Correct Choice Type Problems

1. D	**2.** C	**3.** D	**4.** C	**5.** A	**6.** D	**7.** D	**8.** D	**9.** C	**10.** B
11. A									

Multiple Correct Choice Type Problems

1. A, D	**2.** B, D	**3.** B, C	**4.** A, C, D	**5.** B, D	**6.** A, D

Integer/Numerical Answer Type Questions

1. 1

CHAPTER

3 Electric Current and Circuits

Learning Objectives

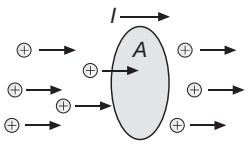
After reading this chapter, you will be able to understand concepts and problems based on:

(a) Current
(b) Ohm's Law
(c) Resistance
(d) Resistance Combinations
(e) Kirchhoff's Laws
(f) Nodal Analysis
(g) Cells
(h) Cell Combinations
(i) Ammeter
(j) Voltmeter
(k) Heating Effects of Current
(l) Wheatstone Bridge
(m) Post Office Box
(n) Potentiometer
(o) RC Circuit and Applications

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

ELECTRIC CURRENT

Whenever a charge moves (i.e. a charge moving with respect to an observer) we get current. So, moving charges (positive and negative) constitute electric current. Flow of electric charge is a direct measure of electric current. Suppose a collection of charges is moving perpendicular to a surface of area A, as shown in figure.



Charges moving through a cross section

The electric current is defined as the rate at which charges flow across any cross-sectional area. If an amount of charge ΔQ passes through a surface in a time interval Δt, then the average current I_{avg} is given by

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad \ldots(1)$$

The SI unit of current is the ampere (A), where $1\ A = 1$ coulomb $\sec^{-1}$.

Common currents range from mega-amperes (in lightning) to nano-amperes (in your nerves).

In the limit $\Delta t \to 0$, the instantaneous current I may be defined as

$$I = \lim_{t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \ldots(2)$$

Since flow has a direction, so we have implicitly introduced a convention that the **direction of current corresponds to the direction in which positive charges are flowing**. This current is called the *Conventional Current*.

The flowing charges inside wires are negatively charged electrons that move in the opposite direction of the current and is called 'Electronic Current'. Electric currents flow in conductors : solids (metals, semiconductors), liquids (electrolytes, ionized) and gases (ionized), but the flow is impeded in non-conductors or insulators.

According to its magnitude and direction we categorise the current in two types.

(a) Direct Current (DC): If the magnitude and direction of current does not vary with time, it is said to be direct current (DC). This type of current is provided by a Cell, a battery or DC dynamo.

(b) Alternating Current (AC): If a current is periodic (with constant amplitude) and has half cycle positive and half negative, it is said to be alternating current (AC). This type of current is generally sinusoidal in nature. An AC dynamo provides this type of current.

Remark(s)

(a) In all electric circuits, the current shown (or the arrows drawn) must represent (or indicate) the "Conventional Current".

(b) A negative charge moving to the right is equivalent to a positive charge moving to the left. So current due to this negative charge is towards the right.

(c) If a charge Q revolves in a circle of radius R with angular frequency ω or frequency f, then equivalent current,

$$I = \frac{\text{Charge Circulating}}{\text{Time to Complete one Revolution}}$$

$$I = \frac{Q}{T} = \frac{Q\omega}{2\pi} = \frac{Qv}{2\pi R}$$

DIFFERENT SITUATIONS PRODUCING CURRENT

Due to Translatory Motion of Charges

(a) If N particles, each having a charge q, pass through a given area in time t, the current is given by

$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{t}$$

(b) If n particles (each having a charge q) pass per second per unit area, then

$$n = \frac{N}{A\Delta t}$$

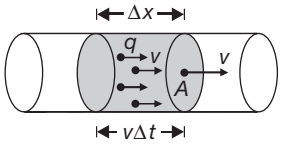
So, the current associated with cross-sectional area A is

$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{\Delta t} = nqA$$

(c) If there are n particles per unit volume in the conductor (each having a charge q) moving with velocity v, then

$$n = \frac{N}{A\Delta x}$$

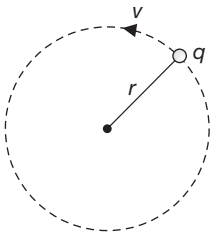
So, the current through cross-sectional area A is



$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{\Delta t} = nqA\frac{\Delta x}{\Delta t} = nqvA$$

Due to Circular Motion of Charge

If a point charge q is moving in a circle of radius r with speed v, then its time period $T = \frac{2\pi}{\omega} = \left(\frac{2\pi r}{v}\right) = \frac{1}{f}$.



So through a given hypothetical cross-section (perpendicular to motion), the current is

$$I = \frac{\text{Charge Circulating}}{\text{Period of One Revolution}}$$

$$\Rightarrow \quad I = \frac{q}{T} = qf = \frac{qv}{2\pi r} = \frac{q\omega}{2\pi}$$

where ω is the angular velocity of the charge.

Due to Convectional Motion of Charge

It is possible that a charged body is transported from one place to another. The **convectional current** is the current which is developed due to the transportation of charge or the mechanical transfer of a charge.

Conceptual Note(s)

Instantaneous current through a cross-section is

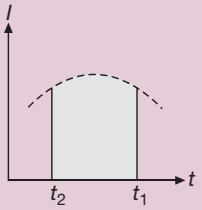
$$I = \frac{dq}{dt}$$

Charge passed through the cross-section in the interval t to $t + dt$

$$dq = Idt$$

Total charge in the interval t_1 to t_2 is given by

$$\Delta q = \int_{t_1}^{t_2} Idt = \text{Area under } I \text{ versus } t \text{ graph in the interval } t_1 \text{ to } t_2 \text{ as shown in figure.}$$



Average current in the interval t_1 to t_2 is

$$I_{av} = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{t_2 - t_1}$$

$$\Rightarrow \quad I_{av} = \frac{\Delta q}{t_2 - t_1} = \frac{\int_{t_1}^{t_2} Idt}{t_2 - t_1} = \frac{\text{Area under } I \text{ versus } t \text{ graph}}{\text{Time interval}}$$

ILLUSTRATION 1

The current in a wire varies with time according to the relation $I = a + bt^2$, where current I is in ampere and time t is in second and $a = 4$ A, $b = 2$ As^{-2}.

(a) How many coulomb pass a cross-section of the wire in the time interval between $t = 5$ s and $t = 10$ s ?

(b) What constant current could transport the same charge in same time interval ?

SOLUTION

(a) $$\Delta q = \int_5^{10} I\, dt = \int_5^{10} \left(4 + 2t^2\right) dt$$

$$\Rightarrow \quad \Delta q = \left| 4t + \frac{2}{3}t^3 \right|_5^{10} = 4(10-5) + \frac{2}{3}(1000-125)$$

$$\Delta q = 603.33 \text{ C}$$

(b) $$I_e = \frac{\Delta q}{\Delta t} = \frac{603.33}{10-5} = 120.67 \text{ A}$$

ILLUSTRATION 2

In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius 5×10^{-11} m, at a speed of 2.2×10^6 ms^{-1}. What is the current associated with electron motion ?

SOLUTION

The time taken to complete one rotation is

$$T = \frac{2\pi r}{v}$$

Therefore, the current is

$$I = \frac{q}{t} = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 2.2 \times 10^6}{2 \times 3.14 \times 5 \times 10^{-11}} = 1.12 \text{ mA}$$

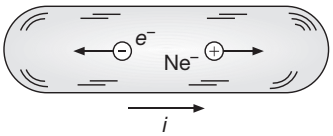
ILLUSTRATION 3

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} C. Calculate the current in the discharge tube.

SOLUTION

Since current is actually due to the flow of positive charges. Also we know that negative charge moving to the left is equivalent to positive charge moving to the right, so

$$\text{Net current } i = i_+ + i_- = \frac{(n_+)(q_+)}{t} + \frac{(n_-)(q_-)}{t}$$



$$\Rightarrow \quad i = \frac{(n_+)e}{t} + \frac{(n_-)e}{t}$$

$$\Rightarrow \quad i = \left(2.9 \times 10^{18} + 1.2 \times 10^{18}\right) \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad i = 0.66 \text{ A}$$

ILLUSTRATION 4

A long cylinder with uniformly charged surface and cross-sectional radius $a = 1$ cm moves with a constant velocity $v = 10\ \text{ms}^{-1}$ along its axis. An electric field strength at the surface of the cylinder is equal to $E = 0.9\ \text{kVcm}^{-1}$. Find the resulting convection current, that is, the current caused by mechanical transfer of a charge.

SOLUTION

The convection current is

$$I = \frac{dq}{dt} \qquad \ldots(1)$$

where, $dq = \lambda dx$, λ is the linear charge density

But, from the Gauss's Law, electric field at the surface of the cylinder,

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

Substituting the value of λ and subsequently of dq in equation (1), we get

$$I = \frac{dq}{dt} = \frac{\lambda dx}{dt}$$

$$\Rightarrow \quad I = \frac{2E\pi\varepsilon_0 a dx}{dt}$$

$$\Rightarrow \quad I = 2\pi\varepsilon_0 Eav \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

Substituting the given values, we get $I = 0.5\ \mu\text{A}$

Test Your Concepts-I

Based on Current Definition

(Solutions on page H.198)

1. In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path 5.29×10^{-11} m from the proton.
   (a) Show that the speed of the electron is $2.19\times10^{6}\ \text{ms}^{-1}$.
   (b) What is the effective current associated with this orbiting electron?

2. The quantity of charge q (in coulombs) that has passed through a surface of area 2 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds.
   (a) What is the instantaneous current through the surface at $t = 1$ s?
   (b) What is the value of the current density?

3. An electric current is given by the expression $I(t) = 100\sin(120\pi t)$, where I is in ampere and t is in second. What is the total charge carried by the current from $t = 0$ to $t = \left(\frac{1}{240}\right)$s?

4. A very small earthed conducting sphere is at a distance a from a point charge q_1 and at a distance b from a point charge $q_2\,(a < b)$. At a certain instant, the sphere starts expanding so that its radius grows according to the law $R = vt$. Determine the time dependence $I(t)$ of the current in the earthing conductor, assuming that the point charges and the centre of the sphere are at rest, and in due time the initial point charges get into the expanding sphere without touching it (through small holes).

5. The current in a wire varies with time according to the relation

$$I = (3A) + \left(2\frac{A}{s}\right)t$$

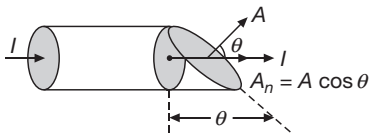
   (a) How many coulomb of charge passes a cross-section of the wire in the time interval between $t = 0$ and $t = 5$ s?
   (b) What constant current would transport the same charge in the same time interval?

6. The gap between two plane plates of a capacitor equal to d is filled with a gas. One of the plates emits n_0 electrons per second, which while moving in an electric field, ionize the gas molecules. This way each electron produces α new electrons (and ions) along a unit length of its path. Find the electronic current at the opposite plate, neglecting the ionization of gas molecules by the ions so formed. Take charge on an electron as e.

7. A long conductor of charge q, with charge density λ is moving with a velocity 2v parallel to its own axis. Find the convectional current due to motion of conductor.

CURRENT DENSITY $(\vec{j})$

The current density at any point inside a conductor is defined as a vector quantity whose magnitude is equal to current per unit infinitesimal area at that point, the area held normal to the direction of flow of current.



Current density $\vec{J}$ points along the direction of current flow.
If A_n is small area normal to current I, then

$$\text{Current density } J = \frac{I}{A_n}$$

If the plane of the small area A is not normal to current, but makes an angle θ with the to current, then

$$J = \frac{I}{A_n} = \frac{I}{A\cos\theta}$$

The unit of current density is Am^{-2}.
From above, we have

$$I = JA\cos\theta = \vec{J}\cdot\vec{A}$$

OHM'S LAW AND ELECTRICAL RESISTANCE

On applying a potential difference across a conductor, a current I is set up in the conductor. According to Ohm's Law "Under given physical conditions the potential difference (V) applied across a conductor produces a proportionate amount of current (I) in the conductor" i.e.,

$$V \propto I$$

$$\Rightarrow \quad V = IR$$

where the constant R is called the electrical resistance of the given conductor.
The unit of resistance R is volt/ampere.

$$1\ \text{VA}^{-1} = 1\ \text{ohm}(\Omega)$$

RESISTIVITY (ρ), CONDUCTIVITY (σ) AND CONDUCTANCE (G)

For a given conductor of uniform cross-section A and length l, it has been observed that the electrical resistance R is

(a) directly proportional to length l and
(b) inversely proportional to cross-sectional area A

$$\Rightarrow \quad R \propto \frac{l}{A}$$

$$\Rightarrow \quad R = \frac{\rho l}{A} \qquad \ldots(1)$$

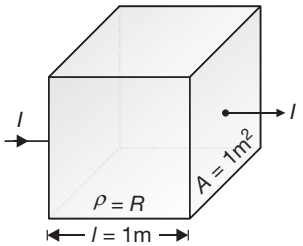
where ρ is a constant of proportionality called the **specific resistance** or **resistivity of the metal of the conductor** at a given temperature. Since

$$\rho = \frac{RA}{l}$$

If $l = 1\ \text{m}$, $A = 1\ \text{m}^2$, then $\rho = R$ (numerically).

So, the specific resistance of the material of the conductor is defined as the resistance offered by the conductor of length 1 m and cross sectional area $1\ \text{m}^2$, when current flows normal to the area.

Alternatively, the specific resistance of a material is defined as the resistance offered by the opposite faces of a cube of that material of side 1 m. The unit of resistivity is ohm metre (Ωm).



The reciprocal of resistivity (ρ) is called the **conductivity** (σ)

$$\Rightarrow \quad \sigma = \frac{1}{\rho}$$

The unit of conductivity is mho metre^{-1} $(\Omega^{-1}\ \text{m}^{-1})$
Ohm's Law in alternative form may be expressed as

$$J = \sigma E$$

where J = Current Density and E = Electric Field Strength.

Conductance (G) is the reciprocal of resistance

$$\Rightarrow \quad G = \frac{1}{R}$$

Unit of conductance is siemen or mho $(= \Omega^{-1})$.

ILLUSTRATION 5

A 3000 km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the cable. Use $3 \times 10^{-6}\ \Omega$ cm for the resistivity of the copper.

SOLUTION

The resistance R of a conductor is related to the resistivity ρ by $R = \frac{\rho l}{A}$, where l and A are the length of the conductor and the cross-sectional area, respectively. Since the cable consists of $N = 7$ copper wires, the total cross sectional area is

$$A = N\pi r^2 = N\frac{\pi d^2}{4} = 7\frac{\pi(0.073\ \text{cm})^2}{4}$$

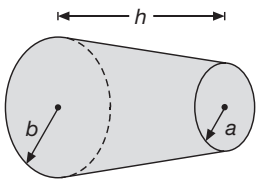
The resistance then becomes

$$R = \frac{\rho l}{A} = \frac{(3\times10^{-6}\ \Omega\text{cm})(3\times10^{8}\ \text{cm})}{\frac{7\pi(0.073\ \text{cm})^2}{4}} = 3.1\times10^{4}\ \Omega$$

$$\Rightarrow \quad R = 31\ \text{k}\Omega$$

ILLUSTRATION 6

Consider a material of resistivity ρ in a shape of a truncated cone of altitude h, and radii a and b, for the right and the left ends, respectively, as shown in the figure.



A truncated cone

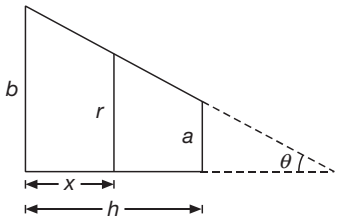
Assuming that the current is distributed uniformly throughout the cross-section of the cone, what is the resistance between the two ends?

SOLUTION

Consider a thin disk of radius r at a distance x from the left end. From the figure shown, we have

$$\frac{b-r}{x} = \frac{b-a}{h}$$

$$\Rightarrow \quad r = (a-b)\frac{x}{h} + b$$



Since resistance R is related to resistivity ρ by $R = \frac{\rho l}{A}$, where l is the length of the conductor and A is the cross section, the contribution to the resistance from the disk having a thickness dy is

$$dR = \frac{\rho\, dx}{\pi r^2} = \frac{\rho\, dx}{\pi\left[b + \frac{(a-b)x}{h}\right]^2}$$

Straight forward integration then yields

$$R = \int_0^h \frac{\rho\, dx}{\pi\left[b + \frac{(a-b)x}{h}\right]^2} = \frac{\rho h}{\pi ab}$$

$$\Rightarrow \quad R = \frac{\rho h}{\pi ab}$$

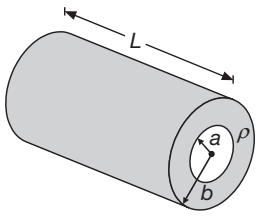
where we have used

$$\int \frac{du}{(\alpha u + \beta)^2} = -\frac{1}{\alpha(\alpha u + \beta)}$$

Note that if $b = a$, we get $R = \frac{\rho h}{\pi a^2} = \frac{\rho l}{A}$

ILLUSTRATION 7

Consider a hollow cylinder of length L and inner radius a and outer radius b, as shown in figure. The material has resistivity ρ.



A hollow cylinder

(a) Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?

(b) If instead the potential difference is applied between the inner and outer surfaces so that current flows radially outward, what is the resistance measured?

SOLUTION

(a) When a potential difference is applied between the ends of the cylinder, current flows parallel to the axis. In this case, the cross-sectional area is $A = \pi(b^2 - a^2)$, and the resistance is given by

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(b^2 - a^2)}$$

(b) Consider a differential element which is made up of a thin cylinder of inner radius r and outer radius $r + dr$ and length L. Its contribution to the resistance of the system is given by

$$dR = \frac{\rho\, dl}{A} = \frac{\rho\, dr}{2\pi r L}$$

where $A = 2\pi rL$ is the area normal to the direction of current flow. The total resistance of the system becomes

$$R = \int_a^b \frac{\rho dr}{2\pi rL} = \frac{\rho}{2\pi L}\log_e\left(\frac{b}{a}\right)$$

RECASTING A WIRE OF GIVEN MASS

Consider a conductor of mass m, density d, resistivity ρ to be recast to a wire of length l, cross-sectional area A (or radius r), then we have

$$R = \frac{\rho l}{A} \qquad \ldots(1)$$

Since, m = volume × density = Ald $\ldots(2)$

$$\Rightarrow \quad A = \frac{m}{ld}$$

Then from (1), we get

$$R = \frac{\rho l}{(m/ld)} = \left(\frac{\rho d}{m}\right) l^2 \qquad \ldots(3)$$

As ρ, d, m are constants, so we have

$$R \propto l^2$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 \qquad \ldots(4)$$

So, we conclude that if a given mass of wire is stretched (or drawn) to n times its original length, then the new resistance is n^2 times the original resistance of wire. Hence

$$R_{\text{new}} = n^2 R_{\text{old}}$$

Further if we put value of l from (2) i.e. $l = \frac{m}{Ad}$ in (1), then we get

$$R = \frac{\rho m}{AdA} = \left(\frac{m\rho}{d}\right)\left(\frac{1}{A^2}\right)$$

$$\Rightarrow \quad R \propto \frac{1}{A^2} \qquad \ldots(5)$$

Since $A = \pi r^2$

$$\Rightarrow \quad R \propto \frac{1}{r^4}$$

Remark(s)

Thus if given mass of material is stretched to make its length ℓ, then $R \propto \ell^2$ and if the given mass of material is stretched to make its radius r, then

$$R \propto \frac{1}{r^4}$$

or $$\frac{R_2}{R_1} = \left(\frac{r_1}{r_2}\right)^4$$

So, we conclude that if a given mass of wire is stretched (or draw) such that new radius of wire is n times its old radius, then

$$R_{\text{new}} = \frac{1}{n^4} R_{\text{old}}$$

FRACTIONAL CHANGE IN RESISTANCE

The resistance of a wire depends on its length L, radius r and cross-sectional area A (where $A = \pi r^2$). If the wire is stretched by a small amount, its resistance R changes.

CASE-1: Suppose the length L increases by a small amount $\Delta L (\ll L)$. Then, we can write

$$R = \frac{\rho L}{A} = \rho\left(\frac{L^2}{V}\right)$$

{as AL = volume, V = constant}

Hence fractional change in resistance with a small change in length is

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L}$$

CASE-2: Suppose the wire is stretched so that the cross-sectional area A reduces by small amount $\Delta A (\ll A)$. Then we can write

$$R = \frac{\rho L}{A} = \rho\left(\frac{V}{A^2}\right)$$

$$\Rightarrow \quad \frac{\Delta R}{R} = -2\frac{\Delta A}{A}$$

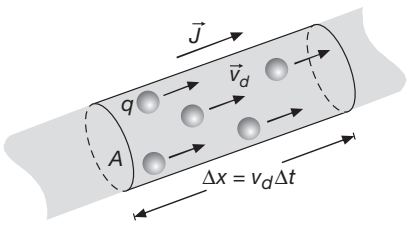
CASE-3: Suppose the wire is stretched so that the radius reduces by $\Delta r (\ll r)$. Then, we can write

$$R = \frac{\rho L}{A} = \rho\left(\frac{LA}{A^2}\right) = \rho\frac{V}{\left(\pi r^2\right)^2} = \left(\frac{\rho V}{\pi^2}\right)\frac{1}{r^4}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = -4\frac{\Delta r}{r}$$

CURRENT DENSITY AND DRIFT VELOCITY

To relate current, a macroscopic quantity, to the microscopic motion of the charges, let's examine a conductor of cross-sectional area A, as shown in figure.



A microscopic picture of current flowing in a conductor

Let the total current through a surface be written as

$$I = \int \vec{J} \cdot d\vec{A} \qquad \ldots(1)$$

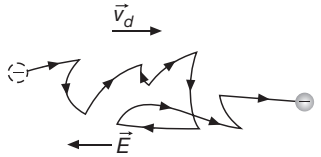
where $\vec{J}$ is the current density (the SI unit of current density are Am^{-2}). If q is the charge of each carrier, and n is the number of charge carriers per unit volume (also called "Charge Carrier Density"), the total amount of charge in this section is then

$$\Delta Q = q(nA\Delta x).$$

Suppose that the charge carriers move with a speed v_d, then the displacement in a time interval Δt will be $\Delta x = v_d \Delta t$. So, by definition of average current, we get

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqAv_d \qquad \ldots(2)$$

The speed v_d (along with the random speed) at which the charge carriers are moving is known as the **drift speed**. Physically, v_d is the average speed of the charge carriers inside a conductor when an external electric field is applied. Actually an electron inside the conductor does not travel in a straight line. Instead, its path is rather erratic, as shown in figure.



Motion of an electron in a conductor

From the above equations (1) and (2), the current density $\vec{J}$ can be written as

$$\vec{J} = nq\vec{v}_d \qquad \ldots(3)$$

Thus, we see that $\vec{J}$ and $\vec{v}_d$ point in the same direction for positive charge carriers, in opposite directions for negative charge carriers.

To find the drift velocity of the electrons, we first note that an electron in the conductor experiences an electric force $\vec{F}_e = -e\vec{E}$ which gives an acceleration

$$\vec{a} = \frac{\vec{F}_e}{m_e} = -\frac{e\vec{E}}{m_e}$$

Let the velocity of a given electron immediate after a collision be $\vec{u}$. The velocity of the electron immediately before the next consecutive collision is then given by

$$\vec{v} = \vec{u} + \vec{a}t = \vec{u} + \left(-\frac{e\vec{E}}{m_e} \right) t$$

where t is the time between two consecutive collisions. The average of $\vec{v}$ over all time intervals is

$$\langle \vec{v} \rangle = \langle \vec{u} \rangle - \frac{e\vec{E}}{m_e} \langle t \rangle$$

This average, $\langle v \rangle$ is equal to the drift velocity $\vec{v}_d$. Since in the absence of electric field, the velocity of the electron is completely random, it follows that $\langle \vec{u} \rangle = \vec{0}$. If $\tau = \langle t \rangle$ is the average characteristic time between successive collisions (the average relaxation time or the mean free time), we have

$$\vec{v}_d = \langle \vec{v} \rangle = -\left(\frac{e\vec{E}}{m_e} \right) \tau$$

The current density, from equation (3) becomes

$$\vec{J} = -ne\vec{v}_d = -ne\left(-\frac{e\vec{E}}{m_e} \tau \right) = \left(\frac{ne^2\tau}{m_e} \right) \vec{E} \qquad \ldots(4)$$

Note that $\vec{J}$ and $\vec{E}$ will be in the same direction for either negative or positive charge carriers.

Also, we compare (4) with $\vec{J} = \sigma\vec{E}$, then we get

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\Rightarrow \quad \rho = \frac{m_e}{ne^2\tau}$$

Remark(s)

(a) $\vec{J}$ and $\vec{v}_d$ point in the same direction for positive charge carriers, in opposite directions for negative charge carriers.

(b) $\vec{J}$ and $\vec{E}$ will be in the same direction for either negative or positive charge carriers.

OHM'S LAW: REVISITED

In many materials, the current density is linearly dependent on the external electric field $\vec{E}$ with their relation expressed as

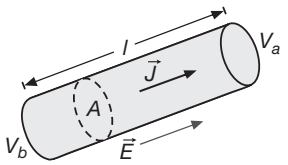
$$\vec{J} = \sigma\vec{E}$$

where σ is called the conductivity of the material. The above equation is known as the **Microscopic Ohm's Law**. A material that obeys this relation is said to be **ohmic**, otherwise, the material is **non-ohmic**.

Also, we have seen that the conductivity σ is given by

$$\sigma = \frac{ne^2\tau}{m_e}$$

To obtain a more useful form of Ohm's Law for practical applications let us consider a segment of straight wire of length l and cross-sectional area A, as shown in figure.



A uniform conuctor of lenght l and potential difference $\Delta V = V_b - V_a$

Suppose a potential difference $\Delta V = V_b - V_a$ is applied across the ends of the wire, creating an electric field $\vec{E}$ and hence make a current I to flow through it. Assuming $\vec{E}$ to be uniform, then

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = El$$

The current density can then be written as

$$J = \sigma E = \sigma \left(\frac{\Delta V}{l} \right)$$

where $J = \frac{I}{A}$. The potential difference becomes

$$\Delta V = \frac{l}{\sigma} J = \left(\frac{l}{\sigma A} \right) I = RI$$

where $R = \frac{\Delta V}{I} = \frac{l}{\sigma A}$, is the resistance of the conductor.

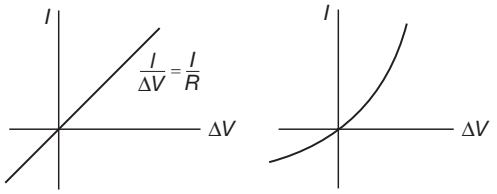
The equation $\Delta V = IR$ is the **"macroscopic"** version of the **Ohm's Law**.

The SI unit of R is the ohm (Ω, the Greek letter Omega), where

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

Once again, a material that obeys the above relation is **ohmic**, and **non-ohmic** if the relation is not obeyed.

Most metals, with good conductivity and low resistivity, are ohmic. We shall focus mainly on **ohmic materials**.



Ohmic vs. Non-ohmic behaviour

The resistivity ρ of a material is defined as the reciprocal of conductivity

$$\rho=\frac{1}{\sigma}=\frac{m_e}{ne^2\tau}$$

From the above equations, we see that ρ can be related to the resistance R of an object by

$$\rho=\frac{E}{J}=\frac{\frac{\Delta V}{l}}{\frac{I}{A}}=\frac{RA}{l}$$

$$\Rightarrow \quad R=\frac{\rho l}{A} \text{ where } \rho=\frac{m_e}{ne^2\tau}$$

MOBILITY

Conductivity arises from mobile charge carriers. These mobile charge carriers are electrons in case of metal, electrons and positive ions in case of an ionized gas and both positive and negative ions in case of an electrolyte. In semiconductor's material such as germanium and silicon, conduction is partly due to electrons and partly due to electron vacancies called holes. Holes are sites of missing electrons which acts like positive charges.

The mobility μ of a charge carrier is defined as the drift velocity per unit electric field.

$$\mu=\frac{v_d}{E}=\frac{|v_d|}{E}$$

Mobility is positive for both electrons and holes, although their drift velocities are opposite to each other. The electric conductivity for a semiconductor containing electrons and holes as charge carriers can be expressed as

$$\sigma=e\left(n_e\mu_e+n_h\mu_h\right)$$

where μ_e and μ_h are electron and hole mobilities and n_e and n_h are electron and hole concentrations (in charge carriers per unit volume) also called as electron and hole density.

Also, $v_d=\frac{q\tau E}{m}$ $\qquad \left\{\because\ v_d=a\tau=\left(\frac{qE}{m}\right)\tau\right\}$

$$\Rightarrow \quad \mu=\frac{v_d}{E}=\frac{q\tau}{m}$$

Hence, $\mu_e=\frac{e\tau_e}{m_e}$ and $\mu_h=\frac{e\tau_h}{m_h}$

τ_e and τ_h are relaxation time for electrons and holes, respectively. m_e and m_h refer to mass of electron and holes respectively. Charge on either of the carriers is e.

ILLUSTRATION 8

The resistivity of sea water is about $25\,\Omega\text{cm}$. The charge carriers are chiefly Na^+ and Cl^- ions, and of each there are about $3\times10^{20}\ \text{cm}^{-3}$. If we fill a plastic tube 2 m long with sea water and connect a 12 V battery to the electrodes at each end, what is the resulting average drift velocity of the ions, in cms^{-1}?

SOLUTION

The current in a conductor of cross sectional area A is related to the drift speed v_d of the charge carriers by

$$I=neAv_d$$

where n is the number of charges per unit volume. We can then rewrite the Ohm's Law as

$$V=IR=\left(neAv_d\right)\left(\frac{\rho l}{A}\right)=nev_d\left(\rho l\right)$$

$$\Rightarrow \quad v_d=\frac{V}{ne\rho l}$$

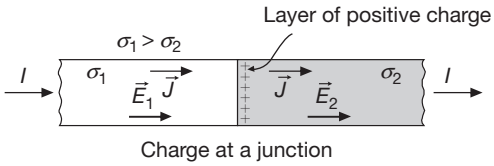
Substituting the values, we get

$$v_d=\frac{12\text{ V}}{\left(6\times10^{20}\ \text{cm}^{-3}\right)\left(1.6\times10^{-19}\ \text{C}\right)\left(25\ \Omega\ \text{cm}\right)\left(200\ \text{cm}\right)}$$

$$\Rightarrow \quad v_d=2.5\times10^{-5}\,\frac{\text{Vcm}}{\text{C}\Omega}=2.5\times10^{-5}\ \text{cms}^{-1}$$

ILLUSTRATION 9

Show that the total amount of charge at the junction of the two materials in figure is $\varepsilon_0 I\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$, where I is the current flowing through the junction, and σ_1 and σ_2 are the conductivities for the two materials.



Charge at a junction

SOLUTION

In a steady state of current flow, the normal component of the current density $\vec{J}$ must be the same on both sides of the junction. Since $J=\sigma E$, we have $\sigma_1 E_1 = \sigma_2 E_2$

$$\Rightarrow \quad E_2 = \left(\frac{\sigma_1}{\sigma_2}\right)E_1$$

Let the charge on the interface be q_{enc}, then from the Gauss's Law, we have

$$\oint \vec{E}\cdot d\vec{A} = (E_2 - E_1)A = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad E_2 - E_1 = \frac{q_{enc}}{A\varepsilon_0}$$

Substituting the expression for E_2 from above then

$$q_{enc} = \varepsilon_0 A E_1\left(\frac{\sigma_1}{\sigma_2}-1\right) = \varepsilon_0 A\sigma_1 E_1\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$$

Since the current is $I = JA = (\sigma_1 E_1)A$. So, the amount of charge on the interface becomes

$$q_{enc} = \varepsilon_0 I\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$$

Test Your Concepts-II

Based on Resistance, Resistivity and Ohm's Law

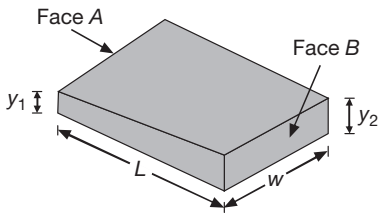
(Solutions on page H.199)

1. A close analogy exists between the flow of energy by heat because of a temperature difference and the flow of electric charge because of a potential difference. The energy *dQ* and the electric charge *dq* can both be transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness *dx*, area *A*, and electrical conductivity σ, with a potential difference *dV* between opposite faces. Show that the current $I = \frac{dq}{dt}$ is given by the equation on the left below:

Charge Conduction	Thermal Conduction
$\frac{dq}{dt} = \sigma A\left\lvert\frac{dV}{dx}\right\rvert$	$\frac{dQ}{dt} = kA\left\lvert\frac{dT}{dx}\right\rvert$

In the analogous thermal conduction equation on the right, the rate of energy flow $\frac{dQ}{dt}$ (in SI units of joules per second) is due to a temperature gradient $\frac{dT}{dx}$, in a material of thermal conductivity *k*. State analogous rules relating the direction of the electric current to the change in potential, and relating the direction of energy flow to the change in temperature.

2. Material with uniform resistivity ρ is formed into a wedge as shown in figure. Show that the resistance between face *A* and face *B* of this wedge is



$$R = \rho\frac{L}{w(y_2 - y_1)}\log_e\left(\frac{y_2}{y_1}\right)$$

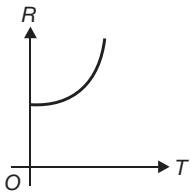
3. A piece of wire of uniform cross section has a resistance 8 Ω. If the length of the wire is doubled and its area of cross section is increased four times,

calculate the new resistance. Neglect the temperature variation of resistance.

4. **(a)** If a copper wire is stretched to make it 0.1% longer, keeping volume constant, what is the percentage change in its resistance?
   (b) What is percentage change in its resistance if radius is increased by 1% and area is increased by 1%?
5. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?
6. The region between two concentric conducting spheres with radii a and b is filled with a material with resistivity ρ.
   (a) Show that the resistance between the spheres is given by
   $$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$
   (b) Find the current density as a function of radius, in terms of the potential difference V_{ab} between the spheres.

VARIATION OF RESISTANCE WITH TEMPERATURE

The resistance of a conductor varies with temperature. The graph of variation of resistance of pure metal with temperature is shown.



Mathematically the dependence of (R) on temperature (T) is expressed as

$$R_T = R_0\left(1 + \alpha T + \beta T^2\right) \quad \ldots(1)$$

where α and β are **Temperature Coefficients of Resistance (TCR)**. The values of α and β vary from metal to metal.

If temperature T is not sufficiently large (as in most practical cases), then equation (1) may be expressed as

$$R_T = R_0(1 + \alpha T) \quad \ldots(2)$$

The constant α is called the **Temperature Coefficient of Resistance (TCR)** of the material. α is positive for metals and negative for semiconductors and electrolytes. If R_1 and R_2 are the resistances of the same specimen at temperatures T_1 °C and T_2 °C, then

$$R_1 = R_0\left(1 + \alpha T_1\right) \quad \ldots(3)$$

$$R_2 = R_0\left(1 + \alpha T_2\right) \quad \ldots(4)$$

So, the temperature coefficient of resistance is given by

$$\alpha = \left|\frac{R_2 - R_1}{R_1 T_2 - R_2 T_1}\right|$$

The resistivity of a material actually varies with temperature T. For metals, the variation is linear over a large range of T

$$\rho_T = \rho_0\left(1 + \alpha T\right)$$

where α is the temperature coefficient of resistivity. Typical values of ρ, σ and α (at 20 °C) for different types of materials are given in the table below.

Material	Resistivity ρ (Ωm)	Conductivity σ $(\Omega m)^{-1}$	Temperature coefficient α $(°C)^{-1}$
Elements			
Silver	1.59×10^{-8}	6.29×10^{7}	0.0038
Copper	1.72×10^{-8}	5.81×10^{7}	0.0039
Aluminium	2.82×10^{-8}	3.55×10^{7}	0.0039
Tungsten	5.6×10^{-8}	1.8×10^{7}	0.0045
Iron	10×10^{-8}	1×10^{7}	0.0050
Platinum	10.6×10^{-8}	1×10^{7}	0.0039
Alloys			
Brass	7×10^{-8}	1.4×10^{7}	0.002
Manganin	44×10^{-8}	0.23×10^{7}	1×10^{-5}
Nichrome	100×10^{-8}	0.1×10^{7}	0.0004

(*Continued*)

Material	Resistivity ρ (Ωm)	Conductivity σ $(\Omega m)^{-1}$	Temperature coefficient α $(°C)^{-1}$
Semiconductors			
Carbon (graphite)	3.5×10^{-5}	2.9×10^{4}	–0.0005
Germanium (pure)	0.46	2.2	–0.048
Silicon (pure)	640	1.6×10^{-3}	–0.075
Insulators			
Glass	$10^{10}-10^{14}$	$10^{-14}-10^{-10}$	
Sulfur	10^{15}	10^{-15}	
Quartz (fused)	75×10^{16}	1.33×10^{-18}	

Variation of Resistivity of Metals with Temperature

Since, $R=\dfrac{m\ell}{ne^2\tau A}$

and $\rho=\dfrac{m}{ne^2\tau}$

The number density of free electrons, n, is practically independent of temperature for most metals. However, an increase of temperature increases the amplitude of vibration of atoms and also the average speed of the free electrons. Therefore the average relaxation time τ decreases and hence resistivity increases with temperature.

If ρ_0 and ρ_T are the values of resistivity of a material at 0°C and T°C respectively then over a temperature range that is not too large, we have approximately,

$$\rho_T=\rho_0(1+\alpha T)$$

ILLUSTRATION 10

The temperature coefficient of resistivity α is given by $\alpha=\left(\dfrac{1}{\rho}\right)\dfrac{d\rho}{dT}$, where ρ is the resistivity at temperature T.

(a) Assume that α is not constant and is given by $\alpha=-\dfrac{a}{T}$, where T is the absolute temperature where a is a constant, show that the resistivity ρ is given by $\rho=\dfrac{b}{T^n}$, where b is another constant.

(b) Using the values $\rho=3.5\times10^{-5}\ \Omega m$ and $\alpha=-5\times10^{-4}\ (C°)^{-1}$ for graphite at 293 K, determine a and b.

(c) Using your result from part (b), determine the resistivity of graphite at –196 °C and 300 °C. (Remember to express T on absolute scale).

SOLUTION

(a) $\alpha=\dfrac{1}{\rho}\dfrac{d\rho}{dT}$

$$\Rightarrow\quad \frac{d\rho}{\rho}=\alpha\,dT=-a\frac{dT}{T}$$

Let $\rho=\rho_0$ at $T=T_0$, then

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho}=-a\int_{T_0}^{T}\frac{dT}{T}$$

$$\Rightarrow\quad \log_e\left(\frac{\rho}{\rho_0}\right)=-a\log_e\left(\frac{T}{T_0}\right)=\log_e\left(\frac{T_0}{T}\right)^a$$

$$\Rightarrow\quad \rho=\left(\rho_0T_0^a\right)\frac{1}{T^a}=\frac{b}{T^a}$$

Here, $b=\rho_0T_0^a$

(b) Given that $\rho_0=3.5\times10^{-5}\ \Omega m$ and

$$\alpha_0=-5\times10^{-4}\,(°C)^{-1}\ \text{at}\ T_0=293\ K$$

$$a=-\alpha_0T_0 \qquad \left\{\text{as } \alpha=-\frac{a}{T}\right\}$$

$$\Rightarrow\quad a\approx0.15$$

and $b=\rho_0T_0^a=(3.5\times10^{-5})(293)^{0.15}$

$$\Rightarrow\quad b=8\times10^{-5}\ \Omega mK^{0.15}$$

{be careful while taking units of a and b}

(c) Now at $T=-196+273=77$ K,

$$\rho=\frac{b}{T^a}=\frac{8\times10^{-5}}{(77)^{0.15}}\approx4.3\times10^{-5}\ \Omega m$$

Similarly at $T=300+273=573$ K

$$\rho=\frac{8\times10^{-5}}{(573)^{0.15}}\approx3\times10^{-5}\ \Omega m$$

COLOUR CODE FOR CARBON RESISTANCES

Carbon resistances find their wide use in electronic circuits at low voltages due to following reasons.

(a) They may have values ranging from few ohms to 100 MΩ .
(b) They are made up of small handy sizes.
(c) They are quite cheap.

A Colour Code is used to indicate the resistance and its percentage reliability. The carbon resistor has a set of concentric rings of various colours on it with their significance in the table.

Colour	Number	Multiplier	% Tolerance
Black (B)	0	$10^0 = 1$	Gold 5%
Brown (B)	1	10^1	Silver 10%
Red (R)	2	10^2	No Colour 20%
Orange (O)	3	10^3	
Yellow (Y)	4	10^4	
Green (G)	5	10^5	
Blue (B)	6	10^6	
Violet (V)	7	10^7	
Grey (G)	8	10^8	
White (W)	9	10^9	

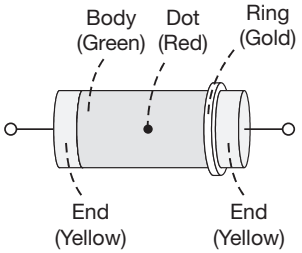
The colour bands are formed from left to right. The first three bands $(A, B$ and $C)$ give the value of resistance. The colours of first and second bands indicate the first and second significant digits while the colour of third band gives decimal multiplier (i.e., the number of zeros which follow the first two digits). The colour of fourth band represents its tolerance. Absence of any colour means a tolerance of 20%.

e.g. if A is red, B is green, C is grey and there is no colour at D, then

$$R = 25 \times 10^8 \ \Omega \pm 20\%$$

The table may be memorised by the mnemonic
BB ROY of **G**reat **B**ritain has a **V**ery **G**ood **W**ife.
Capital letters represent the first letter of colour (*from Darkest to the lightest colour*).
Also you may try to remember the following mnemonic
Black **B**ears **R**oar, **O**rangutans **Y**ell, **G**oats **B**leat **V**iolently, **G**o **W**eep.

Another way of representation is the Dot-Body convention in which the body of the resistance is given one colour, the ends are given the other second same colour and a dot is marked over the body. A ring (silver or gold) on one side indicates the tolerance. The colour code table remains the same.

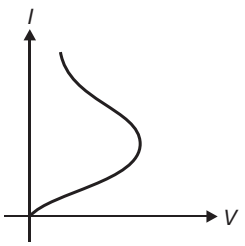


Colour of body describes the first significant digit.
Identical end colours give the second significant figure.
Colour of dot gives the decimal multiplier.
The colour of the ring gives the tolerance.
So, for the colours specified in the figure the resistance has a value

$$R = 54 \times 10^2 \ \Omega \pm 5\%$$

THERMISTOR

A thermistor is a heat sensitive resistor usually made of semiconductor material. The temperature coefficient of a thermistor is negative but is unusually large, of the order of $-0.04(^\circ\text{C})^{-1}$. The V-I curve of a thermistor is unusual and is shown in Figure.



Uses of Thermistor

(a) They are used for resistance thermometers in very low temperature measurement of the order of 10 K .

(b) They may be used to safeguard electronic circuits against current jumps because initially (when cold) thermistor has high resistance and its resistance drops appreciably when it warms up.

Test Your Concepts-III

Based on Variation of Resistance with Temperature

(Solutions on page H.200)

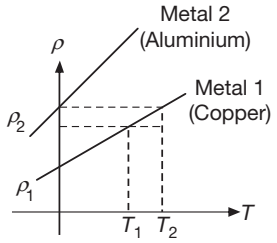
1. When the temperature of a heating element changes, its resistance also changes and so does the temperature coefficient. Suppose the temperature varies linearly with time and is given by $T(°C)=(20+10t)$ where t is time in second. The temperature coefficient of the material is $0.0065\ (°C)^{-1}$ at 0°C. If the initial resistance of the heating element is $2r$, then express the resistance as a function of time.

2. Obtain a general relationship between α_0 and α_T, the respective temperature coefficients at 0°C and T°C.

3. Obtain a general relationship between α_1 and α_2, the respective temperature coefficients at T_1°C and T_2°C.

4. Obtain the expression for the ratio of resistances of a coil at two temperatures T_1 and T_2 assuming that the only other given quantity is the temperature coefficient α_0 at 0°C.

5. It has been experimentally found that the resistivity of conducting materials such as copper and aluminium varies linearly with temperature. Obtain a relation between resistivities of a metal ρ_1 and ρ_2 at two temperatures T_1 and T_2 respectively.

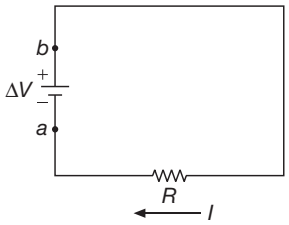


6. When a metal rod is heated, not only its resistance but also its length and area of cross-section changes. Find the percent change in R, ℓ and A of a copper wire for a temperature rise of 1°C. Coefficient of linear expansion for copper is $1.7\times10^{-5}\ (°C)^{-1}$ and its thermal coefficient of resistance is $3.9\times10^{-3}\ (°C)^{-1}$.

7. Two coils connected in series have resistance of 600 Ω and 300 Ω and temperature co-efficient of $0.001\ (°C)^{-1}$ and $0.004\ (°C)^{-1}$ respectively at 20°C. Find resistance of the combination at a temperature of 50°C. What is the effective temperature co-efficient of combination?

ELECTRICAL ENERGY AND POWER

Consider a circuit consisting of a battery and a resistor with resistance R .



Let the potential difference between two points a and b be $\Delta V = V_b - V_a > 0$. If a charge Δq is moved from a through the battery, its electric potential energy is increased by $\Delta U = \Delta q \Delta V$. On the other hand, as the charge moves across the resistor, the potential energy is decreased due to collisions with atoms in the resistor. If we neglect the internal resistance of the battery and the connecting wires, upon returning to a the potential energy of Δq remains unchanged.

Thus, the rate of energy loss through the resistor is given by

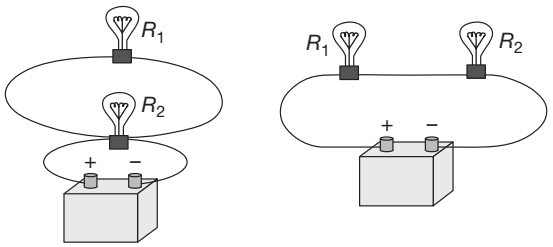
$$P = \frac{\Delta U}{\Delta t} = \left(\frac{\Delta q}{\Delta t}\right)\Delta V = I\Delta V$$

This is precisely the power supplied by the battery. Using $\Delta V = IR$, the above equation can be rewritten as

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

Introduction to Electrical Circuits

Electrical circuits connect power supplies to loads such as resistors, motors, heaters, or lamps.



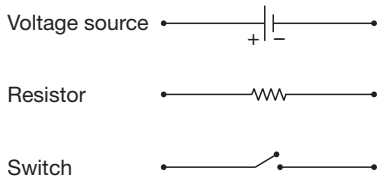
Elements connected (a) in parallel and (b) in series

The connection between the supply and the load is made with the help of wires (that are often called **leads**), or with many kinds of **connectors** and **terminals**. Energy is delivered from the source to the user on demand at the flick of a switch. Sometimes many circuit elements are connected to the same lead, which is called a **common lead** for those elements. Various parts of the circuits are called **circuit elements**, which can be in series or in parallel, as we have already seen in the case of capacitors.

Elements are said to be in parallel when they are connected across the same potential difference. Generally, loads are connected in parallel across the power supply.

On the other hand, when the elements are connected one after another, so that the current passes through each element without any branches, the elements are in series.

There are pictorial diagrams that show wires and components roughly as they appear, and schematic diagrams that use conventional symbols, somewhat like road maps. Some frequently used symbols are shown here for the purpose of convenience.



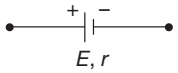
Often, in circuits we observe a switch to be connected in series. When the switch is open the load is disconnected and when the switch is closed, the load is connected.

One can have closed circuits, through which current flows, or open circuits in which there are no currents. Usually by accident, wires may touch, causing a short circuit. Most of the current flows through the short, very little will flow through the load. This may burn out a piece of electrical equipment such as a transformer. To prevent damage, a fuse or circuit breaker is put in series. When there is a short the fuse blows, or the breaker opens.

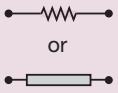
In electrical circuits, a point (or some common lead) is chosen as the ground. This point is assigned an arbitrary voltage, usually zero, and the voltage V at any point in the circuit is defined as the voltage difference between that point and ground.

Active and Passive Elements in Electric Circuits

An **electric circuit** is an arrangement of active and passive elements, having one or more closed paths for electric current to flow. It is due to the active elements (such as cells, batteries, dynamos, generators, etc.) that the current is forced to flow in a closed path. These supply energy to the circuit. The active element in a dc circuit is a **cell** or a **battery**. Its circuit symbol is



On the contrary, the passive elements either consume or store electric energy. There are three basic passive elements.

	1	2	3
Passive element (property)	Resistor (Resistance)	Capacitor (Capacitance)	Inductor (Inductance)
Symbol	R	C	L
Units	Ω	F	H
Circuit Symbol	 or		
***V-I* Relationship**	$V = IR$	$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
Energy consumed/ Stored	I^2Rt	$\frac{1}{2}CV^2$	$\frac{1}{2}LI^2$
Characteristic property	Converts electric energy into heat	Opposes variations in voltage	Opposes variations in current

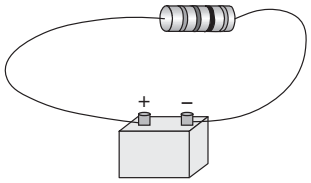
ELECTROMOTIVE FORCE (EMF)

We are aware of the fact that energy has to be supplied to maintain a constant current in a closed circuit. The source of energy is commonly referred to as the **electromotive force**, or the **emf** (symbol E). The ability of a cell or a battery to make the current flow in the circuit is measured in terms of its **electromotive** force or emf. Batteries, solar cells and thermocouples are some examples of emf source. They can be thought of as a "charge pump" that moves charges from lower potential to the higher potential. Mathematically emf is defined as

$$E = \frac{W}{q} \qquad \ldots(1)$$

So, emf is the work done to move a unit positive charge in the direction of the applied external filed or from higher potential to the lower potential. SI unit for E is volt (V).

Consider a simple circuit consisting of a battery as the emf source and a resistor of resistance R, as shown in figure.

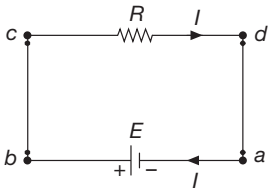


A simple circuit consisting of a battery and a resistor

Assuming that the battery has no internal resistance, the potential difference ΔV (or terminal voltage) between the positive and the negative terminals of the battery is equal to the emf E. To drive the current around the circuit, the battery undergoes a discharging process which converts chemical energy to emf. The current I is found by noting that no work is done in moving a charge q around a closed loop due to the conservative nature of the electrostatic force. So,

$$W = q\oint \vec{E}\cdot d\vec{l} = 0 \qquad \ldots(2)$$

Consider the loop $abcda$ and the point a in figure be the point of start.

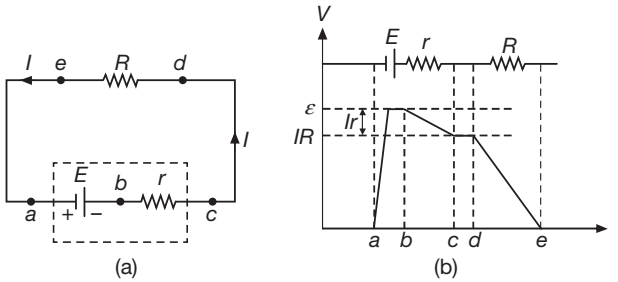


When crossing from the negative to the positive terminal, the potential increases by E. On the other hand, as we cross the resistor, the potential decreases by an amount IR, and the potential energy is converted into thermal energy in the resistor. *Assuming that the connecting wire carries no resistance* so, upon completing the loop the net change in potential difference must be zero. Hence, we must have

$$E - IR = 0 \qquad \ldots(3)$$

$$\Rightarrow \quad I = \frac{E}{R} \qquad \ldots(4)$$

However in reality a battery always carries an internal resistance r and the potential difference across the



(a) Circuit with an emf source having an internal resistance *r* and a resistor of resistance *R*.
(b) Change in electric potential around the circuit.

battery terminals, also called as the **Terminal Potential Difference (TPD)**, becomes

$$\Delta V = E - Ir \qquad \ldots(5)$$

Since there is no net change in potential difference around a closed loop *abcdea*, we have

$$E - Ir - IR = 0 \qquad \ldots(6)$$

$$\Rightarrow \quad I = \frac{E}{R + r} \qquad \ldots(7)$$

Remark(s)

(a) Figure (b) depicts the change in electric potential as we traverse the circuit clockwise. From the figure, we see that the highest voltage is immediately after the battery. The voltage drops as each resistor is crossed. Note that the voltage is essentially constant along the wires. This is because the wires have a negligibly small resistance compared to the resistors.

(b) For a source with emf *E*, the power or the rate at which energy is delivered is

$$P = IE = I(IR + Ir) = I^2R + I^2r$$

That the power of the source emf is equal to the sum of the power dissipated in both the internal and load resistance is required by energy conservation.

(c) *V* (potential difference across the terminals of the cell) may be less than *E*, equal to *E* or greater than *E*.

(i) $V < E$, when the circuit draws current *I* from the cell and in that case $V = E - Ir$.

(ii) $V = E$, when the circuit draws no current from the cell i.e. when $I = 0$.

(iii) $V > E$, when the cell is being charged and in that case $V = E + Ir$.

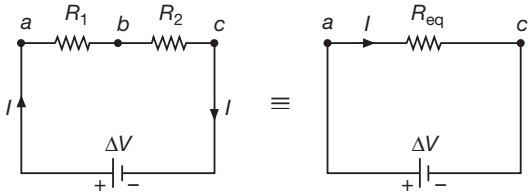
RESISTORS IN SERIES AND IN PARALLEL

Series Combination

The two resistors R_1 and R_2 in figure are connected in series to a voltage source ΔV. By current conservation, the same current I is flowing through each resistor.

The total voltage drop from a to c across both elements is the sum of the voltage drops across the individual resistors

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$



(a) Resistors in series (b) Equivalent circuit

The two resistors in series can be replaced by one equivalent resistor R_{eq} (figure (b)) with the identical voltage drop $\Delta V = IR_{eq}$ which implies that

$$R_{eq} = R_1 + R_2$$

The above argument can be extended to N resistors placed in series. The equivalent resistance is just the sum of the original resistances,

$$R_{eq} = R_1 + R_2 + ... = \sum_{i=1}^{N} R_i$$

(a) Notice that if one resistance R_1 is much larger than the other resistances R_i, then the equivalent resistance R_{eq} is approximately equal to the largest resistor R_1.

(b) The current flowing in each resistor is same.

(c) The total potential difference V across the combination is equal to the sum of the potential difference across individual resistances i.e.,

$$V = V_1 + V_2 + V_3$$

such that $V_1 : V_2 : V_3 \equiv R_1 : R_2 : R_3$

(d) The equivalent or effective resistance (R_S) of the combination is equal to the sum of individual resistances i.e.,

$$R_S = R_1 + R_2 + R_3$$

(e) Net Conductance (G) is

$$\frac{1}{G_S} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

(f) Net Resistance R_s has a value greater than the largest resistance of the combination.

(g) $V_1 = \left(\frac{R_1}{R_S}\right)V$

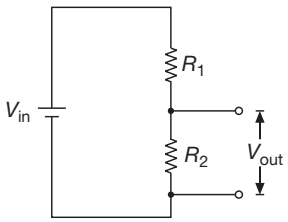
$V_2 = \left(\frac{R_2}{R_S}\right)V$

$V_3 = \left(\frac{R_3}{R_S}\right)V$

where $R_S = R_1 + R_2 + R_3$

Voltage Divider Circuit

Consider a voltage source V_{in} that is connected in series to two resistors, R_1 and R_2.



Voltage divider

The voltage difference, V_{out}, across resistor R_2 will be less than V_{in}. This circuit is called a voltage divider. From the loop rule,

$$V_{in} - IR_1 - IR_2 = 0$$

So the current in the circuit is given by

$$I = \frac{V_{in}}{R_1 + R_2}$$

Thus the voltage difference, V_{out}, across resistor R_2 is given by

$$V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_{in}$$

Note that the ratio of the voltages characterizes the voltage divider and is determined by the resistors

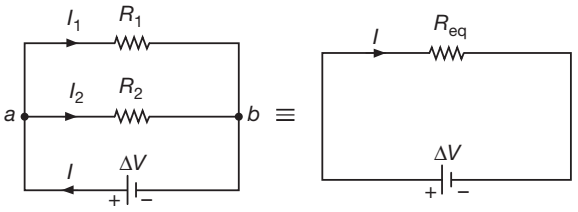
$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

Parallel Combination

Next let's consider two resistors R_1 and R_2 that are connected in parallel across a voltage source ΔV.

By current conservation, the current I that passes through the voltage source must divide into a current I_1 that passes through resistor R_1 and a current I_2 that passes through resistor R_2. Each resistor individually satisfies Ohm's Law, $\Delta V_1 = I_1 R_1$ and $\Delta V_2 = I_2 R_2$. However, the potential across the resistors are the same, $\Delta V_1 = \Delta V_2 = \Delta V$. Current conservation then implies

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



(a) Two resistors in parallel (b) Equivalent resistance

The two resistors in parallel can be replaced by one equivalent resistor R_{eq} with $\Delta V = IR_{eq}$. Comparing these results, the equivalent resistance for two resistors that are connected in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

This result easily generalizes to N resistors connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... = \sum_{i=1}^{N} \frac{1}{R_i}$$

(a) The potential difference (V) across each resistor is the same.

(b) The current is different in different resistances such that the total current flowing in the combination is shared by the individual resistances in the inverse ratio of their resistances i.e.,

$$I = I_1 + I_2 + I_3$$

such that $I_1 : I_2 : I_3 \equiv \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$

(c) The equivalent or effective resistance (R_P) of the combination is given by

$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

(d) Effective conductance G_P is the sum of conductance of individual resistors i.e.,

$$G_P=G_1+G_2+G_3$$

(e) Net Resistance R_P has a value smaller than the least resistance of the combination.

(f) $I_1=\left(\frac{R_P}{R_1}\right)I,\ I_2=\left(\frac{R_P}{R_2}\right)I$ and $I_3=\left(\frac{R_P}{R_3}\right)I$ where

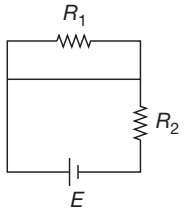
$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

(g) When one resistance R_1 is much smaller than the other resistances, then the equivalent resistance R_{eq} is approximately equal to the smallest resistor R_1. In the case of two resistors,

$$R_{eq}=\frac{R_1R_2}{R_1+R_2}\approx\frac{R_1R_2}{R_2}=R_1$$

This means that almost all of the current that enters the node point will pass through the branch containing the smallest resistance. So, when a short develops across a circuit, all of the current passes through this path of nearly zero resistance. **Hence SHORT CIRCUITING a branch implies taking the resistance of that branch to be zero or close to zero.**

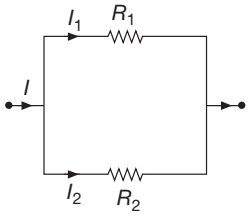
Shown here is an electric circuit whose two points are directly connected by a conducting wire.



Here we say that the resistance R_1 in the circuit is short circuited. Under such condition, both points are at same potential and hence the potential difference across R_1 is zero. Hence, no current will flow through R_1 and the current through R_2 is $I=\frac{E}{R_2}$.

Current Divider

Consider two resistors connected in parallel as shown. From figure, we have



$$I=I_1+I_2 \qquad \ldots(1)$$

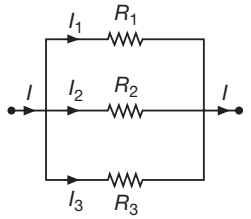
$$I_1R_1=I_2R_2 \qquad \ldots(2)$$

On solving equations (1) and (2), we get

$$I_1=\left(\frac{R_2}{R_1+R_2}\right)I \text{ and } I_2=\left(\frac{R_1}{R_1+R_2}\right)I$$

So, we observe that the division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Similarly, consider three resistors connected in parallel as shown.



From figure, we have

$$I=I_1+I_2+I_3$$

$$I_1R_1=I_2R_2=I_3R_3$$

and $$R_{eq}=\frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_3R_1}$$

$$I_1=I\left[\frac{R_2R_3}{R_1R_2+R_2R_3+R_3R_1}\right]$$

$$\Rightarrow\quad I_1=\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right)I=\left(\frac{\frac{1}{R_1}}{\frac{1}{R_P}}\right)I$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$\Rightarrow \quad I_2 = \left(\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right)I = \left(\frac{\frac{1}{R_2}}{\frac{1}{R_P}}\right)I$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

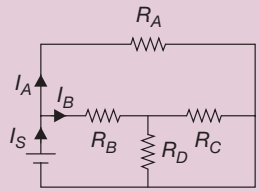
$$\Rightarrow \quad I_3 = \left(\frac{\frac{1}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right)I = \left(\frac{\frac{1}{R_3}}{\frac{1}{R_P}}\right)I$$

It is easy to remember the expressions for I_1, I_2 and I_3 just by noticing which resistance is missing in the term at the numerator.

Word of Advice

Be **careful** in applying current division. Without parallel resistors, there is no way that the current division can be applied. To say

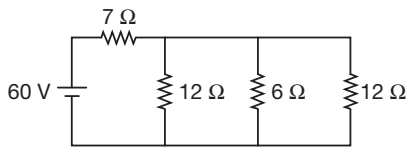
$I_A = I_S \dfrac{R_B}{R_A + R_B}$ is **wrong**.



Remember, parallel resistor must be the resistors connected between the same pair of terminals.

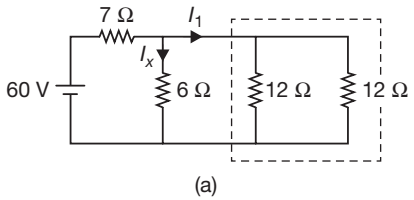
ILLUSTRATION 11

In the given network, calculate the current through $6\ \Omega$ resistor.

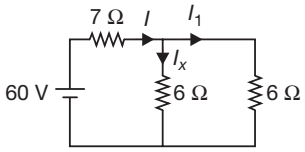


SOLUTION

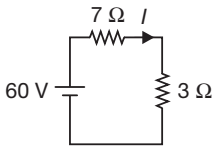
In this problem, our aim is to calculate only the current through $6\ \Omega$ resistor, we can do it **easily** by using current divider concept.



(a)



(b)



(c)

We first calculate the current I supplied by the battery. For this we rearrange the network and simplify it step by step as follows:

(a) We interchange the position of $6\ \Omega$ and $12\ \Omega$ resistors. This will not make any difference, as the two resistors are in parallel [Fig. (a)].

(b) We then combine the two $12\ \Omega$ resistors to give a single resistor of $6\ \Omega$ [Fig. (b)]

(c) We again combine the two $6\ \Omega$ resistors to give a single resistor of $3\ \Omega$ [Fig. (c)].

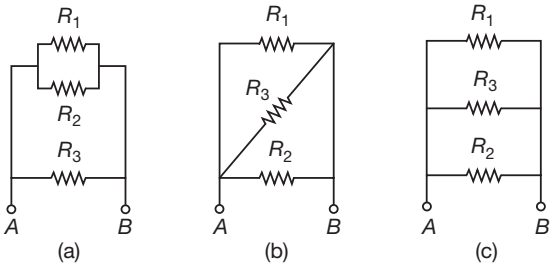
(d) The current I is clearly given as $I = \dfrac{60}{7+3} = 6\ \text{A}$

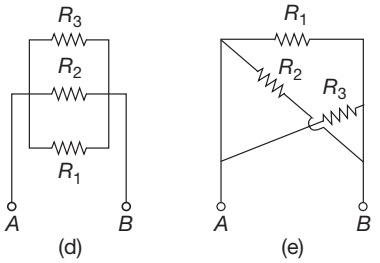
(e) Using Fig. (b), and the current divider concept, we get the current through $6\ \Omega$ resistor as

$$I_x = I\left(\frac{6}{6+6}\right) = 3\ \text{A}$$

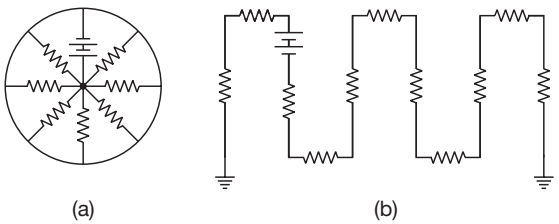
NETWORK ANALYSIS

(a) The first step in analyzing a circuit is to simplify it to simplest equivalent configuration. In figure, each of the circuits has the same equivalent resistance between A and B, in each version, R_1, R_2 and R_3 are in parallel.

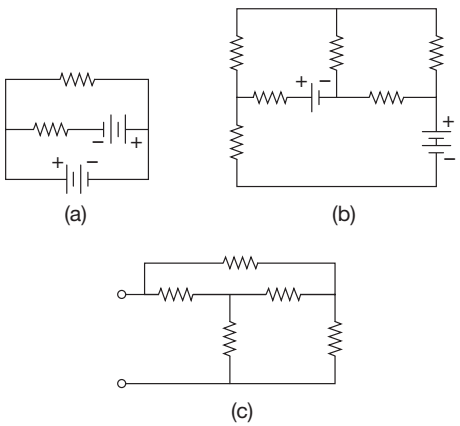


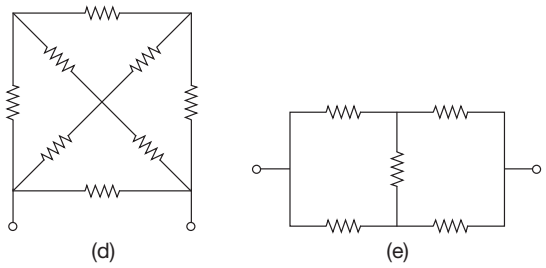


(b) All the resistors in figure (a) are in parallel arrangement. All the resistors in figure (b) are in series arrangement.

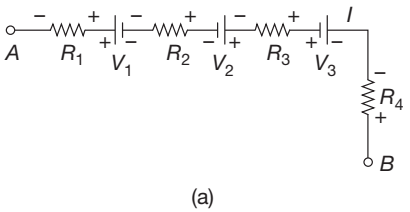


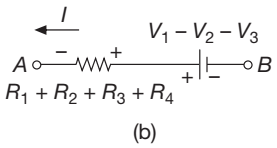
(c) Figure shows several circuits in which the circuit elements are neither in series nor in parallel.





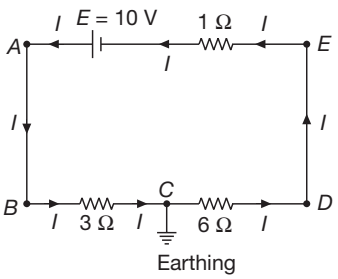
(d) If the same current passes through every resistor in a given branch, respective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.





EARTHING OR GROUNDING IN AN ELECTRICAL CIRCUIT

If a certain point in an electrical circuit is earthed, its potential is taken as zero. Consider the following circuit. When only one point is earthed, then there is no effect on current flowing in the circuit.



Using *KVL* in *ABCDEA*,

$$-3I - 6I - I + 10 = 0$$

$\Rightarrow \quad I = 1$ A

Now, $V_C = 0$ V (because C is grounded)

Using KVL, $V_B - 3I = V_C$

$\Rightarrow \quad V_B = V_C + 3I$

$\Rightarrow \quad V_B = 3$ V

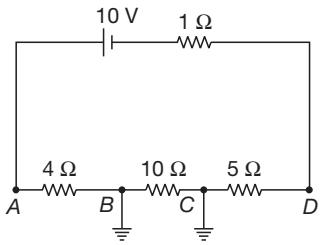
Similarly $V_C - 6I = V_D$

$\Rightarrow \quad V_D = -6$ V

Hence, earthing point gives us the reference potential i.e., zero potential and potentials of different points can then be calculated with respect to this new reference.

ILLUSTRATION 12

In the electrical circuit shown, points B and C are earthed. Find the potentials of points A and D.



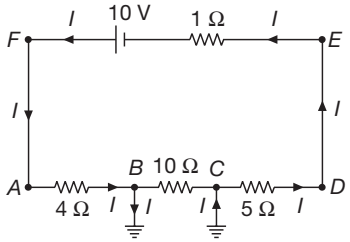
SOLUTION

Since B and C both are earthed, so

$$V_B = V_C = 0$$

$\Rightarrow \quad V_B - V_C = 0$

$\Rightarrow$ Current through 10 Ω is zero



Applying *KJL* to closed loop *ABCDEFA*, we get

$$-4I + 0 - 5I - (1)I + 10 = 0$$

$\Rightarrow \quad I = 1$ A

Now, $V_A - V_B = 4I = 4$ V

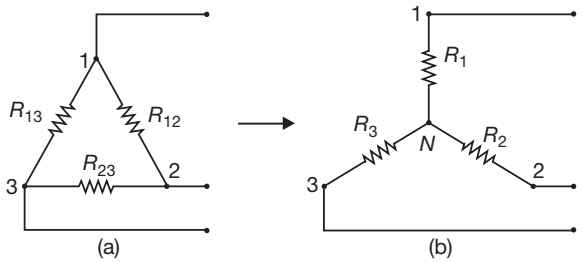
$\Rightarrow \quad V_A = 4$ V

Also, $V_C - V_D = 5I$

$\Rightarrow \quad V_D = -5$ V

DELTA TO STAR OR DELTA-STAR TRANSFORMATION

Suppose we are given three resistances R_{12}, R_{23} and R_{13} connected in delta fashion between terminals 1, 2 and 3 as in figure (a). So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in figure (b).



These two arrangements will be electrically equivalent because resistance as measured between any pair of terminals is the same in both the arrangements.

$$R_1 = \frac{R_{12}R_{13}}{R_{12} + R_{23} + R_{13}}$$

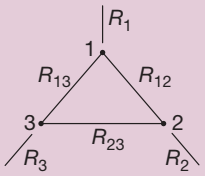
$$R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{13}}$$

and $\quad R_3 = \dfrac{R_{13}R_{23}}{R_{12} + R_{23} + R_{13}}$

Problem Solving Technique(s)

How to Remember (Delta to Star)?

Resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.



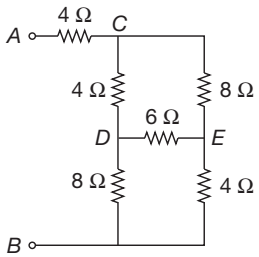
$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{23}+R_{13}}$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{13}} \quad \text{and}$$

$$R_3 = \frac{R_{13}R_{23}}{R_{12}+R_{23}+R_{13}}$$

ILLUSTRATION 13

Find the input resistance of the circuit between the points A and B of figure.



SOLUTION

For finding R_{AB}, we will convert the delta CDE of figure into its equivalent star as shown in figure.

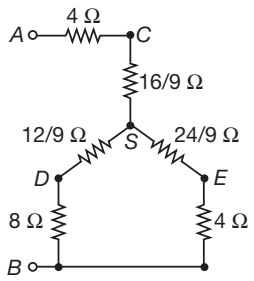
$$R_{CS} = 8\times\frac{4}{18} = \frac{16}{9}\ \Omega\ ;\ R_{ES} = 8\times\frac{6}{18} = \frac{24}{9}\ \Omega$$

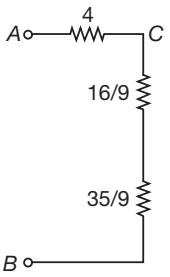
$$R_{DS} = 6\times\frac{4}{18} = \frac{12}{9}\ \Omega$$

The two parallel resistances between S and B can be reduced to a single resistance of $\frac{35}{9}\ \Omega$

As seen from figure

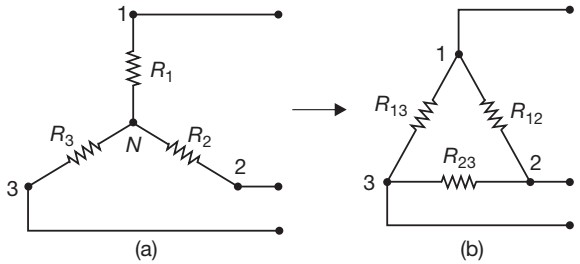
$$R_{AB} = 4+\left(\frac{16}{9}\right)+\left(\frac{35}{9}\right) = \frac{87}{9}\ \Omega$$





STAR TO DELTA OR STAR-DELTA TRANSFORMATION

Suppose we are given three resistances R_1, R_2 and R_3 connected in star fashion between terminals 1, 2 and 3 as shown in figure (a). So far as the respective terminals are concerned, these three resistances can now be replaced by the three resistances R_{12}, R_{23} and R_{13} connected in the delta network as shown in figure (b).



These two arrangements will be electrically equivalent because resistance as measured between any pair of terminals is the same in both the arrangements.

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

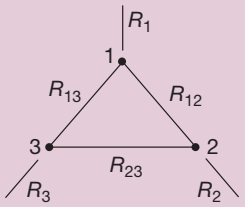
$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

and $$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

Problem Solving Technique(s)

How to Remember (Star to Delta)?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divided by the third star resistance.



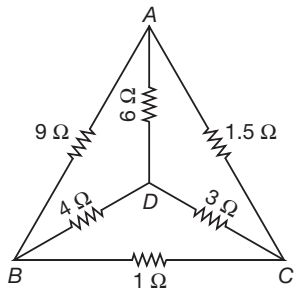
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3},$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \text{ and}$$

$$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

ILLUSTRATION 14

A network of resistances is formed as follows:



(a)

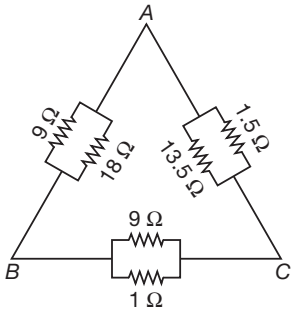
$AB = 9\ \Omega$; $BC = 1\ \Omega$; $CA = 1.5\ \Omega$ forming a delta and $AD = 6\ \Omega$, $BD = 4\ \Omega$ and $CD = 3\ \Omega$ forming a star. Compute the network resistance measured between

(a) A and B,
(b) B and C, and
(c) C and A.

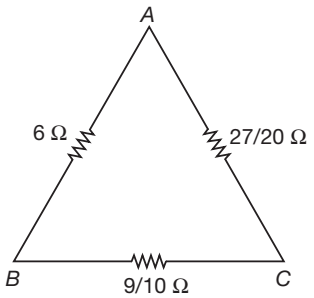
SOLUTION

The star of figure (a) may be converted into the equivalent delta and combined in parallel with the given delta ABC. The three equivalent delta resistances of the given star become as shown in figure (b).

When combined together, the final circuit is as shown in figure (c).



(b)



(c)

(a) As seen, there are two parallel paths across points A and B.

(i) First, directly from A to B having a resistance of $6\ \Omega$, and

(ii) the other via C having a total resistance of

$$\left(\frac{27}{20} + \frac{9}{10}\right) = 2.25\ \Omega$$

$$\Rightarrow \quad R_{AB} = \frac{6 \times 2.25}{(6 + 2.25)} = \frac{18}{11}\ \Omega$$

(b) $$R_{BC} = \frac{\frac{9}{10} \times \left(6 + \frac{27}{20}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{10}\right)} = \frac{441}{550}\ \Omega$$

(c) $$R_{CA} = \frac{\frac{27}{20} \times \left(6 + \frac{9}{10}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{20}\right)} = \frac{621}{550}\ \Omega$$

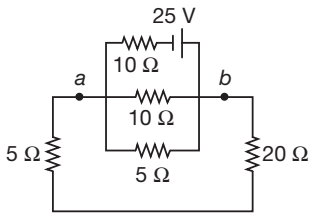
Test Your Concepts-IV

Based on Series and Parallel Combination of Resistances

(Solutions on page H.201)

1. Four copper wires of equal length are connected in series. Their cross-sectional areas are 1 cm^2, 2 cm^2, 3 cm^2, and 5 cm^2. A potential difference of 120 V is applied across the combination. Determine the voltage across the wire having the area 2 cm^2.

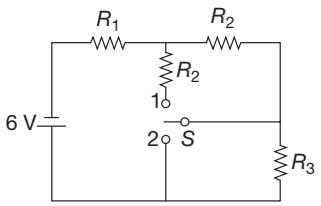
2. Consider the circuit shown in figure. Find



(a) the current in the 20 Ω resistor and

(b) the potential difference between points *a* and *b*.

3. A 6 V battery supplies current to the circuit shown in figure. When the double-throw switch *S* is open, as shown in the figure, the current in the battery is 1 mA. When the switch is closed in position 1, it is observed that the current in the battery is 1.2 mA and when the switch is closed in position 2, it is observed that the current in the battery is 2 mA. Find the resistances R_1, R_2 and R_3.

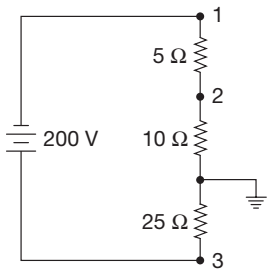


4. Two resistors connected in series have an equivalent resistance of 690 Ω. When they are connected in parallel, their equivalent resistance is 150 Ω. Find the resistance of each resistor.

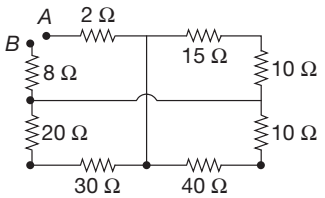
5. In the circuit shown, find the magnitude and polarity of the voltages mentioned below:

(i) V_1, (ii) V_2, (iii) V_3,

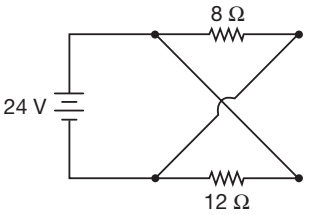
(iv) V_{3-2}, (v) V_{1-2}, (vi) V_{1-3}.



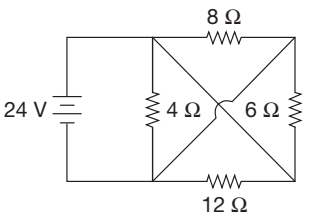
6. In the circuit shown, calculate the net resistance between the points *A* and *B*.



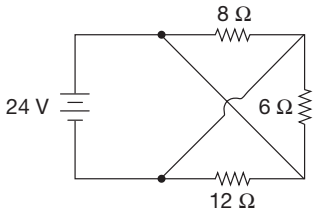
7. In the circuit shown, find the current supplied by the battery.



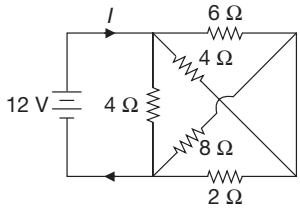
8. Find the current through the battery and the equivalent resistance of the network shown.



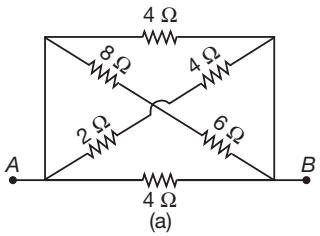
9. Find the total equivalent circuit resistance and current through the battery in the circuit shown.



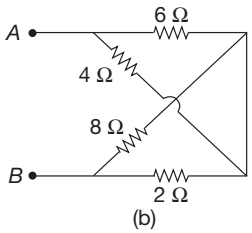
10. Compute the value of battery current I shown in figure.



11. Find the equivalent resistance between A and B in the following circuits:

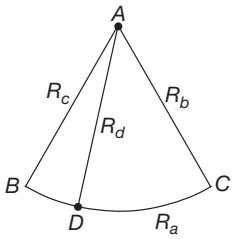


(a)

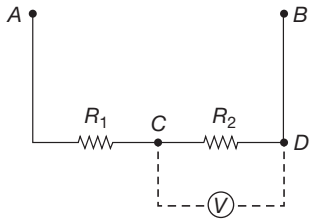


(b)

12. Three wires having resistance R_a, R_b and R_c are joined in the form of a triangle as shown. Another wire is joined at A and it makes a free sliding contact with BC. The resistance of this wire is R_d. Find the maximum resistance of the circuit (assume all wires to be uniform). The arc BC is a part of a circle whose centre is at A.

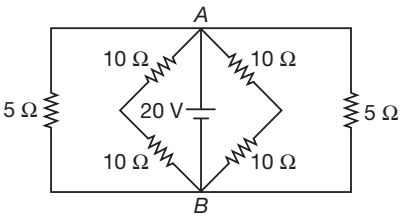


13. Resistances R_1 and R_2, each 60 Ω, are connected in series. The potential difference between points A and B is 120 V. Find the reading of a voltmeter connected to points C and D if its internal resistance is $r = 120\ \Omega$.

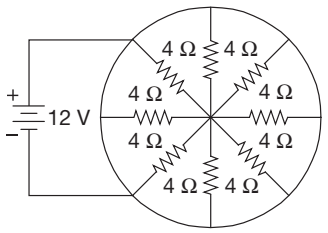


14. Show by diagram, how can we use a rheostat as the potential divider?

15. In the figure shown, determine the current through each 5 Ω resistor and the battery.

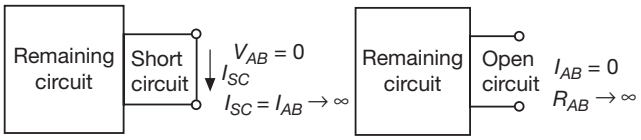


16. In figure shown we wish to determine current in one of the 4 Ω resistors in the circuit.



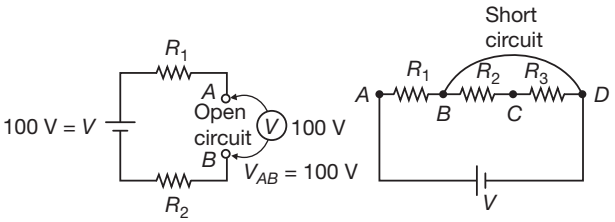
SHORT AND OPEN CIRCUITS

When two points of a circuit are connected together by a conducting wire, they are said to be **short circuited**. The connecting wire is assumed to have zero resistance. No voltage can exist across a short, secondly current through it is very large (theoretically infinity).



Two points are said to be open circuited when there is not direct connection between them, a break in the continuity of circuit exists. Due to this break the resistance between the two points is infinite and there is no flow of current between the two points.

Please note that the entire applied voltage is felt across the open, i.e., across terminals A and B, so, $V_{AB} = 100$ V.

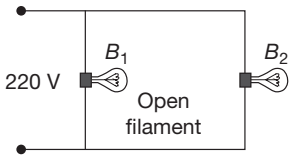


In the circuit shown if B and D are shorted, then

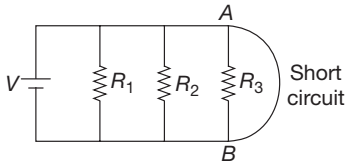
$$R_{eq} = R_1$$

$$\Rightarrow \quad I = \frac{V}{R_{eq}} = \frac{V}{R_1} \text{ and } V_B = V_D$$

Bulb B_1 acts as open circuit, bulb B_1 will not glow. However, other bulb B_2 remains connected across the voltage supply, it will operate normally.



Short circuit across R_3 shorts R_1 and R_2 as well, short across one branch in parallel means short across all branches. There is no current in shorted resistors. The shorted components are not damaged, they will function normally when short circuit is removed.



In figure shown, short circuit across R_3 may short out R_2 but not R_1, because it is protected by R_4.

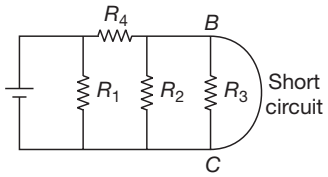
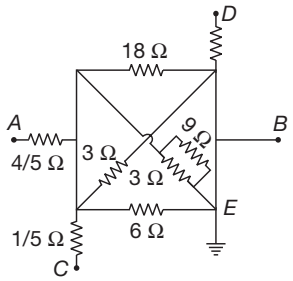


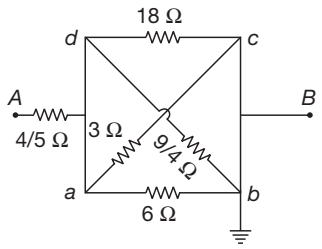
ILLUSTRATION 15

In figure shown, if a battery is connected between points A and B, emf $E = 18$ V, what is current through it?

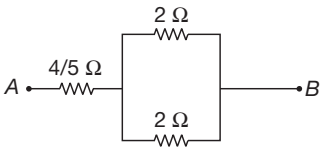


SOLUTION

The resistors at C and D are redundant, they are open branches, therefore no current through them. The point E is grounded, but no current will leak to ground, no return path for the current. Points a and d are at the same potential, similarly points b and c are at the same potential, because these points are connected by conducting wires without any circuit element.



Notice the $6\ \Omega$ and $3\ \Omega$ resistors, similarly $18\ \Omega$, $\frac{9}{4}\ \Omega$ resistors, these pairs are a parallel arrangement.



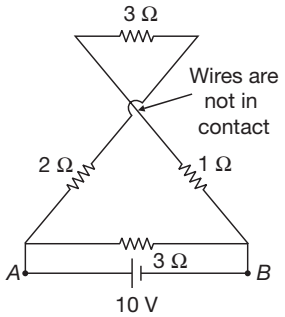
(Parallel of $3\ \Omega$ and $6\ \Omega$)

Thus the equivalent resistance of the circuit between A and B is $1.8\ \Omega$. The current supplied by battery.

$$I = \left(\frac{18}{1.8}\right) \text{A} = 10 \text{ A}$$

ILLUSTRATION 16

In figure shown, determine the current provided by the battery.



SOLUTION

Just flip the circuit about the dotted line shown in the figure. Now the circuit is simple with equivalent resistance $2\ \Omega$ and current $I = \left(\frac{10}{2}\right) \text{A} = 5 \text{ A}$.

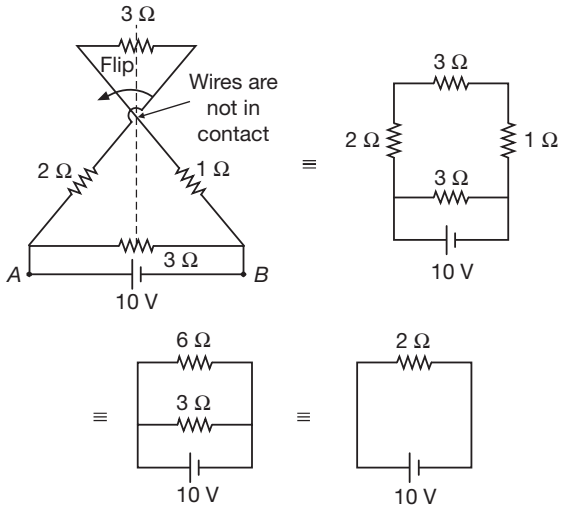
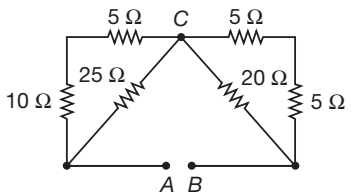


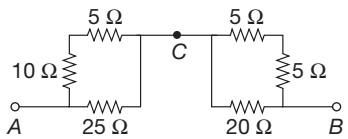
ILLUSTRATION 17

Find the equivalent resistance across AB of the circuit given.



SOLUTION

The circuit is redrawn to make it easily evaluable.



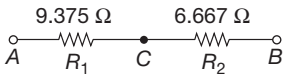
The left block is equivalent to $15\ \Omega$ and $25\ \Omega$ in parallel, and its resistance is given as

$$R_1 = \frac{25 \times 15}{25 + 15} = 9.375\ \Omega$$

The right block is equivalent to $10\ \Omega$ and $20\ \Omega$ in parallel, and its resistance is given as

$$R_2 = \frac{10 \times 20}{10 + 20} = \frac{200}{30} = 6.667\ \Omega$$

The circuit now reduces to two resistors in series, as shown.



Hence, the total resistance,

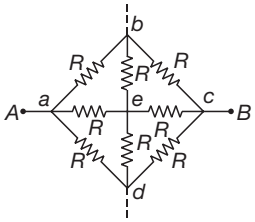
$$R = R_1 + R_2 = 9.375 + 6.667 = 16.042\ \Omega$$

PRINCIPLE OF SYMMETRY

If a network is symmetrical on both sides of a line, all the points lying on the line will have same potential. Hence, no current will flow in a resistance connected between two such points. Such resistances can be ignored while calculating the equivalent resistance. This simplifies the network. Or, else all the points on this line may be treated as shorted.

SYMMETRICAL CIRCUITS

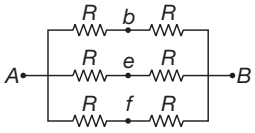
When a circuit is symmetrical about a line (By symmetry we mean that two parts are mirror images of each others), then the potential and current must also be symmetrical.



(a)

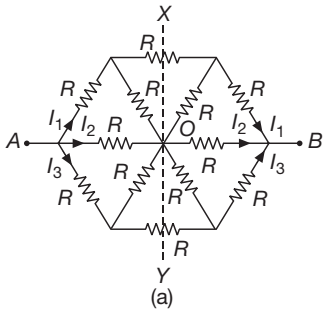
Therefore, currents in ab and ad are same. Currents in dc and bc are same. Potentials of the points b, e and d are same. The equivalent circuit is redrawn, the equivalent resistance is $\frac{2}{3}R$. Note that there is no current in branches be and ef.

Another symmetry is visible along line bd.



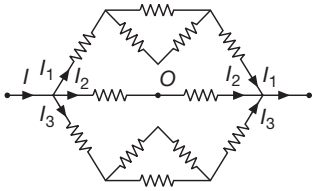
(b)

The current flow is not a mirror image in branches ab and bc because the flow is in same direction. This is called asymmetric condition. The special thing about this asymmetry is that current incoming at b is equal to outgoing current, similar situation exists at b and d also. Thus resistors in branches be and de are ineffective. In the following figure there is a symmetry along the xy.



(a)

The current I_2 reaching O is equal to outgoing current, which simply means there is no mingling of current from upper branch and lower branch into middle branch and hence we can remove the connection of the respective upper and lower branches as shown.

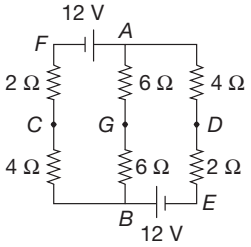


(b)

The resulting circuit is simple enough and the equivalent resistance is $\frac{4R}{5}$.

ILLUSTRATION 18

Find the current in the branch AB of the circuit. Also find the current in the branches BF and EA.

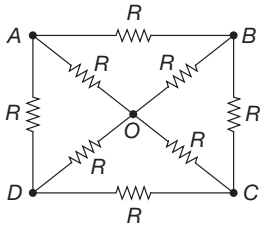


SOLUTION

Referring to figure we see that symmetry demands that current only circulates in outer branch. Points A and B are at the same potential because the circuit is symmetrical. Therefore, no current can go across the resistors in that branch. The current through BF and EA is 2 A.

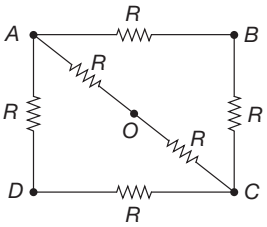
ILLUSTRATION 19

Calculate effective resistance between points A and C for the networks shown.



SOLUTION

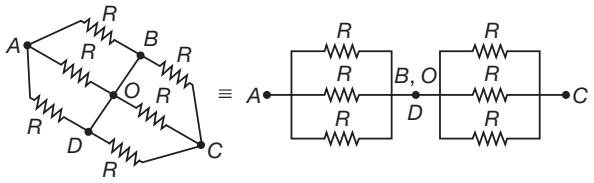
The two sides of the line BOD, are symmetrical. Thus, if the points A and C are connected to a source of emf, points B, O and D of the network will have same potential. Therefore, no current flows through R_{BO} and R_{DO}. These resistances can be ignored. The network then reduces to that shown in figure.



Thus, between points A and C, we now have three branches in parallel, each having resistance of $2R$.

$$\Rightarrow \quad R_{eq} = \frac{2R}{3}$$

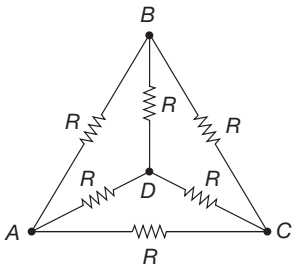
Alternately, since the points B, O and D are at same potential, we can treat them shorted.



$$\Rightarrow \quad R_{eq} = \frac{R}{3} + \frac{R}{3} = \frac{2R}{3}$$

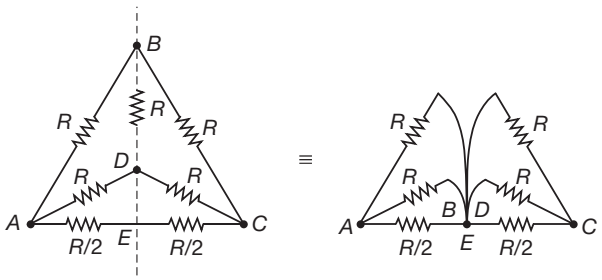
ILLUSTRATION 20

Calculate the effective resistance between points A and C, by applying symmetry principle.



SOLUTION

Break the branch AC into two resistors in series, each $\frac{R}{2}$, and consider the dotted line passing through B, D and E. The network on the two sides of this line is symmetrical. Hence, one can short-circuit the points B, D and E and calculate $(R_{eq})_{AE}$.



Now, we find that resistances R, R and $\frac{R}{2}$ are in parallel across A and E. The parallel combination of R and R gives $\frac{R}{2}$. This $\frac{R}{2}$ in parallel with $\frac{R}{2}$ gives $\frac{R}{4}$. Hence,

$$(R_{eq})_{AE} = \frac{R}{4}$$

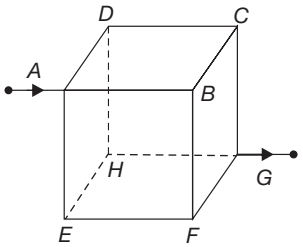
$$\Rightarrow \quad (R_{eq})_{AC} = 2(R_{eq})_{AE} = 2\left(\frac{R}{4}\right) = \frac{R}{2}$$

IDENTICAL POTENTIAL POINTS

In some networks, you may not find symmetry on the two sides of a line. But, you may find that the network contains some set of points having identical potential. Such a set of points can be joined together to make the network simple.

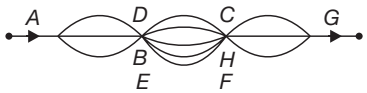
ILLUSTRATION 21

Twelve equal wires of resistances R each are joined up to form the edges of a cube, as shown in figure. The cube is connected into a circuit across the diagonal AG. Find the equivalent resistance of the network.



SOLUTION

Let us search the points of identical potential. Since the three edges of the cube from A, viz., AB, AD and AE are identical in all respects in the circuit, the points B, D and E are at the same potential. Similarly, for the point G, the sides GC, GH and GF are symmetrical and the points C, H and F are at the same potential.



Next, to simplify the circuit, we bring together the points, B, D and E and also C, H and F.

Now, it becomes obvious that

the resistance between A and D = R/3

the resistance between D and C = R/6

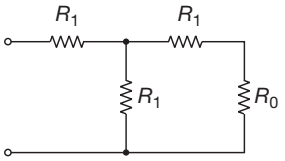
the resistance between C and G = R/3

Thus, the circuit is equivalent to three resistances of value. R/3, R/6 and R/3, in series, and hence the net resistance between A and G is

$$R_{AG} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5R}{6}$$

ILLUSTRATION 22

Consider the circuit shown in figure. For a given resistance R_0, what must be the value of R_1 so that the equivalent resistance between the terminals is equal to R_0?



SOLUTION

The equivalent resistance, R', due to the three resistors on the right is

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_0 + R_1} = \frac{R_0 + 2R_1}{R_1(R_0 + R_1)}$$

$$\Rightarrow \quad R' = \frac{R_1(R_0 + R_1)}{R_0 + 2R_1}$$

Since R' is in series with the fourth resistor R_1, the equivalent resistance of the entire configuration becomes

$$R_{eq} = R_1 + \frac{R_1(R_0 + R_1)}{R_0 + 2R_1} = \frac{3R_1^2 + 2R_1R_0}{R_0 + 2R_1}$$

If $R_{eq} = R_0$, then

$$R_0(R_0 + 2R_1) = 3R_1^2 + 2R_1R_0$$

$$\Rightarrow \quad R_0^2 = 3R_1^2$$

$$\Rightarrow \quad R_1 = \frac{R_0}{\sqrt{3}}$$

INFINITE LADDER AND GRID

Some networks make a ladder (or a grid) and extend to infinity. To reduce such networks we use the following steps.

STEP-1: Let us assume the total resistance of the infinite network to be X (say).

STEP-2: Now just retain one basic repetitive unit and we observe the remaining circuit to be the same as the original circuit. So, resistance of this left out circuit must be X.

STEP-3: Now the equivalent circuit, is the combination of basic unit and original repetitive circuit of resistance X, such that the net resistance of the entire circuit is X.

The following illustrations are done on the basis of these three steps.

ILLUSTRATION 23

(a) Find effective resistance between points A and B of an infinite chain of resistors joined as shown in Figure 1.

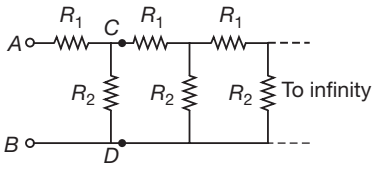


Figure 1

(b) For what value of R_0 in the circuit shown in Figure 2 will the net effective resistance is independent of the number of cells in the chain?

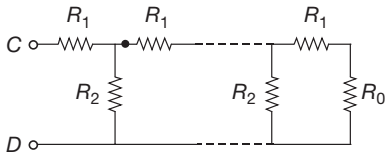


Figure 2

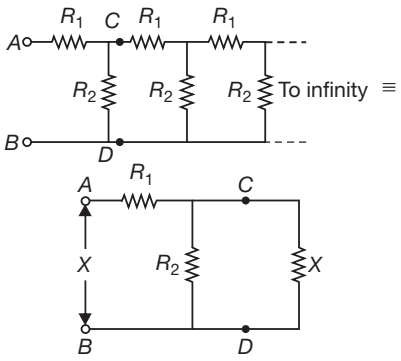
SOLUTION

(a) Suppose the effective resistance between A and B is X. Applying the steps discussed, we get

$$\Rightarrow \quad X = R_1 + [R_2 \,||\, X] = R_1 + \frac{R_2 X}{(R_2 + X)}$$

$$\Rightarrow \quad X^2 - R_1 X - R_1 R_2 = 0$$

$$\Rightarrow \quad X = \frac{1}{2}\left(R_1 \pm \sqrt{(R_1^2 + 4R_1 R_2)}\right)$$



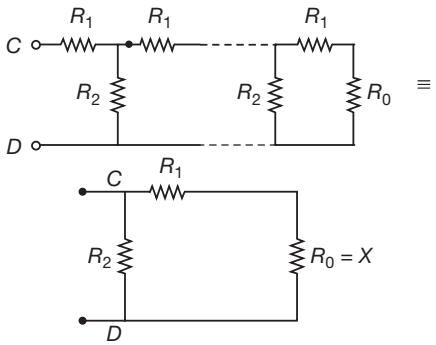
And as resistance can not be negative, we have

$$X = \frac{R_1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

However, if $R_1 = R_2 = R$ we get

$$X = \frac{R}{2}(1 + \sqrt{5}) = 1.6\ \text{R}$$

(b) Suppose there are n sections between points A and B and the network is terminated by R_0 with equivalent resistance X. Now, if we add one more to the network between the C and D, the equivalent resistance of the network X will be independent of number of cells if the resistance between points C and D still remains R_0 (or X). So, the circuit reduces to an equivalent circuit as shown.



$$\Rightarrow \quad R_1 + \frac{R_2 R_0}{(R_2 + R_0)} = R_0$$

$$\Rightarrow \quad X = R_0 = \frac{1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

If $R_1 = R_2 = R$, then we get

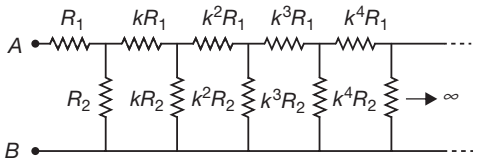
$$X = R_0 = \frac{R}{2}(1 + \sqrt{5}) = 1.6\,R$$

Problem Solving Technique(s)

Whenever each element of the circuit is multiplied by a factor, say k, then the equivalent resistance is also multiplied by the same factor.

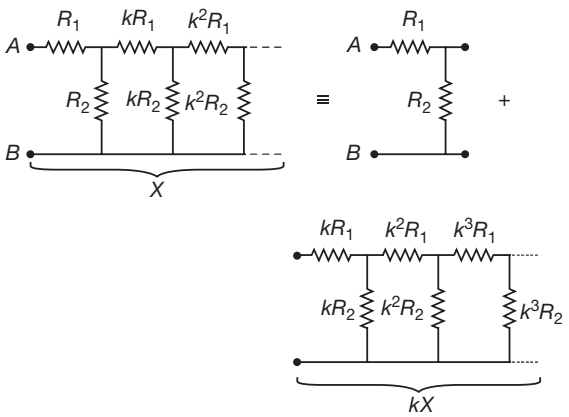
ILLUSTRATION 24

The circuit diagram shown consists of a large number of elements (each element has two resistors R_1 and R_2). The resistances of the resistors in each subsequent element differs by a factor of $k = \frac{1}{2}$ from the resistances of the resistors in the previous elements. Find the equivalent resistance between A and B shown in figure.

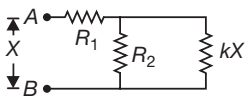


SOLUTION

When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times. Let the equivalent resistance between A and B be X.



So, equivalent circuit becomes



For $k = \frac{1}{2}$

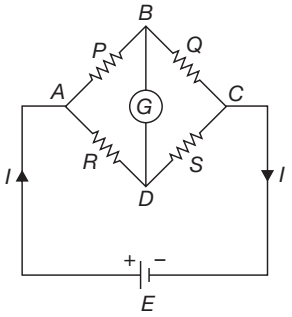
$$\Rightarrow \quad X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6\,R_1R_2}}{2}$$

BALANCED WHEATSTONE BRIDGE LIKE SITUATIONS

Condition of balance is

$$\frac{P}{Q} = \frac{R}{S}$$

So, to conclude



(a) When battery and galvanometer arms of a Wheatstone's Bridge are interchanged, the balance position remains undisturbed while sensitivity of bridge changes.

(b) When Wheatstone's Bridge is balanced, the resistance in arm BD may be ignored while calculating the equivalent resistance of bridge between A and C. So, P and Q are in series, R and S are in series and both in parallel to each other.

$$\Rightarrow \quad \frac{1}{R_{net}} = \frac{1}{P+Q} + \frac{1}{R+S}$$

Three other common forms of balanced Wheatstone's Bridge are shown here.

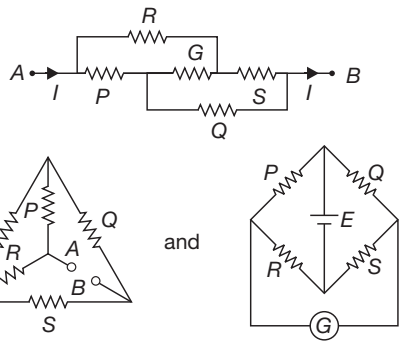
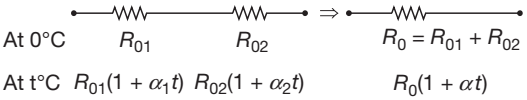


ILLUSTRATION 25

Two resistors with temperature coefficients of resistance α_1 and α_2 have resistances R_{01} and R_{02} at 0 °C. Find the temperature coefficient of the compound resistor consisting of the two resistors connected, (a) in series (b) in parallel

SOLUTION

In Series:



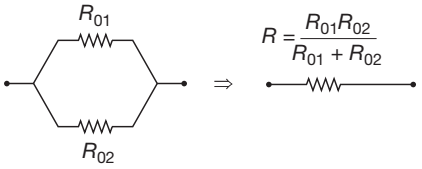
$$R_{01}(1+\alpha_1 t)+R_{02}(1+\alpha_2 t)=R_0(1+\alpha t)$$

$$\Rightarrow\ R_{01}(1+\alpha_1 t)+R_{02}(1+\alpha_2 t)=(R_{01}+R_{02})(1+\alpha t)$$

$$\Rightarrow\ R_{01}+R_{01}\alpha_1 t+R_{02}+R_{02}\alpha_2 t=R_{01}+R_{02}+(R_{01}+R_{02})\alpha t$$

$$\Rightarrow\ \alpha=\frac{R_{01}\alpha_1+R_{02}\alpha_2}{R_{01}+R_{02}}$$

In Parallel:



At t °C, $\dfrac{1}{R_0(1+\alpha t)}=\dfrac{1}{R_{01}(1+\alpha_1 t)}+\dfrac{1}{R_{02}(1+\alpha_2 t)}$

$$\Rightarrow\ \frac{R_{01}+R_{02}}{R_{01}R_{02}(1+\alpha t)}=\frac{1}{R_{01}(1+\alpha_1 t)}+\frac{1}{R_{02}(1+\alpha_2 t)}$$

Since $\alpha t \ll 1$, so by using the Binomial expansion, we get

$$\frac{1}{R_{02}}(1-\alpha t)+\frac{1}{R_{01}}(1-\alpha t)=\frac{1}{R_{01}}(1-\alpha_1 t)+\frac{1}{R_{02}}(1-\alpha_2 t)$$

$$\Rightarrow\ \alpha t\left(\frac{1}{R_{01}}+\frac{1}{R_{02}}\right)=\frac{\alpha_1}{R_{01}}t+\frac{\alpha_2}{R_{02}}t$$

$$\Rightarrow\ \alpha=\frac{\alpha_1 R_{02}+\alpha_2 R_{01}}{R_{01}+R_{02}}$$

ILLUSTRATION 26

A rod of length L and cross-section area A lies along the x-axis between $x=0$ and $x=L$. The material obeys Ohm's Law and its resistivity varies along the rod according to,

$$\rho(x)=\rho_0 e^{-x/L}$$

The end of the rod at $x=0$ is at a potential V_0 and it is zero at $x=L$

(a) Find the total resistance of the rod and the current in the wire.

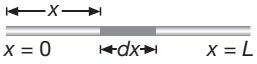
(b) Find the electric potential $V(x)$ in the rod as a function of x.

SOLUTION

(a) Resistance of elementary section dx at $x=x$ is,

$$dR=\frac{\rho(x)dx}{A} \qquad \left\{\because\ R=\rho\frac{l}{A}\right\}$$

$$\Rightarrow\ dR=\frac{\rho_0 e^{-\frac{x}{L}}dx}{A}$$



Since all such elements are in series, hence

$$R=\int_0^L dR=\frac{\rho_0}{A}\int_0^L e^{-\frac{x}{L}}dx=\frac{\rho_0 L}{A}(1-e^{-1})$$

$$\Rightarrow\ R=\frac{\rho_0 L}{A}\left(1-\frac{1}{e}\right)$$

Current in the wire is given by

$$I=\frac{V_0}{R}=\frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)$$

(b) $dV = IdR$

$$dV=\frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)\frac{\rho_0 e^{-x/L}}{A}\cdot dx$$

$$dV=\frac{V_0}{L}\left(\frac{e}{e-1}\right)e^{-x/L}dx$$

$$\Rightarrow\ \int_{V_0}^{V(x)} dV=\frac{V_0}{L}\left(\frac{e}{e-1}\right)\int_0^x e^{-x/L}dx$$

$$\Rightarrow\ V(x)-V_0=V_0\left(\frac{e}{1-e}\right)(1-e^{-x/L})$$

$$\Rightarrow\ V(x)=\frac{V_0(e^{-x/L}-e^{-1})}{1-e^{-1}}$$

Test Your Concepts-V

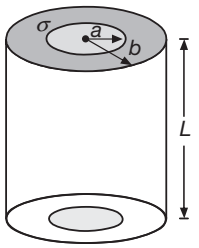
Based on Series and Parallel Combination of Resistances

(Solutions on page H.203)

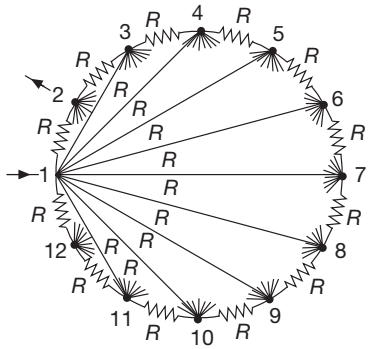
1. The space between two coaxial cylinders, whose radii are a and b (where $a < b$) is filled with a conducting medium. The specific conductivity of the medium is σ.

(a) Compute the resistance between the cylinders in the radial direction. Assume that the cylinders are very long compared to their radii, i.e., $L \gg b$, where L is the length of the cylinders.

(b) Calculate the resistance, assuming σ varies as $\sigma(r) = \dfrac{\sigma_0}{r}$

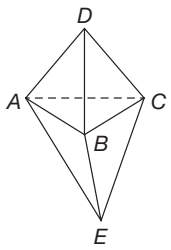


2. **(a)** A network of 12 resistors each of value $R = 6\ \Omega$ are interconnected as shown in figure, being placed along the sides of a regular dodecagon. Each of the terminals 1, 2, 3, …, 12 has been connected to each of the 9 terminals (other than nearest) directly by insulated wires each of resistance R, there being 9 wires from each terminal making 108 wire connections totally. [Only one set of 9 wires, from terminal 1 have been shown]. Find the equivalent resistance of the network when the current enters at the terminal 1 and leaves at terminal 2.



(b) If the above network were generalised so that there are n $(n = even)$ resistors each of resistance R placed along the sides of a regular n sided polygon and if each terminal point of a resistor were connected by $(n - 2)$ insulated wires each of resistance R, directly to the $(n - 2)$ terminals, other than its, nearest terminals, find the equivalent resistance across any two terminals of the network (i.e., current entering at one of the two terminals and leaving by the other.

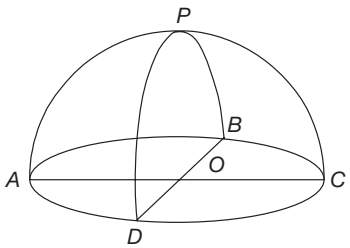
3. Calculate the equivalent resistance of the triangular bipyramid between the points specified. Assume the resistance of each branch to be R.



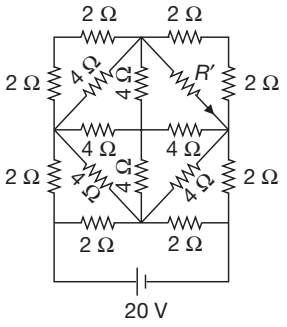
(a) A and C

(b) D and E

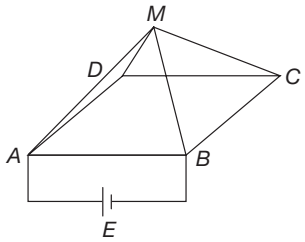
4. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length λ. Calculate the equivalent resistance across OP.



5. In the circuit shown, all the side resistances are of value 2 Ω and all inner resistances are of value 4 Ω. Find the current via branch AB.



6. A square pyramid, having vertex at *M* is formed by joining 8 equal resistances *R* across the edges. The square base of the pyramid has the corners at *A*, *B*, *C* and *D*. Calculate the current in the edge



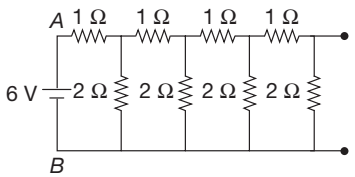
(a) *MC* if an ideal cell of emf *E* is connected across the adjacent corners *A* and *B*.

(b) *MA* if an ideal cell of emf *E* is connected across the opposite corners *A* and *C*.

7. A length $4a$ of uniform wire having resistance per unit length λ, is bent in the form of a square and the opposite angular points are formed with straight pieces of the same wire which are in contact at the intersection. A given current enters at the intersection of diagonals and leaves at an angular point. Find the resistance of the whole network.

8. An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistances, as shown in figure. (1987; 7M)

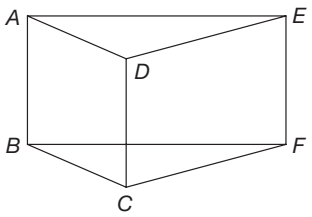
The 6 volt battery between *A* and *B* has negligible internal resistance.



(a) Show that the effective resistance between *A* and *B* is 2 Ω

(b) What is the current that passes through the 2 Ω resistance nearest to the battery?

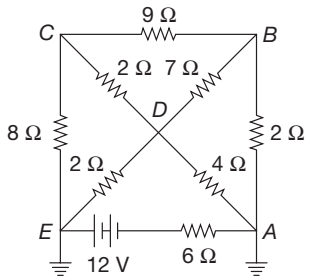
9. Nine wires each of resistance *r* are connected to make a prism as shown in figure. Find the equivalent resistance of the arrangement across.



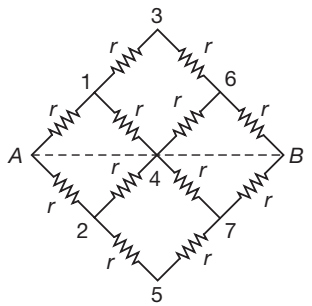
(a) AD

(b) AB

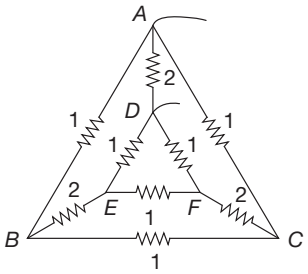
10. In figure, determine the current through 6 Ω and 9 Ω resistor.



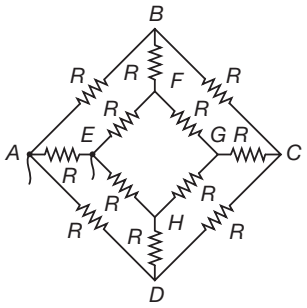
11. Determine equivalent resistance between *A* and *B*.



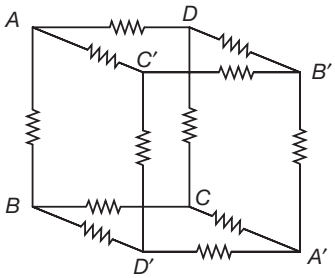
12. In the figure shown, the resistances specified are in ohms. Determine the equivalent resistance between points *A* and *D*.



13. In the network shown in figure all the resistances are equal. Determine equivalent resistance between *A* and *E*.

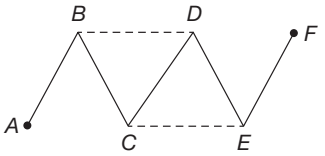


14. Twelve equal resistors each $R\ \Omega$ are connected to form the edges of a cube. Find the equivalent resistance of the network:

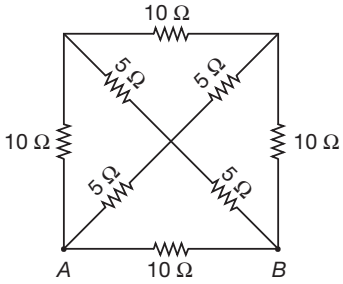


(a) when current enters at *A* and leaves at *A′*.
(b) when current enters at *A* and leaves at *B′*.
(c) when current enters at *A* and leaves at *B*.

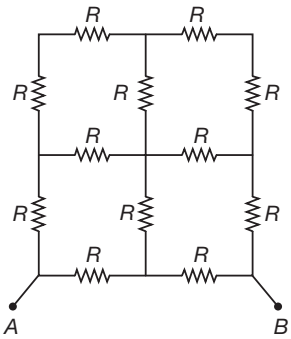
15. What will be the change in the resistance of a circuit between *A* and *F* consisting of five identical conductors if two similar conductors added as shown by the dashed line in figure.



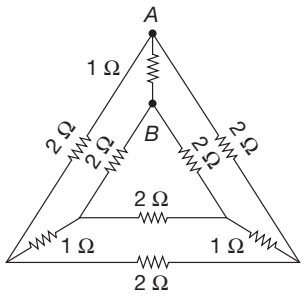
16. Find the equivalent resistance between *A* and *B* in the following circuit:



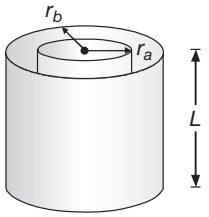
17. Find the equivalent resistance between *A* and *B* in the following circuit:



18. Find the equivalent resistance between *A* and *B* in the following circuit:



19. An oceanographer is studying the variation of the ion concentration in sea water with depth. This is done by lowering into the water a pair of concentric metallic cylinders at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a, outer radius r_b and length L much larger than r_b. A potential difference ΔV is applied between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water. Find the



(a) resistance of the water between the cylinders in terms of L, ρ, r_a and r_b.

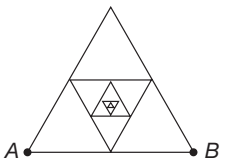
(b) resistivity of the water, ρ in terms of the measured quantities L, r_a, r_b, ΔV and I.

20. The cross section area and length of a cylindrical conductor are A and ℓ, respectively. The specific conductivity varies as $\sigma(x) = \sigma_0 \dfrac{1}{\sqrt{x}}$, where x is the distance along the axis of the cylinder from one of its ends.

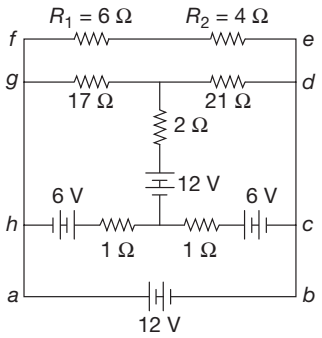
(a) Compute the resistance of the system along the cylindrical axis.

(b) Compute the current density if the potential drop along the cylinder is V_0. What is the electric field at each point in the cylinder in the case described?

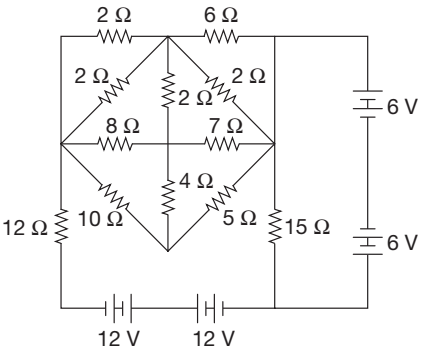
21. A frame made of thin homogeneous wire is shown in figure. Assume that the number of successively embedded equilateral triangle with sides decreasing by half tends to infinity. The side AB has a resistance R_0. Find the equivalent resistance between A and B.



22. In the circuit shown in figure, determine the current and potential drop across resistor R_1.



23. Determine the current through the 8 Ω resistor in figure.



KIRCHHOFF'S CIRCUIT RULES

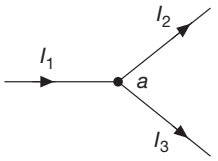
In analyzing circuits, there are two fundamental (Kirchhoff's) rules

Junction Rule

At any point where there is a junction between various current carrying branches, by current conservation the sum of the currents into the node must equal the sum of the currents out of the node (otherwise charge would build up at the junction)

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

In other words this law states that current or charge cannot accumulate at a junction. This law is in accordance with the Law of Conservation of Charge.



Kirchhoff's junction rule

According to the junction rule, the three currents shown here are related by

$$I_1 = I_2 + I_3$$

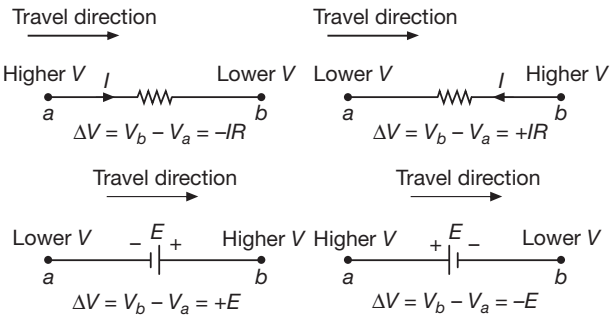
Loop Rule

The sum of the voltage drops ΔV, across any circuit elements that form a closed circuit is zero.

$$\sum_{\text{closed loop}} \Delta V = 0$$

This rule is simply saying that there cannot be a potential difference between a point and itself and is also in accordance with the Law of Conservation of Energy.

The rules for determining ΔV across a resistor and a battery with a designated travel direction are shown below:



Convention for determining ΔV

Note that the choice of travel direction is arbitrary. The same equation is obtained whether the closed loop is traversed clockwise or counter clockwise. Also, remember that a closed loop must start and end at the same point.

Problem Solving Technique(s)

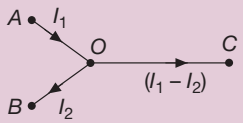
Applying Kirchhoff's Rules

In this chapter we have seen how Kirchhoff's rules can be used to analyze multiloop circuits. The steps are summarized below:

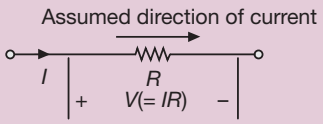
(a) Draw a circuit diagram, and label all the quantities, both known and unknown. The number of unknown quantities is equal to the number of linearly independent equations we must look for.

(b) Assign a direction to the current in each branch of the circuit. (If the actual direction is opposite to what you have assumed, your result at the end will be a negative number).

(c) Apply *KCL* to each junction. Often, it is found more convenient to apply *KCL* while marking the currents in the branches. This reduces the number of unknowns to be solved. For example, if we have assumed currents in the branches *AO* and *OB* as I_1 and I_2, we need not mark the current in branch *OC* as I_3. Instead we apply *KCL* and say that the current in branch *OC* is $(I_1 - I_2)$.



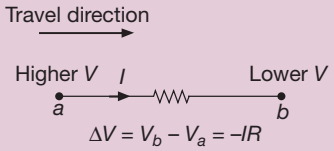
(d) Mark the polarity of the voltage across each element. In a resistor, the polarity depends upon the assumed direction of current. The end into which the current enters is marked positive.

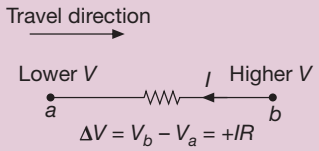


(e) Apply the loop rule to the loops until the number of independent equations obtained is the same as the number of unknowns. For example, if there are three unknowns, then we must write down three linearly independent equations in order to have a unique solution.

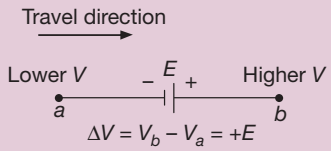
Traverse the loops using the convention below for ΔV

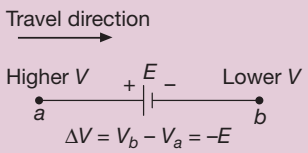
Resistor



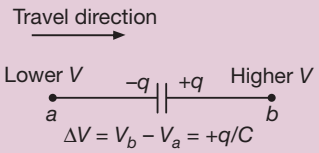


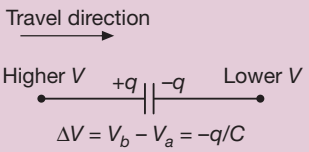
EMF Source





Capacitor

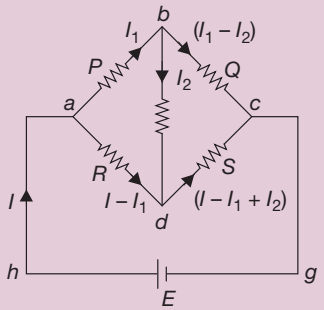




The same equation is obtained whether the closed loop is traversed clockwise or counterclockwise. Try to take as many equations as the number of variables.

Start from any point and mentally go around the loop in the designated direction. Write down directly the voltage of each element whose positive (+) terminal is entered, and write down the negative of every voltage whose negative (–) terminal is entered.

(f) If necessary, choose other loops and repeat above steps until you get as many independent equations as the unknowns. The set of equations obtained will be independent provided each new loop equation contains a voltage change not included in a previous equation. The easiest way to ensure the independency of equations is to select all meshes (just like the panes of a window) as the loops for writing *KVL*. You may have to select loops as per the number of variables to be calculated. Another thing we must keep in mind while selecting loops is that we must select the loops independently. Suppose we require three loops to be selected and we select a bigger loop along with two loops (these two loops collectively form the bigger loop) and write three equations. We will notice that actually we may not be getting three equations but instead we shall be getting only two equations. So avoid taking such type of loops and try to identify and take independent loops. Suppose, in this circuit, we take the currents as shown. Hence we observe that at least three equations are required to get I, I_1 and I_2.



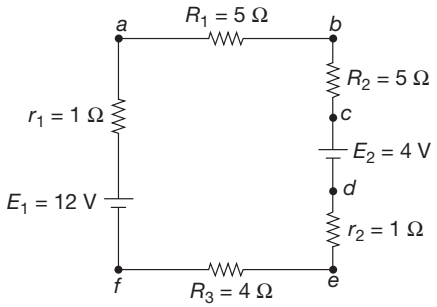
Please be careful and take loops as *abda*, *bcdb* (or *abcda*) because *abcda* = *abda* + *bcdb*. So these are just two loops. The third loop must be *abcgha* (or *adcgha* or *adbcgha* or *abdcgha*).

So, while taking the loops, keep in mind to take unique loops. Avoid taking big loops made from smaller loops (for which equations have already been taken) because the big loop made from the combination of small loops will again fetch you the identical equations which cannot be used as such.

(g) Solve the simultaneous equations to obtain the solutions for the unknowns.

ILLUSTRATION 27

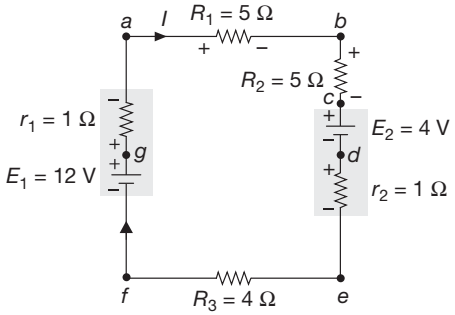
In the arrangement shown, find the



(a) current I in the circuit,
(b) potential at each of the labelled points a, b, c, d, e, assuming potential at f to be zero,
(c) power input and output in the circuit.

SOLUTION

(a) First we assume that current is clockwise. Now we apply KVL in the assumed direction of current.



Circuit element	Changes in potential	Sign
$R_1\ (a \to b)$	Drop IR_1	minus
$R_2\ (b \to c)$	Drop IR_2	minus
$E_2\ (c \to d)$	Drop E_2	minus
$r_2\ (d \to e)$	Drop Ir_2	minus
$R_3\ (e \to f)$	Drop IR_3	minus
$E_1\ (f \to g)$	Gain E_1	plus
$r_1\ (g \to a)$	Drop Ir_1	minus

Always remember the signs of emf are independent of the current.
Hence, we have

$$-IR_1 - IR_2 - E_2 - Ir_2 - IR_3 + E_1 - Ir_1 = 0$$

$$I = \frac{E_1 - E_2}{R_1 + R_2 + R_3 + R_4 + R_5}$$

Note that if E_2 is greater than E_1, we get a negative value of I, which shows that assumed direction of current is wrong.

$$I = \frac{12 - 4}{5 + 5 + 4 + 1 + 1} = 0.5 \text{ A}$$

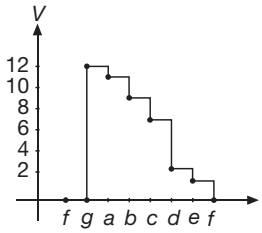
(b) Now we determine the potential at each labelled point in the circuit.

$$V_g = V_f + E_1 = 0 + 12 = 12 \text{ V}$$
$$V_a = V_g - Ir_1 = 12 - (0.5)(1) = 11.5 \text{ V}$$
$$V_b = V_a - IR_1 = 11.5 - (0.5)(5) = 9 \text{ V}$$
$$V_c = V_b - IR_2 = 9 - (0.5)(5) = 6.5 \text{ V}$$
$$V_d = V_c - E_2 = 6.5 - 4 = 2.5 \text{ V}$$
$$V_e = V_d - Ir_2 = 2.5 - (0.5)(1) = 2 \text{ V}$$
$$V_f = V_e - IR_3 = 2 - (0.5)(4) = 0$$

Note that, here we have assigned f to be at zero potential however, we can choose any point of the circuit to be at zero potential and then determine the potentials of the other points relative to it. The zero potential point is indicated by the ground symbol ⏚ at point f.

(c) Power delivered by source of emf E_1,

$$P_{E_1} = E_1 I = (12)(0.5) = 6 \text{ W}$$



Power dissipated in resistors,

$$P_R = I^2 (R_1 + R_2 + R_3 + r_1 + r_2)$$
$$P_R = (0.5)^2 (5 + 5 + 4 + 1 + 1) = 4 \text{ W}$$

The power consumed by battery 2 in getting charged,

$$P_{E_2} = E_2 I = 4(0.5) = 2 \text{ W}$$

Note that battery E_1 is discharging, the current comes out of its positive terminal. The terminal voltage across its terminals is

$$V_1 = E_1 - Ir_1$$

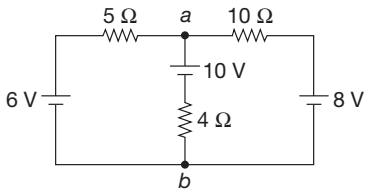
$$\Rightarrow \quad V_1 = 12 - (0.5 \times 1)$$

$$\Rightarrow \quad V_1 = 11.5 \text{ V}$$

The battery E_2 is charging, the current goes in from its positive terminal. The terminal voltage across it $V_2 = E_2 + Ir_2 = 4 + (0.5)(1) = 4.5$ V. The terminal voltage is greater than the emf of the battery.

ILLUSTRATION 28

Calculate the currents through each resistance in the given circuit. Also calculate the potential difference between points a and b.

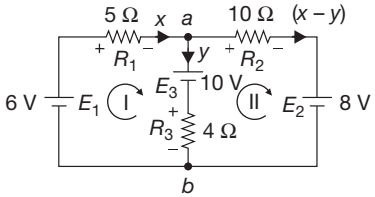


SOLUTION

Let the current through R_1 be x and through R_3 be y, as shown.

Applying KCL at junction a, the current through R_2 will be $(x - y)$.

According to the assumed direction of currents, we mark the polarity of voltage drops across different resistances.



Writing KVL for loop I, we get

$$xR_1 + E_3 + yR_3 - E_1 = 0$$

$$\Rightarrow \quad 5x + 10 + 4y - 6 = 0$$

$$\Rightarrow \quad 5x + 4y = -4 \qquad \ldots(1)$$

Writing KVL for loop II, we get

$$(x - y)R_2 + E_2 - yR_3 - E_3 = 0$$

$$\Rightarrow \quad (x - y)10 + 8 - 4y - 10 = 0$$

$$\Rightarrow \quad 10x - 14y = 2 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = -\frac{24}{25} \text{ A}, \quad y = -\frac{5}{11} \text{ A} \quad \text{and} \quad x - y = \frac{1}{55} \text{ A}$$

The signs indicate that the directions of x and y were assumed incorrectly, whereas the assumed direction of current x-y through R_2 was correct.

Potential difference between a and b,

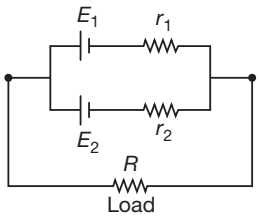
V_{ab} = Potential of a with respect to b is $V_a - V_b$. So,

$V_a - V_b$ = Potential rise when we go from b to a

$$\Rightarrow \quad V_a - V_b = yR_3 + E_3 = -\frac{5}{11} \times 4 + 10 = \frac{90}{11} \text{ V}$$

ILLUSTRATION 29

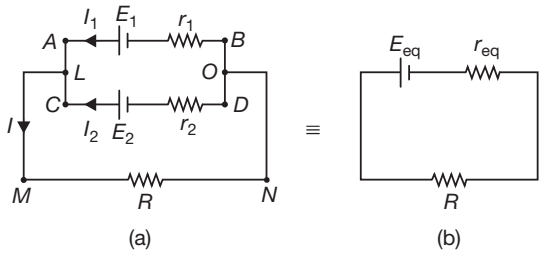
Two dissimilar cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 respectively are connected in parallel across a load resistance R, as shown. Find the emf and internal resistance of the equivalent cell of this combination.



SOLUTION

Let I_1 and I_2 be the currents supplied by the two cells. Let the current through load resistance R be I. According to KCL at junction L,

$$I = I_1 + I_2 \qquad \ldots(1)$$



For loop $ALMNOBA$, writing KVL equation,

$$IR + I_1 r_1 - E_1 = 0$$

$$\Rightarrow \quad I_1 = \frac{E_1 - IR}{r_1} \qquad \ldots(2)$$

Similarly, for loop $CLMNODC$, the KVL equation,

$$IR + I_2 r_2 - E_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{E_2 - IR}{r_2} \qquad \ldots(3)$$

Substituting the value of I_1 and I_2 from equation (2) and (3) into (1), we get

$$I = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right] - IR\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$\Rightarrow \quad I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right] = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]$$

$$\Rightarrow \quad I\left[R + \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}\right] = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]\left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \qquad \ldots(4)$$

Now if E_{eq} and r_{eq} are the equivalent emf and internal resistance of a single cell as shown in Fig. (3.139(b)), then according to loop rule, we have

$$-IR - Ir_{eq} + E_{eq} = 0$$

$$\Rightarrow \quad I\left(R + r_{eq}\right) = E_{eq} \qquad \ldots(5)$$

So, comparing equations (4) and (5), we get

$$r_{eq} = \left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \text{ and } E_{eq} = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]\left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \qquad \ldots(6)$$

$$\Rightarrow \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \text{ and } E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \qquad \ldots(7)$$

Problem Solving Technique(s)

(a) If in the circuit battery E_2 is reversed $E_2 \Rightarrow -E_2$, equation (7) reduces to

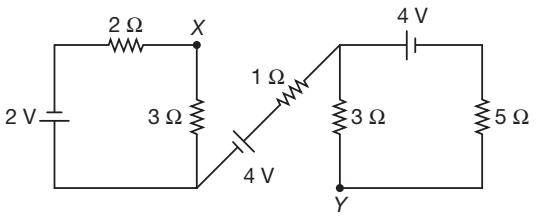
$$r_{eq} = \left[\frac{r_1 r_2}{r_1 + r_2}\right] \text{ and } E_{eq} = \frac{E_1 r_2 - E_2 r_1}{\left(r_1 + r_2\right)}$$

(b) If large number of dissimilar cells are joined in parallel, in the light of equation (6), we have

$$r_{eq} = \left[\sum \frac{1}{r_i}\right]^{-1} \text{ and } E_{eq} = \left[\sum \frac{E_i}{r_i}\right]\left[\sum \frac{1}{r_i}\right]^{-1}$$

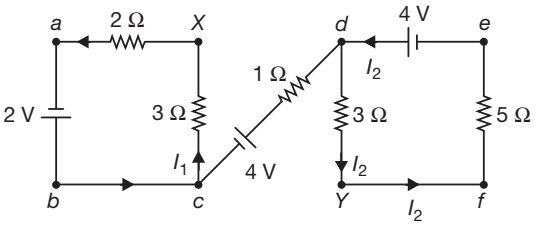
ILLUSTRATION 30

In the given network, determine the potential of point X with respect to the point Y.



SOLUTION

There are two closed circuits or loops. Let the current in the two loops be I_1 and I_2 as shown.



Note that no current can flow through branch cd, since it is not a part of any closed loop. The whole current I_1 goes round the left-hand loop, and whole current I_2 goes round the right-hand loop.

$$\therefore \quad I_1 = \frac{2 \text{ V}}{2\ \Omega + 3\ \Omega} = 0.4 \text{ A}$$

$$\text{and} \quad I_2 = \frac{4 \text{ V}}{3\ \Omega + 5\ \Omega} = 0.5 \text{ A}$$

The potential of point X with respect to point Y is written as V_{XY}. If we traverse any path starting from point Y and end at point X, the net potential rise will be V_{XY}. Now, let us find V_{XY}, by traversing the path $YdcX$,

$$V_{XY} = 3I_2 + 1(0) - 4 - 3I_1$$

$$V_{XY} = 3(0.5) + 0 - 4 - 3(0.4) = 1.5 - 4 - 1.2 = -3.7 \text{ V}$$

It means that point X is 3.7 V lower in potential than the point Y. We can also write

$$V_{YX} = -V_{XY} = -(-3.7) \text{ V} = 3.7 \text{ V}$$

It means point Y is 3.7 V higher in potential than the point X.

Note that whichever path we traverse from Y to X, the result should be the same, that is, –3.7 V. Let us check if by traversing the path $YfedcbaX$,

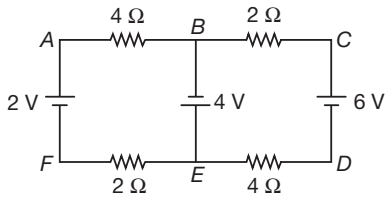
$$V_{XY} = 0 - 5I_2 + 4 + 1(0) - 4 + 0 - 2 + 2I_1$$

$$V_{XY} = 0 - 5(0.5) + 4 + 1(0) - 4 + 0 - 2 + 2(0.4)$$

$$V_{XY} = -2.5 + 4 - 4 - 2 + 0.8 = 4.8 - 8.5 = -3.7 \text{ V}$$

ILLUSTRATION 31

Find currents in different branches of the electric circuit shown in figure. Also find the potential difference between points F and C.



SOLUTION

In this Illustration there are three wire segments $EFAB$, BE and $BCDE$ and two junctions at B and E. Therefore we have three unknowns I, I_1 and I_2 and hence we require three equations. One equation will be obtained by applying Kirchhoff's Junction Law (either at B or at E) and the remaining two equations, we get from the Second Law (Loop Law). Consider three loops $ABEFA$, $ACDFA$ and $BCDEB$. But we have to choose any two of them. Further, we can choose any arbitrary directions of I, I_1 and I_2.

Applying Kirchhoff's First Law (Junction Law) at junction B,

$$I = I_1 + I_2 \qquad \ldots(1)$$

Applying Kirchhoff's Second Law in Loop $ABEFA$,

$$-4I + 4 - 2I + 2 = 0 \qquad \ldots(2)$$

$$\Rightarrow \quad I_1 = 6 \text{ A}$$

Applying Kirchhoff's Second Law in Loop $BCDEB$,

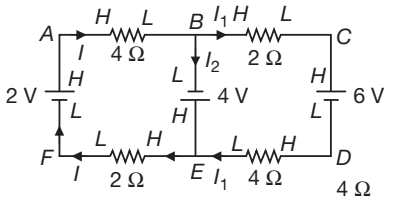
$$-2I_1 - 6 - 4I_1 - 4 = 0 \qquad \ldots(3)$$

$$\Rightarrow \quad I_1 = -\frac{10}{6} = -\frac{5}{3} \text{ A}$$

Using (1), we get

$$I_1 = -\frac{5}{3} \text{ A}$$

Here, negative sign of I_1 implies that current I_1 is in the direction opposite to that assumed by us in the figure i.e., it is from C to B (and not from B to C).



To find the potential difference between any two points of a circuit let us reach from one point to the other via any path of the circuit. It is advisable to choose a path in which we come across the least number of resistors preferably a path which has no resistance.

Let us reach from F to C via A and B,

$$V_F + 2 - 4I - 2I_1 = V_C$$

$$\Rightarrow \quad V_F - V_C = 4I + 2I_1 - 2$$

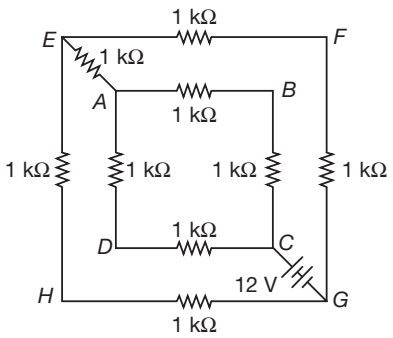
Substituting, $I = 1$ A and $I_1 = -\frac{5}{3}$ A, we get

$$V_F - V_C = -\frac{4}{3} \text{ V}$$

Here negative sign implies that $V_F < V_C$ or F is at a lower potential than C.

ILLUSTRATION 32

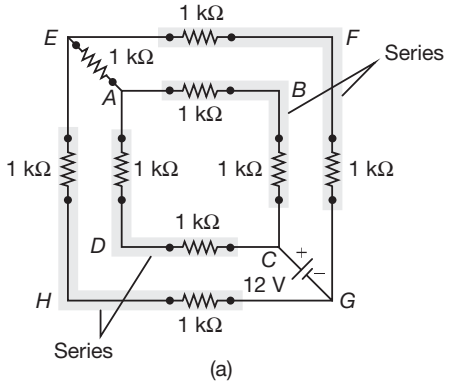
In the arrangement shown, find the



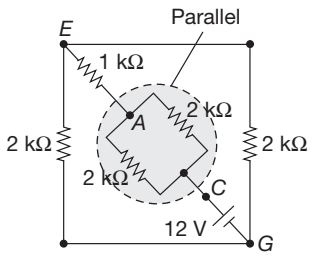
(a) equivalent resistance between C and G.
(b) current provided by the source.
(c) voltage across points G and E.

SOLUTION

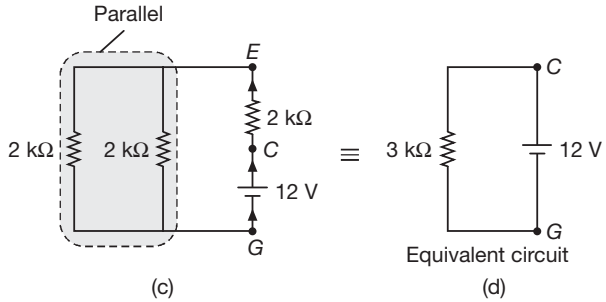
(a) Imagine the wires to be flexible, lift up the inside square with the resistors along with the source attached and then following figures (a), (b), (c) and (d) to arrive at equivalent circuit. The equivalent resistance is $3\ \text{k}\Omega$.



(a)



(b)



(b) Since $V = IR_{\text{eq}}$, $I = \dfrac{V}{R_{\text{eq}}} = 4\ \text{mA}$

(c) Start at point G, assign it a potential V_G, proceed toward E along any path. When you reach point E after adding potential drops and gains you get potential of E

$$V_G + 12 - IR = V_E$$

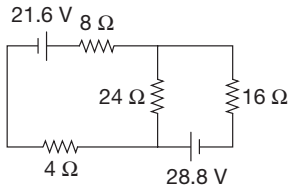
$$\Rightarrow \quad V_E - V_G = 12 - IR$$

$$\Rightarrow \quad V_E - V_G = 12 - \left(4 \times 10^{-3} \times 2 \times 10^{3}\right) = 4\ \text{V}$$

THE PRINCIPLE OF SUPERPOSITION OR SUPERPOSITION THEOREM

Complex network problems can sometimes be solved easily by using the principle of superposition. This principle essentially states that when a number of emf's act in a network, the solution is the same as the superposition of the solutions for one emf acting at a time, the others being shorted.

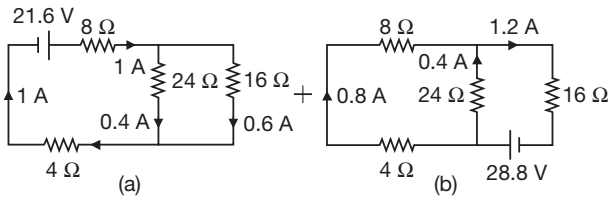
Figure shows a network with two loops.



The currents in various branches can be calculated using Kirchhoff's Laws. We can get the same solution by considering only one battery at a time and then superposing the two solutions. If a battery has an internal resistance, it must be left in place when the emf of the battery is removed. Figure shows how the superposition principle can be applied to the present problem.

The current values in figure (a) and (b) are easily verified. For example when the 21.6 V battery alone is acting, the total resistance in the circuit is,

$$8+\frac{24\times16}{24+16}+4=21.6\ \Omega$$



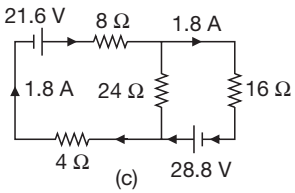
This makes the total current $\frac{21.6\ \text{V}}{21.6\ \Omega}=1\ \text{A}$. This current splits between $16\ \Omega$ and $24\ \Omega$ in the ratio $3:2$.

Similarly, the total resistance when only the 28.8 V battery is acting is,

$$16+\frac{24\times12}{24+12}=24\ \Omega$$

Therefore, the total current is $\frac{28.8\ \text{V}}{24\ \Omega}=1.2\ \text{A}$

The superposition principle shows that there is no current in the $24\ \Omega$ resistance.



Only a current of 1.8 A flows through the outer loop. All these conclusions can also be verified by analyzing the circuit using Kirchhoff's Laws.

Problem Solving Technique(s)

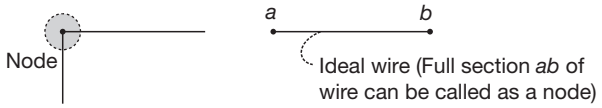
(a) If there are a number of emfs acting simultaneously in any linear network, then each emf acts independent of the others, i.e., as if the other emfs did not exist. The value of current in any conductor is the algebraic sum of the currents due to each emf. Similarly, voltage across any conductor is the algebraic sum of the voltages which each emf would have produced while acting singly.

(b) This theorem is applicable only to linear networks where current is linearly related to voltage as per Ohm's Law.

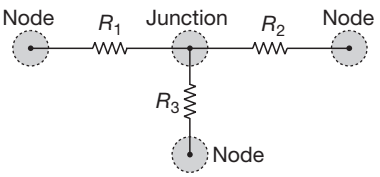
(c) Please note that while applying the superposition theorem only the current sources (or batteries) and the currents have to be superimposed. **Do not superimpose the resistances, because then you will be getting inaccurate results.**

NODAL ANALYSIS IN CIRCUITS

Node is a point at which two or more elements are joined together. For two nodes to be different, their voltages must be different. A conductor with a substantially zero resistance is considered to be a node for the purpose of analysis. So, in circuit diagrams where connections are done using ideal wires (i.e. wires with zero resistance), a node may consist of the entire section of wire between elements, not just a single point.



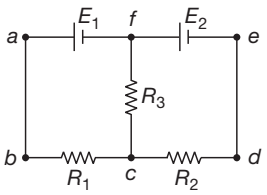
Junction is a point where three or more branches meet together as shown in figure.



Conceptual Note(s)

A junction can also be called a node but a node may or may not be a junction.

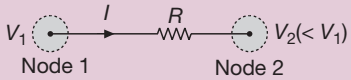
Think about two wires/branches meeting at a point, which makes it a Node, whereas think about three wires meeting at a point which may be called a Node (two or more branches meeting at a point) as well as a Junction (three or more branches meeting at a point).



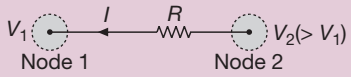
In the above figure we can say that points a, b, c, d, e, f are **Nodes** and point c and f are called **Junctions**. (or can also be called as Nodes)

Conceptual Note(s)

A current through a resistor can be calculated by using node-voltages as follows:



$$I = \frac{V_1 - V_2}{R} \quad (\text{when } V_1 > V_2)$$



$$I = \frac{V_2 - V_1}{R} \quad (\text{when } V_2 > V_1)$$

Problem Solving Technique(s)

The nodal analysis is a systematic way of applying Kirchhoff's Circuit Laws (*KCL*) at each essential node of a circuit and represents the branch current in terms of the node voltages. This will give us a set of equations that we solve together to find the node voltages. Once we find the node voltages, we can use them to calculate any other currents or voltages of interest.

STEP-1: Identify all the essential nodes and select one of them as a reference node i.e. the node at zero potential i.e. 0 V. This reference node (can generally be taken as the ground) has usually most elements tied to it.

STEP-2: Now assign voltages (with respect to the reference node) to all the nodes except the reference node.

STEP-3: Apply the *KCL* at each node except the reference node and write the equations.

In this step, for each node, we assume that all branch currents are either leaving from the node (or entering the node), and then we describe the branch current(s) in term of node voltages.

Always keep in mind that a Junction may be called a node but a node may or may not be a junction.

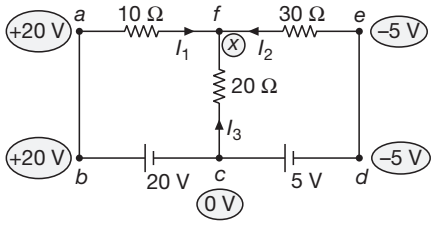
STEP-4: Solve the system of linear equations obtained to find the unknown node voltage(s) and hence the current through different branches of the circuit.

NODAL ANALYSIS METHOD

To solve problems on the basis of nodal analysis, we use the following steps.

STEP-1: Assign potential to every node of circuit. For doing this, we assign zero potential to any one of the node of the circuit. This Node (which is assigned zero potential) is called as **Reference Node** OR **Reference Junction**.

For example, in the circuit shown, we assign zero potential to the point c.



Then, $V_c = 0$ V, $V_b = +20$ V, $V_d = -5$ V. Since ab and de are perfectly conducting sections of the circuit.

So, $V_a = V_b = +20$ V, $V_e = V_d = -5$ V.

Let the potential of node/junction f be x.

STEP-2: Now at junction f, apply *KJL* i.e., $\Sigma I = 0$ where $I = \frac{\Delta V}{R}$. Assuming all currents I_1, I_2 and I_3 to enter the junction, we get

$$I_1 + I_2 + I_3 = 0$$

$$\Rightarrow \quad \frac{20 - x}{10} + \frac{-5 - x}{30} + \frac{0 - x}{20} = 0$$

$$\Rightarrow \quad 6(20 - x) + 2(-5 - x) + 3(-x) = 0$$

$$\Rightarrow \quad 120 - 6x - 10 - 2x - 3x = 0$$

$$\Rightarrow \quad 11x = 110$$

$$\Rightarrow \quad x = 10 \text{ V}$$

So, potential of junction f is $V_f = x = 10$ V

$$\Rightarrow \quad I_1 = \frac{20-x}{10} = \frac{20-10}{10} = 1 \text{ A}$$

$$\Rightarrow \quad I_2 = \frac{-5-x}{30} = \frac{-5-10}{30} = -\frac{1}{2} \text{ A}$$

(Negative sign with I_2 implies that its direction is opposite to the direction as assigned in the circuit i.e., actually from f to e)

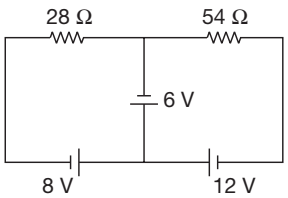
$$\Rightarrow \quad I_3 = -\frac{x}{20} = -\frac{10}{20} = -\frac{1}{2} \text{ A}$$

$$\Rightarrow \quad I_3 = \frac{1}{2} A \left(\text{from } f \text{ to } c\right)$$

Also, we observe that $I_1 = I_2 + I_3$

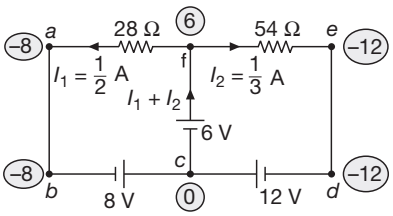
ILLUSTRATION 33

Using Nodal Analysis method, calculate the current through the resistors 28 Ω, 54 Ω and 6 V battery.



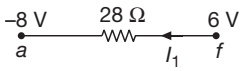
SOLUTION

Let us redraw the circuit and take a zero reference point in the circuit. So let potential of the point c be taken as zero. Then with respect to this point potential of point b is –8 V, point d is –12 V, point f is 6 V.

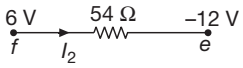


Also sections ab and de are perfectly conducting, so, $V_a = V_b = -8$ V, $V_e = V_d = -12$ V.

A careful observation of this circuit reveals that



$$I_1 = \frac{6-(-8)}{28} = \frac{1}{2} \text{ A}$$



$$I_2 = \frac{6-(-12)}{54} = \frac{1}{3} \text{ A}$$

So, current through 6 V battery is

$$\left(I_1 - I_2\right) = \frac{5}{6} \text{ A (from } e \text{ to } f\text{)}$$

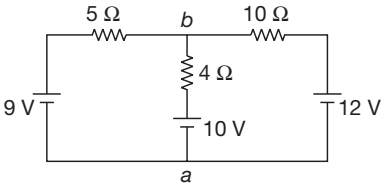
$$I_1 = I_{28\,\Omega} = \frac{1}{2} \text{ A (from } f \text{ to } a\text{)}$$

$$I_2 = I_{54\,\Omega} = \frac{1}{3} \text{ A (from } f \text{ to } e\text{) and}$$

$$I_{6\text{ V battery}} = I_1 + I_2 = \frac{5}{6} \text{ A (from } c \text{ to } f\text{)}$$

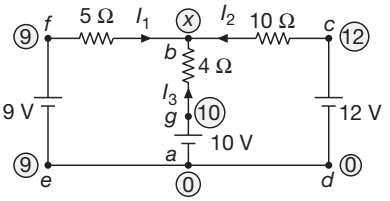
ILLUSTRATION 34

Using Nodal Analysis method, calculate the current through 5 Ω, 4 Ω, 10 Ω resistors. Also calculate potential difference between the points a and b.

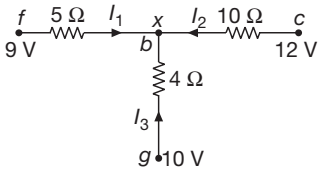


SOLUTION

Let $V_a = 0$ and the circuit is redrawn as shown.



Let x be the potential at junction b.



Applying *KJL* at *b*, we have

$$\Sigma I_b = 0$$

$$\Rightarrow \quad I_1 + I_2 + I_3 = 0$$

$$\Rightarrow \quad \frac{9-x}{5} + \frac{12-x}{10} + \frac{10-x}{4} = 0$$

$$\Rightarrow \quad 4(9-x) + 2(12-x) + 5(10-x) = 0$$

$$\Rightarrow \quad 36 - 4x + 24 - 2x + 50 - 5x = 0$$

$$\Rightarrow \quad 11x = 110$$

$$\Rightarrow \quad x = 10 \text{ V}$$

$$\Rightarrow \quad I_1 = \frac{9-x}{5} = \frac{9-10}{5} = -\frac{1}{5} \text{ A}$$

$$\Rightarrow \quad I_1 = \frac{1}{5} \text{ A (from } b \text{ to } f\text{)}$$

$$\Rightarrow \quad I_2 = \frac{12-x}{10} = \frac{12-10}{10} = \frac{1}{5} \text{ A}$$

$$\Rightarrow \quad I_2 = \frac{1}{5} \text{ A (from } c \text{ to } b\text{)}$$

$$\Rightarrow \quad I_3 = \frac{10-x}{4} = \frac{10-10}{4} = 0 \text{ A}$$

Since no current flows through the branch ab, so

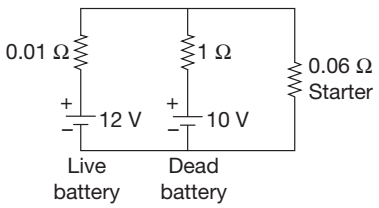
$$V_a - V_b = V_{ab} = 10 \text{ V}$$

Test Your Concepts-VI

Based on Kirchhoff's Laws and Nodal Analysis

(Solutions on page H.211)

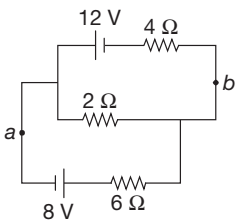
1. A dead battery is charged by connecting it to the live battery of another car with jumper cables. Determine the current in the starter and in the dead battery.



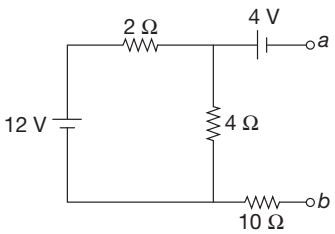
2. For the circuit shown in figure, calculate

(a) The current in the 2 Ω resistor and

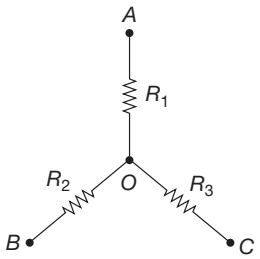
(b) The potential difference between points *a* and *b*.



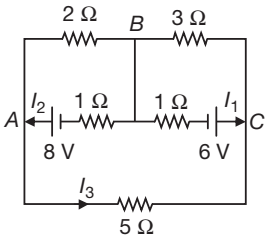
3. Calculate the potential difference between points *a* and *b* in figure and identify which point is at the higher potential.



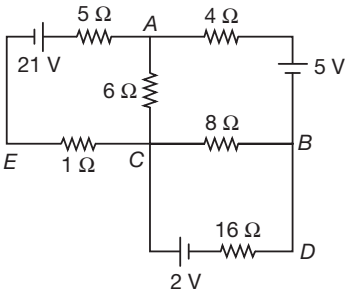
4. A circuit has a section *ABC* as shown in figure. If the potentials at points *A*, *B* and *C* are V_1, V_2 and V_3 respectively, calculate the potential at point *O*.



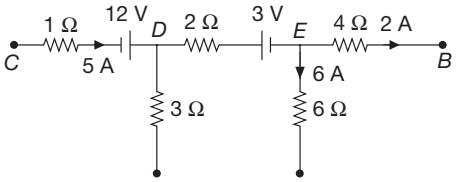
5. In the circuit shown in figure, calculate the currents I_1, I_2 and I_3. Also find out the potential differences between the points *A* and *B* and *B* and *C*.



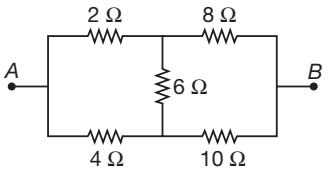
6. Find the current in each branch of the circuit shown in figure.



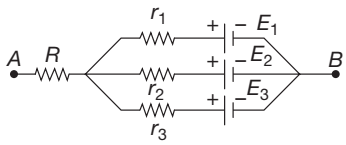
7. Figure shows part of a circuit. Calculate the power dissipated in 3 Ω resistance. What is the potential difference $V_C - V_B$?



8. Find the equivalent resistance between *A* and *B* in the following circuit:



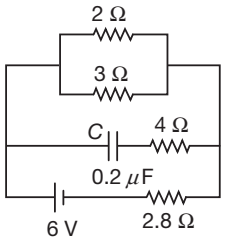
9. In the circuit shown in figure $E_1 = 3\text{ V}$, $E_2 = 2\text{ V}$, $E_3 = 1\text{ V}$ and $R = r_1 = r_2 = r_3 = 1\,\Omega$



(a) Find the potential difference between the points *A* and *B* and the currents through each branch.

(b) If r_2 is short circuited and the point *A* is connected to point *B*, find the currents through E_1, E_2, E_3 and the resistor *R*.

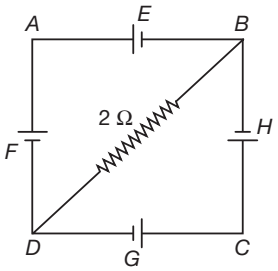
10. Calculate the steady state current in the 2 Ω resistor shown in the circuit (shown in figure). The internal resistance of the battery is negligible and the capacitance of the condenser *C* is 0.2 μF.



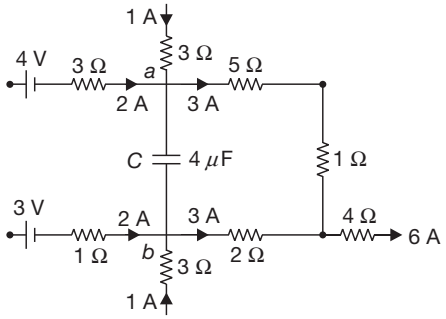
11. In the circuit shown in figure *E*, *F*, *G*, *H* are cells of emf 2 V, 1 V, 3 V and 1 V respectively, and their internal resistances are 2 Ω, 1 Ω, 3 Ω and 1 Ω respectively. Calculate

(a) the potential difference between *B* and *D* and

(b) the potential difference across the terminals of each cells *G* and *H*.



12. A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances etc., is shown in the figure. Calculate the energy stored in the capacitor $C(4\ \mu\text{F})$.

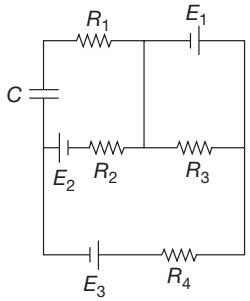


13. In the given circuit:

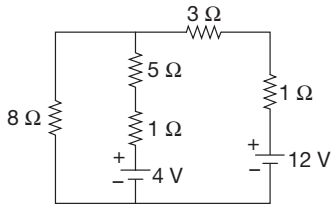
$E_1 = 3E_2 = 2E_3 = 6$ V, $R_1 = 2R_4 = 6\ \Omega$

$R_3 = 2R_2 = 4\ \Omega$, $C = 5\ \mu$F

Find the current in R_3 and the energy stored in the capacitor.



14. Determine the current in each branch of the circuit shown in Figure.



PRIMARY AND SECONDARY CELLS/BATTERIES

A battery consists of a series or parallel combination of two or more similar cells. Cells or batteries can be divided into **primary** and **secondary** types. The secondary cell is rechargeable, whereas the primary is not. The battery used in a car is of secondary type, since it can be recharged. But the cells used in a torch are primary type, as they cannot be recharged.

The resistance offered by the electrolyte to the flow of current through the cell is called the internal resistance of the cell. The internal resistance of a cell depends on

(a) the distance between the electrodes $(r \propto d)$

(b) the area of the electrodes $\left[r \propto \left(\frac{1}{A}\right)\right]$

(c) the concentration of the electrolyte $(r \propto C)$

(d) the absolute temperature of the electrolyte $\left[r \propto \left(\frac{1}{T}\right)\right]$

Internal resistance is different for different types of cells and even for a given type of cell it varies from cell to cell.

IDEAL VOLTAGE SOURCE

An ideal voltage source is one that can maintain constant voltage across its terminals whatever be the current drawn from it. The internal resistance of an ideal cell is zero. In practice, no cell can be ideal.

PRACTICAL VOLTAGE SOURCE

It is due to the internal resistance that the source does not behave as an ideal voltage source. It is usual practice to show a practical voltage source as consisting of an ideal voltage source of voltage E in series with a resistance r. E is the emf of the cell and r is its internal resistance. If such a cell is connected to load of resistance R, the current supplied by the cell is

$$I = \frac{\text{total emf in the circuit}}{\text{total resistance}} = \frac{E}{R+r}$$

Therefore, the potential difference that is now existing between the terminals A and B of the cell is given as

$$V = E - Ir$$

This quantity V is called the **terminal voltage** of the cell.

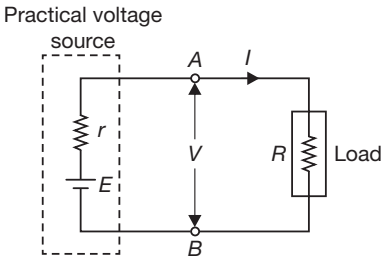
DISCHARGING OF CELL

When current is drawn from a cell, the potential difference across it is less than the emf of the cell and is given by

$$\boxed{V = E - Ir}$$

If the load resistance i.e. $R \to \infty$ (i.e., when the terminals A and B are open-circuited), then $V \to E$. Thus, the emf of a cell is nothing but its terminals voltage when its terminals are open-circuited.

Suppose a battery of emf E and internal resistance r is connected to a box which contains sources with greater emfs.



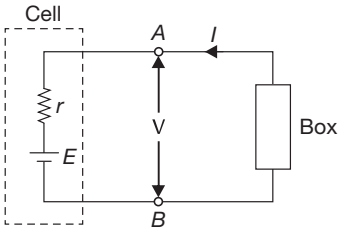
Then the current I may flow into the positive terminal of the battery (instead of its flowing out of the battery). The battery does not supply energy to the box; on the other hand the box supplies energy to the battery. Now, the terminal voltage is given as

$$V = E + Ir$$

CHARGING OF CELL

When a cell is being charged, the voltage across its terminals is greater than its emf and is given by

$$\boxed{V = E + Ir}$$



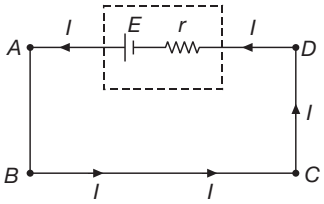
SHORT-CIRCUITING OF A CELL

The terminals A and D of a cell are connected by a conducting wire of zero resistance.

Using KVL in $ABCDA$, $-Ir + E = 0$

$$\Rightarrow \quad E = Ir$$

$$\Rightarrow \quad I = \frac{E}{r}$$



Terminal potential difference across the cell is

$$V_{AD} = E - Ir = E - \left(\frac{E}{r}\right) r$$

$$\Rightarrow \quad V_{AD} = 0$$

INTERNAL RESISTANCE OF CELL (r)

The internal resistance of a cell is the resistance offered by the electrolyte of the cell to the flow of current between its electrodes. It is denoted by r. The internal resistance of a cell

(a) varies directly as concentration of the electrolyte solution of the cell.

(b) varies directly as the separation between electrodes i.e., length of electrolyte solution between electrodes.

(c) varies inversely as the area of immersed electrodes.

(d) is independent of the material of electrodes.

Remark(s)

The internal resistance of a circuit is always in series with external resistance of the circuit.

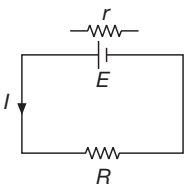
TERMINAL POTENTIAL DIFFERENCE ACROSS A CELL

If a cell of e.m.f. E, internal resistance r sends a current I, through an external resistance, then terminal potential difference

$$V = IR = E - Ir$$

In general the terminal potential difference (T.P.D.) is the potential difference across the external resistance of the circuit.

where $I = \dfrac{E}{R+r}$



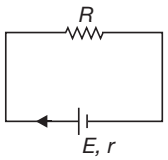
During the process of charging of a battery current is driven into a battery in the reverse direction. In such a case positive charge enters the battery at the positive terminal and leaves the battery from the negative terminal. So, when a cell is being charged by an external battery, then terminal potential difference

$$V = E + Ir$$

MAXIMUM POWER TRANSFER THEOREM

The power consumption across the external resistance of the circuit is maximum when the net external resistance of the circuit equals the net internal resistance.

Consider the circuit shown. The power consumed across R is I^2R, where $I = \dfrac{E}{R+r}$.



$$\Rightarrow \quad P = \frac{E^2R}{(R+r)^2}$$

$$\Rightarrow \quad P = \frac{E^2R}{(R-r)^2 + 4Rr}$$

For P to be MAXIMUM

$$R - r = 0$$

$$\Rightarrow \quad R = r$$

So, we observe that for maximum power consumption across the external resistance, we must have External Resistance = Internal Resistance of the battery.

$$\Rightarrow \quad P_{max} = \frac{E^2}{4R} = \frac{E^2}{4r}$$

This is an example of "impedance matching", in which the variable resistance R is adjusted so that the power delivered to it is maximized. The behaviour of P as a function of R is depicted in figure below.

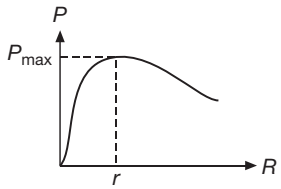
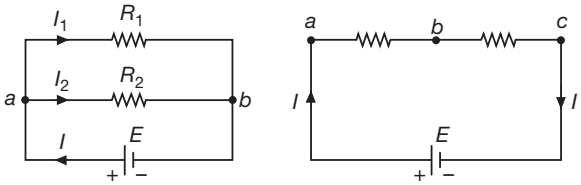


ILLUSTRATION 35

Shown here are two resistors with resistances R_1 and R_2 connected in parallel and in series. The battery has a terminal voltage of E.



Suppose R_1 and R_2 are connected in parallel, then

(a) find the power delivered to each resistor.

(b) show that the sum of the power used by each resistor is equal to the power supplied by the battery.

Suppose R_1 and R_2 are now connected in series, then

(c) find the power delivered to each resistor.

(d) show that the sum of the power used by each resistor is equal to the power supplied by the battery.

Which configuration, parallel or series, uses more power?

SOLUTION

(a) When two resistors are connected in parallel, the current through each resistor is

$$I_1 = \frac{E}{R_1} \text{ and } I_2 = \frac{E}{R_2}$$

and the power delivered to each resistor is given by

$$P_1 = I_1^2 R_1 = \frac{E^2}{R_1} \text{ and } P_2 = I_2^2 R_2 = \frac{E^2}{R_2}$$

The results indicate that the smaller the resistance, the greater the amount of power delivered. If the loads are the light bulbs, then the one with smaller resistance will be brighter since more power is delivered to it.

(b) The total power delivered to the two resistors is

$$P_R = P_1 + P_2 = \frac{E^2}{R_1} + \frac{E^2}{R_2} = \frac{E^2}{R_{eq}}$$

where $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\Rightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

is the equivalent resistance of the circuit. On the other hand, the total power P supplied by the battery is $P = IE$, where $I = I_1 + I_2$, as seen from the figure. Thus,

$$P = I_1 E + I_2 E = \left(\frac{E}{R_1}\right)E + \left(\frac{E}{R_2}\right)E$$

$$P = \frac{E^2}{R_1} + \frac{E^2}{R_2} = \frac{E^2}{R_{eq}} = P_R$$

The above results are in accordance with Law of Conservation of Energy.

(c) When the two resistors are connected in series, the equivalent resistance becomes

$$R'_{eq} = R_1 + R_2$$

and the currents through the resistors are

$$I_1 = I_2 = I = \frac{E}{R_1 + R_2}$$

Therefore, the power delivered to each resistor is

$$P_1 = I_1^2 R_1 = \left(\frac{E}{R_1 + R_2}\right)^2 R_1$$

and $P_2 = I_2^2 R_2 = \left(\frac{E}{R_1 + R_2}\right)^2 R_2$

Contrary to what we have seen in the parallel case, when connected in series, the greater the resistance, the greater the fraction of the power delivered. Once again, if the loads are light bulbs, the one with greater resistance will be brighter.

(d) The total power delivered to the resistors is

$$P'_R = P_1 + P_2 = \left(\frac{E}{R_1 + R_2}\right)^2 R_1 + \left(\frac{E}{R_1 + R_2}\right)^2 R_2$$

$$P'_R = \frac{E^2}{R_1 + R_2} = \frac{E^2}{R'_{eq}}$$

On the other hand, the power supplied by the battery is

$$P' = IE = \left(\frac{E}{R_1 + R_2}\right)E = \frac{E^2}{R_1 + R_2} = \frac{E^2}{R'_{eq}}$$

Again, we see that $P' = P'_R$, as required by energy conservation

Comparing the results obtained in (b) and (d), we see that

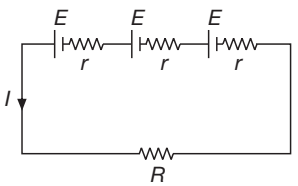
$$P = \frac{E^2}{R_1} + \frac{E^2}{R_2} > \frac{E^2}{R_1 + R_2} = P'$$

$$\Rightarrow \quad P > P'$$

which means that the parallel connection uses more power because of the fact that the equivalent resistance of two resistors connected in parallel is always smaller than that connected in series.

IDENTICAL CELLS IN SERIES

In this arrangement the positive terminal of one cell is connected to negative terminal of the other in succession. Figure represents n cells, each of e.m.f. E and internal resistance r connected in series and an external resistance R is connected across the combination.



Since all the cells are in series, so
Net e.m.f. $= nE$
Similarly all resistances are in series, so
Net internal resistance $= nr$

$\Rightarrow$ Total resistance of circuit $= R + nr$

$$\Rightarrow \quad \text{Current } I = \frac{\text{Net e.m.f.}}{\text{Net resistance}}$$

$$I = \frac{nE}{R + nr}$$

(a) If $R >> nr$, then

$$I = n\frac{E}{R} = n \text{ (current due one cell)}$$

(b) If $R << nr$, then

$$I = \frac{E}{r} \approx \text{ current due to one cell}$$

So, when net external resistance >> net internal resistance, then to get the maximum current, the cells must be connected in series for maximum current, the cells should be connected in series, when net external resistance >> net internal resistance.

Conceptual Note(s)

Note that if in series grouping of m cells, x cells are reversed i.e. x cells are being charged, then

$$E_{eq} = (m - x)E - xE = (m - 2x)E \text{ and } r_{eq} = mr$$

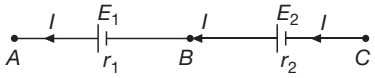
$$\Rightarrow \quad I = \frac{(m - 2x)E}{R + mr}$$

NON-IDENTICAL CELLS IN SERIES

E_1 and E_2 are emf of two cells used in series as shown in figure. If r_1 and r_2 are their respective internal resistances and I is the current flowing from C to A, potentials at points A, B and C respectively be V_A, V_B and V_C, then

$$V_{AB} = V_A - V_B = E_1 - Ir_1 \text{ and}$$

$$V_{BC} = V_B - V_C = E_2 - Ir_2$$

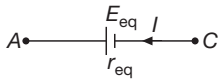


Potential difference between terminals A and C is

$$V_{AC} = V_A - V_C = V_A - V_B + V_B - V_C$$

$$\Rightarrow \quad V_{AC} = (E_1 + E_2) - I(r_1 + r_2) \qquad \ldots(1)$$

We can replace the above combination by a single cell of emf E_{eq} and internal resistance r_{eq} as shown below.



Since, $V_{AC} = E_{eq} - Ir_{eq}$ …(2)

Comparing (1) and (2), we get

$$E_{eq} = E_1 + E_2 \text{ and}$$

$$r_{eq} = r_1 + r_2$$

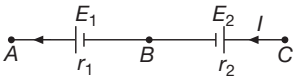
(a) If n cells are connected correctly in series, then

$$E_{eq} = E_1 + E_2 + ... + E_n \text{ and}$$

$$r_{eq} = r_1 + r_2 + ... + r_n$$

If cells are identical then $E_{eq} = nE$ and $r_{eq} = nr$.

(b) If cells of emf E_1 and E_2 are connected wrongly as shown below



then, $V_{AC} = E_1 - Ir_1 - E_2 - Ir_2$

$$\Rightarrow \quad V_{AC} = (E_1 - E_2) - I(r_1 + r_2)$$

$$\Rightarrow \quad V_{AC} = E_{eq} - Ir_{eq}$$

Thus, equivalent emf $E_{eq} = E_1 - E_2$ and internal resistance $r_{eq} = r_1 + r_2$

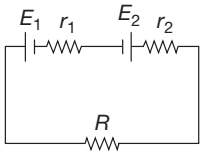
(c) If out of N identical cells, n are wrongly connected then

$$E_{eq} = (N - n)E - nE = (N - 2n)E$$

and $r_{eq} = Nr$

ILLUSTRATION 36

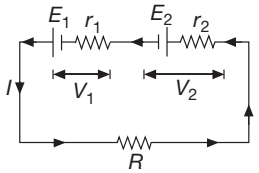
In the circuit shown in figure, $E_1 = 10$ V, $E_2 = 4$ V, $r_1 = r_2 = 1\ \Omega$ and $R = 2\ \Omega$. Find the potential difference across battery 1 and battery 2.



SOLUTION

Net emf of the circuit $= E_1 - E_2 = 6$ V

Total resistance of the circuit $= R + r_1 + r_2 = 4\ \Omega$



$\therefore$ Current in the circuit

$$I = \frac{\text{net emf}}{\text{total resistance}} = \frac{6}{4} = 1.5 \text{ A}$$

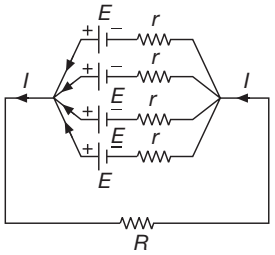
Now, $V_1 = E_1 - Ir_1 = 10 - (1.5)(1) = 8.5$ V

and $V_2 = E_2 + Ir_2 = 4 + (1.5)(1) = 5.5$ V

IDENTICAL CELLS IN PARALLEL

In this arrangement the positive terminals of all cells are connected to one point and negative terminals to the other point. Figure represents m cells, each of e.m.f. E and internal resistance r, connected in parallel and an external resistance R is connected across the combination.

Since all the cells are in parallel, so net e.m.f. equals to the e.m.f. due to a single cell.



$\Rightarrow$ Net e.m.f. $= E$

Similarly all internal resistances are in parallel, so

$$\text{Net internal resistance} = R_{\text{int}} = \frac{r}{m}$$

$$\Rightarrow \quad \text{Total resistance of circuit} = R + \frac{r}{m}$$

$$\Rightarrow \quad \text{Current } I = \frac{E}{R + \left(\frac{r}{m}\right)}$$

(a) If $R << \frac{r}{m}$, then

$$I = \frac{mE}{r} = m(\text{current due to one cell})$$

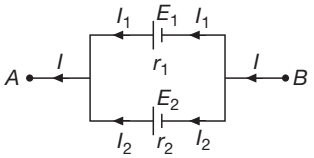
(b) If $R >> \frac{r}{m}$, then

$$I = \frac{E}{R} \approx \text{current due to one cell}.$$

So, when Net Internal Resistance >> Net External Resistance, then to get maximum current, the cells must be connected in parallel.

NON-IDENTICAL CELLS IN PARALLEL

Consider two cells of emf E_1 and E_2 connected in parallel across AB.



If I_1 and I_2 are the currents leaving positive electrodes of both cells, then total current from B to A.

$$I = I_1 + I_2 \qquad \ldots(1)$$

V_A and V_B are potentials at A and B respectively, then potential difference between A and B, can be written as

$$V = V_A - V_B = E_1 - I_1 r_1 \qquad \ldots(2)$$

and $$V = V_A - V_B = E_2 - I_2 r_2 \qquad \ldots(3)$$

Using equation (1), (2) and (3), we get

$$I = I_1 + I_2$$

$$\Rightarrow \quad I=\frac{E_1-V}{r_1}+\frac{E_2-V}{r_2}=\left(\frac{E_1}{r_1}+\frac{E_2}{r_2}\right)-V\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

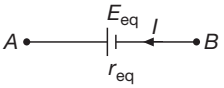
$$\Rightarrow \quad V=\frac{E_1r_2+E_2r_1}{r_1+r_2}-I\left(\frac{r_1r_2}{r_1+r_2}\right) \qquad \ldots(4)$$

We can replace the given combination by a single cell of emf equation and internal resistance r_{eq} across AB as shown in the figure.
We have,

$$V_{AB}=V_A-V_B=E_{eq}-Ir_{eq} \qquad \ldots(5)$$

Comparing (4) and (5), we have

$$E_{eq}=\frac{E_1r_2+E_2r_1}{r_1+r_2}$$



and $r_{eq}=\dfrac{r_1r_2}{r_1+r_2}$

$$\Rightarrow \quad \frac{E_{eq}}{r_{eq}}=\frac{E_1}{r_1}+\frac{E_2}{r_2}, \text{ where } \frac{1}{r_{eq}}=\frac{1}{r_1}+\frac{1}{r_2}$$

(a) If n cells of emf E_1, E_2,……, E_n and internal resistances r_1, r_2,……, r_n are in parallel, then

$$\frac{E_{eq}}{r_{eq}}=\frac{E_1}{r_1}+\frac{E_2}{r_2}+\ldots\ldots+\frac{E_n}{r_n}$$

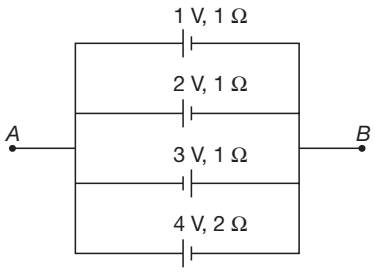
where, $\dfrac{1}{r_{eq}}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\ldots..+\dfrac{1}{r_n}$

(b) For identical cells, $r_1=r_2=\ldots\ldots=r_n=r$ (say) and $E_1=E_2=\ldots\ldots=E_n=E$ (say), then

$$r_{eq}=\frac{r}{n} \text{ and } E_{eq}=E$$

ILLUSTRATION 37

Calculate equivalent emf of four different cells connected in parallel as shown.



SOLUTION

Since $E_{eq}=\dfrac{\dfrac{E_1}{r_1}+\dfrac{E_2}{r_2}+\dfrac{E_3}{r_3}+\dfrac{E_4}{r_4}}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+\dfrac{1}{r_4}}$

$$\Rightarrow \quad E_{eq}=\frac{\dfrac{1}{1}+\dfrac{2}{1}-\dfrac{3}{1}+\dfrac{4}{2}}{\dfrac{1}{1}+\dfrac{1}{1}+\dfrac{1}{1}+\dfrac{1}{2}}=\frac{2}{\left(\dfrac{7}{2}\right)}=\frac{4}{7}\text{ V}$$

The equivalent cell arrangement between A and B is shown for reference.

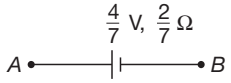
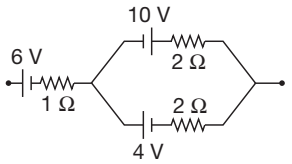


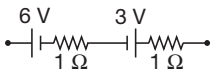
ILLUSTRATION 38

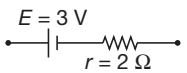
Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



SOLUTION

The given combination consists of two batteries in parallel and resultant of these two in series with the third one.





For parallel combination we can apply,

$$E_{eq}=\frac{\dfrac{E_1}{r_1}-\dfrac{E_2}{r_2}}{\dfrac{1}{r_1}+\dfrac{1}{r_2}}=\frac{\dfrac{10}{2}-\dfrac{4}{2}}{\dfrac{1}{2}+\dfrac{1}{2}}=3\text{ V}$$

Further, $\dfrac{1}{r_{eq}}=\dfrac{1}{r_1}+\dfrac{1}{r_2}=\dfrac{1}{2}+\dfrac{1}{2}=1$

$$\Rightarrow \quad r_{eq}=1\ \Omega$$

Now this resistance is in series with the third resistance so the equivalent emf of these two is $(6-3)\ \text{V} = 3\ \text{V}$ and the internal resistance will be $(1+1) = 2\ \Omega$.

Conceptual Note(s)

The equivalent emf can also be found by another method. Suppose we wish to find the equivalent emf of the circuit. Here we shall be using the fact that when no current is drawn by the cell then, we must have

$$E = V$$

However we may note that the current in the internal circuit may be non zero. This current is,

$$\text{I} = \frac{10+4}{2+1} = \frac{14}{3}\ \text{A}$$

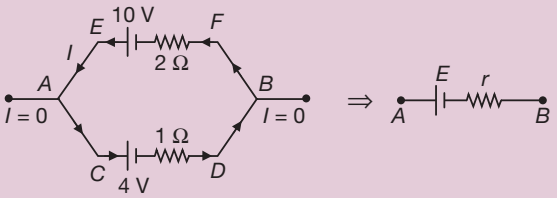
For path ACDB, we have

Now, $V_A + 4 - 1\left(\frac{14}{3}\right) - V_B = 0$

$$\Rightarrow\quad V_A - V_B = \frac{14}{3} - 4 = \frac{2}{3}\ \text{V}$$

$$\Rightarrow\quad E = V_A - V_B = V = \frac{2}{3}\ \text{V}$$

$$\Rightarrow\quad E = \frac{2}{3}\ \text{V}$$



The identical result can also be obtained for the path AEFB for which we will get

$$V_A - 10 + 2\left(\frac{14}{3}\right) + V_B = 0$$

$$\Rightarrow\quad E = V_A - V_B = \frac{2}{3}\ \text{V}$$

Further, since, $V_A - V_B$ is positive, i.e., $V_A > V_B$. So, we conclude that A is connected to the positive terminal of the battery and B to the negative.

Internal resistance of the equivalent battery is found by the same routine procedure. For example, here 2 Ω and 1 Ω resistances are in parallel. Hence, their combined resistance is

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

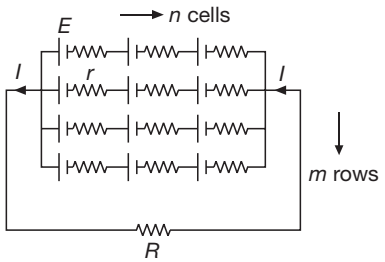
$$\Rightarrow\quad r = \frac{2}{3}\ \Omega$$

MIXED GROUPING OF IDENTICAL CELLS

In this arrangement a total of N cells are used. This arrangement comprises of m rows of cells in parallel, each row containing n cells in series. So, $N = mn$. The e.m.f. of each cell is E and internal resistance of each cell is r. The combination is connected to external resistance R.

Net e.m.f. $= nE$

Net internal resistance $R_{\text{int}} = \frac{nr}{m}$



Net resistance of circuit $= R + \frac{nr}{m}$

$$\Rightarrow\quad \text{Current } I = \frac{nE}{R + \frac{nr}{m}} = \frac{NE}{mR + nr}$$

$$\Rightarrow\quad I = \frac{NE}{\left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnRr}}$$

For maximum current

$$R = \frac{nr}{m}$$

$$\Rightarrow\quad R_{\text{ext}} = R_{\text{int}}$$

Thus for maximum current, the cells should be connected in mixed grouping when external resistance

R = net internal resistance i.e.,

$$R_{ext} = R_{int} = \frac{nr}{m}$$

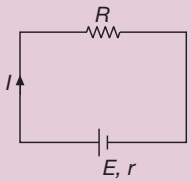
and $$I_{max} = \frac{1}{2}\frac{mE}{r} = \frac{1}{2}\frac{nE}{R}$$

Conceptual Note(s)

Please note that, for the current in the circuit to be maximum or for maximum power to be consumed by the external resistance R, following cases exist.

(a) In the first case, we arrange the cells in such a manner that current and power in the circuit should be maximum. And this happens when we arrange the cells in such a manner that total internal resistance comes out to be equal to total external resistance i.e., load resistance.

(b) In the second case, when a battery of emf E, internal resistance r is connected in series with a load of resistance R, then



$$I = \frac{E}{R+r}$$

$$P_R = I^2R = \left(\frac{E}{R+r}\right)^2 R$$

(i) Now if r can be varied keeping E and R are fixed, then both I and P_R are maximum when $r = 0$.

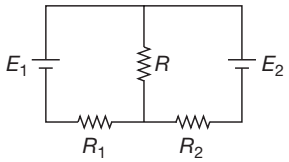
(ii) However, if R can be varied keeping E and r are fixed. Then, current in the circuit is maximum when $R = 0$ (i.e., the battery is shorted), but P_R will be maximum, when

$R = r$

$\Rightarrow$ External Resistance = Internal Resistance

ILLUSTRATION 39

In the circuit shown in figure, the emfs of batteries are E_1 and E_2 which have internal resistances R_1 and R_2. At what value of the resistance R will the thermal power generated in it be the highest? What it is?



SOLUTION

The two batteries are in parallel. Thermal power generated in R will be maximum when,
Total internal resistance = total external resistance

$$\Rightarrow \quad R = \frac{R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad E_{eq} = \frac{\left(\frac{E_1}{R_1}+\frac{E_2}{R_2}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \left(\frac{E_1R_2+E_2R_1}{R_1+R_2}\right)$$

Since, $$R_{net} = \frac{2R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad I = \frac{E_{eq}}{R_{net}} = \frac{E_1R_2+E_2R_1}{2R_1R_2}$$

Maximum power through R.

$$P_{max} = I^2R = \frac{(E_1R_2+E_2R_1)^2}{4R_1R_2(R_1+R_2)}$$

Test Your Concepts-VII

Based on EMF, Internal Resistance and Combination of Cells

(Solutions on page H.215)

1. An automobile battery has an emf of 12.6 V and an internal resistance of 0.08 Ω. The headlights together present equivalent resistance 5 Ω (assumed constant). What is the potential difference across the headlight bulbs when
(a) they are the only load on the battery and
(b) the starter motor is operated, taking an additional 35 A from the battery?

2. Under what circumstances can the terminal potential difference of a battery exceeds its emf?

3. Two batteries having the same emf E but different internal resistances r_1 and r_2 are connected in series with an external resistor R. For what value of R does the potential difference between the terminals of the first battery become zero?

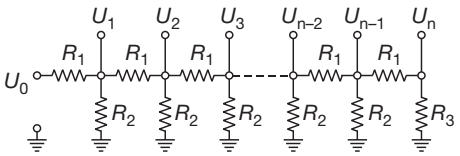
4. The emf of a storage battery is 90 V before charging and 100 V after charging. When charging began the current was 10 A. What is the current at the end of charging if the internal resistance of the storage battery during the whole process of charging may be taken as constant and equal to 2 Ω?

5. A battery is made by joining m rows of identical cells in parallel each row consists n cells joined in series. This battery sends a maximum current I in a given external circuit. Now the cells are so arranged that instead of m rows, n rows are joined in parallel and each row consists of m cells joined in series. Show that current through the same external circuit is now given by $I' = \dfrac{2mn}{m^2 + n^2} I$.

6. We are made available with 4 batteries each of emf 2 V and supplying a current of 2 A. Show by a circuit diagram how these can be arranged to supply 4 A of current at 4 V.

7. The potential difference across a battery is 8.5 V when there is a current of 3 A in the battery from the negative to the positive terminals. When the current is 2 A in the reverse direction, the potential difference becomes 11 V. What is the emf and the internal resistance of the battery ?

8. The output voltage can be reduced in the output circuits of generators as desired by means of an attenuator designed as the voltage divider shown in figure.



A special selector switch makes it possible to connect the output terminal either to the point with a potential U_0 produced by the generator, or to any of the points U_1, U_2 ..., U_n, each having a potential k times smaller $(k > 1)$ than the previous one. The second output terminal and the lower ends of the resistances are earthed.
Find the ratio between the resistances $R_1 : R_2 : R_3$ with any number of cells in the attenuator.

AMMETER

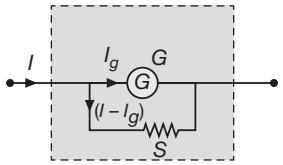
Measurement of Current: The instrument used for measuring current in a circuit is called **ammeter**. It is basically a moving coil galvanometer in which a low resistance wire (shunt) is connected in parallel with the coil. The shunt is connected for two reasons.

(a) To increase the range of current measurement: The moving coil galvanometer is a very sensitive device and its coil gets a full scale deflection for a very small current $(\sim 1\ \text{mA})$. By connecting a shunt of suitable resistance, the excess current passes through it, without damaging the galvanometer.

(b) To reduce the error of measurement: Generally the galvanometer coil has a fairly high resistance (10 to 1000 Ω) and therefore it will give error in the measurement of current-the measured current will be less than the actual current. By connecting a low resistance wire in parallel with the coil, the effective resistance of the ammeter can be made small and hence the error is reduced.

SHUNT RESISTANCE TO CONVERT A GALVANOMETER INTO AN AMMETER OF DESIRED RANGE

If I is the range of the ammeter, I_g is the galvanometer current for full scale deflection, G is the resistance of the coil and S is the shunt resistance then, clearly,



Ammeter

$$(I-I_g)S = I_gG$$

$$\Rightarrow \quad S = \frac{I_gG}{I-I_g}$$

The resistance of the ammeter so obtained is

$$R_A = \frac{GS}{G+S}$$

Generally $S \ll G$, so

$$G+S \simeq G$$

$$R_A \simeq S$$

i.e., ammeter is a low resistance device and hence it is always connected in series in a circuit.

Remark(s)

(a) The reading of an ammeter is always lesser than the actual current in the circuit. If V is the potential difference across a resistance R, the true current is $I = \left(\frac{V}{R}\right)$. However, when an ammeter of resistance r is used to measure it, the reading will be

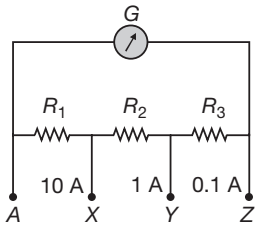
$$I' = \frac{V}{(R+r)}$$

which is less than the true current I

(b) Smaller the resistance of an ammeter, the more accurate will be its reading. An ammeter is said to be ideal if its resistance (r) is zero. However, ideal ammeter cannot be realized in practice.

ILLUSTRATION 40

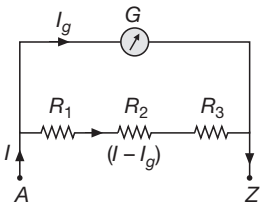
The galvanometer shown in figure has resistance $50\ \Omega$ and current required for full scale



deflection is 1 mA. Find the resistances R_1, R_2 and R_3 required to convert it into ammeter having ranges as indicated.

SOLUTION

For the range 0.1 A, R_1, R_2 and R_3 are in series combination.

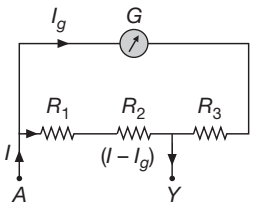


So, equating potential difference across galvanometer and series combination of R_1, R_2 and R_3, we get

$$I_gG = (I-I_g)(R_1+R_2+R_3)$$

Hence $$R_1+R_2+R_3 = \frac{I_gG}{(I-I_g)} = \frac{10^{-3}\times 50}{(0.1-10^{-3})} = \frac{50}{99}\ \Omega \quad \ldots(1)$$

For range 1 A, (R_1+R_2) is in parallel to $(G+R_3)$



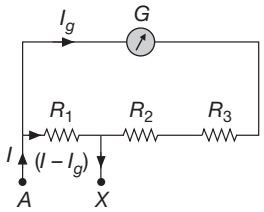
$$\Rightarrow \quad I_g(G+R_3) = (I-I_g)(R_1+R_2)$$

$$\Rightarrow \quad I_g(G+R_1+R_2+R_3) = I(R_1+R_2)$$

Substituting for I_g, G, I and $(R_1+R_2+R_3)$, we have

$$R_1+R_2=10^{-3}\left(\frac{50+\frac{50}{99}}{1}\right)=\frac{5}{99}\ \Omega \qquad \ldots(2)$$

For range 10 A, R_1 is in parallel to $(G+R_2+R_3)$



So, $I_g(G+R_2+R_3)=(I-I_g)R_1$

$$\Rightarrow \quad R_1=\frac{I_g(G+R_1+R_2+R_3)}{I}$$

$$\Rightarrow \quad R_1=\frac{10^{-3}\left(50+\frac{50}{99}\right)}{10}=\frac{5}{990}\ \Omega=\frac{1}{198}\ \Omega \qquad \ldots(3)$$

So, $R_2=\frac{5}{99}-\frac{5}{990}=\frac{1}{22}\ \Omega$

$$\Rightarrow \quad R_3=\frac{50}{99}-(R_1+R_2)$$

$$\Rightarrow \quad R_3=\frac{50}{99}-\frac{5}{99}=\frac{45}{99}$$

$$\Rightarrow \quad R_3=\frac{15}{33}\ \Omega$$

Hence, $R_1=\frac{1}{198}\ \Omega$, $R_2=\frac{1}{22}\ \Omega$ and $R_3=\frac{15}{33}\ \Omega$

VOLTMETER

Measurement of Potential Difference: The most commonly used instrument for measurement of potential difference is voltmeter. It is a moving coil galvanometer to which a high resistance is connected in series. The high resistance is connected for two reasons.

(a) To increase the range of measurement: Since the galvanometer has a full scale deflection for very small currents and hence it can measure only a small potential difference. However, if a large resistance is connected in series with the coil, any desired potential difference can be measured.

(b) To reduce the error of measurement: The voltmeter is connected in parallel with that part of the circuit across which the potential difference is to be measured. Therefore its own resistance should be very high, otherwise it will change the current in (and hence the potential difference across) that part of the circuit. The measured potential difference will be less than the actual value. By connecting a high resistance in series with the coil, we can minimize this error.

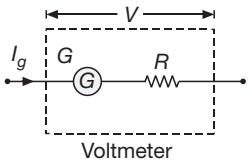
SERIES RESISTANCE TO CONVERT A GALVANOMETER INTO A VOLTMETER OF DESIRED RANGE

If V is the desired range and R is the additional series resistance, then

$$V=I_g(G+R)$$

$$\Rightarrow \quad R=\frac{V}{I_g}-G$$

Voltmeter is a high resistance device and hence it is always connected in parallel across a circuit whose voltage is to be measured.



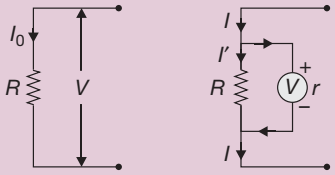
Remark(s)

(a) The reading of a voltmeter is always lesser than the true value. If a current I_0 is passing through a resistance R, the true value $V=I_oR$. However, when a voltmeter having resistance r is connected across R, the current through R will become

$$I'=\frac{r}{(R+r)}I_o$$

and so $V'=I'R=\frac{V}{[1+(R/r)]}$

When the voltmeter is connected across R, its reading will also be V' which is less than V.



(b) Greater the resistance of voltmeter, the more accurate will be its reading. A voltmeter is said to be **ideal** if its resistance r is infinite. An ideal voltmeter draws no current from the circuit element for its operation.

ILLUSTRATION 41

How can we make a galvanometer with $G = 20\ \Omega$ and $I_g = 1\ \text{mA}$ into a voltmeter with a maximum range of 10 V?

SOLUTION

Using $R = \dfrac{V}{I_g} - G$

We have, $R = \dfrac{10}{10^{-3}} - 20 = 9980\ \Omega$

Thus, a resistance of $9980\ \Omega$ is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

Please note that at full scale deflection current through the galvanometer, the voltage drop across the galvanometer

$$V_g = I_g G = 20 \times 10^{-3}\ \text{V} = 0.02\ \text{V}$$

and the voltage drop across the series resistance R is,

$$V = I_g R = 9980 \times 10^{-3}\ \text{V} = 9.98\ \text{V}$$

Conceptual Note(s)

(a) If an ammeter of range I and resistance G is to be converted into a voltmeter of range V, then the resistance R to be connected in series is

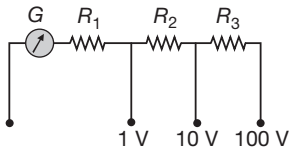
$$R = \frac{V}{I} - G$$

(b) If the voltmeter of range V and resistance G is to be converted into an ammeter of range I, then shunt S to be connected in parallel is

$$S = \frac{VG}{IG - V}$$

ILLUSTRATION 42

A galvanometer has an internal resistance of $50\ \Omega$ and current required for full scale deflection is 1 mA. Find the series resistances required (as shown in figure) to use it as a voltmeter with different ranges, as indicated in figure.



SOLUTION

For range 1 V, galvanometer and R, are in series

$$I_g = \frac{V}{(G + R_1)}$$

$$\Rightarrow \quad 10^{-3} = \frac{1}{(50 + R_1)}$$

$$\Rightarrow \quad 50 + R_1 = 1000$$

$$\Rightarrow \quad R_1 = 1000 - 50 = 950\ \Omega$$

For range 10 V, galvanometer and R_2, R_3 are in series.

$$10^{-3} = \frac{10}{(G + R_1 + R_2)}$$

$$\Rightarrow \quad G + R_1 + R_2 = \frac{10}{10^{-3}} = 10 \times 10^3$$

$$\Rightarrow \quad R_2 = 10000 - (50 + 950) = 9000$$

$$R_2 = 9\ \text{k}\Omega$$

and for range 100 V, galvanometer, R_1, R_2 and R_3 are in series.

$$10^{-3} = \frac{100}{(G + R_1 + R_2 + R_3)}$$

$$\Rightarrow \quad G + R_1 + R_2 + R_3 = \frac{100}{10^{-3}} = 100 \times 10^3$$

$$\Rightarrow \quad R_3 = 100\times10^3 - (G + R_1 + R_2)$$

$$\Rightarrow \quad R_3 = 100\times10^3 - 10\times10^3$$

$$\Rightarrow \quad R_3 = 90\times10^3 = 90\ \text{k}\Omega$$

ILLUSTRATION 43

The scale of galvanometer is divided into 150 equal divisions. The galvanometer has the current sensitivity of 10 divisions per mA and the voltage sensitivity of 2 divisions per mV. How can you convert this galvanometer into a/an

(a) ammeter of 6 A per division
(b) voltmeter of 1 V per division

Calculate the galvanometer resistance G, value of shunt S to be connected to convert into ammeter and value of series resistance R to be connected to convert into voltmeter.

SOLUTION

Since galvanometer resistance is

$$G = \frac{\text{Full scale voltage}}{\text{Full scale current}}$$

Also, full scale voltage =

$$\frac{\text{No. of divisions}}{\text{Voltage sensitivity}} = \frac{150}{2} = 75\ \text{V}$$

and full scale current =

$$\frac{\text{No. of divisions}}{\text{Current sensitivity}} = \frac{150}{10} = 15\ \text{mA}$$

$$= 15\times10^{-3}\ \text{A}$$

$$\Rightarrow \quad G = \frac{75}{15\times10^{-3}} = 5000\ \Omega$$

$$\Rightarrow \quad S = \frac{I_g G}{I - I_g} = \frac{15\times10^{-3}\times5000}{150\times6 - 15\times10^{-3}} = 83\ \text{m}\Omega$$

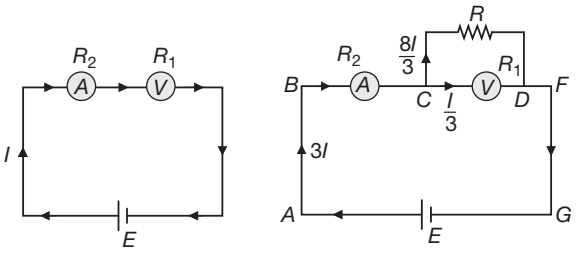
$$\Rightarrow \quad R = \frac{V}{I_g} - G = \frac{150}{15\times10^{-3}} - 5 = 5000\ \Omega$$

ILLUSTRATION 44

A voltmeter of resistance R_1 and an ammeter of resistance R_2 are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third. Find R_1 and R_2 in terms of R.

SOLUTION

Let E be the emf of the battery



In the first case let I be the current in the circuit, then

$$E = I(R_1 + R_2) \qquad \ldots(1)$$

In the second case main current increases three times while current through voltmeter will reduce to $\frac{I}{3}$. Hence, the remaining $3I - \frac{I}{3} = \frac{8I}{3}$ passes through R as shown in figure.

$$V_C - V_D = \left(\frac{I}{3}\right)R_1 = \left(\frac{8I}{3}\right)R$$

$$\Rightarrow \quad R_1 = 8R$$

Applying Kirchhoff's Second Law in Loop $ABFGA$,

$$E = 3I(R_2) + \left(\frac{I}{3}\right)(R_1) = I\left(3R_2 + \frac{R_1}{3}\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$R_1 + R_2 = 3R_2 + \frac{R_1}{3}$$

$$\Rightarrow \quad 2R_2 = \frac{2R_1}{3}$$

$$\Rightarrow \quad R_2 = \frac{R_1}{3} = \frac{8R}{3}$$

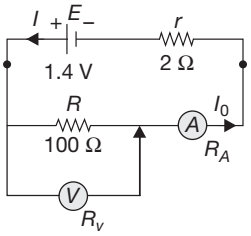
ILLUSTRATION 45

A battery of emf 1.4 V and internal resistance $2\ \Omega$ is connected to a resistor of $100\ \Omega$. In order to measure the current through the resistance and the potential difference across its ends, an ammeter is connected in series with it and a voltmeter is connected across its ends. The resistance of the ammeter is $\frac{4}{3}\ \Omega$ and that of the voltmeter is $200\ \Omega$. What are the readings of the two instruments? What would be their reading if they were ideal instruments?

SOLUTION

Let R_A and R_V be the resistances of the ammeter and voltmeter respectively. Then the total resistance across the emf E is

$$R_{eq} = \frac{RR_V}{R+R_V} + R_A + r = \frac{100\times 200}{100+200} + \frac{4}{3} + 2 = 70\ \Omega$$



Therefore, the current

$$I_o = \frac{E}{R_{eq}} = \frac{1.4}{70} = 0.02\ \text{A}$$

This is the current through ammeter. Hence, the reading of ammeter is 0.02 A

Reading of voltmeter is the potential difference across its terminals. So reading of voltmeter is

$$V = I_o\left(\frac{RR_V}{R+R_V}\right) = 0.02\left(\frac{100\times 200}{100+200}\right) = 1.33\ \text{V}$$

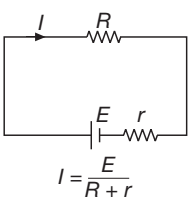
If the ammeter and the voltmeter were **ideal**, $R_A = 0$ and $R_V \to \infty$. Then,

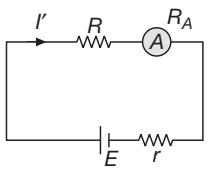
The reading of ammeter $= \frac{E}{r+R} = \frac{1.4}{2+100} = 0.0137\ \text{A}$

The reading of voltmeter $= I_o R = \frac{1.4}{102}\times 100 = 1.37\ \text{V}$

ERROR IN THE MEASUREMENT BY AMMETER

To measure current flowing in a certain branch, we connect an ammeter in that branch. Consider the following circuit. To measure I, we connect an ammeter in the wire.





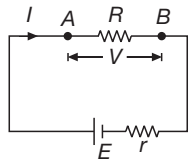
Connecting the ammeter changes the current. Let R_A is the resistance of the ammeter, then

$$I' = \frac{E}{R+R_A+r}$$

Clearly $I' < I$. So an ammeter reads less than actual value. Error in the reading $= I' - I$

ERROR IN THE MEASUREMENT BY VOLTMETER

Consider the same circuit as before. Now, we wish to find out the potential drop across the external resistance R. To measure the potential difference, a voltmeter of resistance R_V is connected between A and B.



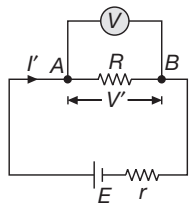
Current through the cell is

$$I = \frac{E}{R+r}$$

Now, $V = V_{AB} = IR$

$$\Rightarrow \quad V = V_{AB} = \frac{ER}{(R+r)} = \frac{E}{1+\frac{r}{R}}$$

Now, $R' = \frac{R\times R_V}{R+R_V} = \frac{R}{1+\frac{R}{R_V}} < R$



Current through the cell is

$$I' = \frac{E}{R'+r},\ R' = \frac{R\times R_V}{R+R_V}$$

$$\Rightarrow \quad V' = I'R' = \frac{ER'}{(R'+r)} = \frac{E}{1+\frac{r}{R'}}$$

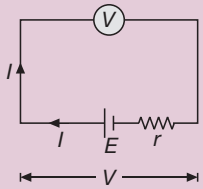
As $R' < R$, $V' < V$ (error in the reading is $V' - V$). So a voltmeter reads less than the actual value.

Conceptual Note(s)

(a) An ideal ammeter should have zero resistance, but a non-ideal ammeter has small but non-zero resistance.

(b) An ideal voltmeter should have infinite resistance, but a non-ideal voltmeter has large but not infinite resistance.

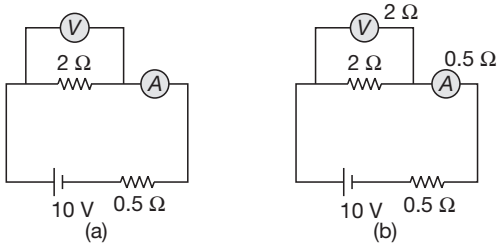
(c) A non-ideal voltmeter (i.e., a practical voltmeter) always draws some current from the circuit and there is always a potential drop across a non-ideal ammeter (i.e., a practical ammeter).



Therefore, when it is used to measure the emf of a cell, it will read $V = E - Ir$. As $I \neq 0$ therefore, reading of a voltmeter will not be equal to emf of the cell.

ILLUSTRATION 46

In the electrical circuits shown in figure (a) and figure (b), the instruments are ideal in figure (a) while their resistances are marked in figure (b). Determine their readings.



SOLUTION

For figure (a), we have

$$I = \frac{10}{2 + 0.5} = 4 \text{ A}$$

So, reading of ammeter is 4 A

and reading of voltmeter is $IR = 8$ V

For figure (b), we have

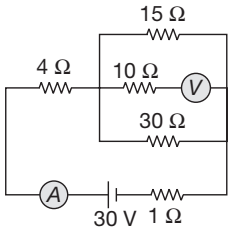
$$I' = \frac{10}{R' + 0.5 + 0.5}, \text{ where } R' = \frac{2 \times 2}{2 + 2} = 1\ \Omega$$

$$\Rightarrow \quad I' = \frac{10}{2} = 5 \text{ A}$$

So, reading of ammeter is 5 A and reading of voltmeter is $I'R' = 5 \times (1) = 5$ V

ILLUSTRATION 47

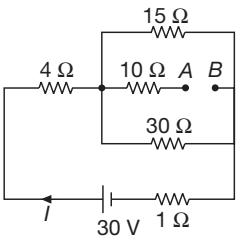
In the electrical circuit shown, the instruments are ideal. Determine their readings.



SOLUTION

Since the ideal ammeter has zero resistance and an ideal voltmeter has infinite resistance.

So, no current flows through the voltmeter. The given circuit can be redrawn as shown in figure.



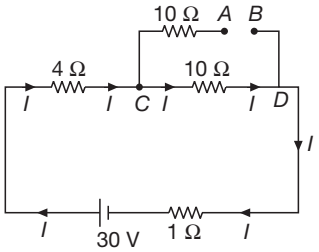
The effective external resistance of the circuit is

$$R_{\text{net}} = 4 + \frac{15 \times 30}{15 + 30} = 14\ \Omega$$

$$\Rightarrow \quad I = \frac{30}{14 + 1} = 2 \text{ A}$$

Hence, reading of ammeter is 2 A

Since 15 Ω and 30 Ω are in parallel, so their effective resistance is 10 Ω as shown in figure.



The reading of voltmeter is the potential difference between A and B. Since $V_{AB} = I(10) = (2)(10) = 20$ V.

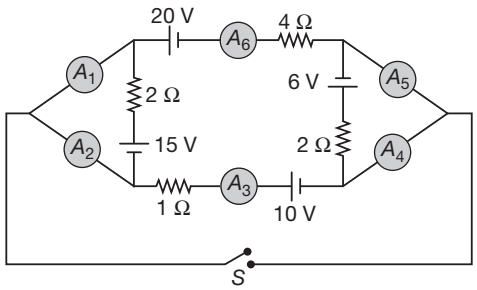
So, reading of voltmeter is 20 V

ILLUSTRATION 48

In the circuit shown, all the ammeters are ideal.

(a) If the switch S is open, find the reading of all ammeters and the potential difference across the switch.

(b) If the switch S is closed, find the current through all ammeters and the switch also.



SOLUTION

Using the loop current method.

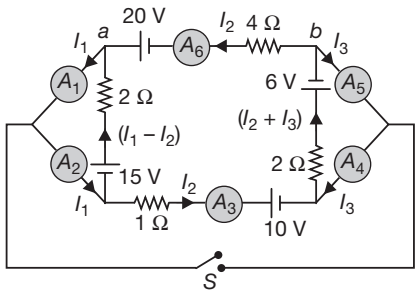
(a) $-2(I_1 - I_2) + 15 = 0$ …(1)

$-4I_2 + 20 - 2(I_2 - I_1) - 15 - I_2 - 10 - 2(I_2 + I_3) - 6 = 0$

$\Rightarrow \quad 2I_1 - 9I_2 - 2I_3 - 11 = 0$ …(2)

$-2(I_2 + I_3) - 6 = 0$

$\Rightarrow \quad I_2 + I_3 + 3 = 0$ …(3)



Solving these three equation, we get

$I_1 = 9.5$ A

$I_2 = 2$ A and $I_3 = -5$ A

Ammeter	A_1	A_2	A_3	A_4	A_5	A_6
Reading (Amp)	9.5	9.5	2	5	5	2

P.D. across switch $= 10 + (1)I_2 = 10 + 2 = 12$ V

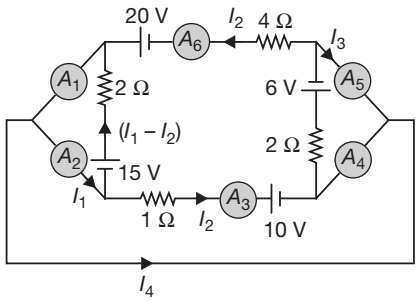
(b) When switch is closed

$-2(I_1 - I_2) + 15 = 0$ …(1)

$2I_1 - 9I_2 - 2I_3 - 11 + I_4 = 0$ …(2)

$I_2 + I_3 + 3 = 0$ …(3)

$10 - (I_4 - I_2) = 0$ …(4)



Solving these four equation, we get

$I_1 = 12.5$ A

$I_2 = 5$ A

$I_3 = -8$ A

$I_4 = 15$ A

Ammeter	A_1	A_2	A_3	A_4	A_5	A_6
Reading (Amp)	12.5	2.5	10	7	8	5

And the current through switch is 15 A.

Test Your Concepts-VIII

Based on Galvanometer, Voltmeter and Ammeter

(Solutions on page H.216)

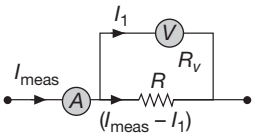
1. Assume that a galvanometer has an internal resistance of 60 Ω and requires a current of 0.5 mA to produce full scale deflection. What resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter that has a full scale deflection for a current of 0.1 A?

2. Design a multirange ammeter capable of full scale deflection for 25 mA, 50 mA, and 100 mA. Assume the meter movement is a galvanometer that has a resistance of 25 Ω and gives a full scale deflection for 1 mA.

3. Design a multirange voltmeter capable of full scale deflection for 20 V, 50 V and 100 V. Assume the meter movement is a galvanometer that has a resistance of 60 Ω and gives a full scale deflection for a current of 1 mA.

4. A particular galvanometer serves as a 2 V full scale voltmeter when a 2500 Ω resistor is connected in series with it. It serves as a 0.5 A full scale ammeter when a 0.22 Ω resistor is connected in parallel with it. Determine the internal resistance of the galvanometer and the current required to produce full scale deflection.

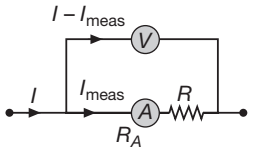
5. To measure the resistance R of a resistor, a voltmeter of resistance R_v is placed across a resistor and an ammeter is placed in series with the combination as shown in figure



(a) Find the resistance R in terms of the measured readings on the ammeter I_{meas} and voltmeter V_{meas}

(b) Discuss the result for $R_v \gg \dfrac{V_{meas}}{I_{meas}}$

6. To measure the resistance R of a resistor, an ammeter of resistance R_A is placed in series with the resistor and the voltmeter is placed across the series combination as shown in figure.

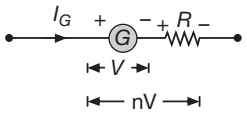


(a) Find the resistance R in terms of the measured readings on the ammeter I_{meas} and voltmeter V_{meas}

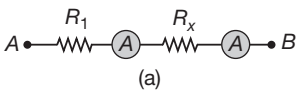
(b) Discuss the result for $\dfrac{V_{meas}}{I_{meas}} \gg R_A$.

7. What shunt resistance is required to make the 1 mA, 20 Ω galvanometer into an ammeter with a range of 0 A to 50 mA?

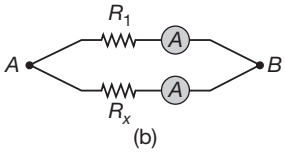
8. A voltmeter has a resistance G ohm and range V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.



9. Consider two different ammeters in which the currents are proportional to the respective deflections of the needle. The first ammeter is connected to a resistor of resistance R_1 and the second to a resistor of unknown resistance R_x. Firstly, the ammeters are connected in series between points A and B (as shown in figure). In this case the readings of the ammeters are n_1 and n_2. Then the ammeters are connected in parallel between A and B (as shown in figure) and indicate N_1 and N_2. Determine the unknown resistance R_x of the second resistor.



(a)



(b)

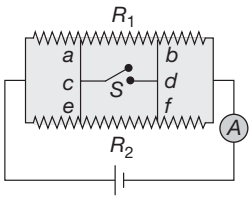
10. An ammeter is connected to measure the current intensity in a circuit with a resistance R. What relative error ε will be made if connection of the ammeter does not change the current intensity in the circuit? The voltage across the ends of the circuit is kept constant.

11. Determine the voltage across a resistance R using a voltmeter connected to its ends. What relative error, ε will be made if the readings of the voltmeter are taken as the voltage applied before it was switched on? The current intensity in the circuit is constant.

12. What resistance r should be used to shunt a galvanometer with an internal resistance of $R = 10\ \text{k}\Omega$ to reduce its sensitivity $n = 50$ times?

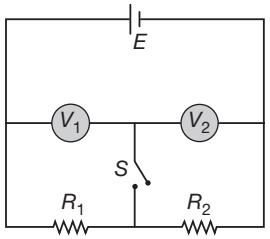
13. An ammeter and a voltmeter are connected in series to a battery of emf $E = 6$ V. When a certain resistance is connected in parallel with the voltmeter, the reading of the latter decreases two times, whereas the readings of the ammeter increase the same number of times. Find the voltmeter readings after the connection of the resistance.

14. Two conductors ace and bdf are connected at equal potentials on resistors R_1 and R_2. Will there any currents flowing through them and through the cd section if the switch S is closed. Will this lead to a change in the reading of an ammeter?



15. A cell of emf 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and to an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance the ammeter reading is 0.04 A. Find the voltage read by the voltmeter and its resistance. Had the voltmeter been an ideal one what would have been its reading?

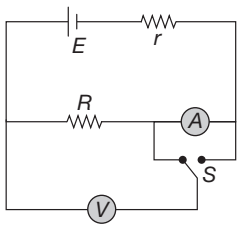
16. In the circuit shown in figure, V_1 and V_2 are two voltmeters having resistances 6 kΩ and 4 kΩ respectively. The emf of the battery is 250 V, having negligible internal resistance. Two resistances $R_1 = 4\ \text{k}\Omega$ and $R_2 = 6\ \text{k}\Omega$ are also connected in the circuit as shown. Find the reading of the voltmeters V_1 and V_2 when



(a) the switch S is open and

(b) the switch S is closed.

17. The emf E and the internal resistance r of the battery shown in figure are 4.3 V and 1 Ω respectively. The external resistance R is 50 Ω. The resistances of the ammeter and voltmeter are 2 Ω and 200 Ω respectively.



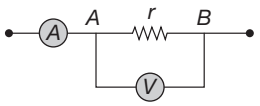
(a) Find the readings of the two meters.

(b) The switch is thrown to the other side. What will be the readings of the two meters now?

18. A moving coil galvanometer of resistance 20 Ω gives a full scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale. But the shunt of 0.005 Ω only is available. What resistance should be connected in series with the galvanometer coil?

19. A resistance box, a battery and a galvanometer of resistance G Ω are connected in series. If the galvanometer is shunted by resistance of S Ω, find the change in resistance in the box required to maintain the current from the battery unchanged.

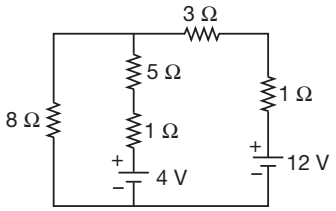
20. Determine the resistance r if an ammeter shows a current of $I = 5$ A and a voltmeter 100 V. The internal resistance of the voltmeter is $R = 2500$ Ω.



21. Two resistors, 400 Ω, and 800 Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 Ω resistance is used for this purpose. What will be the reading in the ammeter? Similarly, if a voltmeter of 1000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in the voltmeter?

22. Draw the circuit for experimental verification of Ohm's Law using a source of variable d.c. voltage, a main resistance of 100 Ω , two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter.

23. In figure, show how to add just enough ammeters to measure every different current. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.



HEATING EFFECT OF CURRENT

When a charge dq passes across a potential difference V, the work done dW is given by,

$$dW = Vdq$$

This work represents the loss of potential energy of charges. The flow of charge dq in time dt is equivalent to current I, where

$$I = \frac{dq}{dt}$$

$$\Rightarrow \quad dq = Idt$$

$$\Rightarrow \quad dW = VIdt$$

If constant current I passes through conductor for time t under a potential difference V, then

$$W = VIt \qquad \ldots(1)$$

According to Ohm's Law, $V = IR$

$$\Rightarrow \quad \text{Work done } = I^2Rt = \frac{V^2}{R}t = VIt \qquad \ldots(2)$$

This work is converted into the energy of random thermal motion of molecules of the conductor. That is the electric current through a conductor produces thermal energy in the conductor and the conductor gets heated. This phenomenon is called **Joule's heating effect of current**.

If V is in volt, I in amp, R in ohm, then Joule's heat is equivalent to

$$W = VIt = I^2Rt = \frac{V^2}{R}t \text{ joule}$$

OR

Heat produced Q is given by

$$Q = \frac{W}{J} = \frac{VIt}{J} = \frac{I^2Rt}{J} = \frac{\left(\frac{V^2}{R}\right)t}{J} \text{ cal}$$

POWER

The rate of work done in an electrical circuit is called the **power** and is dissipated in the form of heat.
Power dissipated,

$$P = \frac{dW}{dt} = VI = I^2R = \frac{V^2}{R}$$

The SI unit of power is watt (W)

$$1 \text{ W} = 1 \text{ Js}^{-1}$$

ELECTRIC ENERGY

The usual unit of energy is joule but for convenience electrical energy (measured as a large unit) is measured in kilowatthour (kWh).

$$1\ \text{kWh} = P(\text{in kilowatt}) \times (t \text{ in hour})$$

$$1\ \text{kWh} = 1000 \times 3600 \text{ joule}$$

$$1\ \text{kWh} = 3.6 \times 10^6 \text{ joule}$$

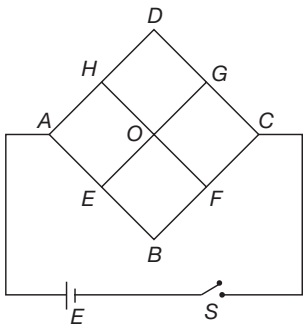
In houses the electric appliances (e.g., bulbs, refrigerator, T.V., cooler, heater etc.) are connected in parallel and the electric energy consumed is measured in kilowatthour (kWh).

$$\text{Number of units consumed } (N) = \frac{\text{watt} \times \text{hours}}{1000}$$

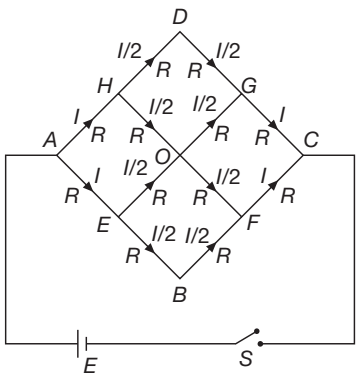
$$\Rightarrow N = \frac{(\text{Power Consumed})(\text{Time of Consumption})}{1000}$$

ILLUSTRATION 49

In the given figure each of the segments has resistance R. EMF of battery is E and internal resistance is negligible. Find ratio of power generated in AE to that in HO.



SOLUTION



$$\frac{P_{AE}}{P_{HO}} = \frac{I^2 R}{\left(\frac{I}{2}\right)^2 R} = 4$$

SPECIFICATION OF A BULB OR OTHER ELECTRIC APPLIANCES

Every electrical appliance like Bulb, Heater, Geyser etc. has wattage and voltage printed on it. These are called rated values or the specified values. When a bulb has specified voltage V and power P, then resistance R and maximum allowed current I may be determined.

$$\text{Power}(P) = \text{Voltage}(V) \times \text{Current}(I)$$

$\Rightarrow$ Maximum allowed current in bulb is

$$I = \frac{P}{V}$$

Resistance of its filament is

$$R = \frac{V}{I} = \frac{V}{P/V} = \frac{V^2}{P}$$

The more the current flowing through a bulb the more is its glow.

The bulbs and other electric appliances are manufactured for parallel combination. If they are connected in series, the effect is reversed. For example if two bulbs of 50 W, 200 W are given, then for parallel combination of the bulbs, the 200 W bulb would glow more brightly. However, for the series combination of the bulbs, the 50 W bulb would glow more brightly as more current flows through it.

Conceptual Note(s)

(a) S.I. unit of power is Js^{-1} and $1\,Js^{-1} = 1$ watt

(b) Rated values of Voltage and Power: Every electrical appliance like Bulb, Heater, Geyser etc. has wattage and voltage printed on it. These are called rated values.

FOR EXAMPLE: A 40 W, 220 V bulb has a rated power $P_R = 40$ W and the rated voltage $V_R = 220$ V. This simply means that when this bulb is connected across the rated voltage i.e. 220 V, then the power consumed across it will also be the rated power that is 40 W. So,

when $V_{\text{applied}} < V_{\text{rated}}$, then $P_{\text{consumed}} < P_{\text{rated}}$ and

when $V_{\text{applied}} > V_{\text{rated}}$, then $P_{\text{consumed}} > P_{\text{rated}}$

(c) Resistance of electrical appliance: If variation of resistance with temperature is neglected then resistance of any electrical appliance can be

calculated by rated power and rated voltage i.e. by using $R = \frac{V_R^2}{P_R}$.

(d) It is observed that for a bulb

Brightness $\propto P_{\text{consumed}}$

(e) Power consumed and illumination: An electrical appliance (Bulb, heater, ... etc.) consume rated power P_R only when the applied voltage V_A is equal to the rated voltage V_R. So

(i) If $V_A = V_R$, then $P_{\text{consumed}} = P_R$

(ii) If $V_A < V_R$, then $P_{\text{consumed}} = \frac{V_A^2}{R}$

Since, $R = \frac{V_R^2}{P_R}$ and hence

$$\Rightarrow \quad P_{\text{consumed}} = \left(\frac{V_A^2}{V_R^2}\right) P_R \propto (\text{Brightness})$$

POWER TRANSFORMATION RULE

CASE-1: Bulbs Connected in Series

When bulbs/appliances having rated voltage V, and rated powers P_1, P_2, P_3, ... are connected in series across a voltage supply V, then the power consumption across the series combination of the appliances will be

$$P_s = \frac{V^2}{R_s},$$

where $R_s = R_1 + R_2 + R_3$ and

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3}$$

$$\Rightarrow \quad P_s = \frac{V^2}{\left(\frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}\right)}$$

$$\Rightarrow \quad \frac{1}{P_s} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

CASE-2: Bulbs Connected in Parallel

When bulbs/appliances having rated voltage V, and rated powers P_1, P_2, P_3, ... are connected in parallel across a voltage supply V, then the power consumption across the parallel combination of the appliances will be

$$P_p = \frac{V^2}{R_p},$$

where $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ and

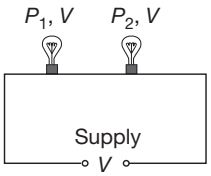
$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3}$$

$$\Rightarrow \quad P_s = V^2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\Rightarrow \quad P_s = P_1 + P_2 + P_3$$

COMBINATION OF BULBS

Series Combination



(a) Total power consumed $\frac{1}{P_{\text{total}}} = \frac{1}{P_1} + \frac{1}{P_2} + ...$

(b) If n bulbs are identical, $P_{\text{total}} = \frac{P}{n}$

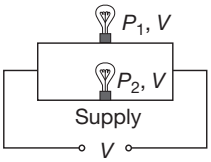
(c) P_{consumed} (Brightness) $\propto V \propto R \propto \frac{1}{P_{\text{rated}}}$

i.e. in series combination, the bulb of lesser wattage will give more bright light and potential difference appeared across it will be more.

Parallel Combination

(a) Total power consumed

$$P_{\text{total}} = P_1 + P_2 + P_3 ..... + P_n$$



(b) If n identical bulbs are in parallel, then $P_{\text{total}} = nP$

(c) P_{consumed} (Brightness) $\propto P_R \propto i \propto \dfrac{1}{R}$

i.e. in parallel combination, bulb of greater wattage will give more bright light and more current will pass through it.

Conceptual Note(s)

(a) If $V_{\text{Applied}} < V_{\text{Rated}}$ then percentage drop in output power of electrical device is

$$\%\text{age drop} = \frac{(P_R - P_{\text{consumed}})}{P_R} \times 100$$

(b) Different bulbs

$\Rightarrow$ Resistance $R_{25} > R_{100} > R_{1000}$

$\Rightarrow$ Thickness of filament $t_{1000} > t_{100} > t_{40}$

$\Rightarrow$ Brightness $B_{1000} > B_{100} > B_{25}$

(c) Necessary series resistance to glow a bulb of rated power P_R, if $V_{\text{Applied}} > V_{\text{Rated}}$ is

$$R = \left(\frac{V_{\text{Applied}} - V_{\text{Rated}}}{P_R}\right) \times V_R$$

(d) When some potential difference applied across the conductor then collision of free electrons with ions of the lattice result's in conversion of electrical energy into heat energy

(e) If a heating coil of resistance R, (length ℓ) consumed power P, when voltage V is applied to it then by keeping V constant if it is cut in n equal parts then resistance of each part will be $\dfrac{R}{n}$ and from $P_{\text{consumed}} \propto \dfrac{1}{R}$, power consumed by each part $P' = nP$.

(f) In series a device of higher power rating consumes less power.

(g) Consider that n bulbs are connected in series across V volt supply. If one bulb gets fused and $(n-1)$ bulbs are again connected in series across same supply, the illumination will be more with $(n-1)$ bulbs than with n bulbs, but risk of fusing of bulbs will increases.

(h) When a heavy current appliance such us motor, heater or geyser is switched on, it will draw a heavy current from the source so that terminal voltage of source decreases. Hence power consumed by the bulb decreases, so the light of bulb becomes less.

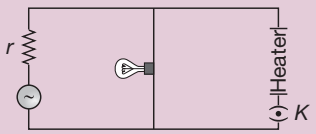


ILLUSTRATION 50

A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of wire is raised by same amount ΔT in the same time t. Find the value of N.

SOLUTION

Since we have

$$\text{Heat} = mc\Delta T = i^2Rt$$

CASE-1: Length (L), Resistance R and mass m

CASE-2: Length $(2L)$, Resistance $2R$ and mass $2m$

$$\Rightarrow \quad \frac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \frac{i_1^2 R_1 t_1}{i_2^2 r_2 t_2}$$

$$\Rightarrow \quad \frac{mS\Delta T}{2mS\Delta T} = \frac{i_1^2 Rt}{i_2^2 2Rt}$$

$$\Rightarrow \quad i_1 = i_2$$

$$\Rightarrow \quad \frac{(3E)^2}{12} = \frac{(NE)^2}{2R}$$

$$\Rightarrow \quad N = 6$$

TIME RELATION FOR SERIES AND PARALLEL COMBINATION

Let an electrical appliance having rated power P_1 can generate energy E in time t_1, then $E = P_1 t_1$.

Similarly another electrical appliance having rated power P_2 can generate same energy E in time t_2, then $E = P_2 t_2$.

Now when these appliances are connected across the same voltage source

CASE-1: In Series

Then power generated is

$$\frac{1}{P_S} = \frac{1}{P_1} + \frac{1}{P_2}$$

So time required to generate the same energy E is

$$t_S = \frac{E}{P_S} = \frac{E}{P_1} + \frac{E}{P_2} = \frac{E}{\left(\frac{E}{t_1}\right)} + \frac{E}{\left(\frac{E}{t_2}\right)} = t_1 + t_2$$

$$\Rightarrow \quad \boxed{t_S = t_1 + t_2}$$

CASE-2: In Parallel

Then power generated is

$$P_P = P_1 + P_2$$

So, time required to generate the same energy E is

$$t_P = \frac{E}{P_P} = \frac{E}{P_1 + P_2} = \frac{E}{\frac{E}{t_1} + \frac{E}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

$$\Rightarrow \quad \boxed{\frac{1}{t_P} = \frac{1}{t_1} + \frac{1}{t_2}}$$

ILLUSTRATION 51

An electric tea kettle has a multi-position switch and two heating coils. When only one of the coils is switched on, the well-insulated kettle makes a full pot of water to a boil in a time interval Δt. When only the other coil is switched on, it requires a time interval of $2\Delta t$ to boil the same amount of water. Find the time interval required to boil the same amount of water if both coils are switched while being used

(a) in a parallel connection and
(b) in a series connection.

SOLUTION

A certain quantity of energy $\Delta E_{\text{int}} = P(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $P = I\Delta V = \frac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus comparing coils 1 and 2, we have for the energy

$$H = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$$

$$\Rightarrow \quad R_2 = 2R_1$$

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{2R_1}} = \frac{2R_1}{3}$$

and $H = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_p}{\frac{2R_1}{3}}$

$$\Rightarrow \quad \Delta t_p = \frac{2\Delta t}{3}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1$

$= 3R_1$ and $\frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_s}{3R_1}$

$$\Rightarrow \quad \Delta t_s = 3\Delta t$$

CHARACTERISTICS OF A FUSE

Fuse is used with the main electrical circuit for the safety of electrical appliances.

A fuse wire must have high resistance and low melting point. Hence, generally it is made of tin-lead alloy.

Let R be the resistance, ρ resistivity, l length, a cross-sectional area and I ampere be its current carrying capacity.

When the fuse is safe, then for its steady state temperature, heat produced per second must be equal to heat radiated by it per second. Heat produced in fuse wire per second

$$H = \frac{Q}{t} = I^2 R = I^2 \left(\frac{\rho l}{A}\right)$$

$$H = \frac{I^2 \rho l}{\pi r^2} \text{ joule sec}^{-1} \qquad \ldots(1)$$

If e is emissivity of fuse material of radius r and T is the excess safe temperature of wire above surroundings then according to **Newton's Law of Cooling**, the energy radiated per second

$$H = e(2\pi rl)T \qquad \ldots(2)$$

For steady state

$$e(2\pi rl)T = I^2 \frac{\rho l}{\pi r^2}$$

$$\Rightarrow \quad T = \frac{I^2\rho}{2\pi^2 e r^3} \qquad \ldots(3)$$

So, we observe that the steady state temperature of a fuse is independent of its length.
Hence length is immaterial for an electric fuse.
For a given material of a fuse wire

$$I^2 \propto r^3$$

ILLUSTRATION 52

What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases down to zero

(a) uniformly during a time interval t_0 ?
(b) halving its value every t_0 seconds?

SOLUTION

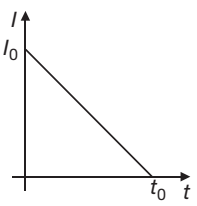
Heat generated in a resistance is given by,

$$H = I^2Rt$$

We can directly use this formula provided I is constant. Here, I is varying. So, first we will calculate I at any time t, then find a small heat dH in a short interval of time dt. Then by integrating it with proper limits we can obtain the total heat produced.

(a) The corresponding $I-t$ graph will be a straight line with I decreasing from a peak value (say I_0) to zero in time t_0. $I-t$ equation will be as

$$\Rightarrow \quad I = I_0 - \left(\frac{I_0}{t_0}\right)t \qquad (y = -mx + c) \qquad \ldots(1)$$



Here, I_0 is unknown, which can be obtained by using the fact that area under I-t graph gives the flow of charge. Hence,

$$q = \frac{1}{2}(t_0)(I_0)$$

$$\Rightarrow \quad I_0 = \frac{2q}{t_0}$$

Substituting in (1), we get,

$$I = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right)$$

$$\Rightarrow \quad I = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Now at time t, heat produced in a short interval dt is,

$$dH = I^2R\,dt$$

$$\Rightarrow \quad dH = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R dt$$

So, total heat produced is $H = \int_0^{t_0} dH$

$$\Rightarrow \quad H = \int_0^{t_0}\left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\,dt$$

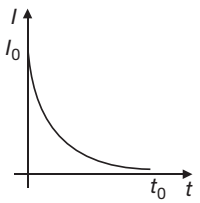
$$\Rightarrow \quad H = \frac{4}{3}\frac{q^2R}{t_0}$$

(b) Here, current decreases from some peak value (say I_0) to zero exponentially with half life t_0. I-t equation in this case will be

$$I = I_0e^{-\lambda t}$$

Here, $\lambda = \dfrac{\log_e(2)}{t_0}$

Now, $q = \int_0^{\infty} I\,dt = \int_0^{\infty} I_0e^{-\lambda t}dt = \left(\frac{I_0}{\lambda}\right)$



$$\Rightarrow \quad I_0 = \lambda q$$

$$\Rightarrow \quad I = (\lambda q)e^{-\lambda t}$$

$$\Rightarrow \quad dH = I^2 R dt = \lambda^2 q^2 e^{-2\lambda t} R dt$$

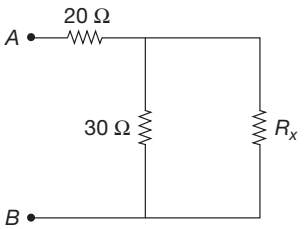
$$\Rightarrow \quad H = \int_0^\infty dH = \lambda^2 q^2 R \int_0^\infty e^{-2\lambda t} dt = \frac{q^2 \lambda R}{2}$$

Substituting $\lambda = \dfrac{\log_e(2)}{t_0}$, we have

$$H = \frac{q^2 R \log_e(2)}{2t_0}$$

ILLUSTRATION 53

A circuit shown in the figure has resistances 20 Ω and 30 Ω. At what value of resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance? The voltage between points A and B is supposed to be constant in this case.

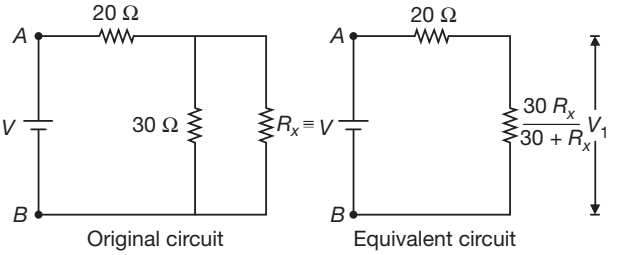


SOLUTION

V = Constant

Now, voltage across R_x is equal to the voltage across the 30 Ω resistor, because both are in parallel. Hence, if V_1 is the voltage across R_x, then

$$V_1 = V_{30\,\Omega} = \left(\frac{\dfrac{30R_x}{30+R_x}}{\dfrac{30R_x}{30+R_x} + 20} \right) V = \left(\frac{30R_x}{50R_x + 600} \right) V$$



Now, power generated in R_x is

$$P = \frac{V_1^2}{R_x} = \frac{900 R_x V^2}{(50R_x + 600)^2}$$

For P to be constant, we have

$$\frac{dP}{dR_x} = 0$$

$$\Rightarrow \quad \frac{(50R_x + 600)^2 (900V^2) - (1800)(50)(R_x V^2)(50R_x + 600)}{(50R_x + 600)^4} = 0$$

$$\Rightarrow \quad 50R_x + 600 - 100R_x = 0$$

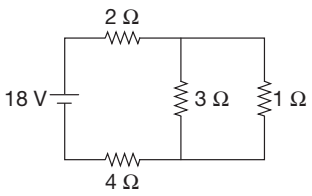
$$\Rightarrow \quad R_x = 12\ \Omega$$

Test Your Concepts-IX

Based on Heating Effects and Power Consumption

(Solutions on page H.220)

1. Calculate the power delivered to each resistor in the circuit shown.



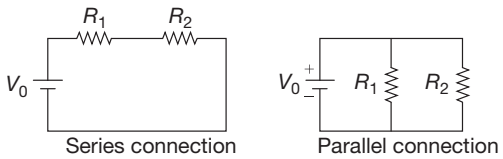
2. An electric teakettle has a multiposition switch and two heating coils. When only one of the coils is switched on, the well-insulated kettle makes a full pot of water to a boil in a time interval Δt. When only the other coil is switched on, it requires a time interval of $2\Delta t$ to boil the same amount of water. Find the time interval required to boil the same amount of water if both coils are switched while being used

(a) in a parallel connection and
(b) in a series connection.

3. When two unknown resistors are connected in series with a battery, the battery delivers total power P_s and carries a total current of I. For the same total current, a total power P_p is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

4. Two resistors R_1 and R_2 are in parallel with each other. Together they carry total current I.
(a) Determine the current in each resistor.
(b) Prove that this division of the total current I between the two resistors results in less power delivered to the combination than any other division. In other words prove the general principle that current in a direct current circuit distributes itself so that the total power delivered to the circuit is a minimum.

5. Consider an ideal battery with emf V_0 and two resistors with resistances R_1 and R_2. For the power absorbed by resistors to be maximum, should they be connected in series or in parallel, as in figure.

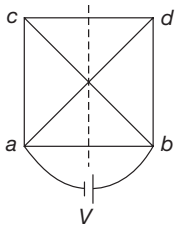


6. Two resistors are made of identical material, have the same length but one has the diameter twice that of the other. Determine the ratio of powers dissipated in the two resistors when
(a) each resistor is being connected across the same voltage source i.e., in parallel
(b) they are connected in series

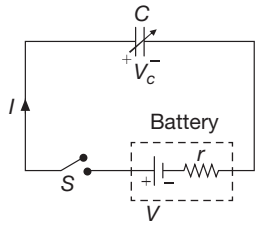
7. A 100 W, 110 V light bulb has a filament made of an alloy having a temperature coefficient of $0.0055\ °C^{-1}$ at 0°C. The normal operating temperature of the bulb is 2000°C. How much current will the bulb draw at the instant it is turned on when the room temperature is 20°C and 2000°C?

8. **(a)** Two wires made of same tinned copper alloy having equal cross-sectional area but of different lengths are to be used as fuses. Show that the fuses will melt at the same value of current which is independent of the length of the wires.
(b) A fuse wire of radius 0.1 mm blows when a current of 10 A passes through it. What should be the radius of a fuse wire made of the same material which will blow at a current of 20 A?

9. In the circuit shown in figure *abcd* is a square. All the wires forming the square and its diagonals are homogeneous and have same cross-section. Find the ratio of power dissipated in resistors *ab* and *cd*.

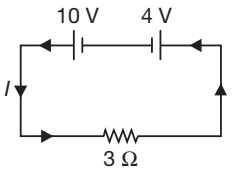


10. The variable capacitor in figure is connected to a battery of emf V and internal resistance r. The current in the circuit is kept constant by changing the capacitance. (Assume that we are able to increase the capacitance indefinitely in order to accomplish this).



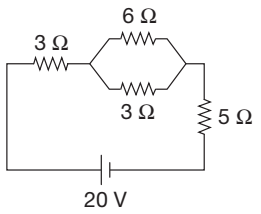
(a) Calculate the power supplied by the battery.
(b) Compare the rate at which energy is supplied by the battery with the rate of change of the energy stored in the capacitor.

11. In the circuit shown in figure, find the power

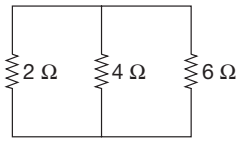


(a) supplied by 10 V battery
(b) consumed by 4 V battery and
(c) dissipated in 3 Ω resistance.

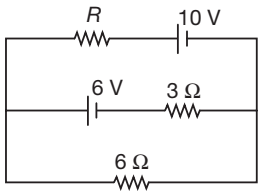
12. In the circuit shown in figure, find the heat developed across each resistance in $t = 2$ s.



13. In which branch of the circuit shown, a 11 V battery be inserted so that it dissipates minimum power. What will be the current through the 2 Ω resistance for this position of the battery?



14. In a circuit shown in figure if the internal resistances of the sources are negligible then at what value of resistance R will the thermal power generated in it be the maximum. What is its value?



15. The current I through a rod of a certain metallic oxide is given by $I = 0.2\,V^{1/2}$, where V is the potential difference across the rod. The rod is connected in series with a resistance to a 6 V battery of negligible internal resistance. What value should the series resistance have so that:

(a) the current in the circuit is 0.4 A.

(b) the power dissipated in the rod is twice that dissipated in the resistance.

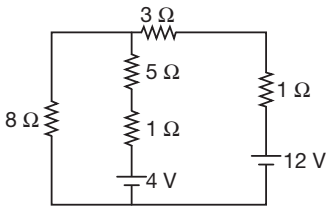
16. A storage battery with emf 2.6 V loaded with external resistance produces a current 1 A. In this case the potential difference between the terminals of the storage battery equals 2 V. Find the thermal power generated in the battery and the power developed in it by electric forces.

17. The circuit considered shown in figure is connected for a duration of 2 minute.

(a) Find the energy delivered by each battery.

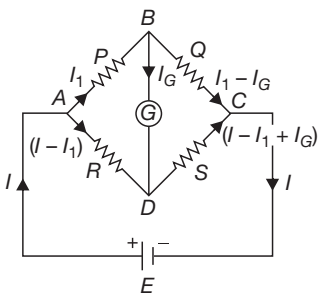
(b) Find the energy delivered to each resistor.

(c) Identify the types of energy transformations that occur in the operation of the circuit and the total amount of energy involved in each type of transformation.



WHEATSTONE BRIDGE: CONDITION OF BALANCE

The Wheatstone's Bridge is shown in figure.



P, Q, R and S are four resistances, G is galvanometer and E is a battery. The Wheatstone's Bridge is said to be balanced when no current flows in galvanometer. So,

Potential of B = Potential of D.

The condition of balance is best achieved by applying Kirchhoff's Laws

Applying Kirchhoff's 2nd Law to loop ABDA

$$-I_1P - I_GG + (I - I_1)R = 0 \qquad \ldots(1)$$

Applying Kirchhoff's 2nd Law to loop BCDB

$$-(I_1 - I_G)Q + S(I - I_1 + I_G) + I_GG = 0 \qquad \ldots(2)$$

For balance $I_G = 0$, so that (1) and (2) give

$$I_1 P = (I - I_1) R \quad \text{and} \qquad \ldots(3)$$

$$I_1 Q = (I - I_1) S \qquad \ldots(4)$$

Dividing (3) by (4), we get
Condition of balance is

$$\frac{P}{Q} = \frac{R}{S}$$

Remark(s)

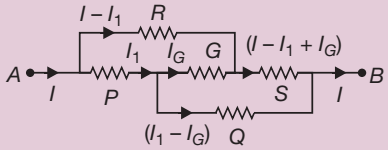
So, to conclude

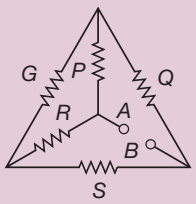
(a) When battery and galvanometer arms of a Wheatstone's Bridge are interchanged, the balance position remains undisturbed while sensitivity of bridge changes.

(b) When Wheatstone's Bridge is balanced, the resistance in arm *BD* may be ignored while calculating the equivalent resistance of bridge between *A* and *C*. So, *P* and *Q* are in series, *R* and *S* are in series and both in parallel to each other.

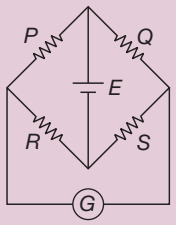
$$\Rightarrow \quad \frac{1}{R_{net}} = \frac{1}{P+Q} + \frac{1}{R+S}$$

Three other common forms of balanced Wheatstone's Bridge are shown here.





and



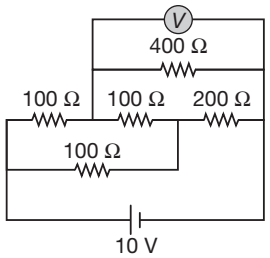
(c) If the bridge is not balanced, then the current will flow

(i) from *D* to *B* if $\frac{P}{Q} > \frac{R}{S}$

(ii) from *B* to *D* if $\frac{P}{Q} < \frac{R}{S}$

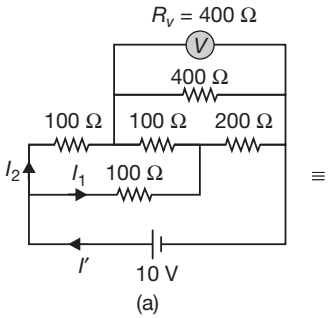
ILLUSTRATION 54

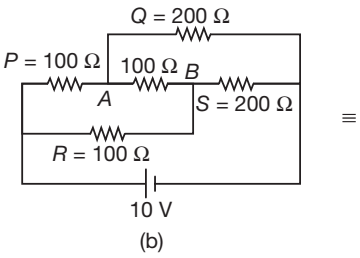
An electrical circuit is shown in figure. Calculate the potential difference across the resistor of $400\ \Omega$ as will be measured by the voltmeter V of resistance $400\ \Omega$ either by applying Kirchhoff's rules or otherwise.



SOLUTION

The given circuit actually forms a balanced Wheatstone bridge (including the voltmeter) as shown in figure.





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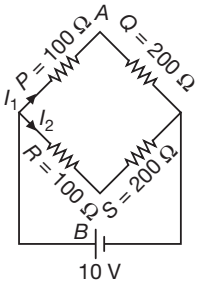


Here, we see that $\frac{P}{Q} = \frac{R}{S}$ {Bridge is balanced}

Therefore, resistance between A and B can be ignored and equivalent simple circuit can be drawn as shown in figure (c).

The voltmeter will read the potential difference across resistance Q. So, currents I_1 and I_2 are given by

$$I_1 = I_2 = \frac{10}{100+200} = \frac{1}{30} \text{ A}$$

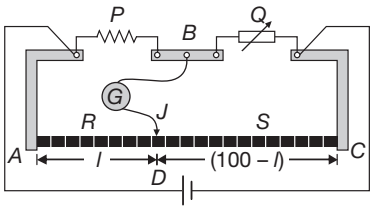
$\Rightarrow$ Potential difference across voltmeter is

$$\Delta V = I_1 Q = (200)\left(\frac{1}{30}\right) \text{V} = \frac{20}{3} \text{ V}$$

So, reading of voltmeter is $\frac{20}{3}$ V

THE METRE BRIDGE

The metre bridge is the practical application of the Wheatstone network principle in which the ratio of two of the resistances, say R and S, is deduced from the ratio of their balancing lengths. AC is a 1 m long uniform wire. If $AD = l$ cm, then $DC = (100 - l)$ cm
Since Resistance $\propto$ Length



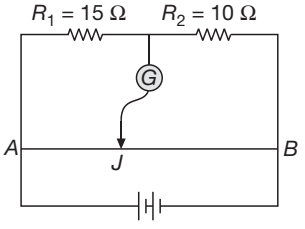
$$\Rightarrow \quad \frac{P}{Q} = \frac{l}{100-l}$$

If P is known then Q can be determined.

At the balancing point galvanometer G gives no deflection at all. At A and C, the galvanometer must have deflections in opposite direction because, then only zero deflection can be expected when the jockey (J) attached to the galvanometer is moved from A to C.

ILLUSTRATION 55

In the meter bridge circuit shown in figure. Calculate the length AJ for null deflection in galvanometer.



SOLUTION

Let $AJ = l$ cm, then $JB = (100 - l)$ cm
At zero deflection of Galvanometer

$$\frac{R_1}{R_2} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100-l}$$

$$\Rightarrow \quad \frac{15}{10} = \frac{l}{100-l}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{l}{100-l}$$

$$\Rightarrow \quad 300 - 3l = 2l$$

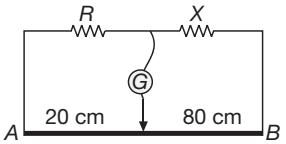
$$\Rightarrow \quad 5l = 300$$

$$\Rightarrow \quad l = 60 \text{ cm}$$

ILLUSTRATION 56

In a meter bridge, null point is found to be at 20 cm. When the known resistance R is shunted by 10 Ω resistance, null point is found to be shifted by 10 cm. Find the unknown resistance X.

SOLUTION



$$\frac{R}{20} = \frac{X}{80}$$

$$\Rightarrow \quad \frac{R}{X} = \frac{1}{4} \qquad \ldots(1)$$

Now shunting R by 10 Ω, i.e., 10 Ω connected in parallel to R, then new resistance becomes $\frac{10R}{10+R}$ (which is less than R) so new balance length is $20 - 10 = 10$ cm. Hence

$$\frac{\left(\frac{10R}{10+R}\right)}{10} = \frac{X}{90}$$

$$\Rightarrow \quad \frac{10}{10+R} = \frac{4}{9}$$

$$\Rightarrow \quad 90 = 40 + 4R$$

$$\Rightarrow \quad R = 12.5 \text{ Ω}$$

$$\Rightarrow \quad X = 4R = 50 \text{ Ω}$$

ILLUSTRATION 57

If resistance R_1 in resistance box is $300\ \Omega$, then the balanced length is found to be 75 cm from end A. The diameter of unknown wire is 1 mm and length of the unknown wire is 31.4 cm. Find the specific resistance of the unknown wire.

SOLUTION

Since, $\frac{R}{X}=\frac{l}{100-l}$

$\Rightarrow \quad X=\left(\frac{100-l}{l}\right)R$

$\Rightarrow \quad X=\left(\frac{100-75}{75}\right)(300)=100\ \Omega$

where $X=\frac{\rho l}{A}=\frac{\rho l}{\left(\frac{\pi d^2}{4}\right)}$

$\Rightarrow \quad \rho=\frac{\pi d^2 X}{4l}$

$\Rightarrow \quad \rho=\frac{\left(\frac{22}{7}\right)\left(10^{-3}\right)^2(100)}{(4)(0.314)}$

$\Rightarrow \quad \rho=2.5\times10^{-4}\ \Omega\text{m}$

END CORRECTIONS IN METRE BRIDGE

In meter bridge, some extra length (under the metallic strips) comes at points A and C. Therefore, some additional length x_1 and x_2 should be included at the ends. So, x_1 and x_2 are called the end corrections. Hence, in place of l we use $l+x_1$ and in place of $(100-l)$ we use $100-l+x_2$. To find x_1 and x_2, we use known resistors R_1 and R_2 in place of R and X and suppose we get null point length equal to l_1 Then,

$$\frac{R_1}{R_2}=\frac{l_1+x_1}{100-l_1+x_2} \qquad \ldots(1)$$

Now, we interchange the positions of R_1 and R_2 and suppose the new null point length is l_2. Then,

$$\frac{R_2}{R_1}=\frac{l_2+x_1}{100-l_2+x_2} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$x_1=\frac{R_2l_1-R_1l_2}{R_1-R_2}$$

and $\quad x_2=\frac{R_1l_1-R_2l_2}{R_1-R_2}-100$

ILLUSTRATION 58

When we use $100\ \Omega$ and $200\ \Omega$ in place of R and X we get null point deflection at $l=33$ cm. On interchanging the resistors, the null point length is found to be 67 cm. Calculate the end corrections x_1 and x_2.

SOLUTION

$$x_1=\frac{R_2l_1-R_1l_2}{R_1-R_2}$$

$\Rightarrow \quad x_1=\frac{(200)(33)-(100)(67)}{100-200}$

$\Rightarrow \quad x_1=1$ cm

$$x_2=\frac{R_1l_1-R_2l_2}{R_1-R_2}-100$$

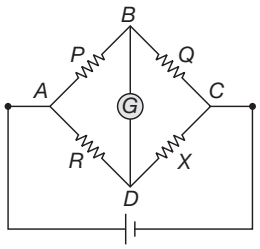
$\Rightarrow \quad x_2=\frac{(100)(33)-(200)(67)}{100-200}-100$

$\Rightarrow \quad x_2=1$ cm

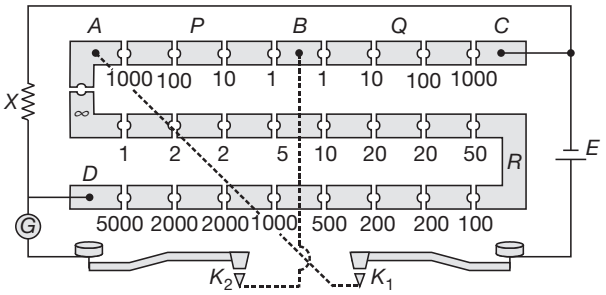
POST OFFICE BOX

Post office box also works on the principle of Wheatstone's bridge. In a Wheatstone's bridge circuit, when $\frac{P}{Q}=\frac{R}{X}$, then the bridge is said to be balanced. So, unknown resistance X is given by

$$X=\frac{Q}{P}R$$



It is observed that P and Q are set in arms AB and BC, where we can have the ratio $\frac{Q}{P}$ can be set by using $10\ \Omega$, $100\ \Omega$ or $1000\ \Omega$ resistances.



These arms are called ratio arm, initially we take $Q = 10\ \Omega$ and $P = 10\ \Omega$ to set $\frac{Q}{P} = 1$. The unknown resistance (X) is connected between C and D, the battery is connected across A and C.

Now, adjust resistance in part A to D such that the bridge gets balanced. For this, keep on increasing the resistance with $1\ \Omega$ interval, check the deflection in galvanometer by first pressing key K_1 then galvanometer key K_2.

Suppose at $R = 4\ \Omega$, we get deflection towards left and at $R = 5\ \Omega$, we get deflection towards right. Then, we can say that for balanced condition, R should lie between $4\ \Omega$ to $5\ \Omega$.

Now, $X = \frac{Q}{P}R = \frac{10}{10}R = R = 4\ \Omega$ to $5\ \Omega$.

To get closer value of X, in the second observation, let us choose $\frac{Q}{P} = \frac{1}{10}$ i.e., $\left(\frac{P = 100}{Q = 10}\right)$

Suppose, now at $R = 42$ we get deflection towards left and at $R = 43\ \Omega$ deflection is towards right.

So R lies between $42\ \Omega$ and $43\ \Omega$

Now, $X = \frac{Q}{P}R = \frac{10}{100}R = \frac{1}{10}R$

$\Rightarrow$ R lies between $4.2\ \Omega$ and $4.3\ \Omega$

Now, to get further closer value take $\frac{Q}{P} = \frac{1}{100}$ and so on.

The observation table is shown below

S. No.	Resistance in ratio arm		Resistance in arm *AD* (is *R* ohm)	Direction of deflection	Unknown resistance $X = \frac{Q}{P} \times R$ (ohm)
	AB (is *P* ohm)	*BC* (is *Q* ohm)			
1.	10	10	4	Left	4 to 5
			5	Right	
2.	100	10	40	Left (large)	4.2 to 4.3
			50	Right (large)	
			42	Left	
			43	Right	
3.	1000	10	420	Left	4.25
			424	Left	
			425	No deflection	
			426	Right	

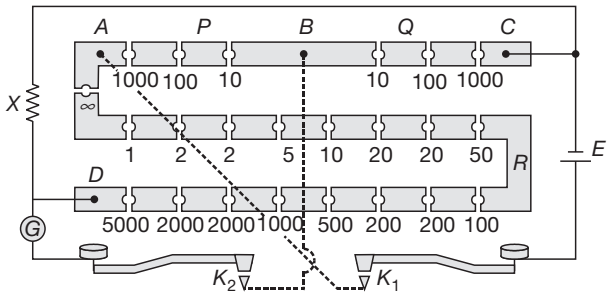
So, the correct value of X is $4.25\ \Omega$

Conceptual Note(s)

To locate the null point, deflection battery key (K_1) is pressed slightly earlier than the galvanometer key (K_2) because if galvanometer key K_2 is pressed first, then just after closing the battery key K_1, the current suddenly increases and due to self-induction, a large back emf is generated in the galvanometer, which may damage the galvanometer.

ILLUSTRATION 59

Calculate the maximum and minimum values of unknown resistance X, which can be determined using the post office box shown in the figure?



SOLUTION

$$X = \frac{QR}{P}$$

So, maximum value of x is given by

$$X_{max} = \frac{Q_{max} R_{max}}{P_{min}}$$

$$\Rightarrow \quad X_{max} = \frac{1000}{10}(11110)$$

$$\Rightarrow \quad X_{max} = 1111\ \text{k}\Omega$$

$$X_{min} = \frac{Q_{min} R_{min}}{P_{max}}$$

$$\Rightarrow \quad X_{min} = \frac{(10)(1)}{1000}$$

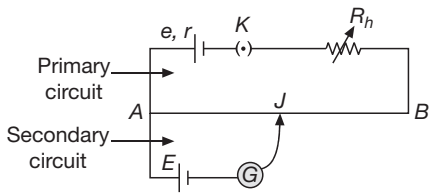
$$\Rightarrow \quad X_{min} = 0.01\ \Omega$$

POTENTIOMETER

Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

Circuit Diagram

Potentiometer consists of a long resistive wire AB of length L (about 6 m to 10 m long) made up of manganin or constantan and a battery of known voltage e and internal resistance r called supplier battery or driver cell. Connection of these two forms primary circuit.



J = Jockey
K = Key
$R_{AB} = R$ = Resistance of potentiometer wire
ρ = Specific resistance of potentiometer wire
R_h = Variable resistance which controls the current through the wire AB

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows :

(a) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(b) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(c) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(d) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not slide while in contact with the wire.

(e) The diameter of potentiometer wire must be uniform everywhere.

Potential Gradient (x)

Potential difference (or fall in potential) per unit length of wire is called potential gradient i.e.,

$$x = \frac{V}{L}\frac{\text{volt}}{\text{m}} \quad \text{where}$$

$$V = IR_{AB} = \left(\frac{e}{R + R_h + r}\right)R$$

So, potential gradient x is given by

$$x = \frac{V}{L} = \frac{IR}{L} = \frac{I\rho}{A} = \frac{e}{(R + R_h + r)}\frac{R}{L}$$

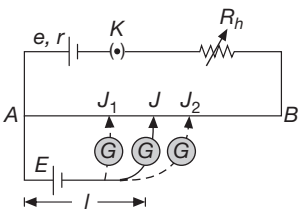
(a) The potential gradient directly depends upon the
- (i) resistance per unit length $\left(\frac{R}{L}\right)$ of potentiometer wire.
- (ii) radius of potentiometer wire (i.e., Area of cross-section)
- (iii) specific resistance of the material of potentiometer wire (i.e., ρ)
- (iv) current flowing through potentiometer wire (I)

(b) Potential gradient indirectly depends upon the
- (i) emf of battery in the primary circuit (i.e., e)
- (ii) resistance of rheostat in primary circuit (i.e., R_h)

Working

Suppose the jockey is made to touch a point J on wire. Then potential difference between A and J will be $V = xl$. At this length (l), two potential difference values are obtained.

(a) V due to battery e and
(b) E due to unknown cell



If $V > E$, then current will flow in galvanometer circuit in one direction, shown as

If $V < E$, then current will flow in galvanometer circuit in opposite direction, shown as

If $V = E$, then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length, shown as

In balanced condition $E = xl$

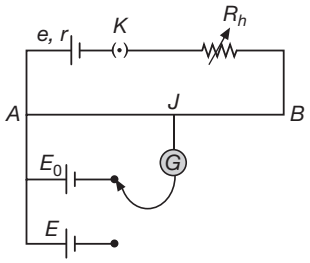
$$\Rightarrow \quad E = xl = \frac{V}{L}l = \frac{IR}{L}l = \left(\frac{e}{R + R_h + r}\right)\frac{R}{L} \times l$$

If V is constant then $L \propto l$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{L_1}{L_2} = \frac{l_1}{l_2}$$

Standardization of Potentiometer

The process of determining potential gradient experimentally is known as **standardization of potentiometer**.



Let the balancing length for the standard emf E_0 is l_0, then by the principle of potentiometer we have $E_0 = xl_0$

$$\Rightarrow \quad x = \frac{E_0}{l_0}$$

Sensitivity of Potentiometer

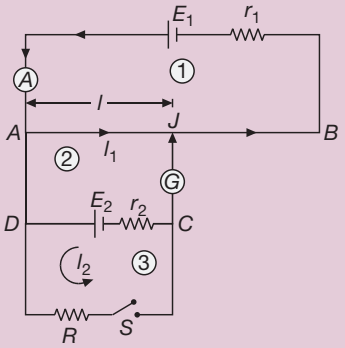
A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(a) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(b) In order to increase the sensitivity of potentiometer, the
- (i) resistance in primary circuit will have to be decreased.
- (ii) length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

Conceptual Note(s)

(a) In the potentiometer arrangement shown,



under balanced condition (when $I_G = 0$) Loop ① and Loop ③ are independent of each other. All problems in this condition can be solved by a single equation. i.e.,

$$V_{AJ} = V_{DC}$$

$$\Rightarrow \quad I_1 R_{AJ} = E_2 - I_2 r_2$$

$$\Rightarrow \quad I_1 \lambda \ell = E_2 - I_2 r_2 \qquad \ldots(1)$$

where, λ is the resistance per unit length of potentiometer wire AB. The length ℓ is called the balance point length. Currents I_1 and I_2 are independent with each other. Current $I_2 = 0$, if switch is open.

(b) From equation (1), the null point length is given by

$$\ell = \frac{E_2 - I_2 r_2}{I_1 \lambda}$$

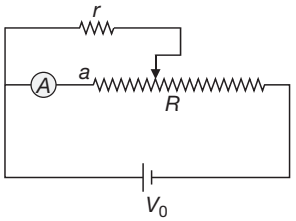
Now, suppose E_1 is increased then I_1 will also increase and null point length ℓ will decrease.

(c) Under balanced condition, a part of potential difference of E_1 is balanced by the lower circuit. So, normally $E_2 < E_1$ for taking balance point length. Similarly, we observe that $V_{AJ} = V_{DC}$. Also, $V_A = V_D$ and $V_J = V_C$, so we conclude that positive terminals of both batteries should be on same side and negative terminals on the other side.

(d) If we do not get any balanced condition $(I_G \neq 0)$, then the given circuit is simply a three loop problem, which can be solved by applying Kirchhoff's Laws.

ILLUSTRATION 60

A constant voltage V_0 is applied to a potentiometer of resistance R connected to an ammeter. A constant resistor r is connected to the sliding contact of the potentiometer and the fixed end of the potentiometer. How will the reading of ammeter vary as the sliding contact is moved from one end of the potentiometer to the other. The resistance of ammeter is assumed to be negligible.



SOLUTION

If x is the resistance of the potentiometer between point a and the sliding contact, the total resistance between a and the sliding contact is $\frac{rx}{r+x}$, while the resistance of the entire circuit is $(R-x)+\frac{rx}{r+x}$. The current supplied by the source is

$$I = \frac{V_0}{R - x + \frac{rx}{r+x}}$$

The potential difference between the sliding contact and point a is

$$V = \frac{V_0 rx}{(R-x)(r+x)+rx} = \frac{V_0 rx}{Rx - x^2 + Rr}$$

The current passing through ammeter is,

$$I_a = \frac{V_0 r}{Rx - x^2 + Rr} \qquad \ldots(1)$$

To find the extremum (maximum, minimum or constant)

$$\frac{dI_a}{dx} = 0$$

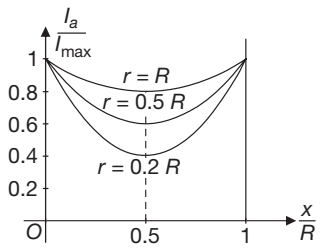
$$\Rightarrow \quad V_0 r\left[\frac{-R+2x}{(Rx - x^2 + Rr)^2}\right] = 0$$

$$\Rightarrow \quad x = \frac{R}{2}$$

Substituting x in equation (1) we find the minimum current

$$I_{\min} = \frac{V_0 r}{R\left(r + \frac{R}{4}\right)}$$

Thus, as the sliding contact is moved the current through the ammeter passes through a minimum and the smaller the r the deeper the minimum. At $x = 0$ and $x = R$, a current of $I_{\max} = \frac{V_0}{R}$ passes through the ammeter. The $\frac{I_a}{I_{\max}}$ versus $\frac{x}{R}$ curves for several values of $\frac{r}{R}$ are shown in figure.



APPLICATIONS OF POTENTIOMETER

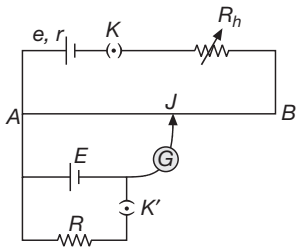
To Determine the Internal Resistance of a Primary Cell

(a) Initially, in the secondary circuit the key K' remains open and balancing length $l_1 = l_{\text{key open}}$ is obtained. Since cell E is in open circuit so it's emf balances for a length l_1 i.e.,

$$E = xl_1 \qquad \ldots(1)$$

(b) Now key K' is closed so cell E comes in closed circuit. If the process of balancing repeated again then potential difference V balances on length $l_2 = l_{\text{key closed}}$ i.e.,

$$V = xl_2 \qquad \ldots(2)$$



(c) Since we know that the internal resistance is given by

$$r = \left(\frac{E}{V} - 1\right)R$$

$$\Rightarrow \quad r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R$$

ILLUSTRATION 61

In a potentiometer circuit to find the internal resistance r of the cell of emf E, when K is open, the balance point is obtained at 60 cm. When K is closed, the balance point is obtained at 50 cm. What is the value of r? If the cell is shorted by a $10\ \Omega$ resistor, then calculate the internal resistance of the cell.

SOLUTION

$l_1 = 60$ cm, when K is open and
$l_2 = 50$ cm, when K is closed

Since $r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R$

$$\Rightarrow \quad r = \left(\frac{l_1}{l_2} - 1\right)R = \left(\frac{60}{50} - 1\right)10\ \Omega = \frac{1}{5} \times 10\ \Omega = 2\ \Omega$$

ILLUSTRATION 62

When a resistor of $5\ \Omega$ is connected across a cell its terminal potential difference is balanced by 150 cm of potentiometer wire and when a resistor of $10\ \Omega$ is connected across the cell, then the terminal potential difference is balanced by 175 cm of the potentiometer wire. Calculate the internal resistance of the cell.

SOLUTION

Since the same cell is used in both cases, so $l_{\text{key open}} = l$ will remain the same for both.
For the first case, we have $l_{\text{key closed}} = l_1$ and $R = R_1$. So

$$r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R = \left(\frac{l}{l_1} - 1\right)R_1 \qquad \ldots(1)$$

For the second case, we have $l_{\text{key closed}} = l_2$ and $R = R_2$. So

$$r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R = \left(\frac{l}{l_2} - 1\right)R_2 \qquad \ldots(2)$$

From (1), we get

$$r\left(\frac{l_1}{R_1}\right)=l-l_1 \qquad \ldots(3)$$

From (2), we get

$$r\left(\frac{l_2}{R_2}\right)=l-l_2 \qquad \ldots(4)$$

Subtracting (4) from (3), we get

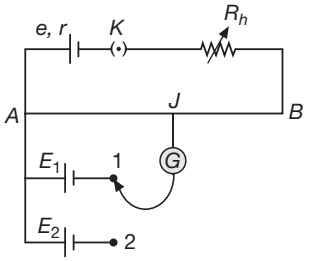
$$r\left(\frac{l_1}{R_1}-\frac{l_2}{R_2}\right)=l_2-l_1$$

$$\Rightarrow \quad r=\frac{l_2-l_1}{\left(\frac{l_1}{R_1}-\frac{l_2}{R_2}\right)}=\frac{175-150}{\frac{150}{5}-\frac{175}{10}}=\frac{25}{12.5}=2$$

$$\Rightarrow \quad r=2\ \Omega$$

Comparison of EMF's of Two Cell

Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1=xl_1$ and $E_2=xl_2$



$$\Rightarrow \quad \frac{E_1}{E_2}=\frac{l_1}{l_2}$$

Let $E_1>E_2$ and both are connected in series. If balancing length is l_1, when cells assist each other and is l_2, when they oppose each other (as shown), then

$$(E_1+E_2)=xl_1$$

$$(E_1-E_2)=xl_2$$

$$\Rightarrow \quad \frac{E_1+E_2}{E_1-E_2}=\frac{l_1}{l_2}$$

$$\Rightarrow \quad \frac{E_1}{E_2}=\frac{l_1+l_2}{l_1-l_2}$$

ILLUSTRATION 63

In a potentiometer arrangement a cell of emf 1.5 V gives a balance point at 30 cm length of wire. Now, when the cell is replaced by another cell, the balance point shifts to 50 cm. Calculate the emf of second cell?

SOLUTION

$E_1=1.5$ V, $l_1=30$ cm

$E_2=?$, $l_2=50$ cm

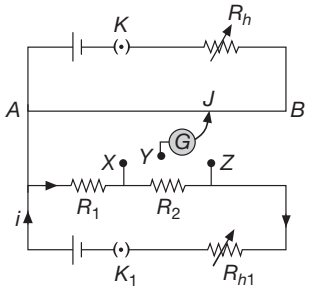
Using the formula for comparison of emf of cells by potentiometer, we have

$$\frac{E_2}{E_1}=\frac{l_2}{l_1}$$

$$\Rightarrow \quad E_2=\frac{l_2}{l_1}\times E_1=\frac{50}{30}\times 1.5\ \text{V}=2.5\ \text{V}$$

Comparison of Resistances

Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance R_1+R_2 (when YZ is connected) is l_2.



Then $IR_1=xl_1$ and $I(R_1+R_2)=xl_2$

$$\Rightarrow \quad \frac{R_2}{R_1}=\frac{l_2-l_1}{l_1}$$

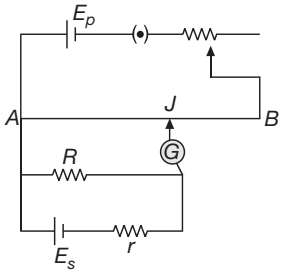
ILLUSTRATION 64

One of the circuits for the measurement of resistance using a potentiometer is shown. The galvanometer is connected and zero deflection is observed at length $AJ=30$ cm. In second case the secondary cell is changed.

Take $E_S=10$ V and $r=1\ \Omega$ in 1st reading

and $E_S=5$ V and $r=2\ \Omega$ in 2nd reading.

In second case, the zero deflection is observed at length $AJ = 10$ cm. Calculate the resistance R (in ohm).



SOLUTION

At zero deflection, we have

$$\frac{V}{l} = \frac{I_1 R}{30} = \frac{I_2 R}{10} \qquad \ldots(1)$$

Now $I_1 = \dfrac{E_{S_1}}{r_1 + R}$

$\Rightarrow \quad I_1 = \dfrac{10}{1+R}$

$$I_2 = \frac{E_{S_2}}{r_2 + R}$$

$\Rightarrow \quad I_2 = \dfrac{5}{2+R}$

Substituting in (1), we get

$$\frac{10R}{(1+R)\times 30} = \frac{5R}{(2+R)\times 10}$$

$\Rightarrow \quad 4 + 2R = 3 + 3R$

$\Rightarrow \quad R = 1\ \Omega$

Calibration of Ammeter

Checking the correctness of ammeter readings with the help of potentiometer is called **calibration of ammeter.**

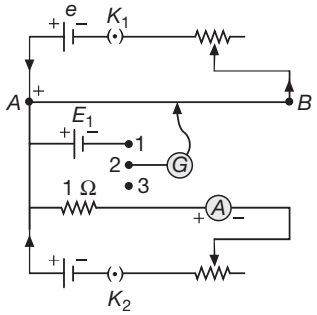
(a) In the process of calibration of an ammeter the current flowing in a circuit is measured by an ammeter and the same current is also measured with the help of potentiometer. By comparing both the values, the error(s) in the ammeter readings can be determined.

(b) For the calibration of an ammeter, $1\ \Omega$ standard resistance coil is specifically used in the secondary circuit of the potentiometer, because the potential difference across $1\ \Omega$ is equal to the current flowing through it. So, from Ohm's Law we get $V = I$

(c) If the balancing length for the emf E_0 is l_0 then

$$E_0 = xl_0$$

$\Rightarrow \quad x = \dfrac{E_0}{l_0}$ (Process of standardisation)



(d) Let I' be the current that flows through $1\ \Omega$ resistance, thus giving the potential difference as $V' = I'(1) = xl_1$ where l_1 is the balancing length. So error can be found as

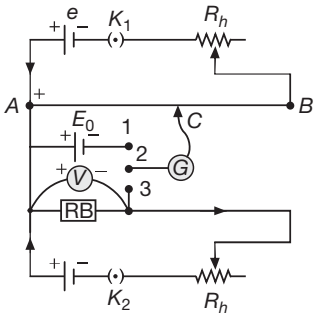
$$\Delta I = I - I' = I - xl_1 = I - \left(\frac{E_0}{l_0}\right) l_1$$

Calibration of Voltmeter

(a) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of potential difference by potentiometer.

(b) If l_0 is balancing length for E_0, the emf of standard cell (by connecting 1 and 2 of bi-directional key), then $x = \dfrac{E_0}{l_0}$.

(c) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3), then $V' = xl_1 = \left(\dfrac{E_0}{l_0}\right) l_1$



If the voltmeter reading is V then the error will be $(V - V')$ which may be positive, negative or zero.

DIFFERENCE BETWEEN VOLTMETER AND POTENTIOMETER

Voltmeter	Potentiometer
It's resistance is high but finite.	It's resistance is infinite.
It draws some current from source of emf.	It does not draw any current from the source of unknown emf.
The potential difference measured by it is lesser than the actual potential difference.	The potential difference measured by it is equal to actual potential difference.
Its sensitivity is low.	Its sensitivity is high.
It is a versatile instrument.	It measures only emf or potential difference.
It is based on deflection method.	It is based on zero deflection method.

ILLUSTRATION 65

A cell having a steady emf of 2 V is connected across the potentiometer wire of length 10 m. The potentiometer wire is of manganin and having resistance of $11.5\ \Omega m^{-1}$. An another cell gives a null point at 6.9 m. If a resistance of $5\ \Omega$ is put in series with the potentiometer wire, find the new position of the null point.

SOLUTION

Length of potentiometer wire $= 10$ m

So, fall in potential per meter (also known as potential gradient) $= \frac{2}{10} = 0.2\ Vm^{-1}$

Since the cell gives a null point at 6.9 m, hence its emf is,

$$E = (0.2)(6.9) = 1.38\ V$$

Resistance of potentiometer wire $= 11.5 \times 10 = 115\ \Omega$. When a resistance of $5\ \Omega$ is connected, total resistance becomes $R_{net} = 115 + 5 = 120\ \Omega$ and

$$\text{Current, } I = \frac{2}{120}\ A$$

So, new potential gradient = (current) (resistance per unit length)

$$\Rightarrow \quad \text{Potential Gradient} = \left(\frac{2}{120}\right)(11.5) = \frac{23}{120}\ Vm^{-1}$$

Let l' be the position of new null point, then

$$\left(\frac{23}{120}\right) l' = E = 1.38$$

$$\Rightarrow \quad l' = \frac{1.38 \times 120}{23} = 7.2\ m$$

ILLUSTRATION 66

In a potentiometer circuit, the emf of driver cell is $2\ V$ and internal resistance is $0.5\ \Omega$. The potentiometer wire is 1 m long. It is found that a cell of emf $1\ V$ and internal resistance $0.5\ \Omega$ is balanced against $60\ cm$ length of the wire. Calculate the resistance of potentiometer wire.

SOLUTION

Let resistance of potentiometer wire be $R_{AB} = R$. At balance, we have

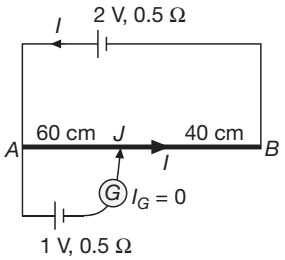
$$I = \frac{2}{R + 0.5}$$

$$\Rightarrow \quad V_{AB} = IR_{AB} = \frac{2R}{R + 0.5} \qquad \ldots(1)$$

Since, $V_{AJ} = 1\ V$ $\{\because$ There is no deflection$\}$

Across $60\ cm$ length of potentiometer wire, potential difference is $1\ V$

So, across 1 cm of potentiometer wire, potential difference is $\frac{1}{60}$ V



Hence, across 100 cm of potentiometer wire, potential difference is

$$V_{AB} = \left(\frac{1}{60}\right)100 = \frac{5}{3} \text{ V} \qquad \ldots(2)$$

Equating (1) and (2), we get

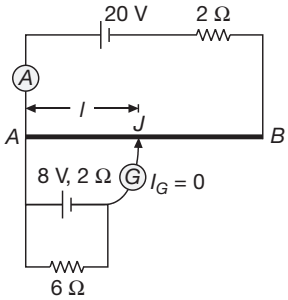
$$\frac{5}{3} = \frac{2R}{R+0.5}$$

$$\Rightarrow \quad 5R + 2.5 = 6R$$

$$\Rightarrow \quad R = 2.5\ \Omega$$

ILLUSTRATION 67

In the figure shown, wire AB has a length of 100 cm and resistance 8 Ω. Find the balance point length l.

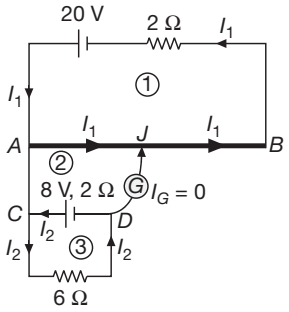


SOLUTION

For condition of balance, we have $I_G = 0$. So loops ① and ③ are independent of each other. Hence we observe that

$$V_{AJ} = V_{CD}$$

$$\Rightarrow \quad I_1 R_{AJ} = E - I_2 r \qquad \ldots(1)$$



where $E = 8$ V, $r = 2\ \Omega$

$$I_1 = \left(\frac{20}{2+8}\right) = 2 \text{ A}$$

$$R_{AJ} = \left(\frac{R_{AB}}{l_{AB}}\right) l_{AJ} = \left(\frac{8}{100}\right) l = \frac{2l}{25}$$

$$I_2 = \frac{8}{2+6} = 1 \text{ A}$$

Substituting these values in equation (1), we get

$$2\left(\frac{2l}{25}\right) = 8 - (1)(2)$$

$$\Rightarrow \quad \frac{4l}{25} = 6$$

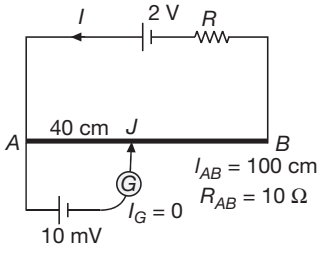
$$\Rightarrow \quad 4l = 150$$

$$\Rightarrow \quad l = \frac{150}{4} = 37.5 \text{ cm}$$

ILLUSTRATION 68

A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance R and a cell of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of R?

SOLUTION



For the condition of balance i.e., when $I_G = 0$, we have

$$I = \frac{2}{R + R_{AB}} = \frac{2}{R + 10}$$

$$\Rightarrow \quad V_{AB} = IR_{AB} = \left(\frac{20}{R+10}\right)V$$

Since $V_{AJ} = 10 \text{ mV} = 10 \times 10^{-3} \text{ V} = 10^{-2} \text{ V}$ …(1)

For a length of 100 cm, potential difference across AB is

$$V_{AB} = \frac{20}{R+10}$$

For a length of 1 cm, potential difference is

$$= \left(\frac{20}{R+10}\right)\frac{1}{100}$$

For a length of 40 cm, potential difference is

$$V_{AJ} = \frac{20}{100(R+10)} \times 40$$

$$\Rightarrow \quad \left[\frac{20}{10(R+10)}\right]40 = 10^{-2} \qquad \{\because \text{ of (1)}\}$$

$$\Rightarrow \quad 800 = R + 10$$

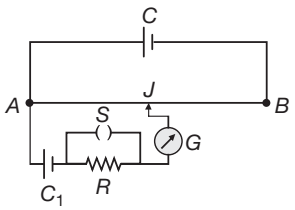
$$\Rightarrow \quad R = 790 \ \Omega$$

Test Your Concepts-X

Based on Wheatstone Bridge and Potentiometer

(Solutions on page H.226)

1. Figure shows a potentiometer using a cell C of emf 2 V and internal resistance 0.4 Ω connected to a resistor wire AB. A standard cell of constant emf of 1.02 V gives a balance point at 67.3 cm length of the wire. A very high resistance R = 600 kΩ is put in series with the standard cell. This resistance is shorted by inserting switch S when close to the balance point. The standard cell is then replaced by a cell of unknown emf E and the null point turns out to be 82.3 cm length of the wire.



(a) What is the value of E?

(b) What is the purpose of using the high resistance R?

(c) Is the null point affected by this high resistance?

(d) Is the null point affected by the internal resistance of the cell C?

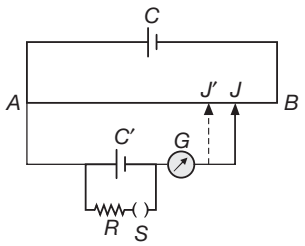
(e) Would this method work if:

(i) the internal resistance of cell C were higher than the resistance of wire AB and

(ii) the emf of cell C were 1 V instead of 2 V?

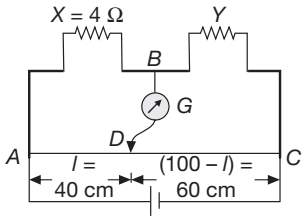
(f) Would the circuit work well for determining extremely small emfs of the order of a few millivolts? What modification do you suggest in the circuit.

2. Figure shows a potentiometer circuit for determining the internal resistance of a cell. When switch S is open, the balance point is found to be at 76.3 cm of the wire. When switch S is closed and the value of R is 4 Ω, the balance point shifts to 60 cm. Find the internal resistance of cell C'.



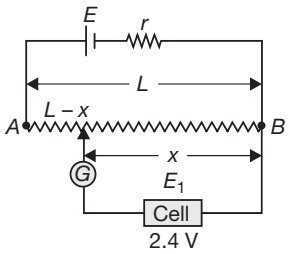
3. Figure shows a metre bridge consisting of two resistances X and Y together in parallel with a metre-long constantan wire AC of uniform cross-section. D is a movable contact that can slide along the wire AC. The resistors X, Y and resistances of segments AD and DC of the wire constitute the four arms of the bridge. The length of wire AC is 100 cm. X is a standard 4 Ω resistor and Y is a coil of wire. With Y immersed in melting ice the null point is found to be at a distance of 40 m from point A. When the

coil Y is heated to 100°C, a 100 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil.

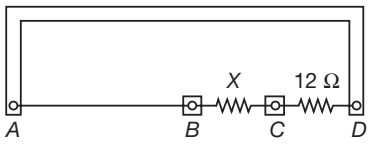


4. In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of the cell are connected across 0.52 m of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω a balance is obtained when the cell is connected across 0.4 m of the wire. Find the internal resistance of the cell.

5. A battery of emf 4 V is connected across a 10 m long potentiometer wire having a resistance per unit length 1.6 Ωm^{-1}. A cell of emf 2.4 V is connected so that its negative terminal is connected to the low potential end of the potentiometer wire and the other end is connected through a galvanometer to a sliding contact along the wire. It is found that the no-deflection point occurs against the balancing length of 8 m. Calculate the internal resistance of the 4 V battery.



6. A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.

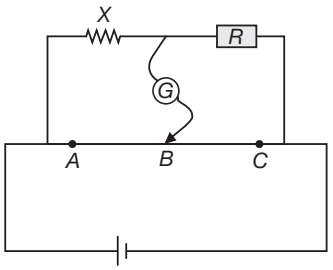


(a) Are there positive and negative terminals on the galvanometer?

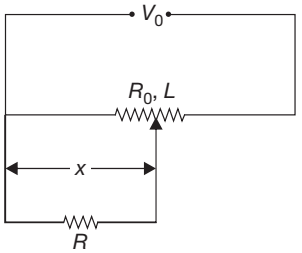
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

7. R_1, R_2, R_3 are different values of R. A, B, C are the null points obtained corresponding to R_1, R_2 and R_3 respectively. For which resistor, the value of X will be the most accurate and why?

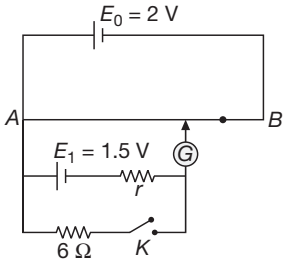


8. Figure illustrates potentiometer circuit by means of which we can vary a voltage V applied to a certain device possessing a resistance R. The potentiometer wire has a length L and a resistance R_0. A voltage V_0 is applied to the terminals of wire. Find the voltage V fed to the device as a function of distance x. Analyse the case $R \gg R_0$.



9. For the potentiometer arrangement to measure the internal resistance of a cell, when the switch is open in lowermost loop the balance point length is 60 cm. When the switch is closed with a known resistance of $R = 4\ \Omega$, the balance point length decreases to 40 cm. Calculate the internal resistance of the unknown battery.

10. For the potentiometer arrangement shown in the figure, the balance point is obtained at a distance 75 cm from *A* when the key *K* is open.



The second balance point is obtained at 60 cm from *A* when the key *K* is closed. Find the internal resistance (in Ω) of the battery E_1.

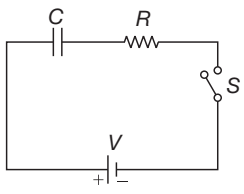
RC CIRCUIT AND APPLICATIONS

RC CIRCUIT

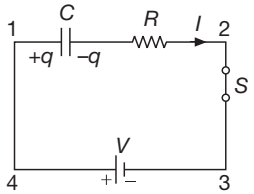
Charging a Capacitor

Consider the circuit shown below. The capacitor is connected to a DC voltage source of voltage V. At time $t = 0$, the switch S is closed. The capacitor initially is uncharged,

$$q(\text{initial}) = q(\text{at } t = 0) = 0$$



(a)
RC circuit diagram for $t < 0$



(b)
Circuit diagram for $t < 0$

In particular, for $t < 0$, there is no voltage across the capacitor so the capacitor acts like a short circuit. At $t = 0$, the switch is closed and current begins to flow and has a value

$$I_0 = \frac{V}{R}$$

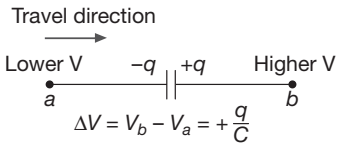
At this instant, the potential difference from the battery terminals is the same as that across the resistor. This initiates the charging of the capacitor. As the capacitor starts to charge, the voltage across the capacitor increases in time. At any instant, the voltage across the capacitor is

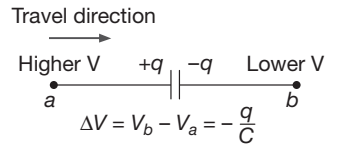
$$V_C(t) = \frac{q(t)}{C}$$

Applying Kirchhoff's loop rule to 12341 shown in figure (b), we obtain

$$V - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad V - R\frac{dq}{dt} - \frac{q}{C} = 0 \qquad \left\{\because I = \frac{dq}{dt}\right\}$$





Kirchhoff's rule for capacitors

Since I must be the same in all parts of the series circuit, the current across the resistance R is equal to the rate of increase of charge on the capacitor plates. The current flow in the circuit will continue to decrease because the charge already present on the capacitor makes it harder to put more charge on the capacitor. Once the charge on the capacitor plates reaches its maximum value q_0 the current in the circuit will drop to zero.

Thus, the charging capacitor satisfies a first order differential equation that relates the rate of change of charge to the charge on the capacitor

$$\frac{dq}{dt} = \frac{1}{R}\left(V - \frac{q}{C}\right)$$

This equation can be solved by the method of separation of variables. The first step is to separate terms involving charge and time, (this means putting terms involving dq and q on one side of the equality sign and terms involving dt on the other side),

$$\frac{dq}{\left(V - \frac{q}{C}\right)} = \frac{1}{R}dt$$

$$\Rightarrow \quad \frac{dq}{CV - q} = \frac{1}{RC}dt$$

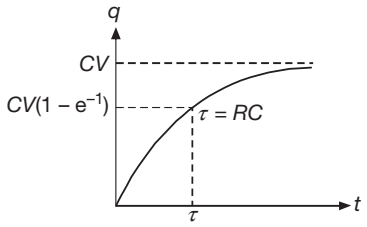
Integrating both sides of the above equation, we get

$$\int_0^q \frac{dq}{CV - q} = \frac{1}{RC}\int_0^t dt$$

$$\Rightarrow \quad \log_e\left(\frac{CV-q}{CV}\right)=-\frac{t}{RC}$$

$$\Rightarrow \quad q(t)=CV\left(1-e^{-t/RC}\right)=q_0\left(1-e^{-t/RC}\right)$$

where $q_0=CV$ is the maximum amount of charge stored on the plates. The time dependence of $q(t)$ is plotted in figure below:



Once we know the charge on the capacitor we also can determine the voltage across the capacitor,

$$V_C(t)=\frac{q(t)}{C}=V\left(1-e^{-t/RC}\right)$$

The graph of voltage as a function of time has the same form as shown in figure. From the figure, we see that after a sufficiently long time the charge on the capacitor approaches the value

$$q(t\to\infty)=CV=q_0$$

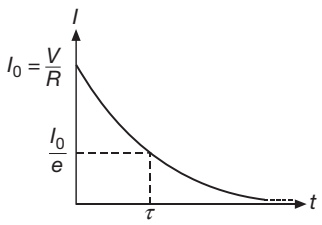
At that time, the voltage across the capacitor is equal to the applied voltage source and the charging process effectively ends,

$$V_C=\frac{q(t\to\infty)}{C}=\frac{q_0}{C}=V \text{ (Battery Voltage)}$$

The current that flows in the circuit is given by

$$I(t)=\frac{dq}{dt}=\left(\frac{V}{R}\right)e^{-t/RC}=I_0e^{-t/RC}$$

The coefficient in front of the exponential (I_0) is equal to the initial current that flows in the circuit when the switch was closed at $t=0$. The graph of current as a function of time is shown in figure below:



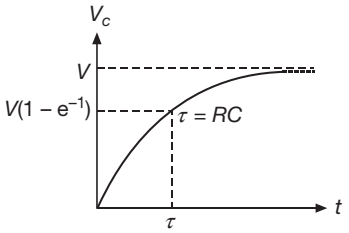
The current in the charging circuit decreases exponentially in time, $I(t)=I_0e^{-t/RC}$. This function is often written as $I(t)=I_0e^{-t/\tau}$ where $\tau=RC$ is called the time constant. The SI unit of τ is second, as can be seen from the dimensional analysis

$$[\tau]=[RC]=[\Omega][F]=\left(\frac{[V]}{[A]}\right)\left(\frac{[C]}{[V]}\right)$$

$$[\tau]=\frac{[C]}{[A]}=\frac{[C]}{[C]/[s]}=[s]$$

The time constant τ is a measure of the decay time for the exponential function. This decay rate satisfies the following property

$$I(t+\tau)=I(t)e^{-1}$$



which shows that after one time constant τ has elapsed, the current falls off by a factor of $e^{-1}=0.368$, as indicated in figure above. Similarly, the voltage across the capacitor (shown in figure) can also be expressed in terms of the time constant τ

$$V_C(t)=V\left(1-e^{-t/\tau}\right)$$

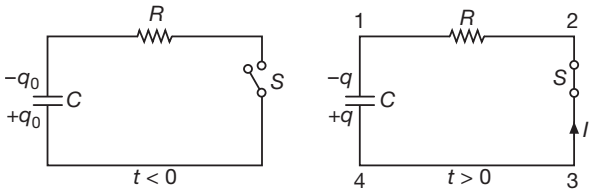
Notice that initially at time $t=0$, $V_C(t=0)=0$.

After one time constant τ has elapsed, the potential difference across the capacitor plates has increased by a factor $\left(1-e^{-1}\right)=0.632$ of its final value

$$V_C(\tau)=V\left(1-e^{-1}\right)=0.632V$$

Discharging a Capacitor

Suppose initially the capacitor has been charged to some value q_0. For $t<0$, the switch is open and the potential difference across the capacitor is given by $V_C=\frac{q_0}{C}$. On the other hand, the potential difference across the resistor is zero because there is no current flow, that is, $I=0$. Now suppose at $t=0$ the switch is closed (shown in figure). The capacitor will begin to discharge.



Discharging the RC circuit

The charged capacitor is now acting like a voltage source to drive current around the circuit. When the capacitor discharges (electrons flow from the negative plate through the wire to the positive plate), the voltage across the capacitor decreases. The capacitor is losing strength as a voltage source. Applying the Kirchhoff's loop rule to 12341, the equation that describes the discharging process is given by

$$IR - \frac{q}{C} = 0$$

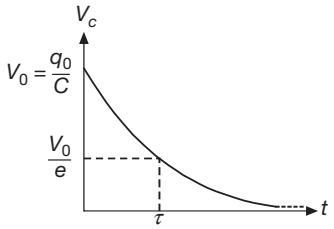
The current that flows away from the positive plate is proportional to the charge on the plate,

$$I = -\frac{dq}{dt}$$

The negative sign in the equation is an indication that the rate of change of the charge is proportional to the negative of the charge on the capacitor. This is due to the fact that the charge on the positive plate is decreasing as more positive charges leave the positive plate. Thus, charge satisfies a first order differential equation

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

This equation when integrated by using the method of separation of variables



$$\frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_{q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

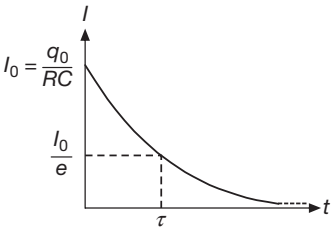
$$\Rightarrow \quad \log_e\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$$

$$\Rightarrow \quad q(t) = q_0 e^{-t/RC}$$

The voltage across the capacitor is then

$$V_C(t) = \frac{q(t)}{C} = \left(\frac{q_0}{C}\right)e^{-t/RC}$$

A graph of voltage across the capacitor vs. time for the discharging capacitor is shown in figure.



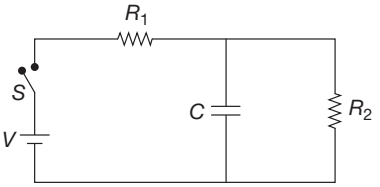
The current also exponentially decays in the circuit as can be seen by differentiating the charge on the capacitor

$$I = -\frac{dq}{dt} = \left(\frac{q_0}{RC}\right)e^{-t/RC}$$

A graph of the current flowing in the circuit as a function of time also has the same form as the voltage graph depicted in figure.

ILLUSTRATION 69

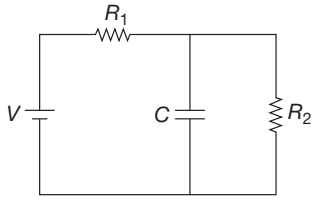
At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_0(1 - e^{-\alpha t})$. Obtain the value of Q_0 and α in the given circuit parameters.



SOLUTION

Q_0 is the steady state charge stored in the capacitor.

$Q_0 = C$ (Potential difference across capacitor in steady state)



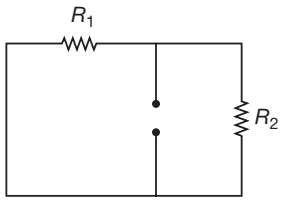
$\Rightarrow \quad Q_0 = C$ (Steady state current through R_2) (R_2)

$$\Rightarrow \quad Q_0 = C\left(\frac{V}{R_1 + R_2}\right)R_2$$

$$\Rightarrow \quad Q_0 = \frac{CVR_2}{R_1 + R_2}$$

Since, $\alpha = \frac{1}{\tau_C} = \frac{1}{CR_{net}}$

where, R_{net} is the equivalent resistance across capacitor after short circuiting the battery. So,

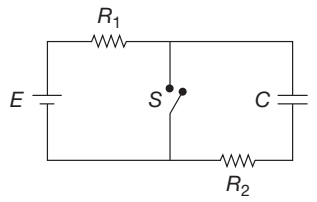


$$R_{net} = \frac{R_1R_2}{R_1 + R_2} \qquad \{\because R_1 \text{ and } R_2 \text{ are in parallel}\}$$

$$\Rightarrow \quad \alpha = \frac{1}{C\left(\frac{R_1R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{CR_1R_2}$$

ILLUSTRATION 70

In the circuit in figure, suppose the switch has been open for a very long time. At time $t = 0$, it is suddenly closed.



(a) What is the time constant before the switch is closed?

(b) What is the time constant after the switch is closed?

(c) Find the current through the switch as a function of time after the switch is closed.

SOLUTION

(a) Before the switch is closed, the two resistors R_1 and R_2 are in series with the capacitor.
Since the equivalent resistance is $R_{eq} = R_1 + R_2$, the time constant is given by

$$\tau = R_{eq}C = (R_1 + R_2)C$$

The amount of charge stored in the capacitor is

$$q(t) = CE(1 - e^{-t/\tau})$$

(b) After the switch is closed, the closed loop on the right becomes a decaying RC circuit with time constant $\tau' = R_2C$. Charge begins to decay according to

$$q'(t) = CEe^{-t/\tau'} = CEe^{-t/R_2C}$$

(c) The current passing through the switch consists of two sources :

(i) the steady current I_1 from the left circuit given by $I_1 = \frac{E}{R_1}$ and

(ii) the decaying current I_2 from the RC circuit, given by

$$I'(t) = \frac{dq'}{dt} = -\left(\frac{CE}{\tau'}\right)e^{-t/\tau'} = -\left(\frac{E}{R_2}\right)e^{-t/R_2C}$$

The negative sign in $I'(t)$ indicates that the direction of flow is opposite of the charging process. Since both I_1 and I' move downward across the switch, the total current is given by

$$I(t) = I_1 + I'(t) = \frac{E}{R_1} + \left(\frac{E}{R_2}\right)e^{-t/R_2C}$$

Problem Solving Technique(s)

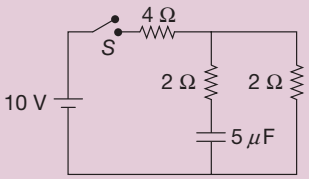
(a) An uncharged capacitor offers zero resistance to the current in a circuit, i.e., the branch containing the uncharged capacitor can be assumed to be short circuited in terms of the capacitance. That is the capacitor can just be thought of being absent initially.

(b) A fully charged capacitor offers an infinite resistance to the current and hence no current will pass through the branch that contains a fully charged capacitor.

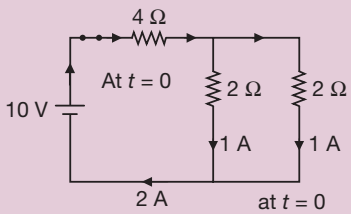
(c) While solving problems, whenever we come across the branch that contains a fully charged capacitor then we can omit that branch to calculate the net resistance of the circuit.

(d) Please be alert that no current will pass through the branch of the circuit that contains a fully charged capacitor However, this does not imply that potential across the capacitor is zero. The potential across the capacitor will be $\Delta V = \pm\frac{Q}{C}$.

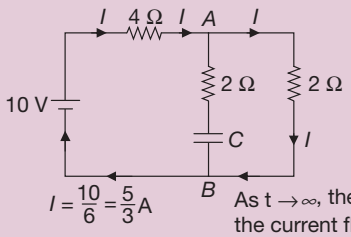
For instance, consider the circuit shown.



The circuit at time $t = 0$, when switch is closed, becomes as shown in figure.



When the capacitor becomes fully charged i.e., when $t \to \infty$, the circuit in the figure becomes as if no current passes through the branch containing the capacitor.



As t $\to \infty$, the capacitor *C* blocks the current flow in the branch *AB*

Equivalent Time Constant

To find the equivalent time constant of a circuit, follow the steps mentioned here :

STEP-1: Short-circuit the battery i.e., just remove the battery and join the ends across which it was connected.

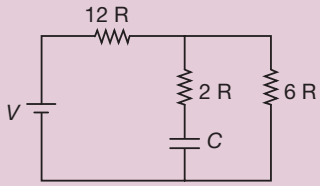
STEP-2: Find net resistance across the capacitor (say it R_{net}), Just think the capacitor is not there and the two plates of the capacitor had been converted to the ends across which we wish to find the net equivalent resistance R_{net} of the circuit.

OR

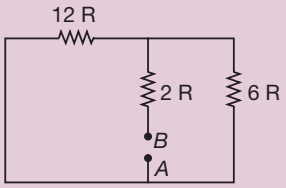
For doing this just remove the capacitor and make the points across which the capacitor was connected as the current inlet and outlet points.

STEP-3: Then, the capacitive time constant is $\tau = (R_{net})C$

For example, in the circuit shown in figure, after short circuiting the battery $6R$ and $12R$



are in parallel, so their combined resistance is $\frac{(12R)(6R)}{12R+6R} = 4R$. Now this $4R$ is in series with the remaining $2R$.



STEP-1: Battery Shorted

STEP-2: Capacitor removed to find equivalent resistance across *A* and *B*.

Hence, $R_{net} = 4R + 2R = 6R$

$\Rightarrow \quad \tau = (R_{net})C = 6RC$

Alternate Method of Finding Current in the Circuit and Charge on the Capacitor at Any Time *t*:

In a complicated *RC* circuit it is easy to find current in the circuit and charge stored in the capacitor at time $t = 0$ and $t \to \infty$ by conditions which we have already discussed. But to find the current and charge as function of time *t* following steps may be followed.

(a) Find equivalent time constant (τ) of the circuit.

(b) Find steady state charge q_0 (at time $t \to \infty$) on the capacitor.

(c) Charge on the capacitor at any time *t* is,

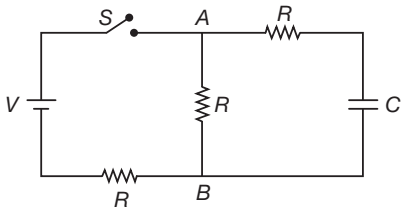
$$q = q_0\left(1 - e^{-t/\tau}\right)$$

By differentiating it w.r.t. time we can find current through the capacitor at time *t*. Then by using Kirchhoff's Laws we can calculate currents in other parts of the circuit also.

Otherwise we can also find the current in the circuit as shown and calculated in the following ILLUSTRATION because, it may not always be easy and convenient for us to find the current through the capacitor as a function of time.

ILLUSTRATION 71

In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.



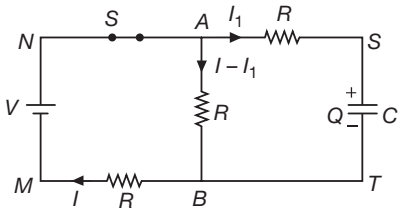
(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

SOLUTION

Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t. Therefore,

$$I_1 = \frac{dQ}{dt}$$



(a) Applying Kirchhoff's Second Law in Loop *MNABM*

$$V = (I - I_1)R + IR$$

$$\Rightarrow \quad V = 2IR - I_1R \qquad \ldots(1)$$

Similarly, applying Kirchhoff's Second Law in Loop *MNSTM*, we have

$$V = I_1R + \frac{Q}{C} + IR \qquad \ldots(2)$$

Eliminating I from equations (1) and (2), we get

$$V = 3I_1R + \frac{2Q}{C}$$

$$\Rightarrow \quad 3I_1R = V - \frac{2Q}{C}$$

$$\Rightarrow \quad I_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \quad \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R}$$

$$\Rightarrow \quad \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}\left(1 - e^{-2t/3RC}\right)$$

(b) $I_1 = \dfrac{dQ}{dt} = \dfrac{V}{3R}e^{-2t/3RC}$

From equation (1), we get

$$I = \frac{V + I_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

So, current through AB is given by

$$I_2 = I - I_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$I_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

$$I_2 = \frac{V}{2R} \text{ as } t \to \infty$$

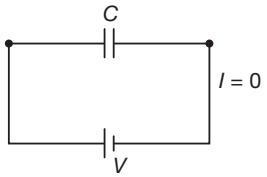
Problem Solving Technique(s)

In the above ILLUSTRATION, please note that it is advisable to take the current in the branch containing the capacitor as I_1 because then, directly we get $I_1 = \frac{dQ}{dt}$ otherwise we had to take

$$(I - I_1) = \frac{dQ}{dt}$$

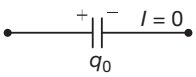
LEAKAGE CURRENT THROUGH A CAPACITOR

In ideal situation, when the space between the capacitor is filled with a dielectric (insulator) then no current flows through it when it is connected to a battery as shown in figure below



(a) Ideal Situation

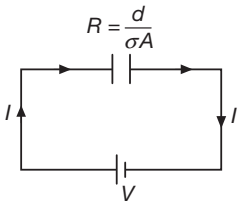
Another ideal case where no current flows through the capacitor is when a capacitor is charged and the charge on the capacitor is left over for a longer duration of time, as shown.



(b) Ideal Situation

However, in both the cases discussed, we get some small amount of current flowing through the capacitor. This non zero current (of the order of microampere) is called the leakage current. This is due to the fact that every insulator has some conductivity (or very lightly conducting), on account of which some current flows through the capacitor which is connected to battery as in Figure (a). Similarly when the capacitor is charged and left over, then the charge does not sustain (at its value) over a longer duration of time and starts discharging. This situation can be thought of as being equivalent to the case of discharging of a capacitor in an RC series circuit. In both the cases discussed above, we calculate the resistance of the dielectric using the Ohm's Law, according to which, we get

$$R = \frac{l}{\sigma A} \qquad \{\sigma = \text{specific conductance}\}$$



(c) Non-Ideal Practical Situation

Here $l = d$ (the separation between the plates of the capacitor)

$$\Rightarrow \quad R = \frac{d}{\sigma A}$$

Thus, the leakage current in the circuit shown in figure is

$$I = \frac{V}{R}$$

(d) Non-Ideal Practical Situation

Similarly, if the capacitor is given a charge q_0 at time $t = 0$, and left over, then after time t the charge that will remain on it is q, given by

$$q = q_0 e^{-t/\tau} \qquad \{\text{discharging of a capacitor}\}$$

Here, $\tau = RC = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{d}{\sigma A}\right)$

$$\Rightarrow \quad \tau = \frac{K\varepsilon_0}{\sigma}$$

Remark(s)

(a) Dielectric leakage occurs in a capacitor as the result of LEAKAGE CURRENT through the dielectric. Normally it is assumed that the dielectric will effectively prevent the flow of current through the capacitor. Although the resistance of the dielectric is extremely high, a minute amount of current does flow. Ordinarily this current is so small that for all practical purposes it is ignored. However, if the leakage through the dielectric is abnormally high, there will be a rapid loss of charge and an overheating of the capacitor.

(b) The power loss of a capacitor is determined by loss in the dielectric. If the loss is negligible and the capacitor returns the total charge to the circuit, it is considered to be a perfect capacitor with a power loss of zero.

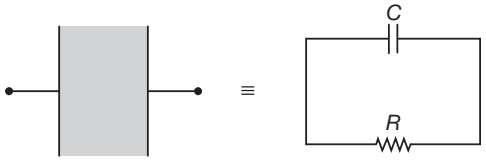
(c) The dielectric used inside the capacitor is not a perfect insulator resulting in a very small current flowing or "leaking" through the dielectric when applied to a constant supply voltage. This small current flow (in the region of micro amperes) is called the **Leakage Current**. This **leakage current** is a result of electrons physically making their way through the dielectric medium, around its edges or across the leads. The leakage current of a capacitor is sometimes called the **insulation resistance**.

ILLUSTRATION 72

A leaky parallel plane capacitor is filled completely with a material having dielectric constant $k = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ \text{m}^{-1}$. If the charge on the plane at instant $t = 0$ is $q = 8.85$ mC, then calculate the leakage current at the instant $t = 12$ s.

SOLUTION

The problem deals with discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.



$$R = \frac{\rho d}{A} = \frac{d}{\sigma A}$$

$\{$where ρ = resistivity & σ = conductivity$\}$

and $C = \dfrac{K\varepsilon_o A}{d}$

The capacitive time constant of the circuit is

$$\tau = CR = \frac{K\varepsilon_o}{\sigma}$$

Substituting the values, we have

$$\tau = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98\ \text{s}$$

Charge at any time decrease exponentially as

$$q = q_o e^{-t/\tau}$$

where $q_o = 8.85 \times 10^{-6}$ C is the charge at time $t = 0$ Therefore, discharging (leakage) current at time t is given by

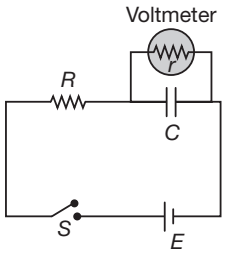
$$I = \left(-\frac{dq}{dt}\right) = \frac{q_o}{\tau} e^{-t/\tau}$$

So, the current at $t = 12$ s is

$$I = \frac{\left(8.85 \times 10^{-6}\right)}{5.98} e^{-12/5.98} = 0.198 \times 10^{-6}\ \text{A} = 0.198\ \mu\text{A}$$

ILLUSTRATION 73

A digital voltmeter of internal resistance r is used to measure the voltage across a capacitor after the switch in Figure is closed. Because the meter has finite resistance, part of the current supplied by the battery passes through the meter.



(a) Apply Kirchhoff's rules to this circuit, and use the fact that $I_C = \dfrac{dq}{dt}$ to show that this leads to the differential equation

$$R_{eq}\frac{dq}{dt}+\frac{q}{C}=\frac{r}{R+r}E$$

where $R_{eq}=\dfrac{rR}{R+r}$

(b) Show that the solution to this differential equation is

$$q=\frac{CEr}{R+r}\left(1-e^{-\frac{t}{R_{eq}C}}\right)$$

and that the voltage across the capacitor as a function of time is

$$V_C=\frac{Er}{R+r}\left(1-e^{-\frac{t}{R_{eq}C}}\right)$$

(c) If the capacitor is fully charged, and the switch is then opened, how does the voltage across the capacitor behave in this case?

SOLUTION

(a) Let I represent the current in the battery and I_C the current charging the capacitor. Then $I-I_C$ is the current in the voltmeter. The loop rule applied to the inner loop is $+E-IR-\dfrac{q}{C}=0$. The loop rule for the outer perimeter is $E-IR-(I-I_C)r=0$. With $I_C=\dfrac{dq}{dt}$, this becomes $E-IR-Ir+\dfrac{dq}{dt}r=0$. Between the two loop equations we eliminate $I=\dfrac{E}{R}-\dfrac{q}{RC}$ by substitution to obtain

$$E-(R+r)\left(\frac{E}{R}-\frac{q}{RC}\right)+\frac{dq}{dt}r=0$$

$$\Rightarrow\quad E-\left(\frac{R+r}{R}\right)E+\left(\frac{R+r}{RC}\right)q+r\frac{dq}{dt}=0$$

$$\Rightarrow\quad -\left(\frac{r}{R+r}\right)E+\frac{q}{C}+\left(\frac{Rr}{R+r}\right)\frac{dq}{dt}=0$$

This is the required differential equation

(b) From above, we get

$$\frac{dq}{dt}=\frac{E}{R}-\frac{R+r}{RrC}q=-\frac{R+r}{RrC}\left(q-\frac{ErC}{R+r}\right)$$

$$\Rightarrow\quad \int_0^q\frac{dq}{q-ErC/(R+r)}=-\frac{R+r}{RrC}\int_0^t dt$$

$$\Rightarrow\quad \log_e\left(q-\frac{ErC}{R+r}\right)\Big|_0^q=-\frac{R+r}{RrC}t\Big|_0^t$$

$$\Rightarrow\quad \log_e\left(\frac{q-ErC/(R+r)}{-ErC/(R+r)}\right)=-\left(\frac{R+r}{RrC}\right)t$$

$$\Rightarrow\quad q-\frac{ErC}{R+r}=-\frac{ErC}{R+r}\exp\left[\left(-\frac{R+r}{RrC}\right)t\right]$$

$$\Rightarrow\quad q=\frac{r}{R+r}CE\left(1-e^{-\frac{t}{R_{eq}C}}\right)\text{ where }R_{eq}=\frac{Rr}{R+r}$$

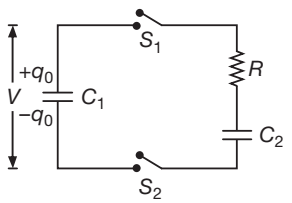
The voltage across the capacitor is

$$V_C=\frac{q}{C}=\frac{Er}{R+r}\left(1-e^{-t/R_{eq}C}\right)$$

(c) As $t\to\infty$ the capacitor voltage approaches $\dfrac{Er}{R+r}(1-0)=\dfrac{Er}{R+r}$. If the switch is then opened, the capacitor discharges through the voltmeter. Its voltage decays exponentially according to $\dfrac{Er}{R+r}e^{-\frac{t}{rC}}$.

ILLUSTRATION 74

The capacitor C_1 in figure initially carries a charge q_0. When the switches S_1 and S_2 are shut, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which initially does not carry any charge.



(a) Find the charges deposited on the capacitors and the current through R as a function of time.

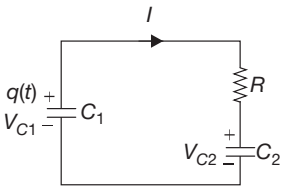
(b) What is the heat lost in the resistor after a long time of closing the switch?

SOLUTION

(a) Suppose at a moment t the charge deposited on C_1 is $q(t)$. Then,

$$V_{C_1}=\frac{q(t)}{C_1} \text{ and } V_{C_2}=\frac{q_0-q(t)}{C_2}$$

$$V_R=IR \text{ where } I=-\frac{dq}{dt}$$



Applying KVL, we get

$$\frac{q}{C_1}-\frac{(q_0-q)}{C_2}=IR=-R\frac{dq}{dt}$$

$$\Rightarrow \quad R\frac{dq}{dt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)q=\frac{q_0}{C_2}$$

Put $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$, we get

$$R\frac{dq}{dt}+\frac{q}{C}=\frac{q_0}{C_2}$$

$$\Rightarrow \quad R\frac{dq}{dt}=\frac{q_0}{C_2}-\frac{q}{C}$$

$$\Rightarrow \quad R\frac{dq}{dt}=\frac{1}{C}\left(\frac{C}{C_2}q_0-q\right)$$

$$\Rightarrow \quad \int_{q_0}^{q}\frac{dq}{q-\frac{C}{C_2}q_0}=-\int_0^t\frac{dt}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{q-\frac{C}{C_2}q_0}{q_0-\frac{C}{C_2}q_0}\right)=-\frac{t}{RC}$$

$$\Rightarrow \quad q-\frac{C}{C_2}q_0=q_0\left(1-\frac{C}{C_2}\right)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q=q(t)=q_0\left[\left(1-\frac{C}{C_2}\right)e^{-\frac{t}{RC}}+\frac{C}{C_2}\right] \text{ where}$$

$$C=\frac{C_1C_2}{C_1+C_2}$$

$$\Rightarrow \quad I(t)=-\frac{dq}{dt}=\frac{q_0}{RC_1}e^{-\frac{t}{RC}}$$

Charge on C_2,

$$q_2=q_0-q(t)=q_0\frac{C}{C_1}\left(1-e^{-\frac{t}{RC}}\right)$$

So, we observe that the charge on q_1 decays and that on q_2 grows exponentially.

(b) Electrostatic energy at $t=0$ is

$$U(0)=\frac{q_0^2}{2C_1}$$

Electrostatic energy at $t\to\infty$ is final energy given by

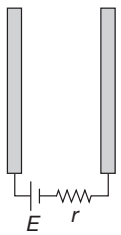
$$U_f=U(\infty)=\frac{q_0^2}{2(C_1+C_2)}$$

Since, Heat Loss $=-\Delta U=U_i-U_f$

$$\Rightarrow \quad -\Delta U=U(0)-U(\infty)=\frac{q_0^2C_2}{2C_1(C_1+C_2)}$$

ILLUSTRATION 75

Two large conducting plates are arranged to form a parallel-plate capacitor of capacitance C. The plates are given charges Q_1 and Q_2 respectively. The plates are then connected to the terminals of a battery of emf E and internal resistance r at $t=0$. Find the charges on the four surfaces as function of time.



SOLUTION

When the battery is connected, the charges on the inner surfaces will always be equal and opposite. Since the electric field inside the plates will be zero, the cancellation of electric field must be accomplished by the charges on the outer surfaces. For this, they have to be of same magnitude and nature. It thus follows

that the charges on the outer surfaces will always be what they initially were, i.e., $\frac{Q_1+Q_2}{2}$.

Let q be the charges at time t on the inner surfaces. Applying KVL,

$$-\frac{q}{C}-Ir+E=0$$

$$\Rightarrow \quad \frac{dq}{E-\frac{q}{C}}=\frac{dt}{r}$$

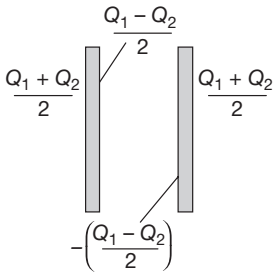
$$\int_{\frac{Q_1-Q_2}{2}}^{q}\frac{dq}{CE-q}=\int_0^t\frac{dt}{rC}$$

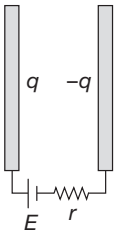
$$-\log_e(CE-q)\Big|_{\frac{Q_1-Q_2}{2}}^{q}=\frac{t}{rC}$$

$$\Rightarrow \quad \log_e\left[\frac{CE-q}{CE-\left(\frac{Q_1-Q_2}{2}\right)}\right]=-\frac{t}{rC}$$

$$\Rightarrow \quad \frac{CE-q}{CE-\left(\frac{Q_1-Q_2}{2}\right)}=e^{-\frac{t}{rC}}$$

$$\Rightarrow \quad CE-q=\left(CE-\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{rC}}$$

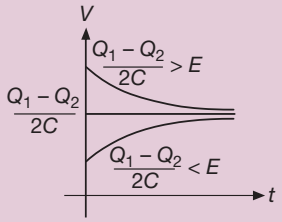




$$\Rightarrow \quad q=CE\left(1-e^{-\frac{t}{rC}}\right)+\left(\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{rC}}$$

Observation

We can interpret this result as a superposition of discharging and charging of a capacitor. Also note that in steady state, the charge on the inner surfaces will be CE and $-CE$ respectively, regardless of the initial value. In particular if $\frac{Q_1-Q_2}{2C}$ (initial potential difference between the plates) is equal to E, there is no charge flow through the battery. If $\frac{Q_1-Q_2}{2C}>E$, an extra charge is given to the battery and if $\frac{Q_1-Q_2}{2C}<E$, the battery supplies the charge to the capacitor. This all is shown in the graph shown in figure, wherein potential difference across the plates has been plotted as function of time.



Test Your Concepts-XI

Based on RC Circuit

(Solutions on page H.228)

1. Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t)=I_0e^{-\frac{t}{\tau}}$ where I_0 is the initial current (at $t=0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. Calculate the charge passes through this point between

(a) $t=0$ and $t=\tau$. **(b)** $t=0$ and $t=10\tau$.

(c) $t=0$ and $t\to\infty$.

2. The dielectric material between the plates of a parallel plate capacitor always has some nonzero conductivity σ. Let A represent the area of each plate and d the distance between them. Let κ represent the dielectric constant of the material

(a) Show that the resistance R and the capacitance C of the capacitor are related by $RC=\frac{\kappa\varepsilon_0}{\sigma}$.

(b) Find the resistance between the plates of a 14 nF capacitor with a fused quartz dielectric having $\kappa=3.78$

3. A 2 nF capacitor with an initial charge of 5.1 μC is discharged through a 1.3 kΩ resistor.

(a) Calculate the current in the resistor 9 μs after the resistor is connected across the terminals of the capacitor.

(b) What charge remains on the capacitor after 8 μs?

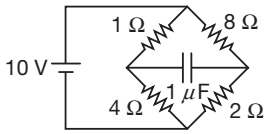
(c) What is the maximum current in the resistor?

4. A fully charged capacitor stores energy U_0. How much energy remains when its charge has decreased to half its original value?

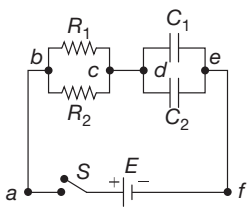
5. The circuit in figure has been connected for a long time.

(a) What is the voltage across the capacitor?

(b) If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?



6. The circuit in figure contains two resistors, R_1 = 2 kΩ and R_2 = 3 kΩ, and two capacitors, C_1 = 2 μF and C_2 = 3 μF, connected to a battery with emf E = 120 V. Charge is on either capacitor before switch S is closed is zero. Determine the charges q_1 and q_2 on capacitors C_1 and C_2, respectively, after the switch is closed.



7. During the process of charging of a capacitor, how long does it take for the current to decay to half its value?

8. At what time the charge on the capacitor in a discharging circuit equal to one half its initial value?

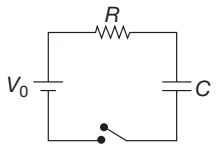
9. In a discharging RC circuit, at what time is the electrical potential energy becomes half of its initial value?

10. (a) Calculate the power absorbed by a charging capacitor in the series RCcircuit as a function of time.

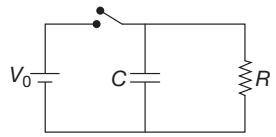
(b) Integrate $P(t)$ from $t = 0$ to $t \to \infty$ to show that the energy stored in the capacitor is $\left(\frac{1}{2}\right)CV_0^2$, when it is fully charged.

11. A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ m^{-1}$. If the charge on the plate at the instant $t = 0$ is $q = 8.85$ μC, then calculate the leakage current at $t = 12$ s.

12. A 10 μF capacitor is charged through a resistance of 0.1 MΩ from a battery of 1.5 V. Calculate the time required for the capacitor to get charged upto 0.75 V for situations shown.

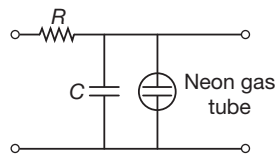


Situation 1

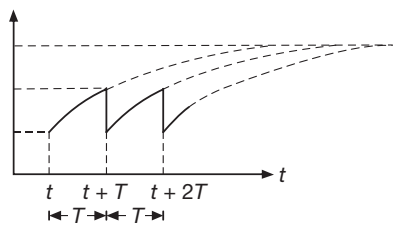


Situation 2

13. Figure (a) shows a circuit in which time interval of flashes of a neon gas tube is controlled by a capacitor. When cool, the gas is a good insulator. The tube "fires" (ionizes and emits light) when the potential difference across it reaches the firing value V_f. Its resistance becomes very small and so the capacitor rapidly discharges through it. As the potential difference drops, the gas cools down and becomes an insulator. The extension potential difference is V_e. At this stage the capacitor again starts to charge. The variation of potential difference is shown in Figure (b). Determine the time period T of flashing of the neon gas tube.

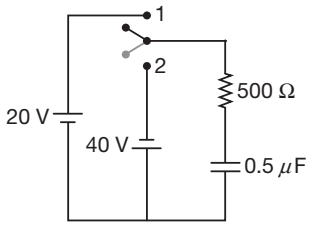


(a)

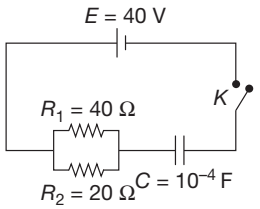


(b)

14. In the given circuit, the switch is closed in the position 1 at $t = 0$ and then moved to 2 after 250 μs. Derive an expression for current as a function of time for $t > 0$. Also, plot the variation of current with time.



15. The circuit shown in figure is closed at time $t = 0$. Calculate the total amount of heat generated in R_2 during the time in which the capacitor is charged to a voltage of 20 V.



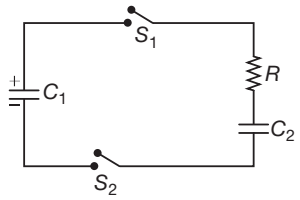
16. A battery consists of 2 identical cells, each of e.m.f. E, connected in series. The battery is used to charge a capacitor, with capacitance C, through a resistor R.

(a) by connecting it across the terminals of the entire battery and

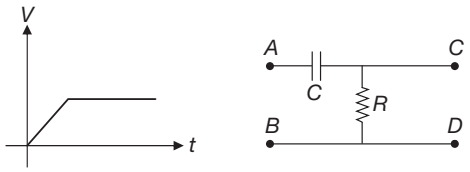
(b) by connecting it first across a single cell, then across two cells until it is again fully charged.

Determine, in each case, the amount of energy transferred from the battery, which is not stored in the capacitor. What has happened to this energy?

17. The switch S is closed at $t = 0$. The capacitor C is uncharged but C_0 has a charge $Q_0 = 2$ μC at $t = 0$. If $R = 100$ Ω, $C = 2$ μF, $C_0 = 2$ μF, $E = 4$ V. Calculate $I(t)$ in the circuit.



18. A time varying voltage is applied to the clamps A and B such that voltage across the capacitor plates is as shown in the figure. Plot the time dependence of voltage across the terminals of the resistance C and D.



19. A circuit consists of a source of a constant emf E a resistance R and a capacitor with capacitance C connected series. The internal resistance of the source is negligible. At a moment $t = 0$, the capacitance of the capacitor is abruptly decreased η-fold. Find the current flowing through the circuit as a function of time t.

20. The gap between the plates of a parallel plate capacitor is filled with glass of resistivity $\rho = 10^{11}$ Ωm. The capacitance of the capacitor $C = 4$ nF. Find the leakage current of the capacitor when a voltage $V = 2$ kV is applied across it. (Dielectric constant of glass $K = 6$).

SOLVED PROBLEMS

PROBLEM 1

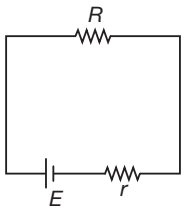
A battery has an open circuit potential difference of 6 V between its terminals. When a load resistance of 60 Ω is connected across the battery, the total power dissipated by the battery is 0.4 W. What should be the load resistance R, so that maximum power will be dissipated in R. Calculate this power. What is the total power supplied by the battery when such a load is connected?

SOLUTION

When the circuit is open, $V = E$

$\Rightarrow \quad E = 6$ V

Let r be the internal resistance of the battery. Then the power supplied by the battery in this case is,



$$P = \frac{E^2}{R+r}$$

Substituting the values, we have

$$0.4 = \frac{(6)^2}{60+r}$$

$\Rightarrow \quad r = 30\ \Omega$

According to Maximum Power Transfer Theorem, maximum power is dissipated in the circuit when, net external resistance is equal to net internal resistance. So, we have

$$R = r$$

$\Rightarrow \quad R = 30\ \Omega$

Further, total power supplied by the battery under this condition is,

$$P_{\text{Total}} = \frac{E^2}{R+r} = \frac{(6)^2}{30+30} = 0.6 \text{ W}$$

Out of this 0.6 W, half of the power is dissipated in R and half in r. Therefore, maximum power dissipated in R would be

$$P_{\text{MAX}} = \frac{0.6}{2} = 0.3 \text{ W}$$

PROBLEM 2

A conductor has a temperature independent resistance R and a total heat capacity C. At the moment $t = 0$ it is connected to a dc voltage V. Find the time dependence of the conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$ where k is a constant, T_0 is the environmental temperature (equal to conductor's temperature at the initial moment).

SOLUTION

Here energy is being generated in the resistance at a rate of $\frac{V^2}{R}$. Of which part of energy is being lost in the environment and the rest is utilized in raising the temperature of conductor. So, applying the Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Energy}\\ \text{supplied}\\ \text{by the dc}\\ \text{source per}\\ \text{unit time}\end{pmatrix} = \begin{pmatrix}\text{Energy lost}\\ \text{in the}\\ \text{environment}\\ \text{per}\\ \text{unit time}\end{pmatrix} + \begin{pmatrix}\text{Energy used}\\ \text{in raising the}\\ \text{temperature of}\\ \text{the conductor}\\ \text{per unit time}\end{pmatrix}$$

Hence, $\frac{V^2}{R} = k(T - T_0) + C\left(\frac{dT}{dt}\right)$

$\Rightarrow \quad C\left(\frac{dT}{dt}\right) = \frac{V^2}{R} - k(T - T_0)$

$\Rightarrow \quad \frac{dT}{\frac{V^2}{R} - k(T - T_0)} = \frac{dt}{C}$

Integrating the above expression, we get

$$\int_{T_0}^{T} \frac{dT}{\frac{V^2}{R} - k(T - T_0)} = \int_0^t \frac{dt}{C}$$

Solving this equation, we get

$$T = T_0 + \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{C}}\right)$$

PROBLEM 3

A battery of emf 2 V and negligible internal resistance is connected across a uniform wire of length 10 m and resistance 30 Ω. The appropriate terminals of a cell of emf 1.5 V and internal resistance 1 Ω is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire.

(a) What length of the wire will be required to produce zero deflection of the galvanometer?

How will the balancing change,

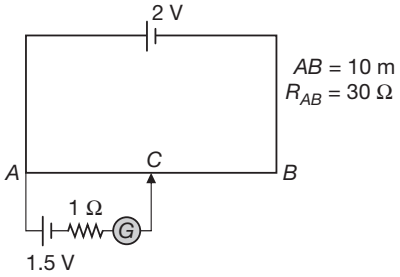
(b) when a coil of resistance 5 Ω is placed in series with the battery.

(c) When the cell of 1.5 V is shunted with 5 Ω resistor?

SOLUTION

(a) Potential gradient across wire AB is

$$\frac{\Delta V}{\Delta l} = \frac{2}{10} = 0.2 \text{ Vm}^{-1}$$



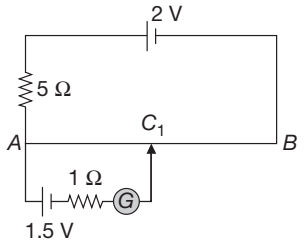
Now, $V_{AC} = 1.5$ V

$$\Rightarrow \quad \left(\frac{\Delta V}{\Delta l}\right)(AC) = 1.5$$

$$\Rightarrow \quad (0.2)(AC) = 1.5 = E$$

$$\Rightarrow \quad AC = 7.5 \text{ m}$$

(b) $V_{AB} = \left(\frac{R_{AB}}{R_{AB}+5}\right)\times 2 = \left(\frac{30}{30+5}\right)\times 2 = \frac{12}{7}$ V



Now, the potential gradient across AB is

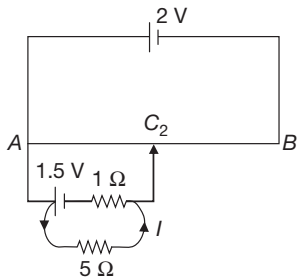
$$\left(\frac{\Delta V}{\Delta l}\right)_{AB} = \frac{12}{70} \text{ voltm}^{-1}$$

Since, $V_{AC_1} = 1.5 \text{ V} = E$

$$\Rightarrow \quad \left(\frac{12}{70}\right)(AC_1) = 1.5$$

$$\Rightarrow \quad AC_1 = 8.75 \text{ m}$$

(c) V_{AC_2} = Voltage drop across 1.5 V battery



$$\Rightarrow \quad (0.2)(AC_2) = \left(\frac{5}{5+1}\right)(1.5)$$

$$\Rightarrow \quad AC_2 = 6.25 \text{ m}$$

PROBLEM 4

A 10 m long potentiometer wire has resistance 10 Ω and is connected to an accumulator of 2 V and internal resistance zero. Two resistance boxes B_1 and B_2 are connected in series with the accumulator. A cell of emf 1.018 V and a shunted galvanometer are connected in parallel across the box B_1. Reading of galvanometer is zero.

(a) For what values of resistances R_1 and R_2 both having integral values will be required from the boxes B_1 and B_2 to maintain a potential gradient of 10^{-3} Vm^{-1} in the potentiometer wire.

(b) Determine the length of potentiometer wire that balances the thermo emf of iron-copper couple at 300 °C which develops 1.7×10^{-5} volt °C^{-1}.

SOLUTION

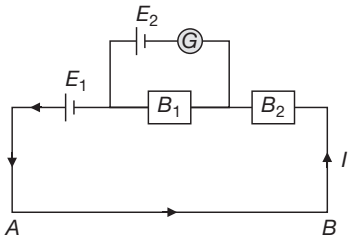
According to the statement, we are provided with the following data

$$E_1 = 2\text{ V}, E_2 = 1.018\text{ V}, AB = 10\text{ m and } R_{AB} = 10\ \Omega$$

(a) Potential gradient across potentiometer wire $AB = 10^{-3}$ Vm^{-1}

$$\therefore \quad V_{AB} = (10^{-3})(10) = 10^{-2}\text{ V}$$

$$I = \frac{V_{AB}}{R_{AB}} = \frac{10^{-2}}{10} = 10^{-3}\text{ A} \qquad \ldots(1)$$



Further, potential difference across $B_1 = E_2 = 1.018$ V

$$\Rightarrow \quad IR_1 = 1.018\text{ V}$$

$$\Rightarrow \quad R_1 = \frac{1.018}{I} = 1018\ \Omega$$

$$\text{Also, } I = \frac{E_1}{R_1 + R_2 + R_{AB}}$$

$$\Rightarrow \quad 10^{-3} = \frac{2}{1018 + R_2 + 10}$$

$$\Rightarrow \quad R_2 = 972\ \Omega$$

(b) $V = (1.7\times10^{-5})(300) = 5.1\times10^{-3}$ V

$$\Rightarrow \quad l = \frac{V}{\text{Potential gradient}} = \frac{5.1\times10^{-3}}{10^{-3}} = 5.1\text{ m}$$

PROBLEM 5

It is desired to send a current of 8 A through a circuit whose resistance is 5 Ω. Find the least number of cells which must be used for this purpose and how should they be connected. The emf of each cell is 2 V and the internal resistance is 0.5 Ω.

SOLUTION

Let N = total number of cells required to be grouped in m rows, each row carrying n cells $= mn$

Since, we know that for n cells in series connected in m rows in parallel, the current in the external circuit is given by

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} = \frac{NE}{mR + nr}$$

$$\Rightarrow \quad 8 = \frac{2N}{5m + \frac{0.5N}{m}}$$

$$\Rightarrow \quad 20m^2 + 2N = Nm \qquad \ldots(1)$$

For least value of N, we have

$$\frac{dN}{dm} = 0$$

Differentiating equation (1), we have

$$40m + 2\frac{dN}{dm} = m\frac{dN}{dm} + N$$

$$\Rightarrow \quad \frac{dN}{dm} = \frac{N - 40m}{2 - m} = 0$$

$$\Rightarrow \quad N = 40m \qquad \ldots(2)$$

Since $N = mn$ $\qquad \ldots(3)$

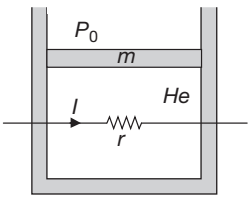
Solving equations (1), (2) and (3), we get

$$n = 40 \text{ and } m = 4$$

$$\Rightarrow \quad N = mn = 160$$

PROBLEM 6

A resistance coil of resistance r connected to an external battery, is placed inside an adiabatic cylinder fitted with a frictionless piston of mass m and same area A. Initially cylinder contains one mole of ideal gas He. A current I flows through the coil such that temperature of gas varies as $T = T_0 + at + bt^2$, keeping pressure constant with time t. Atmosphere pressure above piston is P_0. Find



(a) Current I flowing through the coil as function of time and
(b) Speed of piston as function of time.

SOLUTION

Heat produced by coil inside the cylinder in time dt is

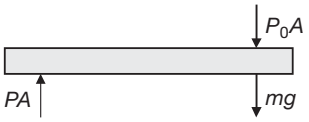
$$dQ = I^2 r\,dt \qquad \ldots(1)$$

(a) From First Law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W \qquad \ldots(2)$$

$$\Rightarrow \quad I^2 r dt = C_v dT + PdV$$

$$\Rightarrow \quad I^2 r = C_v \frac{dT}{dt} + R\frac{dT}{dt} = C_p \frac{dT}{dt}$$



As $T = T_0 + at + bt^2$

$$\Rightarrow \quad \frac{dT}{dt} = (a + 2bt)$$

$$\Rightarrow \quad I = \sqrt{\frac{5R}{2r}(2bt + a)} \qquad \left(C_p = \frac{5R}{2}\right)$$

(b) $PV = RT$

$$\Rightarrow \quad PdV = RdT \qquad \text{(pressure is constant)}$$

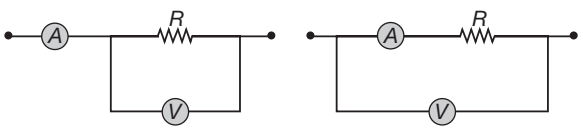
$$\Rightarrow \quad PAdx = RdT$$

$$\Rightarrow \quad \text{Velocity } v = \frac{dx}{dt} = \frac{R}{PA}\left(\frac{dT}{dt}\right)$$

$$\Rightarrow \quad v = \frac{R}{PA}(2bt + a) \qquad (\text{where } P_0A + mg = PA)$$

PROBLEM 7

The value of a resistor R is to be determined using the ammeter-voltmeter setup shown in figure. The ammeter has a resistance of 0.5 Ω, and the voltmeter has a resistance of 20000 Ω.



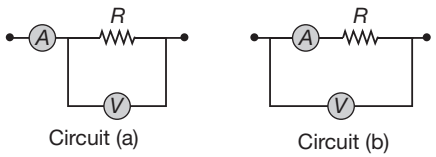
Within what range of actual values of R will the measured values be correct to within 5% if the measurement is made using the circuit shown in figure.

SOLUTION

Let R_m = measured value, R = actual value,
I_R = current through the resistor R
I = current measured by the ammeter.

(a) When using circuit (a), $I_R R = \Delta V = 20000(I - I_R)$

$$\Rightarrow \quad R = 20000\left[\frac{I}{I_R} - 1\right]$$



But since $I = \frac{\Delta V}{R_m}$ and $I_R = \frac{\Delta V}{R}$, we have $\frac{I}{I_R} = \frac{R}{R_m}$

$$\Rightarrow \quad R = 20000\frac{(R - R_m)}{R_m} \qquad \ldots(1)$$

When $R > R_m$, we require $\left(\frac{R - R_m}{R}\right) \le 0.05$

$$\Rightarrow \quad R_m \ge R(1 - 0.05) \text{ and from (1) we get}$$

$$R \le 1050\ \Omega.$$

(b) When using circuit (b), we have

$$I_R R = \Delta V - I_R(0.5\ \Omega)$$

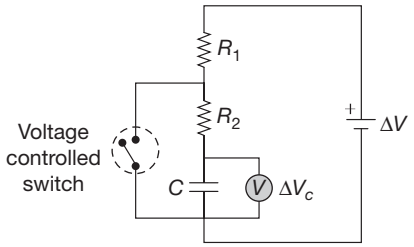
But since $I_R = \frac{\Delta V}{R_m}$ and $R_m = (0.5 + R)$ $\qquad \ldots(2)$

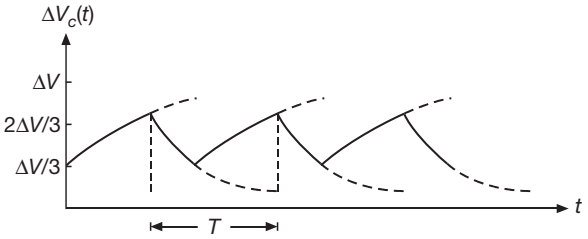
When $R_m > R$, we require $\frac{(R_m - R)}{R} \le 0.05$

From (2) we find $R \ge 10\ \Omega$

PROBLEM 8

The switch in figure closes when $\Delta V_c > \frac{2\Delta V}{3}$ and opens when $\Delta V_c < \frac{\Delta V}{3}$ The voltmeter reads a voltage as plotted in figure. What is the period T of the waveform in terms of R_1, R_2 and C ?

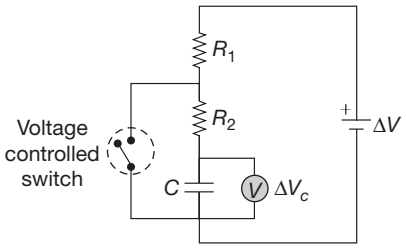




SOLUTION

Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C

$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right] e^{-\frac{t}{R_2C}}$$



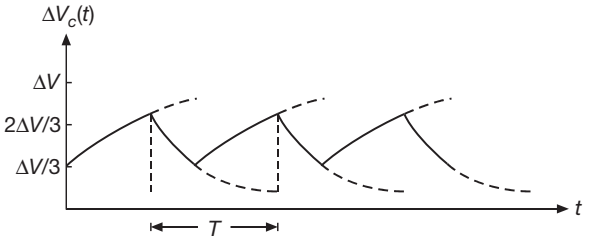
Let, $\Delta V_C(t) = \frac{1}{3}\Delta V$, at time t_1, then

$$\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right] e^{-\frac{t_1}{R_2C}}$$

$$\Rightarrow \quad e^{-\frac{t_1}{R_2C}} = \frac{1}{2} \quad \Rightarrow \quad t_1 = R_2C \log_e 2$$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1 + R_2)C$, so

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right] e^{-\frac{t}{(R_1+R_2)C}}$$



Let $\Delta V_C(t) = \frac{2}{3}\Delta V$, in further time t_2, then

$$\Rightarrow \quad \frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-\frac{t_2}{(R_1+R_2)C}}$$

$$\Rightarrow \quad e^{-\frac{t_2}{(R_1+R_2)C}} = \frac{1}{2}$$

$$\Rightarrow \quad t_2 = (R_1 + R_2)C \log_e 2$$

$$\Rightarrow \quad T = t_1 + t_2 = (R_1 + 2R_2)C \log_e 2$$

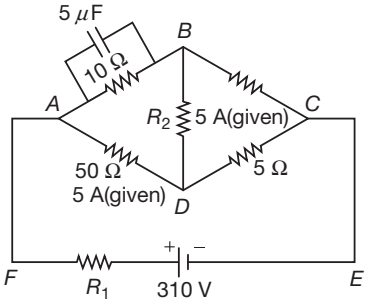
PROBLEM 9

Figure shows a circuit in steady state. If charge on the capacitor is 1000 μC, find

(a) The battery current

(b) The resistance R_1, R_2, R_3.

The current in 50 Ω resistor and R_2 is given to be 5 A.



SOLUTION

(a) The potential difference across 5 μC capacitor, is

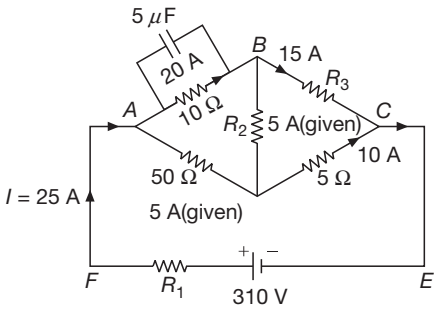
$$V = \frac{Q}{C} = \left(\frac{1000}{5}\right) \text{V} = 200 \text{ V}$$

The 10 Ω resistor and capacitor are in parallel arrangement and therefore current I through 10 Ω resistor is given by

$$I = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

(b) From KCL at junction A we get

$I = 25$ A



Now we apply KVL in the loop $ABDA$,

$$-(10)(20) - 5R_2 + (50)(5) = 0$$

$$\Rightarrow \quad R_2 = 10\ \Omega$$

Apply KVL in loop $ADCEFA$, we get

$$-(50)(5) - (5)(10) + 310 - 25R_1 = 0$$

$$\Rightarrow \quad R_1 = \frac{10}{25}\ \Omega = 0.4\ \Omega$$

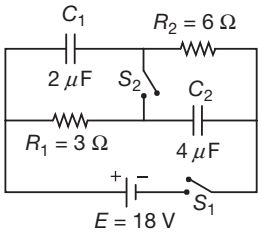
Apply KVL in loop $ABCDA$,

$$-(20)(10) - 15R_3 + (10)(5) + (50)(5) = 0$$

$$\Rightarrow \quad R_3 = \frac{100}{15} = 6.66\ \Omega$$

PROBLEM 10

In the circuit shown, initially the switches are open and the capacitors are uncharged. Switches S_1 and S_2 are closed simultaneously at $t = 0$.



(a) Obtain the expression for current through switch S_2 as function of time.

(b) The switch S_2 is opened after a long time interval. Find the heat dissipated in resistors and the charge flowing through S_1.

SOLUTION

(a) The distribution of current in the circuit is shown in the figure. Apply KVL to loop $abefa$, we get

$$-\frac{q_1}{C_1} + (I_2 + I_3 - I_1)R_1 = 0 \qquad \ldots(1)$$

Apply KVL to loop $bcdeb$, we get

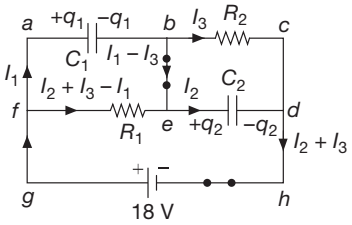
$$-I_3R_2 + \frac{q_2}{C_2} = 0 \qquad \ldots(2)$$

Apply KVL to loop $achga$, we get

$$I_3R_2 + \frac{q_1}{C_1} = E \qquad \ldots(3)$$

The current through capacitor C_1 is given by

$$I_1 = \frac{dq_1}{dt}$$



The current through capacitor C_2 is given by

$$I_2 = \frac{dq_2}{dt}$$

From equations (2) and (3), we get, at any instant,

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} = E \qquad \ldots(4)$$

From equation (1), we get

$$I_2 - I_1 = \frac{q_1}{R_1C_1} - I_3$$

$$\Rightarrow \quad \frac{d}{dt}(q_2 - q_1) = \frac{q_1}{C_1R_1} - \frac{q_2}{C_2R_2} \qquad \ldots(5)$$

$$\Rightarrow \quad \frac{d}{dt}\left[q_2 - C_1\left(E - \frac{q_2}{C_2}\right)\right] = \frac{1}{C_1R_1}\left[C_1\left(E - \frac{q_2}{C_2}\right)\right] - \frac{q_2}{C_2R_2}$$

$$\Rightarrow \quad \left(1 + \frac{C_1}{C_2}\right)\frac{dq_2}{dt} = \frac{E}{R_1} - \frac{q_2}{C_2}\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$$

$$\Rightarrow \quad \left(\frac{C_1+C_2}{\cancel{C_2}}\right)\left(\frac{dq_2}{dt}\right)=\frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}{\cancel{C_2}}\left[\frac{EC_2}{R_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}-q_2\right]$$

$$\Rightarrow \quad \frac{dq_2}{dt}=\frac{1}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}\left[\frac{EC_2R_2}{R_1+R_2}-q_2\right]$$

$$\Rightarrow \quad \int_0^{q_2}\frac{dq_2}{\left(\frac{EC_2R_2}{R_1+R_2}-q_2\right)}=\int_0^{t}\frac{dt}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow \quad -\log_e\left(\frac{EC_2R_2}{R_1+R_2}-q_2\right)\Bigg|_0^{q_2}=\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow \quad \log_e\left(\frac{\frac{EC_2R_2}{R_1+R_2}-q_2}{\frac{EC_2R_2}{R_1+R_2}}\right)=-\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow \quad \frac{EC_2R_2}{R_1+R_2}-q_2=\left(\frac{EC_2R_2}{R_1+R_2}\right)\exp\left[-\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}\right]$$

$$\Rightarrow \quad q_2=\left(\frac{EC_2R_2}{R_1+R_2}\right)\left(1-e^{-\frac{t}{RC}}\right) \qquad \ldots(6)$$

where $R=\dfrac{R_1R_2}{R_1+R_2}$ and $C=C_1+C_2$ …(7)

Substituting the value of q_2 from above in equation (4), we get

$$q_1=C_1E-\frac{C_1}{C_2}\left(\frac{EC_2R_2}{R_1+R_2}\right)\left(1-e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad q_1=C_1E-\frac{C_1ER_2}{R_1+R_2}\left(1-e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad q_1=C_1E\left[1-\frac{R_2}{R_1+R_2}(1-e)^{-\frac{t}{RC}}\right] \qquad \ldots(8)$$

where $R=\dfrac{R_1R_2}{R_1+R_2}$ and $C=C_1+C_2$

Now, $I_1=\dfrac{dq_1}{dt}=-\left(\dfrac{C_1E}{RC}\right)\left(\dfrac{R_2}{R_1+R_2}\right)e^{-\frac{t}{RC}}$

$$\Rightarrow \quad I_1=-\frac{C_1E}{\left(\frac{R_1\cancel{R_2}}{\cancel{R_1+R_2}}\right)(C_1+C_2)}\left(\frac{\cancel{R_2}}{\cancel{R_1+R_2}}\right)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I_1=-\frac{C_1E}{R_1(C_1+C_2)}e^{-\frac{t}{RC}}$$

Similarly,

$$I_2=\frac{dq_2}{dt}=\frac{EC_2\cancel{R_2}}{\left(\cancel{R_1+R_2}\right)}\frac{1}{(C_1+C_2)\left(\frac{R_1\cancel{R_2}}{\cancel{R_1+R_2}}\right)}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I_2=\frac{EC_2}{R_1(C_1+C_2)}e^{-\frac{t}{RC}}$$

Let us first now substitute the values of all given in equations (6), (7), (8) to get

$$q_2=\frac{(18)(4)(6)}{(3+6)}\left(1-e^{-\frac{t}{(2)(6)}}\right)$$

$$\Rightarrow \quad q_2=48\left(1-e^{-\frac{t}{12}}\right) \qquad (\text{in } \mu\text{C}) \qquad \ldots(9)$$

Similarly

$$q_1=(2)(18)-\frac{(2)(18)(6)}{(3+6)}\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow \quad q_1=36-24\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow \quad q_1=12+24e^{-\frac{t}{12}}$$

$$\Rightarrow \quad q_1=12\left(1+2e^{-\frac{t}{12}}\right) \qquad (\text{in } \mu\text{C}) \qquad \ldots(10)$$

Please note that as per equation (4), we must have $\frac{q_1}{C_1}+\frac{q_2}{C_2}=E$ (at any instant) and as per the relation the above two equations satisfy it at all the instants of time ranging from $t=0$ to $t\to\infty$. Finally, from (9) and (10), we get

$$I_1=\frac{dq_1}{dt}=-2e^{-\frac{t}{12}} \quad \text{(in A)}$$

$$I_2=\frac{dq_2}{dt}=4e^{-\frac{t}{12}} \quad \text{(in A)}$$

The current through the switch S_2 is given by

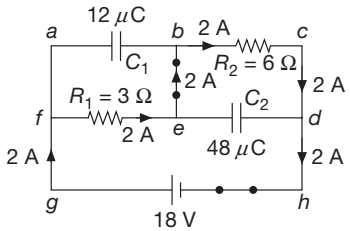
$$I=I_1-I_3=-\frac{q_1}{R_1C_1}+I_2$$

$$\Rightarrow \quad I=-\frac{12\left(1+2e^{-\frac{t}{12}}\right)}{(3)(2)}+48\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow \quad I=-2\left(1+2e^{-\frac{t}{12}}\right)+4e^{-\frac{t}{12}}$$

$$\Rightarrow \quad I=-2 \text{ A}$$

(b) In the steady state, no current will pass through the capacitors. The current and charges on capacitor will be as shown.



Total resistance of circuit is

$$R=9\ \Omega$$

So, current is given by

$$I'=\frac{18}{9}=2 \text{ A}$$

Potential difference across C_1 is

$$V_f-V_e=I'R_1=6 \text{ V}$$

Charge on C_1 is

$$Q_1=(6)(2)=12\ \mu\text{C}$$

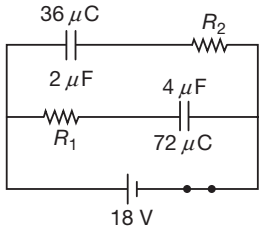
Potential difference across C_2 is

$$V_b-V_c=(6)(2)=12 \text{ V}$$

Charge on C_2 is

$$Q_2=(12)(4)=48\ \mu\text{C}$$

After a long time, when switch S_2 is open and switch S_1 still remains closed then no current is drawn from battery in steady state. Both capacitors are in parallel arrangement with battery, so as to given equivalent capacitance of $(2+4)=6\ \mu\text{F}$



So, $Q_{\text{total}}=\left(C_{\text{eq}}\right)(V)=(6)(18)$

$$\Rightarrow \quad Q_{\text{total}}=108\ \mu\text{C}$$

Hence $Q_{2\ \mu\text{F}}=(2\ \mu\text{F})(18\text{ V})=36\ \mu\text{C}$ and

$Q_{4\ \mu\text{F}}=(4\ \mu\text{F})(18\text{ V})=72\ \mu\text{C}$

Hence, the charge flowing through S_1 is

$$\Delta Q=Q_f-Q_i$$

$$\Rightarrow \quad \Delta Q=(36+72)-(12+48)$$

$$\Rightarrow \quad \Delta Q=48\ \mu\text{C}$$

Total heat dissipated in the resistors is

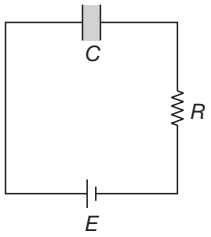
$$\Delta H=\begin{pmatrix}\text{Initial}\\ \text{Energy}\end{pmatrix}+\begin{pmatrix}\text{Work done by}\\ \text{battery when}\\ \text{charge 48 } \mu\text{C}\\ \text{flows through}\\ \text{battery after}\\ \text{switch } S_2\\ \text{is opened}\end{pmatrix}-\begin{pmatrix}\text{Final}\\ \text{Energy}\end{pmatrix}$$

$$\Rightarrow \quad \Delta H=\frac{1}{2}\left(\frac{Q_1^2}{C_1}\right)+\frac{1}{2}\left(\frac{Q_2^2}{C_2}\right)+E(\Delta Q)-\frac{1}{2}(C_1+C_2)V^2$$

$$\Rightarrow \quad \Delta H=136\ \mu\text{J}$$

PROBLEM 11

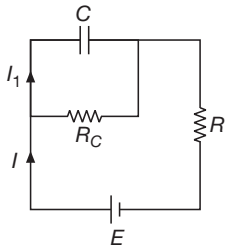
Consider a parallel plate capacitor of capacitance C with partially conducting medium between its plates having a resistance R_C. If this capacitor is connected to a battery of emf E and a resistor R as shown. Find the



(a) charge on the capacitor as a function of time.
(b) current through 'R' as a function of time.

SOLUTION

The equivalent circuit can be drawn as shown here.



Applying Kirchhoff's Law for the two loops,

$$(I - I_1)R_C + IR = E \qquad \ldots(1)$$

$$\frac{q}{C} = R_C(I - I_1) \qquad \ldots(2)$$

(a) Eliminating I between (1) and (2), we get

$$\frac{q}{C}\left(1 + \frac{R}{R_C}\right) + I_1 R = E$$

For capacitor C, $\frac{dq}{dt} = I_1$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{CE - q(1+\alpha)}{RC} \qquad \left\{\because \alpha = \frac{R}{R_C}\right\}$$

(b) Solving, we get $q = \frac{EC}{1+\alpha}\left(1 - e^{-\frac{t(1+\alpha)}{RC}}\right)$

Adding (1) and (2), we get

$$\frac{Q}{C} + IR = E$$

$$\Rightarrow \quad I = \frac{CE - q}{RC}$$

Substituting for q, we get

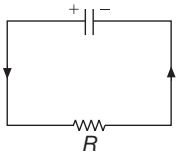
$$I = \frac{E}{R}\left(1 - \frac{1}{1+\alpha}\left(1 - e^{-\frac{t(1+\alpha)}{RC}}\right)\right)$$

PROBLEM 12

A capacitor initially given a charge Q_0 is connected across a resistor R at $t = 0$. The separation d between the plates changes according to the relation

$$d = \frac{d_0}{(1+t)} \qquad (0 \le t < 1)$$

A small bulb is connected across the plates of the capacitor which lights when potential difference across the plates of the capacitor reaches V_0. Find the



(a) variation of charge with time
(b) time when the bulb will light?

SOLUTION

(a) Capacitance at $t = 0$ is given by

$$C_0 = \frac{\varepsilon_0 A}{d_0}$$

and at time t is $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d_0}(1+t) = C_0(1+t)$

Using Kirchhoff's Law, we get

$$\frac{q}{C} - IR = 0$$

$$\Rightarrow \quad \frac{q}{C_0(1+t)} + R\frac{dq}{dt} = 0$$

$$\Rightarrow \quad \frac{dq}{q} = -\frac{1}{RC_0}\frac{dt}{(1+t)}$$

$$\Rightarrow \quad \log_e q\Big|_{Q_0}^{q} = -\frac{1}{RC_0}\log_e(1+t)\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right) = -\frac{1}{RC_0}\log_e(1+t)$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\log_e(1+t)^{-\frac{1}{RC_0}}$$

$$\Rightarrow \quad q=Q_0(1+t)^{-\frac{1}{RC_0}}$$

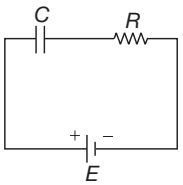
(b) $V=\dfrac{q}{C}=\dfrac{Q_0(1+t)^{-\frac{1}{RC_0}}}{C_0(1+t)}$

$$\Rightarrow \quad V=\frac{Q_0}{C_0}(1+t)^{-\frac{1}{RC_0}+1}$$

$$\Rightarrow \quad t=1-\left(\frac{V_0C_0}{Q_0}\right)^{-\left(\frac{RC_0}{RC_0+1}\right)}$$

PROBLEM 13

In the following RC circuit, the capacitor is in the steady state. The initial separation of the capacitor plates is x_0. If at $t=0$, the separation between the plates starts changing so that a constant current flows through R, find the velocity of the moving plates as a function of time. The plate area is A.



SOLUTION

Let q be the instantaneous charge on the capacitor when a steady current I flows through the circuit. Applying KVL on the circuit, we have

$$E=\frac{q}{C}+IR$$

$$\Rightarrow \quad E=\frac{qx}{\varepsilon_0 A}+IR \qquad \ldots(1)$$

At the instant separation between the plates of the capacitor is x, its capacity $C=\dfrac{\varepsilon_0 A}{x}$

Differentiating equation (1) with respect to time, we get

$$0=\frac{q}{\varepsilon_0 A}\left(\frac{dx}{dt}\right)+\frac{Ix}{\varepsilon_0 A}+0 \qquad \left\{\because I=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad q=-\frac{Ix}{\left(\dfrac{dx}{dt}\right)} \qquad \left(\text{where}\ \frac{dx}{dt}=\text{velocity}\right) \qquad \ldots(2)$$

From equation (2), substituting the value of q in equation (1), we have

$$E=-I\frac{x^2}{\varepsilon_0 A\left(\dfrac{dx}{dt}\right)}+IR$$

$$\Rightarrow \quad \frac{dx}{dt}=v=-\left(\frac{I}{\varepsilon_0 A}\right)\left(\frac{x^2}{E-IR}\right) \qquad \ldots(3)$$

$$\Rightarrow \quad -\frac{dx}{x^2}=\left(\frac{I}{\varepsilon_0 A}\right)\left(\frac{dt}{E-IR}\right) \qquad \ldots(4)$$

Integrating the above expression w.r.t. time, we get

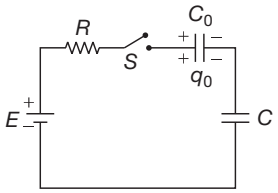
$$\frac{1}{x}-\frac{1}{x_0}=\left(\frac{I}{\varepsilon_0 A(E-IR)}\right)t \qquad \ldots(5)$$

From equations (3) and (5), we get

$$v=\frac{I}{\varepsilon_0 A(IR-E)}\frac{1}{\left[\left(\dfrac{I}{\dfrac{\varepsilon_0 A}{E-IR}}\right)t+\dfrac{1}{x_0}\right]^2}$$

PROBLEM 14

The switch S is closed at $t=0$. The capacitor C is uncharged but C_0 has a charge q_0 at $t=0$. Calculate the current $I(t)$ in the circuit.



SOLUTION

Let q_0 and q be the instantaneous charges on C_0 and C respectively. Applying KVL to the circuit, we have

$$\frac{q_0}{C_0}+\frac{q}{C}+IR=E \qquad \ldots(1)$$

Differentiating this equation, we get

$$\frac{1}{C_0}\frac{dq_0}{dt}+\frac{1}{C}\frac{dq}{dt}+R\frac{dI}{dt}=0$$

$$\Rightarrow \quad I\left(\frac{1}{C_0}+\frac{1}{C}\right)=-R\frac{dI}{dt} \qquad \left\{\text{where } I=\frac{dq_0}{dt}=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \frac{dI}{I}=-\frac{dt}{RC_{eq}} \qquad \left\{\text{where } C_{eq}=\frac{C_0C}{C+C_0}\right\}$$

Integrating this expression, we have

$$\int_{I_0}^{I(t)}\frac{dI}{I}=-\int_0^t\frac{dt}{RC_{eq}}$$

$$\Rightarrow \quad \log_e I\Big|_{I_0}^{I(t)}=-\frac{t}{RC_{eq}}$$

$$\Rightarrow \quad I(t)=I_0e^{-\frac{t}{RC_{eq}}} \qquad \ldots(2)$$

where I_0 is the initial current.

Further, $I_0R+\frac{q_0}{C_0}=E$

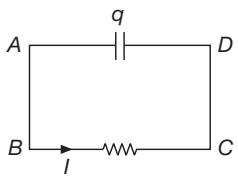
$$\Rightarrow \quad I_0=\frac{\left(E-\frac{q_0}{C_0}\right)}{R} \qquad \ldots(3)$$

Substituting I_0 from equation (3) into equation (2), we get

$$I(t)=\frac{1}{R}\left[E-\frac{q_0}{C_0}\right]e^{-\frac{t}{RC_{eq}}}$$

PROBLEM 15

Plates of a parallel plate capacitor initially charged to Q_0 are connected to a variable resistance R whose resistance is given by $R=R_0+\alpha t$. At time $t=0$, $R=R_0$. Find the current through the capacitor as function of time.



SOLUTION

The resistance varies according to the relation given by

$$R=R_0+\alpha t$$

Applying Kirchhoff's Law to the loop $ABCDA$, at time t, when charge on capacitor is q, we have

$$\frac{q}{C}-IR=0 \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{q}{C}-R\frac{dq}{dt}=0 \qquad \left\{\because\ I=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \int_{Q_0}^{q}\frac{dq}{q}=\int_0^t\frac{dt}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\int_0^t\frac{dt}{(R_0+\alpha t)C}$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\frac{1}{\alpha C}\log_e(R_0+\alpha t)\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\frac{1}{\alpha C}\log_e\left(\frac{R_0+\alpha t}{R_0}\right)$$

$$\Rightarrow \quad \frac{q}{Q_0}=\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

$$\Rightarrow \quad q=Q_0\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

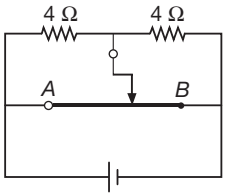
Since $I=\frac{q}{CR}$

$$\Rightarrow \quad I=\frac{Q_0}{RC}\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

$$\Rightarrow \quad I=\frac{Q_0}{C(R_0+\alpha t)}\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

PROBLEM 16

The wire AB of a meter bridge changes linearly from radius r to $2r$ from left end to right end. Where should the free end of the galvanometer be connected on AB so that the deflection in the galvanometer is zero?



SOLUTION

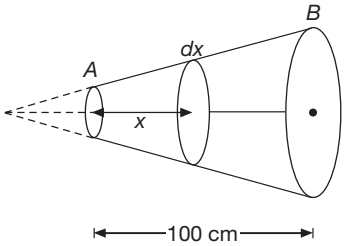
Let the galvanometer be connected at a point $x = x_1$ from end A where $x = 0$.

Let R_1 = resistance of left part, i.e., AX_1 and

R_2 = resistance of right part, i.e., X_1B

Length = 100 cm = 1 m

Consider an element of thickness dx at a distance x from end A and of radius r_x.



Thus, $r_x = \left(r + \frac{r}{1}x\right) = r(1+x)$

Resistance of this element will be, $dR_x = \frac{\rho\, dx}{\pi r_x^2}$

$$R_1 = \int_0^1 \frac{\rho\, dx}{\pi(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[1 - \frac{1}{1+x_1}\right]$$

$$R_2 = \int_1^{\infty} \frac{\rho\, dx}{(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

For null point of zero deflection,

$$\frac{R_1}{R_2} = \frac{4}{4}$$

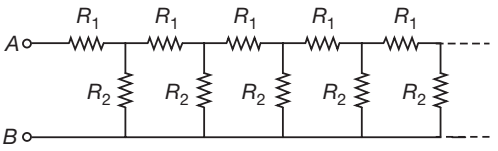
$$\Rightarrow \quad 1 - \frac{1}{1+x_1} = \frac{1}{1+x_1} - \frac{1}{1+1}$$

$$\Rightarrow \quad x_1 = \frac{1}{3}\text{ m} = 33.33\text{ cm}$$

PROBLEM 17

Consider an infinite ladder of network shown. A voltage is applied between points A and B. If the voltage is halved after each section, find the ratio $\frac{R_1}{R_2}$.

Suggest a method to terminate it after a few sections without introducing much error in its attenuation.

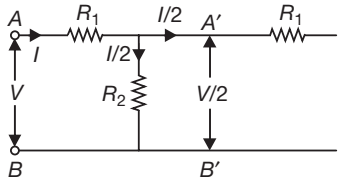


SOLUTION

Voltage across $AB = V$

Voltage across $A'B' = \frac{V}{2}$

Voltage across $R_2 = \frac{V}{2}$



Now from Kirchhoff's Law it is obvious that voltage across must be $\left(V - \frac{V}{2}\right) = \frac{V}{2}$

Now when the voltage is halved then the current is also halved. So, current in R_2 is half of that in R_1

$$\Rightarrow \quad IR_1 = \left(\frac{I}{2}\right)R_2$$

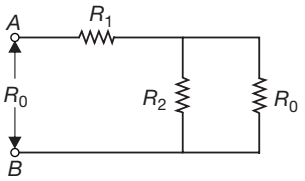
$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{1}{2}$$

Now the attenuation produced by circuit on termination by a resistance will not be affected if equivalent resistance R becomes independent of number of sections in the circuit. This is only possible if the terminating resistance R_0 is itself equal to equivalent resistance as shown. The equivalent resistance of R_0 and R_2 is

$$R' = \frac{R_0 R_2}{R_0 + R_2}$$

R_1 is in series with it, so equivalent resistance between A and B is

$$R_{eq} = R_1 + R' = R_1 + \frac{R_0 R_2}{R_0 + R_2}$$



According to proposition we have

$$R_{eq} = R_0$$

$$R_0 = R_1 + \frac{R_0 R_2}{R_0 + R_2}$$

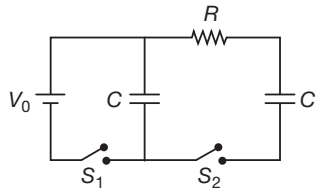
Solving for R_0, we get

$$R_0 = \frac{R_1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

Hence, the circuit may be terminated after a few sections if resistance R_0 is connected in parallel as shown in figure.

PROBLEM 18

In a circuit shown in figure the capacitance of each capacitor is equal to C and the resistance equal to R. One of the capacitance was connected to a voltage V_0 by closing switch S_1. Now at $t = 0$, the switch S_1 is opened and S_2 is closed. Calculate



(a) the current I in the circuit as a function of time t

(b) the amount of heat generated provided the dependence of current $I(t)$ on time t is known.

SOLUTION

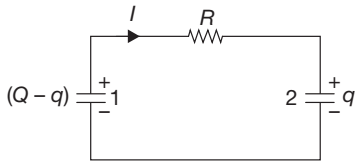
(a) Initial charge on capacitor '1' is $Q = CV_0$

Let charge q flow in the current in time t.

Then current $I = \frac{dq}{dt}$

After time t, charge on capacitor '1' $= (Q - q)$

Charge on capacitor 2 is q



According to Kirchhoff's Loop Law, we have

$$\frac{Q-q}{C} - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad IR = \frac{Q-2q}{C}$$

$$\Rightarrow \quad R\frac{dq}{dt} = \frac{Q-2q}{C}$$

$$\Rightarrow \quad \frac{dq}{Q-2q} = \frac{dt}{RC}$$

$$\Rightarrow \quad \int_0^q \frac{dq}{Q-2q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad \left(\frac{\log_e(Q-2q)}{-2}\right)\Bigg|_0^q = \frac{t}{RC}$$

$$\Rightarrow \quad \log_e(Q-2q) - \log_e Q = -\frac{2t}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{(Q-2q)}{Q}\right) = -\frac{2t}{RC}$$

$$\Rightarrow \quad 1 - \frac{2q}{Q} = e^{-2t/RC}$$

$$\Rightarrow \quad \frac{2q}{Q} = 1 - e^{-2t/RC}$$

$$\Rightarrow \quad q = \frac{Q}{2}\left[1 - e^{-2t/RC}\right]$$

Since, $Q = CV_0$

$$\Rightarrow \quad q = \frac{CV_0}{2}\left[1 - e^{-2t/RC}\right]$$

Current $I = \frac{dq}{dt} = \frac{CV_0}{2}\left[0 + \left(\frac{2}{RC}\right)e^{-2t/RC}\right]$

$$\Rightarrow \quad I = \frac{V_0}{R}e^{-2t/RC}$$

(b) Heat produced is given by

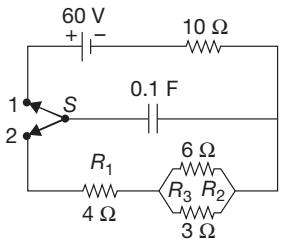
$$H = \int_0^\infty I^2 R dt = \int_0^\infty \left(\frac{V_0}{R} e^{-2t/RC} \right)^2 R dt$$

$$\Rightarrow \quad H = \frac{V_0^2}{R} \int_0^\infty e^{-4t/RC} dt = \frac{V_0^2}{R} \left(\frac{e^{-4t/RC}}{-\frac{4}{RC}} \right) \Bigg|_0^\infty$$

$$\Rightarrow \quad H = -\frac{CV_0^2}{4} \left(e^{-\infty} - e^{-0} \right) = \frac{1}{4} CV_0^2$$

PROBLEM 19

A two way switch S is used in the circuit as shown in figure. First the capacitor is charged by putting the switch in position 1. Now in switch is thrown to position 2. Calculate the heat generated in each resistor.



SOLUTION

Initially the switch S is in position 1. Therefore battery of 60 V is in the circuit. The capacitor begins to collect charge. When capacitor is fully charged, the steady state is reached. In steady state there is no current in $R = 10\ \Omega$ resistance.

Energy stored by capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} 0.1 \times (60)^2 = 180 \text{ J}$$

When switch S is thrown to position 2, the capacitor starts discharging through three resistances $4\ \Omega$ and $6\ \Omega$ and $3\ \Omega$ in parallel.

Let I_1 be the instantaneous current in $4\ \Omega$, I_2 current in $6\ \Omega$ and I_3 current in $3\ \Omega$, then

$$I_2 + I_3 = I_1 \text{ and } 6I_2 = 3I_3$$

$$\Rightarrow \quad I_3 = 2I_2$$

Also, $I_2 + 2I_2 = I_1$

$$\Rightarrow \quad I_2 = \frac{I_1}{3} \text{ and so } I_3 = \frac{2I_1}{3}$$

The ratio of currents

$$I_1 : I_2 : I_3 \equiv I_1 : \frac{I_1}{3} : \frac{2}{3} I_1 = 1 : \frac{1}{3} : \frac{2}{3} = 3 : 1 : 2$$

This ratio remains constant throughout.

Therefore ratio of heat energy dissipated in $R_1 : R_2$ and R_3 is

$$H_1 : H_2 : H_3 = (3)^2 \times 4 : (1)^2 \times 6 : (2)^2 \times 3$$

$$\Rightarrow \quad H_1 : H_2 : H_3 = 6 : 1 : 2$$

If K is ratio constant, then

$$H_1 = 6K,\ H_2 = K \text{ and } H_3 = 2K$$

Also, $H_1 + H_2 + H_3 = 180 \text{ J}$

$$\Rightarrow \quad 6K + K + 2K = 180 \text{ J}$$

$$\Rightarrow \quad K = \frac{180}{9} = 20$$

Heat generated in $R_1 = 4\ \Omega$ is

$$H_1 = 6K = 6 \times 20 = 120 \text{ J}$$

Heat generated in $R_2 = 6\ \Omega$ is

$$H_2 = K = 20 \text{ J}$$

Heat generated in $R_3 = 3\ \Omega$ is

$$H_3 = 2K = 2 \times 20 = 40 \text{ J}$$

During discharging of capacitor, no current flows in $R = 10\ \Omega$, so heat generated in $10\ \Omega$ is zero.

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A conductor of area of cross-section A having charge carriers, each having a charge q is subjected to a potential V. The number density of charge carriers in the conductor is n and the charge carriers along with their random motion are moving with a velocity v. A current I flows in the conductor. If $\vec{j}$ is the current density, then

(A) $|\vec{j}| = nqV$, in the direction of current flow.

(B) $|\vec{j}| = nqv$, in the direction opposite to current flow.

(C) $|\vec{j}| = nqV$, in the direction perpendicular to current flow.

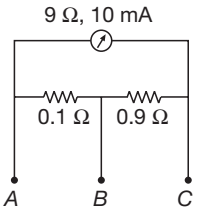
(D) $|\vec{j}| = nqv$, in the direction of current flow.

2. An electron in a hydrogen atom is considered to be revolving around a proton with a velocity $\frac{e^2}{\hbar}$ in a circular orbit of radius $\frac{\hbar^2}{me^2}$. If I is the equivalent current, then

(A) $I = \frac{me^5}{2\pi\hbar^3}$ (B) $I = \frac{me^5}{4\pi\hbar^3}$

(C) $I = \frac{2me^5}{\pi\hbar^3}$ (D) $I = \frac{me^5}{\pi\hbar^3}$

3. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The meter gives full scale deflection for current I when A and B are used as its terminals, i.e. current enters at A and leaves at B (C is left isolated). The value of I is



(A) 100 mA (B) 900 mA

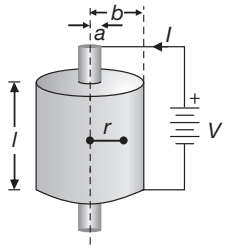
(C) 1 A (D) 1.1 A

4. The charge on a capacitor decreases η times in time t, when it discharges through a circuit with a time constant τ.

(A) $t = \eta\tau$ (B) $t = \tau\log_e\eta$

(C) $t = \tau(\log_e\eta - 1)$ (D) $t = \tau\log_e\left(1 - \frac{1}{\eta}\right)$

5. A metal rod of radius a is concentric with a metal cylindrical shell of radius b and length l. The space between rod and cylinder is tightly packed with a high resistance material of resistivity ρ. A battery having a terminal voltage V is connected across the combination as shown. Neglect resistance of rod and cylinder. If I is the total current in the circuit then,



(A) $I = \frac{lV}{\rho}$ (B) $I = \frac{2\pi lV}{\rho(\ln b - \ln a)}$

(C) $I = \frac{4\pi lV}{\rho(\ln b - \ln a)}$ (D) $I = \frac{lV}{4\pi\rho(\ln b - \ln a)}$

6. N identical light bulbs, each designed to draw P power from a certain voltage supply, are joined in series across that supply. The total power which they will draw is

(A) NP (B) P

(C) $\frac{P}{N}$ (D) $\frac{P}{N^2}$

7. A battery E is connected in series to an ammeter and a voltmeter such that the respective readings are A and V. Now a resistance is joined in parallel with the voltmeter. Then

(A) A increases, V decreases

(B) A decreases, V increases

(C) A and V both decrease

(D) A and V both increase

8. A conductor of area of cross section A having charge carriers, each having a charge q is subjected to a potential V. The number density of charge carriers in the conductor is n and the charge carriers (along with their random motion) are moving with a velocity v. If σ is the conductivity of the conductor and τ is the average relaxation time, then

(A) $\tau = \dfrac{m}{nq^2\sigma}$ (B) $\tau = \dfrac{m\sigma}{nq^2}$

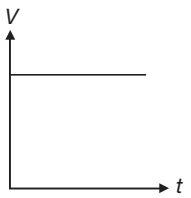
(C) $\tau = \dfrac{2m\sigma}{nq^2}$ (D) $\tau = \dfrac{1}{2}\dfrac{m\sigma}{nq^2}$

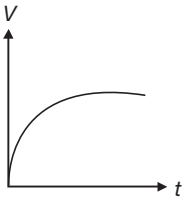
9. A long conductor of charge q, with charge density λ is moving with a velocity $2v$ parallel to its own axis. The conventional current due to motion of conductor is I. Then

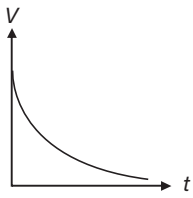
(A) $I = \dfrac{q}{t}$ (B) $I = 2\lambda v$

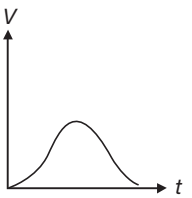
(C) $I = \lambda v$ (D) $I = 3\lambda v$

10. An ideal battery is connected to a capacitor through a voltmeter. The reading V of the voltmeter is plotted against time t. The V-t graph for this is given by

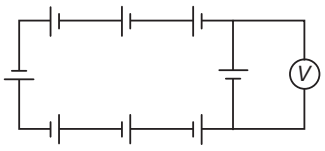
(A) 

(B) 

(C) 

(D) 

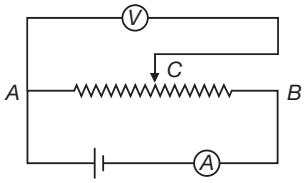
11. In the circuit shown each battery is $5V$ and has an internal resistance of $0.2\ \Omega$. The reading of the voltmeter is V. Then V equals



(A) 5 V (B) 10 V

(C) 15 V (D) ZERO

12. In the given circuit, as the sliding contact C is moved from A to B



(A) the readings of both the ammeter and the voltmeter remain constant

(B) the readings of both the ammeter and the voltmeter increase

(C) the reading of the ammeter remains constant but that of the voltmeter increases

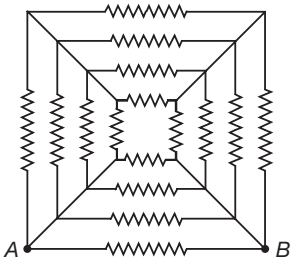
(D) the reading of the ammeter remains constant but that of the voltmeter decreases

13. Two resistances R_1 and R_2 are made of different materials. The temperature coefficient of the material of R_1 is α and that of material of R_2 is $-\beta$. The resistance of the series combination of R_1 and R_2 will not change with temperature if $\dfrac{R_1}{R_2}$ equals

(A) $\dfrac{\alpha}{\beta}$ (B) $\dfrac{\alpha+\beta}{\alpha-\beta}$

(C) $\dfrac{\alpha^2+\beta^2}{2\alpha\beta}$ (D) $\dfrac{\beta}{\alpha}$

14. Sixteen resistors each of resistance $16\ \Omega$ are connected in the circuit as shown. The net resistance between AB is



(A) $1\ \Omega$ (B) $2\ \Omega$

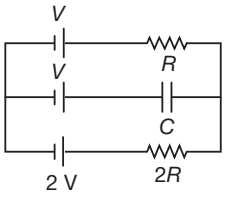
(C) $3\ \Omega$ (D) $4\ \Omega$

15. When an unknown resistance is connected across a series combination of two identical batteries, each of 1.5 V, the current through the resistor is 1.0 A. When it is connected across a parallel combination of the same batteries, the current through it is 0.6 A. The internal resistance of each battery is

(A) $\frac{1}{5}\ \Omega$ (B) $\frac{1}{4}\ \Omega$

(C) $\frac{1}{3}\ \Omega$ (D) $\frac{1}{2}\ \Omega$

16. In the circuit shown in figure, with steady current, the potential drop across the capacitor must be



(A) V (B) $\frac{V}{2}$

(C) $\frac{V}{3}$ (D) $\frac{2V}{3}$

17. A galvanometer with 50 divisions on the scale has a resistance of 25 Ω. A current of 2×10^{-4} A gives a deflection of one scale division. The additional series resistance required to convert it into a voltmeter reading up to 25 V is

(A) 1200 Ω (B) 1225 Ω

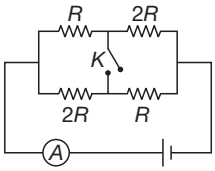
(C) 2475 Ω (D) 2500 Ω

18. A cell has an emf of 1.5 V. When short circuited, it gives a current of 3 A. The internal resistance of the cell is

(A) 0.5 Ω (B) 2.0 Ω

(C) 4.5 Ω (D) $\frac{1}{4.5}\ \Omega$

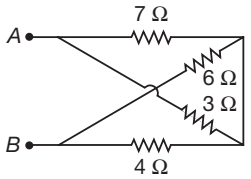
19. The ratio of currents as measured by ammeter in two cases (when the key is open and when the key is closed) is



(A) $\frac{9}{8}$ (B) $\frac{10}{11}$

(C) $\frac{8}{9}$ (D) None of these

20. The equivalent resistance between A and B (of the circuit shown) is



(A) 4.5 Ω (B) 12 Ω

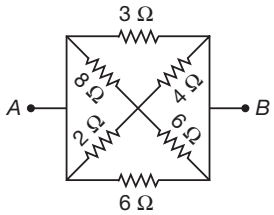
(C) 5.4 Ω (D) 20 Ω

21. When a resistor of 20 Ω is connected across a battery, the current is 0.5 A. When a resistor of 10 Ω is connected across the same battery, the current is 0.8 A. The emf and internal resistance of the battery are

(A) 10 V, $\frac{10}{3}\ \Omega$ (B) 20 V, $\frac{20}{3}\ \Omega$

(C) $\frac{40}{3}$ V, $\frac{10}{3}\ \Omega$ (D) $\frac{40}{3}$ V, $\frac{20}{3}\ \Omega$

22. In the network shown, the equivalent resistance between A and B is



(A) $\frac{4}{3}\ \Omega$ (B) $\frac{3}{4}\ \Omega$

(C) $\frac{24}{17}\ \Omega$ (D) $\frac{17}{24}\ \Omega$

23. A voltmeter of range 1 V has a resistance of 1000 Ω. To extend the range to 10 V, the additional series resistance required is

(A) $\frac{1000}{9}\ \Omega$ (B) 1000 Ω

(C) 9000 Ω (D) 10000 Ω

24. A primary cell has an emf of 1.5 V. When a 5 Ω resistor is connected across it, the current is 0.2 A. The internal resistance of the cell is

(A) 0.5 Ω (B) 1.25 Ω

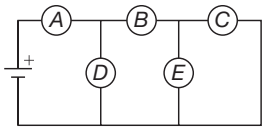
(C) 2.5 Ω (D) 3.0 Ω

25. Three resistors of 2 Ω, 3 Ω and 5 Ω are connected in parallel across a battery of 10 V and of negligible internal resistance. The potential difference across the 3 Ω resistor is

(A) 2 V (B) 3 V
(C) 5 V (D) 10 V

26. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and an accumulator of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a 40 cm length of the potentiometer wire. The value of the external resistance is
(A) 395 Ω (B) 790 Ω
(C) 405 Ω (D) 810 Ω

27. In the circuit diagram shown in figure, a fuse bulb can cause all other bulbs to go out. Identify the bulb

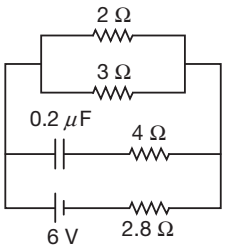


(A) B (B) C
(C) A (D) D or E

28. Two coils when connected in series have an equivalent resistance of 18 Ω and when connected in parallel have a resistance of 4 Ω. The possible values of resistances for the coils is/are
(A) $R_1 = 4\ \Omega;\ R_2 = 14\ \Omega$
(B) $R_1 = 2\ \Omega,\ R_2 = 16\ \Omega$
(C) $R_1 = 12\ \Omega;\ R_2 = 6\ \Omega$
(D) $R_1 = 8\ \Omega,\ R_2 = 10\ \Omega$

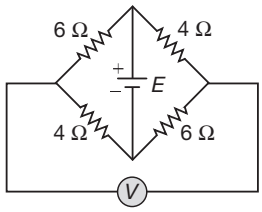
29. If a wire is stretched to make it 0.1% longer, the percentage change in its resistance would be
(A) ZERO (B) 0.1%
(C) 0.2% (D) 0.4%

30. In the given circuit the steady state current through the 2 Ω resistor is



(A) 0.6 A (B) 0.9 A
(C) 1.2 A (D) 1.5 A

31. In the circuit shown, the voltmeter has a large resistance. The emf of the cell is E. The reading of the voltmeter is

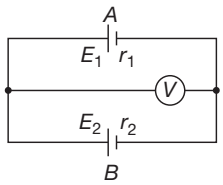


(A) zero (B) $\frac{E}{10}$
(C) $\frac{E}{5}$ (D) $\frac{E}{2}$

32. N identical cells, each of emf E and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is ΔV. Then
(A) $\Delta V = E$ (B) $\Delta V = \frac{E}{N}$
(C) $\Delta V = \left(\frac{N-1}{N}\right)E$ (D) $\Delta V = 0$

33. A non-conducting ring of radius R has two positive point charges lying diametrically opposite to each other, each of magnitude Q. The ring rotates with an angular velocity ω. If I is the equivalent current then,
(A) $I = Q\omega$ (B) $I = \frac{2Q\omega}{\pi}$
(C) $I = \frac{Q\omega}{\pi}$ (D) $I =$ ZERO

34. Two cells A and B of emf 1.3 V and 1.5 V respectively are arranged as shown in figure. The voltmeter reads 1.45 V. The voltmeter is assumed to be ideal. Then



(A) $r_1 = 2r_2$ (B) $r_1 = 3r_2$
(C) $r_2 = 2r_1$ (D) $r_2 = 3r_1$

35. A straight conductor of uniform cross-section carries a time varying current which varies at the rate $\frac{dI}{dt} = \dot{I}$. If s is the specific charge that is carried by each charge carrier of the conductor and l is the length of the conductor then the total force experienced by all the charge carriers per unit length of the conductor due to their drift velocities only is

(A) $F = \dot{I}s$ (B) $F = \dfrac{\dot{I}}{2\sqrt{Is}}$

(C) $F = \dfrac{\dot{I}}{s}$ (D) $F = \dfrac{2I\dot{I}}{s}$

36. A milliammeter of range 10 mA has a coil of resistance 1 Ω. To use it as an ammeter of range 1 A, the required shunt must have a resistance of

(A) $\dfrac{1}{101}\Omega$, connected in series

(B) $\dfrac{1}{100}\Omega$, connected in parallel

(C) $\dfrac{1}{99}\Omega$, connected in parallel

(D) $\dfrac{1}{9}\Omega$, connected in series

37. To use the milliammeter of PROBLEM 36 as a voltmeter of range 10 V, a resistance R is placed in series with it. The value of R is

(A) 9 Ω, connected in parallel

(B) 99 Ω, connected in series

(C) 999 Ω, connected in series

(D) 1000 Ω, connected in parallel

38. N identical cells, each of emf E and internal resistance r, are joined in series to form a closed circuit. It is observed that mistakenly one cell has been joined with its polarity reversed. The potential difference across each cell, except the wrongly connected one is ΔV. Then

(A) $\Delta V = \dfrac{2E}{N}$ (B) $\Delta V = \left(\dfrac{N-1}{N}\right)E$

(C) $\Delta V = \left(\dfrac{N-2}{N}\right)E$ (D) $\Delta V = \left(\dfrac{2N}{N-2}\right)E$

39. A rod of length L has charge distributed on it such that linear charge density $\lambda = kx$, k is a constant and x is the distance of a point from the end of the rod where no charge exists. The rod has a mass m and the mass distribution is uniform. The rod is now rotated about an axis passing through the end of the rod where no charge exists and this axis is perpendicular to the length of the rod. If I is the equivalent current then,

(A) $I = \dfrac{kL^2\omega}{\pi}$ (B) $I = \pi kL\omega$

(C) $I = \dfrac{kL^2\omega}{4\pi}$ (D) $I =$ zero

40. N identical cells, each of emf E_0 and internal resistance r_0, are joined in series. Out of these N cells, n cells are wrongly connected. Assuming $n < \dfrac{1}{2}N$, E and r to be the respective net emf and internal resistance of the resulting battery, then

(A) $E = (N-n)E_0,\ r = Nr_0$

(B) $E = (N-n)E_0,\ r = (N-n)r_0$

(C) $E = (N-2n)E_0,\ r = (N-2n)r_0$

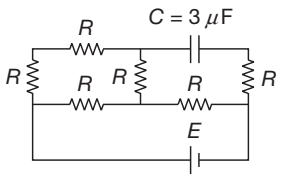
(D) $E = (N-2n)E_0,\ r = Nr_0$

41. A block of metal is made in the cuboid form with all edges of unequal length. The shortest length is one-third the longest one. If R_{max} and R_{min} are the maximum and minimum resistance between parallel faces then,

(A) $\dfrac{R_{max}}{R_{min}} = 4$ (B) $\dfrac{R_{max}}{R_{min}} = 9$

(C) $\dfrac{R_{max}}{R_{min}} = 3$ (D) Data Insufficient

42. In the circuit, the potential difference across the capacitor is 10 V. Each resistance is of 3 Ω. The cell is ideal. The emf of the cell is



(A) 14 V (B) 16 V

(C) 18 V (D) 24 V

43. Three copper wires have their lengths in the ratio 5 : 3 : 1 and their masses are in the ratio 1 : 3 : 5. Their electrical resistance will be in the ratio

(A) 5 : 3 : 1 (B) 1 : 3 : 5

(C) 125 : 15 : 1 (D) 1 : 15 : 125

44. It is observed that a milliammeter of range 10 mA gives full-scale deflection for a current of 100 mA, when a shunt of 0.1 Ω is connected in parallel to it. The coil of the milliammeter has a resistance of

(A) 0.11 Ω (B) 1.1 Ω

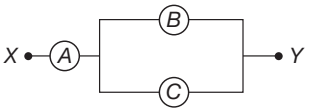
(C) 1 Ω (D) 0.9 Ω

45. The external resistance of a circuit is η times higher than the internal resistance of the source. The ratio of the potential difference across the terminals of the source to its e.m.f. is

(A) $\frac{1}{\eta}$ (B) η

(C) $\frac{1+\eta}{\eta}$ (D) $\frac{\eta}{1+\eta}$

46. Three voltmeters A, B and C, connected as shown, have resistances R, $\frac{3}{2}R$ and $3R$ respectively. When some potential difference is applied between X and Y, the respective voltmeter readings are V_A, V_B and V_C. Then



(A) $V_A \neq V_B = V_C$ (B) $V_B \neq V_A = V_C$

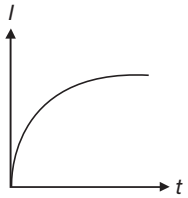
(C) $V_A = V_B \neq V_C$ (D) $V_A = V_B = V_C$

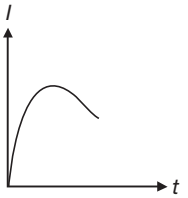
47. The terminal potential difference of a cell is 2.2 V (when circuit is open) and reduces to 1.8 V (when the cell is connected to a resistance $R = 5\ \Omega$). The internal resistance of cell (r) is

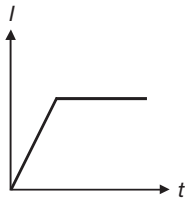
(A) $\frac{10}{9}\ \Omega$ (B) $\frac{9}{10}\ \Omega$

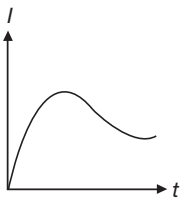
(C) $\frac{11}{9}\ \Omega$ (D) $\frac{5}{9}\ \Omega$

48. The current flowing through a running electric heater, I, is plotted against the time t. Taking into account the variation of resistance with temperature, which of the following curve best represents the variation of I with t?

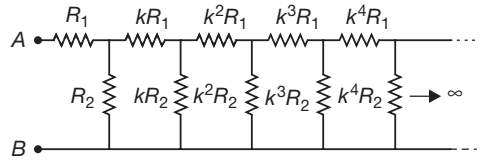
(A) 

(B) 

(C) 

(D) 

49. The circuit diagram shown consists of a large number of elements (each element has two resistors R_1 and R_2). The resistances of the resistors in each subsequent element differs by a factor of $k = \frac{1}{2}$ from the resistances of the resistors in the previous elements. The equivalent resistance between A and B shown in figure is



(A) $\frac{R_1 - R_2}{2}$

(B) $\frac{(R_1 - R_2) + \sqrt{6R_1R_2}}{2}$

(C) $\frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

(D) None of these

50. A capacitor of capacitance C has charge Q. It is connected to an identical capacitor through a resistance. The heat produced in the resistance is

(A) $\frac{1}{2}\left(\frac{Q^2}{C}\right)$

(B) $\frac{1}{4}\left(\frac{Q^2}{C}\right)$

(C) $\frac{1}{8}\left(\frac{Q^2}{C}\right)$

(D) dependent on the value of the resistance.

51. A and B are two points on a uniform ring of resistance R. The $\angle ACB = \theta$, where C is the centre of the ring. The equivalent resistance between A and B is

(A) $\frac{R}{4\pi^2}(2\pi - \theta)\theta$ (B) $R\left(1 - \frac{\theta}{2\pi}\right)$

(C) $R\left(\frac{\theta}{2\pi}\right)$ (D) $R\left(\frac{2\pi - \theta}{4\pi}\right)$

52. A copper wire of length l and radius r is nickel-plated till its final radius is $2r$. If the resistivities of the copper and nickel are ρ_C and ρ_N, then the equivalent resistance of wire is

(A) $\frac{\rho_C l}{\pi r^2}$ (B) $\frac{\rho_N l}{4\pi r^2}$

(C) $\frac{l}{\pi r^2\left[\frac{1}{\rho_C} + \frac{3}{\rho_N}\right]}$ (D) $\frac{l}{\pi r^2}\left(\rho_C + \frac{\rho_N}{3}\right)$

53. A battery of e.m.f. E, internal resistance r is applied in the circuit in a closed loop. The current I in the circuit goes from the positive terminal of the battery to the negative terminal inside it. The potential drop across the battery is

(A) $V = E - Ir$ (B) $V = E + Ir$

(C) $V = +Ir$ (D) $V = -Ir$

54. Carbon resistors used in electronic circuits are marked for their resistance values and tolerance by a colour scheme. A given resistor has colour scheme brown, black, green and gold. Its value in Ω is

(A) $3.2 \times 10^5 \pm 5\%$ (B) $1.0 \times 10^6 \pm 10\%$

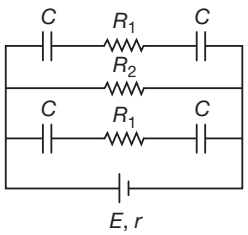
(C) $1.0 \times 10^6 \pm 5\%$ (D) $1.0 \times 10^3 \pm 5\%$

55. Two wires of the same material and the same length are connected to the two gaps of a metre bridge. The balancing length measured from left is 25 cm. The ratio radius of the wire is the left gap to that in the right gap is

(A) 1.73 (B) 3

(C) 0.577 (D) $\frac{1}{3}$

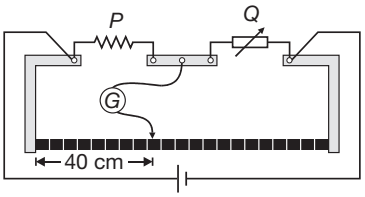
56. In the adjoining circuit $E = 5\,\text{V}$, $r = 1\,\Omega$, $R_1 = 1\,\Omega$, $R_2 = 4\,\Omega$, $C = 3\,\mu\text{F}$. Then the numerical value of charge on each plate of capacitors is



(A) $3\,\mu\text{C}$ (B) $6\,\mu\text{C}$

(C) $12\,\mu\text{C}$ (D) $24\,\mu\text{C}$

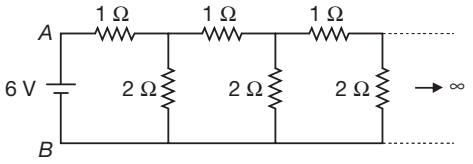
57. In a metre bridge, the gaps are closed by two resistances P and Q and the balance point is obtained at 40 cm. When Q is shunted by a resistance of 10 Ω, the balance point shifts to 50 cm. The values of P and Q are



(A) $\frac{10}{3}\,\Omega, 5\,\Omega$ (B) $20\,\Omega, 30\,\Omega$

(C) $10\,\Omega, 15\,\Omega$ (D) $5\,\Omega, \frac{15}{2}\,\Omega$

58. An infinite ladder network is constructed with 1 Ω and 2 Ω resistors as shown. The effective resistance between A and B is



(A) 1 Ω (B) 2 Ω

(C) 3 Ω (D) 4 Ω

59. In PROBLEM 58, the current through the 2 Ω resistor nearest to the battery is

(A) 1.5 A (B) 2.0 A

(C) 3.0 A (D) 4.0 A

60. Three conductors draw respectively currents of 1 A, 2 A and 4 A when connected in turn across a battery. If they are connected in series across the same battery, The current drawn will be

(A) $\frac{2}{7}$ A (B) $\frac{3}{7}$ A

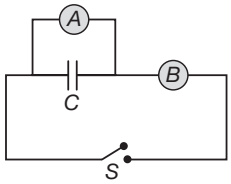
(C) $\frac{4}{7}$ A (D) $\frac{5}{7}$ A

61. Two resistances r_1 and $r_2 (r_1 < r_2)$ are connected in parallel. The equivalent resistance R is

(A) $R > r_1 + r_2$ (B) $r_2 < R < r_1 + r_2$

(C) $r_1 < R < r_2$ (D) $R < r_1$

62. In the circuit shown, the capacitor having capacitance C is connected to two voltmeters A and B. Voltmeter A being ideal, having infinite resistance, while voltmeter B has a resistance R. The capacitor is charged and then the switch S is closed. Both A and B show equal readings at time t. Then this is possible for



(A) $t \to \infty$ (B) $t = RC$

(C) $t = RC\log_e(2)$ (D) all the t values

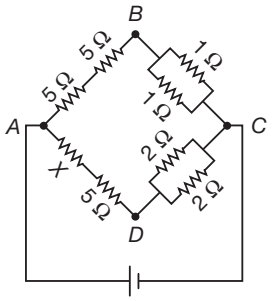
63. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

(A) 1154 K (B) 1100 K
(C) 1400 K (D) 1127 K

64. Two resistors of 500 Ω and 300 Ω are connected in series with a battery of emf 20 V. A voltmeter of resistance 500 Ω is used to measure the potential difference across the 500 Ω resistor. The error in the measurement is

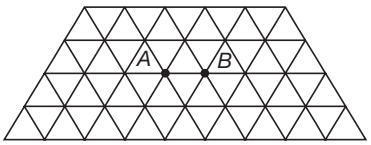
(A) 1.4 V (B) 2.4 V
(C) 3.4 V (D) 4.4 V

65. In the given bridge the value of X for which the potential difference between the points B and D will be zero, is



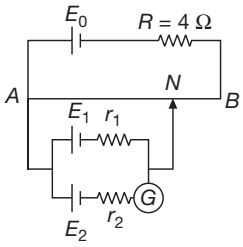
(A) 5 Ω (B) 10 Ω
(C) 15 Ω (D) 20 Ω

66. There is an infinite wire grid with cells in the form of equilateral triangles. The resistance of each wire between neighbouring joint connections is R_0. The net resistance of the whole grid between the points A and B as shown is



(A) R_0 (B) $\frac{R_0}{2}$
(C) $\frac{R_0}{3}$ (D) $\frac{R_0}{4}$

67. A battery of emf $E_0 = 12$ V is connected across a 4 m long uniform wire having resistance 4 Ωm^{-1}. The cell of small emfs $E_1 = 2$ V and $E_2 = 4$ V having internal resistance 2 Ω and 6 Ω respectively are connected as shown in the figure. If galvanometer shows no deflection at the point N, the distance of point N from the point A is equal to



(A) $\frac{5}{3}$m (B) $\frac{4}{3}$m
(C) $\frac{3}{2}$m (D) None of these

68. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V respectively. If a resistance is now joined in parallel with the voltmeter,

(A) both A and V will increase.
(B) both A and V will decrease.
(C) A will decrease, V will increase.
(D) A will increase, V will decrease.

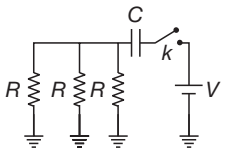
69. A nonconducting ring of radius R has charge Q distributed unevenly over it. If it rotates with an angular velocity ω, the equivalent current will be

(A) ZERO (B) $Q\omega$
(C) $\frac{Q\omega}{2\pi}$ (D) $\frac{Q\omega}{2\pi R}$

70. Two wires of the same metal has the same length but their cross-sections are in the ratio 3 : 1. They are joined in series. The resistance of the thicker wire is 10 Ω. The total resistance of the combination will be

(A) 40 Ω (B) $\frac{40}{3}$ Ω
(C) $\frac{5}{2}$ Ω (D) 100 Ω

71. The switch shown in the figure is closed at $t = 0$. The charge on the capacitor as a function of time is given by

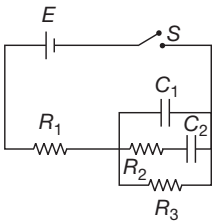


(A) $CV\left(1-e^{-\frac{t}{RC}}\right)$ (B) $3CV\left(1-e^{-\frac{t}{RC}}\right)$
(C) $CV\left(1-e^{-\frac{3t}{RC}}\right)$ (D) $CV\left(1-e^{-\frac{t}{3RC}}\right)$

72. Two electric bulbs A and B having power ratings P_A and P_B, $(<P_A)$ respectively are designed for the same voltage. They are joined in series across a supply of voltage V. Then

(A) A and B will draw the same power.
(B) A will draw more power than B.
(C) B will draw more power than A.
(D) the ratio of powers drawn by them will depend on V.

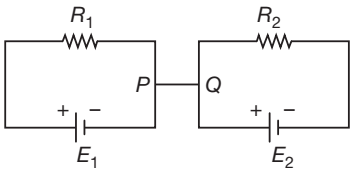
73. In the circuit diagram, the current through the battery immediately after the switch S is closed is



(A) zero (B) $\frac{E}{R_1}$

(C) $\frac{E}{R_1+R_2}$ (D) $\frac{E}{R_1+\frac{R_2R_3}{R_2+R_3}}$

74. In the circuit shown, the conductor PQ is of negligible resistance. Then



(A) current will flow through PQ if $\frac{E_1+E_2}{R_1+R_2}\neq\frac{|E_1-E_2|}{R_1-R_2}$

(B) current will flow through PQ if $E_1\neq E_2$

(C) no current will flow through PQ

(D) current will flow through PQ if $\frac{E_1}{R_1}\neq\frac{E_2}{R_2}$

75. A fuse wire of radius r, resistivity ρ, length l has a current I passing through it. If ε is the emissivity of the fuse wire and T is the excess safe temperature above the surroundings then

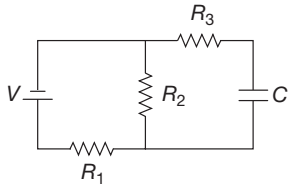
(A) $T=\frac{I^2\rho}{\pi^2\varepsilon r^3}$ (B) $T=\frac{I^2\rho}{2\pi^2\varepsilon r^3}$

(C) $T=\frac{2I^2\rho}{\pi^2\varepsilon r^3}$ (D) $T=\frac{4I^2\rho}{\pi^2\varepsilon r^3}$

76. A battery is formed by connecting N identical cells together. On short circuiting the terminals of the battery a current I flows in the circuit. To maximise the value of I,

(A) two rows of $\frac{N}{2}$ cells each are connected in parallel.
(B) $\sqrt{N}$ rows of $\sqrt{N}$ cells each are connected in parallel, given that $\sqrt{N}$ is an integer.
(C) all the cells are connected in series.
(D) all the cells are connected in parallel.

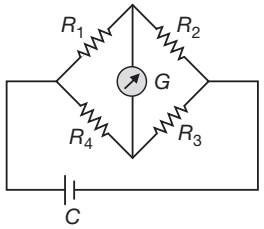
77. In the circuit shown in figure, the steady state voltage drop across the capacitor is V_C. Then



(A) $V_C=\frac{VR_1}{R_2+R_3}$ (B) $V_C=\frac{VR_2}{R_1+R_3}$

(C) $V_C=\frac{VR_1}{R_1+R_2}$ (D) $V_C=\frac{VR_2}{R_1+R_2}$

78. In a balanced Wheatstone bridge shown, if the positions of the battery E and the galvanometer G are interchanged, then G will show zero deflection



(A) only if $\frac{R_1}{R_3}=\frac{R_2}{R_4}$.

(B) only if $R_1=R_3$ and $R_2=R_4$.

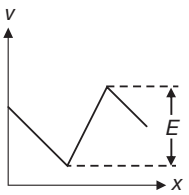
(C) only if all the resistances are equal.
(D) in all cases.

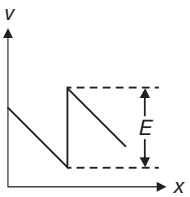
79. A cell develops the same power across two resistors r_1 and r_2 when connected separately. If r is the internal resistance of the cell then

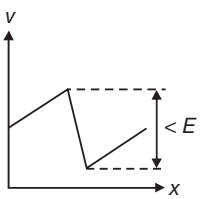
(A) $r=\frac{1}{2}\sqrt{r_1r_2}$ (B) $r=\sqrt{r_1r_2}$

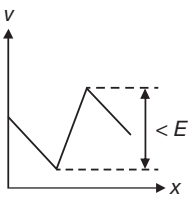
(C) $r=\frac{1}{2}(r_1+r_2)$ (D) $r=r_1+r_2$

80. A uniform conductor has its two free ends joined to a cell of emf E and some non zero internal resistance. Consider a point P right at the middle of the conductor. Let us now move away from the midpoint of the conductor in the direction of the current and then return to P. During this process we study the variation of the potential V at every point on the path and then plot it against the distance covered (x). The curve that best represents the variation of V with x is

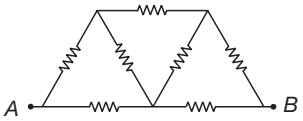
(A) 

(B) 

(C) 

(D) 

81. Seven resistors each of resistance R, are connected as shown in figure. The equivalent resistance between A and B is



(A) $\frac{4}{3}R$ (B) $\frac{3}{2}R$

(C) $7R$ (D) $\frac{8}{7}R$

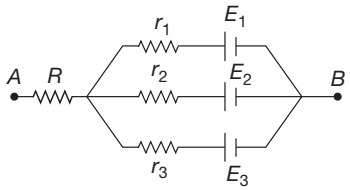
82. A cell of emf E and internal resistance r has its terminals connected to a resistance R such that the potential difference between the terminals is 1.6 V when $R = 4\ \Omega$, and 1.8 V when $R = 9\ \Omega$. Then,

(A) $E = 1$ V, $r = 1\ \Omega$ (B) $E = 2.5$ V, $r = 0.5\ \Omega$

(C) $E = 2$ V, $r = 1\ \Omega$ (D) $E = 2$ V, $= 2\ \Omega$

83. In the network shown the potential difference between A and B for the values

$R = r_1 = r_2 = r_3 = 1\ \Omega$, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V is



(A) 1 V (B) 2 V

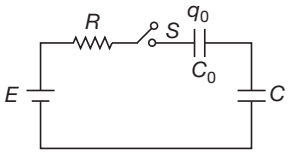
(C) 3 V (D) 4 V

84. The capacitive growth current and the capacitive decay current in series RC circuit are I_g and I_d respectively. If I_0 is the value of growth current as $t \to \infty$ then at any instant of time

(A) $I_g = I_d$ (B) $I_g = I_0$

(C) $I_g = -I_d$ (D) $I_d = -I_0$

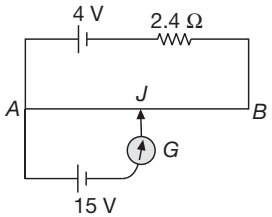
85. In the circuit shown, the switch S is closed initially. The capacitor C is uncharged but C_0 has a charge q_0 at $t = 0$. The capacitive current immediately after switch S is closed is I_C. Then



(A) $I_C = 0$ (B) $I_C \to \infty$

(C) $I_C = \dfrac{\left(E - \frac{q_0}{C_0}\right)}{R}$ (D) $I_C = \dfrac{E}{R}$

86. A simple potentiometer circuit is shown in figure.

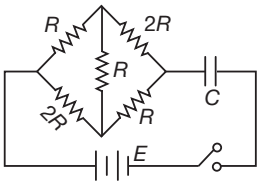


The internal resistance of 4 V battery is negligible. AC is the uniform wire of length 100 cm and resistance 2 Ω. What would be the length of AB for which galvanometer shows zero deflection

(A) 82.5 cm (B) 41.50 cm

(C) 165 cm (D) 50 cm

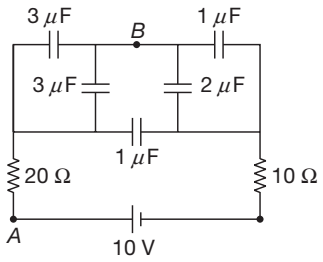
87. In the circuit shown the steady state value is reached at time t_0. Then



(A) $t_0 \to \infty$ (B) $t_0 = 7RC$

(C) $t_0 = \frac{7}{5}RC$ (D) $t_0 = \frac{5}{7}RC$

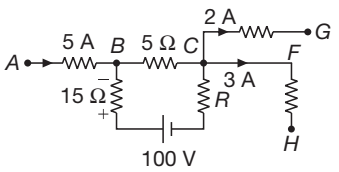
88. In the circuit shown in figure, the potential difference between the points A and B in the steady state is



(A) zero (B) 6 V

(C) 4 V (D) $\frac{10}{3}$ V

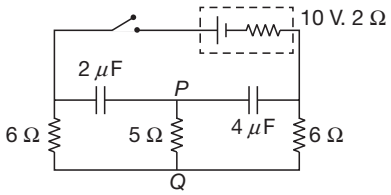
89. In the circuit shown, the voltage drop across the 15 Ω resistor is 30 V having the polarity as indicated. The ratio of potential difference across 5 Ω resistor and resistance R is



(A) $\frac{2}{7}$ (B) 0.4

(C) $\frac{5}{7}$ (D) 1

90. In the circuit shown in figure, the capacitors are initially uncharged. The current through resistor PQ just after closing the switch is



(A) 2 A from P to Q

(B) 2 A from Q to P

(C) 6 A from P to Q

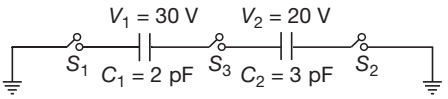
(D) zero

91. In a metre bridge, a standard resistor of R Ω is connected in the left gap and the two wires A and B are connected one after the other in the right gap. The balancing length measured from the left is 50 cm for either of them. If the two wires are connected in series and put in the right gap, balancing length measured from left would be (in cm)

(A) 25 (B) 33.3

(C) 66.7 (D) 75

92. For the circuit shown in figure, which of the following statement(s) is true?



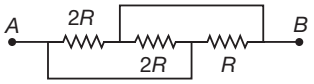
(A) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V.

(B) With S_3 closed, $V_1 = V_2 = 25$ V.

(C) With S_1 and S_2 closed $V_1 = V_2 = ZERO$.

(D) With S_1 and S_3 closed $V_1 = 30$ V, $V_2 = 20$ V.

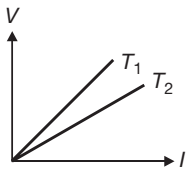
93. The equivalent resistance between points A and B in the circuit shown is



(A) R (B) $\frac{R}{2}$

(C) $\frac{R}{4}$ (D) $\frac{R}{8}$

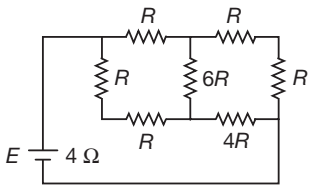
94. The voltage V and current I graph for a conductor at two different temperatures T_1 and T_2 are shown in figure. The relation between T_1 and T_2 is



(A) $T_1 > T_2$ (B) $T_1 < T_2$

(C) $T_1 = T_2$ (D) $T_1 \approx T_2$

95. A battery of internal resistance 4 Ω is connected to a network of resistances as shown. In order that the maximum power can be delivered to the network, the value of R in ohm should be



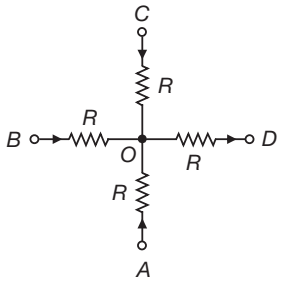
(A) $\frac{4}{9}$ (B) 2

(C) $\frac{8}{3}$ (D) 18

96. Two cells, having emfs E_1 and $E_2 (E_1 > E_2)$, when placed in series produce null deflection at a distance of 204 cm in a potentiometer. When placed in opposition, they produce null deflection at a distance of 36 cm. If $E_2 = 1.4$ V, E_1 is

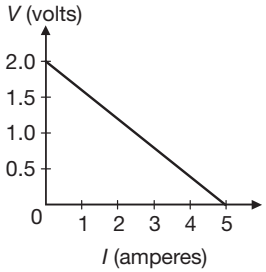
(A) 14 V (B) 10 V
(C) 4.2 V (D) 2 V

97. The given four terminal network is part of a larger circuit. The points A, B, C are at the same potential. The potential difference between any one of A, B or C and D is 40 V. The potential difference between A and O is



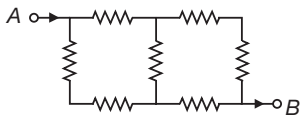
(A) 10 V (B) 15 V
(C) 18 V (D) 20 V

98. For a cell, the graph between the potential difference (V) across the terminals of the cell and the current (I) drawn from the cell is shown in the figure. The e.m.f. and the internal resistance of the cell are



(A) 2 V, 0.5 Ω (B) 2 V, 0.4 Ω
(C) >2 V, 0.5 Ω (D) >2 V, 0.4 Ω

99. Seven resistors each of value 5 Ω are connected as shown. The equivalent resistance between the points A and B is



(A) 5 Ω (B) 7 Ω
(C) 14 Ω (D) 35 Ω

100. A galvanometer can be converted into a voltmeter or an ammeter by using either of the two resistances R_1 and $R_2 (R_1 \gg R_2)$

(A) R_1 in series with the galvanometer for voltmeter and R_2 in parallel for ammeter

(B) R_1 in parallel with the galvanometer for voltmeter and R_2 in series for ammeter

(C) R_2 in series with the galvanometer for voltmeter and R_1 in parallel for ammeter

(D) R_2 in parallel with the galvanometer for voltmeter and R_1 in series for ammeter

101. All the edges of a block with parallel faces are unequal. Its longest edge is thrice its shortest edge. The ratio of the maximum to minimum resistance between parallel faces is

(A) 3
(B) 6
(C) 9
(D) indeterminate unless the length of the third edge is specified

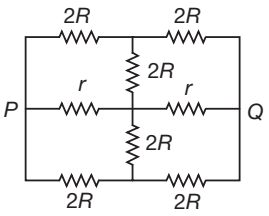
102. A 2.0 V potentiometer is used to determine the internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 75 cm. When a resistor of 10 Ω is connected across the cell, the balance point shifts to 60 cm. The internal resistance of the cell is

(A) 1.5 Ω (B) 2.5 Ω
(C) 3.5 Ω (D) 4.5 Ω

103. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with resistance of 2950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the above series resistance should be

(A) 4450 Ω (B) 5050 Ω
(C) 5550 Ω (D) 6050 Ω

104. The effective resistance between points P and Q of the electrical circuit shown in figure is



(A) $\frac{2Rr}{R+r}$ (B) $\frac{8R(R+r)}{3R+r}$

(C) $2r+4R$ (D) $\frac{5R}{2}+2r$

105. Five cells, each of e.m.f. E and internal resistance r, are connected in series. If by mistake, one of the cells is connected wrongly, the equivalent e.m.f. and internal resistance of the combination are

(A) $5E$, $5r$ (B) $3E$, $5r$

(C) $5E$, $3r$ (D) $3E$, $3r$

106. The e.m.f. and the internal resistance of a source which is equivalent to two batteries which are connected in parallel having e.m.f.'s E_1 and E_2 and internal resistances r_1 and r_2 respectively are E and r.

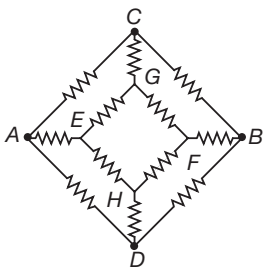
(A) $E=\frac{E_1r_1+E_2r_2}{(r_1+r_2)}$; $r=r_1+r_2$

(B) $E=\frac{E_1r_2+E_2r_1}{(r_1+r_2)}$; $r=r_1+r_2$

(C) $E=\frac{E_1r_2+E_2r_1}{(r_1+r_2)}$; $r=\frac{r_1r_2}{r_1+r_2}$

(D) $E=\frac{E_1r_1+E_2r_2}{(r_1+r_2)}$; $r=\frac{r_1r_2}{r_1+r_2}$

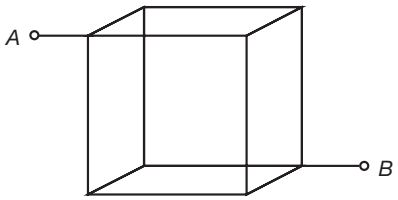
107. Twelve identical resistors each of value $1\,\Omega$ are connected as shown. Net resistance between C and D is R



(A) $R=\frac{7}{6}\,\Omega$ (B) $R=\frac{4}{3}\,\Omega$

(C) $R=1\,\Omega$ (D) $R=\frac{3}{4}\,\Omega$

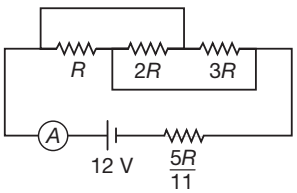
108. Twelve wires, each of resistance R, are connected to form a cube as shown in the figure. The effective resistance between A and B is



(A) $\frac{R}{6}$ (B) $\frac{R}{3}$

(C) $\frac{5R}{3}$ (D) $\frac{5R}{6}$

109. In the circuit shown the reading of ammeter is 2 A. The ammeter has negligible resistance. The value of R equals.



(A) $2\,\Omega$ (B) $4\,\Omega$

(C) $6\,\Omega$ (D) $8\,\Omega$

110. The deflection in a galvanometer falls from 50 divisions to 20 divisions when a $12\,\Omega$ shunt is applied. The galvanometer resistance is

(A) $18\,\Omega$ (B) $24\,\Omega$

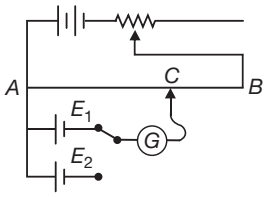
(C) $30\,\Omega$ (D) $36\,\Omega$

111. A straight conductor of uniform cross-section carries a current I. If s be the specific charge of an electron. The momentum of all the free electrons per unit length of the conductor, due to their drift velocities only, is

(A) Is (B) $\frac{I}{s}$

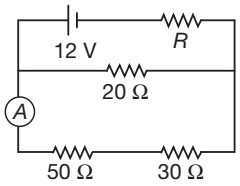
(C) $\sqrt{\frac{I}{s}}$ (D) $\left(\frac{I}{s}\right)^2$

112. The circuit shown here is used to compare the e.m.f.'s of the cells E_1 and $E_2(E_1>E_2)$. When the galvanometer is connected to E_1, the null point is at C. When the galvanometer is connected to E_2, the null point will be



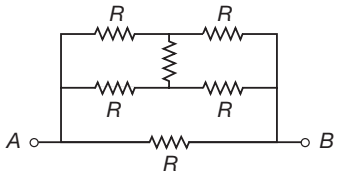
(A) to the left of C (B) to the right of C
(C) at C itself (D) no where on AB

113. In the given circuit, the ammeter A, assumed to have negligible resistance, reads 0.1 A. The value of R is



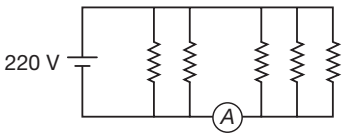
(A) 6 Ω (B) 8 Ω
(C) 16 Ω (D) 20 Ω

114. In the network shown below, the equivalent resistance between A and B is



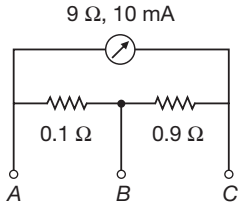
(A) $\frac{R}{2}$ (B) R
(C) $2R$ (D) $4R$

115. Five identical resistors, each of value 1100 Ω, are connected to a 220 V battery as shown. The reading of the ideal ammeter A is



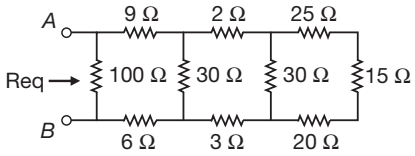
(A) $\frac{1}{5}$ A (B) $\frac{2}{5}$ A
(C) $\frac{3}{5}$ A (D) $\frac{4}{5}$ A

116. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The metre gives full-scale deflection for current I when A and B are used as its terminals. The value of I is



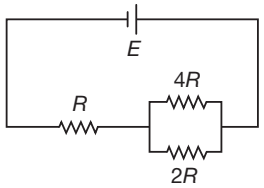
(A) 100 mA (B) 900 mA
(C) 1 A (D) 1.1 A

117. In the given three section network, the equivalent resistance between the points A and B is



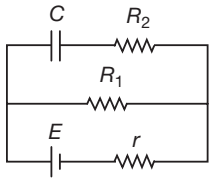
(A) 42.64 Ω (B) 32.42 Ω
(C) 22.26 Ω (D) 12.13 Ω

118. In the network, the current flowing through the resistance $2R$ as shown in the figure is



(A) $\frac{E}{R}$ (B) $\frac{E}{7R}$
(C) $\frac{2E}{7R}$ (D) $\frac{2E}{R}$

119. The numerical value of charge on either plate of capacitor C shown in figure



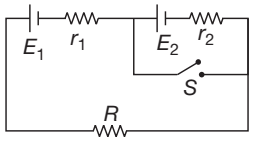
(A) CE (B) $\frac{CER_1}{R_1+r}$
(C) $\frac{CER_2}{R_2+r}$ (D) $\frac{CER_1}{R_2+r}$

120. 24 identical cells, each of internal resistance 0.5 Ω, are arranged in a parallel combination of n rows, each row containing m cells in series. The combination is connected across a resistor of 3 Ω. In order to send maximum current through the resistor, we should have

(A) $m = 12$, $n = 2$ (B) $m = 8$, $n = 3$

(C) $m = 2$, $n = 12$ (D) $m = 3$, $n = 8$

121. Switch S is closed at time $t = 0$. Which one of the following statements is correct?



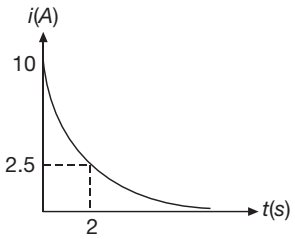
(A) Current in the resistance R increases if $E_1 r_2 < E_2 (R + r_1)$

(B) Current in the resistance R increases if $E_1 r_2 > E_2 (R + r_1)$

(C) Current in the resistance R decreases if $E_1 r_2 > E_2 (R + r_1)$

(D) Current in the resistance R decreases if $E_1 r_2 = E_2 (R + r_1)$

122. The figure shows a graph of the current in a discharging circuit of a capacitor through a resistor of resistance 10 Ω.



(A) The initial potential difference across the capacitor is 100 V

(B) The capacitance of the capacitor is $\frac{1}{10 \ln 2}$ F

(C) The total heat produced in the circuit will be $\frac{500}{\ln 2}$ J

(D) All of the above

123. Two cells of emf E_1 and E_2 are to be compared in a potentiometer $(E_1 > E_2)$. When the cells are used in series correctly, the balancing length obtained is 400 cm. When they are used in series but E_2 is connected with reverse polarities, the balancing length obtained is 200 cm. Ratio of emf of cells is

(A) 3 : 2 (B) 3 : 1

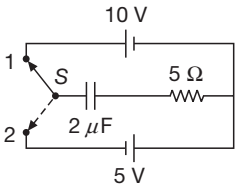
(C) 4 : 1 (D) 4 : 3

124. The charge flowing through a resistor R varies as $Q(t) = \alpha t - \beta t^2$. The total heat produced in R is

(A) $\frac{\alpha^3 R}{\beta}$ (B) $\frac{\alpha^3 R}{2\beta}$

(C) $\frac{\alpha^3 R}{3\beta}$ (D) $\frac{\alpha^3 R}{6\beta}$

125. In the circuit shown in figure switch S is thrown to position 1 at $t = 0$. When the current in the resistor is 1 A, it is shifted to position 2. The total heat generated in the circuit after shifting to position 2 is



(A) zero (B) 625 μJ

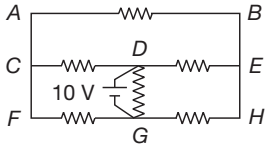
(C) 100 μJ (D) None of these

126. In the metre bridge arrangement, an unknown resistance R_1 is connected in series with or resistance of 10 Ω. When the combination is connected to left gap of the metre bridge and R_2 is connected to right gap, balance point from left end is at 50 cm. When 10 Ω resistor is removed, the balance point shifts to 40 cm. The value of R_1 is

(A) 20 Ω (B) 10 Ω

(C) 60 Ω (D) 40 Ω

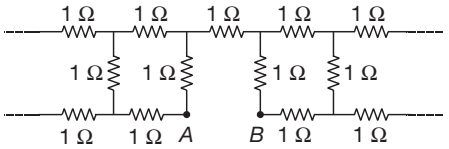
127. All resistances shown in circuit are 2 Ω each. The current in the resistance between D and E is



(A) 5 A (B) 2.5 A

(C) 1 A (D) 7.5 A

128. Each resistor shown in figure is an infinite network of resistance 1 Ω. The effective resistance between points A and B is



(A) less than $1\ \Omega$

(B) $1\ \Omega$

(C) more than $1\ \Omega$ but less than $3\ \Omega$

(D) $3\ \Omega$

129. Two wires A and B made of same material and having their lengths in the ratio $6:1$ are connected in series. The potential difference across the wires are 3 V and 2 V respectively. If r_A and r_B are the radii of A and B respectively, then $\frac{r_B}{r_A}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

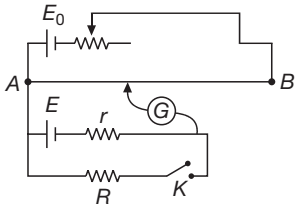
(C) 1 (D) 2

130. A leaky parallel capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\text{m}^{-1}$. Charge on the plate at instant $t = 0$ is $q = 8.885\ \mu\text{C}$. Then, time constant of leaky capacitor is

(A) 3 s (B) 4 s

(C) 5 s (D) 6 s

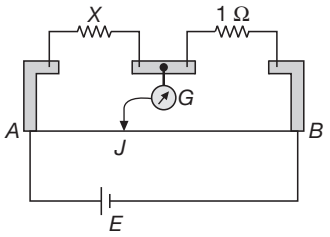
131. The given figure represents an arrangement of potentiometer for the calculation of internal resistance (r) of the unknown battery (E). The balance length is 70.0 cm with the key opened and 60.0 cm with the key closed. R is $132.40\ \Omega$. The internal resistance (r) of the unknown cell will be



(A) $22.1\ \Omega$ (B) $113.5\ \Omega$

(C) $154.5\ \Omega$ (D) $10\ \Omega$

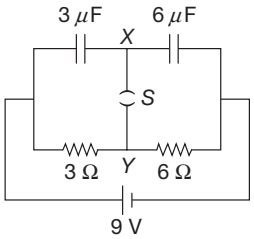
132. In the metre bridge, the balancing length $AJ = l = 20$ cm The unknown resistance X is equal to



(A) $0.25\ \Omega$ (B) $0.8\ \Omega$

(C) $0.2\ \Omega$ (D) $0.16\ \Omega$

133. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



(A) zero (B) $54\ \mu\text{C}$

(C) $27\ \mu\text{C}$ (D) $81\ \mu\text{C}$

134. Consider a capacitor – charging circuit. Let Q_1 be the charge given to the capacitor in time interval of 20 ms and Q_2 be the charge given in the next time interval of 20 ms. Let $10\ \mu\text{C}$ charge be deposited in a time interval t_1 and the next $10\ \mu\text{C}$ charge is deposited in the next time interval t_2. Then,

(A) $Q_1 > Q_2,\ t_1 > t_2$ (B) $Q_1 > Q_2,\ t_1 < t_2$

(C) $Q_1 < Q_2,\ t_1 > t_2$ (D) $Q_1 < Q_2,\ t_1 < t_2$

135. The temperature coefficient of resistance of a conductor varies as $\alpha(T) = 3T^2 + 2T$. If R_0 is resistance at $T = 0$ and R be resistance at T then

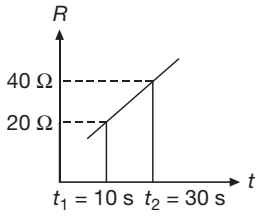
(A) $R = R_0(6T + 2)$

(B) $R = 2R_0(3 + 2T)$

(C) $R = R_0(1 + T^2 + T^3)$

(D) $R = R_0(1 - T + T^2 + T^3)$

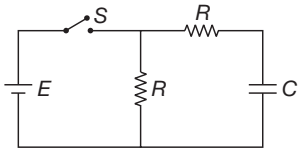
136. A source of emf $E = 10$ V and having negligible internal resistance is connected to a variable resistance. The resistance varies as shown in figure. The total charge that has passed through the resistor R during the time interval from t_1 to t_2 is



(A) $40\log_e 4$ (B) $30\log_e 3$

(C) $20\log_e 2$ (D) $10\log_e 2$

137. A capacitor C is connected to two equal resistances as shown in the figure. Consider the following statements.



(1) At the time of charging of capacitor time constant of the circuit is $2CR$
(2) At the time of discharging of the capacitor the time constant of the circuit is CR
(3) At the time of discharging of the capacitor the time constant of the circuit is $2CR$
(4) At the time of charging of capacitor the time constant of the circuit is CR

(A) Statements (1) and (2) only are correct
(B) Statements (2) and (3) only are correct
(C) Statements (3) and (4) only are correct
(D) Statements (1) and (3) only are correct.

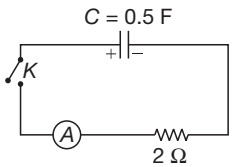
138. The mean free path of charge carriers in a current carrying conductor is

(A) $\frac{2I\tau}{nqA}$ (B) $\frac{j\tau}{nq}$

(C) $\frac{2j\tau}{nq}$ (D) $\frac{I\tau}{2nqA}$

(All symbols carry their usual meaning).

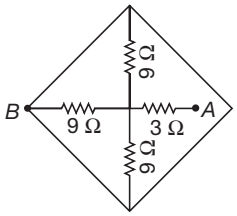
139. A charged capacitor is allowed to discharge through a resistor by closing the key at the instant $t = 0$. At the instant $t = (\ln 4)\mu s$, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to



(A) 0.5 Ω (B) 1 Ω

(C) 2 Ω (D) 4 Ω

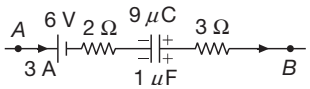
140. The equivalent resistance between the points A and B for the circuit shown in figure is



(A) 3 Ω (B) 6 Ω

(C) 9 Ω (D) 12 Ω

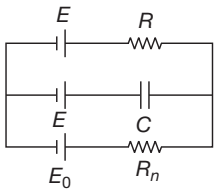
141. The potential difference $V_A - V_B$ between points A and B for the circuit segment shown in figure at the given instant is



(A) 12 V (B) –12 V

(C) 6 V (D) –6 V

142. For the circuit shown in the figure, find the charge stored on capacitor in steady state.



(A) $\frac{RC}{R+R_0}E$ (B) $\frac{RC}{R_0}(E-E_0)$

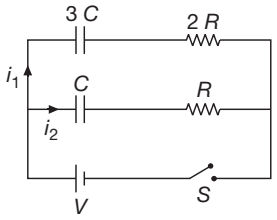
(C) zero (D) $\frac{RC}{R+R_0}(E-E_0)$

143. A cell of internal resistance r, when applied across an external resistance R draws a current I. The current drawn in the circuit is maximum when

(A) $R = r$ (B) $R \ll r$

(C) $R \gg r$ (D) $R = 2r$

144. In the circuit shown, switch S is closed at $t = 0$. Let i_1 and i_2 be the current at any finite time t, then the ratio $\frac{i_1}{i_2}$



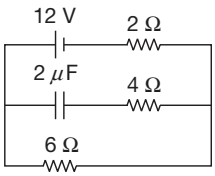
(A) is constant
(B) increases with time
(C) decreases with time
(D) first increases and then decreases

145. 24 cells each of internal resistance $1\,\Omega$ are arranged such that n cells in series are connected in m rows in parallel across a load resistance of $10\,\Omega$. The power consumption across the load is maximum for

(A) $n=24,\ m=1$ (B) $n=12,\ m=2$

(C) $n=8,\ m=3$ (D) $n=6,\ m=4$

146. For the circuit arrangement shown in figure, in the steady state condition charge on the capacitor is



(A) $12\,\mu C$ (B) $14\,\mu C$

(C) $20\,\mu C$ (D) $18\,\mu C$

147. A uniform wire of resistance $4\,\Omega$ is bent into a circle of radius r. A specimen of the same wire is connected along the diameter of the circle. What is the equivalent resistance across the ends of this wire?

(A) $\frac{4}{(4+\pi)}\Omega$ (B) $\frac{3}{(3+\pi)}\Omega$

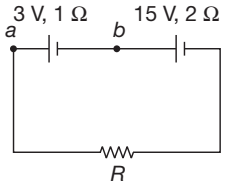
(C) $\frac{2}{(2+\pi)}\Omega$ (D) $\frac{1}{(1+\pi)}\Omega$

148. 9 identical light bulbs, each designed to extract power P from a certain voltage supply are connected in series across that supply. The total power drawn by them is

(A) $9P$ (B) P

(C) $\frac{P}{9}$ (D) $\frac{P}{81}$

149. Two batteries one of the emf $3\,V$, internal resistance $1\,\Omega$ and the other of emf $15\,V$, internal resistance $2\,\Omega$ are connected in series with a resistance R as shown. If the potential difference between points a and b is zero, the resistance R in Ω is



(A) 5 (B) 7

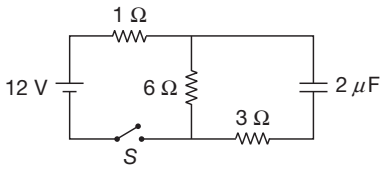
(C) 3 (D) 1

150. Six identical resistors each of value $1\,\Omega$ are joined to form a hexagon. Further six more resistors of the same value are connected between the six vertices and the centre of the hexagon. The equivalent resistance between any two vertices lying opposite to each other is

(A) $\frac{5}{4}\Omega$ (B) $\frac{4}{5}\Omega$

(C) $\frac{3}{5}\Omega$ (D) $\frac{4}{3}\Omega$

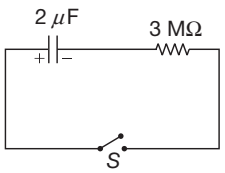
151. When the switch is closed, the initial current through the $1\,\Omega$ resistor is



(A) 2 A (B) 4 A

(C) 3 A (D) 6 A

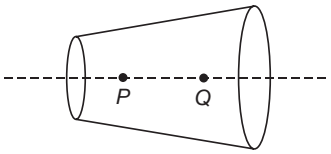
152. In the circuit shown in figure, the capacitor is charged with a cell of $5\,V$. If the switch is closed at $t=0$, then at $t=12\,s$, charge on the capacitor is



(A) $(0.37)10\,\mu C$ (B) $(0.37)^2 10\,\mu C$

(C) $(0.63)10\,\mu C$ (D) $(0.63)^2 10\,\mu C$

153. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and Q is v_P and v_Q.



(A) $v_P = v_Q$ (B) $v_P < v_Q$

(C) $v_P > v_Q$ (D) Data Insufficient

154. A moving coil galvanometer is converted into an ammeter reading up to 0.03 A by connecting a shunt of resistance $\frac{r}{4}$. What is the maximum current which can be sent through this galvanometer, if no shunt is used. (Here, r = resistance of galvanometer)

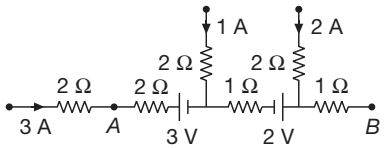
(A) 0.004 A (B) 0.005 A

(C) 0.006 A (D) 0.008 A

155. Kirchhoff's junction law and loop law are manifestations of conservation of

(A) charge only.
(B) energy only.
(C) charge and energy respectively.
(D) energy and charge respectively.

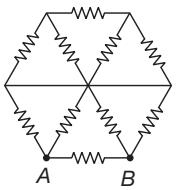
156. The potential difference between points A and B, in a section of a circuit shown, is



(A) 5 volt (B) 1 volt

(C) 10 volt (D) 17 volt

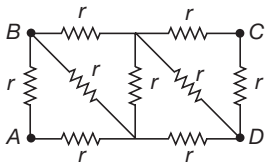
157. In the network shown in figure, each resistance is R. The equivalent resistance between points A and B is



(A) $\frac{20}{11}R$ (B) $\frac{19}{20}R$

(C) $\frac{8}{15}R$ (D) $\frac{R}{2}$

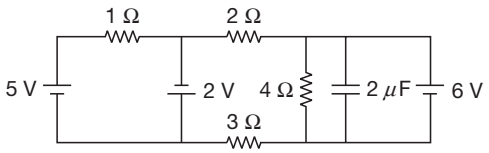
158. For the circuit shown the equivalent resistance between A and C is



(A) $\frac{12}{11}r$ (B) $\frac{13}{11}r$

(C) $\frac{14}{11}r$ (D) $\frac{15}{11}r$

159. The current in $1\ \Omega$ resistance and charge stored in the capacitor are



(A) 4 A , 6 μC (B) 7 A , 12 μC

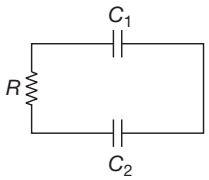
(C) 4 A , 12 μC (D) 7 A , 6 μC

160. The space between two concentric shells of radii a and b $(a<b)$ is filled with a medium of resistivity ρ. The resistance offered by the medium to the flow of current when a source is connected across the two spheres is

(A) $\frac{\rho}{4\pi a}$ (B) $\frac{\rho}{4\pi b}$

(C) $\frac{\rho}{4\pi(a-b)}$ (D) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

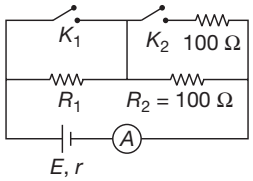
161. A capacitor C_1 is charged to a potential V and connected to another capacitor in series with a resistor R as shown. It is observed that heat H_1 is dissipated across resistance R, till the circuit reaches steady state. Same process is repeated using resistance of $2R$. If H_2 is heat dissipated in this case, then



(A) $\frac{H_2}{H_1}=1$ (B) $\frac{H_2}{H_1}=4$

(C) $\frac{H_2}{H_1}=\frac{1}{4}$ (D) $\frac{H_2}{H_1}=2$

162. In the circuit shown, when keys K_1 and K_2 both are closed, the ammeter reads I_0. But when K_1 is open and K_2 is closed, the ammeter reads $\frac{I_0}{2}$. Assuming that ammeter resistance is much less than R_2, the values of r and R_1 in Ω are



(A) 25, 50 (B) 25, 100
(C) 0, 100 (D) 0, 50

163. In a series RC circuit if we assume that the key K is opened as soon as the steady state is reached in the circuit as a result of which the charge had build up. Due to this a pulse is obtained at a regular interval t_0.

(A) $t_0 \to \infty$ (B) $t_0 = 5\,RC$
(C) $t_0 = 10\,RC$ (D) $t_0 = 15\,RC$

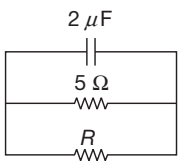
164. Potentiometer wire of length 1 m is connected in series with 490 Ω resistance and 2 V battery. If 0.2 mVcm^{-1} is the potential gradient, then resistance of the potentiometer wire is approximately

(A) 4.9 Ω (B) 7.9 Ω
(C) 5.9 Ω (D) 6.9 Ω

165. In a series RC circuit a steady state charge of 10 μC is established in time of 10 ms. If 1 ms is the time constant of the circuit and 3 μC is the charge at any instant for the growth part then the decay charge in the circuit is

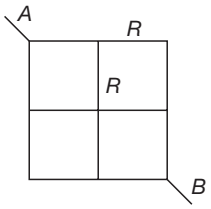
(A) $q_d = 3\ \mu C$ (B) $q_d = 7\ \mu C$
(C) $q_d = -3\ \mu C$ (D) $q_d = -7\ \mu C$

166. A capacitor of capacitance 2 μF is charged to a potential difference of 5 V. Now the charging battery is disconnected and the capacitor is connected in parallel to a resistor of 5 Ω and another unknown resistor of resistance R as shown in figure. If the total heat produced in 5 Ω resistance is 10 μJ, then the unknown resistance R is equal to



(A) 10 Ω (B) 15 Ω
(C) $\frac{10}{3}$ Ω (D) 7.5 Ω

167. The equivalent resistance between the points A and B is (R is the resistance of each side of smaller square)

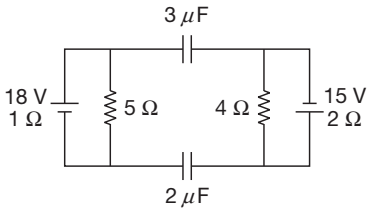


(A) R (B) $\frac{3R}{2}$
(C) $2R$ (D) $\frac{R}{2}$

168. A capacitor is charged from a cell with the help of a resistor. The circuit has a time constant τ. The capacitor collects 10% of the steady charge at time t given by

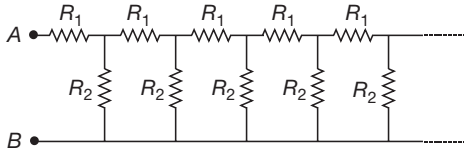
(A) $\tau \ln(1.1)$ (B) $\tau \ln\left(\frac{10}{9}\right)$
(C) $\tau \ln(0.9)$ (D) $\tau \ln(0.1)$

169. Two cells, two resistors and two capacitors are connected as shown in figure, The charge on 2 μF capacitor is



(A) 30 μC (B) 20 μC
(C) 25 μC (D) 48 μC

170. Consider an infinite ladder network shown in figure. A voltage V is applied between the points A and B. This applied value of voltage is halved after each section.



(A) $\frac{R_1}{R_2}=1$ (B) $\frac{R_1}{R_2}=\frac{1}{2}$

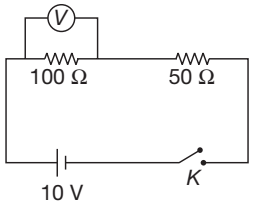
(C) $\frac{R_1}{R_2}=2$ (D) $\frac{R_1}{R_2}=3$

171. In order to increase the resistance of a given wire of uniform cross-section to four times its value, a fraction of its length is stretched uniformly till the full length of the wire becomes $\frac{3}{2}$ times the original length. What is the value of this fraction?

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$

(C) $\frac{1}{16}$ (D) $\frac{1}{6}$

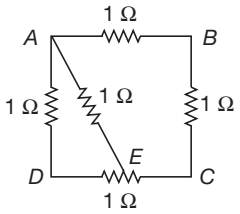
172. In the given circuit, the voltmeter records 5 volt . The resistance of the voltmeter in Ω is



(A) 200 (B) 100

(C) 10 (D) 50

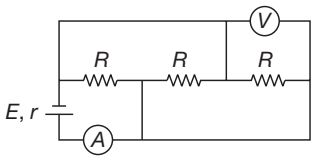
173. ABCD is a square where each side is a uniform wire of resistance $1\ \Omega$. A point E lies on CD such that if a uniform wire of resistance $1\ \Omega$ is connected across AE and constant potential difference is applied across A and C then B and E are equipotential.



(A) $\frac{CE}{ED}=1$ (B) $\frac{CE}{ED}=2$

(C) $\frac{CE}{ED}=\frac{1}{\sqrt{2}}$ (D) $\frac{CE}{ED}=\sqrt{2}$

174. In the circuit shown in the figure ammeter and voltmeter are ideal. If $E=4$ V , $R=9\ \Omega$ and $r=1\ \Omega$, then readings of ammeter and voltmeter are



(A) 1 A , 3 V (B) 2 A , 3 V

(C) 3 A , 4 V (D) 4 A , 4 V

175. A capacitor of capacitance C has a charge Q_0 . This capacitor is now connected to another identical capacitor through a resistance R . The heat produced in the resistance is

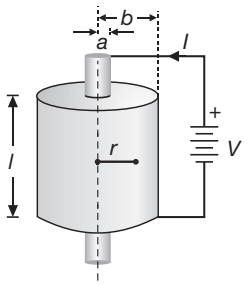
(A) $\frac{Q_0^2}{C}$ (B) $\frac{Q_0^2}{2C}$

(C) $\frac{Q_0^2}{4C}$ (D) $\frac{Q_0^2}{8C}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

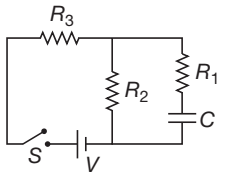
1. A metal rod of radius a is concentric with a metal cylindrical shell of radius b and length l . The space between rod and cylinder is tightly packed with a high resistance material of resistivity ρ . A battery having a terminal voltage V is connected across the combination as shown. Neglect resistance of rod and cylinder. If I is the total current in the circuit then, J is the current density and E is the electric field, then



(A) $J = \dfrac{I}{2\pi rl}$ (B) $E = \dfrac{\rho I}{2\pi rl}$

(C) $J = \dfrac{I}{\pi r^2}$ (D) $E = \text{ZERO}$

2. In the network shown, the capacitor C is initially uncharged. The time constant of the circuit is τ and the charge on C at time t after the switch S is closed is q. Then



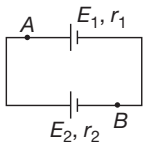
(A) $\tau = CR_1$

(B) $\tau = C\left(R_1 + \dfrac{R_2R_3}{R_2 + R_3}\right)$

(C) $q = \dfrac{CVR_2}{R_2 + R_3}\left(1 - e^{-t/\tau}\right)$

(D) $q = \dfrac{CVR_1}{R_1 + R_2}\left(1 - e^{-t/\tau}\right)$

3. Two batteries of having emfs E_1 and E_2 and internal resistances r_1 and r_2 respectively are joined as shown. V_A and V_B are the potentials at A and B respectively.



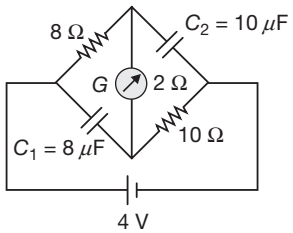
(A) The potential difference across one battery will be greater than its emf.

(B) $V_A - V_B = \dfrac{(E_1r_2 + E_2r_1)}{(r_1 + r_2)}$.

(C) The potential difference across both the battery will be equal.

(D) One battery will continuously supply energy to the other.

4. In the circuit shown,



the battery is ideal and has an emf $E = 2$ V. The resistance of the coil of the galvanometer G is $1\,\Omega$. The current that flows through the galvanometer G is I, the potential difference across the capacitors C_1 and C_2, in steady state is ΔV_1 and ΔV_2 respectively. Then

(A) $I = 0$ (B) $I = \dfrac{1}{5}$ A

(C) $\Delta V_1 = 1.6$ V (D) $\Delta V_2 = 2.4$ V

5. For a metallic conductor

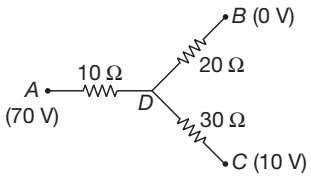
(A) the number density of charge carriers is 10^{28} m^{-3}.

(B) the number density of charge carriers is 10^{26} m^{-3}.

(C) the drift velocity of electrons is 1 cms^{-1}.

(D) the drift velocity of electrons is 10 cms^{-1}.

6. In the network shown, points A, B and C are at potentials $V_A = 70$ V, $V_B = 0$, $V_C = 10$ V and V_D respectively. Then



(A) $V_D = 40$ V

(B) The currents in the sections AD, DB, DC are in the ratio $3:2:1$.

(C) The currents in the sections AD, DB, DC are in the ratio $1:2:3$.

(D) The network draws a total power of 200 W.

7. A long round conductor of cross-sectional area A is made of a material whose resistivity depends on the radial distance r from the axis of the conductor as $\rho = \dfrac{\alpha}{r^2}$, α is a constant. The total resistance per unit length of the conductor is R and the electric field strength in the conductor due to which a current I flows in it is E. Then

(A) $R = \dfrac{2\pi\alpha}{A^2}$ (B) $R = \dfrac{4\pi\alpha}{A^2}$

(C) $E = \dfrac{2\pi\alpha I}{A^2}$ (D) $E = \dfrac{4\pi\alpha I}{A^2}$

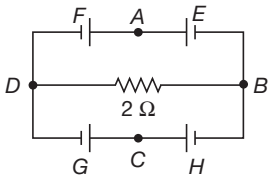
8. A voltmeter and an ammeter having resistances R_V and R_A respectively are connected in series to an ideal battery of emf E. The voltmeter reading is V, the ammeter reading is I, the potential difference across the ammeter is ΔV_A and the total resistance of the

circuit is R. Which of the following statement(s) seem to be correct?

(A) $V < E$ (B) $R_V = \dfrac{V}{I}$

(C) $\Delta V_A = E - V$ (D) $R_A + R_V = \dfrac{E}{I}$

9. In the circuit shown E, F, G and H are cells of e.m.f. 2 V, 1 V, 3 V and 1 V respectively and their internal resistances are $2\,\Omega$, $1\,\Omega$, $3\,\Omega$ and $1\,\Omega$ respectively.



(A) $V_D - V_B = -\dfrac{2}{13}\text{ V}$

(B) $V_D - V_B = \dfrac{2}{13}\text{ V}$

(C) $V_G = \dfrac{21}{13}\text{ V}$ = Potential difference across G.

(D) $V_H = \dfrac{19}{13}\text{ V}$ = Potential difference across H.

10. A uniform wire of resistance R is shaped into a regular, n even sided polygon. The equivalent resistance between any two corners can have

(A) the maximum value $\dfrac{R}{4}$

(B) the maximum value $\dfrac{R}{n}$

(C) the minimum value $R\left(\dfrac{n-1}{n^2}\right)$

(D) the minimum value $\dfrac{R}{n}$

11. A voltmeter and an ammeter are joined in series to an ideal battery such that the respective readings shown by them are V and A. A resistance equal to the resistance of the ammeter is now joined in parallel to the ammeter. Then,

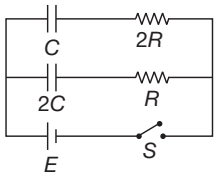
(A) V will not change.

(B) V will increase slightly.

(C) A will become exactly half of its initial value.

(D) A will become slightly more than half of its initial value.

12. In the circuit shown in the figure, switch S is closed at time $t = 0$. Select the correct statements.



(A) Rate of increase of charge is same in both the capacitors

(B) Ratio of charge stored in capacitors C and $2C$ at any time t would be $1:2$

(C) Time constants of both the capacitors are equal

(D) Steady state charges on capacitors C and $2C$ are in the ratio of $1:2$

13. A number of resistors R_1, R_2, R_3, ... are connected in parallel such that R_p is the equivalent resistance of parallel combination. R_2 is the resistance of least magnitude. A current I is flowing in the circuit due to a potential V applied across the circuit. I_1, I_2, I_3, ... are the currents in resistors R_1, R_2, R_3, ... respectively and V_1, V_2, V_3 ..... are potentials across R_1, R_2, R_3,..... respectively.

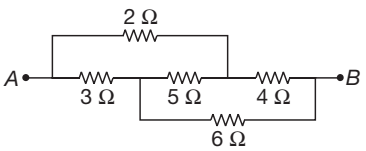
(A) $V_1 = V_2 = V_3 = ...$

(B) $I_1 = \left(\dfrac{R_p}{R_1}\right)I\ ;\ I_2 = \left(\dfrac{R_p}{R_2}\right)I\ ;\ I_3 = \left(\dfrac{R_p}{R_3}\right)I\ ;.....$

(C) $R_p > R_2$

(D) $R_p < R_2$

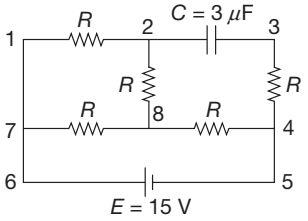
14. In the circuit shown, the points A and B are connected across a battery of emf E. The equivalent resistance between A and B is R_{AB}, the potential difference and the current across the $5\,\Omega$ resistor are ΔV and I respectively.



(A) $I = \dfrac{5E}{18}$ (B) $\Delta V = 0$

(C) $R_{AB} = \dfrac{10}{3}\,\Omega$ (D) $R_{AB} = \dfrac{18}{5}\,\Omega$

15. In the circuit shown, the battery is ideal, with emf $E = 15$ V and it sends a current I in the circuit. All resistors are identical and each resistor has resistance $R = 3\ \Omega$. The potential difference across the capacitor in steady state is V_C, then



(A) $I = 1$ A (B) $V_C = 9$ V

(C) $V_C = 12$ V (D) $I = 3$ A

16. A number of resistors R_1, R_2, R_3, ... are connected in series such that R_s is the equivalent resistance of series combination. A current I is flowing in the circuit due to a potential V applied across the circuit. V_1, V_2, V_3, ... are potentials across R_1, R_2, R_3, ... respectively.

(A) Same current I will flow through each resistor.

(B) $V_1 + V_2 + V_3 + ... = V$

(C) $V_1 = \left(\frac{R_1}{R_s}\right)V\ ;\ V_2 = \left(\frac{R_2}{R_s}\right)V\ ;\ V_3 = \left(\frac{R_3}{R_s}\right)V\ ;$ .....

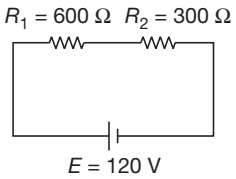
(D) Data Insufficient

17. A vacuum diode consists of plane parallel electrodes separated by a distance d and each having an area A. On applying a potential V to the anode with respect to the cathode a current I flows through the diode. Assume that the electrons are emitted with zero velocity and they do not change the field between the electrodes. The electron velocity is v and the charge density is ρ at any point between the electrodes at a distance x from the cathode . If I is the equivalent current, m is the mass of each charge carrier, then

(A) $v = \sqrt{\frac{2eVx}{md}}$ (B) $\rho = I\sqrt{\frac{md}{2eVxA^2}}$

(C) $v = \sqrt{\frac{eVx}{2md}}$ (D) $\rho = I\sqrt{\frac{2md}{eVxA^2}}$

18. In the circuit, the battery connected is ideal. A voltmeter of resistance $600\ \Omega$ is connected one by one across R_1 and R_2, giving readings of V_1 and V_2 respectively.



(A) $V_2 = 40$ V (B) $V_1 = 80$ V

(C) $V_2 = 30$ V (D) $V_1 = 60$ V

19. In household electric circuits, the electrical appliance and the usage of the switch with the appliance are governed by the following facts.

(A) All electric appliances drawing power are joined in parallel.

(B) A switch may be either in series or in parallel with the appliance which it controls.

(C) If a switch is in parallel with an appliance, it will draw power when the switch is in the 'off' position (open).

(D) If a switch is in parallel with an appliance, the fuse will blow (burn out) when the switch is put 'on' (closed).

20. A piece of germanium (material used for making semiconductors) and a piece of copper (material used for making conducting wires) are cooled from room temperature to 85 K . R_1 and R_2 be the resistances of the pieces respectively and α_1 and α_2 are the temperature coefficients of resistance of materials respectively.

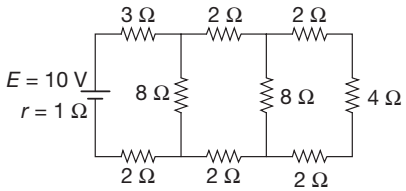
(A) R_1 decreases and R_2 increases

(B) R_1 increases and R_2 decreases

(C) $\alpha_1 < 0$, $\alpha_2 > 0$

(D) $\alpha_1 > 0$, $\alpha_2 > 0$

21. In the circuit shown, the cell has emf $E = 10$ V and internal resistance $= 1\ \Omega$.



(A) The current through the $3\ \Omega$ resistor is 1 A.

(B) The current through the $3\ \Omega$ resistor is 0.5 A.

(C) The current through the $4\ \Omega$ resistor is 0.5 A.

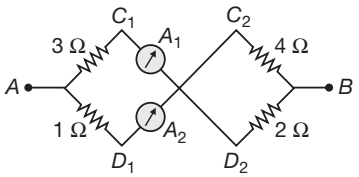
(D) The current through the 4Ω resistor is 0.25 A.

22. Two identical fuses are rated at 10 A. When joined in series the combination acts as a fuse of rating I_1 and when joined in parallel the combination acts as a fuse of rating I_2. Then

(A) $I_1 = 10$ A (B) $I_1 = 20$ A

(C) $I_2 = 20$ A (D) $I_2 = 5$ A

23. The ammeters, A_1 and A_2, each of resistance $5\ \Omega$ are connected as shown. An ideal cell of emf 10 V is applied between A and B, then



(A) the current drawn from the cell is 1 A

(B) the reading of A_1 is 1 A

(C) the reading of A_2 is 1 A

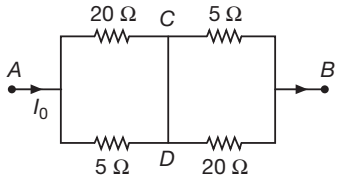
(D) for C_1 joined to C_2 and D_1 joined to D_2, the ammeter readings will become equal

24. A leaky parallel plate capacitor is filled completely with a material having dielectric constant k, electrical conductivity σ. The charge on the plate initially is q_0 and the leakage current at time t is I. The time constant of the circuit is τ. Then

(A) $\tau = \dfrac{\varepsilon_0 k}{\sigma}$ (B) $I = \dfrac{q_0\sigma}{\varepsilon_0 k}$

(C) $I = I_0 e^{\left(-\frac{t\sigma}{\varepsilon_0 k}\right)}$ (D) $I_0 = -\dfrac{q_0\sigma}{\varepsilon_0 k}$

25. A current I_0 enters the network at A and leaves at B, when some potential difference is maintained across the points A and B. If R_{AB} is the equivalent resistance between A and B, V_C and V_D be the potentials at the points C and D respectively and I be the current flowing through the branch CD, then which of the following is correct?



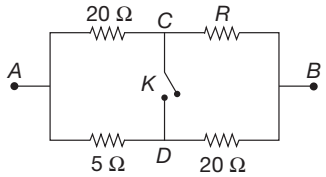
(A) $V_C = V_D$

(B) $R_{AB} = 8\ \Omega$

(C) $I = \dfrac{3}{5} I_0$, from D to C

(D) $I = 0$

26. When the switch K is open, the equivalent resistance between A and B is $20\ \Omega$. Which of the following statement(s) is/are correct?



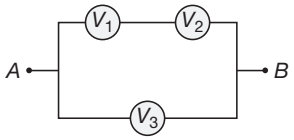
(A) $R = 80\ \Omega$

(B) No current flows through K when it is closed

(C) The powers dissipated in R and in the $5\ \Omega$ resistor are always equal

(D) The powers dissipated in the two $20\ \Omega$ resistors are unequal

27. Three voltmeters V_1, V_2 and V_3, all having different resistances, are connected between two points A and B, such that V_1 and V_2 are in series which in turn fall to be in parallel with V_3. When some potential difference is applied across A and B, the respective readings of the voltmeters are V_1, V_2 and V_3. Which of the following mathematical relations support the argument given here?



(A) $V_1 = V_2$ (B) $V_1 \neq V_2$

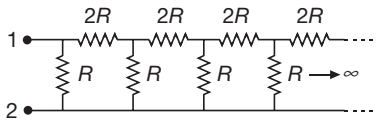
(C) $V_1 + V_2 = V_3$ (D) $V_1 + V_2 > V_3$

28. A battery of emf E, internal resistance r drives a current I through an external resistance R. The battery supplies a power P and produces per second a heat H across the external circuit and a heat h across the internal circuit of the battery. Then

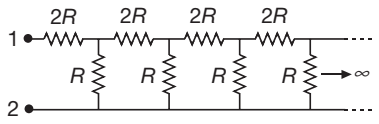
(A) $P = EI$ (B) $P = \dfrac{E^2}{R+r}$

(C) $H = \dfrac{EIR}{R+r}$ (D) $h = \dfrac{E^2 r}{(R+r)^2}$

29. Two circuits (as shown in figure) are called 'Circuit A' and 'Circuit B'. The equivalent resistance of 'Circuit A' is x and that of 'Circuit B' is y between 1 and 2.



Circuit A

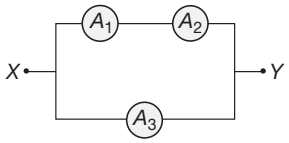


Circuit B

(A) $y > x$ (B) $y = (\sqrt{3}+1)R$

(C) $xy = 2R^2$ (D) $x - y = 2R$

30. Three ammeters A_1, A_2 and A_3 having resistances R_1, R_2 and R_3 respectively are connected between two points X and Y, such that A_1 and A_2 are in series which in turn fall to be in parallel with A_3. When some potential difference is applied across the terminals X and Y, the respective readings of the ammeters are I_1, I_2 and I_3. Which of the following mathematical relations support the argument given here?



(A) $I_1 = I_2$ (B) $I_1R_1 + I_2R_2 = I_3R_3$

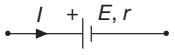
(C) $\frac{I_2}{I_3} = \frac{R_3}{R_1}$ (D) $\frac{I_1}{I_3} = \frac{R_3}{R_1 + R_2}$

31. A circuit has an equivalent resistance R_0. A voltmeter of resistance R_v is applied across the circuit to measure the potential drop across R_0. The new equivalent resistance of the circuit is

(A) R_0 $(\text{for } R_0 \ll R_v)$ (B) $\frac{R_0R_v}{R_0 + R_v}$

(C) $R_0 + R_v$ (D) Data Insufficient

32. A current I is being driven through a battery of emf E and internal resistance r, as shown. Then which of the following statement(s) best suit the above mentioned condition?



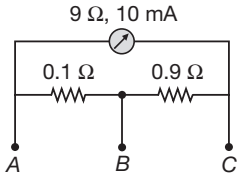
(A) The battery absorbs energy at the rate of EI.

(B) The battery stores chemical energy at the rate of $(EI - I^2r)$.

(C) The potential difference across the battery is $E + Ir$.

(D) Some heat is produced in the battery.

33. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The milliammeter gives full scale deflection for current I when A and B are used as its terminals. It gives a full scale deflection again for a current I' when A and C are used as terminals. Then



(A) $I' = 111$ mA (B) $I' = 900$ mA

(C) $I = 1$ A (D) $I = 1.1$ A

34. A cube is made of twelve identical wires each of resistance R_0. The equivalent resistance between the two points lying on the diagonal corners of cube is x and the equivalent resistance between the two opposite corners of a face of cube is y.

(A) $\frac{x}{y} = \frac{10}{9}$ (B) $x - y = \frac{R_0}{12}$

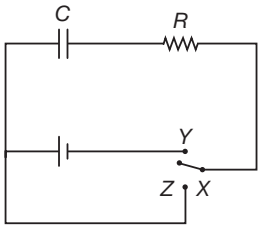
(C) $\frac{x}{y} = \frac{10}{7}$ (D) $x - y = \frac{R_0}{4}$

35. Two electric bulbs, A and B, with respective ratings 25 W, 220 V and 100 W, 220 V are connected in series across a voltage source of 220 V. The bulbs A and B draw powers P_A and P_B respectively. Then

(A) $P_A = 16$ W (B) $P_A = 4$ W

(C) $P_B = 4$ W (D) $P_B = 16$ W

36. In the circuit shown, that contains a capacitor C and a resistor R, the capacitor C is uncharged initially. When the terminal X is joined to the terminal Y for a long time, it is observed that a heat H_1 is produced in the resistor. When the terminal X is joined to the terminal Z, again for a long time, it is observed that a heat H_2 is produced in the resistor. Also, energy supplied by the battery during the process of charging is H. Then



(A) $H_1 = H_2 = \frac{H}{2}$

(B) $H_1 + H_2 = 2H$

(C) $H_1 + H_2 = H$

(D) The maximum energy stored in C at any time is H_1.

37. Two sources of current of equal e.m.f. are connected in series and have different internal resistances r_1 and r_2. An external resistance R is connected in series with two sources of e.m.f.

(A) $R = (r_1 - r_2)$, for terminal potential difference to be zero across source 1.

(B) $R = \sqrt{r_1 r_2}$, for terminal potential difference to be zero across both sources.

(C) $R = (r_2 - r_1)$, for terminal potential difference to be zero across source 2.

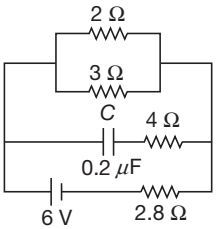
(D) Data insufficient to arrive at a conclusion.

38. Two heaters A and B, designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in times t_A and t_B respectively. When connected together across the same source, they produce H amount of heat in time T (when in series) and t (when in parallel). Then

(A) $T = 2(t_A + t_B)$ (B) $T = t_A + t_B$

(C) $t = \frac{t_A t_B}{2(t_A + t_B)}$ (D) $t = \frac{t_A t_B}{t_A + t_B}$

39. In the circuit shown,



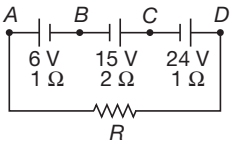
(A) a current of 0.9 A flows through 2 Ω resistor when steady state is reached.

(B) a potential drop of 4.2 V appears across the resistance of 2.8 Ω.

(C) a potential drop of 1.8 V appears across the capacitor C.

(D) a potential of 4.2 V appears across the capacitor C.

40. In the given circuit the point A is 9 V higher than point B.



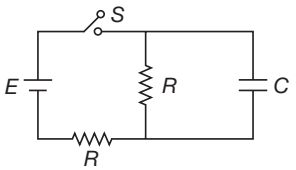
(A) $R = 1\ \Omega$

(B) $R = 7\ \Omega$

(C) Potential difference between B and D is 30 V.

(D) Potential difference between B and C is 15 V.

41. In the circuit shown in figure, the switch S is closed at $t = 0$. The voltage across the capacitor C at time t after the switch S is closed is V. The voltage as $t \to \infty$ is V_0.



(A) $V = \frac{E}{3}(1 - e^{-3t/RC})$ (B) $V_0 = \frac{E}{3}$

(C) $V = \frac{E}{2}(1 - e^{-2t/RC})$ (D) $V_0 = \frac{E}{2}$

42. Two heaters designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in time t_1 and t_2 respectively. When used together across the same source, they produce H amount of heat in time t

(A) If they are in series, $t = t_1 + t_2$

(B) If they are in series, $t = 2(t_1 + t_2)$

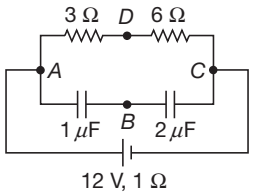
(C) If they are in parallel, $t = \frac{t_1 t_2}{(t_1 + t_2)}$

(D) If they are in parallel, $t = \frac{t_1 t_2}{2(t_1 + t_2)}$

43. The charge, Q flowing in a conductor varies with time in accordance with the relation $Q = at - bt^2$. If I be the current at an instant of time, then I

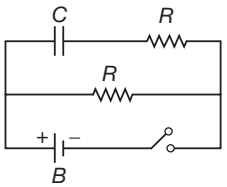
(A) decreases linearly with time.
(B) reaches a maximum and then decreases.
(C) falls to zero after time $t = \frac{1}{2}\frac{a}{b}$.
(D) charges at a rate $-2b$.

44. In the circuit shown in figure, the charge on each capacitor is Q and the potential difference between the points B and D is V.



(A) $Q = 7.2\ \mu C$ (B) $Q = 7.2$ mC
(C) $V = 3.6$ V (D) $V = -3.6$ V

45. In the circuit shown, when the switch is closed, the capacitor charges with a time constant t_1 and when the switch is opened (after the capacitor has been charged) then the capacitor discharges with a time constant t_2. Then



(A) $\frac{t_1}{t_2} = 2$ (B) $t_1 + t_2 = 3RC$
(C) $\frac{t_1}{t_2} = 1$ (D) $t_1 + t_2 = 2RC$

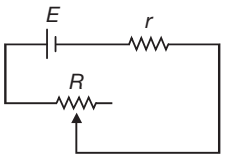
46. A capacitor is charged and then made to discharge through a resistance. The time constant is τ. In time t_1 the potential difference across the capacitor decreases by 10% and in time t_2 the potential difference across the capacitor falls to 10% of its initial value. Then

(A) $t_1 = \tau \log_e\left(\frac{9}{8}\right)$ (B) $t_1 = \tau \log_e\left(\frac{10}{9}\right)$
(C) $t_2 = 2.303\tau$ (D) $t_2 = 0.693\tau$

47. A capacitor charges from a cell through a resistance. The time constant is τ. The capacitor collects 10% of its final charge in a time t_1 and after a time t_2, the charge on the capacitor becomes 10% less than its final charge. Then

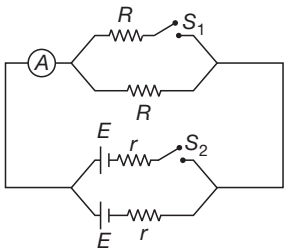
(A) $t_2 - t_1 = \tau \log_e\left(\frac{10}{9}\right)$ (B) $t_1 + t_2 = 2\tau \log_e\left(\frac{10}{3}\right)$
(C) $t_1 + t_2 = 2\tau \log_e(3)$ (D) $t_2 - t_1 = 2\tau \log_e(3)$

48. A battery of e.m.f. E and internal resistance r is connected to a variable resistor R as shown. Which one of the following is true?



(A) Potential difference across the terminals of the battery is maximum where $R = r$
(B) Power delivered to resistor is maximum when $R = r$
(C) Current in the circuit is maximum when $R = r$
(D) Current in the circuit is maximum when $R >> r$

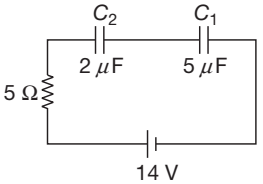
49. In the circuit shown in figure, reading of ammeter will



(A) increase if S_1 is closed
(B) decrease if S_1 is closed
(C) increase if S_2 is closed
(D) decrease if S_2 is closed

50. When a resistance of $9.5\ \Omega$ is connected across a battery, the voltage across the resistance is 11.4 V. If the resistance connected across the same battery is $11.5\ \Omega$, the voltage across the resistance is 11.5 V
(A) The emf of the battery is 12.0 V
(B) The internal resistance of the battery is $0.5\ \Omega$
(C) The internal resistance of the battery is $11.45\ \Omega$
(D) The emf of the battery is 11.5 V

51. In the given circuit is steady state



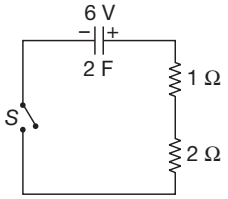
(A) the potential difference across C_1 is 10 V

(B) the potential difference across C_2 is 10 V

(C) the charge on C_1 is 20 μC

(D) the charge on C_2 is 8 μC

52. A capacitor of 2 F (practically not possible to have a capacity of 2 F) is charged by a battery of 6 V . The battery is removed and circuit is made as shown. Switch is closed at time $t = 0$. Choose the correct options.



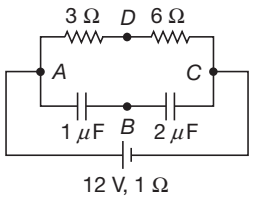
(A) At time $t = 0$ current in the circuit is 2 A

(B) At time $t = (6 \ln 2)$ second, potential difference across capacitor is 3 V

(C) At time $t = (6 \ln 2)$ second, potential difference across 1 Ω resistance is 1 V.

(D) At time $t = (6 \ln 2)$ second, potential difference across 2 Ω resistance is 2 V.

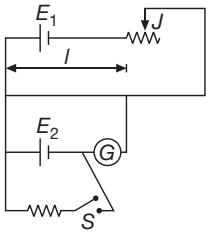
53. In the circuit shown in figure, the charge on each capacitor is Q and the potential difference between the points B and D is V .



(A) $Q = 7.2\ \mu C$ (B) $Q = 7.2$ mC

(C) $V = 3.6$ V (D) $V = -3.6$ V

54. In the potentiometer experiment shown in figure, the null point length is l . Choose the correct options given below.



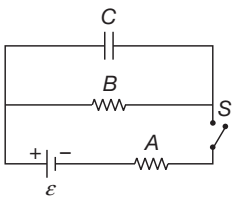
(A) If jockey J is shifted towards right, l will increase

(B) If value of E_1 is increased, l is decreased

(C) If value of E_2 is increased, l is increased

(D) If switch S is closed, l will decrease

55. In the circuit shown, A and B are equal resistances. When S is close, the capacitor C charges from the cell of emf ε and reaches a steady state.



(A) During charging, more heat is produced in A than in B

(B) In steady state, heat is produced at the same rate in A and B

(C) In the steady state, energy stored in C is $\frac{1}{4}C\varepsilon^2$

(D) In the steady state energy stored in C is $\frac{1}{8}C\varepsilon^2$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** In a wire of non-uniform cross-section, the current is same everywhere.
Statement-2: The current in a wire is due to the drift of electrons along the wire.

2. **Statement-1:** A conductor carrying electric current becomes electrically charged.
Statement-2: A conductor carrying electric current contains same number of positive and negative charges and is electrically neutral.

3. **Statement-1:** Electric field outside the conducting wire which carries a constant current is zero.
Statement-2: Net charge on conducting wire is zero.

4. **Statement-1:** Direction of electronic current can not be from negative potential to positive potential.
Statement-2: Direction of current is opposite to flow of electrons.

5. **Statement-1:** Insulators do not allow flow of current through them.
Statement-2: Insulators have no free charge carriers.

6. **Statement-1:** If there is current in a wire potential drop has to be there.
Statement-2: If potential drop is zero, the resistance may be zero.

7. **Statement-1:** Constant potential difference is applied across a conductor. If temperature of conductor is increased, the drift speed of electrons will decrease.
Statement-2: Resistivity increases with increase in temperature.

8. **Statement-1:** When a conductor is stretched to double its length, its resistance will get doubled.
Statement-2: Resistance is directly proportional to the length of a conductor.

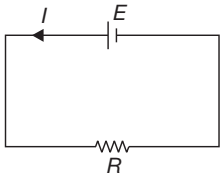
9. **Statement-1:** The drift velocity of electrons in a metallic wire decreases when the temperature of the wire is increased.
Statement-2: On increasing temperature, conductivity of metallic wire decreases.

10. **Statement-1:** Voltmeter always reads the e.m.f. of a cell if it is connected across the terminals of a cell.
Statement-2: Terminal potential of a cell is given by $V = E - Ir$.

11. **Statement-1:** A voltmeter is an inherently inaccurate instrument.
Statement-2: A voltmeter is always connected in parallel in a circuit.

12. **Statement-1:** Potential measured by a voltmeter across a wire is always less than actual potential difference across it.
Statement-2: Finite resistance of voltmeter changes current flowing through the resistance across which potential difference is to be measured.

13. **Statement-1:** In a simple battery circuit, the point at the lowest potential is the positive terminal of the battery.



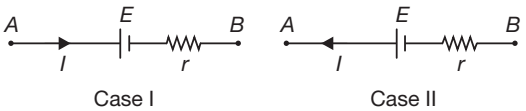
Statement-2: The current flows towards the point of lowest potential of the battery from the outside in the same manner as it flows in a circuit from positive to the negative terminal.

14. **Statement-1:** If potential difference between two points is zero and resistance between those points is zero, current may flow between the points.
Statement-2: Kirchhoff's 1st Law is based on conservation of charge.

15. **Statement-1:** Internal resistance of the battery is connected in parallel to it in an electrical circuit.
Statement-2: Heat generated in battery is due to internal resistance.

16. **Statement-1:** Two bulbs of 25 W and 100 W rated at 200 V are connected in series across 200 V supply. Ratio of powers of both the bulb in series is 2 : 1.
Statement-2: In series, current in both bulbs is same, therefore power depends on resistance of bulb.

17. **Statement-1:** Consider the two situations shown in the figure. Potential difference between points A and B, in Case-1 is more as compared to Case-2.



Statement-2: In Case-1 $V_A - V_B = E + Ir$

In Case-2 $V_A - V_B = E - Ir$

18. **Statement-1:** Since all the current coming to our house returns to power house (Since current travels in a closed loop). So there is no need to pay the electricity bill.
Statement-2: The electricity bill is paid for the electrical energy used and not for the current.

19. **Statement-1:** When current through a bulb is increased by 2% power increases by 4%.

Statement-2: Current passing through the bulb is inversely proportional to its resistance.

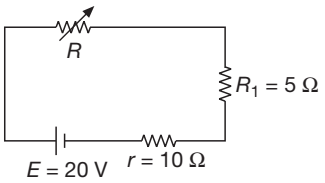
20. **Statement-1:** The e.m.f. of the driver cell in the potentiometer experiment should be greater than the e.m.f. of the cell to be determined.
Statement-2: The fall of potential across the potentiometer wire should not be less than e.m.f. of the cell to be determined.

21. **Statement-1:** When a cell is charged by connecting its positive electrode with positive terminal of the charger battery then potential difference across the electrodes of cell will be smaller than the EMF of cell.
Statement-2: Potential difference across electrodes in a cell providing electric current I is $E - Ir$ where E is the EMF and r internal resistance.

22. **Statement-1:** In metre bridge experiment, a high resistance is always connected in series with a galvanometer.
Statement-2: As resistance increases, current through the circuit increases.

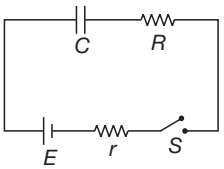
23. **Statement-1:** In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance
Statement-2: Resistance of a metal increases with increase in temperature.

24. **Statement-1:** For zero value of R in circuit, power transfer in external resistance will be maximum.



Statement-2: Since $R_1 < r$ in the given circuit. So, power transfer in external resistance will be maximum when $R = 0$.

25. **Statement-1:** The switch S shown in the figure is closed at $t = 0$, Initial current flowing through battery is $\frac{E}{R+r}$.



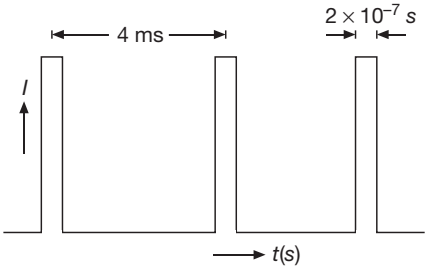
Statement-2: Initially capacitor was uncharged, so resistance offered by capacitor at $t = 0$ is zero.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

In a certain particle accelerator, electrons emerge with an energy of 40 MeV. The electrons emerge not in a steady stream but rather in pulses at the rate of 250 pulses/s. This corresponds to a time interval between pulses of 4 ms. Each pulse has a duration of 200 ns and the electrons in the pulse constitute a current of 250 mA. The current is zero between pulses. Based on the information supplied, answer the following questions.



1. How many electrons are delivered by the accelerator per pulse?
(A) 2.12×10^{11} (B) 21.2×10^{10}
(C) 3.13×10^{11} (D) 31.3×10^{10}

2. The average current per pulse delivered by the accelerator is
(A) 0.005% of peak current
(B) 0.05% of peak current
(C) 0.001% of peak current
(D) 0.01% of peak current

3. The peak power delivered by the electron beam is
(A) 1 MW (B) 10 MW (C) 100 MW (D) 100 kW

4. The average power (instead of the peak power) delivered by the electron beam is
(A) 500 MW (B) 50 MW (C) 25 W (D) 500 W

Comprehension 2

A galvanometer (coil resistance 99 Ω) is converted into an ammeter using a shunt of 1 Ω and connected as shown in figure (a). The ammeter reads 3 A. The same galvanometer is converted into a voltmeter by connecting a resistance of 101 Ω in series. This voltmeter is connected as shown in figure (b).

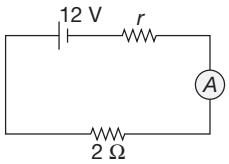


Figure (a)

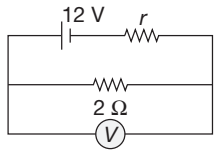


Figure (b)

Its reading is found to be $\frac{4}{5}$ times the full scale reading.

Based on the facts and figures provided answer the following questions.

5. The internal resistance r of the cell is

(A) 10.1 Ω (B) 1 Ω

(C) 1.01 Ω (D) 11.1 Ω

6. The range of the ammeter and voltmeter respectively is

(A) 5 A, 9 V (B) 50 A, 9 V

(C) 5 A, 9.95 V (D) 5.95 A, 9 V

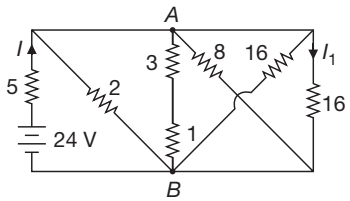
7. The full scale deflection current in the galvanometer is given by

(A) 5 A (B) 0.05 A

(C) 0.5 A (D) 50 A

Comprehension 3

Based on the circuit shown, answer the following questions. Assume that all the resistances have value in ohm.



8. The current I in the circuit is given by

(A) 2 A (B) 4 A

(C) 6 A (D) 8 A

9. The current I_1 is given by

(A) 1 A (B) $\frac{1}{2}$ A

(C) $\frac{1}{4}$ A (D) $\frac{1}{8}$ A

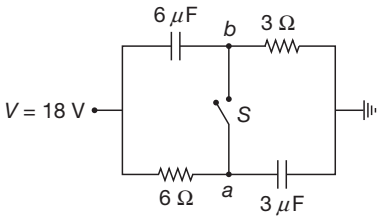
10. The potential difference across the points AB is V_{AB}. Then

(A) $V_{AB} = 4$ V (B) $V_{AB} = 8$ V

(C) $V_{AB} = 6$ V (D) $V_{AB} = 0$

Comprehension 4

Based on the circuit diagram shown, answer the following questions.



11. The potential of point a with respect to point b in figure when switch S is open is

(A) $V_{ab} = 9$ V (B) $V_{ab} = 18$ V

(C) $V_{ab} = -9$ V (D) $V_{ab} = -18$ V

12. The final potential of point b with respect to ground when switch S is closed is

(A) 18 V (B) 9 V

(C) 6 V (D) 3 V

13. The change in the value of charge on 3 μF capacitor is Δq and on 6 μF capacitor is $\Delta q'$, when the switch S is closed . Then

(A) $\Delta q = \Delta q' = -36\ \mu C$

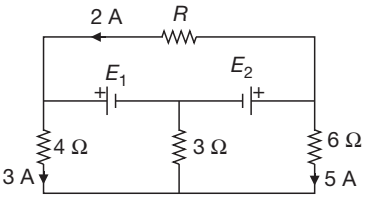
(B) $\Delta q = 36\ \mu C,\ \Delta q' = -36\ \mu C$

(C) $\Delta q = \Delta q' = -18\ \mu C$

(D) $\Delta q' = 18\ \mu C,\ \Delta q = -18\ \mu C$

Comprehension 5

Based on the circuit diagram shown answer the following questions.



14. The current in the 3 Ω resistor is

(A) 2 A (B) 4 A

(C) 8 A (D) 10 A

15. The respective E_1 and E_2 values, in volt, are given by
(A) 34, 56 (B) 36, 54
(C) 54, 36 (D) 56, 34

16. The resistance R, in ohm is
(A) 3 Ω (B) 6 Ω
(C) 7.5 Ω (D) 9 Ω

Comprehension 6

A perfect insulator is filled between the plates of a capacitor. For practical purposes this insulator is assumed to be perfect when its resistance is taken to be very large or infinite. Due to this, if a capacitor is connected to a battery, no current flows through it in steady state. However, an insulator must have some finite resistance which can be calculated by using the relation $R = \frac{\rho l}{A}$. So, due to the finite value of the resistance a non zero current must pass through the resistance. This small non zero current is called leakage current. Consider a leaky parallel plate capacitor is filled completely with material having dielectric constant $K = 18$ and its resistivity is $(4\pi \times 10^3)\Omega$m. The charge on the plate at instant $t = 0$ is 2 μC. Based on the above facts and the data provided answer the following questions.

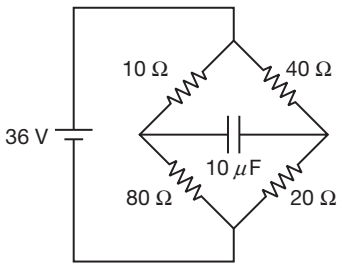
17. Time constant of circuit is
(A) 3 μs (B) 2 μs
(C) 9 μs (D) 1 μs

18. Charge on the capacitor at $t = 2$ μs is
(A) 0.54 μC (B) 0.37 μC
(C) 1 μC (D) 0.74 μC

19. Leakage current at $t = 4$ μs is
(A) 0.48 μA (B) 0.36 μA
(C) 0.24 μA (D) 0.13 μA

Comprehension 7

Consider the circuit shown here. Based on the circuit and the basic knowledge of the Kirchhoff's Laws, answer the following questions.



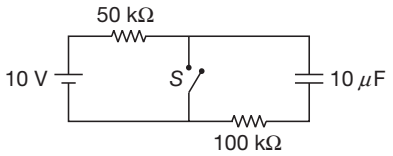
20. In steady state, the potential difference across the capacitor is given by
(A) zero (B) 10 V
(C) 20 V (D) 25 V

21. The capacitor current as a function of time, when the battery is disconnected is given by
(A) $100\ \exp\left(-\frac{3t}{1000}\right)$ mC
(B) $100\ \exp\left(-\frac{t}{1000}\right)$ μC
(C) $200\ \exp\left(-\frac{3t}{1000}\right)$ μC
(D) $200\ \exp\left(-\frac{t}{1000}\right)$ mC

22. The time taken by the capacitor to discharge until the potential difference across it is 1 V
(A) 17.66 ms (B) 1.766 ms
(C) 1766 ms (D) 1 ms

Comprehension 8

In the circuit shown, the switch S has been open for a long time. It is then suddenly closed. The time constant of the circuit before the switch is closed is τ_1 and that after the switch is closed τ_2. Assuming the switch to be closed just when $t = 0$, give answers to the following questions.



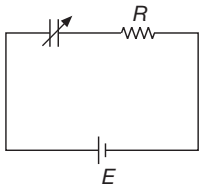
23. The value of τ_1 is
(A) 0.5 s (B) 1 s
(C) 1.5 s (D) 2 s

24. The value of τ_2 is
(A) 0.5 s (B) 1 s
(C) 1.5 s (D) 2 s

25. The current in the switch as a function of time is
(A) $100(2 + e^{-t})$ mA (B) $100(2 - e^{-t})$ mA
(C) $100(2 + e^{-t})$ μA (D) $100(2 - e^{-t})$ μA

Comprehension 9

A certain circuit consists of a variable capacitor, a resistor R and an emf source of emf E as shown in figure. Initially the capacitance of the capacitor is 10 μF which is abruptly changed to 5 μF at $t = 0$. If $E = 10$ V and $R = 50$ Ω, then answer the following questions



26. The current in the circuit just after the capacitance has been changed is

(A) $\frac{1}{5}$ A (B) $\frac{2}{5}$ A

(C) $\frac{3}{5}$ A (D) $\frac{4}{5}$ A

27. The current in the circuit as function of time is

(A) $0.2\exp(-2000t)$ (B) $0.2\exp(-4000t)$

(C) $0.1\exp(-2000t)$ (D) $0.1\exp(-4000t)$

28. The charge on the capacitor as function of time is

(A) $50\left(1+e^{-2000t}\right)$ (B) $5\left(1+e^{-4000t}\right)$

(C) $50\left(1+e^{-4000t}\right)$ (D) $5\left(1+e^{-2000t}\right)$

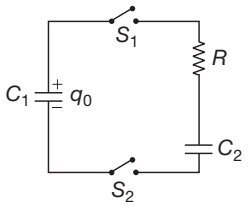
29. The heat produced in the resistor as function of time is

(A) $25\left(1-e^{-8000t}\right)$ mJ (B) $250\left(1-e^{-4000t}\right)$ μJ

(C) $250\left(1-e^{-8000t}\right)$ μJ (D) $25\left(1-e^{-4000t}\right)$ mJ

Comprehension 10

The capacitor C_1 in the figure shown initially carries a charge q_0. When the switches S_1 and S_2 are closed, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which is initially uncharged.



30. The charge flown through wires as a function of time t is

(A) $q_0 e^{-\frac{t}{RC}} + \frac{C}{C_2} q_0$ (B) $\frac{q_0 C}{C_1} \times \left[1 - e^{-\frac{t}{RC}}\right]$

(C) $q_0 \frac{C}{C_1} e^{-\frac{t}{CR}}$ (D) $q_0 e^{-\frac{t}{RC}}$

where, $C = \frac{C_1 C_2}{C_1 + C_2}$

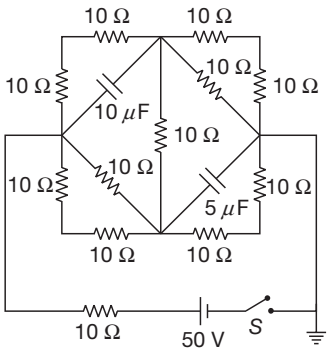
31. The total heat dissipated in the circuit during the discharging process of C_1 is

(A) $\frac{q_0^2}{2C_1^2} \times C$ (B) $\frac{q_0^2}{2C}$

(C) $\frac{q_0^2 C_2}{2C_1^2}$ (D) $\frac{q_0^2}{2C_1 C_2}$

Comprehension 11

In the circuit shown in Figure, the capacitors are initially uncharged. Based on the facts & figure provided, answer the following questions.



32. The initial value of the battery current when the switch S is closed is

(A) 2 A (B) 4 A

(C) 6 A (D) 8 A

33. The battery current after a long time is

(A) $\frac{33}{70}$ A (B) $\frac{73}{30}$ A

(C) $\frac{70}{33}$ A (D) None of these

34. The charges on the 10 μF and 5 μF capacitors after the steady state is reached are q_{10} and q_5 respectively. Then

(A) $q_5 = q_{10} = \frac{2500}{33}$ μC

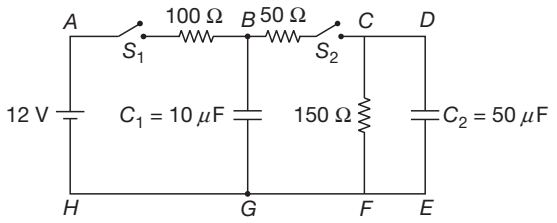
(B) $q_5 = q_{10} = \frac{5000}{33}$ μC

(C) $q_5 = \frac{2500}{33}$ μC, $q_{10} = \frac{5000}{33}$ μC

(D) $q_5 = \frac{5000}{33}$ μC, $q_{10} = \frac{2500}{33}$ μC

Comprehension 12

In the circuit shown the capacitors are initially uncharged. The switch S_2 is closed and then switch S_1 is closed. Based on the facts & figure provided, answer the following questions.



35. What is the battery current immediately after S_1 is closed?
(A) 1.2 A (B) 12 A
(C) 0.12 A (D) None of these

36. What is battery current a long time after both switches are closed?
(A) 0.08 A (B) 0.04 A
(C) 0.02 A (D) 0.01 A

37. What is final voltage across C_1?
(A) 2 V (B) 4 V
(C) 6 V (D) 8 V

38. What is final voltage across C_2?
(A) 2 V (B) 4 V
(C) 6 V (D) 8 V

39. Switch S_2 is opened again after a long time. The current, (in A) in the 150 Ω resistor as a function of time is
(A) $0.02\exp\left(-\frac{400t}{3}\right)$ (B) $-0.04\exp\left(\frac{200t}{3}\right)$
(C) $-0.04\exp\left(-\frac{400t}{3}\right)$ (D) $-0.02\exp\left(-\frac{400t}{3}\right)$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	r	●	●
B	p	●	●	s	t
C	●	●	r	s	t
D	p	q	r	●	●

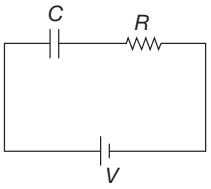
1. **COLUMN-I** contains some of the laws while **COLUMN-II** contains the corresponding expression. Match the Laws in **COLUMN-I** with the expressions in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Ohm's Law	(p) $\Sigma V = 0$ in a closed mesh
(B) Kirchhoff's Junction Law	(q) $R = \frac{V}{I}$ under same physical conditions
(C) Kirchhoff's Loop Law	(r) $W = I^2RT$ joule, heat produced in a resistance
(D) Joule's Law	(s) $\Sigma I = 0$ at any junction

2. Three wires of same material are connected in parallel to a battery. The length ratio of wires is 1 : 2 : 3 and the ratio of their area of cross-section is 2 : 4 : 1. Match the ratio of quantities in **COLUMN-I** with their respective answers in **COLUMN-II**

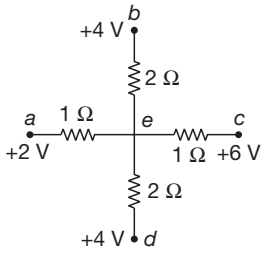
COLUMN-I	COLUMN-II
(A) Resistance	(p) 1 : 1 : 1
(B) Current	(q) 6 : 6 : 1
(C) Power	(r) 1 : 6 : 6
(D) Conductivity	(s) 1 : 1 : 6

3. Consider a series RC circuit. Match the quantities in **COLUMN-I** with the quantities in **COLUMN-II**



COLUMN-I	COLUMN-II
(A) Charging current at time $t = 0$	(p) $\frac{1}{2}CV^2$
(B) Discharging current at $t = 0$	(q) Maximum
(C) While charging energy stored	(r) Capacitor becomes short circuit
(D) While charging energy dissipated as heat	(s) Exponential Law

4. For the circuit shown in figure, match the two columns.



COLUMN-I	COLUMN-II
(A) Current in wire *ae*	(p) 1 A
(B) Current in wire *be*	(q) 2 A
(C) Current in wire *ce*	(r) 0.5 A
(D) Current in wire *de*	(s) None of these

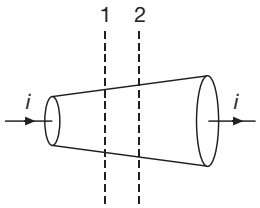
5. Consider two identical cells each of emf E and internal resistance r connected to a load resistance R

COLUMN-I	COLUMN-II
(A) For maximum power transfer to load if cells are connected in series	(p) $\frac{E^2}{4r}$
(B) For maximum power transfer to load if cells are connected in parallel	(q) $\frac{E^2}{2r}$

(*Continued*)

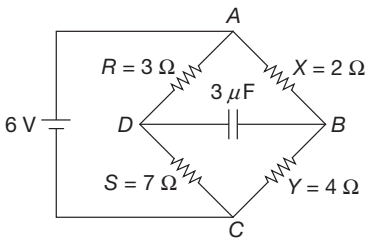
COLUMN-I	COLUMN-II
(C) For series combination of cells	(r) $E_{eq} = E, r_{eq} = \frac{r}{2}$
(D) For parallel connection of cells	(s) $E_{eq} = 2E, r_{eq} = 2r$

6. Current i is flowing through a wire of non-uniform cross-section as shown. Match the following two columns.



COLUMN-I	COLUMN-II
(A) Current density	(p) is more at 1
(B) Electric field	(q) is more at 2
(C) Resistance per unit length	(r) is same at both sections 1 and 2
(D) Potential difference per unit length	(s) data is insufficient

7. With reference to the circuit diagram shown match the following



COLUMN-I	COLUMN-II
(A) Potential difference, in volt, across A and D	(p) zero
(B) Potential difference, in volt, across capacitor	(q) $\frac{9}{5}$
(C) Value of Y, in ohm, for which no energy is stored across capacitor	(r) $\frac{1}{5}$

(*Continued*)

COLUMN-I	COLUMN-II
(D) Steady state current, in ampere, in the branch containing the capacitor is	(s) 14
	(t) $\frac{14}{3}$

8. Match the consumption of power given in **COLUMN-II** to the corresponding appliance(s) given in **COLUMN-I**.

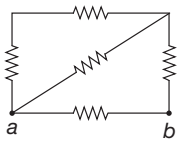
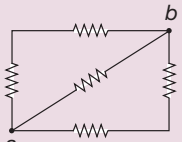
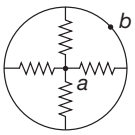
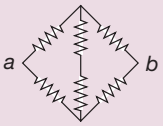
COLUMN-I	COLUMN-II
(A) 100 W, 220 V bulb connected across 220 V supply	(p) 300 W
(B) 200 W, 220 V bulb connected across 220 V supply	(q) 66.7 W
(C) Above 100 W and 200 W bulbs connected in series across a 220 V supply	(r) 100 W
(D) Above 100 W, 200 W bulbs connected in parallel across a 220 V supply	(s) 200 W

9. Two bulbs A and B consume same power P when operated at voltage V_A and V_B respectively. Bulbs can be connected with a dc voltage source. Match the quantities for respective combinations in **COLUMN-I** to their appropriate answers in **COLUMN-II**.

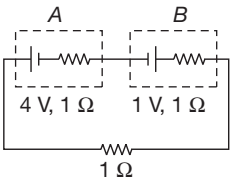
COLUMN-I	COLUMN-II
(A) In series connection, the ratio of potential difference across A and B	(p) $\frac{R_A}{R_B}$
(B) In series connection, the ratio of power consumed by A and B	(q) $\frac{V_A^2}{V_B^2}$
(C) In parallel connection, the ratio of current in A and B	(r) $\frac{R_B}{R_A}$
(D) In parallel connection, the ratio of power consumed in A and B	(s) $\frac{V_B^2}{V_A^2}$

(*Continued*)

10. **COLUMN-I** represents a combination of resistors of equal identical resistors of resistance r while **COLUMN-II** represents the equivalent resistance across (a) and (b). Match identities in **COLUMN-I** to their corresponding match(es) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) $\frac{r}{4}$
(B) 	(q) r
(C) 	(r) $\frac{r}{2}$
(D) 	(s) $\frac{5}{8}r$

11. In the circuit shown in figure, match the following two columns

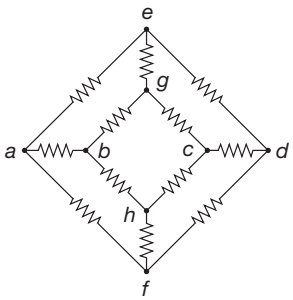


COLUMN-I	COLUMN-II (In SI Units)
(A) Potential difference across battery A	(p) zero

(*Continued*)

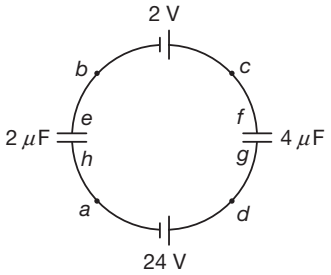
COLUMN-I	COLUMN-II (In SI Units)
(B) Potential difference across battery B	(q) 1
(C) Net power supplied/ consumed by A	(r) 2
(D) Net power supplied/ consumed by B	(s) 3

12. For the given network if R_{ij} is the equivalent resistance between the points i, j of the circuit. Then match the resistances in **COLUMN-I** with their respective equivalent values in **COLUMN-II**



COLUMN-I	COLUMN-II
(A) R_{ab}	(p) zero
(B) R_{bc}	(q) $\frac{3R}{4}$
(C) R_{ac}	(r) $\frac{5R}{6}$
(D) R_{ch}, when points c and h are shorted	(s) $\frac{7R}{12}$

13. With reference to the circuit diagram shown match the following



COLUMN-I	COLUMN-II
(A) $v_d - v_c$ (in volt)	(p) –8
(B) $v_a - v_b$ (in volt)	(q) –4
(C) work done by 12 V cell is (in μJ)	(r) –192
(D) work done by 24 V cell is (in μJ)	(s) 384

(*Continued*)

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

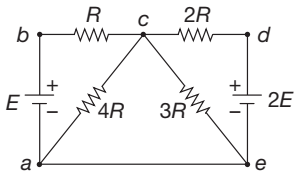
1. An aluminium wire having a cross-sectional area of 4×10^{-6} m^2 carries a current of 5 A. Find the drift speed of the electrons in the wire, in μms^{-1}. The density of aluminium is 2.7 gcm^{-3}. Assume that one conduction electron is supplied by each atom.

2. A carbon wire and a Nichrome wire are connected in series, so that the same current exists in both wires. If the combination has a resistance of 10 kΩ at 0 °C, what is the resistance of each wire, in ohm, at 0 °C, so that the resistance of the combination does not change with temperature?

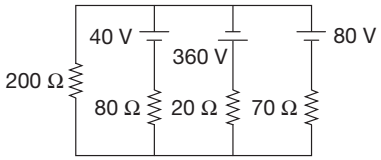
3. A high-voltage transmission line with a diameter of 2 cm and a length of 200 km carries a steady current of 1000 A. If the conductor is copper wire with a free charge density of 8.49×10^{28} electron / m^3, how long does it take one electron to travel the full length of the line? Give your answer in years.

4. The potential difference across the filament of a lamp is maintained at a constant level while equilibrium temperature is being reached. It is observed that the steady-state current in the lamp is only one tenth of the current drawn by the lamp when it is first turned on. If the temperature coefficient of resistivity for the lamp at 20 °C is 0.00450 $(°C)^{-1}$, and if the resistance increases linearly with increasing temperature, what is the final operating temperature, in °C, of the filament?

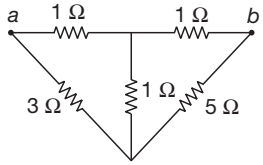
5. Taking $R = 1\ k\Omega$ and $E = 250$ V in figure, determine the direction and magnitude of the current, in mA, in the horizontal wire between a and e.



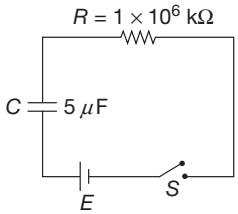
6. In the circuit of figure, determine the current, in ampere, in each resistor and the voltage across the 200 Ω resistor.



7. For the network shown in figure, show that the resistance $R_{ab} = \left(1 + \frac{x}{17}\right)\Omega$. Find x



8. Consider a series RC circuit (see figure) for which $R = 1\ M\Omega$, $C = 5\ \mu F$ and $E = 30$ V. Find



(a) the time constant of the circuit, in second.

(b) the maximum charge, in μC, on the capacitor after the switch is closed.

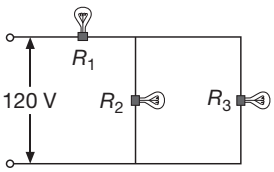
(c) find the current in the resistor 10 s after the switch is closed, in μA to the nearest integer.

9. A capacitor in an RC circuit is charged to 60% of its maximum value in 0.9 s. What is the time constant of the circuit? (Express your result in second to the nearest integer).

10. A $10\ \mu F$ capacitor is charged by a 10 V battery through a resistance R. The capacitor reaches a potential difference of 4 V in a time 3 s after charging begins. Find R, in kΩ.

11. When two unknown resistors are connected in series with a battery, the battery delivers 225 W and carries a total current of 5 A. For the same total current, 50 W is delivered when the resistors are connected in parallel. Determine the values of the two resistors (in ohm).

12. Three 60 W, 120 V light bulbs are connected across a 120 V power source, as shown in figure. Find

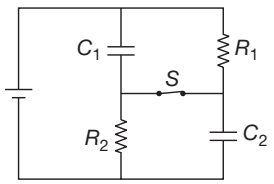


(a) the total power delivered to the three bulbs (in watt) and

(b) the voltage across each (in volt).

Assume that the resistance of each bulb is constant.

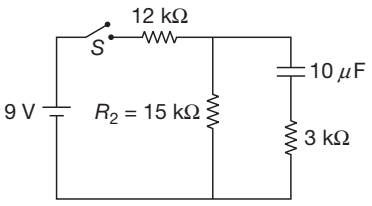
13. Switch S has been closed for a long time, and the electric circuit shown in figure carries a constant current. Take $C_1 = 3\ \mu F$, $C_2 = 6\ \mu F$, $R_1 = 4\ k\Omega$, and $R_2 = 7\ k\Omega$. The power delivered to R_2 is 2.4 W.



(a) Find the charge on C_1, in μC.

(b) Now the switch is left open for a long time. Find the change in the value of charge on C_2, in μC.

14. In figure, suppose the switch has been closed for a time sufficiently long for the capacitor to become fully charged. Find



(a) the steady-state current in each resistor, in μA.
(b) the charge Q on the capacitor, in μC.
(c) the switch is now opened at $t = 0$. Write an equation for the current I through R_2 as a function of time.
(d) the time interval required for the charge on the capacitor to fall to one-fifth its initial value, in millisecond.

15. A 75 W tungsten light bulb has a resistance of 190 Ω when switched on and 15 Ω when switched off. Estimate the temperature, in kelvin of the filament when the bulb is switched on. Given $\alpha = 4.5 \times 10^{-3}$ °C^{-1}

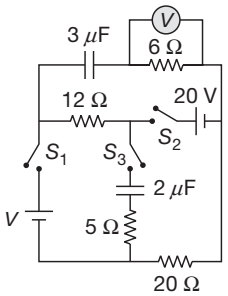
16. Two wires of different materials x and y have resistances per unit lengths, 100 Ω(km)$^{-1}$, 50 Ω(km)$^{-1}$ and temperature coefficients 0.0025 (°C)$^{-1}$, 0.00075 (°C)$^{-1}$ respectively. It is desired to make a coil having 1000 Ω resistance and a temperature coefficient of 0.001(°C)$^{-1}$ by using suitable lengths of the two wires in series. Calculate their respective lengths in metre.

17. A block of metal is heated directly by dissipating power in the internal resistance of block. Because of temperature rise, the resistance increases exponentially with time and is given by $R(t) = 0.5e^{2t}$, where t is in second. The block is connected across a 110 V source and dissipates 7644 J heat energy over a certain period of time. Calculate this period of time (in millisecond).

18. An electric kettle has two coil. When one coil is switched on, it takes 15 minutes to boil water and when the second coil is switched on it takes double the time for the same job to be done. How long, in minutes will it take to boil water when both the coils are used in
(a) series?
(b) parallel?

19. A wire of length 1 m and radius 10^{-3} m is carrying a heavy current and is assumed to radiate as a black body. At equilibrium its temperature is 900 K while that of the surroundings is 300 K. The resistivity of the material of the wire at 300 K is $\pi^2 \times 10^{-8}$ Ωm and its temperature coefficient of resistance is 7.8×10^{-3} per °C. Find the current, in ampere in the wire. Stefan's constant, $\sigma = 5.68 \times 10^{-8}$ Wm^{-2}K^{-4}

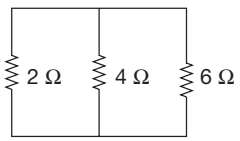
20. With only switch S_1 closed in the circuit shown,



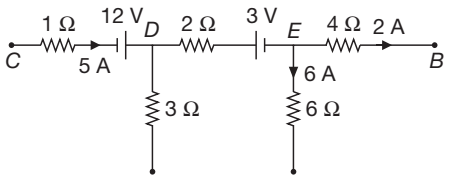
(a) what is the steady state reading of the voltmeter (in volt)?
(b) what is the charge on the 3 μF capacitor (in μC)?
(c) how much power does the 12 V battery supply in the steady state after all the switches are closed (in watt)?
(d) what is the charge on 2 μF capacitor (in μC)?

21. 200 identical electrical bulbs, each having resistance 300 Ω, are connected in parallel to a current source of emf 200 V and internal resistance 0.5 Ω.
(a) What is the power expended, in watt on each bulb?
(b) The fractional change in power expended on each bulb when one bulb burns out is f. Find $10000f$.

22. In which branch of the circuit shown in figure, an 11 V battery be inserted so that it dissipates minimum power. What will be the current, in ampere, through the 2 Ω resistance for this position of the battery?



23. Figure shows part of a circuit. Calculate the power, in ohm, dissipated in $3\ \Omega$ resistance. What is the potential difference $V_C - V_B$?

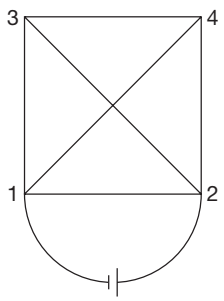


24. An ammeter and a voltmeter are connected in series to a battery with an emf $E = 6$ V. When a certain resistance is connected in parallel with the voltmeter, the reading of the latter decreases two times, whereas the reading of the ammeter increase the same number of times. Find the voltmeter reading, in volt, after the connection of the resistance.

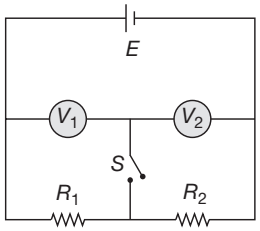
25. A voltmeter of resistance R_1 and an ammeter of resistance R_2 are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third. Find R_1 and R_2 when $R = 3\ \Omega$.

26. The emf of a storage battery is 90 V before charging and 100 V after charging. When charging began the current was 10 A. What is the current, in ampere, at the end of charging if the internal resistance of the storage battery during the whole process of charging may be taken as constant and equal to $2\ \Omega$?

27. The circuit shown in figure is made of a homogeneous wire of uniform cross-section. 1234 is a square. The ratio $\frac{Q_{12}}{Q_{34}}$ of the amounts of heat liberated per unit time in conductor 1-2 and 3-4 is found to be $11x + y\sqrt{2x}$, where x and y are non zero positive integers. Find the values of x and y.



28. In the circuit shown in figure, V_1 and V_2 are two voltmeters having resistances $6000\ \Omega$ and $4000\ \Omega$ respectively. E.M.F. of the battery is 250 V, having negligible internal resistance. Two resistances R_1 and R_2 are $4000\ \Omega$ and $6000\ \Omega$ respectively. Find the reading of the voltmeters V_1 and V_2, in volt, when

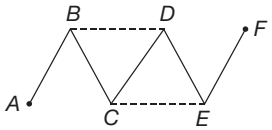


(a) the switch S is open and

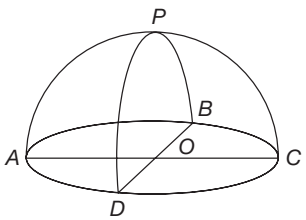
(b) the switch S is closed.

29. It is required to send a current of 8 A through a circuit whose resistance is $5\ \Omega$. What is the least number of cells which must be used for this purpose and how should they be connected? The emf of each cell is 2 V and the internal resistance is $0.5\ \Omega$.

30. What will be the change in the resistance, in ohm, of a circuit between A and F consisting of five identical conductors each of resistance $2\ \Omega$ if two similar conductors added as shown by the dashed line in figure.

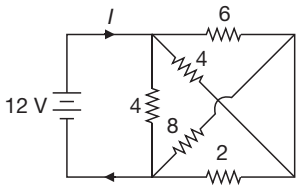


31. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length $\lambda = \left(\frac{64}{2+\pi}\right)\frac{1}{a}$. Find the equivalent resistance, in ohm, across OP.



32. Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section $1\ \text{cm}^2$ and length 10 km. Free electron density of copper is $n = 8.5 \times 10^{28}$ per m^3. How long will it take the electric charge to travel from one end of the conductor to the other. Give your answer in years.

33. Compute the value of battery current I, in ampere, shown in figure. All resistances are in ohm.



34. Calculate the potential, in volt, of points A, B, C and D shown in Figure 1. What would be the new potential values, in volt, if polarity of 6 V battery is reversed as shown in Figure 2? All resistances are in Ω.

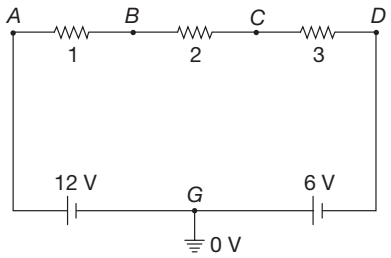


Figure 1

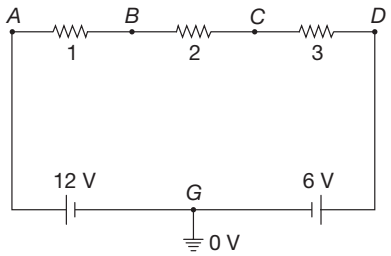


Figure 2

35. Twelve cells each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with the others. The current 3 A when the cells and battery aid each other and 2 A when the cells and battery oppose each other. How many cells in the battery are wrongly connected?

36. A parallel plate capacitor has plate of area $10\ \text{cm}^2$ separated by a distance of 1 mm. It is filled with the dielectric mica the resistivity of mica is $1\times10^3\ \Omega\text{m}$, the leakage current through the capacitor is found to be $*\times10^{-13}$ ampere, where $*$ is not readable. Find the value of $*$.

37. The gap between the plates of a parallel plate capacitor is filled up with an inhomogeneous poorly conducting medium whose conductivity varies linearly in a direction perpendicular to the plates from $\sigma=10^{-12}\ \Omega^{-1}\text{m}^{-1}$ to $2\times10^{-12}\ \Omega^{-1}\text{m}^{-1}$. Each plate has an area $230\ \text{cm}^2$ and the separation between the plates is $d=2$ mm. Find the current, in nanoampere, flowing through the capacitor due to a voltage $V=300$ V.

38. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. Calculate the resistance R that one must be put in series with the bulb, so that the bulb delivers 500 W. Give your answer in ohm.

39. A fuse of lead wire has an area of cross-sectional $0.2\ \text{mm}^2$. On short circuiting, the current in the fuse wire reaches 30 A. How long after the short-circuiting, will fuse begin to melt? Give your answer in millisecond.

For lead, specific heat $0.032\ \text{calg}^{-1}(^\circ\text{C})^{-1}$, melting point is $327\ ^\circ\text{C}$, density is $11.34\ \text{gcm}^{-3}$ and resistivity is $22\times10^{-6}\ \Omega\text{cm}$. The initial temperature of wire is 20°C. Neglect heat losses.

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1. **[Online April 2019]**

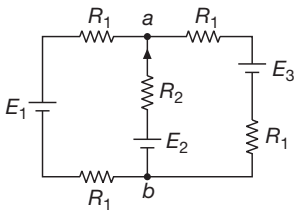
A $200\ \Omega$ resistor has a certain colour code. If one replaces the red colour by green in the code, the new resistance will be

(A) $400\ \Omega$ (B) $500\ \Omega$

(C) $300\ \Omega$ (D) $100\ \Omega$

2. **[Online April 2019]**

For the circuit shown, with $R_1=1.0\ \Omega$, $R_2=2.0\ \Omega$, $E_1=2$ V and $E_2=E_3=4$ V, the potential difference between the points a and b is approximately (in V)



(A) 2.7 (B) 3.7

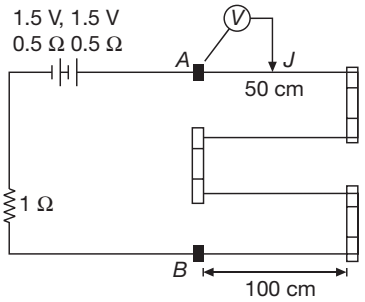
(C) 2.3 (D) 3.3

3. **[Online April 2019]**
A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when

(A) $R = 1000r$ (B) $R = r$

(C) $R = 2r$ (D) $R = 0.001r$

4. **[Online April 2019]**
In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01\ \Omega\text{cm}^{-1}$. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be

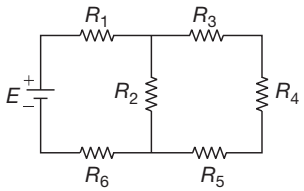


(A) 0.75 V (B) 0.50 V

(C) 0.20 V (D) 0.25 V

5. **[Online April 2019]**
In the figure shown, what is the current (in Ampere) drawn from the battery? You are given

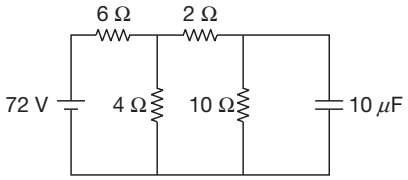
$R_1 = 15\ \Omega,\ R_2 = 10\ \Omega,\ R_3 = 20\ \Omega,\ R_4 = 5\ \Omega,\ R_5 = 25\ \Omega,$
$R_6 = 30\ \Omega,\ E = 15$ V



(A) $\frac{13}{24}$ (B) $\frac{9}{32}$

(C) $\frac{20}{3}$ (D) $\frac{7}{18}$

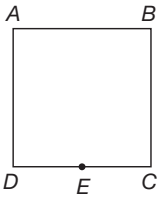
6. **[Online April 2019]**
Determine the charge on the capacitor in the following circuit



(A) 200 μC (B) 60 μC

(C) 10 μC (D) 2 μC

7. **[Online April 2019]**
A wire of resistance R is bent to form a square $ABCD$ as shown in the figure. The effective resistance between E and C is (E is mid-point of arm CD)



(A) $\frac{3}{4}R$ (B) R

(C) $\frac{1}{16}R$ (D) $\frac{7}{64}R$

8. **[Online April 2019]**
A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to

(A) 10 V (B) 20 V

(C) 15 V (D) 40 V

9. **[Online April 2019]**
A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be

(A) $\frac{5}{3}\ \Omega$ (B) $\frac{5}{2}\ \Omega$

(C) $\frac{7}{2}\ \Omega$ (D) $\frac{12}{5}\ \Omega$

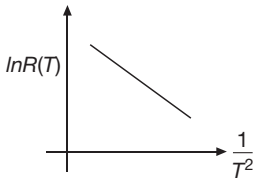
10. **[Online April 2019]**
The resistance of a galvanometer is 50 Ω and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range $0 - 0.5$ A?

(A) 0.2 Ω (B) 0.002 Ω
(C) 0.5 Ω (D) 0.02 Ω

11. [Online April 2019]
In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), its approximate resistivity is $\left(m_e = 9.1\times10^{-31}\text{ kg}\right)$

(A) 10^{-5} Ωm (B) 10^{-6} Ωm
(C) 10^{-7} Ωm (D) 10^{-8} Ωm

12. [Online April 2019]
In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that

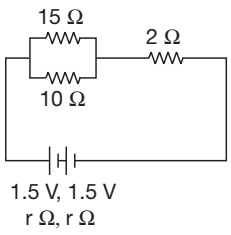


(A) $R(T)=\frac{R_0}{T^2}$ (B) $R(T)=R_0e^{-\frac{T_0^2}{T^2}}$
(C) $R(T)=R_0e^{-\frac{T^2}{T_0^2}}$ (D) $R(T)=R_0e^{\frac{T^2}{T_0^2}}$

13. [Online April 2019]
A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0 – 5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0 – 10 mA is

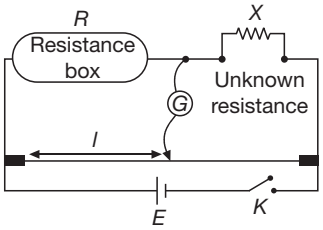
(A) 200 Ω (B) 500 Ω
(C) 100 Ω (D) 10 Ω

14. [Online April 2019]
In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is



(A) 0.5 Ω (B) 0 Ω
(C) 1.5 Ω (D) 1 Ω

15. [Online April 2019]
In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.



S. No.	$R(\Omega)$	l(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?
(A) 1 (B) 2
(C) 3 (D) 4

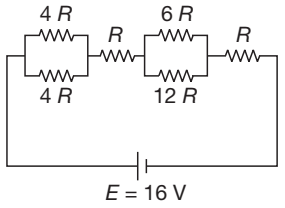
16. [Online April 2019]
A current of 5 A passes through a copper conductor (resistivity $=1.7\times10^{-8}$ Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} ms^{-1}.

(A) 1.3 m^{2}V^{-1}s^{-1} (B) 1.8 m^{2}V^{-1}s^{-1}
(C) 1.5 m^{2}V^{-1}s^{-1} (D) 1.0 m^{2}V^{-1}s^{-1}

17. [Online April 2019]
Space between two concentric conducting spheres of radii a and $b(b>a)$ is filled with a medium of resistivity ρ. The resistance between the two spheres will be

(A) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (B) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$
(C) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (D) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

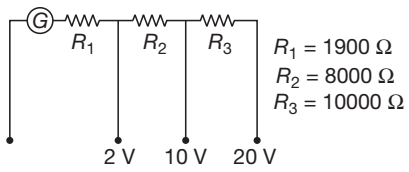
18. [Online April 2019]
The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 W. The value of R is

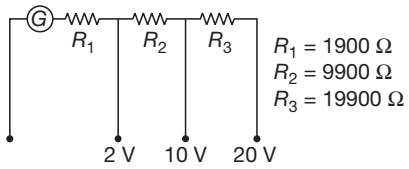


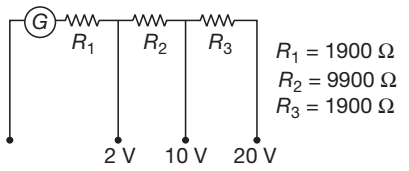
(A) 8 Ω (B) 1 Ω
(C) 16 Ω (D) 6 Ω

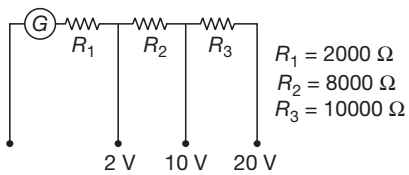
19. [Online April 2019]

A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division . It is to be converted to a voltmeter with three ranges, of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is

(A)


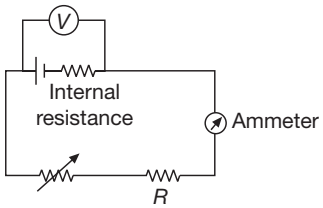
(B)


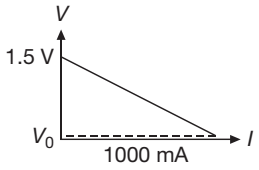
(C)


(D)


20. [Online April 2019]

To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained





If V_0 is almost zero, identify the correct statement

(A) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω
(B) The emf of the battery is 1.5 V and the value of R is 1.5 Ω
(C) The value of the resistance R is 1.5 Ω
(D) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

21. [Online April 2019]

A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0\left(I_0 > I_g\right)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to $V\left(V = GI_0\right)$ by connecting a series resistance R_V to it. Then,

(A) $R_A R_V = G^2$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

(B) $R_A R_V = G^2\left(\dfrac{I_g}{I_0 - I_g}\right)$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_0 - I_g}{I_g}\right)^2$

(C) $R_A R_V = G^2$ and $\dfrac{R_A}{R_V} = \dfrac{I_g}{\left(I_0 - I_g\right)}$

(D) $R_A R_V = G^2\left(\dfrac{\left(I_0 - I_g\right)}{I_g}\right)$ and $\dfrac{R_A}{R_V} = \left(\dfrac{I_g}{\left(I_0 - I_g\right)}\right)^2$

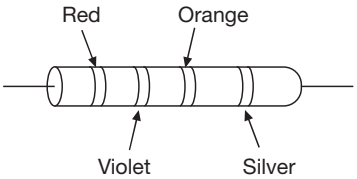
22. [Online January 2019]

A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is

(A) 0.5% (B) 2.0%
(C) 2.5% (D) 1.0%

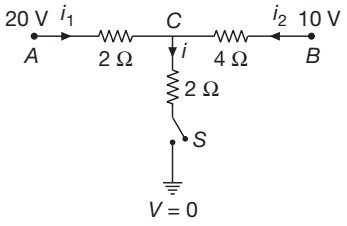
23. [Online January 2019]

A resistance is shown in the figure. Its value and tolerance are given respectively by



(A) 27 kΩ, 20% (B) 270 Ω, 5%
(C) 27 kΩ, 10% (D) 270 Ω, 10%

24. [Online January 2019]
When the switch S, in the circuit shown, is closed, then the value of current i will be

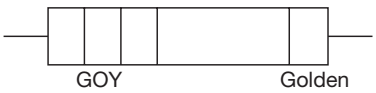


(A) 2 A (B) 5 A
(C) 4 A (D) 3 A

25. [Online January 2019]
Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9×10^{28} m^{-3} the value of v in mms^{-1} is close to (Take charge of electron to be $=1.6\times10^{-19}$ C)

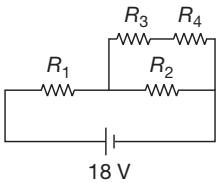
(A) 0.02 (B) 0.2
(C) 3 (D) 2

26. [Online January 2019]
A carbon resistance has a following colour code. What is the value of the resistance?



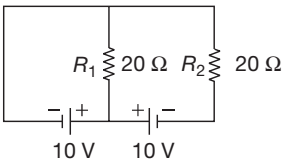
(A) 6.4 MΩ ± 5% (B) 5.3 MΩ ± 5%
(C) 64 kΩ ± 10% (D) 530 kΩ ± 5%

27. [Online January 2019]
In the given circuit the internal resistance of the 18 V cells is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be



(A) 230 Ω (B) 450 Ω
(C) 550 Ω (D) 300 Ω

28. [Online January 2019]
In the given circuit the cells have zero internal resistance. The currents (in amperes) passing through resistance R_1 and R_2 respectively, are



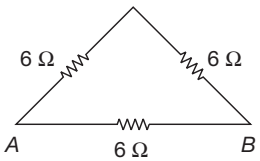
(A) 1, 2 (B) 0, 1
(C) 0.5, 0 (D) 2, 2

29. [Online January 2019]
A 2 W carbon resistor is colour coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is

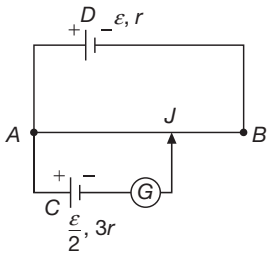
(A) 20 mA (B) 0.4 mA
(C) 100 mA (D) 63 mA

30. [Online January 2019]
A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

(A) 4 Ω (B) 12 Ω
(C) 8 Ω (D) 2 Ω



31. [Online January 2019]
A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\frac{\varepsilon}{2}$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is

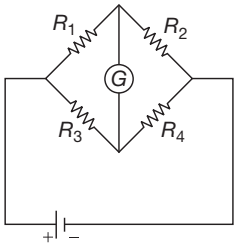


(A) $\frac{11}{12}L$ (B) $\frac{11}{24}L$

(C) $\frac{5}{12}L$ (D) $\frac{13}{24}L$

32. [Online January 2019]

The Wheatstone bridge shown in figure here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are $80\ \Omega$ and $40\ \Omega$ respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be

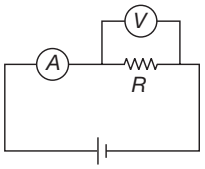


(A) Brown, Blue, Black (B) Red, Green, Brown

(C) Grey, Black, Brown (D) Brown, Blue, Brown

33. [Online January 2019]

The actual value of resistance R, shown in the figure is $30\ \Omega$. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is



(A) $570\ \Omega$ (B) $600\ \Omega$

(C) $350\ \Omega$ (D) $35\ \Omega$

34. [Online January 2019]

A current of $2\ \text{mA}$ was passed through an unknown resistor which dissipated a power of $4.4\ \text{W}$. Dissipated power when an ideal power supply of $11\ \text{V}$ is connected across it is

(A) $11\times10^{-5}\ \text{W}$ (B) $11\times10^{5}\ \text{W}$

(C) $11\times10^{-3}\ \text{W}$ (D) $11\times10^{-4}\ \text{W}$

35. [Online January 2019]

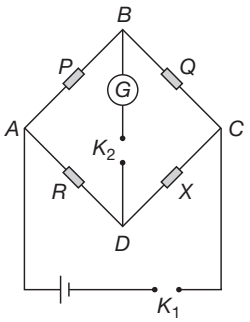
Two equal resistances when connected in series to a battery, consume electric power of $60\ \text{W}$. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be

(A) 60 W (B) 30 W

(C) 120 W (D) 240 W

36. [Online January 2019]

In a Wheatstone bridge (see figure), resistances P and Q are approximately equal. When $R = 400\ \Omega$, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is $405\ \Omega$. The value of X is close to

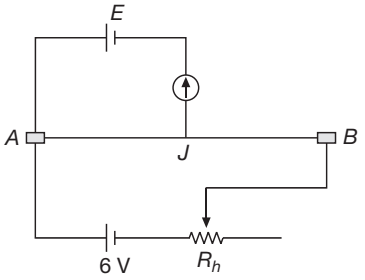


(A) $404.5\ \Omega$ (B) $401.5\ \Omega$

(C) $402.5\ \Omega$ (D) $403.5\ \Omega$

37. [Online January 2019]

The resistance of the metre bridge AB in given figure is $4\ \Omega$. With a cell of emf $E = 0.5\ \text{V}$ and rheostat resistance $R_h = 2\ \Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $E = E_2$ the same null point J is found for $R_h = 6\ \Omega$. The emf E_2 is



(A) 0.6 V (B) 0.5 V

(C) 0.3 V (D) 0.4 V

38. [Online January 2019]

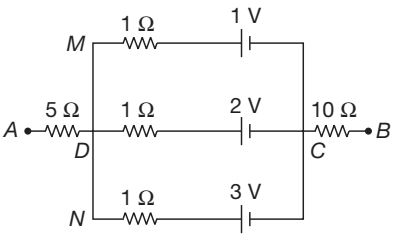
A galvanometer having a resistance of $20\ \Omega$ and 30 divisions on both sides has figure of merit 0.005 ampere/division . The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt , is:

(A) $100\ \Omega$ (B) $125\ \Omega$

(C) $80\ \Omega$ (D) $120\ \Omega$

39. [Online January 2019]

In the circuit shown, the potential difference between A and B is:

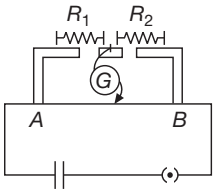


(A) 6 V (B) 3 V

(C) 2 V (D) 1 V

40. [Online January 2019]

In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a $10\ \Omega$ resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1+10)\ \Omega$ such that the null point shifts back to its initial position is:

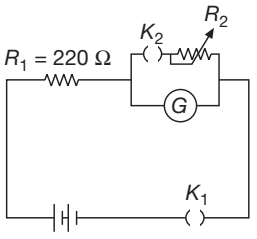


(A) $60\ \Omega$ (B) $30\ \Omega$

(C) $40\ \Omega$ (D) $20\ \Omega$

41. [Online January 2019]

The galvanometer deflection, when key K_1 is closed but K_2 is open, equal θ_0 (see figure). On closing K_2 also and adjusting R_2 to $5\ \Omega$, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then given by (neglect the internal resistance of battery)

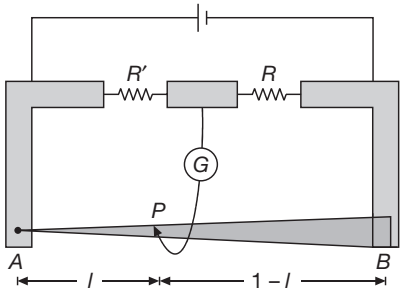


(A) $22\ \Omega$ (B) $25\ \Omega$

(C) $5\ \Omega$ (D) $12\ \Omega$

42. [Online January 2019]

In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{dl}$ of its resistance R with length l is $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?



(A) 0.2 m (B) 0.35 m

(C) 0.25 m (D) 0.3 m

43. [Online January 2019]

An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance $5\ \Omega$. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is

(A) $480\ \Omega$ (B) $490\ \Omega$

(C) $495\ \Omega$ (D) $395\ \Omega$

44. [Online January 2019]

Two electric bulbs, rated at $(25\ \text{W}, 220\ \text{V})$ and $(100\ \text{W}, 220\ \text{V})$, are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then

(A) $P_1 = 9\ \text{W}$, $P_2 = 16\ \text{W}$ (B) $P_1 = 4\ \text{W}$, $P_2 = 16\ \text{W}$

(C) $P_1 = 16\ \text{W}$, $P_2 = 9\ \text{W}$ (D) $P_1 = 16\ \text{W}$, $P_2 = 4\ \text{W}$

45. [Online January 2019]

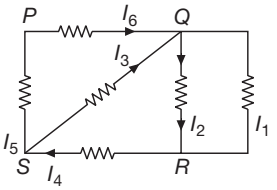
A galvanometer, whose resistance is $50\ \Omega$, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of

(A) $6250\ \Omega$ (B) $250\ \Omega$

(C) $200\ \Omega$ (D) $6200\ \Omega$

46. [Online January 2019]

In the given circuit diagram, the currents, $I_1 = -0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are:



(A) 1.1 A , 0.4 A , 0.4 A

(B) 1.1 A , –0.4 A , 0.4 A

(C) 0.4 A , 1.1 A , 0.4 A

(D) –0.4 A , 0.4 A , 1.1 A

47. [2018]

Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of $10\ \Omega$. The internal resistances of the two batteries are $1\ \Omega$ and $2\ \Omega$ respectively. The voltage across the load lies between

(A) 11.6 V and 11.7 V (B) 11.5 V and 11.6 V

(C) 11.4 V and 11.5 V (D) 11.7 V and 11.8 V

48. [2018]

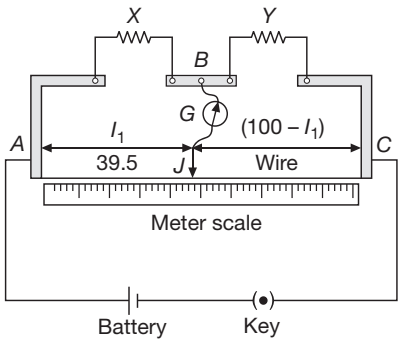
On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combinations is $1\ \text{k}\Omega$. How much was the resistance on the left slot before the interchange?

(A) $990\ \Omega$ (B) $505\ \Omega$

(C) $550\ \Omega$ (D) $910\ \Omega$

49. [Online 2018]

In a meter bridge, as shown in the figure, it is given that resistance $Y = 12.5\ \Omega$ and that the balance is obtained at a distance 39.5 cm from end A (by Jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2?



(A) 19.15 Ω and 60.5 cm

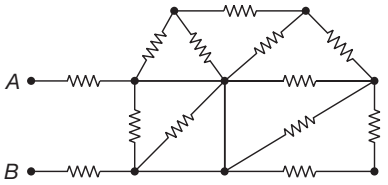
(B) 8.16 Ω and 60.5 cm

(C) 8.16 Ω and 39.5 cm

(D) 19.15 Ω and 39.5 cm

50. [Online 2018]

In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is



(A) $2R$ (B) $\frac{5R}{2}$

(C) $3R$ (D) $\frac{5R}{3}$

51. [Online 2018]

A constant voltage is applied between two ends of a metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be

(A) Increased 8 times (B) Unchanged

(C) Doubled (D) Halved

52. [Online 2018]

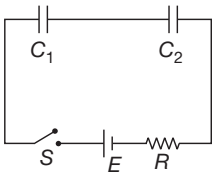
A copper rod of cross-sectional area A carries a uniform current I through it. At temperature T, if the volume charge density of the rod is ρ, how long will the charges take to travel a distance d?

(A) $\frac{\rho dA}{I}$ (B) $\frac{\rho dA}{IT}$

(C) $\frac{2\rho dA}{I}$ (D) $\frac{2\rho dA}{IT}$

53. [Online 2018]

In the following circuit, the switch S is closed at $t = 0$. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \frac{C_1C_2}{C_1 + C_2}\right)$



(A) $C_{eq}E\left[1 - \exp\left(-\frac{t}{RC_{eq}}\right)\right]$

(B) $C_1E\left[1 - \exp\left(-\frac{tR}{C_1}\right)\right]$

(C) $C_{eq}E\exp\left(-\frac{t}{RC_{eq}}\right)$

(D) $C_2E\left[1 - \exp\left(-\frac{t}{RC_2}\right)\right]$

54. [Online 2018]

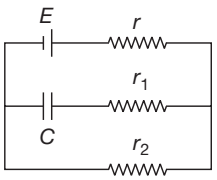
A heating element has a resistance of 100 Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500 °C more than room temperature. What is the temperature coefficient of resistance of the heating element?

(A) 1×10^{-4} °C^{-1} (B) 2×10^{-4} °C^{-1}

(C) 0.5×10^{-4} °C^{-1} (D) 5×10^{-4} °C^{-1}

55. [2017]

In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be

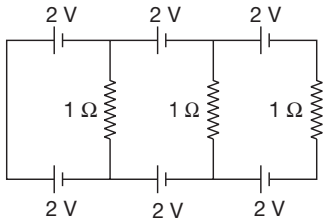


(A) CE (B) $CE\frac{r_1}{(r_2 + r)}$

(C) $CE\frac{r_2}{(r + r_2)}$ (D) $CE\frac{r_1}{(r_1 + r)}$

56. [2017]

In the following circuit the current in each resistance is



(A) 1 A (B) 0.25 A

(C) 0.5 A (D) 0 A

57. [2017]

Which of the following statement is false?

(A) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

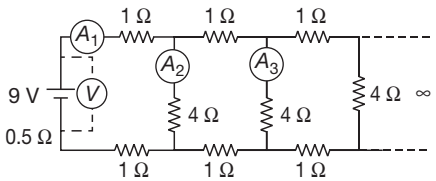
(B) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

(C) A rheostat can be used as a potential divider.

(D) Kirchhoff's second law represents energy conservation.

58. [Online 2017]

A 9 V battery with internal resistance of 0.5 Ω is connected across an infinite network as shown in the figure. All ammeters A_1, A_2, A_3 and voltmeter V are ideal. Choose correct statement.



(A) Reading of V is 9 V

(B) Reading of A_1 is 2 A

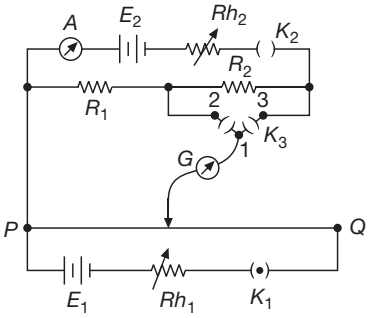
(C) Reading of V is 7 V

(D) Reading of A_1 is 18 A

59. [Online 2017]

A potentiometer PQ is set up to compare two resistances as shown in the figure. The ammeter A in the circuit reads 1 A when two way key K_3 is open. The balance point is at a length l_1 cm from P when two way key K_3 is plugged in between 2 and 1, while the balance points is at a length l_2 cm from P when key K_3

is plugged in between 3 and 1. The ratio of two resistances $\frac{R_1}{R_2}$, is found to be

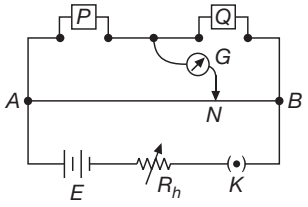


(A) $\frac{l_2}{l_2 - l_1}$ (B) $\frac{l_1}{l_2 - l_1}$

(C) $\frac{l_1}{l_1 + l_2}$ (D) $\frac{l_1}{l_1 - l_2}$

60. [Online 2017]

In a meter bridge experiment resistances are connected as shown in the figure. Initially resistance $P = 4\ \Omega$ and the neutral point N is at 60 cm from A. Now an unknown resistance R is connected in series to P and the new position of the neutral point is at 80 cm from A. The value of unknown resistance R is

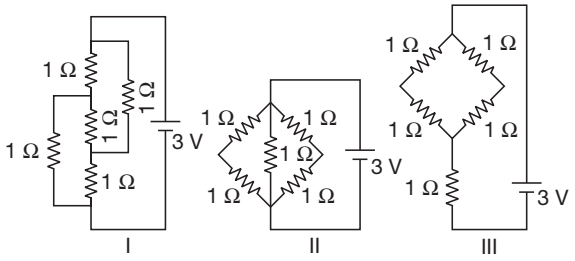


(A) $\frac{20}{3}\ \Omega$ (B) $\frac{33}{5}\ \Omega$

(C) $6\ \Omega$ (D) $7\ \Omega$

61. [Online 2017]

The figure shows three circuits I, II and III which are connected to a 3 V battery. If the powers dissipated by the configurations I, II and III are P_1, P_2 and P_3 respectively, then



(A) $P_3 > P_2 > P_1$ (B) $P_2 > P_1 > P_3$

(C) $P_1 > P_3 > P_2$ (D) $P_1 > P_2 > P_3$

62. [Online 2017]

A uniform wire of length l and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be

(A) 400 Ω (B) 100 Ω

(C) 200 Ω (D) 1600 Ω

63. [2016]

The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by

(A) Linear decrease for Cu, linear decrease for Si.

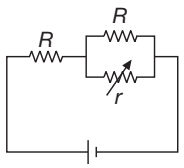
(B) Linear increase for Cu, linear increase for Si.

(C) Linear increase for Cu, exponential increase for Si.

(D) Linear increase for Cu, exponential decrease for Si.

64. [Online 2016]

In the circuit shown, the resistance r is a variable resistance. If for $r = fR$, the heat generation in r is maximum then the value of f is



(A) $\frac{1}{2}$ (B) 1

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

65. [Online 2016]

The resistance of an electrical toaster has a temperature dependence given by $R(T) = R_0\left[1 + \alpha(T - T_0)\right]$ in its range of operation. At $T_0 = 300$ K, $R = 100\ \Omega$ and at $T = 500$ K, $R = 120\ \Omega$. The toaster is connected to a voltage source at 200 V and its temperature is raised at a constant rate from 300 to 500 K in 30 s. The total work done (in kJ) in raising the temperature is

(A) $40\ln\left(\frac{6}{5}\right)$ (B) $20\ln\left(\frac{3}{2}\right)$

(C) $60\ln\left(\frac{6}{5}\right)$ (D) $40\ln\left(\frac{15}{13}\right)$

66. [2015]

When $5V$ potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is

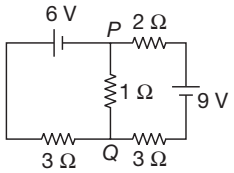
2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(A) $1.6\times10^{-8}\ \Omega$m (B) $1.6\times10^{-7}\ \Omega$m

(C) $1.6\times10^{-6}\ \Omega$m (D) $1.6\times10^{-5}\ \Omega$m

67. [2015]

In the circuit shown, the current in the $1\ \Omega$ resistor is



(A) 1.3 A, from P to Q

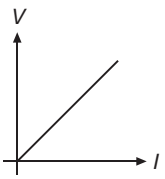
(B) 0 A

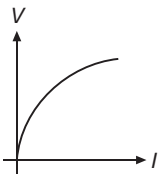
(C) 0.13 A, from Q to P

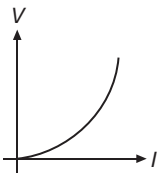
(D) 0.13 A, from P to Q

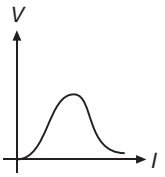
68. [Online 2015]

Suppose the drift velocity v_d in a material varied with the applied electric field E as $V_d \propto \sqrt{E}$. Then V-I graph for a wire made of such a material is best given by

(A) 

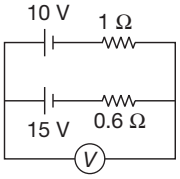
(B) 

(C) 

(D) 

69. [Online 2015]

A 10 V battery with internal resistance $1\ \Omega$ and a 15 V battery with internal resistance $0.6\ \Omega$ are connected in parallel to a voltmeter (shown in figure). The reading in the voltmeter will be close to

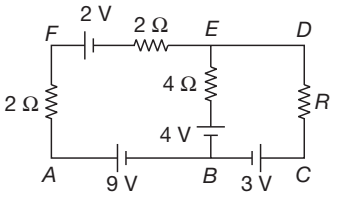


(A) 11.9 V (B) 12.5 V

(C) 13.1 V (D) 24.5 V

70. [Online 2015]

In the electric network shown, when no current flows through the $4\ \Omega$ resistor in the arm EB, the potential difference between the points A and D will be



(A) 3 V (B) 4 V

(C) 5 V (D) 6 V

71. [2014]

In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be

(A) 8 A (B) 10 A

(C) 12 A (D) 14 A

72. [2013]

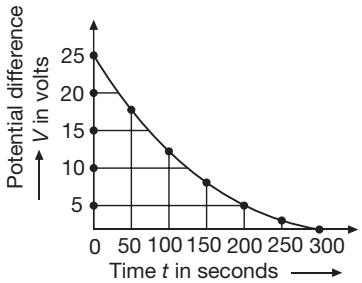
The supply voltage to a room is 120 V. The resistance of the lead wires is $6\ \Omega$. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(A) zero volt (B) 2.9 volt

(C) 13.3 volt (D) 10.04 volt

73. [2012]

The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant t of this circuit lies between



(A) 0 and 50 sec (B) 50 sec and 100 sec

(C) 100 sec and 150 sec (D) 150 sec and 200 sec

74. [2012]

Two electric bulbs marked 25 W – 220 V and 100 W – 220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

(A) 100 W (B) 25 W
(C) neither (D) Both

75. [2011]
If a wire is stretched to make it 0.1% longer, its resistance will
(A) increase by 0.05% (B) increase by 0.2%
(C) decrease by 0.2% (D) decrease by 0.05%

76. [2010]
Two conductors have the same resistance at $0\ ^\circ C$ but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(A) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (B) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(C) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (D) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

77. [2009]
This question contains Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement 1: The temperature dependence of resistance is usually given as $R=R_0(1+\alpha\Delta t)$. The resistance of a wire changes from $100\ \Omega$ to $150\ \Omega$ when its temperature is increased from $27\ ^\circ C$ to $227\ ^\circ C$. This implies that $\alpha=2.5\times10^{-3}\ ^\circ C^{-1}$.

Statement 2: $R=R_0(1+\alpha\Delta t)$ is valid only when the change in the temperature ΔT is small and $\Delta R=(R-R_0)\ll R_0$.

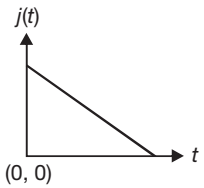
(A) Statement 1 is true, Statement 2 is false.
(B) Statement 1 is true, Statement 2 is true; Statement 2 is the correct explanation of Statement 1.
(C) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of Statement 1.
(D) Statement 1 is false, Statement 2 is true.

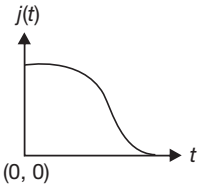
ARCHIVE: JEE ADVANCED

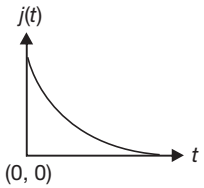
Single Correct Choice Type Problems

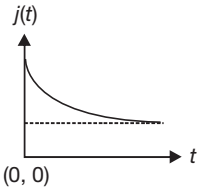
In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2016]
An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t=0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

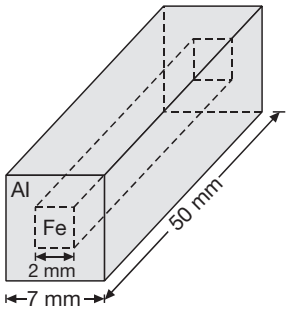
(A) 

(B) 

(C) 

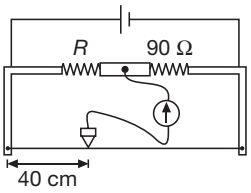
(D) 

2. [JEE (Advanced) 2015]
In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7\times10^{-8}\ \Omega m$ and $10\times10^{-7}\ \Omega m$, respectively. The electrical resistance between the two faces P and Q of the composite bar is



(A) $\frac{2475}{64}\ \mu\Omega$ (B) $\frac{1875}{64}\ \mu\Omega$
(C) $\frac{1875}{49}\ \mu\Omega$ (D) $\frac{2475}{132}\ \mu\Omega$

3. [JEE (Advanced) 2014]
During an experiment with a meter bridge, the galvanometer shows a null point when the jockey is pressed at 40 cm using a standard resistance of $90\ \Omega$, as show in the scale used in the meter bridge is 1 mm. The unknown resistance is

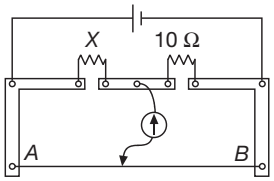


(A) $60 \pm 0.15\ \Omega$ (B) $135 \pm 0.56\ \Omega$

(C) $60 \pm 0.25\ \Omega$ (D) $135 \pm 0.23\ \Omega$

4. [IIT-JEE 2011]

A meter bridge is set-up as shown in figure, to determine an unknown resistance X using a standard $10\ \Omega$ resistor. The galvanometer shows null point when tapping key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is

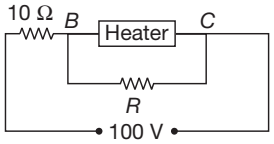


(A) $10.2\ \Omega$ (B) $10.6\ \Omega$

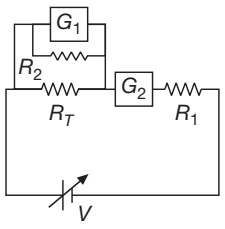
(C) $10.8\ \Omega$ (D) $11.1\ \Omega$

5. [IIT-JEE 2010]

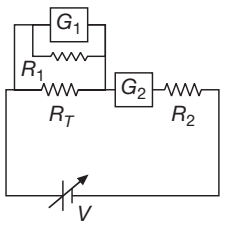
To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2 and a variable voltage source V. The correct circuit to carry out the experiment is



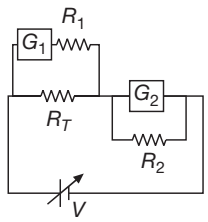
(A)



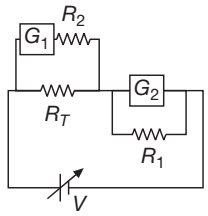
(B)



(C)

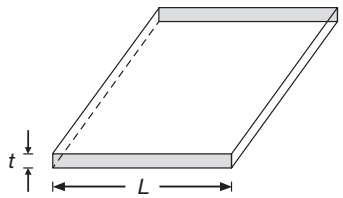


(D)



6. [IIT-JEE 2010]

Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



(A) directly proportional to L

(B) directly proportional to t

(C) independent of L

(D) independent of t

7. [IIT-JEE 2010]

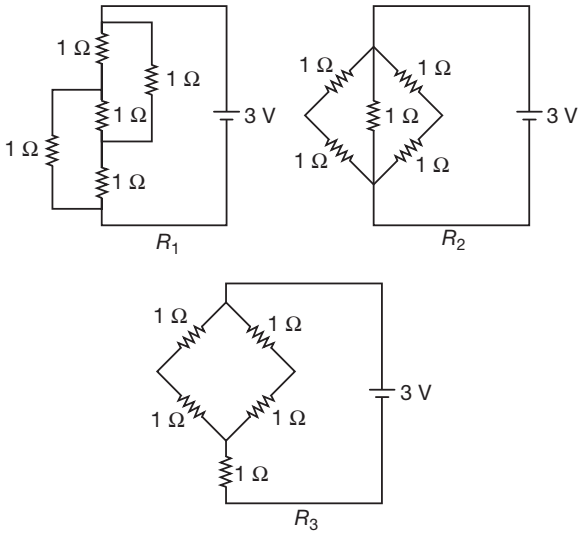
Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

(A) $\dfrac{1}{R_{100}} = \dfrac{1}{R_{40}} + \dfrac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$

(C) $R_{100} > R_{60} > R_{40}$ (D) $\dfrac{1}{R_{100}} > \dfrac{1}{R_{60}} > \dfrac{1}{R_{40}}$

8. [IIT-JEE 2008]

Figure shows three resistor configurations R_1, R_2 and R_3 connected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then



(A) $P_1 > P_2 > P_3$ (B) $P_1 > P_3 > P_2$

(C) $P_2 > P_1 > P_3$ (D) $P_3 > P_2 > P_1$

9. [IIT-JEE 2007]

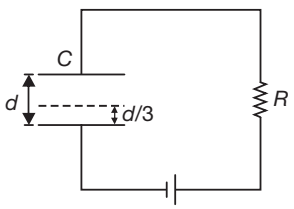
A resistance of $2\ \Omega$ is connected across one gap of a metre bridge (the length of the wire is 100 cm) and an unknown resistance, greater than $2\ \Omega$, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

(A) $3\ \Omega$ (B) $4\ \Omega$

(C) $5\ \Omega$ (D) $6\ \Omega$

10. [IIT-JEE 2007]

A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time is

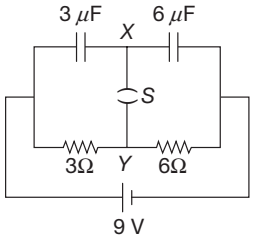


(A) $\frac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(C) $\frac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

11. [IIT-JEE 2007]

A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

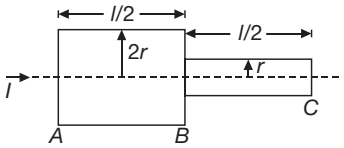


(A) zero (B) 54 μC

(C) 27 μC (D) 81 μC

12. [IIT-JEE 2006]

Two bars of radius r and $2r$ are kept in contact as shown. An electric current I is passed through the bars. Which one of the following is correct?



(A) Heat produced in bar BC is 4 times the heat produced in bar AB

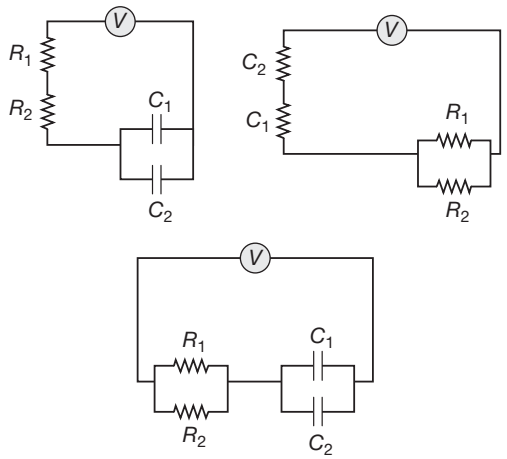
(B) Electric field in both halves is equal

(C) Current density across AB is double that of across BC

(D) Potential difference across AB is 4 times that of across BC

13. [IIT-JEE 2006]

Find the time constant for the given RC circuits in correct order



$R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 4\ \mu$F, $C_2 = 2\ \mu$F

(A) $18, 4, \frac{8}{9}$ (B) $18, \frac{8}{9}, 4$

(C) $4, 18, \frac{8}{9}$ (D) $4, \frac{8}{9}, 18$

14. [IIT-JEE 2005]

A moving coil galvanometer of resistance $100\ \Omega$ is used as an ammeter using a resistance $0.1\ \Omega$. The maximum deflection current in the galvanometer is $100\ \mu A$. Find the minimum current in the circuit, so that the ammeter shown maximum deflection

(A) 100.1 mA (B) 1000.1 mA

(C) 10.01 mA (D) 1.01 mA

15. [IIT-JEE 2005]

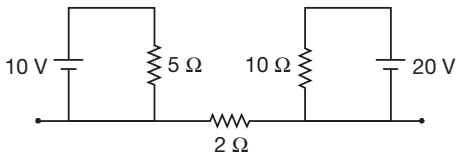
A rigid container with thermally insulated walls contains a coil of resistance $100\ \Omega$, carrying current 1 A. Change in internal energy after 5 minute will be

(A) zero (B) 10 kJ

(C) 20 kJ (D) 30 kJ

16. [IIT-JEE 2005]

Find out the value of current through $2\ \Omega$ resistance for the given circuit.



(A) 5 A (B) 2 A

(C) zero (D) 4 A

17. [IIT-JEE 2005]

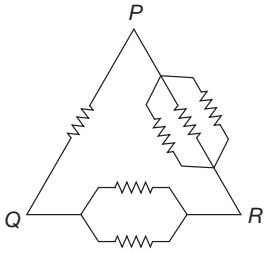
A $4\ \mu F$ capacitor, a resistance of $2.5\ M\Omega$ is in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor: $[\text{Given } \log_e(2)=0.693]$

(A) 13.86 s (B) 6.93 s

(C) 7 s (D) 14 s

18. [IIT-JEE 2004]

Six equal resistances are connected between points P, Q and R as shown in the figure. Then, the net resistance will be maximum between

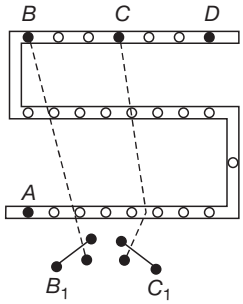


(A) P and Q (B) Q and R

(C) P and R (D) any two points

19. [IIT-JEE 2004]

For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between

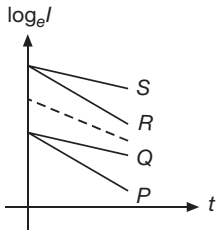


(A) B and C (B) C and D

(C) A and D (D) B_1 and C_1

20. [IIT-JEE 2004]

A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of $\log_e I$ with respect to time. If the resistance is changed to $2x$, the new graph will be



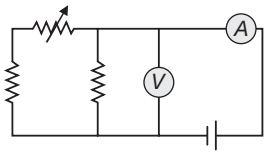
(A) P (B) Q

(C) R (D) S

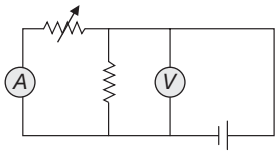
21. [IIT-JEE 2003]

Express which of the following set-up can be used to verify Ohm's Law?

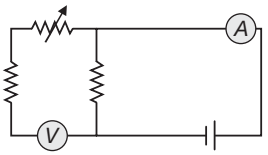
(A)



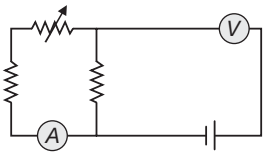
(B)



(C)

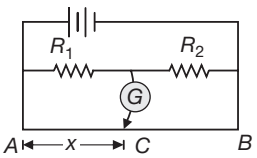


(D)



22. [IIT-JEE 2003]

In the shown arrangement of the experiment of the meter bridge if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

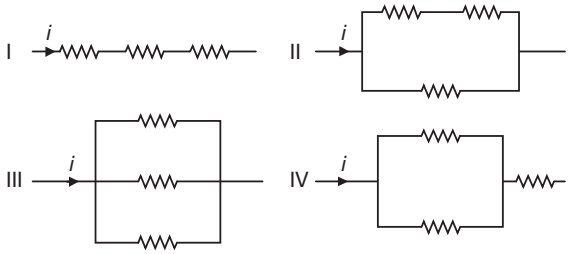


(A) x (B) $\frac{x}{4}$

(C) $4x$ (D) $2x$

23. [IIT-JEE 2003]

The three resistances of equal value are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation

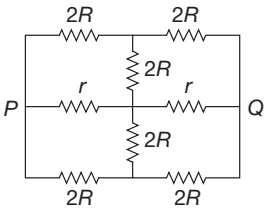


(A) III < II < IV < I (B) II < III < IV < I

(C) I < IV < III < II (D) I < III < II < IV

24. [IIT-JEE 2002]

The effective resistance between points P and Q of the electrical circuit shown in figure is

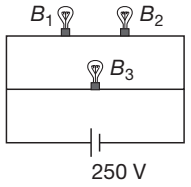


(A) $\frac{2Rr}{R+r}$ (B) $\frac{8R(R+r)}{3R+r}$

(C) $2r+4R$ (D) $\frac{5R}{2}+2r$

25. [IIT-JEE 2002]

A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3 respectively. Then

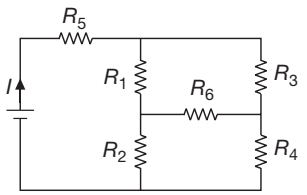


(A) $W_1 > W_2 = W_3$ (B) $W_1 > W_2 > W_3$

(C) $W_1 < W_2 = W_3$ (D) $W_1 < W_2 < W_3$

26. [IIT-JEE 2001]

In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then, the resistance values must satisfy



(A) $R_1R_2R_5 = R_3R_4R_6$

(B) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}$

(C) $R_1R_4 = R_2R_3$

(D) $R_1R_3 = R_2R_4$

27. [IIT-JEE 2001]

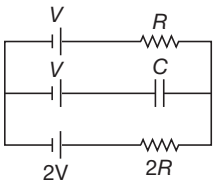
A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

(A) 4 (B) 6

(C) 8 (D) 9

28. [IIT-JEE 2001]

In the circuit shown in figure, with steady current, the potential drop across the capacitor must be

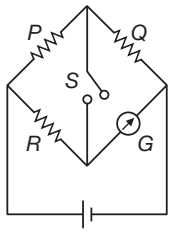


(A) V (B) $\frac{V}{2}$

(C) $\frac{V}{3}$ (D) $\frac{2V}{3}$

29. [IIT-JEE 1999]

In the circuit shown $P \neq R$, the reading of galvanometer is same with switch S open or closed. Then

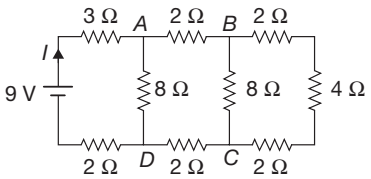


(A) $I_R = I_G$ (B) $I_P = I_G$

(C) $I_R = I_P$ (D) $I_Q = I_R$

30. [IIT-JEE 1998]

In the circuit shown in figure, the current through



(A) the 3 Ω resistor is 0.5 A.

(B) the 3 Ω resistor is 0.25 A.

(C) the 4 Ω resistor is 0.5 A.

(D) the 4 Ω resistor is 0.25 A.

31. [IIT-JEE 1997]

A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are

(A) current, electric field and drift speed

(B) drift speed only

(C) current and drift speed

(D) current only

32. [IIT-JEE 1997]

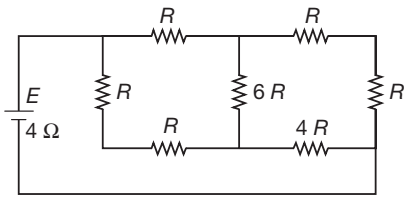
A parallel combination of $0.1\,\text{M}\,\Omega$ resistor and a $10\,\mu\text{F}$ capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged upto 0.75 V is approximately (in second)

(A) ∞ (B) $\log_e 2$

(C) $\log_{10} 2$ (D) zero

33. [IIT-JEE 1995]

A battery of internal resistance $4\,\Omega$ is connected to the network of resistances as shown. In order that the maximum power can be delivered to the network, the value of R in Ω should be



(A) $\frac{4}{9}$ (B) 2

(C) $\frac{8}{3}$ (D) 18

34. [IIT-JEE 1993]

Read the following statements carefully :

Y: The resistivity of semiconductor decreases with increase of temperature.

Z: In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature

Select the correct statement(s) from the following :

(A) Y is true but Z is false

(B) Y is false but Z is true

(C) Both Y and Z are true

(D) Y is true and Z is the correct reason for Y

35. [IIT-JEE 1988]

A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(A) each of them increases.

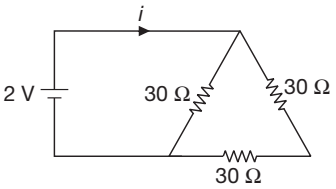
(B) each of them decreases.

(C) copper increases and that of germanium decreases.

(D) copper decreases and that of germanium increases.

36. [IIT-JEE 1983]

The current i in the circuit (see figure) is

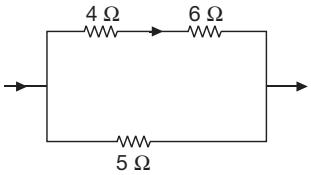


(A) $\frac{1}{45}$ A (B) $\frac{1}{15}$ A

(C) $\frac{1}{10}$ A (D) $\frac{1}{5}$ A

37. [IIT-JEE 1981]

In the circuit shown in figure the heat produced in the $5\ \Omega$ resistor due to the current flowing through it is 10 calories per second.



The heat generated in the $4\ \Omega$ resistor is

(A) $1\ \text{cals}^{-1}$ (B) $2\ \text{cals}^{-1}$

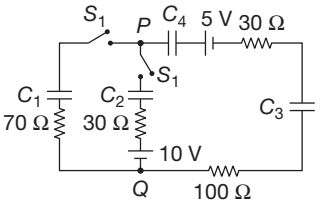
(C) $3\ \text{cals}^{-1}$ (D) $4\ \text{cals}^{-1}$

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\ \mu\text{F}$, $C_2 = 30\ \mu\text{F}$ and $C_3 = C_4 = 80\ \mu\text{F}$. Which of the following statement(s) is/are correct?



(A) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be $4\ \text{V}$.

(B) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across $30\ \Omega$ resistor (between points P and Q) will be $0.2\ \text{A}$ (round off to 1st decimal place).

(C) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be $25\ \text{mA}$.

(D) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between point P and Q will be $10\ \text{V}$.

2. [JEE (Advanced) 2019]

Two identical moving coil galvanometers have $10\ \Omega$ resistance and full scale deflection at $2\ \mu\text{A}$ current. One of them is converted into a voltmeter of $100\ \text{mV}$ full scale reading and the other into an Ammeter of $1\ \text{mA}$ full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000\ \Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct?

(A) The measured value of R will be $978\ \Omega < R < 982\ \Omega$

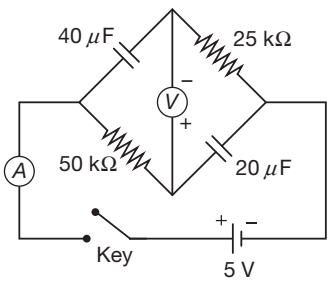
(B) The resistance of the ammeter will be $0.02\ \Omega$ (round off to 2nd decimal place)

(C) If the ideal cell is replaced by a cell having internal resistance of $5\ \Omega$ then the measured value of R will be more than $1000\ \Omega$

(D) The resistance of the voltmeter will be $100\ \text{k}\Omega$

3. [JEE (Advanced) 2016]

In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true?



(A) The voltmeter display $-5\ \text{V}$ as soon as the key is pressed and displays $+5\ \text{V}$ after a long time.

(B) The voltmeter will display $0\ \text{V}$ at time $t = \ln 2$ seconds.

(C) The current in the ammeter becomes $\frac{1}{e}$ of the initial value after 1 second.

(D) The current in the ammeter becomes zero after a long time.

4. [JEE (Advanced) 2016]

An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform.

(B) The resistance over small sections of the filament decreases with time.

(C) The filament emits more light at higher band of frequencies before it breaks up.

(D) The filament consumes less electrical power towards the end of the life of the bulb.

5. [JEE (Advanced) 2016]

Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < \frac{R}{2}$, which of the following statement(s) about any one of the galvanometers is(are) true?

(A) The maximum voltage range is obtained when all the components are connected in series.

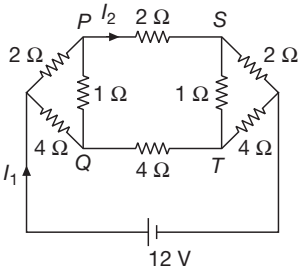
(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer.

(C) The maximum current range is obtained when all the components are connected in parallel.

(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

6. [IIT-JEE 2012]

For the resistance network shown in the figure, choose the **correct** option(s)



(A) The current through PQ is zero

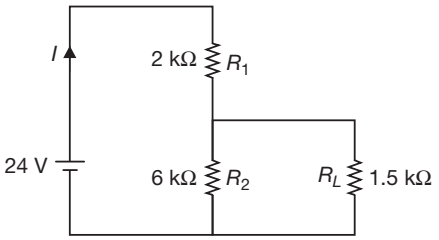
(B) $I_1 = 3$ A

(C) The potential at S is less than that at Q

(D) $I_2 = 2$ A

7. [IIT-JEE 2009]

For the circuit shown in the figure



(A) the current I through the battery is 7.5 mA

(B) the potential difference across R_L is 18 V

(C) ratio of powers dissipated in R_1 and R_2 is 3

(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

8. [IIT-JEE 1999]

When a potential difference is applied across, the current passing through

(A) an insulator at 0 K is zero

(B) a semiconductor at 0 K is zero

(C) a metal at 0 K is finite

(D) a p-n diode at 300 K is finite, if it is reverse biased

9. [IIT-JEE 1991]

A microammeter has a resistance of 100 Ω and full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s)

(A) 50 V range with 10 kΩ resistance in series

(B) 10 V range with 200 kΩ resistance in series

(C) 5 mA range with 1 Ω resistance in parallel

(D) 10 mA range with 1 Ω resistance in parallel

10. [IIT-JEE 1989]

Capacitor C_1 of capacitance 1 μF and capacitor C_2 of capacitance 2 μF are separately charged fully by a common battery. The two capacitors are then separated allowed to discharge through equal resistors at time $\tau = 0$

(A) the current in each of the two discharging circuits is zero at $t = 0$

(B) the currents in the two discharging circuits at $t = 0$ are equal but not zero

(C) the currents in the two discharging circuits at $t = 0$ are unequal

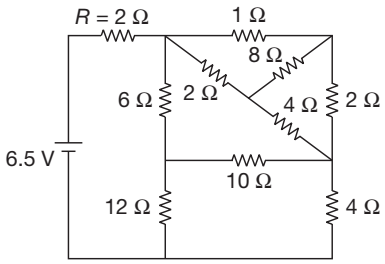
(D) the capacitors C_1 and C_2 lose their 50% charge in same duration

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

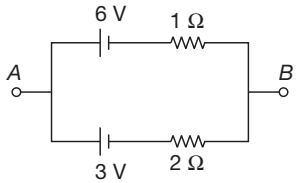
1. [JEE (Advanced) 2015]

In the following circuit, the current through the resistor $R(=2\ \Omega)$ is I amperes. The value of I is



2. [IIT-JEE 2011]

Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volt is



3. [JEE (Advanced) 2014]

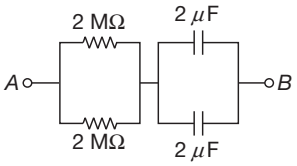
A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a $4990\ \Omega$ resistance, it can be converted into a voltmeter of range $0-30$ V. If connected to a $\frac{2n}{249}\ \Omega$ resistance, it becomes an ammeter of range 0-1.5 A. The value of n is

4. [IIT-JEE 2010]

When two identical batteries of internal resistance $1\ \Omega$ each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\, J_2$ then the value of R in Ω is

5. [IIT-JEE 2010]

At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in second) does the voltage across them become 4 V? [Take: $\ln 5 = 1.6$, $\ln 3 = 1.1$]



Assertion and Reasoning Type Problems

For the following Assertion-Reason type questions given below, choose the correct option:

(A) Both statement I & II are correct and statement II is correct explanation of statement I.
(B) Both statement I & II are correct and statement II is not correct explanation of statement I.
(C) Statement I is true and statement II is false.
(D) Statement II is correct and statement I is false.

1. [IIT-JEE 2008]

Statement-1: In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance

Statement-2: Resistance of a metal increases with increase in temperature.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Current Definition)

1. (b) 1.05 mA
2. (a) 17 A
   (b) 85 kAm^{-2}
3. 0.265 C
4. $I(t)=\begin{cases} -v\left(\frac{q_1}{a}+\frac{q_2}{b}\right), & t<\frac{a}{v}, \\ -v\frac{q_2}{b}, & \frac{a}{v}\le t<\frac{b}{v}, \\ 0, & t\ge\frac{b}{v}. \end{cases}$
5. (a) 40 C
   (b) 10 A
6. $en_0e^{\alpha d}$
7. $2\lambda v$

Test Your Concepts-II (Based on Resistance, Resistivity and Ohm's Law)

1. $\sigma A\left|\frac{dV}{dx}\right|$
2. $\frac{\rho L}{w(y_2-y_1)}\log_e\left(\frac{y_2}{y_1}\right)$
3. 4 Ω
4. (a) 0.2%
   (b) 4%
6. (b) $\frac{abV_{ab}}{\rho(b-a)r^2}$

Test Your Concepts-III (Based on Variation of Resistance with Temperature)

1. $(2+0.115t)\ \Omega$
2. $\frac{\alpha_0}{1+\alpha_0 T}$
3. $\frac{\alpha_1}{1+\alpha_1(T_2-T_1)}$
4. $1+\alpha_0(T_1-T_2)$
5. $\rho_2=\rho_1\left[1+\frac{m}{\rho_1}(T_2-T_1)\right]$
6. Percentage change in Length = 0.0017%
   Percentage change in Area = 0.0034%
   Percentage change in Resistance = 0.39%
7. 0.002 $(°C)^{-1}$

Test Your Concepts-IV (Based on Series and Parallel Combination of Resistances)

1. 29.5 V
2. (a) 227 mA
   (b) 5.68 V
3. $R_1=1\ k\Omega$, $R_2=2\ k\Omega$, $R_3=3\ k\Omega$
4. 470 Ω and 220 Ω
5. (i) –75 V
   (ii) –50 V
   (iii) 125 V
   (iv) 175 V
   (v) –25 V and
   (vi) –200 V
6. 22.5 Ω
7. 5 A
8. $\frac{8}{5}\ \Omega$; 15 A
9. $\frac{8}{3}\ \Omega$; 9 A
10. 6 A
11. (a) $\frac{42}{31}\ \Omega$
    (b) 4 Ω
12. $(R_{AD})_{MAX}=\frac{R_d(R_d+2R_b)(R_a+2R_c)}{4R_d(R_a+R_b+R_c)+(R_a+2R_b)(R_a+2R_c)}$
13. 48 V
15. 4 A; 10 A
16. zero

Test Your Concepts-V (Based on Series and Parallel Combination of Resistances)

1. (a) $\frac{1}{2\pi\sigma L}\log_e\left(\frac{b}{a}\right)$

(b) $\frac{b-a}{2\pi L\sigma_0}$

2. (a) 1 Ω

(b) $\frac{2R}{n}$

3. (a) $\frac{2R}{5}$

(b) $\frac{2R}{3}$

4. $\frac{(2+\pi)a\lambda}{8}$

5. 0.833 A

6. (a) $\frac{E}{8R}$

(b) $\frac{E}{2R}$

7. $\left(\frac{\sqrt{2}}{2\sqrt{2}+1}\right)\lambda a$

8. (a) 2Ω

(b) 1.5 A

9. (a) $\frac{8}{15}r$

(b) $\frac{3}{5}r$

11. $\frac{3r}{2}$

12. 1 Ω

13. $\frac{7R}{2}$

14. (a) $\frac{5}{6}R$

(b) $R_{AB'}=\frac{3R}{4}$

(c) $\frac{7}{12}$ R

15. The new equivalent resistance in the circuit has become 0.6 times.

16. $\frac{25}{6}$ Ω

17. $\frac{5R}{4}$

18. $\frac{5}{7}$ Ω

19. (a) $\frac{\rho}{2\pi L}\log_e\left(\frac{r_b}{r_a}\right)$

(b) $\rho=\frac{2\pi L\Delta V}{I\log_e\left(\frac{r_b}{r_a}\right)}$

20. (a) $\frac{2\sqrt{\ell}}{3A\sigma_0}$

(b) $\frac{3V_0\sqrt{x}}{2\ell^{\frac{3}{2}}}$

21. $R_{AB}=\left(\frac{\sqrt{7}-1}{3}\right)R_0$

22. 1.2 A , 7.2 V

23. zero

Test Your Concepts-VI (Based on Kirchhoff's Laws and Nodal Analysis)

1. 0.283 A, in the dead battery

171 A, in the starter

2. (a) 909 mA

(b) –1.82 V

3. 4 V and a is at the higher potential.

4. $\frac{\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$

5. $I_1=\frac{108}{77}$ A, $I_2=\frac{212}{77}$ A and $I_3=\frac{10}{77}$ A

$\frac{404}{77}$ V, $-\frac{354}{77}$ V

6. Current through 1 Ω and 5 Ω is $I_1=2$ A

Current through 6 Ω is $I_1-I_2=1.5$ A

Current through 8 Ω is $I_2-I_3=0.25$ A

Current through 4 Ω is $I_2=0.5$ A and

Current through 16 Ω is $I_3=0.25$ A

7. 20 V

8. $\frac{122}{21}\ \Omega$

9. (a) 2 V, 1 A, 0, 1 A

(b) 1 A, 2 A, 1 A, 2 A

10. 0.9 A

11. (a) $\frac{2}{13}$ V

(b) $\frac{21}{13}$ V, $\frac{19}{13}$ V

12. 0.288 mJ

13. 0.2 A, 1.44×10^{-5} J

14. 846 mA, 462 mA, 1.31 A

Test Your Concepts-VII (Based on Emf, Internal Resistance and Combination of Cells)

1. (a) 12.4 V

(b) 9.65 V

3. $(r_1 - r_2)$

4. 110 V, 5 A

7. 10 V and 0.5 Ω

8. $(k-1)^2 : k : (k-1)$

Test Your Concepts-VIII (Based on Galvanometer, Voltmeter and Ammeter)

1. 0.302 Ω

4. 145 Ω, 0.756 mA

5. (a) $\frac{V_{\text{meas}}}{\left(I_{\text{meas}} - \frac{V_{\text{meas}}}{R_v}\right)}$

(b) $R = \frac{V_{\text{meas}}}{I_{\text{meas}}}$

6. (a) $\left(\frac{V_{\text{meas}}}{I_{\text{meas}}} - R_A\right)$

(b) $R \simeq \frac{V_{\text{meas}}}{I_{\text{meas}}}$

7. 0.408 Ω

8. $(n-1)G$

9. $\frac{R_1 n_2 N_1}{n_1 N_2}$

10. $\frac{R_0}{R+R_0}$

11. $\frac{R}{R+R_0}$

12. 204 Ω

13. 2 V

14. Increase

15. 3.2 V, 3.238 V

16. (a) 100 V

(b) 125 V

17. (a) 4 V

(b) 4.2 V, 0.08 A

18. 79.995 Ω

19. $\frac{G^2}{G+S}$

20. 20.16 Ω

21. 4.96 mA, 1.58 V

Test Your Concepts-IX (Based on Heating Effects and Power Consumption)

1. 14.2 W in 2 Ω, 1.33 W in 3 Ω, 4 W in 1 Ω

2. (a) $\frac{2\Delta t}{3}$

(b) $3\Delta t$

3. $\frac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}, \frac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

4. (a) $\frac{IR_2}{R_1+R_2}, \frac{IR_1}{R_1+R_2}$

5. In parallel

6. (a) 4

(b) $\frac{1}{4}$

7. $I_{20} = 9.82$ A

$I_{2000} = 0.91$ A

8. (b) 0.16 mm

10. (a) VI

(b) $P = 2P_c$

11. (a) 20 W
(b) 8 W
(c) 12 W

12. 40 J

13. 13.5 W, 1 W, 2 W

14. 2 Ω, 4.5 W

15. (a) 5 Ω
(b) 5 Ω

16. 2 W, 0.6 W

17. (a) By 4 V battery –222 J
By 12 V battery 1.88 kJ
(b) By 8 Ω resistor 687 J
By 5 Ω resistor 128 J
By 1 Ω resistor 25.6 J
By 3 Ω resistor 616 J
By 1 Ω resistor 205 J
(c) 1.66 kJ from chemical to electrical
1.66 kJ from electrical to internal

Test Your Concepts-X (Based on Wheatstone Bridge and Potentiometer)

1. (a) 1.25 V
(c) no effect
(d) no effect
(e) (i) not work
(ii) not work

2. 1.1 Ω

3. 6.3×10^{-4} K^{-1}

4. 1.5 Ω

5. $\frac{16}{3}$ Ω

6. (a) no
(c) 8 Ω

7. B

8. $\frac{V_0Rx}{RL+R_0x\left(1-\frac{x}{L}\right)}$, $V=\frac{V_0x}{L}$ for $R\gg R_0$

Test Your Concepts-XI (Based on RC Circuit)

1. (a) $(0.632)I_0\tau$
(b) $(0.99995)I_0\tau$
(c) $I_0\tau$

2. (b) 1.79×10^{15} Ω

3. (a) –61.6 mA
(b) 0.235 μC
(c) 1.96 A

4. $\frac{1}{4}U_0$

5. (a) 6 V
(b) 8.29 μs

6. $240\left(1-e^{-\frac{1000t}{6}}\right)$ μC, $360\left(1-e^{-\frac{1000t}{6}}\right)$ μC

7. 0.693τ

8. 0.693τ

9. 0.34τ

10. (a) $\frac{V^2}{R}e^{-\frac{2t}{RC}}$

11. 0.198 μA

12. SITUATION 1: $t=0.693$ s
SITUATION 2: Charged instantaneously

13. $RC\log_e\left(\frac{V_0-V_e}{V_0-V_f}\right)$

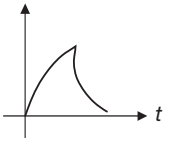
14. $\begin{cases}40e^{-4000t} & 0<t<250\ \mu s\\ -286e^{-4000t} & t>250\ \mu s\end{cases}$

15. 40 mJ

16. (a) $2CE^2$
(b) CE^2

17. $\left(0.03e^{-10^4t}\right)$ A

18. V_{CD}



19. $\frac{E}{R}(\eta-1)e^{-\frac{\eta t}{CR}}$

20. 1.5 μA

Single Correct Choice Type Questions

1. D	2. A	3. C	4. B	5. B	6. C	7. A	8. B	9. B	10. C
11. D	12. C	13. D	14. C	15. C	16. C	17. C	18. A	19. C	20. A
21. D	22. A	23. C	24. C	25. D	26. B	27. C	28. C	29. C	30. B
31. C	32. D	33. C	34. B	35. C	36. C	37. C	38. A	39. C	40. D
41. B	42. A	43. C	44. D	45. D	46. D	47. A	48. D	49. C	50. B
51. A	52. C	53. B	54. C	55. A	56. B	57. A	58. B	59. A	60. C
61. D	62. D	63. B	64. C	65. C	66. C	67. D	68. D	69. C	70. A
71. C	72. C	73. B	74. C	75. B	76. D	77. D	78. D	79. B	80. D
81. D	82. C	83. B	84. C	85. C	86. A	87. B	88. D	89. D	90. D
91. B	92. B	93. B	94. A	95. B	96. D	97. A	98. B	99. B	100. A
101. C	102. B	103. A	104. A	105. B	106. C	107. D	108. D	109. C	110. A
111. B	112. A	113. B	114. A	115. C	116. C	117. C	118. C	119. B	120. A
121. B	122. D	123. B	124. D	125. C	126. A	127. B	128. C	129. B	130. D
131. A	132. A	133. C	134. B	135. C	136. D	137. C	138. B	139. C	140. B
141. A	142. D	143. A	144. B	145. B	146. D	147. A	148. C	149. C	150. B
151. B	152. B	153. C	154. C	155. C	156. D	157. D	158. D	159. B	160. D
161. A	162. D	163. C	164. A	165. B	166. C	167. B	168. B	169. A	170. B
171. B	172. B	173. D	174. A	175. C					

Multiple Correct Choice Type Questions

1. A, B	2. B, C	3. A, B, C, D	4. B, D	5. A, C
6. A, B, D	7. A, C	8. A, B, C, D	9. A, C	10. A, C
11. B, D	12. B, C, D	13. A, B, D	14. B, D	15. C, D
16. A, B, C	17. A, B	18. C, D	19. A, C, D	20. B, C
21. A, D	22. A, C	23. B, C, D	24. A, C, D	25. A, B, C
26. A, B, C, D	27. B, C	28. A, B, C, D	29. A, B, C, D	30. A, B, D
31. A, B	32. A, B, C, D	33. A, C	34. A, B	35. A, C
36. A, C, D	37. A, C	38. B, D	39. A, B, D	40. B, C
41. C, D	42. A, C	43. A, C, D	44. A, D	45. A, B
46. B, C	47. B, D	48. B, C	49. A, C	50. A, B
51. B, C	52. A, B, C, D	53. A, D	54. A, B, C, D	55. A, B, D

Reasoning Based Questions

1. B	2. D	3. A	4. D	5. A	6. D	7. B	8. D	9. B	10. D
11. B	12. A	13. D	14. B	15. D	16. D	17. A	18. D	19. B	20. A
21. D	22. C	23. D	24. C	25. A					

Linked Comprehension Type Questions

1. C	2. A	3. B	4. D	5. C	6. C	7. B	8. B	9. C	10. A
11. B	12. C	13. A	14. C	15. B	16. D	17. B	18. D	19. D	20. C

21. C	**22.** B	**23.** C	**24.** B	**25.** C	**26.** A	**27.** B	**28.** C	**29.** C	**30.** B
31. A	**32.** B	**33.** C	**34.** C	**35.** C	**36.** B	**37.** D	**38.** C	**39.** C	

Matrix Match/Column Match Type Questions

1. A → (q)	B → (s)	C → (p)	D → (r)
2. A → (s)	B → (q)	C → (q)	D → (p)
3. A → (q, r, s)	B → (q, s)	C → (p, s)	D → (p, s)
4. A → (q)	B → (s)	C → (q)	D → (s)
5. A → (q)	B → (q)	C → (s)	D → (r)
6. A → (p)	B → (p)	C → (p)	D → (p)
7. A → (q)	B → (r)	C → (t)	D → (p)
8. A → (r)	B → (s)	C → (q)	D → (p)
9. A → (p, q)	B → (p, q)	C → (r, s)	D → (r, s)
10. A → (s)	B → (r)	C → (p)	D → (q)
11. A → (s)	B → (r)	C → (s)	D → (r)
12. A → (s)	B → (q)	C → (r)	D → (p)
13. A → (q)	B → (p)	C → (r)	D → (s)

Integer/Numerical Answer Type Questions

1. 130	**2.** 5560, 4440	**3.** 27	**4.** 2020	**5.** 50
6. 1 in 200 Ω, 4 in 70 Ω, 3 in 80 Ω, 8 in 20 Ω		**7.** 10	**8.** (a) 5 (b) 150 (c) 4	**9.** 1
10. 587	**11.** 6, 3	**12.** (a) 240, 360, 40 (b) 80, 40, 40		**13.** (a) 222 (b) 444
14. (a) 333 (b) 50 (c) 278 (d) 290		**15.** 2885	**16.** 6250, 7500	**17.** 500
18. (a) 45 (b) 10	**19.** 36	**20.** (a) zero (b) 36 (c) 2 (d) 30		**21.** (a) 75 (b) 25
22. 1	**23.** 20	**24.** 2	**25.** 24, 8	**26.** 5
27. 1, 6	**28.** (a) 100 (b) 125	**29.** 160	**30.** 4	**31.** 8
32. 431	**33.** 6	**34.** (a) 12, 9, 3, –6 (b) 12, 11, 9, 6		**35.** 1
36. 6	**37.** 5	**38.** 20	**39.** 95	

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1. B	**2.** D	**3.** B	**4.** D	**5.** B	**6.** A	**7.** D	**8.** B	**9.** A	**10.** A
11. D	**12.** B	**13.** (*)	**14.** A	**15.** D	**16.** D	**17.** A	**18.** A	**19.** A	**20.** A
21. A	**22.** D	**23.** C	**24.** B	**25.** A	**26.** D	**27.** D	**28.** C	**29.** A	**30.** A
31. D	**32.** D	**33.** A	**34.** A	**35.** D	**36.** C	**37.** C	**38.** C	**39.** C	**40.** A
41. A	**42.** C	**43.** D	**44.** D	**45.** C	**46.** A	**47.** B	**48.** C	**49.** B	**50.** A
51. A	**52.** A	**53.** A	**54.** B	**55.** C	**56.** D	**57.** B	**58.** B	**59.** B	**60.** A
61. B	**62.** D	**63.** D	**64.** A	**65.** C	**66.** D	**67.** C	**68.** C	**69.** C	**70.** C
71. C	**72.** D	**73.** C	**74.** B	**75.** B	**76.** A	**77.** A			

(*) None of the options given in exam is correct

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Single Correct Choice Type Problems

1. C	**2.** B	**3.** C	**4.** B	**5.** C	**6.** C	**7.** D	**8.** C	**9.** A	**10.** A
11. C	**12.** A	**13.** B	**14.** A	**15.** D	**16.** C	**17.** A	**18.** A	**19.** C	**20.** B
21. A	**22.** A	**23.** A	**24.** A	**25.** D	**26.** C	**27.** B	**28.** C	**29.** A	**30.** D
31. D	**32.** D	**33.** B	**34.** C	**35.** D	**36.** C	**37.** B			

Multiple Correct Choice Type Problems

1. A, C	**2.** A, B	**3.** A, B, C, D	**4.** C, D	**5.** B, C
6. A, B, C, D	**7.** A, D	**8.** A, B, D	**9.** B, C	**10.** B

Integer/Numerical Answer Type Questions

1. 1	**2.** 5	**3.** 5	**4.** 4	**5.** 2

Assertion and Reasoning Type Problems

1. D

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HINTS AND EXPLANATIONS

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CHAPTER 1: ELECTROSTATICS

Test Your Concepts-I (Based on Coulomb's Law)

1. $F_{air} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

Let us now replace the slab of thickness t with an equivalent amount of air such that two charges placed in air at separation x will experience the same force if the separation between them completely has a slab of dielectric constant K between them. In such a case

$$\frac{q_1 q_2}{4\pi\varepsilon_0 x^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K t^2}$$

$\Rightarrow$ Equivalent separation in air $= x = t\sqrt{K}$

So, now the effective separation between the charges is

$$r' = r - t + x = r - t + t\sqrt{K}$$

$$\Rightarrow \quad r' = r - t + t\sqrt{K}$$

So, $F' = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r'^2}$

$$\Rightarrow \quad F' = \frac{q_1 q_2}{4\pi\varepsilon_0 \left(r - t + t\sqrt{K}\right)^2}$$

Now, as per the problem we have $t = \dfrac{r}{2}$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left(r - \frac{r}{2} + \frac{r}{2}\sqrt{K}\right)^2}$$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q_1 q_2}{\left(1+\sqrt{K}\right)^2 r^2}\right]$$

$$\Rightarrow \quad \frac{F'}{F_{air}} = \frac{4}{\left(1+\sqrt{K}\right)^2} = \frac{4}{9}$$

$$\Rightarrow \quad 1 + \sqrt{K} = 3$$

$$\Rightarrow \quad \sqrt{K} = 2$$

$$\Rightarrow \quad K = 4$$

2. 63.5 g of Cu contains 6×10^{23} Cu atoms

1 g of Cu contains $\dfrac{6\times10^{23}}{63.5}$ Cu atoms

10 g of Cu contains $\left[\dfrac{6\times10^{23}}{63.5}\times10\right]$ Cu atoms

Since for every 1000 atoms, an electron is transferred,

So, $n = \dfrac{6\times10^{23}\times10}{63.5\times1000}$

$$\Rightarrow \quad q = ne = \frac{6\times10^{24}}{63500}\times1.6\times10^{-19}\ \text{C}$$

$$\Rightarrow \quad q = \frac{1920}{127}\ \text{C}$$

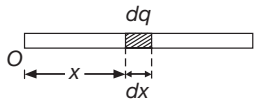
Now, one piece of Cu has positive charge and the other has a negative charge, so both will attract each other with a force of

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\Rightarrow \quad F = \frac{(9\times10^9)\left(\frac{1920}{127}\right)^2}{\left(\frac{1}{100}\right)^2} \cong 2.0\times10^{16}\ \text{N}$$

$$\Rightarrow \quad F = 2\times10^{16}\ \text{N}$$

3. Consider an element of length dx, carrying a charge dq at a distance x from O. Then, by definition



$$dq = \lambda\, dx$$

$$\Rightarrow \quad dq = \lambda_0\left(\frac{x^2}{L}\right)dx$$

$$\Rightarrow \quad \int_0^Q dq = \frac{\lambda_0}{L}\int_0^L x^2 dx$$

$$\Rightarrow \quad Q = \frac{\lambda_0}{L}\left(\frac{L^3}{3}\right)$$

$$\Rightarrow \quad Q = \frac{\lambda_0 L^2}{3}$$

4. Each ball is in equilibrium under the following three forces:

(i) tension (T)

(ii) electric force (F)

(iii) weight (W)

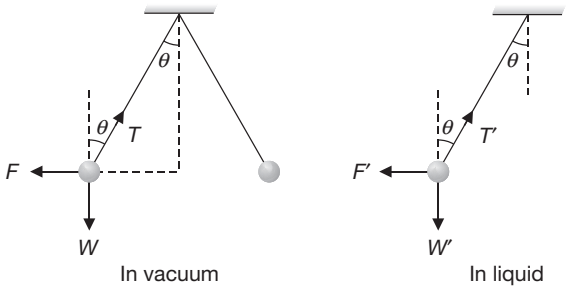
So, Lami's theorem can be applied.

$$T\cos\theta = W \qquad \ldots(1)$$

$$T\sin\theta = F \qquad \ldots(2)$$

$$T'\cos\theta = W' \qquad \ldots(3)$$

$$T'\sin\theta = F' \qquad \ldots(4)$$



So, from (1), (2), (3) and (4), we get

$$\frac{W}{F}=\frac{W'}{F'}$$

where $W'=W-U=V\rho g-V\sigma g$ and $F'=\frac{F}{K}$

$$\Rightarrow \quad \frac{V\rho g}{F}=\frac{V\rho g-V\sigma g}{\frac{F}{K}}$$

$$\Rightarrow \quad K=\frac{\rho}{\rho-\sigma}$$

5. Between Two Electrons:

$$F_e=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \text{ and } F_g=G\frac{m_e^2}{r^2}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\frac{e^2}{m_e^2}\frac{1}{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\left(\frac{1.6\times10^{-19}}{9.1\times10^{-31}}\right)^2\frac{9\times10^9}{6.67\times10^{-11}}$$

$$\Rightarrow \quad \frac{F_e}{F_e}\cong 4\times10^{42}$$

Between Two Protons:

$$F_e=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \text{ and } F_g=\frac{Gm_p^2}{r^2}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\frac{e^2}{m_p^2}\frac{1}{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\left(\frac{1.6\times10^{-19}}{1.67\times10^{-27}}\right)^2\frac{9\times10^9}{6.67\times10^{-11}}$$

$$\Rightarrow \quad \frac{F_e}{F_g}\cong 10^{36}$$

Let the specific charge be $\frac{q}{m}$ for which both the forces become equal, then

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}=\frac{Gm^2}{r^2}$$

$$\Rightarrow \quad \frac{q}{m}=\sqrt{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{q}{m}=8.6\times10^{-11}\ \text{Ckg}^{-1}$$

6. $\vec{F}=\vec{F}_{21}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}-\vec{r}_0\right|^3}\left(\vec{r}-\vec{r}_0\right)$

where $\vec{r}-\vec{r}_0=6\hat{i}-8\hat{j}$

$$\Rightarrow \quad \left|\vec{r}-\vec{r}_0\right|=10\ \text{m}$$

So, $\vec{F}=\frac{50\times10^{-6}\times2\times10^{-6}\times9\times10^9}{(10)^3}\left(6\hat{i}-8\hat{j}\right)$

$$\Rightarrow \quad \vec{F}=9\times10^{-4}\left(6\hat{i}-8\hat{j}\right)\text{ N}$$

$$\Rightarrow \quad \left|\vec{F}\right|=9\times10^{-4}\sqrt{6^2+(-8)^2}$$

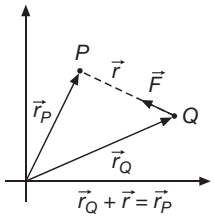
$$\Rightarrow \quad \left|\vec{F}\right|=9\times10^{-3}\text{ N}$$

7. Here, $\vec{r}_P=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{r}_Q=4\hat{i}+6\hat{j}-16\hat{k}$

$$\Rightarrow \quad \vec{r}_P-\vec{r}_Q=-3\hat{i}-4\hat{j}+12\hat{k}$$

$$\Rightarrow \quad \left|\vec{r}_P-\vec{r}_Q\right|=\sqrt{(3)^2+(-4)^2+(12)^2}=13\ \text{m}$$

Since $\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left|\vec{r}_P-\vec{r}_Q\right|^3}\left(\vec{r}_P-\vec{r}_Q\right)$



$$\Rightarrow \quad \vec{F}=\frac{17\times10^{-12}\times9\times10^9}{(13)^3}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}=\frac{17\times9\times10^{-3}}{169\times13}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}=\frac{170\times90\times10^{-5}}{169\times13}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}\cong(1)(7)(10^{-5})\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}\cong\left(-21\hat{i}-28\hat{j}+84\hat{k}\right)\times10^{-5}\text{ N}$$

8. Since $\lambda = \dfrac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2} \text{ (we have done this already)}$$

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{T}{A}\right)}{\left(\frac{\Delta R}{R}\right)}$$

$$\Rightarrow \quad Y = \frac{TR}{A\Delta R}$$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

Putting $q_0 = 10^{-8}$ C, $Q = \pi$ C, $R = 0.1$ m, $A = 10^{-6}$ m^2, $Y = 2\times10^{-11}$ Nm^{-2}

we get $\Delta R = 2.25$ mm

9. DO YOURSELF (same kind of illustration done before) 33×10^{-9} C

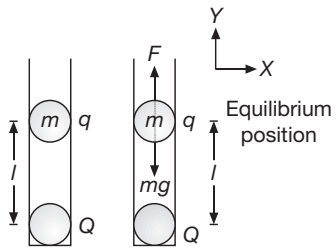
10. In equilibrium position, gravitational force is balanced by Coulomb repulsive force

$$mg = \frac{Qq}{4\pi\varepsilon_0\ell^2}$$

If charge q is displaced in positive y – direction, such that $y \ll \ell$, from Newton's Second Law,

$$\frac{Qq}{4\pi\varepsilon_0(\ell+y)^2} - mg = ma$$

$$\frac{Qq}{4\pi\varepsilon_0\ell^2}\left[\frac{1}{\left(1+\frac{y}{\ell}\right)^2}\right] - mg = ma$$



$$\Rightarrow \quad mg\left(1-\frac{2y}{\ell}\right) - mg = ma$$

$$\Rightarrow \quad a = -\frac{2gy}{\ell}$$

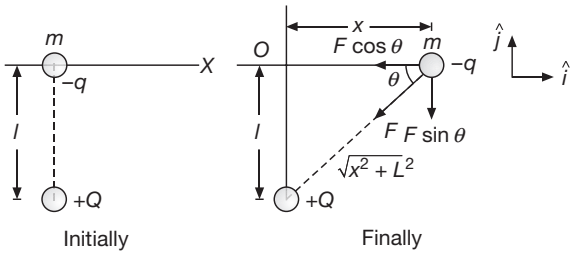
$$\Rightarrow \quad \frac{d^2y}{dt^2} + \frac{2g}{\ell}y = 0$$

Comparing with the standard equation of SHM according to which we have

$$\ddot{y} + \omega^2 y = 0$$

where we get $\omega = \sqrt{\dfrac{2g}{\ell}}$

11. When the bead is displaced through x, then the separation between the two charges becomes $\sqrt{x^2+L^2}$ and at that instant let the Coulomb force of attraction between the two charges be F. We observe that the component of force $F\cos\theta$ restores the charge q to its mean position O.



From Newton's Second Law,

$$-F\cos\theta = ma$$

where the negative sign indicates that the component of force has a restoring nature and restores the charge $-q$ to its mean position.

$$\Rightarrow \quad -\left(\frac{Qq}{4\pi\varepsilon_0(x^2+L^2)}\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right) = ma$$

$$\Rightarrow \quad a = -\left(\frac{Qq}{4\pi\varepsilon_0 m}\right)\left(\frac{x}{(x^2+L^2)^{\frac{3}{2}}}\right)$$

For a small linear displacement, $x \ll L$, we can ignore x^2 in comparison to L^2, so we get

$$a \simeq -\left(\frac{Qq}{4\pi\varepsilon_0 mL^3}\right)x$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Qq}{4\pi\varepsilon_0 mL^3}\right)x = 0$$

Comparing with the standard equation of SHM, which is given by $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mL^3}} = \frac{2\pi}{T} = 2\pi\nu$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mL^3}{Qq}}$$

12. Since the spring is metallic so the charge distributes equally on both the blocks. Hence a charge $\frac{Q}{2}$ resides on each block, due to which the blocks will repel each other with a force given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{L^2} = k\left(L - L_0\right)$$

Solving for Q

$$Q = 2L\sqrt{4\pi\varepsilon_0 k\left(L - L_0\right)}$$

13. Let q_1 and q_2 be the initial charges on the spheres, then according to Coulomb's Law

$$0.108 = 9\times10^9\frac{q_1q_2}{(0.5)^2} \quad \Rightarrow \quad q_1q_2 = 3\times10^{-12} \quad \ldots(1)$$

After connection, the charge flows from one sphere to another till both acquire the same potential (or in this case equal charge). So, final charge on both is

$$q_f = \frac{q_1 - q_2}{2}$$

(as initially they have opposite nature)

Now, again according to Coulomb's Law, the repulsive force is

$$0.036 = 9\times10^9\frac{\left(\frac{q_1 - q_2}{2}\right)^2}{(0.5)^2}$$

$$\Rightarrow \quad q_1 - q_2 = \frac{0.036\times0.25\times4}{9\times10^9}$$

$$\Rightarrow \quad q_1 - q_2 = 2\times10^{-6} \quad \ldots(2)$$

Solving (1) and (2), we get

$$q_1 - \frac{3\times10^{-12}}{q_1} = 2\times10^{-6}$$

$$\Rightarrow \quad q_1^2 - 2\times10^{-6}q_1 - 3\times10^{-12} = 0$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6} \pm \sqrt{4\times10^{-12} + 12\times10^{-12}}}{2}$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6} \pm 4\times10^{-6}}{2}$$

$$\Rightarrow \quad q_1 = 3\times10^{-6}\ \text{C} \ \text{ and } \ q_2 = -1\times10^{-6}\ \text{C}$$

$$\Rightarrow \quad q_1 = 3\ \mu\text{C and } q_2 = -1\ \mu\text{C}$$

14. $q_1 + q_2 = 50\times10^{-6}$

$$\Rightarrow \quad q_1 + q_2 = 5\times10^{-5} \quad \ldots(1)$$

Also $1 = 9\times10^9\frac{q_1q_2}{(2)^2}$

$$\Rightarrow \quad q_1q_2 = \frac{4}{9\times10^9}$$

$$\Rightarrow \quad q_1q_2 = 4.44\times10^{-10} \quad \ldots(2)$$

Put value of q_2 from (2) in (1), we get

$$q_1 + \frac{4.44\times10^{-10}}{q_1} = 5\times10^{-5}$$

$$\Rightarrow \quad q_1^2 - 5\times10^{-5}q_1 + 4.44\times10^{-10} = 0$$

Solving the quadratic, we get

$$q_1 = 3.8\times10^{-5}\ \text{C} = 38\ \mu\text{C}$$

OR $q_1 = 1.2\times10^{-5}\ \text{C} = 12\ \mu\text{C}$

So, we conclude that charge on one sphere is $38\ \mu$C and that on the other is $12\ \mu$C .

15. $F_e = F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

$$\Rightarrow \quad F = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(0.53\times10^{-10}\right)^2}$$

$$\Rightarrow \quad F = 8.2\times10^{-8}\ \text{N}$$

This electrostatic force between the proton and electron, provides the necessary centripetal force to electron to revolve in a circle of radius 0.53 Å.

$$\Rightarrow \quad mr\omega^2 = F$$

$$\Rightarrow \quad a_c = r\omega^2 = \frac{F}{m} = \frac{8.2\times10^{-8}}{9.1\times10^{-31}}$$

$$\Rightarrow \quad a_c = 9\times10^{22}\ \text{ms}^{-2}$$

Further $r\omega^2 = 9\times10^{22}$

$$\Rightarrow \quad \omega^2 = \frac{9\times10^{22}}{0.53\times10^{-10}}$$

$$\Rightarrow \quad \omega = 4.1\times10^{16}\ \text{rads}^{-1}$$

Also, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad T = 1.5\times10^{-16}\ \text{s}$$

16. $F_g = \dfrac{Gm_em_p}{r^2}$ …(1)

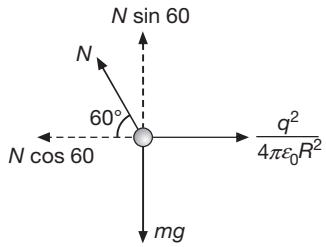
where m_e = mass of electron $= 9.1\times10^{-31}$ kg and m_p = mass of proton $= 1.67\times10^{-27}$ kg

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \ldots(2)$$

$\Rightarrow \quad F_8 \cong 3.6\times10^{-47}$ N and $F_e \cong 8.2\times10^{-8}$ N

So, we observe the gravitational force to be negligible in comparison to the electrostatic force. (But this result is not to be generalised to the solar system). So, this result is true in case of subatomic particles and we can also conclude here that gravitational forces are long range forces having significant values for massive bodies (like the sun-earth system).

17. $N\sin60 = mg$ and $N\cos60 = \dfrac{q^2}{4\pi\varepsilon_0R^2}$



$$\Rightarrow \quad \tan60° = \frac{mg}{\left(\dfrac{q^2}{4\pi\varepsilon_0R^2}\right)}$$

$$\Rightarrow \quad q^2 = \frac{4\pi\varepsilon_0mgR^2}{\sqrt{3}}$$

$$\Rightarrow \quad q = \left(\frac{4\pi\varepsilon_0mgR^2}{\sqrt{3}}\right)^{\frac{1}{2}}$$

18. $\dfrac{q^2}{4\pi\varepsilon_0r^2} = \mu mg$

$$\Rightarrow \quad \frac{9\times10^9\times(10^{-7})^2}{(10\times10^{-2})^2} = \mu\left(\frac{5}{1000}\right)(10)$$

$$\Rightarrow \quad \mu = 0.18$$

19. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q(Q-q)}{r^2}$

For this to be minimum, we have $\dfrac{dF}{dq} = 0$

$$\Rightarrow \quad q = \frac{Q}{2}$$

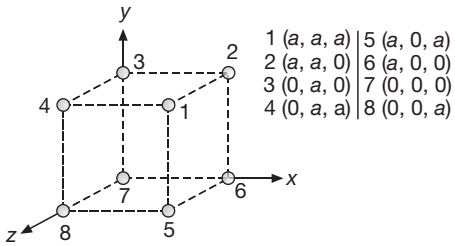
$$\Rightarrow \quad \frac{q}{Q-q} = \frac{Q/2}{Q-Q/2} = 1$$

Test Your Concepts-II (Based on Principle of Superposition)

1. Let us redraw the diagram to locate the coordinates of all the points. Considering origin at 7, we get

Now, according to Principle of Superposition, net force on 1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \vec{F}_{16} + \vec{F}_{17} + \vec{F}_{18}$$



From Coulomb's Law in vector form, we have

$$\vec{F}_{ij} = \frac{1}{4\pi\varepsilon_0}\frac{q_iq_j}{|\vec{r}_i-\vec{r}_j|^3}(\vec{r}_i-\vec{r}_j)$$

So, $\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{a^3}(a\hat{k}) = \left(\dfrac{Q^2}{4\pi\varepsilon_0a^2}\right)\hat{k}$

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{2}a)^3}(a\hat{i}+a\hat{k})$$

$$\vec{F}_{13} = \frac{Q^2}{4\pi\varepsilon_0a^2}\left[\frac{1}{2\sqrt{2}}(\hat{i}+\hat{k})\right]$$

$$\vec{F}_{14} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^3}(a\hat{i}) = \left(\frac{Q^2}{4\pi\varepsilon_0a^2}\right)\hat{i}$$

$$\vec{F}_{15} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^3}(a\hat{j}) = \left(\frac{Q^2}{4\pi\varepsilon_0a^2}\right)\hat{j}$$

$$\vec{F}_{16} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{2}a)^3}(a\hat{j}+a\hat{k})$$

$$\vec{F}_{16} = \frac{Q^2}{4\pi\varepsilon_0a^2}\left[\frac{1}{2\sqrt{2}}(\hat{j}+\hat{k})\right]$$

$$\vec{F}_{17} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{3}a)^3}(a\hat{i}+a\hat{j}+a\hat{k})$$

$$\vec{F}_{17} = \frac{Q^2}{4\pi\varepsilon_0a^2}\left[\frac{1}{3\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})\right]$$

$$\vec{F}_{18} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{2}a)^3}(a\hat{i}+a\hat{j})$$

CHAPTER 1

$$\vec{F}_{18} = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{2\sqrt{2}}\left(\hat{i}+\hat{j}\right)\right]$$

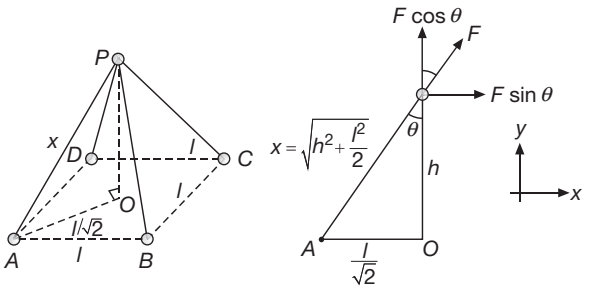
$$\Rightarrow \quad \vec{F}_1 = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\left(\hat{i}+\hat{j}+\hat{k}\right)+\frac{1}{2\sqrt{2}}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)+\frac{1}{3\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)\right]$$

$$\Rightarrow \quad \vec{F}_1 = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\Rightarrow \quad \left|\vec{F}_1\right| = \frac{\sqrt{3}Q^2}{4\pi\varepsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right) \text{ MAGNITUDE}$$

The force on 1 is equally inclined to x, y and z-axis.

2. The force due to one charge on the charge at P is shown here. When all the forces are taken into account, then the sine components cancel each other. So,



$$F_{\text{net}} = F_y = 4F\cos\theta$$

$$\Rightarrow \quad F_{\text{net}} = 4\frac{Q(1)}{4\pi\varepsilon_0\left(\sqrt{h^2+\frac{\ell^2}{2}}\right)^2}\frac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}$$

$$\left\{\because \cos\theta = \frac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}\right\}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{3/2}} \qquad \text{(UPWARDS)}$$

For equilibrium of m, this F_{net} must balance the weight of the particle mg (downwards).

$$\Rightarrow \quad \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{\frac{3}{2}}} = mg$$

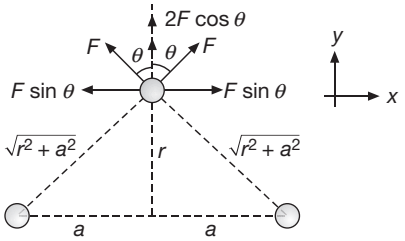
$$\Rightarrow \quad Q = \frac{mg\pi\varepsilon_0}{h}\left(h^2+\frac{\ell^2}{2}\right)^{3/2}$$

3. From the figure drawn, we see that on resolution of the two identical forces, $F\sin\theta$ and $F\sin\theta$ cancel. So, net force equals

$$F_{\text{net}} = 2F\cos\theta$$

$$\Rightarrow \quad F_{\text{net}} = F_y = 2\left[\frac{Qq_0}{4\pi\varepsilon_0\left(r^2+a^2\right)}\right]\cos\theta$$

$$\Rightarrow \quad F_y = \frac{2Qq_0 r}{4\pi\varepsilon_0\left(r^2+a^2\right)^{3/2}}$$



This force is directed radially outwards when q_0 is positive in nature.

F_y will be MAXIMUM, when

$$\frac{d}{dr}\left(F_y\right) = 0$$

$$\Rightarrow \quad \frac{Qq_0}{2\pi\varepsilon_0}\frac{d}{dr}\left[\frac{r}{\left(r^2+a^2\right)^{3/2}}\right] = 0$$

$$\Rightarrow \quad r\left[-\frac{3}{2}\left(r^2+a^2\right)^{-5/2}2r\right]+\left(r^2+a^2\right)^{-3/2} = 0$$

$$\Rightarrow \quad -\frac{3r^2}{r^2+a^2}+1 = 0$$

$$\Rightarrow \quad 3r^2 = r^2+a^2$$

$$\Rightarrow \quad 2r^2 = a^2$$

$$\Rightarrow \quad r = \pm\frac{a}{\sqrt{2}}$$

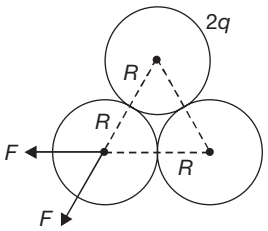
$$\Rightarrow \quad F_{\max} = \frac{Qq_0}{2\pi\varepsilon_0}\frac{\left(\frac{a}{\sqrt{2}}\right)}{\left(\frac{a^2}{2}+a^2\right)^{3/2}}$$

$$\Rightarrow \quad F_{\max} = \frac{Qq_0}{2\pi\varepsilon_0}\frac{a(2)}{3\sqrt{3}a^3}$$

$$\Rightarrow \quad F_{\max} = \frac{Qq_0}{3\sqrt{3}\pi\varepsilon_0 a^2}$$

4. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(2q)^2}{(2R)^2} = \dfrac{q^2}{4\pi\varepsilon_0 R^2}$

$F_{net} = \sqrt{F^2 + F^2 + 2F^2\cos 60}$

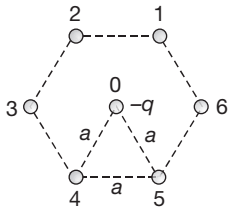


$\Rightarrow \quad F_{net} = \sqrt{3}F = \sqrt{3}\dfrac{q^2}{4\pi\varepsilon_0 R^2}$

$\Rightarrow \quad F_{net} = \dfrac{\sqrt{3}q^2}{4\pi\varepsilon_0 R^2}$

5. Assume, no charge at 6,

$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \vec{F}_{04} + \vec{F}_{05}$



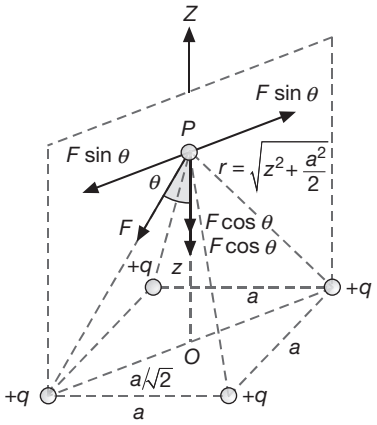
$\Rightarrow \quad |\vec{F}_0| = \dfrac{q^2}{4\pi\varepsilon_0 a^2}$ {from 0 to 3}

6. (a) zero

(b) Assume, a to be the distance between q and Q, then

$F = \dfrac{qQ}{4\pi\varepsilon_0 a^2}$

7. Let us first visualise the situation given in the problem and draw it here. Now let us consider two diagonally opposite charges having charge $+q$ each. Each of the charge will exert an attractive force F on $-Q$ such that the horizontal component of this force F cancels, whereas vertical components add. Similarly for other two diagonally opposite charges the horizontal components cancel and the vertical components still stay so as to make the resultant force as



$F_R = -4F\cos\theta$

$\Rightarrow \quad F_R = -4\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{\left(z^2 + \dfrac{L^2}{2}\right)}\right)\left(\dfrac{z}{\sqrt{z^2 + \dfrac{L^2}{2}}}\right)$

$\Rightarrow \quad F_R = -\dfrac{4Qqz}{4\pi\varepsilon_0\left(z^2 + \dfrac{L^2}{2}\right)^{\frac{3}{2}}}$

For $z \ll L$, we may neglect z^2 term in the denominator so as to get

$F_R = m\ddot{z} = -4(2)^{\frac{3}{2}}\left(\dfrac{Qq}{4\pi\varepsilon_0 a^3}\right)z$

$\Rightarrow \quad \ddot{z} + \left(\dfrac{4(2)^{\frac{3}{2}}Qq}{4\pi\varepsilon_0 ma^3}\right)z = 0$

Comparing with the standard equation of SHM given by $\ddot{z} + \omega^2 z = 0$, we get

$\omega = \sqrt{\dfrac{4(2)^{\frac{3}{2}}Qq}{4\pi\varepsilon_0 ma^3}} = \dfrac{2\pi}{T} = 2\pi\nu$

$\Rightarrow \quad T = 2\pi\sqrt{2\sqrt{2}\left(\dfrac{Qq}{\pi\varepsilon_0 ma^3}\right)}$

8. If we place one more charge q at the $24th$ vertex, the total force on the central charge will add up to zero. So according to the principle of superposition we have

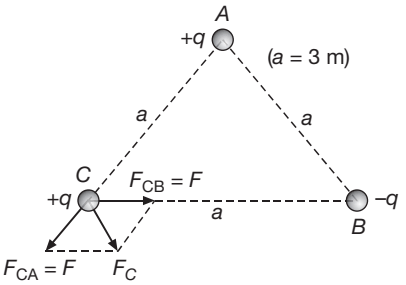
$F = \dfrac{qQ}{4\pi\varepsilon_0 a^2}$

9. $F_{CB} = F_{CA} = F = \dfrac{q^2}{4\pi\varepsilon_0 a^2}$

$F_C^2 = F^2 + F^2 + 2F^2\cos 120$

$\Rightarrow \quad F_C^2 = 2F^2 + 2F^2\left(-\dfrac{1}{2}\right)$

$\Rightarrow \quad F_C^2 = 2F^2 - F^2 = F^2$



$\Rightarrow \quad F_C = F = \dfrac{q^2}{4\pi\varepsilon_0 a^2}$ {parallel to AB}

$\Rightarrow \quad F_C = \dfrac{9\times10^9\times(100\times10^{-6})^2}{9}$

$\Rightarrow \quad F_C = 10\text{ N}$ (parallel to AB)

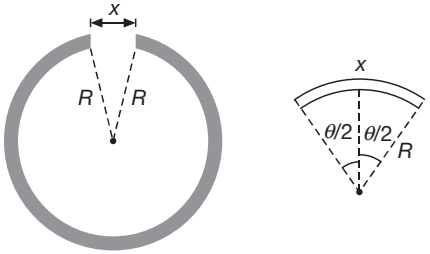
Test Your Concepts-III (Based on Electric Field)

1. $E = \dfrac{q}{4\pi\varepsilon_0 a^2}$

2. Let us assume that the small length is not cut from the loop. In that case, at the centre, $\vec{E}_{\text{complete loop}} = \vec{0}$

$\Rightarrow \quad \vec{E}_{\substack{\text{due to removed}\\\text{portion}}} + \vec{E}_{\substack{\text{due to the}\\\text{remaining wire}}} = \vec{0}$

$\Rightarrow \quad \left|\vec{E}_{\substack{\text{due to the remaining}\\\text{wire}}}\right| = \left|\vec{E}_{\substack{\text{due to removed}\\\text{portion of length } x(\ll R)}}\right|$



Now, the removed portion is actually an arc having charge $q = \left(\dfrac{Q}{2\pi R}\right)x$ and subtending an angle θ at the centre. In that case, we know

$E = \dfrac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\dfrac{\theta}{2}\right)$ (done already)

$\Rightarrow \quad E = \dfrac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\dfrac{x}{2R}\right)$

Since $\dfrac{x}{R} \ll 1$, so

$\sin\left(\dfrac{x}{2R}\right) \simeq \dfrac{x}{2R}$

$\Rightarrow \quad E \simeq \dfrac{Qx}{8\pi^2\varepsilon_0 R^3}$

3. Force on $+2\ \mu\text{C} = qE = (2\times10^{-6})(10^5) = 0.2$ N (due North)

Force on $-5\ \mu\text{C} = (5\times10^{-6})(10^5) = 0.5$ N (due South)

4. DO YOURSELF

$\dfrac{\sqrt{6}q}{4\pi\varepsilon_0\ell^2}$

5. Consider the arm AB. Let us consider an element of length dx charge dq at a distance x from XY. Then torque on this element due to the field E is

$d\tau = x(dqE)$

$\Rightarrow \quad d\tau = x\left(\dfrac{q}{L}dx\right)E$

$\Rightarrow \quad \tau = \int d\tau = \dfrac{qE}{L}\int_0^L x\,dx$

$\Rightarrow \quad \tau = \dfrac{qEL}{2}$

$\tau_{\text{total}} = \dfrac{qEL}{2} + (qE)L + \dfrac{qEL}{2} = (2qE)L$

$\Rightarrow \quad \tau = 2qEL = (mg)\dfrac{L}{2} + (mg)L + mg\left(\dfrac{L}{2}\right)$

$\Rightarrow \quad \tau = 2qEL = 2mgL$

$\Rightarrow \quad E = \dfrac{mg}{q}$

6. Force on the particle at the centre of the ring is zero.

Let $-Q$ be now displaced through $x(\ll R)$ along axis of ring. Then a restoring force F will act on $-Q$, where

$F = -QE_{\substack{\text{due to ring}\\\text{at axis}}}$

$$\Rightarrow \quad F = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

Since $x \ll R$

$$\Rightarrow \quad F = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0 R^3} \qquad \left\{\because \left(R^2+x^2\right)^{3/2} \cong R^3\right\}$$

$$\Rightarrow \quad m\ddot{x} = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0 R^3}$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Q\lambda}{2\varepsilon_0 mR^2}\right)x = 0$$

which can be compared to the standard equation of SHM i.e.,

$$\ddot{x} + \omega^2 x = 0$$

where $\omega = \sqrt{\dfrac{\lambda Q}{2\varepsilon_0 mR^2}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2\varepsilon_0 mR^2}{\lambda Q}}$$

7. (a) The charge Q_2 is negative and the charge Q_1 is positive.

(b) Since the electric field is zero at a distance a from the point 2, therefore

$$\frac{Q_1}{(\ell+a)^2} - \frac{Q_2}{a^2} = 0$$

$$\Rightarrow \quad \left|\frac{Q_1}{Q_2}\right| = \left(\frac{\ell+a}{a}\right)^2$$

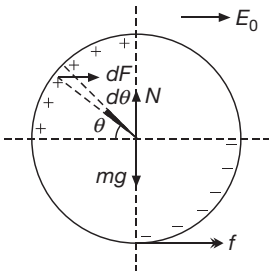
(c) For all points $x > a$,

$$E = \frac{Q_1}{(\ell+x)^2} - \frac{Q_2}{a^2}$$

For max. E, $\dfrac{dE}{dx} = 0 = -\dfrac{2Q_1}{(\ell+x)^3} + \dfrac{2Q_2}{x^3}$

Thus, $x = \dfrac{1}{\left(\dfrac{Q_1}{Q_2}\right)^{1/3} - 1} = b$

8. Consider an infinitesimal element subtending an angle $d\theta$ at the centre and at angle θ as shown in figure.



$dF = \lambda R d\theta E_0$

A force of same magnitude but in opposite direction acts on a corresponding element in the region of negative charge.

Net torque due to both types of charges is

$$d\tau = \left(\lambda R d\theta E_0\right)(2R\sin\theta)$$

$$\Rightarrow \quad \tau = 2\lambda R^2 E_0 \int_0^{\frac{\pi}{2}} \sin\theta\, d\theta$$

$$\Rightarrow \quad \tau = 2\lambda R^2 E_0$$

Equation for pure rolling motion is

$$\Sigma\tau = I\alpha$$

$$\Rightarrow \quad 2\lambda R^2 E_0 - fR = mR^2\alpha \qquad \ldots(1)$$

where $f = ma$ $\qquad \ldots(2)$

and $a = R\alpha$ $\qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$f = \lambda R E_0$ along positive x-axis.

9. (a) $F_{\text{net}} = F = 2\dfrac{Qq}{4\pi\varepsilon_0\left(R^2+x_0^2\right)}\dfrac{x_0}{\sqrt{R^2+x_0^2}}$

$$\Rightarrow \quad F = \frac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}} \qquad \{\text{towards the centre}\}$$

(b) The motion of the bead is periodic between $\pm x_0$

(c) $m\dfrac{d^2x}{dt^2} = -\dfrac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}}$ with $\dfrac{dx}{dt} = 0$ at $x = \pm x_0$

and $x = x_0$ at $t = 0$

(d) $m\dfrac{d^2x}{dt^2} = m\ddot{x} = -\left(\dfrac{2Qq}{4\pi\varepsilon_0 mR^3}\right)x$

$$\Rightarrow \quad \ddot{x} + \left(\frac{2Qq}{4\pi\varepsilon_0 mR^3}\right)x = 0$$

$$\Rightarrow \quad \ddot{x} + \omega^2 x = 0$$

where $\omega = \sqrt{\dfrac{2Qq}{4\pi\varepsilon_0 mR^3}}$ and $T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 mR^3}{2Qq}}$

such that

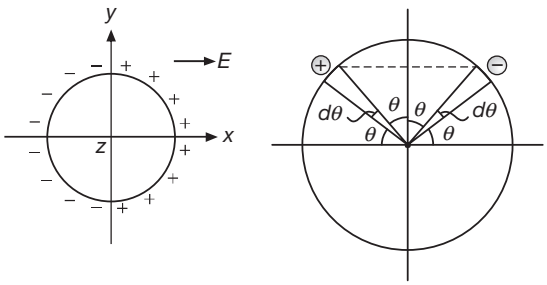
$$x = x_0\cos(\omega t) \text{ and } v = -x_0\omega\sin(\omega t)$$

(e) The velocity of bead will vanish for first time at $t = \dfrac{T}{2}$

$$\Rightarrow \quad t = \sqrt{\frac{4\pi^3\varepsilon_0 mR^3}{2Qq}}$$

CHAPTER 1

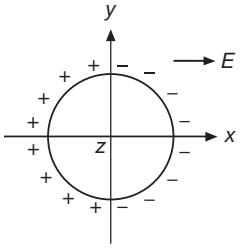
10. Let us first calculate the effective dipole moment of the arrangement.



dp = (Either charge) (Separation between the charges)

$$\Rightarrow \quad dp = \left[\left(\frac{q}{\pi R}\right)(R\,d\theta)\right] 2R\sin\theta$$

$$\Rightarrow \quad dp = \frac{2qR}{\pi}\sin\theta\, d\theta$$



$$\Rightarrow \quad p = \frac{2qR}{\pi}\int_0^{\pi}\sin\theta\, d\theta = -\frac{2qR}{\pi}\left[\cos\theta\Big|_0^{\pi}\right]$$

$$\Rightarrow \quad p = -\frac{2qR}{\pi}(\cos\pi - \cos 0)$$

$$\Rightarrow \quad p = \frac{4qR}{\pi}$$

Since, $T = 2\pi\sqrt{\frac{I}{pE}}$

$$T = 2\pi\sqrt{\frac{mR^2}{\left(\frac{4qR}{\pi}\right)E}} = 2\pi\sqrt{\frac{mR\pi}{4qE}}$$

11. Here we can clearly see in the figure

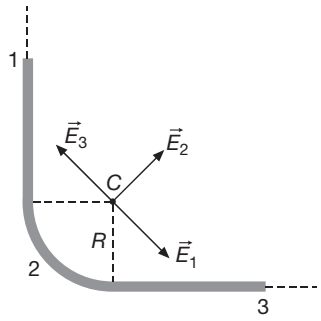
$\vec{E}_1$ (field due to wire 1) cancels out with

$\vec{E}_3$ (field due to wire 3) and

$\vec{E}_2$ (field due to wire 2)

Thus net field is $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$$\vec{E}_{net} = \vec{E}_2 \qquad \{\text{As } \vec{E}_1 = -\vec{E}_2\}$$



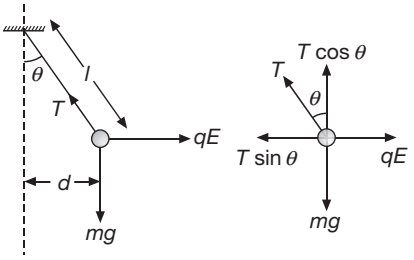
$$\vec{E}_{net} = \frac{2K\left(\lambda\frac{\pi R}{2}\right)\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{2}R^2}$$

$$\vec{E}_{net} = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

12. From figure we can write for equilibrium of ball

$$T\sin\theta = qE$$

$$T\cos\theta = mg$$



$$\Rightarrow \quad \tan\theta = \frac{qE}{mg}$$

$$\Rightarrow \quad \frac{d}{\sqrt{\ell^2 - d^2}} = \frac{qE}{mg}$$

$$\Rightarrow \quad E = \frac{mgd}{q\sqrt{\ell^2 - d^2}}$$

13. Different forces acting on bob are shown in figure From the figure

$$T\sin\alpha = qE\sin\beta \qquad \ldots(1)$$

$$T\cos\alpha + qE\cos\beta = mg$$

$$T\cos\alpha = mg - qE\cos\beta \qquad \ldots(2)$$

From equations (1) and (2), we have

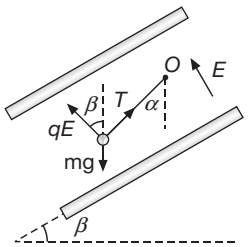
$$\tan\alpha = \frac{qE\sin\beta}{mg - qE\cos\beta}$$

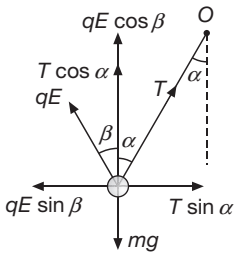
$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{qE\sin\beta}{mg - qE\cos\beta}\right) \qquad \ldots(3)$$

The net acceleration on the bob due to mg and qE is

$$a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2 + 2g\left(\frac{qE}{m}\right)\cos(180 - \beta)}$$

$$\Rightarrow \quad a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2 - 2g\left(\frac{qE}{m}\right)\cos\beta}$$





Time period of pendulum can be given as

$$T = 2\pi\sqrt{\frac{\ell}{a}}$$

$$T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2 - 2g\left(\frac{qE}{m}\right)\cos\beta}}}$$

14. Here acceleration of block is $a = \frac{qE}{m}$

Time taken by block to reach wall

$$d = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$t = \sqrt{\frac{2md}{qE}}$$

Velocity at the time of impact is

$$v = \sqrt{2ad}$$

$$\Rightarrow \quad v = \sqrt{\frac{2qEd}{m}}$$

When the block rebounds, time taken by block to come to rest is given by

$$0 = \sqrt{\frac{2qEd}{m}} - \left(\frac{qE}{m}\right)t$$

$$\Rightarrow \quad t = \frac{\sqrt{\frac{2qEd}{m}}}{\frac{qE}{m}} = \sqrt{\frac{2m}{qE}}$$

Thus time period of oscillations of block is

$$T = 4t = 2\sqrt{\frac{2md}{qE}}$$

Since the restoring force is independent of x, (the displacement from mean position), hence this is not a simple harmonic motion but periodic in nature.

15. Electric Field due to 1 is

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0\ell}(\cos 45° + \cos 45°)\hat{i}$$

$$\Rightarrow \quad E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{2}\lambda}{\ell}\right)\hat{i}$$

Electric Field due to 2 is $E_2 = \frac{1}{4\pi\varepsilon_0}\left(-\frac{4\sqrt{2}\lambda}{\ell}\right)\hat{i}$

Electric Field due to 3 is $E_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{6\sqrt{2}\lambda}{\ell}\right)\hat{j}$

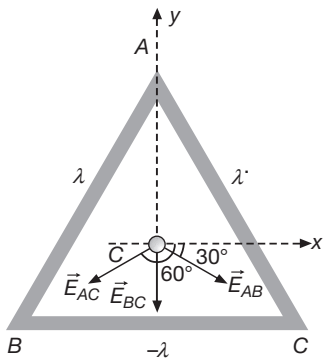
Electric Field due to 4 is $E_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{8\sqrt{2}\lambda}{\ell}\right)\hat{j}$

$$\Rightarrow \quad \vec{E}_{\text{total}} = E = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{2}\lambda}{\ell} - \frac{4\sqrt{2}\lambda}{\ell}\right)\hat{i} + \frac{1}{4\pi\varepsilon_0}\left(\frac{6\sqrt{2}\lambda}{\ell} + \frac{8\sqrt{2}\lambda}{\ell}\right)\hat{j}$$

$$\Rightarrow \quad \vec{E} = \frac{\lambda\sqrt{2}}{4\pi\varepsilon_0\ell}\left(-2\hat{i} + 14\hat{j}\right)$$

16. The electric field strength due to the three rods AB, BC and CA are as shown in figure



$$\vec{E}_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{-2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60°)\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\vec{E}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60°)\left(\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

$$\vec{E}_{BC} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60°)\hat{j}$$

$$\vec{E}_{\text{net}} = \vec{E}_{AC} + \vec{E}_{AB} + \vec{E}_{BC}$$

$$\vec{E}_{\text{net}} = -\left(\frac{3\lambda}{\pi\varepsilon_0\ell}\right)\hat{j}$$

17. Again, Newton's Second Law

$$\sum F_x = -T\sin\theta + qA = 0 \qquad \ldots(1)$$

and $$\sum F_y = +T\cos\theta + qB - mg = 0 \qquad \ldots(2)$$

(a) Substituting $T = \frac{qA}{\sin\theta}$, in equation (2), we get

$$\frac{qA\cos\theta}{\sin\theta} + qB = mg$$

$$\Rightarrow \quad q = \frac{mg}{(A\cot\theta + B)}$$

(b) Substituting this value in equation (1), we get

$$T = \frac{mgA}{(A\cos\theta + B\sin\theta)}$$

18. (a) Each ion moves in a quarter circle. The electric force causes the centripetal acceleration.

$$\sum F = ma$$

$$qE = \frac{mv^2}{R}$$

$$E = \frac{mv^2}{qR}$$

(b) For the x-motion, we have

$$v_x^2 = u_x^2 + 2a_x\left(x_f - x_i\right)$$

$$\Rightarrow \quad 0 = v^2 + 2a_x R$$

$$\Rightarrow \quad a_x = -\frac{v^2}{2R} = \frac{F_x}{m} = \frac{qE_x}{m}$$

$$\Rightarrow \quad E_x = -\frac{mv^2}{2qR}$$

Similarly for the y-motion, we have

$$v^2 = 0 + 2a_y R$$

$$\Rightarrow \quad a_y = +\frac{v^2}{2R} = \frac{qE_y}{m}$$

$$\Rightarrow \quad E_y = \frac{mv^2}{2qR}$$

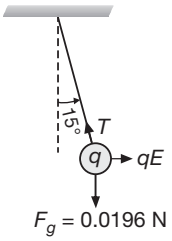
The magnitude of the field E is given by

$$E = \sqrt{E_x^2 + E_y^2}$$

$$\Rightarrow \quad E = \frac{mv^2}{\sqrt{2qR}}$$

at an angle of $135°$ in the counterclockwise direction from the x-axis.

19. From the free-body diagram shown,



$$\sum F_y = 0$$

$$\Rightarrow \quad T\cos(15°) = 1.96\times10^{-2}N$$

$$\Rightarrow \quad T = 2.03\times10^{-2}\text{ N}.$$

Again for the equilibrium, we have

$$\sum F_x = 0$$

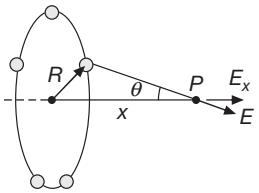
$$\Rightarrow \quad qE = T\sin(15°)$$

$$\Rightarrow \quad q = \frac{T\sin(15°)}{E}$$

$$\Rightarrow \quad q = \frac{(2.03\times10^{-2}\text{ N})\sin(15°)}{1.00\times10^{3}\text{ NC}^{-1}} = 5.25\times10^{-6}C$$

$$\Rightarrow \quad q = 5.25\ \mu\text{C}$$

20. One of the charges creates at P a field $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{n}\right)\frac{1}{R^2 + x^2}$ at an angle θ to the x-axis as shown.



When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is

$$E = \frac{1}{4\pi\varepsilon_0}\left[n\left(\frac{Q}{n}\right)\left(\frac{1}{R^2 + x^2}\right)\cos\theta \right]\hat{i}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{\frac{3}{2}}}\hat{i}$$

This result is identical to the electric field due to a uniformly charged ring at a point lying at its axis.

21. $$E = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dx}{x^2}$$

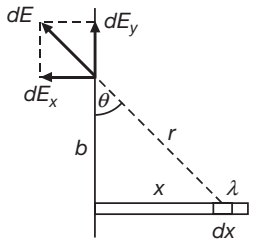
$$E = \frac{1}{4\pi\varepsilon_0}\int_{x_0}^{\infty} \frac{\lambda_0 x_0 dx}{x^3} = \frac{\lambda_0 x_0}{4\pi\varepsilon_0}\int_{x_0}^{\infty} x^{-3} dx$$

$$E = \frac{\lambda_0 x_0}{4\pi\varepsilon_0}\left(-\frac{1}{2x^2}\Big|_{x_0}^{\infty} \right) = \frac{\lambda_0}{8\pi\varepsilon_0 x_0}$$

directed towards the negative x axis.

22. From figure, electric field due to small length element dx at observation point at distance r.

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}$$



and $dE_y = dE\cos\theta$ and $dE_x = dE\sin\theta$

Put $x = b\tan\theta$

$$dx = b\sec^2\theta d\theta$$

Also $r = \frac{b}{\cos\theta} = b\sec\theta$

$$dE_y = \left[\frac{\lambda b\sec^2\theta d\theta}{4\pi\varepsilon_0 b^2\sec^2\theta} \right]\cos\theta$$

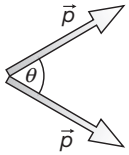
Then $E_y = \frac{\lambda}{4\pi\varepsilon_0 b}\int_0^{\frac{\pi}{2}} \cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 b}$ directed upward

Similarly $E_x = \frac{\lambda}{4\pi\varepsilon_0 b}\int_0^{\frac{\pi}{2}} \sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 b}$ to left.

If rod were infinite in both direction. The answer would become $E_x = 0$, $E_y = \frac{\lambda}{2\pi\varepsilon_0 b}$

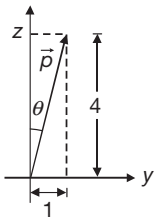
Test Your Concepts-IV (Based on Dipoles and Dipole Moment)

1. $p_{net} = 2p\cos\frac{\theta}{2}$



2.

q_i	x_i	$q_i x_i$	y_i	$q_i y_i$	z_i	$q_i z_i$
$2q$	0	0	a	$2qa$	a	$2qa$
q	0	0	$-a$	$-qa$	a	qa
$-q$	0	0	0	0	$-a$	qa



$$p_x = \sum_i q_i\, x_i = 0$$

$$p_y = \sum_i q_i\, y_i = 2qa - qa$$

$$\Rightarrow \quad p_y = qa$$

$$p_z = \sum_i q_i\, z_i = 2qa + qa + qa$$

$$\Rightarrow \quad p_z = 4qa$$

CHAPTER 1

Since, $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

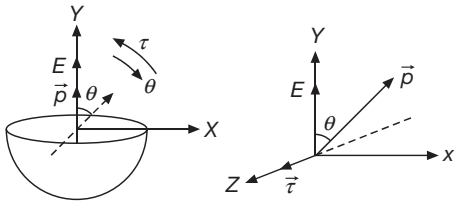
$\Rightarrow \quad \vec{p} = qa\left(\hat{j} + 4\hat{k}\right)$ {in yoz plane}

$\Rightarrow \quad |\vec{p}| = \sqrt{17}qa$

at an angle $\tan^{-1}\left(\frac{1}{4}\right)$ with the z axis.

3. Thus torque experienced by dipole has magnitude

$$\tau = p\left(\frac{\sigma}{4\varepsilon_0}\right)\sin\theta$$



For small angular displacement,

$$\tau = -\left(\frac{\sigma}{4\varepsilon_0}\right)\theta = \mathrm{I}\alpha$$

$$\Rightarrow \quad \alpha = -\left(\frac{\sigma}{4\pi\varepsilon_0\mathrm{I}}\right)\theta$$

On comparing the above expression with standard expression for SHM

$$\alpha = -\omega^2\theta$$

we obtain $\omega = \sqrt{\frac{\sigma}{4\pi\varepsilon_0\mathrm{I}}} = \frac{2\pi}{T}$

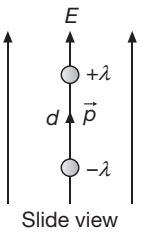
$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0\mathrm{I}}{\sigma}}$$

Potential energy of dipole is $U = -pE\cos\theta$

$$\Rightarrow \quad U = -p\left(\frac{\sigma}{4\pi\varepsilon_0}\right)\cos\theta$$

4. (a) The figure shows a side view of the system, it can be represented as an electric dipole of dipole moment

$$|\vec{p}| = (\lambda L)d$$



The time period of oscillation of dipole is

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{pE}}$$

$$T = 2\pi\sqrt{\frac{md^2/4 + md^2/4}{(\lambda Ld)E}}$$

$$T = 2\pi\sqrt{\frac{md}{2\lambda LE}}$$

(b) Work done required to turn the dipole from initial angle θ_i to θ_f is

$$W = pE\left(\cos\theta_i - \cos\theta_f\right)$$

(i) For $\theta_i = 0$, $\theta_f = \frac{\pi}{2}$,

$$W = (\lambda Ld)E[1-0]$$

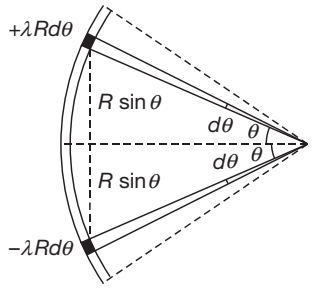
$$W = \lambda LdE$$

(ii) For $\theta_i = 0$, $\theta_f = \pi$,

$$W = \lambda LdE(\cos 0 - \cos\pi)$$

$$W = 2\lambda LdE$$

5. (a) We consider two differential elements on the rod as shown in figure



These elements constitute an electric dipole whose dipole moment is

$$|d\vec{p}| = (\lambda Rd\theta)(2R\sin\theta)$$

Net dipole moment of the system is

$$|\vec{p}| = \int_0^{2\theta_0} (\lambda Rd\theta)2R\sin\theta$$

$$|\vec{p}| = 2\lambda R^2\left(1 - \cos 2\theta_0\right)$$

$$|\vec{p}| = 4\lambda R^2\sin^2\theta_0$$

Time period of oscillation of a dipole in a uniform magnetic field is

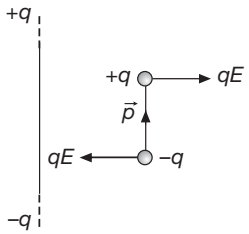
$$T = 2\pi\sqrt{\frac{\mathrm{I}}{pE}}$$

$$T = 2\pi\sqrt{\frac{mR^2}{4\lambda R^2\sin^2\theta_0 E}} = \pi\sqrt{\frac{m}{\lambda\sin^2\theta_0 E}}$$

(b) Work done required to rotate the dipole through $180°$ is $W = pE(\cos\theta_i - \cos\theta_f) = pE(\cos 0 - \cos 180°)$

$$W = 2pE = 2\left(4\lambda R^2\sin^2\theta_0\right)E = 8\lambda R^2 E\sin^2\theta_0$$

6. (a) In this case the resultant force on the dipole is zero as shown in figure (a)



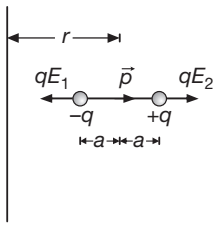
(a)

(b) The electric field at the position of negative charge $(-q)$,

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0(r-a)}$$

The electric field at the position of positive charge

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0(r+a)}$$



(b)

Resultant force on the dipole is

$$F = qE_1 - qE_2$$

$$F = \frac{q\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r-a} + \frac{1}{r+a}\right]$$

$$F = \frac{q\lambda}{2\pi\varepsilon_0}\frac{2a}{(r^2-a^2)}$$

for $r \gg a$, $F \simeq \frac{(2aq)\lambda}{2\pi\varepsilon_0 r^2} = \frac{p\lambda}{2\pi\varepsilon_0 r^2}$

Alternatively, the electric force on an electric dipole in a non-uniform field is given by

$$F = p\frac{dE}{dr}$$

$$F = p\frac{d}{dr}\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)$$

$$F = -\frac{p\lambda}{2\pi\varepsilon_0 r^2}$$

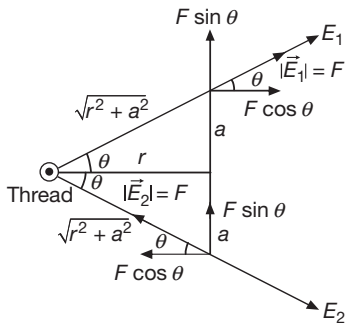
$$\Rightarrow \quad \vec{F} = -\frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2} \qquad \{\text{force is attractive}\}$$

(c) The magnitude of electric field at the position of two charges is

$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$$

and force on the charges is

$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = F = \frac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}$$



(c)

As shown in figure (c), resultant force on the dipole is

$$F_R = 2F\sin\theta$$

$$F_R = \frac{2q\lambda}{2\pi\varepsilon_0\sqrt{r^2+a^2}}\frac{a}{\sqrt{r^2+a^2}}$$

For $r \gg a$ and $2aq = p$,

$$F_R \simeq \frac{p\lambda}{2\pi\varepsilon_0 r^2}$$

Resultant force is parallel to dipole moment

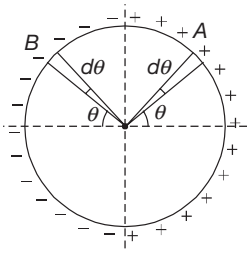
$$\vec{F}_R = \frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2}$$

7. Consider two differential charge elements at A and B as shown in figure. Dipole moment of this pair is

$$dp = \left[\left(\lambda_0\cos\theta\right)Rd\theta\right]2R\cos\theta$$

$$\Rightarrow \quad dp = 2\lambda_0 R^2\cos^2\theta\, d\theta$$

CHAPTER 1



So, the Dipole moment of the charge distribution is

$$p = 2\lambda_0 R^2 \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos^2\theta d\theta = \pi R^2 \lambda_0$$

8. (a) Applying Energy Conservation Principle,

$$\begin{pmatrix}\text{Increase in}\\ \text{kinetic energy of}\\ \text{the dipole}\end{pmatrix} = \begin{pmatrix}\text{decrease in}\\ \text{electrostatic potential}\\ \text{energy of the dipole}\end{pmatrix}$$

Kinetic energy of dipole at distance d from origin $= U_i - U_f$

$$\Rightarrow \quad KE = 0 - \left(-\vec{p}.\vec{E}\right) = \vec{p}.\vec{E} = \left(p\hat{i}\right).\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{d^2}\hat{i}\right)$$

$$\Rightarrow \quad KE = \frac{qp}{4\pi\varepsilon_o d^2}$$

(b) Electric field at origin due to the dipole,

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{2p}{d^3}\hat{i}\ \left(\vec{E}_{axis} \uparrow\uparrow \vec{p}\right)$$

$\therefore$ Force on charge q

$$\vec{F} = q\vec{E} = \frac{pq}{2\pi\varepsilon_o d^3}\hat{i}$$

Test Your Concepts-V (Based on Flux)

1. $\phi_{\text{total}} = \left(\phi_{\text{due to } +q}\right) + \left(\phi_{\text{due to } -q}\right)$

Since, we have seen already that

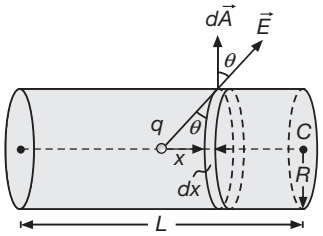
$$\phi = \frac{q}{2\varepsilon_0}(1-\cos\alpha) = \frac{q}{2\varepsilon_0}\left(1 - \frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

$$\Rightarrow \quad \phi_{\text{total}} = 2\phi = \frac{q}{\varepsilon_0}\left(1 - \frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

(Do not expect the answer to come to be zero as one charge is $+q$ and other $-q$. If both had been $+q$ or $-q$ then it would have been zero)

2. For this we consider an infinitesimal strip of width dx on the surface of cylinder as shown. The area of this strip is

$$dA = 2\pi R dx$$



The electric field due to the point charge on the strip is

$$E = \frac{q}{4\pi\varepsilon_0\left(R^2+x^2\right)}$$

If $d\phi$ is the electric flux through the strip, then

$$d\phi = E\,dA\cos\theta$$

$$d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x^2+R^2\right)}\left(2\pi R dx\right)\left(\frac{R}{\sqrt{x^2+R^2}}\right)$$

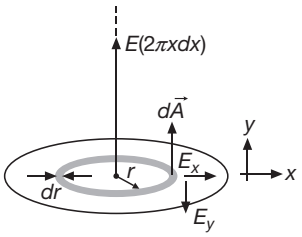
$$\Rightarrow \quad d\phi = \frac{2\pi q R^2}{4\pi\varepsilon_0}\left[\frac{dx}{\left(x^2+R^2\right)^{3/2}}\right]$$

Total flux through the lateral surface of cylinder is given by integrating the above result for the complete lateral surface. So

$$\phi = \int d\phi = \frac{qR^2}{2\varepsilon_0}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{\left(x^2+R^2\right)^{3/2}}$$

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0}\left(\frac{L}{\sqrt{L^2+4R^2}}\right)$$

3. Consider an infinitesimal area element on the surface of the circle.



Let this element have radius r and thickness dr. Then

$$d\vec{A} = 2\pi r dr\hat{j}$$

Since,

$$\vec{E} = E_x\hat{i} + E_y\left(-\hat{j}\right)$$

where $E_x = E_y = \dfrac{\lambda}{4\pi\varepsilon_0 r}$

Since, $d\phi = \vec{E}\cdot d\vec{A}$

$$\Rightarrow \quad d\phi = -2\pi r dr\left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)$$

$$\Rightarrow \quad \phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dr$$

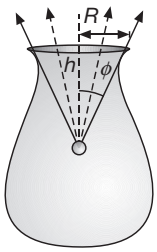
$$\Rightarrow \quad \phi = \frac{\lambda R}{2\varepsilon_0}$$

4. $\phi_{\text{lateral surface}} = \phi = \dfrac{q}{\varepsilon_0} -$ flux through the mouth

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0} - \frac{q}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{h^2+R^2}}\right)$$

$$\Rightarrow \quad \phi = \frac{q}{2\varepsilon_0}\left(1 + \frac{h}{\sqrt{h^2+R^2}}\right)$$

$$\Rightarrow \quad \phi = \left(1 + \frac{\sqrt{3}}{2}\right)\frac{q}{2\varepsilon_0}$$



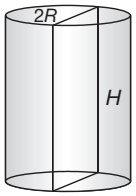
5. The cross sectional area perpendicular to the electric field is

$$A = (2R)H$$

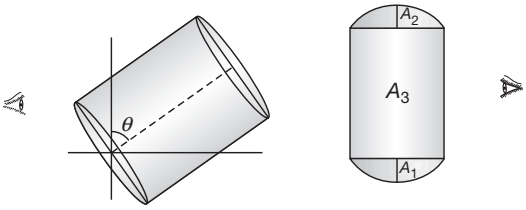
Thus electric flux crossing the cylinder is

$$\phi = EA$$

$$\Rightarrow \quad \phi = E(2RH) = 2HRE$$



6. Looking at the cylinder from $-$x-axis it appears as shown in figure.



Perpendicular component of area $A_\perp = A_1 + A_2 + A_3$

Now $A_1 = A_2$

$$A_1 = \left(\frac{\pi R^2}{2}\right)\sin\theta$$

and $A_3 = 2HR\cos\theta$

Total $\perp$ component of area is

$$A_\perp = \left(\frac{\pi R^2}{2}\right)\sin\theta \times 2 + 2HR\cos\theta$$

$$A_\perp = 2RH\cos\theta + \pi R^2\sin\theta$$

Thus flux passing through cylinder is

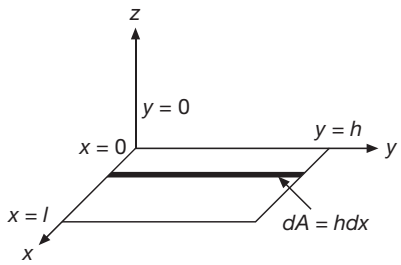
$$\phi = EA_\perp$$

$$\phi = E(2RH\cos\theta + \pi R^2\sin\theta)$$

7. In general, we have

$$E = ay\hat{i} + bz\hat{j} + cx\hat{k}$$

In the xy plane, $z = 0$ and so, $E = ay\hat{i} + cx\hat{k}$



By definition of flux, we have

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = \int \left(ay\hat{i} + cx\hat{k}\right)\cdot \hat{k}dA$$

$$\Rightarrow \quad \Phi_E = ch\int_{x=0}^{\ell} xdx = ch\frac{x^2}{2}\Big|_{x=0}^{\ell} = \frac{ch\ell^2}{2}$$

8. (a) $\Phi_E = EA\cos\theta$

$$\Phi_E = (3.5\times10^3)(0.35\times0.7)\cos0°$$

$$\Phi_E = 858\ \text{Nm}^2\text{C}^{-1}$$

(b) $\theta = 90°$

$$\Rightarrow \quad \Phi_E = 0$$

(c) $\Phi_E = (3.5\times10^3)(0.35\times0.7)\cos60°$

$$\Phi_E = 428.75\ \text{Nm}^2\text{C}^{-1}$$

CHAPTER 1

9. $\Phi_E = EA\cos\theta = (2\times10^4\ \text{NC}^{-1})(18\ \text{m}^2)\cos60° = 180\,\text{kN m}^2\text{C}^{-1}$

10. $\Phi_E = EA\cos\theta$

Since $A = \pi r^2 = \pi(0.4)^2 = 0.504\ \text{m}^2$

$\Rightarrow \quad 5.04\times10^5 = E(0.504)\cos0°$

$\Rightarrow \quad E = 10^6\ \text{NC}^{-1} = 1\ \text{MNC}^{-1}$

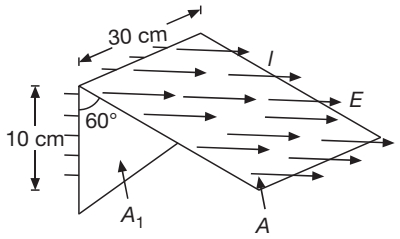
11. (a) $A_1 = (10\ \text{cm})(30\ \text{cm})$

$A_1 = 300\ \text{cm}^2 = 0.03\ \text{m}^2$

$\phi_1 = EA_1\cos\theta$

$\phi_1 = (7.8\times10^4)(0.03)\cos180°$

$\phi_1 = -2.34\ \text{kNm}^2\text{C}^{-1}$



(b) $\phi_2 = EA\cos\theta = (7.8\times10^4)(A)\cos60°$

Now, $A = (30\ \text{cm})(\ell) = (30\ \text{cm})\left(\dfrac{10\ \text{cm}}{\cos60°}\right)$

$\Rightarrow \quad A\cos60 = 30\times10$

$\Rightarrow \quad \phi_2 = 7.8\times10^4\times30\times10 = +2.34\ \text{kNm}^2\text{C}^{-1}$

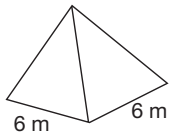
(c) The bottom and the two triangular sides all lie parallel to E, so $\Phi_E = 0$ for each of these. Thus,

$\Phi_{E,\,\text{total}} = -2.34\ \text{kN m}^2\text{C}^{-1} + 2.34\ \text{kN m}^2\ \text{C}^{-1} + 0 + 0 + 0 = 0$

12. $\Phi_E = EA\cos\theta$ through the base

$\Phi_E = (52)(36)\cos180° = -1.87\ \text{kNm}^2\text{C}^{-1}$

Note the same number of electric field lines pass through the base as are passing through the surface of the pyramid excluding the base.



For the inclined surfaces, $\Phi_E = +1.87\ \text{kNm}^2\text{C}^{-1}$

13. The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is given by

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = ERh$$

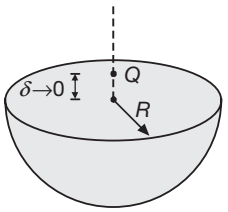
This is the same flux that exits the right side of the cone. Here we must observe that for a uniform field only the cross sectional area matters, note the shape.

14. (a) With δ being very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere or hemisphere

$\Rightarrow \quad \Phi_{\text{curved}} = \int E\cdot dA = E_{\text{local}}A_{\text{hemisphere}}$

$\Rightarrow \quad \Phi_{\text{curved}} = \left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}\right)\left(\dfrac{1}{2}4\pi R^2\right)$

$\Rightarrow \quad \Phi_{\text{curved}} = \dfrac{1}{4\pi\varepsilon_0}Q(2\pi) = +\dfrac{Q}{2\varepsilon_0}$



(b) The closed surface encloses zero charge, so according to Gauss's Law we have

$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0$

$\Rightarrow \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = -\dfrac{Q}{2\varepsilon_0}$

Test Your Concepts-VI (Based on Gauss's Law)

1. Net charge enclosed by the sphere is $q_{\text{enc}} = 0$. Therefore, according to Gauss's Law net flux passing through the sphere is zero.

2. (a) According to Gauss's Law,

$$\phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

(b) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be,

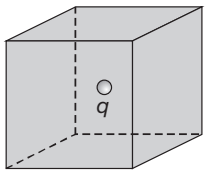
$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\varepsilon_0}$$

We can also do this by another method discussed below :

Here the total solid angle subtended by cube surface at the point charge q is 4π. As q is at centre of cube, we can say the each face of cube subtend equal solid angle at the centre, thus solid angle subtended by each face at point charge is

$$\Omega_{\text{face}} = \frac{4\pi}{6} \text{ steradian}$$

Thus electric flux through each face is



$$\phi = \left(\frac{q}{4\pi\varepsilon_0}\right)\Omega$$

$$\Rightarrow \quad \phi_{\text{face}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{4\pi}{6}\right)$$

$$\Rightarrow \quad \phi_{\text{face}} = \frac{q}{6\varepsilon_0}$$

If charge is not at the centre, the answer of part (a) will remain same while that of part (b) will change.

3. At any point $P(x, y, z)$ on the sphere a unit vector perpendicular to the sphere directed radially outwards is,

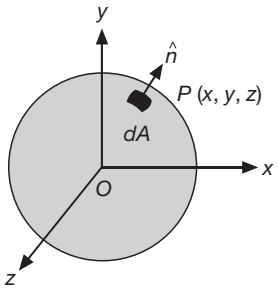
$$\hat{n} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$$\Rightarrow \quad \hat{n} = \frac{x}{R}\hat{i} + \frac{y}{R}\hat{j} + \frac{z}{R}\hat{k} \qquad \{\because\ x^2+y^2+z^2 = R^2\}$$

Let us find the electric flux passing through a small area dA at point P on the sphere, then

$$d\phi = \vec{E}\cdot\hat{n}\,dA = \left\{\frac{\alpha x^2}{R\left(x^2+y^2\right)} + \frac{\alpha y^2}{R\left(x^2+y^2\right)}\right\}dA$$

$$\Rightarrow \quad d\phi = \left(\frac{\alpha}{R}\right)dA$$



Here, we note that $d\phi$ is independent of the coordinates x, y and z. Therefore, total flux passing through the sphere

$$\phi = \int d\phi = \frac{\alpha}{R}\int dA = \left(\frac{\alpha}{R}\right)\left(4\pi R^2\right)$$

$$\Rightarrow \quad \phi = 4\pi\alpha R$$

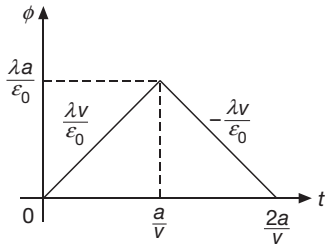
From Gauss's Law, $\phi = \frac{q_{\text{enc}}}{\varepsilon_0}$

$$\Rightarrow \quad (4\pi\alpha R) = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow \quad q_{\text{enc}} = 4\pi\varepsilon_0\alpha R$$

4. At time t, length of rod inside the cube is $\ell = vt$

So, charge enclosed $q_{\text{enc}}(t) = \lambda\ell = \lambda vt$



$$\Rightarrow \text{ Flux } = \phi_1(t) = \frac{1}{\varepsilon_0}\left[q_{\text{enc}}(t)\right] = \frac{\lambda vt}{\varepsilon_0} \text{ \{By Gauss's Law\}}$$

So, flux increases as a function of time

When the rod enters the cube completely, flux is

$$\phi_{\max} = \frac{1}{\varepsilon_0}(\lambda a)$$

and again when the rod starts leaving the cube it starts decreasing as

$$\phi_2(t)=\frac{\lambda}{\varepsilon_0}(a-vt)$$

and ultimately falls to zero as soon as rod is out of the cube

5. Draw a Gaussian cylinder of length ℓ and radius s. For this surface, Gauss's Law states

$$\oint_S E\cdot d\vec{A}=\frac{1}{\varepsilon_0}Q_{\text{enc}}$$

The enclosed charge is

$$Q_{enc}=\int \rho\, dV=\frac{2}{3}\pi k\ell x^3$$

Now, symmetry dictates that $\vec{E}$ must point radially outward, so for the curved portion of the Gaussian cylinder we have

$$\int \vec{E}\cdot d\vec{A}=\int |\vec{E}|dA=|\vec{E}|\int dA=|\vec{E}|2\pi x\ell$$

while the two ends contribute nothing (here $\vec{E}$ is perpendicular to $d\vec{A}$). Thus,

$$|\vec{E}|2\pi s\ell=\frac{1}{\varepsilon_0}\frac{2}{3}\pi k\ell x^3$$

$$\Rightarrow \quad |\vec{E}|=E=\frac{1}{3\varepsilon_0}kx^2$$

6. (a) One half of the total flux created by the charge q goes through the plane. Thus,

$$\phi_{E,\text{ plane}}=\frac{1}{2}\phi_{E,\text{ total}}=\frac{1}{2}\left(\frac{q}{\varepsilon_0}\right)=\frac{q}{2\varepsilon_0}$$

(b) The square looks like an infinite plane to a charge very close to the surface. Hence,

$$\phi_{E,\text{ square}}\approx\phi_{E,\text{ plane}}=\frac{q}{2\varepsilon_0}$$

(c) The plane and the square behave in the same manner for the charge.

7. The total charge enclosed is $Q_{\text{enc}}=Q+(-6q)=Q-6q.$

The total outward flux from the cube is $\phi_E=\dfrac{Q-6q}{\varepsilon_0}$, of which one-sixth goes through each face so , flux through each face is given by

$$(\phi_E)_{\text{one face}}=\frac{Q-6q}{6\varepsilon_0}$$

For, the cube not to be associated with any flux, we must have $q=\dfrac{Q}{6}$

8. The charge density is determined by $Q=\dfrac{4}{3}\pi a^3\rho$

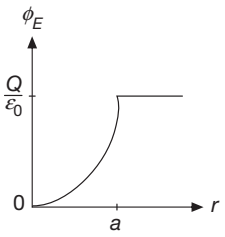
$$\rho=\frac{3Q}{4\pi a^3}$$

(a) The flux is that created by the enclosed charge within radius r.

$$\phi_E=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{4\pi r^3\rho}{3\varepsilon_0}=\left(\frac{4\pi r^3}{3\varepsilon_0}\right)\left(\frac{3Q}{4\pi a^3}\right)=\frac{Qr^3}{\varepsilon_0 a^3}$$

(b) $\phi_E=\dfrac{Q}{\varepsilon_0}$. Note that the answers to parts (a) and (b) agree at $r=a$.

(c)



9. **CASE-1**

When $R\le d$, the sphere encloses no charge and $\phi_E=\dfrac{q_{\text{enc}}}{\varepsilon_0}=0$

CASE-2

When $R>d$, the length of line falling within the sphere is $2\sqrt{R^2-d^2}$. So, according to Gauss's Law

$$\phi_E=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{2\lambda\sqrt{R^2-d^2}}{\varepsilon_0}$$

10. $\oint \vec{E}\cdot d\vec{A}=E(4\pi r^2)=\dfrac{q_{\text{enc}}}{\varepsilon_0}$

(a) For $r>R$, we have

$$q_{\text{enc}}=\int_0^R \rho_0 r^2(4\pi r^2)dr=\frac{4\pi\rho_0 R^5}{5}$$

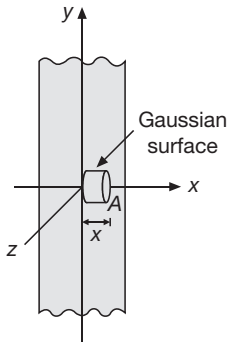
$$\Rightarrow \quad E_{\text{outside}}=\frac{\rho_0 R^5}{5\varepsilon_0 r^2}$$

(b) For $r<R$, we have

$$q_{\text{enc}}=\int_0^r \rho_0 r^2(4\pi r^2)dr=\frac{4\pi\rho_0 r^5}{5}$$

$$\Rightarrow \quad E_{\text{inside}}=\frac{\rho_0 r^3}{5\varepsilon_0}$$

11. Consider the Gaussian surface, a Gaussian pill box as shown



(a) For $x > \frac{d}{2}$

$$dq = \rho dV = \rho A dx = A\rho_0 x^2 dx$$

Since $\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int dq$

$$\Rightarrow \quad EA = \frac{\rho_0 A}{\varepsilon_0}\int_0^{\frac{d}{2}} x^2 dx = \frac{1}{3}\left(\frac{\rho_0 A}{\varepsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$\Rightarrow \quad E = \frac{\rho_0 d^3}{24\varepsilon_0}$$

$$\Rightarrow \quad E = \begin{cases} \frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for} \quad x > \frac{d}{2} \\ -\frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for} \quad x < -\frac{d}{2} \end{cases}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$

$$\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int dq = \frac{\rho_0 A}{\varepsilon_0}\int_0^x x^2 dx = \frac{\rho_0 A x^3}{3\varepsilon_0}$$

$$\Rightarrow \quad E = \begin{cases} \frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for} \quad x > 0 \\ -\frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for} \quad x < 0 \end{cases}$$

12. $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_0^r \frac{\rho_0}{r} 4\pi r^2 dr$

$$\Rightarrow \quad E(4\pi r^2) = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r r dr = \frac{4\pi\rho_0 r^2}{2\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_0}{2\,\varepsilon_0} = \text{constant in magnitude}$$

(The direction is radially outward from the center for positive ρ_0 and radially inward for negative ρ_0)

13. (a) A point mass m creates a gravitational acceleration $\vec{g} = -\frac{Gm}{r^2}\hat{r}$ at a distance r.

The flux of this field through a sphere is

$$\oint \vec{g}\cdot d\vec{A} = -\frac{Gm}{r^2}(4\pi r^2) = -4\pi Gm$$

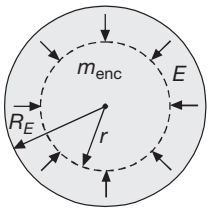
Since the flux is independent of r, so we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\oint \vec{g}\cdot d\vec{A} = -4\pi G\sum m_{enc}$$

(b) Take a spherical Gaussian surface of radius r. The field is inward so

$$\phi = \oint \vec{g}\cdot d\vec{A} = g\left(4\pi r^2\right)\cos 180° = -g\left(4\pi r^2\right)$$

$$\Rightarrow \quad \phi = -g(4\pi r^2)$$



Now, we have

$$m_{enc} = \frac{4}{3}\pi r^3\rho$$

So, according to Gauss's Law, we have

$$\oint \vec{g}\cdot d\vec{A} = -4\pi G\sum m_{enc}$$

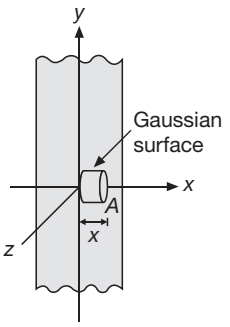
$$\Rightarrow \quad -g(4\pi r^2) = -4\pi G\left(\frac{4}{3}\pi r^3\rho\right)$$

$$\Rightarrow \quad g = \frac{4}{3}\pi G r\rho$$

But $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} \Rightarrow g = \left(\frac{GM_E}{R_E^3}\right)r$, inwards

14. (a) Consider a cylindrical shaped Gaussian surface, also called Gaussian Pill box, perpendicular to the yz plane with one end in the yz plane and the other end containing the point x.

CHAPTER 1



According to Gauss's Law, we have

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

By symmetry, the electric field is zero in the yz plane and is perpendicular to dA over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \Rightarrow \quad EA = \frac{\rho(Ax)}{\varepsilon_0}$$

so that at distance x from the mid-line of the slab,

$$E = \frac{\rho x}{\varepsilon_0}$$

(b) $$a = \frac{F}{m} = \frac{(-e)E}{m} = -\left(\frac{\rho e}{m\varepsilon_0}\right)x$$

The acceleration of the electron is of the form

$$a = -\omega^2 x \text{ with } \omega = \sqrt{\frac{\rho e}{m\varepsilon_0}} = \frac{2\pi}{T}$$

Thus, the motion is simple harmonic with period

$$T = 2\pi\sqrt{\frac{m\varepsilon_0}{\rho e}}$$

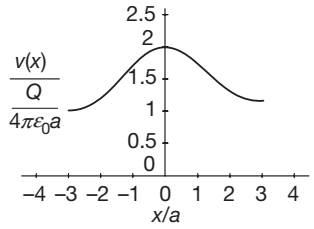
15. DO YOURSELF (Already Discussed)

$$\vec{E} = \frac{\rho\vec{a}}{2\varepsilon_0}$$

Test Your Concepts-VII (Based on Electrostatic Potential and Energy)

1. (a) $$V(x) = \frac{2Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}} = \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{2}{\sqrt{\left(\frac{x}{a}\right)^2+1}}\right)$$

$$\Rightarrow \quad \frac{V(x)}{\left(\frac{Q}{4\pi\varepsilon_0 a}\right)} = \frac{2}{\sqrt{\left(\frac{x}{a}\right)^2+1}}$$

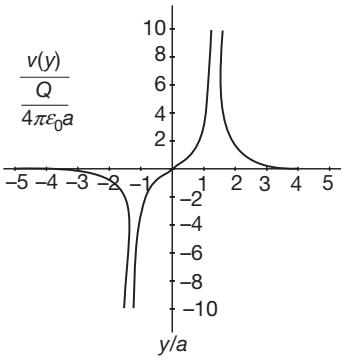


(b) $$V(y) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2}$$

$$\Rightarrow \quad V(y) = \frac{1}{4\pi\varepsilon_0}\left(\frac{+Q}{|y-a|} + \frac{-Q}{|y+a|}\right)$$

$$\Rightarrow \quad V(y) = \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{1}{\left|\frac{y}{a}-1\right|} - \frac{1}{\left|\frac{y}{a}+1\right|}\right)$$

$$\Rightarrow \quad \frac{V(y)}{\left(\frac{Q}{4\pi\varepsilon_0 a}\right)} = \left(\frac{1}{\left|\frac{y}{a}-1\right|} - \frac{1}{\left|\frac{y}{a}+1\right|}\right)$$



2. In this problem,

$$r_{41} = r_{43} = r_{32} = r_{21} = 1 \text{ m}$$

and $$r_{42} = r_{31} = \sqrt{(1)^2+(1)^2} = \sqrt{2} \text{ m}$$

$$\Rightarrow \quad U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_4}{r_{34}} + \frac{q_4q_1}{r_{14}} + \frac{q_2q_4}{r_{24}} + \frac{q_1q_3}{r_{13}}\right]$$

Substituting the proper values with sign in above equation, we get

$$U = (9\times10^9)(10^{-6})(10^{-6})\left[\frac{(1)(2)}{1} + \frac{(2)(-3)}{1} + \frac{(-3)(4)}{1} + \frac{(1)(4)}{1} + \frac{(2)(4)}{\sqrt{2}} + \frac{(1)(-3)}{\sqrt{2}}\right]$$

$$U = (9\times10^{-3})\left[-12 + \frac{5}{\sqrt{2}}\right]$$

$$U = -7.62\times10^{-2} \text{ J}$$

The negative sign of U implies that positive work has to be done by electrostatic forces in assembling these charges at respective positions from infinity.

3. $\Delta V = -14$ V and

$$Q = -N_A e = -(6.02\times10^{23})(1.6\times10^{-19}) = -9.63\times10^{4}\ \text{C}$$

Since $\Delta V = \dfrac{W}{Q}$

$$\Rightarrow \quad W = Q\Delta V = (-9.63\times10^{4}\ \text{C})(-14\ \text{JC}^{-1})$$

$$\Rightarrow \quad W = 1.35\times10^{6}\ \text{J} = 1.35\ \text{MJ}$$

4. The net electric potential at origin is,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right]$$

Substituting the values, we have

$$V = (9\times10^{9})\left(\frac{1}{1}-\frac{2}{2}+\frac{3}{3}\right)\times10^{-6}$$

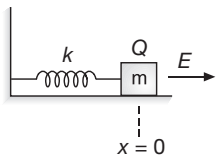
$$\Rightarrow \quad V = 9\times10^{3}\ \text{V}$$

5. (a) Let us consider $V = 0$ at 0. Then at other points

$$V = -Ex \quad \text{and} \quad U_e = QV = -Q(Ex)$$

Between the extreme points of the motion, we have

$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$



$$\Rightarrow \quad 0+0+0 = 0+\frac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$\Rightarrow \quad x_{\max} = \frac{2QE}{k}$$

(b) At equilibrium,

$$\sum F_x = -F_s + F_e = 0$$

$$\Rightarrow \quad kx = QE$$

So the equilibrium position is at $x = \dfrac{QE}{k}$

(c) The block's equation of motion is

$$\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -k\left(x - \frac{QE}{k}\right)$$

Let $X = x - \dfrac{QE}{k}$,

$$\Rightarrow \quad \frac{d^2X}{dt^2} = \frac{d^2X}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2X}{dt^2} = -kX$$

$$\Rightarrow \quad \frac{d^2X}{dt^2} + \frac{k}{m}X = 0$$

$$\Rightarrow \quad \ddot{X} + \omega^2 X = 0$$

This is the equation for simple harmonic motion with $\omega = \sqrt{\dfrac{k}{m}}$

The period of the motion is then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(d) $(K + U_s + U_e)_i + \Delta E_{\text{mech}} = (K + U_s + U_e)_f$

$$\Rightarrow \quad 0+0+0-\mu_k mgx_{\max} = 0+\frac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \frac{2(QE-\mu_k mg)}{k}$$

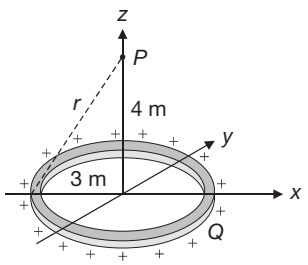
6. The electric potential at point P, at distance r from circumference is

$$V = \frac{Q}{2\pi\varepsilon_0 r}$$

where $r = \sqrt{(3)^2+(4)^2} = 5$ m and

$$Q = 10\ \mu\text{C} = 10^{-5}\ \text{C}$$

$$\Rightarrow \quad V = \frac{(9.0\times10^{9})(10^{-5})}{(5.0)} = 1.8\times10^{4}\ \text{V}$$



CHAPTER 1

7. For the entire motion, $y_f - y_i = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow \quad 0 - 0 = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow \quad a_y = -\frac{2u_y}{t}$

Since, $\sum F_y = ma_y$

$\Rightarrow \quad -mg - qE = -\frac{2mu_y}{t}$

$\Rightarrow \quad E = \frac{m}{q}\left(\frac{2u_y}{t} - g\right)$ OR $\vec{E} = -\frac{m}{q}\left(\frac{2u_y}{t} - g\right)\hat{j}$

For the upward flight, we have

$v_y^2 = u_y^2 + 2a_y\left(y_f - y_i\right)$

$\Rightarrow \quad 0 = u_y^2 + 2\left(-\frac{2u_y}{t}\right)\left(y_{\max} - 0\right)$

$\Rightarrow \quad y_{\max} = \frac{1}{4} u_y t$

Since $\Delta V = -\int_0^{y_{\max}} E dy = +\frac{m}{q}\left(\frac{2u_y}{t} - g\right) y \Big|_0^{y_{\max}}$

$\Rightarrow \quad \Delta V = \frac{m}{q}\left(\frac{2u_y}{t} - g\right)\left(\frac{1}{4} u_y t\right)$

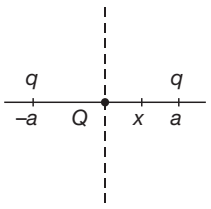
$\Rightarrow \Delta V = \frac{2}{5\times 10^{-6}}\left(\frac{2(20.1)}{4.1} - 9.8\right)\left[\frac{1}{4}(20.1)(4.1)\right] \cong 40\,\text{kV}$

8. Initially, the potential energy of Q is

$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{a} + \frac{Qq}{a}\right]$

$\Rightarrow \quad U_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}$

When Q is displaced by a small amount x to the right, the potential energy of Q becomes



$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(a+x)} + \frac{Qq}{(a-x)}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2Qqa}{(a^2 - x^2)}$

$\Rightarrow \quad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\left(1 - \frac{x^2}{a^2}\right)^{-1}$

Expanding binomially and neglecting higher powers of $\frac{x^2}{a^2}$ as $x \ll a$

$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\left(1 + \frac{x^2}{a^2}\right)$

$\left\{\because \left(1 - \frac{x^2}{a^2}\right)^{-1} \cong 1 + \frac{x^2}{a^2} \text{ for } x^2 \ll a^2\right\}$

So, the change in electrostatic potential energy of the system,

$dU = U_2 - U_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\frac{x^2}{a^2}$

$\Rightarrow \quad dU = \frac{1}{4\pi\varepsilon_0}\frac{2Qqx^2}{a^3}$

9. Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length,

$V = -Ed$ and $U_e = -\lambda LEd$

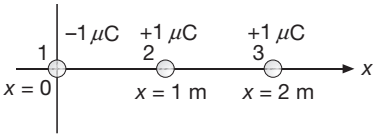
$(K+U)_i = (K+U)_f$

$\Rightarrow \quad 0 + 0 = \frac{1}{2}\mu L v^2 - \lambda LEd$

$\Rightarrow \quad v = \sqrt{\frac{2\lambda Ed}{\mu}}$

$\Rightarrow \quad v = \sqrt{\frac{2(40\times 10^{-6}\,\text{Cm}^{-1})(100\,\text{NC}^{-1})(2\text{m})}{\left(0.1\,\text{kgm}^{-1}\right)}} = 0.4\ \text{ms}^{-1}$

10. (a) The work that must be done on q_3 by an external force is equal to the difference of potential energy U when the charge is at $x = 2$ m and the potential energy when it is at infinity.



$\therefore \quad W_{\text{ext}} = U_f - U_i = U_f - U_\infty$

$V_3 = V_{13} + V_{23} = \frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}$

Now $W_{\infty\to 3} = q_3 V_3$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_2}{r_{23}} + \frac{q_3 q_1}{r_{13}}\right]$$

Substituting the values, we have

$$\Rightarrow \quad W = (9\times 10^9)(10^{-12})\left[\frac{(1)(1)}{(1)} + \frac{(1)(-1)}{(2)}\right]$$

$$\Rightarrow \quad W = 4.5\times 10^{-3}\ \text{J}$$

(b) The total potential energy of the three charges is given by,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3 q_2}{r_{23}} + \frac{q_3 q_1}{r_{13}} + \frac{q_2 q_1}{r_{12}}\right)$$

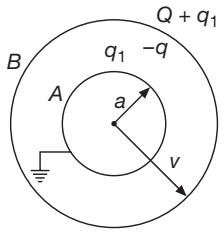
$$\Rightarrow \quad U = (9\times 10^9)\left[\frac{(1)(1)}{(1)} + \frac{(1)(-1)}{(2)} + \frac{(1)(-1)}{(1)}\right](10^{-12})$$

$$\Rightarrow \quad U = -4.5\times 10^{-3}\ \text{J}$$

Since $U < 0$, the system of three charges has lower potential energy then it would if the three charges were infinitely far apart. An external force would have to do negative work to bring the three charges from infinity to assemble this entire arrangement and would have to do positive work to move the three charges back to infinity.

11. When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed.

The charge q' on A induces $-q'$ on inner surface of B and $+q'$ on outer surface of B, in equilibrium the charge distribution is as shown in figure.



$$\begin{pmatrix}\text{Potential due to inner sphere}\end{pmatrix} = \begin{pmatrix}\text{potential due to charge on A}\end{pmatrix} + \begin{pmatrix}\text{potential due to charge on B}\end{pmatrix}$$

$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = V_{\text{Earth}} = 0$$

$$\Rightarrow \quad q' = -Q\left(\frac{a}{b}\right)$$

This implies that a charge $+Q\left(\frac{a}{b}\right)$ has been transferred to earth leaving negative charge on A.

12. (a) $E = \dfrac{|Q|}{4\pi\varepsilon_0 r^2}$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \frac{|V|}{|E|} = \frac{3000\ \text{V}}{500\ \text{Vm}^{-1}} = 6\ \text{m}$$

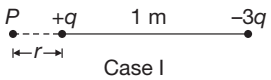
(b) $V = -3000$ V

$$\Rightarrow \quad -3000 = \frac{Q}{4\pi\varepsilon_0 (6)}$$

$$\Rightarrow \quad Q = -\frac{3000\times 6}{9\times 10^9} = -2\ \mu\text{C}$$

13. Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,

CASE-1:

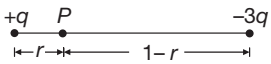


Case I

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 (1+r)} = 0$$

Solving this, we get $r = 0.5$ m

CASE-2:



Case II

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 (1-r)} = 0,$$

which gives $r = 0.25$ m

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$.

14. The original electrical potential energy is

$$U_e = qV = q\left(\frac{q}{4\pi\varepsilon_0 d}\right) = \frac{q^2}{4\pi\varepsilon_0 d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are

CHAPTER 1

$$-k(2d)+\frac{q^2}{4\pi\varepsilon_0(3d)^2}=0$$

$$\Rightarrow \quad k=\frac{q^2}{4\pi\varepsilon_0(18d^3)}$$

In the final configuration the total potential energy is

$$\frac{1}{2}kx^2+qV=\frac{1}{2}\frac{q^2}{4\pi\varepsilon_0(18d^3)}(2d)^2+q\frac{q}{4\pi\varepsilon_0(3d)}=\frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 d}$$

The missing energy must have become internal energy, as the system is isolated

So, $\frac{q^2}{4\pi\varepsilon_0 d}=\frac{4}{9}\left(\frac{q^2}{4\pi\varepsilon_0 d}\right)+\Delta E_{\text{int}}$

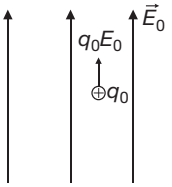
$$\Rightarrow \quad \Delta E_{\text{int}}=\frac{5}{9}\left(\frac{q^2}{4\pi\varepsilon_0 d}\right)$$

15. $\Delta V=V_{2R}-V_0=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+(2R)^2}}-\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

$$\Delta V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left(\frac{1}{\sqrt{5}}-1\right)=-0.553\frac{Q}{4\pi\varepsilon_0 R}$$

16. Electrostatic force F on the test charge,

$F=q_0E_0$ {along positive y-direction}



Since, $W_{i\to f}=-\Delta U$

$$\Rightarrow \quad \Delta U=-W_{i\to f}=-\left[q_0E_0(2a-a)\right]$$

$$\Rightarrow \quad \Delta U=-q_0E_0a$$

Here work done by electrostatic force is positive. Hence the potential energy must be decreasing.

17. $V=\frac{1}{4\pi\varepsilon_0}\int_{\text{all charge}}\frac{dq}{r}$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\int_{3R}^{R}\frac{\lambda dx}{x}+\int_{\text{semicircle}}\frac{\lambda ds}{R}+\int_{R}^{3R}\frac{\lambda dx}{x}\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\lambda\log_e(x)\Big|_{3R}^{R}+\frac{1}{4\pi\varepsilon_0}\left[\frac{\lambda}{R}\pi R+\lambda\log_e x\Big|_{R}^{3R}\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\log_e 3+\lambda\pi+\log_e 3\right]$$

$$\Rightarrow \quad V=\frac{\lambda}{4\pi\varepsilon_0}(\pi+2\log_e 3)$$

18. For an element of area which is a ring of radius r and width dr, $dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+x^2}}$

$dq=\sigma dA=\sigma_0 r(2\pi r dr)$ and

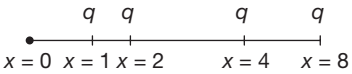
$$V=\sigma_0\left(\frac{2\pi}{4\pi\varepsilon_0}\right)\int_0^R\frac{r^2dr}{\sqrt{r^2+x^2}}$$

$$\Rightarrow \quad V=\left(\frac{\pi\sigma_0}{4\pi\varepsilon_0}\right)\left[R\sqrt{R^2+x^2}+x^2\log_e\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]$$

19. (i) Potential at $x=0$ will be

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\ldots\infty\right)$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{2}+\frac{1}{4}+\ldots.\infty\right]$$



This is an infinite G.P. sum of ∞ terms of G.P. is

$S_n=\frac{a}{1-r}$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{2}}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1/2}\right]=\frac{2q}{4\pi\varepsilon_0}$$

Now electric field

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(1)^2}+\frac{q}{(2)^2}+\frac{q}{(4)^2}+\ldots\infty\right]$$

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\infty\right]$$

{This is also an infinite G.P.}

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-1/4}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{3/4}\right]$$

$$\Rightarrow \quad E=\frac{4q}{4\pi\varepsilon_0\times 3}=\frac{q}{3\pi\varepsilon_0}$$

(ii) When consecutive charges are negative then

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}+\ldots\infty\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{4}+\frac{q}{16}+\ldots\infty\right]-\left[\frac{q}{2}+\frac{q}{8}+\frac{q}{32}+\ldots\infty\right]$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left\{\left[\frac{1}{\left[1-\left(\frac{1}{4}\right)\right]}\right]-\frac{1}{2}\left[\frac{1}{\left[1-\left(\frac{1}{4}\right)\right]}\right]\right\}$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{4}{3}-\frac{2}{3}\right]=\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3}\right)$$

Electric field at $x=0$ is

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(1)^2}-\frac{q}{(2)^2}+\frac{q}{(4)^2}-\frac{q}{(8)^2}+\frac{q}{(16)^2}+\ldots\infty\right]$$

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{16}+\frac{1}{256}+\ldots\right]-\left[\frac{1}{4}+\frac{1}{64}+\frac{1}{1024}+\ldots\right]$$

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{\left[1-\frac{1}{16}\right]}\right\}-\frac{1}{4}\left[\frac{1}{1-\frac{1}{16}}\right]$$

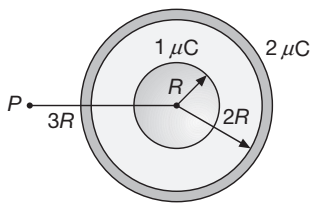
$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[\frac{16}{15}-\frac{1}{4}\frac{16}{15}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{4q}{5}\right]$$

20. Potential at point P

$$9000=\frac{K(1\times10^{-6})}{3R}+\frac{K(2\times10^{-6})}{3R}$$

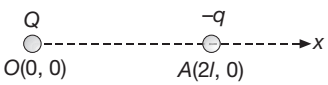
$$3R=3$$

$$R=1\text{ m}$$



21. We can here use Law of Conservation of Energy, according to which

$$(U+K)_{\text{at }\frac{\ell}{4}}+(U+K)_{\text{at }A}$$



$$\Rightarrow \quad \frac{-qQ}{4\pi\varepsilon_0\left(\frac{\ell}{4}\right)}+\frac{1}{2}mv^2=\frac{-qQ}{4\pi\varepsilon_0(2\ell)}+\frac{1}{2}m(0)^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{qQ}{4\pi\varepsilon_0}\left[\frac{4}{\ell}-\frac{1}{2\ell}\right]$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{qQ}{4\pi\varepsilon_0}\left[\frac{7}{2\ell}\right]$$

$$\Rightarrow \quad v=\sqrt{\frac{7Qq}{4\pi\varepsilon_0 m\ell}}$$

22. For potential energy of the system of charges, total number of charge pairs will be ${}^8C_2$ or 28 of these 28 pairs, 12 unlike charges are at a separation 'a', 12 like charges are at separation $\sqrt{2}\,a$ and 4 unlike charges are at separation $\sqrt{3}\,a$. Therefore, the potential energy of the system

$$U=\frac{1}{4\pi\varepsilon_o}\left[\frac{(12)(q)(-q)}{a}+\frac{(12)(q)(q)}{\sqrt{2}\,a}+\frac{(4)(q)(-q)}{\sqrt{3}\,a}\right]$$

$$\Rightarrow \quad U=-5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

The binding energy of this system is therefore,

$$|U|=5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

So, work done by external forces in disassembling, this system of charges is

$$W=5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

23. Let q be the charge on the bubble, then

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

$$\Rightarrow \quad q=4\pi\varepsilon_0 aV$$

After collapsing, let the radius of droplet becomes R. Then equating the volume, we get

$$\left(4\pi a^2\right)t=\frac{4}{3}\pi R^3$$

$$\Rightarrow \quad R=\left(3a^2t\right)^{1/3}$$

Now, potential of new droplet will be $V'=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

Substituting the values, we have

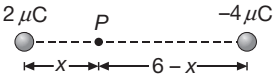
$$V'=\frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 aV}{\left(3a^2t\right)^{1/3}}$$

$$\Rightarrow \quad V'=V\left(\frac{a}{3t}\right)^{1/3}$$

24. There will be two points where electric potential will be zero. One point lies between the charges and other point lies in region outside the charges.

CHAPTER 1

For inside point,



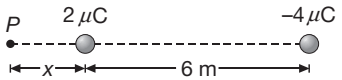
$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\times10^{-6}}{x}+\frac{-4\times10^{-6}}{6-x}\right)=0$$

$$\Rightarrow \quad \frac{2}{x}=\frac{4}{6-x}$$

$$\Rightarrow \quad 12-2x=4x$$

$$\Rightarrow \quad x=2 \text{ m}$$

For outside point



$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\times10^{-6}}{x}+\frac{-4\times10^{-6}}{6+x}\right)=0$$

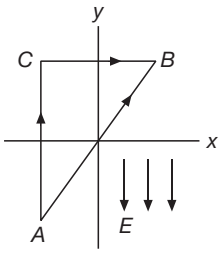
$$\Rightarrow \quad \frac{2}{x}=\frac{4}{6+x}$$

$$\Rightarrow \quad 12+2x=4x$$

$$\Rightarrow \quad x=6 \text{ m}$$

Test Your Concepts-VIII (Based on Equipotential Surfaces)

1. $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{\ell} = -\int_A^C \vec{E}\cdot d\vec{\ell} - \int_C^B \vec{E}\cdot d\vec{\ell}$



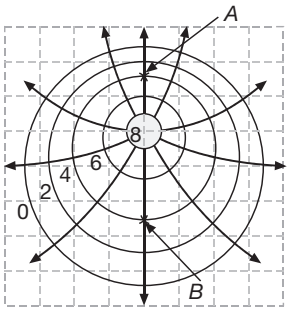
$$\Rightarrow \quad V_B - V_A = \left(-E\cos 180^o\right)\int_{-0.3}^{0.5} dy - \left(E\cos 90^o\right)\int_{-0.2}^{0.4} dx$$

$$\Rightarrow \quad V_B - V_A = (325)(0.8) = +260V$$

2. (a) $E_A > E_B$ since $E = \dfrac{\Delta V}{\Delta \ell}$

(b) $E_B = -\dfrac{\Delta V}{\Delta \ell} = -\dfrac{(6-2)\text{ V}}{0.02\text{ m}} = 200 \text{ NC}^{-1}$ downwards

(c) The figure is shown here with the field lines sketched.



3. For the given charge distribution,

$$V(x,\ y,\ z) = \frac{1}{4\pi\varepsilon_0}\frac{(q)}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{(-2q)}{r_2} \qquad \ldots(1)$$

where $r_1 = \sqrt{(x+R)^2+y^2+z^2}$ and $r_2 = \sqrt{x^2+y^2+z^2}$

The surface on which $V(x,\ y,\ z)=0$

is given by $\dfrac{1}{4\pi\varepsilon_0}q\left(\dfrac{1}{r_1}-\dfrac{2}{r_2}\right)=0 \quad \Rightarrow \quad 2r_1 = r_2$

From (1), we get $4(x+R)^2+4y^2+4z^2 = x^2+y^2+z^2$

$$\Rightarrow \quad x^2+y^2+z^2+\left(\frac{8}{3}R\right)x+\left(\frac{4}{3}R^2\right)=0 \qquad \ldots(2)$$

The general equation for a sphere of radius r centered at $(x_0,\ y_0,\ z_0)$ is

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2-r^2=0$$

$$\Rightarrow \quad x^2+y^2+z^2+(-2x_0)x+(-2y_0)y+$$

$$(-2z_0)z+\left(x_0^2+y_0^2+z_0^2-r^2\right)=0 \quad \ldots(3)$$

Comparing equations (2) and (3), it is seen that the equipotential surface for which $V=0$ is indeed a sphere and that

$$-2x_0=\frac{8}{3}R, -2y_0=0, -2z_0=0, x_0^2+y_0^2+z_0^2-r^2=\frac{4}{3}R^2$$

$$\Rightarrow \quad x_0=-\frac{4}{3}R,\ y_0=z_0=0, \text{ and } r^2=\left(\frac{16}{9}-\frac{4}{3}\right)R^2=\frac{4}{9}R^2$$

The equipotential surface is therefore a sphere centered at $\left(-\dfrac{4}{3}R,\ 0,\ 0\right)$, having a radius $\dfrac{2}{3}R$

4. Let us consider that the equipotential surface corresponding to this situation mentioned lies at a perpendicular distance r from all the charge densities. Then, at the equipotential surface, we have

$\Sigma V = \text{constant}$

$\Rightarrow \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_1}{r}\right)+\frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_2}{r}\right)-\frac{2\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_3}{r}\right)=\text{constant}$

$\Rightarrow \log_e\left(\frac{r_1 r_2}{r^2}\right)-2\log_e\left(\frac{r_3}{r}\right)=\text{constant}$

$\Rightarrow \log_e\left(\frac{r_1 r_2}{r^2}\right)+\log_e\left(\frac{r^2}{r_3^2}\right)=\text{constant}$

$\left\{\because \log_e x = -\log_e\left(\frac{1}{x}\right)\right\}$

$\Rightarrow \log_e\left(\frac{r_1 r_2}{r_3^2}\right)=\text{constant}$

$\left\{\because \log_e m+\log_e m=\log_e(mn)\right\}$

$\Rightarrow \frac{r_1 r_2}{r_3^2}=\text{constant}$

Test Your Concepts-IX (Based on Relation Between Electrostatic Field and Potential)

1. $V = 5x - 3x^2y + 2yz^2$

Since $\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$

$\Rightarrow \vec{E} = (-5+6xy)\hat{i}+(3x^2-2z^2)\hat{j}-4yz\hat{k}$

$\Rightarrow \vec{E}\Big|_{(1,\,0,\,-2)} = -5\hat{i}-5\hat{j}$

$\Rightarrow E = \sqrt{E_x^2+E_y^2+E_z^2}$

$E = \sqrt{(-5)^2+(-5)^2+0^2} = 5\sqrt{2}\ \text{NC}^{-1}$

2. Inside the sphere, $E_x = E_y = E_z = 0$.

Outside, $E_x = -\frac{\partial V}{\partial x}$

$\Rightarrow E_x = -\frac{\partial}{\partial x}\left[V_0 - E_o z + E_o a^3 z\left(x^2+y^2+z^2\right)^{-\frac{3}{2}}\right]$

$\Rightarrow E_x = -\left[E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}(2x)\right]$

$\Rightarrow E_x = 3E_0 a^3 xz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$

$E_y = -\frac{\partial V}{\partial y}$

$\Rightarrow E_y = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-\frac{3}{2}}\right)$

$\Rightarrow E_y = -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}2y$

$\Rightarrow E_y = 3E_0 a^3 yz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$

$E_z = -\frac{\partial V}{\partial z}$

$\Rightarrow E_z = E_0 + E_0 a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$

3. $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$

where, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x+3y-z) = 2$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x+3y-z) = 3$

$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x+3y-z) = -1$

$\Rightarrow \vec{E} = -2\hat{i}-3\hat{j}+\hat{k}$

4. $dV = -\vec{E}\cdot d\vec{\ell}$

$\int_B^A dV = -\int_{(0,\,2,\,4)}^{(2,1,\,0)}\left(x\hat{i}-2y\hat{j}+z\hat{k}\right)\cdot\left(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\right)$

$\Rightarrow V_A - V_B = -\int_{(0,\,2,\,4)}^{(2,\,1,\,0)}(xdx-2ydy+zdz)$

$\Rightarrow V_{AB} = -\left[\left(\frac{x^2}{2}-y^2+\frac{z^2}{2}\right)\Big|_{(0,\,2,\,4)}^{(2,\,1,\,0)}\right]$

$\Rightarrow V_{AB} = 3\ \text{V}$

5. (a) $dV = -\vec{E}\cdot d\vec{\ell}$

$\int_B^A dV = -\int_{(1,\,1,\,1)}^{(0,\,0,\,0)}\left(y\hat{i}+x\hat{j}\right)\cdot\left(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\right)$

CHAPTER 1

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} (ydx + xdy)$$

$$\Rightarrow \quad V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d(xy) \quad \{\text{as } y\,dx + xdy = d(xy)\}$$

$$\Rightarrow \quad V_{AB} = -\left[(xy)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1 \text{ V}$$

(b) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} (3x^2y\hat{i} + x^3\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

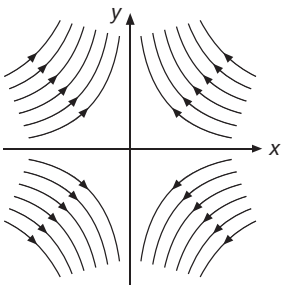
$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} (3x^2\,ydx + x^3\,dy)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} d(x^3y)$$

$$\Rightarrow \quad V_{AB} = -\left[(x^3y)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1 \text{ V}$$

Since in both the cases, the line integral of the field is an exact differential and hence both the fields are conservative in nature.

6. (a) Given, $\varphi = a(x^2 - y^2)$



(a)

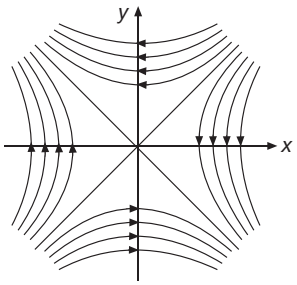
So, $\vec{E} = -\vec{\nabla}\varphi = -2a(x\hat{i} - y\hat{j})$

The sought shape of field lines is as shown in the figure.

(b) Since $\varphi = axy$

So, $\vec{E} = -\vec{\nabla}\varphi = -ay\hat{i} - ax\hat{j}$

The sought shape of field lines is as shown in the figure.



(b)

7. Given , $V = a(x^2 + y^2) + bz^2$

Since, $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow \quad \vec{E} = -(2ax\hat{i} + 2ay\hat{j} + 2bz\hat{k})$$

Hence $|\vec{E}| = 2\sqrt{a^2(x^2 + y^2) + b^2z^2}$

Shape of the equipotential surface i.e. for V = constant

Since, $V = a(x^2 + y^2) + bz^2$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{V}{a}\right)} + \frac{y^2}{\left(\frac{V}{a}\right)} + \frac{z^2}{\left(\frac{V}{b}\right)} = 1$$

$$\Rightarrow \quad \frac{x^2}{\left(\sqrt{\frac{V}{a}}\right)^2} + \frac{y^2}{\left(\sqrt{\frac{V}{a}}\right)^2} + \frac{z^2}{\left(\sqrt{\frac{V}{b}}\right)^2} = 1$$

which is an ellipsoid of revolution with semi-axis $\sqrt{\frac{V}{a}}, \sqrt{\frac{V}{a}}, \sqrt{\frac{V}{b}}$.

8. Given, again

$$dV = -\vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow \quad dV = -(ay\hat{i} + (ax + bz)\hat{j} + by\hat{k})\,(dx\hat{i} + dy\hat{j} + dx\hat{k})$$

$$\Rightarrow \quad dV = -a(ydx + axdy) + b(zdy + ydz)$$

$$\Rightarrow \quad dV = -[ad(xy) + bd(yz)]$$

Integrating we get,

$$V = -(axy + byz) + C$$

Test Your Concepts-X (Based on Motion of Charged Particles in Electric Field)

1. (a) $|\Delta V| = Ed = (6\times10^3 \text{ Vm}^{-1})(0.01 \text{ m}) = 60 \text{ V}$

(b) $\frac{1}{2}mv_f^2 = |q\Delta V|$

$$\Rightarrow \quad \frac{1}{2}(9.1\times10^{-31})v_f^2 = (1.6\times10^{-19})(60)$$

$$\Rightarrow \quad v_f = 4.6\times10^{6}\ \text{ms}^{-1}$$

2. (a) Energy of the proton-field system is conserved as the proton moves from higher to lower potential, which can be defined for this problem as moving from 120 V down to 0 V .

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$\Rightarrow \quad 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\Rightarrow \quad (1.6\times10^{-19})(120) = \frac{1}{2}(1.67\times10^{-27})v_p^2$$

$$\Rightarrow \quad v_p = 1.52\times10^{5}\ \text{ms}^{-1}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V . So,

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$\Rightarrow \quad 0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$\Rightarrow \quad 0 = \frac{1}{2}mv_e^2 + (-e)V$$

$$\Rightarrow \quad \frac{1}{2}(9.1\times10^{-31})v_e^2 = (1.6\times10^{-19})(120)$$

$$\Rightarrow \quad v_e = 6.5\times10^{6}\ \text{ms}^{-1}$$

3. (a) $$U = \frac{10^{-18}}{4\pi\varepsilon_0}\left[\frac{(20)(10)}{0.04} + \frac{(10)(-20)}{0.04} + \frac{(20)(-20)}{0.08}\right]$$

$$\Rightarrow \quad U = 9\times10^{-9}\times100\left[\frac{200}{4} - \frac{200}{4} - \frac{400}{8}\right]$$

$$\Rightarrow \quad U = 9\times10^{-9}\times100\times\left(-\frac{400}{8}\right)$$

$$\Rightarrow \quad U = -4.5\times10^{-5}\ \text{J}$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$V = V_1 + V_2 + V_3$$

$$\Rightarrow \quad V = (9\times10^{9})\left(\frac{20\times10^{-9}}{0.05} + \frac{10\times10^{-9}}{0.03} - \frac{20\times10^{-9}}{0.05}\right)$$

$$\Rightarrow \quad V = 3\times10^{3}\ \text{V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$\Rightarrow \quad 0 + (40\times10^{-9}\,\text{C})(3\times10^{3}\,\text{V}) = \frac{1}{2}(2\times10^{-13}\,\text{kg})v^2 + 0$$

$$\Rightarrow \quad v = \sqrt{\frac{2(1.2\times10^{-4}\ \text{J})}{2\times10^{-13}\ \text{kg}}} = 3.46\times10^{4}\ \text{ms}^{-1}$$

4. Using conservation of energy for the alpha particle-nucleus system,

we have $K_f + U_f = K_i + U_i$.

where $U_i = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{gold}}}{r_i}$

Since $r_i \to \infty$

$$\Rightarrow \quad U_i = 0$$

also $K_f = 0$ because, at turning point, we have $v_f = 0$

so we get, $U_f = K_i$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{gold}}}{r_{\min}} = \frac{1}{2}m_\alpha v_\alpha^2$$

$$\Rightarrow \quad r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2}$$

$$\Rightarrow \quad r_{\min} = \frac{2(9\times10^{9})(2)(79)(1.6\times10^{-19})^2}{(6.67\times10^{-27})(2\times10^{7})^2}$$

$$\Rightarrow \quad r_{\min} = 2.73\times10^{-14}\ \text{m} = 27.3\ \text{fm}$$

5. Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$\Rightarrow \quad 0 + \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L} + \frac{2q^2}{\sqrt{2}L}\right) = 4\left(\frac{1}{2}mv^2\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{2L} + \frac{2q^2}{2\sqrt{2}L}\right)$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{L} = 2mv^2$$

$$\Rightarrow \quad v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mL}}$$

6. The potential created by the ring at the electron's starting point is

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_i^2 + a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$$

CHAPTER 1

while at the center, it is $V_f = \frac{1}{4\pi\varepsilon_0}(2\pi\lambda)$

From Law of Conservation of Energy, we have

$$(U+K)_i = (U+K)_f$$

$$\Rightarrow \quad -eV_i + 0 = -eV_f + \frac{1}{2}m_e v_f^2$$

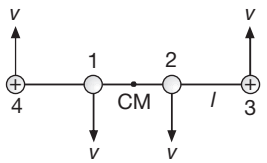
$$\Rightarrow \quad v_f^2 = \frac{2e}{m_e}\left(V_f - V_i\right) = \frac{1}{4\pi\varepsilon_0}\frac{4\pi e\lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$$

$$\Rightarrow \quad v_f^2 = \frac{4\pi(1.6\times10^{-19})(9\times10^{9})(1\times10^{-7})}{9.1\times10^{-31}} \times \left(1 - \frac{0.2}{\sqrt{(0.1)^2 + (0.2)^2}}\right)$$

$$\Rightarrow \quad v_f = 1.45\times10^{7}\ \text{ms}^{-1}$$

7. Initially, since no external horizontal forces act on the set of four balls, so the center of mass of the system stays fixed at the initial location of the center of the square. As the charged balls 4 and 3 swing out and away from each other, balls 1 and 2 move down with equal *y*-components of velocity. The maximum kinetic energy point is illustrated. For system energy to be conserved,

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{3\ell} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$



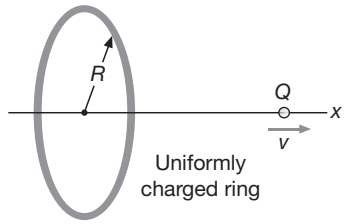
$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3\ell} = 2mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{3m\ell}}$$

8. From Law of Conservation of Energy,

$$K_f + U_f = K_i + U_i$$

$$\Rightarrow \quad \frac{1}{2}Mv_f^2 + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}$$



$$\Rightarrow \quad v_f = \sqrt{\frac{Q^2}{2\pi\varepsilon_0 MR}}$$

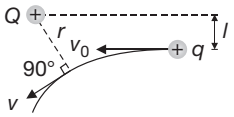
9. By Law of Conservation of Energy,

$$(U+K)_{\substack{\text{initial for}\\(A+B)\text{ system}}} = (U+K)_{\substack{\text{final for}\\(A+B)\text{ system}}}$$

$$\Rightarrow \quad mg\ell + \frac{q^2}{4\pi\varepsilon_0\sqrt{2}\ell} + 0 + 0 = \frac{q^2}{4\pi\varepsilon_0(2\ell - \ell)} + \frac{1}{2}mu^2 + K_A$$

$$\Rightarrow \quad K_A = mg\ell - \frac{1}{2}mu^2 + \frac{q^2}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}\ell}\left(1 - \sqrt{2}\right)$$

10. Let $r_{\min}$ be the distance of closest approach between the two charges. Then, from Conservation of Angular Momentum, we get



$$mv_0\ell = mvr$$

$$\Rightarrow \quad v_0\ell = vr \qquad \ldots(1)$$

From Law of Conservation of Energy,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$r_{\min} = \frac{Qq}{4\pi\varepsilon_0 mv_0^2} + \sqrt{\left(\frac{Qq}{4\pi\varepsilon_0 mv_0^2}\right)^2 + \ell^2}$$

11. Since $|E| = \frac{V}{\ell} = \frac{at}{\ell}$

$$\Rightarrow \quad m\frac{dv}{dt} = \left(\frac{at}{\ell}\right)e$$

$$\Rightarrow \quad \int_0^v dv = \frac{ae}{m\ell}\int_0^t t\,dt$$

$$\Rightarrow \quad v = \left(\frac{ae}{2m\ell}\right)t^2 \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dx}{dt} = \left(\frac{ae}{2m\ell}\right)t$$

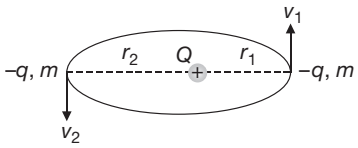
$$\Rightarrow \quad x\Big|_0^{\ell} = \frac{ae}{2m\ell}\left(\frac{t^3}{3}\right)\Big|_0^{t}$$

$$\Rightarrow \quad \ell = \left(\frac{ae}{6m\ell}\right)t^3 \qquad \ldots(2)$$

Substituting value of t from (2) in (1), we get

$$v = \left(\frac{ae}{2m\ell}\right)\left(\frac{6m\ell^2}{ae}\right)^{\frac{2}{3}}$$

12. Using Law of Conservation of Angular Momentum about $+Q$, we get



$$mv_1r_1 = mv_2r_2$$

$$\Rightarrow \quad v_1r_1 = v_2r_2 \qquad \ldots(1)$$

From Law of Conservation of Energy,

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} = \frac{1}{2}mv_2^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2} \qquad \ldots(2)$$

On solving equations (1) and (2), we get

$$v_1 = \sqrt{\frac{Qqr_2}{2\pi\varepsilon_0 mr_1(r_1+r_2)}} \text{ and } v_2 = \sqrt{\frac{Qqr_1}{2\pi\varepsilon_0 mr_2(r_1+r_2)}}$$

13. Since, no external force acts on the system of protons, so by Law of Conservation of Linear Momentum, we get

$$mv_0 = 2mv$$

$$\Rightarrow \quad v = \frac{v_0}{2} \qquad \ldots(1)$$

At the closest distance of approach, both the protons have same velocity, so

By Law of Conservation of Energy, we get

$$\frac{1}{2}mv_0^2 = 2\left[\frac{1}{2}mv^2\right] + \frac{e^2}{4\pi\varepsilon_0 r_{\min}} \qquad \ldots(2)$$

On solving equations (1) and (2) for $r_{\min}$, we get

$$r_{\min} = \frac{e^2}{\pi\varepsilon_0 mv_0^2}$$

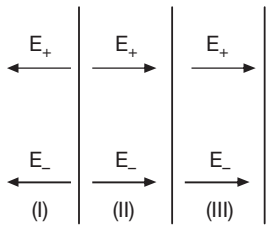
Test Your Concepts-XI (Based on Conductors)

1. The charge divides equally between the identical spheres, with charge $\frac{Q}{2}$ on each. Then they repel like point charges at their centers so, we have

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{(L+R+R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4(L+2R)^2}$$

$$F = \frac{9\times10^9\times(60\times10^{-6})^2}{4\times(2.01)^2} \cong 2\text{ N}$$

2. Since $E = \frac{\sigma}{2\varepsilon_0}$



Let us designate left to right as positive direction. Then

$$E_I = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

$$E_{II} = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

$$E_{III} = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

For $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$, we get

$$E_I = E_{III} = 0 \text{ and } E_{II} = \frac{\sigma}{\varepsilon_0}$$

3. Consider as a Gaussian surface a box with horizontal area A, lying between 500 m and 600 m elevation.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad (+120\text{ NC}^{-1})A + (-100\text{ NC}^{-1})A = \frac{\rho A(100\text{ m})}{\varepsilon_0}$$

$$\Rightarrow \quad \rho = \frac{(20\text{ NC}^{-1})(8.85\times10^{-12}\text{ C}^2\text{Nm}^{-2})}{100\text{ m}}$$

$$\rho = 1.77\times10^{-12}\text{ Cm}^{-3} = 1.77\text{ pCm}^{-3}$$

The charge has to be positive so that it produces the net outward flux of electric field.

CHAPTER 1

4. The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad \sigma = E\varepsilon_0$$

(a) At the point on the surface where the radius of curvature is the greatest, we have

$$\sigma = \varepsilon_0 E_{\min} = (8.85\times10^{-12}\,\text{C}^2\text{Nm}^{-2})(5.6\times10^{4}\,\text{NC}^{-1})$$

$$\Rightarrow \quad \sigma = 496\ \text{nCm}^{-2}$$

(b) At the point on the surface where the radius of curvature is the smallest, we have

$$\sigma = \varepsilon_0 E_{\max} = (8.85\times10^{-12}\,\text{C}^2\text{Nm}^{-2})(11.2\times10^{4}\,\text{NC}^{-1})$$

$$\Rightarrow \quad \sigma = 992\ \text{nCm}^{-2}$$

5. (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is given by

$$\sigma_a = \frac{-q}{4\pi a^2}$$

(b) The outer surface carries charge $Q+q$ with density

$$\sigma_b = \frac{Q+q}{4\pi b^2}$$

6. Charge on the outer surface of the sphere is $-Q$

Charge on the inner surface of the shell is $+Q$

Charge on the outer surface of the shell is $+2Q$

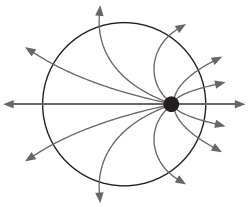
Using Gauss's Law we evaluate the electric field in each region, keeping in mind that the electric field is zero everywhere within conducting materials. Then

Inside the sphere and within the material of the shell, we have $E = 0$

Between the sphere and shell the field is directed radially inwards given by $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

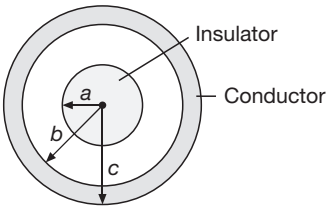
Outside the shell the field is directed radially outwards given by $E = \dfrac{2Q}{4\pi\varepsilon_0 r^2}$

7. An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



8. (a) $\oint \vec{E}\cdot d\vec{A} = E(4\pi r^2) = \dfrac{q_{\text{enc}}}{\varepsilon_0}$

For $r < a$, $q_{\text{enc}} = \rho\left(\dfrac{4}{3}\pi r^3\right)$



$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

For $a < r < b$ and $c < r$, $q_{\text{enc}} = Q$

$$\Rightarrow \quad E = \frac{Q}{4\pi r^2\varepsilon_0}$$

For $b \le r \le c$, $E = 0$ (since $E = 0$ inside a conductor).

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$

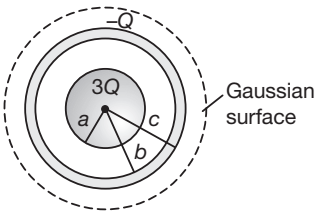
$$\Rightarrow \quad \sigma_1 = \frac{q_1}{4\pi b^2} = \frac{-Q}{4\pi b^2}$$

Let q_2 = induced charge on the outside surface of the hollow sphere. since the hollow sphere is uncharged, we require

$$q_1 + q_2 = 0$$

$$\Rightarrow \quad \sigma_2 = \frac{q_1}{4\pi c^2} = \frac{Q}{4\pi c^2}$$

9. (a) $q_{enc} = +3Q - Q = +2Q$



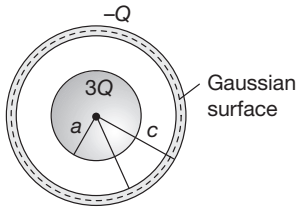
(b) The charge distribution is spherically symmetric and $q_{enc} > 0$. Thus, the field is directed radially outward.

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{2Q}{4\pi\varepsilon_0 r^2} \text{ for } r \geq c.$$

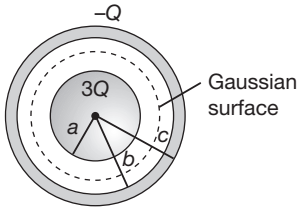
(c) Since all points within this region are located inside conducting material, $E = 0$ for $b < r < c$.

(d) $\phi_E = \int E \cdot dA = 0$

$\Rightarrow \quad q_{enc} = \varepsilon_0 \phi_E = 0$

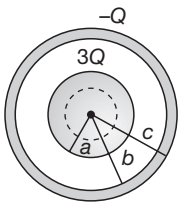


(e) $q_{enc} = +3Q$



(f) $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{3Q}{4\pi\varepsilon_0 r^2}$ (radially outward) for $a \leq r < b$

(g) $q_{enc} = \rho V = \left(\frac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = +3Q\left(\frac{r^3}{a^3}\right)$



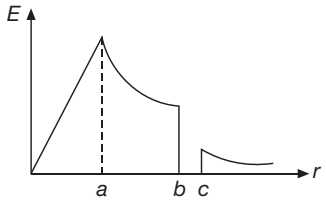
(h) $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{1}{4\pi\varepsilon_0 r^2}\left(+3Q\frac{r^3}{a^3}\right) = \left(\frac{3Q}{4\pi\varepsilon_0 a^3}\right)r$

(radially outwards) for $0 \leq r \leq a$

(i) From part (c), $E = 0$ for $b < r < c$. Thus, for a spherical Gaussian surface with $b < r < c$, we have $q_{enc} = +3Q + q_{inner} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{inner} = -3Q$

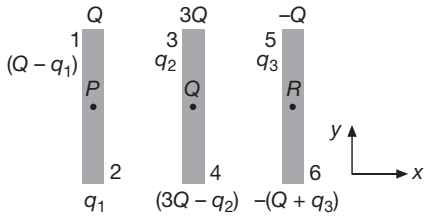
(j) Since the total charge on the conducting shell is $q_{net} = q_{outer} + q_{inner} = -Q$, we have $q_{outer} = -Q - q_{inner} = -Q - (-3Q) = +2Q$

(k) This is shown in the figure.



Test Your Concepts-XII (Based on Charge Distribution)

1. We assume that the charges on surfaces 2, 3 and 5 are q_1, q_2 and q_3 in equilibrium. Following conservation of charge, we see that surfaces 1, 4 and 6 have charges $-Q - q_1$, $3Q - q_2$ and $Q - q_3$ respectively. The electric field inside a metallic plate is zero and hence the fields at points P, Q and R are zero.



Resultant field at P: Considering field to the right as positive, we get

$$E_P = 0$$

$$\frac{1}{2\varepsilon_0 A}[(Q-q_1) - q_1 - q_2 - (3Q-q_2) - q_3 + (Q+q_3)] = 0$$

$\Rightarrow \quad -Q - 2q_1 = 0$

$\Rightarrow \quad q_1 = -\frac{Q}{2}$

Resultant field at Q is $E_Q = 0$

$$\frac{1}{2\varepsilon_0 A}[(Q-q_1) + q_1 + q_2 - (3Q-q_2) - q_3 + (Q+q_3)] = 0$$

$\Rightarrow \quad -Q + 2q_2 = 0$

$\Rightarrow \quad q_2 = +\frac{Q}{2}$

Resultant field at R:

$$E_R = 0$$

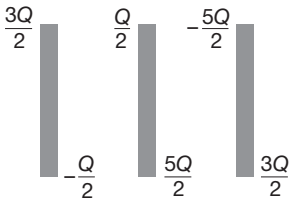
$$\Rightarrow \quad \frac{1}{2\varepsilon_0 A}[(Q-q_1) + q_1 + q_2 + (3Q-q_2) + q_3 + (Q+q_3)] = 0$$

CHAPTER 1

$\Rightarrow \quad 5Q + 2q_3 = 0$

$\Rightarrow \quad q_3 = -\dfrac{5Q}{2}$

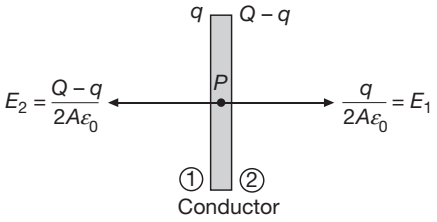
Thus the final charge distribution on all the faces is:



Also note that the faces opposite to each other have opposite charges equal in magnitude.

2. Let a charge q appears on left side of sheet, so a charge $(Q-q)$ will appear on the right side of sheet. Since field at a point P (say) inside the sheet is zero, so

$E_P = 0$



$\Rightarrow \quad \dfrac{q}{2A\varepsilon_o} - \dfrac{Q-q}{2A\varepsilon_o} = 0$

$\Rightarrow \quad \dfrac{2q}{2A\varepsilon_o} = \dfrac{Q}{2A\varepsilon_o}$

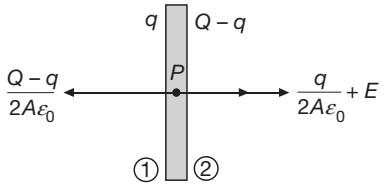
$\Rightarrow \quad q = \dfrac{Q}{2}$ and $(Q-q) = \dfrac{Q}{2}$

So charge is equally distributed on both sides.

3. Let charge on left side of plate be q, then $(Q-q)$ is charge on right side of plate.

Since inside the plate, at point P, we have

$E_P = 0$



$\Rightarrow \quad \dfrac{q}{2A\varepsilon_0} + E = \dfrac{Q-q}{2A\varepsilon_0}$

$\Rightarrow \quad \dfrac{q}{A\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0} - E$

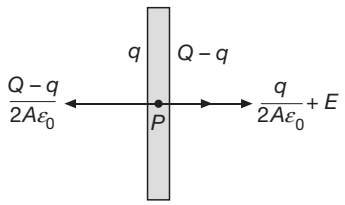
$\Rightarrow \quad q = \dfrac{Q}{2} - \varepsilon_o AE$ and $(Q-q) = \dfrac{Q}{2} + \varepsilon_o AE$

So charge on one side is $\left(\dfrac{Q}{2} - \varepsilon_0 AE\right)$ and other side $\left(\dfrac{Q}{2} + \varepsilon_0 AE\right)$

If sheet is neutral, then $Q = 0$, so charges $-\varepsilon_0 AE$ and $+\varepsilon_0 AE$ appear on left and right side of sheet.

4. Let a charge q appear on left side of plate, so a charge $(Q-q)$ appears on right side of plate.

$E_P = 0$



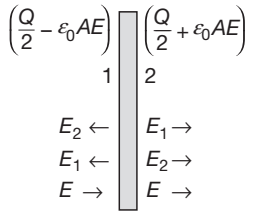
$\Rightarrow \quad \dfrac{q}{2A\varepsilon_0} + E = \dfrac{Q-q}{2A\varepsilon_0}$

$\Rightarrow \quad \dfrac{q}{A\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0} - E$

$\Rightarrow \quad q = \dfrac{Q}{2} - \varepsilon_0 AE$ and $Q - q = \dfrac{Q}{2} + \varepsilon_0 AE$

So charge on one side is $\dfrac{Q}{2} - \varepsilon_0 AE$ and other side $\dfrac{Q}{2} + \varepsilon_0 AE$

Electric field due to conducting sheet at a point outside the sheet $= \dfrac{Q_{net}}{2A\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0}$ and having direction away from plate



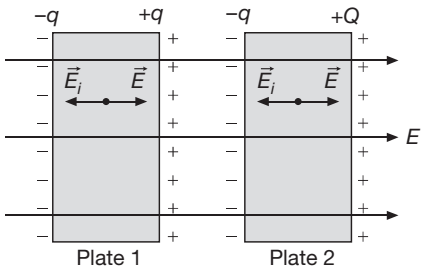
If E_1 and E_2 be electric field due to charges on left and right side of the plate respectively, then

$$E_{\text{left}} = E - E_1 - E_2 = E - \frac{\frac{Q}{2} - \varepsilon_0 AE + \frac{Q}{2} + \varepsilon_0 AE}{2A\varepsilon_0}$$

$$\Rightarrow \quad E_{\text{left}} = E - \frac{Q}{2A\varepsilon_0} \qquad \text{(towards right)}$$

and $E_{\text{right}} = E + E_1 + E_2 = E + \frac{Q}{2A\varepsilon_0}$ (towards right)

5. Since plates are conducting so net electric field inside these plates should be zero. So, electric field due to induced charges (on the surface of the plate) must balance the outside electric field. The magnified view of plates 1 and 2 is shown below:



Let $\vec{E}_i$ be the induced electric field, then

for both plates, inside than $\vec{E}_i + \vec{E} = 0$

$$\Rightarrow \quad \vec{E}_i = -\vec{E} \qquad \ldots(1)$$

Let charge induced on surfaces be $+q$ and $-q$, then

$$\left|\vec{E}_i\right| = \frac{Q}{A\varepsilon_0}$$

Using equation (1), we get

$$\frac{q}{A\varepsilon_0} = E$$

$$\Rightarrow \quad q = AE\varepsilon_0$$

6. Electric field near a large metallic plate is given by $E = \frac{\sigma}{\varepsilon_o}$. In between the plates the two fields will be in opposite direction. Hence

$$E_{\text{net}} = \frac{\sigma_1 - \sigma_2}{\varepsilon_o} = E_o \,(\text{say})$$

Now, $W = (q)(\text{potential difference}) = q(E_o a \cos 45°)$

$$\Rightarrow \quad W = q\left(\frac{\sigma_1 - \sigma_2}{\varepsilon_o}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{(\sigma_1 - \sigma_2)qa}{\sqrt{2}\,\varepsilon_o}$$

Test Your Concepts-XIII (Based on Concept of Self and Interaction Energy)

1. The initial self potential energy of the spherical shell is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1}$$

The final self potential energy of the spherical shell is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

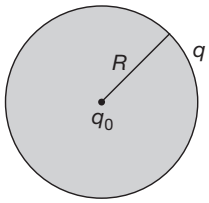
$$\Rightarrow \quad \Delta U = U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

Work done by electrical forces (conservative forces) is given by

$$W = -\Delta U = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

2. The electrical potential energy of the system is :

$$U = \begin{pmatrix}\text{Self potential}\\ \text{energy of shell}\end{pmatrix} + \begin{pmatrix}\text{Interaction energy of}\\ \text{shell and point charge}\end{pmatrix}$$



$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0 R} + \frac{qq_0}{4\pi\varepsilon_0 R}$$

Initial potential energy of system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

Final potential energy of system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{4\pi\varepsilon_0 R_2}$$

Since, work done by a conservative force (electrostatic force) is actually equal to the decrease in electrostatic potential energy. So,

$$W = -\Delta U = -\left(U_f - U_i\right) = U_i - U_f \qquad \ldots(1)$$

On substituting the values of U_i and U_f in equation (1), we get

$$W = \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

CHAPTER 1

3. Electrical field is conservative in nature. For a conservative force,

$$F = \frac{-\partial U}{\partial r}$$

Since, $U = \frac{q^2}{8\pi\varepsilon_0 r}$

$$\Rightarrow \quad F = -\frac{\partial U}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{q^2}{8\pi\varepsilon_0 r}\right) \qquad \left\{\because F = -\frac{\partial U}{\partial r}\right\}$$

$$\Rightarrow \quad F = -\frac{q}{8\pi\varepsilon_0}\frac{\partial}{\partial r}\left(r^{-1}\right) = -\frac{q^2}{8\pi\varepsilon_0}\left(-\frac{1}{r^2}\right) = \frac{q^2}{8\pi\varepsilon_0 r^2}$$

Force per unit area is

$$P = \frac{F}{4\pi r^2} = \frac{q^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2}$$

Since $q = \sigma A = \sigma\left(4\pi r^2\right)$

$$\Rightarrow \quad P = \frac{\sigma^2\left(4\pi r^2\right)^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2} = \frac{\sigma^2}{2\varepsilon_0}$$

4. $W = \int_0^Q V dq$, where $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2R}$$

Single Correct Choice Type Questions

1. By Law of Conservation of Moments about O,

$$\begin{pmatrix}\text{Total Clockwise}\\(\text{CW})\text{ Moments}\end{pmatrix} = \begin{pmatrix}\text{Total Counter}\\\text{Clockwise (CCW)}\\\text{Moments}\end{pmatrix}$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right) + W\left(x - \frac{L}{2}\right) = \frac{2qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right)$$

$$\Rightarrow \quad W\left(x - \frac{L}{2}\right) = \frac{QqL}{8\pi\varepsilon_0 h^2} \quad \Rightarrow \quad x = \frac{QqL + 4\pi\varepsilon_0 h^2 LW}{8\pi\varepsilon_0 h^2 W}$$

Hence, the correct answer is (D).

3. The transfer of electrons from one body to the other produces charge on the body

Hence,

$$\begin{pmatrix}\text{number of electrons}\\\text{given by one body}\end{pmatrix} = \begin{pmatrix}\text{number of electrons}\\\text{obtained by the other}\end{pmatrix}$$

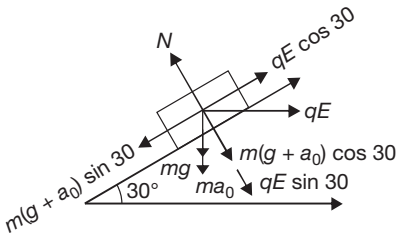
So, mass of negatively charged body slightly increases while mass of positively charged body slightly increases However, the total mass of the system remains the same.

Hence, the correct answer is (D).

4. Net acc of block down the incline is

$$a = \frac{\left(g + a_0\right)}{2} - \frac{\sqrt{3}qE}{2m}$$

$$\Rightarrow \quad a = \frac{1}{2}\left(g + a_0 - \frac{\sqrt{3}qE}{m}\right)$$



Now $\frac{h}{\sin 30} = \frac{1}{2}at^2$

$$\Rightarrow \quad 2h = \frac{1}{4}\left[g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)\right]t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{8h}{g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)}}$$

$$\Rightarrow \quad t = 2\sqrt{\frac{2h}{g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)}}$$

Hence, the correct answer is (C).

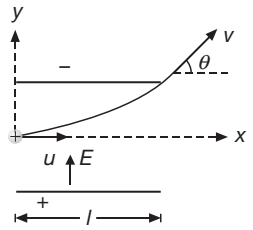
5. $a = \frac{eE}{m}$

Since $\ell = \frac{1}{2}at^2 = \frac{1}{2}a'T^2$

$$\Rightarrow \quad \frac{T}{t} = \sqrt{\frac{a}{a'}} = \sqrt{\frac{qE/m}{qE/M}} = \sqrt{\frac{M}{m}}$$

Hence, the correct answer is (C).

6. $\frac{1}{2}mu^2 = eV$



$$\Rightarrow \quad u = u_x = \sqrt{\frac{2eV}{m}}$$

$$t = \frac{\ell}{u_x} = \ell\sqrt{\frac{m}{2eV}}$$

Since, $a_y = \dfrac{eE}{m}$

$\Rightarrow \quad \dfrac{dv_y}{dt} = \dfrac{eat}{m} \qquad \{\because E = at\}$

$\Rightarrow \quad \displaystyle\int_0^{v_y} dv_y = \frac{ea}{m}\int_0^t t\,dt$

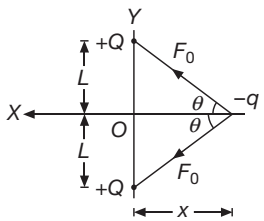
$\Rightarrow \quad v_y = \dfrac{ea}{2m}t^2 = \left(\dfrac{ea}{2m}\right)\left(\dfrac{\ell^2 m}{2eV}\right) = \dfrac{a\ell^2}{4V}$

Since $\tan\theta = \dfrac{v_y}{v_x}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left\{\dfrac{a\ell^2}{4V}\sqrt{\dfrac{m}{2eV}}\right\}$

Hence, the correct answer is (C).

7. The net force F on the particle towards O is the cause of acceleration a.



$F = 2F_0\cos\theta$

$F = \dfrac{2}{4\pi\varepsilon_0}\dfrac{qQ}{(x^2+\ell^2)}\dfrac{x}{\sqrt{x^2+\ell^2}} = \dfrac{Qq}{2\pi\varepsilon_0}\dfrac{x}{(x^2+\ell^2)^{3/2}}$

We observe that F is zero for large values of x and also for $x = 0$.

So, a must increase to a maximum and fall to zero at O

To the right of O, the net force is to the left while motion is to the right. Thus, the direction of a is opposite to the particle's direction of motion and is taken as negative as shown.

Hence, the correct answer is (D).

8. $-\displaystyle\int_{\ell=\infty}^{\ell=0} \vec{E}\cdot d\vec{\ell}$ is the line integral of electric field which is also called electrostatic potential. Since, charge is distributed non-uniformly, so $E \neq 0$ at the centre.

$\Rightarrow \quad -\displaystyle\int_{\ell=\infty}^{\ell=0} \vec{E}\cdot d\vec{\ell} = \frac{q}{4\pi\varepsilon_0 R} = 2$

Hence, the correct answer is (A).

9. If charges were placed at all the corners, the field at the centre would be zero. Hence, the field at the centre due to any one charge is equal (and opposite) to the field due to all the other $(n-1)$ charges.

Hence, the correct answer is (A).

10. Since the potential function is not defined for an infinite conducting sheet, hence to solve this problem we either calculate potential difference or use force equations
Electric field due to an infinite dielectric sheet

$E_1 = \dfrac{\sigma}{2\varepsilon_0}$

Electric field at the axis of a disc of radius R is

$E_2 = \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{x}{\sqrt{x^2+R^2}}\right]$

The resultant electric field is given by

$E = E_1 - E_2 = \dfrac{\sigma}{2\varepsilon_0}\dfrac{x}{\sqrt{x^2+R^2}}$

Force on an electron is

$F = -eE \quad \Rightarrow \quad F = -\dfrac{\sigma ex}{2\varepsilon_0\sqrt{x^2+R^2}}$

$\Rightarrow \quad mv\dfrac{dv}{dx} = -\dfrac{\sigma ex}{2\varepsilon_0\sqrt{x^2+R^2}}$

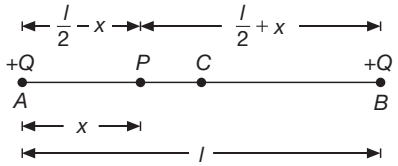
$\Rightarrow \quad m\displaystyle\int_0^v v\,dv = -\frac{\sigma e}{2\varepsilon_0}\int_{\sqrt{3}R}^0 \frac{x}{\sqrt{x^2+R^2}}dx$

$\Rightarrow \quad \dfrac{mv^2}{2} = -\dfrac{\sigma e}{2\varepsilon_0}\left(\sqrt{x^2+R^2}\right)\Big|_{\sqrt{3}R}^0$

$\Rightarrow \quad v = \sqrt{\dfrac{\sigma eR}{m\varepsilon_0}}$

Hence, the correct answer is (B).

11. $E = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{\left(\frac{\ell}{2}-x\right)^2} - \dfrac{1}{\left(\frac{\ell}{2}+x\right)^2}\right]$



The magnitude of E will increase sharply for $x \to 0$ and $x \to \ell$.

Hence, the correct answer is (B).

CHAPTER 1

12. $V=\dfrac{q}{4\pi\varepsilon_0 x_0}\left(1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}.....\right)$

$\Rightarrow \quad V=\dfrac{q}{4\pi\varepsilon_0 x_0}\log_e 2$

Hence, the correct answer is (D).

13. Charge density $\rho=\dfrac{Q}{\dfrac{4}{3}\pi\left(R^3-\left(\dfrac{R}{2}\right)^3\right)}=\dfrac{6Q}{7\pi R^3}$

Using the Superposition Principle,

$$V=\frac{\rho\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 x}-\frac{\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\varepsilon_0\left(x-\frac{R}{2}\right)}=\frac{\rho R^3}{3\varepsilon_0}\frac{(7x-4R)}{4x(2x-R)}$$

At $x=2R$, we get $V=\dfrac{5}{36}\left(\dfrac{\rho R^2}{\varepsilon_0}\right)$

Similarly, $E=\dfrac{\rho\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 x^2}-\dfrac{\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\varepsilon_0\left(x-\frac{R}{2}\right)^2}$

$$E=\frac{\rho R^3}{3\varepsilon_0}\left(\frac{1}{x^2}-\frac{1}{2(2x-R)^2}\right)$$

So, at $x=2R$, we get $E=\dfrac{2\rho R}{27\varepsilon_0}=\dfrac{2}{27}\left(\dfrac{\rho R}{\varepsilon_0}\right)$

Hence, the correct answer is (B).

14. $d=\dfrac{u^2\sin^2\theta}{2a}$ and $\ell=\dfrac{2u^2\sin\theta\cos\theta}{a}$

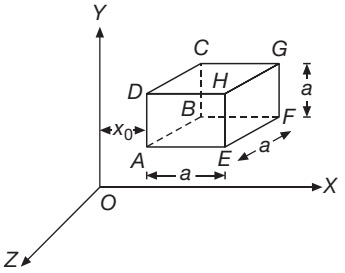
$\Rightarrow \quad \dfrac{d}{\ell}=\dfrac{\tan\theta}{4}$

$\Rightarrow \quad \tan\theta=\dfrac{4d}{\ell}$

Hence, the correct answer is (B).

15. The field at the face $ABCD=\alpha x_0\hat{i}$

So, flux over the face $ABCD=-(\alpha x_0)a^2$



The negative sign arises as the field is directed into the cube

The field at the face $EFGH=\alpha(x_0+a)\hat{i}$

So, flux over the face $EFGH=\alpha(x_0+a)a^2$

The flux over the other four faces is zero as the field is parallel to the surfaces

So, total flux over the cube $=\alpha a^3=\dfrac{q}{\varepsilon_0}$, where q is the total charge inside the cube

$\Rightarrow \quad q=\varepsilon_0\alpha a^3$

Hence, the correct answer is (A).

16. Since, $\vec{E}=-\vec{\nabla}V \quad \Rightarrow \quad \vec{E}=-k\left(y\hat{i}+x\hat{j}\right)$

Further,

$$dW=q_0\vec{E}\cdot d\vec{r}$$

$$\Rightarrow \quad dW=q_0\left[-k\left(y\hat{i}+x\hat{j}\right)\cdot\left(\hat{i}\,dx+\hat{j}\,dy\right)\right]$$

$$\Rightarrow \quad dW=-kq_0(y\,dx+x\,dy)=-kq_0\,d(xy)$$

$$\Rightarrow \quad W=\int dW=-kq_0\int_{(0,\,0)}^{(a,\,a)}d(xy)$$

$$\Rightarrow \quad W=-kq_0a^2$$

Hence, the correct answer is (A).

17. Since, $V=\dfrac{Q}{8\pi\varepsilon_0 R^3}(3R^2-r^2)$

So, at $r=0$, potential at the centre is

$$V_C=\frac{3Q}{8\pi\varepsilon_0 R}$$

As per the question we require a point where

$$V=\frac{V_C}{2}=\frac{3Q}{16\pi\varepsilon_0 R}$$

This point cannot lie inside the sphere where $V\geq\dfrac{Q}{4\pi\varepsilon_0 R}$

Let the point lie outside the sphere, at a distance $r+R$ from the centre. Then,

$$V=\frac{Q}{4\pi\varepsilon_0(R+r)}=\frac{3Q}{16\pi\varepsilon_0 R}$$

$$\Rightarrow \quad R+r=\frac{4}{3}R$$

$$\Rightarrow \quad r=\frac{4R}{3}-R=\frac{R}{3}$$

Distance from the surface is $\frac{R}{3}$

Hence, the correct answer is (D).

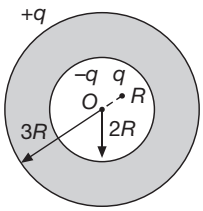
18. $V_f - V_i = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r_f}{r_i}\right)$

$\Rightarrow \quad V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{\sqrt{5^2+12^2}}{\sqrt{1^2+2^2+4^2}}\right)$

$\Rightarrow \quad V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{13}{5}\right)$

Hence, the correct answer is (A).

19. Induced charges will be $-q$ and $+q$ as shown in figure



Potential at centre O is

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{2R} + \frac{q}{3R}\right]$$

$$V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{5q}{6R}\right) = \frac{5q}{24\pi\varepsilon_0 R}$$

Hence, the correct answer is (D).

20. A negative charge, when free to move will always move from lower to higher potential.

Hence, the correct answer is (D).

21. $V_1 = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0\sqrt{2}a}$

$$V_2 = \frac{Q_1}{4\pi\varepsilon_0\sqrt{2}a} + \frac{Q_2}{4\pi\varepsilon_0 a}$$

$\Rightarrow \quad W_{1\to 2} = q(V_2 - V_1)$

$\Rightarrow \quad W_{1\to 2} = \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad W = \frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{4\sqrt{2}\pi\varepsilon_0 a}$

Hence, the correct answer is (B).

22. Charges will be induced as shown

$$V_0 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{1}{2R} - \frac{1}{R}\right]$$

$$V_0 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2R}\right]$$

Hence, the correct answer is (D).

23. For the motion of the electron along Y-axis

$$d = \frac{1}{2}\frac{eE}{m}t^2 \qquad \ldots(1)$$

where E is the electric field due to the charged plate given by

$$E = \frac{\sigma}{\varepsilon_0} \qquad \ldots(2)$$

where σ is the surface density of charge of the plate.

The time taken by the electron is

$$t = \frac{\ell}{u} \qquad \ldots(3)$$

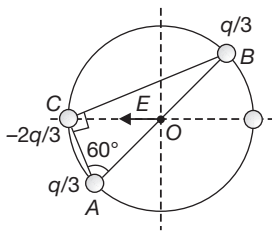
From equations (1), (2) and (3), we obtain

$$d = \frac{1}{2}\frac{e\sigma}{\varepsilon_0 m}\left(\frac{\ell}{u}\right)^2$$

$\Rightarrow \quad \sigma = \frac{2d\varepsilon_0 m u^2}{e\ell^2}$

Hence, the correct answer is (C).

24. Net electric field due to both charges $\frac{q}{3}$, will get cancelled. Electric field due to $\left(-\frac{2q}{3}\right)$ will be directed in –ve axis



$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)}{R^2}$$

$\Rightarrow \quad E = \frac{q}{6\pi\varepsilon_0 R^2}$

$$\text{P.E. of system} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\left(\frac{q}{3}\right)^2}{2R} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60°} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60°}\right)$$

P.E. of system $\neq 0$

Force between B and C

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60°)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\times 2q^2}{9\times 4\times 3R^2}$$

$$\Rightarrow \quad F = \frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2} \qquad \text{(attractive)}$$

$$\Rightarrow \quad F = \frac{1}{54}\frac{q^2}{\pi\varepsilon_0 R^2}$$

Potential at O is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}+\frac{q}{3}-\frac{2q}{3}\right)}{R} = 0$$

Hence, the correct answer is (C).

25. Since both the particles exert equal force on each other, so the angle of deflection for both must be the same i.e. $\alpha = \beta$.

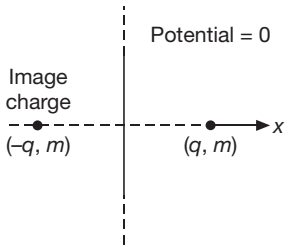
Hence, the correct answer is (C).

27. According to the METHOD OF IMAGES, we place a mirror charge $-q$ of mass m at $x = -d$.

The two charges will move towards each other till they collide i.e., until the charge q reaches the plane. Let at any instant the charge q be at a distance $x(t)$ from the plane. By Law of Conservation of Energy we have

$$\begin{pmatrix}\text{Energy at}\\ t=0 \text{ in}\\ \text{c.g.s. system}\end{pmatrix} = \begin{pmatrix}\text{Energy at any}\\ \text{instant } t \text{ in}\\ \text{c.g.s system}\end{pmatrix}$$

$$\frac{-q^2}{(d+d)} = \frac{-q^2}{2x(t)} + 2\left[\frac{1}{2}m\{\dot{x}(t)\}^2\right]$$



$$\Rightarrow \quad \frac{dx}{dt} = \frac{q}{\sqrt{2md}}\sqrt{\frac{d-x}{x}}$$

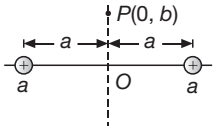
$$\Rightarrow \quad dt = \frac{\sqrt{2md}}{q}\sqrt{\frac{x}{d-x}}dx$$

$$\Rightarrow \quad t = \int dt = \frac{\sqrt{2md}}{q}\int_d^0 \sqrt{\frac{x}{d-x}}dx$$

$$\Rightarrow \quad t = \frac{2}{3q}\sqrt{2md^3}$$

Hence, the correct answer is (A).

28.



$$OP = \sqrt{a^2 + \left(\sqrt{3}a\right)^2} = 2a$$

By Law of Conservation of Energy

$$(U+k)_P = (U+K)_O$$

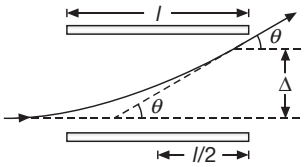
$$\Rightarrow \quad -\frac{2Qq}{4\pi\varepsilon_0(2a)} + 0 = -\frac{2Qq}{4\pi\varepsilon_0 a} + \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{Qq}{4\pi\varepsilon_0 a}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 ma}}$$

Hence, the correct answer is (B).

29. $\tan\theta = \dfrac{\Delta}{\left(\frac{\ell}{2}\right)} = \dfrac{2\Delta}{\ell}$



Hence, the correct answer is (B).

30. In ΔABN $\sin(90-\alpha) = \dfrac{h}{x}$

$$\cos\alpha = \frac{h}{x} \qquad \ldots(1)$$

In ΔAOB $x = 2\ell\sin\left(\dfrac{\theta}{2}\right)$

$$\Rightarrow \quad \sin\left(\frac{\theta}{2}\right) = \frac{x}{2\ell} \qquad \ldots(2)$$

From (1), we get

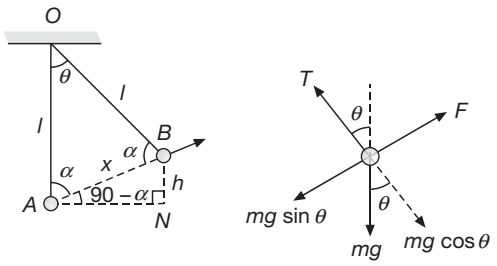
$$\cos\left(90-\frac{\theta}{2}\right) = \frac{h}{x} \qquad \left\{\because\ \alpha = 90-\frac{\theta}{2}\right\}$$

$$\Rightarrow \quad \sin\left(\frac{\theta}{2}\right) = \frac{h}{x} \qquad \ldots(3)$$

From (2) and (3), we get

$$\Rightarrow \quad \frac{x}{2\ell} = \frac{h}{x}$$

$$\Rightarrow \quad x^2 = 2h\ell$$



Also, for equilibrium of q

$$T = mg\cos\theta$$

$$F = mg\sin\theta$$

Since θ is small, so

$$F = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 x^2} = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 (2h\ell)} = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 x} = 2mgh$$

$$\Rightarrow \quad W_{\text{ext}} = mgh + 2mgh = 3mgh$$

Hence, the correct answer is (B).

31. According to option (D), the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up.

Hence, the correct answer is (D).

32. Each of the charged sphere will have potential energy due to its own charge or self energy

$$U_1 = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{r}$$

The mutual potential energy stored in one sphere in the electric field of the other is

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = U_2 \quad \Rightarrow \quad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{R}$$

Therefore the P.E. stored in the system of spheres is

$$U = 2U_1 + U_2 = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{R} + \frac{1}{r}\right]$$

Hence, the correct answer is (A).

33. At the surface of the charged sphere, whether it consists of a single piece or two pieces close together, the electric field strength from Gauss's Law is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

The electric charge per unit surface area is

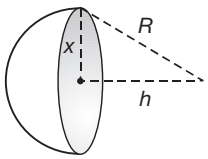
$$\sigma = \frac{Q}{4\pi R^2}$$

This electric field exerts a force $\Delta F = \frac{1}{2}E\Delta Q$ on the charge $\Delta Q = \sigma\Delta A$ which resides on a surface of area ΔA, as illustrated in figure. The reason for the factor of $\frac{1}{2}$ is that the electric field strength is E at the outer surface of the sphere and zero inside and hence its average value comes out to be $\frac{E}{2}$.

The force per unit area exerted by the charges on the pieces of the sphere is therefore

$$\frac{\Delta F}{\Delta A} = \frac{Q^2}{32\pi^2\varepsilon_0 R^4} = p$$

The required force can be considered to be analogous to the force with which a liquid having a pressure p would push apart the two pieces of the sphere and this force is also the product of p and the cross-sectional area of the intersection of the plane and sphere.



So, $F = pa = p(\pi x^2)$

$$\Rightarrow \quad F = p\pi(R^2 - h^2)$$

Hence it follows that the two parts of the sphere can be held together by a force

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^4}(R^2 - h^2)$$

Hence, the correct answer is (D).

34. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2.

Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

Hence, the correct answer is (C).

CHAPTER 1

35. All the three plates will produce electric field at P along negative z-axis. Hence ,

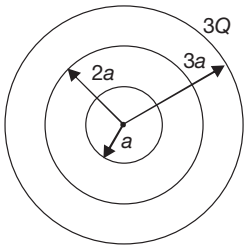
$$\vec{E}_P = \left[\frac{\sigma}{2\varepsilon_o} + \frac{2\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}\right]\left(-\hat{k}\right) = -\frac{2\sigma}{\varepsilon_o}\hat{k}$$

Hence, the correct answer is (B).

36. Since $V = \sum V_i = \frac{1}{4\pi\varepsilon_0}\sum\left(\frac{q_i}{r_i}\right)$

$$\Rightarrow \quad V_{N^{th}} = V_{\substack{\text{at N}^{th}\\ \text{due to 1}}} + V_{\substack{\text{at N}^{th}\\ \text{due to 2}}} + \ldots\ldots + V_{\substack{\text{at N}^{th}\\ \text{due to itself}}}$$

$$\Rightarrow \quad V_{N^{th}} = \frac{Q}{4\pi\varepsilon_0(Na)} + \frac{2Q}{4\pi\varepsilon_0(Na)} + \ldots\ldots + \frac{NQ}{4\pi\varepsilon_0(Na)}$$



$$\Rightarrow \quad V_{N^{th}} = \frac{Q}{4\pi\varepsilon_0(Na)}(1+2+3+\ldots\ldots+N)$$

$$\Rightarrow \quad V_{N^{th}} = \frac{Q}{4\pi\varepsilon_0(Na)}\frac{N(N+1)}{2}$$

$$\left\{\because\ 1+2+3+\ldots\ldots+N = \frac{N(N+1)}{2}\right\}$$

$$\Rightarrow \quad V_{N^{th}} = \frac{Q(N+1)}{8\pi\varepsilon_0 a}$$

Hence, the correct answer is (A).

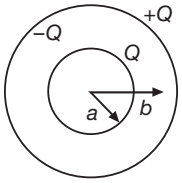
37. Charge on smaller sphere

$$q = C\left(\frac{V}{2}\right) = \frac{4\pi\varepsilon_0 RV}{2}$$

$$\text{Potential difference} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right) = \left(\frac{VR}{2}\right)\left(\frac{1}{2R}\right) = \frac{V}{4}$$

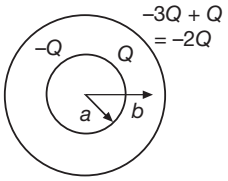
Hence, the correct answer is (B).

38.



$$V = V_{\substack{\text{surface}\\ \text{sphere}}} - V_{\substack{\text{outer}\\ \text{shell}}}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$



$$\text{Potential difference} = V' = V_{\substack{\text{surface}\\ \text{sphere}}} - V_{\substack{\text{outer}\\ \text{shell}}}$$

$$\Rightarrow \quad V' = \left(\frac{Q}{4\pi\varepsilon_0 a} + \frac{-2Q}{4\pi\varepsilon_0 b}\right) - \left(\frac{Q}{4\pi\varepsilon_0 b} + \frac{-2Q}{4\pi\varepsilon_0 b}\right)$$

$$\Rightarrow \quad V' = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = V$$

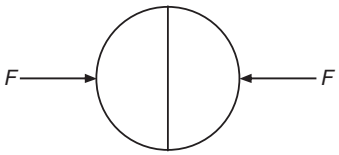
Hence, the correct answer is (A).

39. From Gauss's Law, we have flux equal to

$$\phi = \frac{\Sigma Q_{\text{enc}}}{\varepsilon_0} = \frac{\left(\frac{8C}{4}\right) - 7C + \left(\frac{6C}{2}\right)}{\varepsilon_0} = -\frac{2C}{\varepsilon_0}$$

Hence, the correct answer is (A).

40.



Electrostatics repulsive force :

$$F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$$

$$F = F_{ele} = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

Hence, the correct answer is (A).

41. In equilibrium, $mg = qE$

In absence of electric field, $mg = 6\pi\eta rv$

$$\Rightarrow \quad qE = 6\pi\eta rv$$

$$m = \frac{4}{3}\pi R r^3 d = \frac{qE}{g}$$

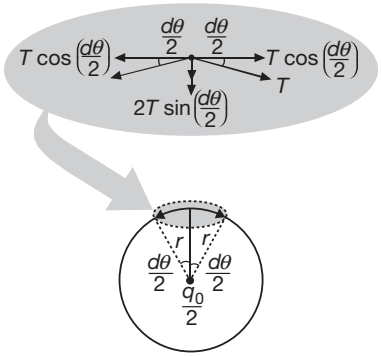
$$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$$

After substituting value we get,

$$q = 8\times10^{-19}\ \text{C}$$

Hence, the correct answer is (D).

42. Let us consider an element of arc length dl having a charge dq. Then $dq = \left(\frac{q_0}{2\pi r}\right)dl$.



If dF is the force of repulsion between the element and the charge $\frac{q_0}{2}$ at the centre, then

$$dF = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_0}{2}\right)\frac{dq}{r^2} \qquad \text{(radially outwards)}$$

For equilibrium to be there

$$dF = 2T\sin\left(\frac{d\theta}{2}\right)$$

$$\Rightarrow \quad \frac{1}{8\pi\varepsilon_0}\frac{q_0 dq}{r^2} \simeq 2T\left(\frac{d\theta}{2}\right)$$

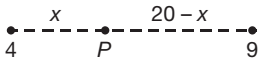
$$\left\{\because \text{ for small angle, } \sin\left(\frac{d\theta}{2}\right) \simeq \frac{d\theta}{2}\right\}$$

$$\Rightarrow \quad \frac{1}{8\pi\varepsilon_0}\frac{q_0\left(\frac{q_0}{2\pi r}\right)dl}{r^2} = T\left(\frac{dl}{r}\right)$$

$$\Rightarrow \quad T = \frac{q_0^2}{16\pi^2\varepsilon_0 r^2}$$

Hence, the correct answer is (B).

43.



$$E_P = 0$$

$$\Rightarrow \quad \frac{4}{x^2} = \frac{9}{(20-x)^2}$$

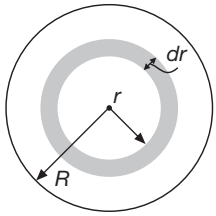
$$\Rightarrow \quad \frac{20-x}{x} = \frac{3}{2}$$

$$\Rightarrow \quad 40 - 2x = 3x$$

$$\Rightarrow \quad x = 8 \text{ cm}$$

Hence, the correct answer is (A).

44. Consider an infinitesimal concentric spherical element of thickness dr, radius r. If dq is the charge on this element, then



$$dq = \rho dV \qquad \{\text{where } dV = 4\pi r^2 dr\}$$

$$\Rightarrow \quad dq = \left(\rho_0 r^3\right)\left(4\pi r^2 dr\right)$$

$$\Rightarrow \quad dq = 4\pi\rho_0 r^5 dr$$

$$\Rightarrow \quad Q = \int_0^Q dq = 4\pi\rho_0 \int_0^R r^5 dr$$

$$\Rightarrow \quad Q = 4\pi\rho_0\left[\frac{r^6}{6}\Big|_0^R\right]$$

$$\Rightarrow \quad Q = \frac{2}{3}\pi\rho_0 R^6$$

Hence, the correct answer is (D).

45. $qE = Mg$

$$q\frac{\sigma}{\varepsilon_0} = Mg$$

$$q\frac{Q}{A\varepsilon_0} = Mg$$

$$\Rightarrow \quad q = \frac{MgA\varepsilon_0}{Q}$$

Hence, the correct answer is (B).

46. $F_{\text{air}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

Let us now replace the slab of thickness t with an equivalent amount of air such that two charges placed in air at separation x will experience the same force if the separation between them completely has a slab of dielectric constant K between them. In such a case

$$\frac{q_1 q_2}{4\pi\varepsilon_0 x^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K t^2}$$

$$\Rightarrow \quad \text{Equivalent separation in air } = x = t\sqrt{K}$$

So, now the effective separation between the charges is

$$r' = r - t + x = r - t + t\sqrt{K}$$

CHAPTER 1

$$\Rightarrow \quad r' = r - t + t\sqrt{K}$$

So, $F' = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r'^2}$

$$\Rightarrow \quad F' = \frac{q_1 q_2}{4\pi\varepsilon_0 \left(r - t + t\sqrt{K}\right)^2}$$

Now, as per the problem we have $t = \dfrac{r}{3}$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{\left(r - \frac{r}{3} + \frac{r}{3}\sqrt{K}\right)^2}$$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0}\left[\frac{9 q_1 q_2}{\left(2 + \sqrt{K}\right)^2 r^2}\right]$$

$$\Rightarrow \quad \frac{F'}{F_{\text{air}}} = \frac{9}{\left(1 + \sqrt{K}\right)^2} = \frac{9}{25}$$

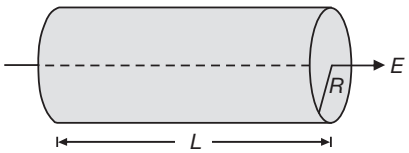
$$\Rightarrow \quad 2 + \sqrt{K} = 5$$

$$\Rightarrow \quad \sqrt{K} = 3$$

$$\Rightarrow \quad K = 9$$

Hence, the correct answer is (C).

47.

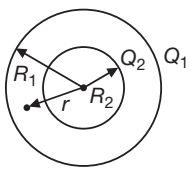


Flux due to curved surface = 0

Since field lines enter at one end and exit at other, so total flux equals zero.

Hence, the correct answer is (D).

48.



$$V_r = \frac{Q_2}{4\pi\varepsilon_0 r} + \frac{Q_1}{4\pi\varepsilon_0 R_1}$$

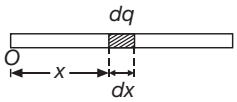
$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{r} + \frac{Q_1}{R_1}\right)$$

Hence, the correct answer is (C).

49. Consider an element of length dx, carrying a charge dq at a distance x from O. Then, by definition

$$dq = \lambda\, dx$$

$$\Rightarrow \quad dq = \lambda_0\left(\frac{x^3}{L}\right)dx$$



$$\Rightarrow \quad \int_O^Q dq = \frac{\lambda_0}{L}\int_O^L x^3 dx$$

$$\Rightarrow \quad Q = \frac{\lambda_0}{L}\left(\frac{L^4}{4}\right)$$

$$\Rightarrow \quad Q = \frac{\lambda_0 L^3}{4}$$

Hence, the correct answer is (C).

51. $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

Hence, the correct answer is (C).

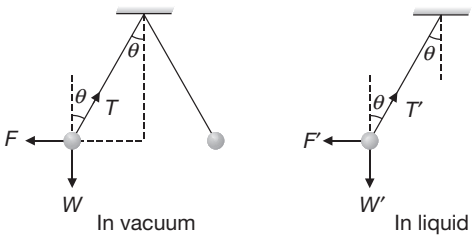
52. Each ball is in equilibrium under the following three forces:

(i) tension (T)

(ii) electric force (F)

(iii) weight (W)

So, Lami's theorem can be applied.



$$T\cos\theta = W \qquad \ldots(1)$$

$$T\sin\theta = F \qquad \ldots(2)$$

$$T'\cos\theta = W' \qquad \ldots(3)$$

$$T'\sin\theta = F' \qquad \ldots(4)$$

So, from (1), (2), (3) and (4), we get

$$\frac{W}{F} = \frac{W'}{F'}$$

where $W' = W - U = V\rho g - V\sigma g$ and $F' = \dfrac{F}{K}$

$$\Rightarrow \quad \frac{\cancel{V}\rho\cancel{g}}{\cancel{F}} = \frac{\cancel{V}\rho\cancel{g} - \cancel{V}\sigma\cancel{g}}{\frac{\cancel{F}}{K}}$$

$$\Rightarrow \quad K = \frac{\rho}{\rho - \sigma}$$

Hence, the correct answer is (D).

53. $V = \dfrac{Q}{4\pi\varepsilon_0 R} - \dfrac{Q}{4\pi\varepsilon_0 (3R)}$

$\Rightarrow \quad V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{2Q}{3R}\right)$

$\Rightarrow \quad Q = (4\pi\varepsilon_0)\left(\dfrac{3VR}{2}\right)$

$\Rightarrow \quad E = \dfrac{Q}{4\pi\varepsilon_0 (3R)^2}$

$\Rightarrow \quad E = \dfrac{4\pi\varepsilon_0\left(\dfrac{3VR}{2}\right)}{4\pi\varepsilon_0 (3R)^2}$

$\Rightarrow \quad E = \dfrac{V}{6R}$

Hence, the correct answer is (D).

55. $\vec{F} = \vec{F}_{21} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}(\vec{r} - \vec{r}_0)$

where $\vec{r} - \vec{r}_0 = 6\hat{i} - 8\hat{j}$

$\Rightarrow \quad |\vec{r} - \vec{r}_0| = 10 \text{ m}$

So, $\vec{F} = \dfrac{50\times10^{-6}\times2\times10^{-6}\times9\times10^{9}}{(10)^3}\left(6\hat{i} - 8\hat{j}\right)$

$\Rightarrow \quad \vec{F} = 9\times10^{-4}\left(6\hat{i} - 8\hat{j}\right) \text{ N}$

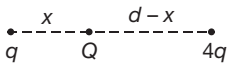
$\Rightarrow \quad |\vec{F}| = 9\times10^{-4}\sqrt{6^2 + (-8)^2}$

$\Rightarrow \quad |\vec{F}| = 9\times10^{-3} \text{ N} = 9 \text{ mN}$

Hence, the correct answer is (B).

56. For equilibrium of Q

$$k\frac{qQ}{x^2} = k\frac{4qQ}{(d-x)^2}$$



$\Rightarrow \quad \dfrac{d-x}{x} = 2$

$\Rightarrow \quad d - x = 2x$

$\Rightarrow \quad x = \dfrac{d}{3}$

For equilibrium of q

$$\frac{kQq}{x^2} + \frac{k4q^2}{d^2} = 0$$

$\Rightarrow \quad \dfrac{Qq}{\left(d^2/9\right)} + \dfrac{4q^2}{d^2} = 0$

$\Rightarrow \quad 9Q + 4q = 0$

$\Rightarrow \quad Q = -\dfrac{4q}{9}$

Hence, the correct answer is (C).

57. Since $\lambda = \dfrac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2} \text{ (we have done this already)}$$

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\dfrac{T}{A}\right)}{\left(\dfrac{\Delta R}{R}\right)}$$

$\Rightarrow \quad Y = \dfrac{TR}{A\Delta R}$

$\Rightarrow \quad \Delta R = \dfrac{TR}{AY} = \dfrac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$

Substituting $q_0 = 10^{-8}$ C, $Q = \pi$ C, $R = 0.1$ m, $A = 10^{-6}$ m^2, $Y = 2\times10^{-11}$ Nm^{-2}, we get

$\Delta R = 2.25$ mm

Hence, the correct answer is (B).

58. Same electric field near surfaces gives

$$\frac{Q_1}{4\pi\varepsilon_0 R_1^2} = \frac{Q_2}{4\pi\varepsilon_0 R_2^2}$$

$\Rightarrow \quad \dfrac{Q_1}{Q_2} = \dfrac{R_1^2}{R_2^2}$

Further

$$\frac{V_1}{V_2} = \frac{Q_1}{Q_2}\frac{R_2}{R_1} = \frac{R_1}{R_2} \qquad \left\{\because V = \frac{Q}{4\pi\varepsilon_0 R}\right\}$$

Hence, the correct answer is (A).

59. Since the spring is metallic so the charge distributes equally on both the blocks. Hence a charge $\dfrac{Q}{2}$ resides on each block, due to which the blocks will repel each other with a force given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\dfrac{Q}{2}\right)\left(\dfrac{Q}{2}\right)}{L^2} = k(L - L_0)$$

Solving for Q

CHAPTER 1

$$Q = 2L\sqrt{4\pi\varepsilon_0 k\left(L - L_0\right)}$$

Substituting $L = \frac{3L_0}{2}$, we get

$$Q = 3L_0\sqrt{2\pi\varepsilon_0 L_0}$$

So, OPTION (A) is correct.

Hence, the correct answer is (A).

60. Velocity of particle at time t is $v = at$ where $a = \frac{F}{m} = \frac{qE}{m}$.

$$\Rightarrow \quad v = \frac{qEt}{m}$$

So, $K.E. = \frac{1}{2}mv^2 = \frac{q^2E^2t^2}{2m}$

Hence, the correct answer is (C).

61. Let q_1 and q_2 be the initial charges on the spheres, then according to Coulomb's Law

$$0.108 = 9\times10^9\frac{q_1q_2}{(0.5)^2}$$

$$\Rightarrow \quad q_1q_2 = 3\times10^{-12} \qquad \ldots(1)$$

After connection, the charge flows from one sphere to another till both acquire the same potential (or in this case equal charge). So, final charge on both is

$$q_f = \frac{q_1 - q_2}{2}$$

(as initially they have opposite nature)
Now, again according to Coulomb's Law, the repulsive force is

$$0.036 = 9\times10^9\frac{\left(\frac{q_1 - q_2}{2}\right)^2}{(0.5)^2}$$

$$\Rightarrow \quad q_1 - q_2 = \frac{0.036\times0.25\times4}{9\times10^9}$$

$$\Rightarrow \quad q_1 - q_2 = 2\times10^{-6} \qquad \ldots(2)$$

Solving (1) and (2), we get

$$q_1 - \frac{3\times10^{-12}}{q_1} = 2\times10^{-6}$$

$$\Rightarrow \quad q_1^2 - 2\times10^{-6}q_1 - 3\times10^{-12} = 0$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6} \pm \sqrt{4\times10^{-12} + 12\times10^{-12}}}{2}$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6} \pm 4\times10^{-6}}{2}$$

$$\Rightarrow \quad q_1 = 3\times10^{-6}\text{ C} \text{ and } q_2 = -1\times10^{-6}\text{ C}$$

$$\Rightarrow \quad q_1 = 3\ \mu\text{C} \text{ and } q_2 = -1\ \mu\text{C}$$

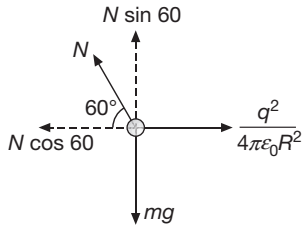
Hence, the correct answer is (B).

62. Since, all A, B, C, D and E lie on an equipotential surface so,

$$W = 0$$

Hence, the correct answer is (C).

63. $N\sin60 = mg$ and $N\cos60 = \frac{q^2}{4\pi\varepsilon_0R^2}$



$$\Rightarrow \quad \tan60° = \frac{mg}{\left(\frac{q^2}{4\pi\varepsilon_0R^2}\right)}$$

$$\Rightarrow \quad q^2 = \frac{4\pi\varepsilon_0mgR^2}{\sqrt{3}}$$

$$\Rightarrow \quad q = \left(\frac{4\pi\varepsilon_0mgR^2}{\sqrt{3}}\right)^{\frac{1}{2}}$$

Hence, the correct answer is (C).

64. $U = 4\left(\frac{-q^2}{4\pi\varepsilon_0a}\right) + 2\left(\frac{q^2}{4\pi\varepsilon_0\sqrt{2}a}\right)$

$$U = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q^2}{a}\left(-4 + \sqrt{2}\right)$$

Hence, the correct answer is (A).

65. $\frac{q^2}{4\pi\varepsilon_0r^2} = \mu mg$

$$\Rightarrow \quad \frac{9\times10^9\times\left(10^{-7}\right)^2}{\left(10\times10^{-2}\right)^2} = \mu\left(\frac{5}{1000}\right)(10)$$

$$\Rightarrow \quad \mu = 0.18$$

Hence, the correct answer is (C).

66. Here we shall use a logical trick. If a charge of $4\ \mu C$ had also been placed at A then

$$\vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = \vec{0}$$

$$\Rightarrow \quad \vec{E}_B + \vec{E}_C + \vec{E}_D = -\vec{E}_A$$

$$\Rightarrow \quad \left|\vec{E}_B+\vec{E}_C+\vec{E}_D\right|=\frac{k\left(4\times10^{-6}\right)}{\left(1/\sqrt{2}\right)^2}$$

$$\Rightarrow \quad \left|\vec{E}_B+\vec{E}_C+\vec{E}_D\right|=\left(9\times10^9\right)\left(4\times10^{-6}\right)(2)$$

$$\Rightarrow \quad \left|\vec{E}_B+\vec{E}_C+\vec{E}_D\right|=7.2\times10^4\ NC^{-1}$$

Hence, the correct answer is (A).

67. $\int \vec{E}\cdot d\vec{A}=\frac{1}{\varepsilon_0}q$

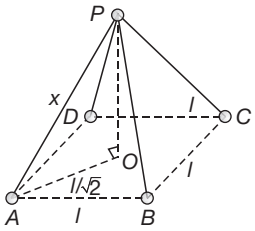
$$\Rightarrow \quad E(2\pi r\ell)=\frac{1}{\varepsilon_0}(\lambda\ell)$$

$$\Rightarrow \quad E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, the correct answer is (C).

68. The force due to one charge on the charge at P is shown here. When all the forces are taken into account, then the sine components cancel each other. So,

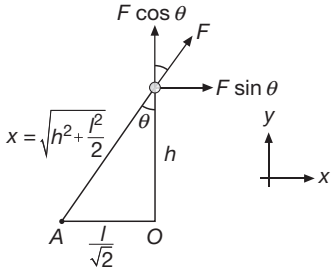
$$F_{\text{net}}=F_y=4F\cos\theta$$



Since, we see that $\cos\theta=\dfrac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}$

$$\Rightarrow \quad F_{\text{net}}=4\frac{Q(1)}{4\pi\varepsilon_0\left(\sqrt{h^2+\frac{\ell^2}{2}}\right)^2}\frac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}$$

$$\Rightarrow \quad F_{\text{net}}=\frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{3/2}} \qquad \text{(UPWARDS)}$$



For equilibrium of m, this F_{net} must balance the weight of the particle mg (downwards).

$$\Rightarrow \quad \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{\frac{3}{2}}}=mg$$

$$\Rightarrow \quad Q=\frac{mg\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{3/2}}{h}$$

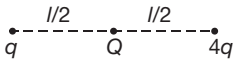
Hence, the correct answer is (B).

69. Field lines always enter or leave a surface normal to it.

Hence, the correct answer is (D).

70. $\left(F_{net}\right)_q=0$

$$\Rightarrow \quad k\frac{Qq}{\left(\frac{\ell}{2}\right)^2}+k\frac{4q^2}{\ell^2}=0$$



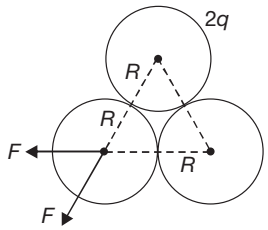
where $k=\dfrac{1}{4\pi\varepsilon_0}$

$$\Rightarrow \quad 4Qq+4q^2=0$$

$$\Rightarrow \quad Q=-q$$

Hence, the correct answer is (A).

71.



$$F=\frac{1}{4\pi\varepsilon_0}\frac{(2q)^2}{(2R)^2}=\frac{q^2}{4\pi\varepsilon_0R^2}$$

$$F_{\text{net}}=\sqrt{F^2+F^2+2F^2\cos60}$$

$$\Rightarrow \quad F_{\text{net}}=\sqrt{3}F=\sqrt{3}\frac{q^2}{4\pi\varepsilon_0R^2}$$

$$\Rightarrow \quad F_{\text{net}}=\frac{\sqrt{3}q^2}{4\pi\varepsilon_0R^2}$$

Hence, the correct answer is (D).

72. According to Gauss Theorem

$$\text{Flux}=\phi=\frac{q}{\varepsilon_0}$$

Hence, the correct answer is (A).

CHAPTER 1

73. Since charge resides only on the outer surface of conductor, So (D) is the correct option.

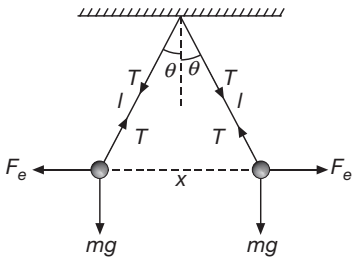
Hence, the correct answer is (D).

74. In this problem, let us first calculate the value of x in terms of other known parameters. Let us consider the particles to be in equilibrium, then

$$T\cos\theta = mg \qquad \ldots(1)$$

$$T\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$



Since $x \ll \ell$

$$\therefore \quad \tan\theta \text{ is very small and hence } \tan\theta \simeq \theta = \frac{x}{2\ell}$$

$$\Rightarrow \quad \frac{x}{2\ell} = \frac{q^2}{4\pi\varepsilon_0 mg\, x^2}$$

$$\Rightarrow \quad x^3 = \frac{q^2\ell}{2\pi\varepsilon_0 mg} \qquad \ldots(3)$$

$$\Rightarrow \quad x = \left(\frac{q^2\ell}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$$

Take derivative of (3) w.r.t. time, we get

$$3x^2\frac{dx}{dt} = \left(\frac{\ell}{2\pi\varepsilon_0 mg}\right)2q\left(\frac{dq}{dt}\right)$$

But according to the problem, $v = \frac{dx}{dt} = \frac{6}{\sqrt{x}}$

$$\Rightarrow \quad 3x^2\frac{6}{\sqrt{x}} = \left(\frac{x^3}{q^2}\right)2q\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \quad \frac{dq}{dt} = 9\left(\frac{q}{x^{3/2}}\right)$$

But $\frac{x^3}{q^2} = \frac{\ell}{2\pi\varepsilon_0 mg}$

$$\Rightarrow \quad \frac{dq}{dt} = 9\sqrt{\frac{2\pi\varepsilon_0 mg}{\ell}}$$

Hence, the correct answer is (D).

75. Since, $\vec{E} = 5\hat{i} + 2\hat{j}$

$$\vec{A} = 2\hat{i}$$

(as area vector is always directed along the outward normal)

So, $\phi = \vec{E}\cdot\vec{A} = 10$ SI units

Hence, the correct answer is (A).

76. Consider an infinitesimal ring element of inner radius r and outer radius $r + dr$ having a charge dq on it. Then

$$dq = \sigma \text{ (Area of infinitesimal ring element)}$$

$$\Rightarrow \quad dq = \sigma(2\pi r dr)$$

But $\sigma = \frac{\sigma_0}{r^2}$

$$\Rightarrow \quad dq = \frac{\sigma_0}{r^2}(2\pi r dr) = 2\pi\sigma_0\frac{dr}{r}$$

$$\Rightarrow \quad q = \int dq = 2\pi\sigma_0\int_a^b \frac{dr}{r} = 2\pi\sigma_0\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (D).

78. Since the spheres attract with a force F, so let q_1 and $-q_2$ be the charges on the spheres, then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2} \qquad \ldots(1)$$

When the spheres are brought in contact, then final charge on both is q (say), given by

$$q = \frac{q_1 + (-q_2)}{2} = \frac{q_1 - q_2}{2}$$

These spheres with new charge q, placed at separation d, again repel with force F. So

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(\frac{q_1 - q_2}{2}\right)^2}{4\pi\varepsilon_0 d^2} \qquad \ldots(2)$$

So, from (1) & (2), we get

$$\frac{\left(\frac{q_1 - q_2}{2}\right)^2}{4\pi\varepsilon_0 d^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2}$$

$$\Rightarrow \quad q_1^2 + q_2^2 - 2q_1 q_2 = 4q_1 q_2$$

$$\Rightarrow \quad q_1^2 + q_2^2 - 6q_1 q_2 = 0$$

$$\Rightarrow \quad \left(\frac{q_1}{q_2}\right)^2 - 6\left(\frac{q_1}{q_2}\right) + 1 = 0$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 2\sqrt{8}}{2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} = 3 \pm \sqrt{8}$$

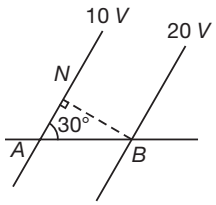
So, ratio between the initial charges is

$$\frac{q_1}{-q_2} = -\left(3 \pm \sqrt{8}\right)$$

$$\Rightarrow \quad \text{Either } \frac{q_1}{-q_2} = -3 - \sqrt{8} \text{ OR } \frac{q_1}{-q_2} = -3 + \sqrt{8}$$

Hence, the correct answer is (C).

79.



Since $|E| = \frac{\Delta V}{\Delta r_\perp}$

In ΔABN

$$\sin(30°) = \frac{BN}{AB}$$

$$\Rightarrow \quad BN = \left(\frac{10}{100}\right)\sin(30°)$$

$$\Rightarrow \quad E = \frac{10}{\left(\frac{10}{100}\right)\sin(30°)}$$

$$\Rightarrow \quad E = 200\ Vm^{-1}$$

E must be perpendicular to the line indicating equipotential surface. Hence it must be directed at $120°$ with X-axis.

Hence, the correct answer is (C).

80. **For Solid Sphere**

$$q_1 = \int \rho dV = \int_0^{R_1} \frac{\rho_0}{r}\left(4\pi r^2 dr\right)$$

$$\Rightarrow \quad q_1 = 4\pi\rho_0 \int_0^{R_1} r dr = 4\pi\rho_0\left(\frac{R_1^2}{2}\right)$$

$$\Rightarrow \quad q_1 = 2\pi R_1^2 \rho_0 \qquad ...(1)$$

For Hollow Sphere

$$q_2 = \left(4\pi R_2^2\right)(-\sigma)$$

$$\Rightarrow \quad q_2 = -4\pi R_2^2 \sigma \qquad ...(2)$$

Since, total charge on the system is zero, so

$$q_1 + q_2 = 0$$

$$\Rightarrow \quad 2\pi R_1^2 \rho_0 - 4\pi R_2^2 \sigma = 0$$

$$\Rightarrow \quad R_1^2 \rho_0 = 2R_2^2 \sigma$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \sqrt{\frac{\rho_0}{2\sigma}}$$

Hence, the correct answer is (C).

81. $\phi_{total} = \frac{q}{\varepsilon_0}$

$$\phi_{face} = \frac{1}{6}\phi_{total} = \frac{q}{6\varepsilon_0}$$

Hence, the correct answer is (A).

82. Since electrostatic force is conservative in nature, so work done by the force in a closed path must be zero.

$$W_{P\to Q} + W_{Q\to R} + W_{R\to S} + W_{S\to P} = 0$$

$$\Rightarrow \quad W_{P\to S} = -W_{S\to P}$$

$$\Rightarrow \quad \begin{pmatrix}\text{Work done in}\\ \text{the process}\end{pmatrix} = W_{P\to Q\to R\to S} = W_{P\to S} = -W_{S\to P}$$

Further $W_{S\to P} = \vec{F}\cdot\overrightarrow{SP}$

$$\Rightarrow \quad W_{S\to P} = \left(qE\hat{i}\right)\cdot\left(a\hat{i} + b\hat{j}\right)$$

$$\Rightarrow \quad W_{S\to P} = qEa$$

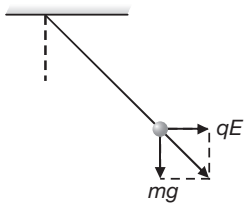
$$\Rightarrow \quad \text{Work done in the process} = -qEa$$

Hence, the correct answer is (B).

83. The velocity of the particle will first increase and then decrease. Thus total potential energy will first decrease and then increase.

Hence, the correct answer is (C).

84. $a_{net} = \frac{\sqrt{m^2g^2 + q^2E^2}}{m}$



CHAPTER 1

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{a_{net}}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

Hence, the correct answer is (B).

85. $q_1 + q_2 = 20\ \mu C$ …(1)

According to Coulomb's Law

$$0.075 = 9\times10^9\ \frac{q_1 q_2}{(3)^2}$$

$$\Rightarrow \quad q_1 q_2 = 75\times10^{-12} \qquad \ldots(2)$$

Since

$$(q_1 - q_2)^2 = (q_1 + q_2)^2 - 4q_1 q_2$$

$$\Rightarrow \quad q_1 - q_2 = \sqrt{400 - 300}\ \mu C$$

$$\Rightarrow \quad q_1 - q_2 = 10\ \mu C \qquad \ldots(3)$$

Solving (1) and (3), we get

$$q_1 = 15\ \mu C\ ,\ q_2 = 5\ \mu C$$

Hence, the correct answer is (B).

86. Electric potential along the x-axis will be zero as $\Sigma q = 0$ and distances of the point from the charges is zero. So,

$$x = y = 0$$

Hence, the correct answer is (C).

88. Whatever may be the charge on the surface of a shell field inside it is always zero.

Hence, the correct answer is (C).

89. Charge on the inner surface will be –q and that on the outer surface will be zero so to make electric field inside the conductor zero and potential of the surface to be zero.

Hence, the correct answer is (D).

90. The potential at the centre of a hollow sphere is equal to the potential at its surface.

Hence, the correct answer is (B).

91. Radius of the semicircle, $R = \frac{L}{\pi}$

Electric potential, $V = \frac{q}{4\pi\varepsilon_o R} = \frac{q}{4\varepsilon_o L}$

Hence, the correct answer is (B).

92. When connected by a long wire then

$$\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{4\pi\varepsilon_0 R_1 V}{4\pi\varepsilon_0 R_2 V}$$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

Hence, the correct answer is (A).

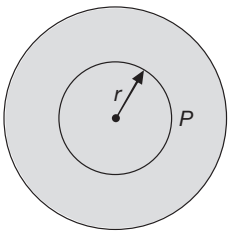
93. $E_1 = \frac{kQ_1}{R_1^2}$ and $E_2 = \frac{kQ_2}{R_2^2}$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{Q_1}{Q_2}\frac{R_2^2}{R_1^2}$$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{R_2}{R_1}$$

Hence, the correct answer is (B).

94. Field at the point P is only due to inner core, so



$$E = \frac{1}{4\pi\varepsilon_o}\frac{\left(\rho\left(\frac{4}{3}\right)\pi r^3\right)}{r^2} = \frac{\rho r}{3\varepsilon_o}$$

Hence, the correct answer is (A).

96. $V_0 = 0$

$$\vec{E}_0 = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D$$

$$\vec{E}_A + \vec{E}_B = \frac{\sqrt{2}q}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{E}_A + \vec{E}_B = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 a^2}\left(-\hat{j}\right)$$

Similarly

$$\vec{E}_C + \vec{E}_D = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 a^2}\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{E}_0 = \frac{4\sqrt{2}q}{4\pi\varepsilon_0 a^2}\left(-\hat{j}\right)$$

$$\Rightarrow \quad |\vec{E}_0| = \left(\frac{4\sqrt{2}q}{a^2}\right)\frac{1}{4\pi\varepsilon_0}$$

Hence, the correct answer is (B).

97. Let $P(x, y)$ be the point where potential is zero.

$$\frac{-2Q}{4\pi\varepsilon_o\sqrt{(x+3a)^2+y^2}} + \frac{Q}{4\pi\varepsilon_o\sqrt{(x-3a)^2+y^2}} = 0$$

$$\Rightarrow \quad (x-5a)^2 + y^2 = 16a^2$$

So, OPTIONS (A) and (B) are correct.
Since potential at the center is positive, OPTION (C) is also correct.

Hence, the correct answer is (D).

98. $W_{A\to B} = Q(V_B - V_A)$

$$W_{A\to C} = Q(V_C - V_A)$$

Since $V_C = V_B$ as both B and C lie on an equipotential surface.

Hence, the correct answer is (B).

99. Kinetic energy of the particle $= \frac{1}{2}mv^2 + qEy$

Hence, the correct answer is (A).

101. The ball will be at equilibrium at an angle θ_0, where

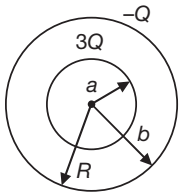
$$\tan\theta_0 = \frac{qE}{mg}$$

The tension will be minimum for

$$\theta = \pi + \theta_0 = \pi + \tan^{-1}\left(\frac{qE}{mg}\right)$$

Hence, the correct answer is (D).

102.



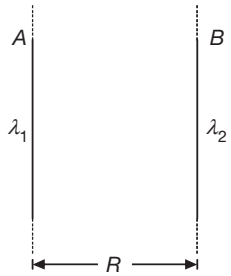
Since field inside the shell is zero. So,

$$E = \frac{3Q}{4\pi\varepsilon_0 R^2}$$

Hence, the correct answer is (C).

103. Electric field due to wire A at a distance R from it is

$$E_1 = \frac{\lambda_1}{2\pi\varepsilon_0 R}$$



Wire B is now in the field of wire A. Let us consider a length ℓ of wire B. Charge on this portion will be $q_2 = \lambda_2\ell$. So, force experienced by this portion of B (of charge q_2) due to field of wire A is

$$F_{21} = q_2E_1 = (\lambda_2\ell)\frac{\lambda_1}{2\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \frac{F_{21}}{\ell} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0 R} = \frac{2\lambda_1\lambda_2}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \frac{F_{21}}{\ell} = K\left(\frac{2\lambda_1\lambda_2}{R}\right)$$

Hence, the correct answer is (B).

104. The motion of the charge will be like projectile with range

$$AB = 2\left(\frac{R}{2}\right) = \frac{v_0^2\sin(2\theta)}{a}$$

So, $\frac{R}{\sin(2\theta)} = \frac{v_0^2}{a}$

For minimum velocity, $\sin(2\theta) = 1$

$$\Rightarrow \quad \frac{v_0^2}{a} = R$$

where $a = \frac{qE}{m} = \frac{q\left(\frac{\rho}{3\varepsilon_0}\frac{R}{2}\right)}{m}$

$$\Rightarrow \quad a = \frac{2\times10^{-4}\times3\varepsilon_0\times10^{-3}}{1\times10^{-3}\times3\varepsilon_0} = 2\times10^{-4}\ \text{ms}^{-2}$$

$$\Rightarrow \quad v_0 = 0.02\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

105. Since, $U = -\vec{p}\cdot\vec{E}$ and

$$\vec{N} = \vec{p}\times\vec{E}$$

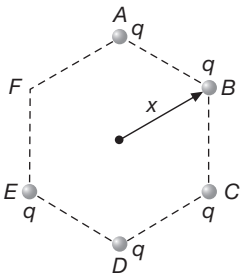
U is MINIMUM when $\theta = 0°$ and for $\theta = 0°$

$$\vec{N} = \vec{0}$$

Hence, the correct answer is (A).

CHAPTER 1

106. Electric fields at the center due to the charges on opposite corners will cancel each other. Therefore at the center net electric field will be due to only that charge which does not has charge on the opposite corner.



$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(2a)^2} = \frac{1}{16\pi\varepsilon_0}\frac{q}{a^2}$$

Hence, the correct answer is (B).

107. Let Q be divided into q and $Q-q$. Then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

For F to be MAXIMUM $\frac{dF}{dq} = 0$

$$\Rightarrow \quad \frac{d}{dq}\left(qQ - q^2\right) = 0$$

$$Q - 2q = 0$$

$$\Rightarrow \quad q = \frac{Q}{2}$$

Hence, the correct answer is (A).

108. Since the point where electric field is zero is nearer to q_1 therefore

$$q_2 > q_1$$

Again electric field is zero between q_1 and q_2, therefore, both charges must be of same nature.

Since electric field due to q_1 is towards right and that of q_2 is towards left therefore q_1 and q_2 are positive charges.

Hence, the correct answer is (A).

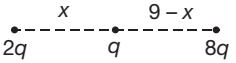
109. Both A and B have same radius and same potential. Hence

$$Q_A = Q_B \qquad \left\{\because V = \frac{Q}{4\pi\varepsilon_0 r} \text{ for both}\right\}$$

Hence, the correct answer is (C).

110. For potential energy to be MINIMUM, the charges with maximum value must be placed the farthest. Also MINIMUM potential energy implies equilibrium i.e.

$$\sum F = 0$$



For equilibrium of q

$$\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{8q^2}{(9-x)^2}$$

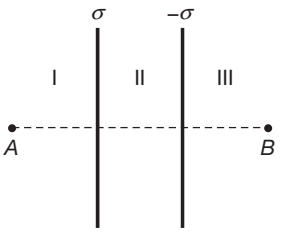
$$\Rightarrow \quad \left(\frac{9-x}{x}\right)^2 = 4$$

$$\Rightarrow \quad 9 - x = 2x$$

$$\Rightarrow \quad x = 3 \text{ cm}$$

Hence, the correct answer is (B).

111. In the region I and III net electric field is zero. In the region II net electric field is



$$E_{II} = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \text{ and is uniform.}$$

Hence, the correct answer is (B).

112. $$V = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots.\right)$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{1-\frac{1}{2}}\right)$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}(2q)$$

$$E = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots.\right)$$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(1-\frac{1}{4}\right)}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q}{3}\right)$$

Hence, the correct answer is (D).

113. $V = \frac{q}{4\pi\varepsilon_0}\left(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...\right)$

$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{1+\frac{1}{2}}\right)$

$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3}\right)$

$E = E_+ - E_-$

$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{16} + ...\right) - \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{4} + \frac{1}{64} + ...\right)$

$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{16}} - \frac{\frac{1}{4}}{1-\frac{1}{16}}\right]$

$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\left(\frac{16}{15} - \frac{4}{15}\right)$

$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\left(\frac{12}{15}\right)$

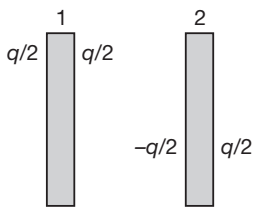
$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{5}\right)$

Hence, the correct answer is (C).

114. Net electric field between the plates is $E = \frac{\sigma}{\varepsilon_0}$

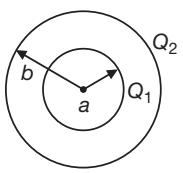
Since $\sigma = \frac{\frac{q}{2}}{A} = \frac{q}{2A}$

$\Rightarrow \quad E = \frac{q}{2\varepsilon_0 A}$



Hence, the correct answer is (A).

115.



$V_{inner} = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0 b}$

$V_{outer} = \frac{Q_2}{4\pi\varepsilon_0 b} + \frac{Q_1}{4\pi\varepsilon_0 b}$

$\Rightarrow \quad V_{inner} > V_{outer}$ as $b > a$

Hence, the correct answer is (B).

117. Electric field at O depends only on the induced charge on the outer surface of the spherical conductor. Shifting of q within the cavity does not change distribution of induced charge over the outer surface.

Hence, the correct answer is (D).

118. $\vec{E} = -\vec{\nabla}V$

$\Rightarrow \quad \vec{E} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$

$\frac{\partial U}{\partial x} = 6 - 8y^2 - 8x$

$\frac{\partial U}{\partial y} = -16xy - 8 + 6z$

$\frac{\partial U}{\partial z} = 6y$

At (0, 0, 0)

$\frac{\partial U}{\partial x} = 6$

$\frac{\partial U}{\partial y} = -8$

$\frac{\partial U}{\partial z} = 0$

$\Rightarrow \quad \vec{E}|_{(0,0,0)} = -6\hat{i} + 8\hat{j}$

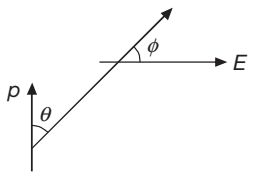
$\Rightarrow \quad |\vec{E}| = 10 \text{ NC}^{-1}$

$\Rightarrow \quad |\vec{F}| = q|\vec{E}|$

$\Rightarrow \quad |\vec{F}| = 20 \text{ N}$

Hence, the correct answer is (D).

119. If angle made by the electric field due to the dipole with the radius vector is ϕ then



$\tan\phi = \frac{1}{2}\tan\theta$

CHAPTER 1

In this equation

$$\phi = \frac{\pi}{2} - \theta$$

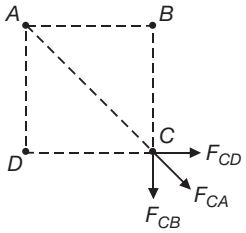
$$\Rightarrow \quad \tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \cot\theta = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\sqrt{2}\right)$$

Hence, the correct answer is (D).

120.



$$F_{\text{net}} = \frac{\sqrt{2}q^2}{4\pi\varepsilon_0 a^2} + \frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2}a\right)^2}$$

$$F_{\text{net}} = \left(\frac{1+2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (C).

121. $V_B - V_0 = -\int E_x dx = -\frac{400}{\sqrt{2}}(0.03)$

$$V_A - V_B = -\int E_y dy = -\frac{400}{\sqrt{2}}(0.02)$$

where V_0 is potential at origin. So,

$$V_A - V_B = \frac{400}{\sqrt{2}}(0.03 - 0.02)$$

$$\Rightarrow \quad V_A - V_B = \frac{4}{\sqrt{2}} = 2.8 \text{ V}$$

Hence, the correct answer is (D).

122. $q_r + q_R = Q$ …(1)

$$\frac{q_r}{q_R} = \frac{4\pi r^2\sigma}{4\pi R^2\sigma} = \frac{r^2}{R^2}$$

$$\Rightarrow \quad \frac{q_r}{q_R} = \frac{r^2}{R^2} \qquad \ldots(2)$$

From (1) & (2)

$$q_r = \frac{Qr^2}{R^2 + r^2} \text{ and } q_R = \frac{QR^2}{R^2 + r^2}$$

So, $V = \frac{q_r}{4\pi\varepsilon_0 r} + \frac{q_R}{4\pi\varepsilon_0 R}$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r+R}{r^2+R^2}\right)$$

Hence, the correct answer is (A).

124. Initially potential difference between the spheres is $V_A - V_B$

Since earthing of B will make potential of B zero but potential difference will not change because potential difference between two concentric spheres depends only on the charge on the inner sphere. Hence

Final Potential Difference = Initial Potential Difference

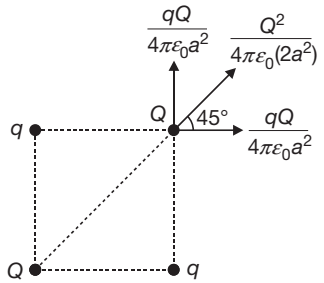
$$\Rightarrow \quad V_A' - V_B' = V_A - V_B$$

$$\Rightarrow \quad V_A' - 0 = V_A - V_B$$

$$\Rightarrow \quad V_A' = V_A - V_B$$

Hence, the correct answer is (C).

125.



For net force on Q to be zero

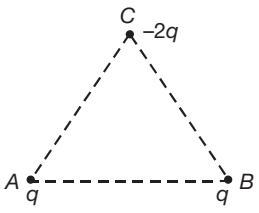
$$\frac{qQ}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)}\cos 45 + \frac{qQ}{4\pi\varepsilon_0 a^2}\cos 90 = 0$$

$$\Rightarrow \quad Q = -2\sqrt{2}\,q$$

Hence, the correct answer is (D).

126. Method 1

Let origin be at A, then co-ordinates of points A, B and C are $(0, 0)$, $(\ell, 0)$ and $\left(\frac{\ell}{2}, \frac{\sqrt{3}\ell}{2}\right)$ respectively.



$$p_x = q(0) + q(\ell) + (-2q)\frac{\ell}{2}$$

$$\Rightarrow \quad p_x = 0$$

$$p_y = q(0) + q(0) + (-2q)\frac{\sqrt{3}\ell}{2}$$

$$\Rightarrow \quad p_y = -\sqrt{3}q\ell$$

$$\Rightarrow \quad p = \sqrt{p_x^2 + p_y^2}$$

$$\Rightarrow \quad p = \sqrt{3}q\ell$$

Method 2 (Shortcut)

$$p = (2q)\left(\frac{\sqrt{3}\ell}{2}\right)$$

because

$$p = \begin{pmatrix}\text{Either}\\ \text{charge}\end{pmatrix}\begin{pmatrix}\text{Distance of}\\ \text{separation}\end{pmatrix}$$

Hence, the correct answer is (C).

128. Angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow \quad L = rp\sin\theta$$

$$\Rightarrow \quad L = x_0(mv)\sin(90°) = x_0 mv$$

Now, at time t, the velocity is given by

$$v = at = \left(\frac{qE_0}{m}\right)t \qquad \left\{\because\ a = \frac{qE_0}{m}\right\}$$

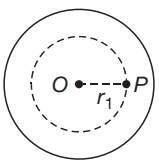
$$\Rightarrow \quad L = x_0 m\left(\frac{qE_0}{m}\right)t = (qE_0x_0)t$$

Since the speed of the particle is increasing with time, therefore angular momentum is increasing with time.

Hence, the correct answer is (C).

129. P is any inside point at distance r_1 from O. Consider a spherical surface of radius r_1 as Gaussian surface. So,

$$\oint_S \vec{E}\cdot\vec{dA} = \frac{q_{enc}}{\varepsilon_0}$$



By symmetry, E at all points on the surface is same and angle between $\vec{E}$ and $\vec{dA}$ is zero everywhere. So,

$$\oint_S \vec{E}.\vec{dA} = EA = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow \quad E\left(4\pi r_1^2\right) = \frac{q_{\text{enc}}}{\varepsilon_0} \qquad \ldots(1)$$

Now, let us calculate q_{enc}. The sphere can be regarded as consisting of a large number of spherical shells. Consider a shell of inner and outer radii r and $r + dr$. Its volume will be $dV = 4\pi r^2 dr$. Charge dq on this shell will be

$$dq = \rho dV = \frac{A}{r}(4\pi r^2 dr) = 4\pi A(rdr)$$

So, total charge enclosed by Gaussian-surface is

$$q_{\text{enc}} = \int dq = 4\pi A\int_0^{r_1} rdr = 4\pi A\left(\frac{r_1^2}{2}\right)$$

$$\Rightarrow \quad q_{\text{enc}} = 4\pi A\int_0^{r_1} rdr = 4\pi A\left(\frac{r_1^2}{2}\right) = 2\pi A r_1^2$$

From Equation (1), we get

$$E\left(4\pi r_1^2\right) = \frac{2\pi A r_1^2}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{A}{2\varepsilon_0}$$

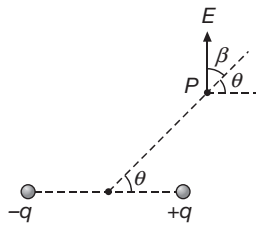
Hence, the correct answer is (D).

130. Direction of electric field with the axis of dipole is given by

$$\theta + \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

Here given that E is along y-axis, so

$$\theta + \tan^{-1}\left(\frac{\tan\theta}{2}\right) = \frac{\pi}{2}$$



$$\Rightarrow \quad \frac{\tan\theta}{2} = \cot\theta$$

$$\Rightarrow \quad \tan\theta = 2\cot\theta$$

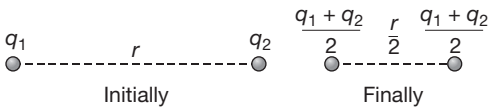
$$\Rightarrow \quad \tan^2\theta = 2$$

$$\Rightarrow \quad \tan\theta = \sqrt{2}$$

Hence, the correct answer is (B).

CHAPTER 1

131.



Initially, $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

After contact i.e., finally, we have

$$F' = \frac{(q_1+q_2)^2}{4\pi\varepsilon_0 r^2}$$

Since, $F' = 4.5F$

$$\Rightarrow \quad \frac{(q_1+q_2)^2}{r^2} = \frac{4.5q_1q_2}{r^2}$$

$$\Rightarrow \quad 2q_1^2 + 2q_2^2 + 4q_1q_2 = 9q_1q_2$$

$$\Rightarrow \quad 2q_1^2 + 2q_2^2 - 5q_1q_2 = 0$$

$$\Rightarrow \quad \frac{q_1^2}{q_2^2} + 2 - 5\left(\frac{q_1}{q_2}\right) = 0$$

$$\Rightarrow \quad 2x^2 - 5x + 2 = 0 \text{, where } x = \frac{q_1}{q_2}$$

$$\Rightarrow \quad 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow \quad 2x(x-2) - 1(x-2) = 0$$

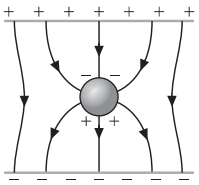
$$\Rightarrow \quad (2x-1)(x-2) = 0$$

$$\Rightarrow \quad x = \frac{1}{2} \text{ or } x = 2$$

$$\Rightarrow \quad q_1 = 2q_2$$

Hence, the correct answer is (A).

132. Due to induction, charges will appear on the surface of sphere as shown in figure. Electric field lines never enter any conducting surface and are perpendicular at the surface.



Hence, the correct answer is (C).

133. $F = \dfrac{k(10)(90)}{r^2}$

Charge on each, when contact is made is given by

$$q_1' = q_2' = \frac{10 + (-90)}{2} = -40\ \mu\text{C}$$

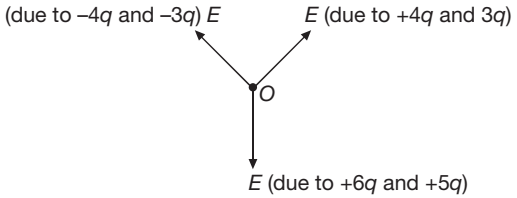
So, new force $F' = \dfrac{k(40)(40)}{r^2}$

$$\Rightarrow \quad \frac{F'}{F} = \frac{40\times 40}{10\times 90} = \frac{16}{9}$$

$$\Rightarrow \quad F' = \frac{16}{9}F$$

Hence, the correct answer is (C).

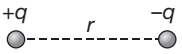
134.



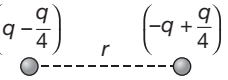
Since all fields are equal in magnitude and are inclined to each other at 120°, so $E_{\text{net}} = 0$

Hence, the correct answer is (D).

135.



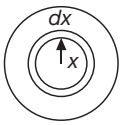
$F = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$ and when 25% of charge of one is transferred to the other, then we have,



$$F' = \frac{\left(\frac{3q}{4}\right)\left(\frac{3q}{4}\right)}{4\pi\varepsilon_0 r^2} = \frac{9}{16}F$$

Hence, the correct answer is (B).

136. We can consider all the charge inside the sphere to be concentrated at the centre of sphere, consider an elementary shell of radius x and thickness dx, then



$$E = \frac{1}{4\pi\varepsilon_0}\frac{\int dq}{r^2} = \frac{\int_0^r 4\pi x^2 dx(\alpha x)}{4\pi\varepsilon_0 r^2} = \frac{\alpha r^2}{4\varepsilon_0}$$

Hence, the correct answer is (B).

137. If 20^{th} charge is also placed, then

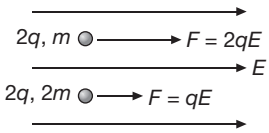
$\vec{F}_{net} = \vec{0}$ (by symmetry)

$$\Rightarrow \quad \vec{F}_{0,1} + \vec{F}_{0,2} + \vec{F}_{0,3} + \ldots + \vec{F}_{0,20} = \vec{0}$$

$$\Rightarrow \quad \left|\vec{F}_{0,\,1}+\vec{F}_{0,\,2}+\ldots+\vec{F}_{0,\,19}\right|=\left|-\vec{F}_{0,\,20}\right|=F_{0,\,20}=\frac{Qq}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (B).

138.



$$\frac{KE_A}{KE_B}=\frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2}$$

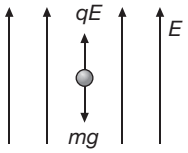
Since, $V_A = u_A + a_A t$

$$\Rightarrow \quad V_A=\frac{2qE}{m}t \text{ and}$$

$$\Rightarrow \quad \frac{KE_A}{KE_B}=\frac{\frac{1}{2}m\left(\frac{2qEt}{m}\right)^2}{\frac{1}{2}(2m)\left(\frac{2qEt}{2m}\right)^2}$$

Hence, the correct answer is (A).

140.



$$\Rightarrow \quad qE = mg$$

$$\Rightarrow \quad neE = mg$$

$$\Rightarrow \quad n=\frac{mg}{eE}=\frac{1.6\times10^{-3}\times9.8}{1.6\times10^{-19}\times10^{9}}$$

$$\Rightarrow \quad n = 9.8\times10^{7}$$

Hence, the correct answer is (A).

141. $F=\dfrac{K(Q)(2Q)}{r^2}$

When allowed to touch, we have

$$F'=\frac{K\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{r^2}$$

$$\Rightarrow \quad F'=\frac{F}{8}$$

Hence, the correct answer is (C).

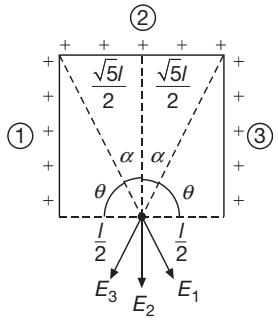
142. Since work done by electric force is equal to the gain in elastic potential energy of the spring, so

$$\frac{1}{2}kx^2 = qEx$$

$$\Rightarrow \quad x=\frac{2qE}{k}$$

Hence, the correct answer is (A).

143.



By Symmetry, $\left|\vec{E}_1\right|=\left|\vec{E}_3\right|$

So, on resolution, their x components will cancel. However their y - components will add up to give

$$(E_{1+3})y=\frac{2\lambda}{4\pi\varepsilon_0\frac{\ell}{2}}(\cos0^\circ-\cos\theta)=\frac{4\lambda}{4\pi\varepsilon_0\ell}\left(1-\frac{1}{\sqrt{5}}\right)$$

Similarly, we have

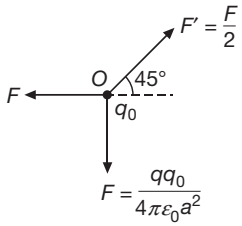
$$E_2=(E_2)_y=\frac{\lambda}{4\pi\varepsilon_0\ell}2\sin\alpha=\frac{2\lambda}{4\pi\varepsilon_0\ell}\left(\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow \quad E_{\text{net}}=(E_{1+3})_y+(E_2)_y=\frac{2\lambda}{4\pi\varepsilon_0\ell}\left(2-\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow \quad E_{\text{net}}=\frac{\lambda}{2\sqrt{5}\pi\varepsilon_0\ell}(2\sqrt{5}-1)$$

Hence, the correct answer is (A).

144.



So, $F_{net}=\sqrt{2}F-\dfrac{F}{2}$

$$\Rightarrow \quad F_{net}=\left(\sqrt{2}-\frac{1}{2}\right)\frac{qq_0}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (C).

145. $mg = qE$

$$\Rightarrow \quad mg=(Ne)E \qquad \{\because q=\text{Ne}\}$$

CHAPTER 1

$$\Rightarrow \quad \frac{4}{3}\pi r^3 \times \rho \times g = NeE$$

$$\Rightarrow \quad N = \frac{4\pi r^3 \rho g}{3eE}$$

Number of moles of the metal in sphere is $n = \frac{m}{A}$

$$\Rightarrow \quad n = \frac{4}{3}\frac{\pi r^3 \rho}{A}$$

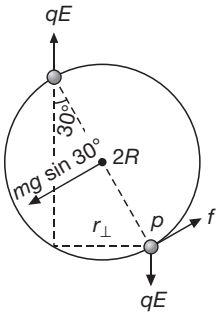
So, number of metal atoms $= \frac{4}{3}\frac{\pi r^3 \rho \times N_A}{A}$

Total number of electrons $= \frac{4}{3}\frac{\pi r^3 \rho N_A \times Z}{A} = N_0$

So, required fraction $= \frac{N}{N_0} = \frac{gA}{eEZN_A}$

Hence, the correct answer is (C).

146.



For system to be in equilibrium, $\tau = 0$

Let us take the torque about the point P, so

$$qE[2R\sin(30°)] = [mg\sin(30°)]R$$

$$\Rightarrow \quad E = \frac{mg}{2q}$$

Hence, the correct answer is (B).

147. Assume a charge q to be placed at the eight corner too. Then by symmetry, we have

$$\vec{E} = \vec{0}$$

$$\Rightarrow \quad \vec{E}_{01} + \vec{E}_{02} + ... + \vec{E}_{08} = \vec{0}$$

$$\Rightarrow \quad \left|\vec{E}_{01} + \vec{E}_{02} + ... + \vec{E}_{07}\right| = \left|\vec{E}_{08}\right| = \frac{q}{4\pi\varepsilon_0\left(\frac{\sqrt{3}\ell}{2}\right)^2}$$

$$\Rightarrow \quad E_{\text{centre}} = E_0 = \frac{q}{3\pi\varepsilon_0\ell^2}$$

Hence, the correct answer is (B).

148.

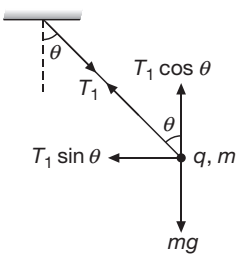


Figure 1

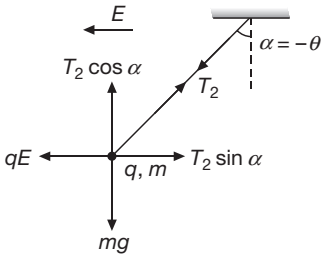


Figure 2

When no field is applied as in Figure 1. Then, we have,

$$T_1\cos\theta = mg \qquad \ldots(1)$$

$$T_1\sin\theta = \frac{mv^2}{r} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg} \qquad \ldots(3)$$

When an external field is applied such that the pendulum makes an angle θ to the opposite side, then $\alpha = -\theta$. From Figure 2, we get

$$T_2\cos\alpha = mg \qquad \ldots(4)$$

$$T_2\sin\alpha - qE = \frac{mv^2}{r}$$

$$\Rightarrow \quad T_2\sin\alpha = \frac{mv^2}{r} + qE \qquad \ldots(5)$$

$$\Rightarrow \quad \tan\alpha = -\tan\theta = \frac{\frac{mv^2}{r} + qE}{mg}$$

$$\Rightarrow \quad -\frac{v^2}{rg} = \frac{\frac{mv^2}{r} + qE}{mg}$$

$$\Rightarrow \quad -\frac{mv^2}{r} = \frac{mv^2}{r} + qE$$

$$\Rightarrow \quad qE = -\frac{2mv^2}{r}$$

$$\Rightarrow \quad E = -\frac{2mv^2}{qr}$$

(negative sign implies opposite direction)

$$\Rightarrow \quad |E| = E = \frac{2mv^2}{qr}$$

Hence, the correct answer is (C).

149. $F = 10 \times 10^{-3} \times 10 \text{ N} = 10^{-1}$

Since $F = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad q = \sqrt{4\pi\varepsilon_0 F r^2}$

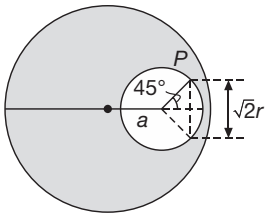
$\Rightarrow \quad q = \sqrt{\dfrac{0.1 \times 0.36}{9 \times 10^9}} = 2 \times 10^{-6} \text{ C}$

Hence, the correct answer is (D).

150. $E = \dfrac{\rho a}{3\varepsilon_0}$

$\Rightarrow \quad F = eE = \dfrac{e\rho a}{3m\varepsilon_0}$

Acceleration $= \dfrac{\rho e a}{3m\varepsilon_0}$



Since $s = \dfrac{1}{2}\left(\dfrac{\rho e a}{3m\varepsilon_0}\right)t^2 = \sqrt{2}r$

$\Rightarrow \quad t = \sqrt{\dfrac{6\sqrt{2}r\varepsilon_0 m}{\rho e a}}$

Hence, the correct answer is (A).

151. $F = \left(\dfrac{\sigma^2}{2\varepsilon_0}\right)A$

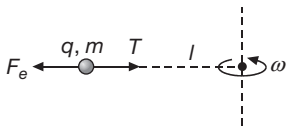
Since a soap bubble has 2 free surfaces, so $F = 2T(2\pi R)$

$\Rightarrow \quad 2[T(2\pi R)] = \dfrac{\sigma^2}{2\varepsilon_0}(\pi R^2)$

$\Rightarrow \quad T = \dfrac{\sigma^2 R}{8\varepsilon_0}$

Hence, the correct answer is (A).

152.



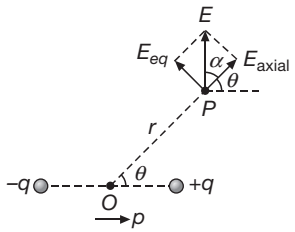
$T - F_e = m\ell\omega^2$

$\Rightarrow \quad T = \dfrac{q^2}{4\pi\varepsilon_0 (2\ell)^2} + m\ell\omega^2$

$\Rightarrow \quad T = \dfrac{q^2}{16\pi\varepsilon_0 \ell^2} + m\ell\omega^2$

Hence, the correct answer is (D).

153. Angle of resultant with x-axis will be $\alpha + \theta$, where $\tan\alpha = \dfrac{1}{2}\tan\theta$



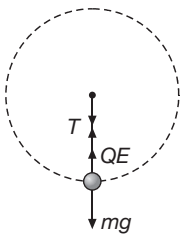
Angle $\phi = \tan^{-1}\left(\dfrac{\tan\theta}{2}\right) + \theta$

$\Rightarrow \quad \phi = \tan^{-1}\left(\dfrac{\tan 45}{2}\right) + \theta$

$\Rightarrow \quad \phi = \tan^{-1}(0.5) + \dfrac{\pi}{4}$

Hence, the correct answer is (B).

154.



Since $(T + QE) - mg = \dfrac{mv^2}{L}$

$\Rightarrow \quad 10mg + QE - mg = \dfrac{mv^2}{L}$

$\Rightarrow \quad 9mg + \dfrac{QE}{m} = \dfrac{mv^2}{L}$

$\Rightarrow \quad v = \sqrt{L\left(9g + \dfrac{QE}{m}\right)}$

Hence, the correct answer is (C).

155. $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad \log_e F = \log_e \dfrac{q_1 q_2}{4\pi\varepsilon_0} - 2\log_e r$

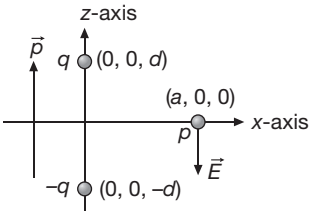
CHAPTER 1

$\Rightarrow \quad y = c + mx$

$\Rightarrow \quad m = -2$

Hence, the correct answer is (A).

157.



Since the point $p\ (a, 0\ 0)$ lies at the equitorial line of dipole, so we have $E = \dfrac{p}{4\pi\varepsilon_0 (r^2 + \ell^2)^{\frac{3}{2}}}$

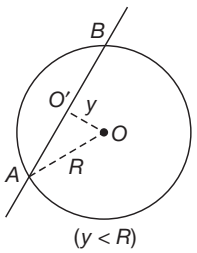
where $p = q(2d)$

$\Rightarrow \quad E = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q \times 2d}{(a^2 + d^2)^{\frac{3}{2}}}$

$\Rightarrow \quad \vec{E} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2qd}{(a^2 + d^2)^{\frac{3}{2}}} (-\hat{k})$

Hence, the correct answer is (C).

158. Electric flux $\oint_S \vec{E} \cdot \vec{dS} = \dfrac{q_{enc}}{\varepsilon_0}$, where q_{enc} is the charge enclosed by the Gaussian-surface which, in the present case, is the surface of given sphere. As shown, length AB of the line lies inside the sphere.



In $\Delta OO'A \quad R^2 = y^2 + (O'A)^2$

$\Rightarrow \quad O'A = \sqrt{R^2 - y^2}$

and $AB = 2\sqrt{R^2 - y^2}$

Charge on length AB is $q_{enc} = 2\sqrt{R^2 - y^2} \times \lambda$

So, electric flux is $\phi = \oint_S \vec{E} \cdot d\vec{A} = \dfrac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$

Hence, the correct answer is (C).

159. $\vec{\tau} = \vec{p} \times \vec{E} = q(2\vec{a}) \times \vec{E}$

Here, $2\vec{a} = (2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k} = \hat{i} - \hat{j} + \hat{k}$

$\vec{E} = 0.20\hat{i}\ \text{Vcm}^{-1} = 20\hat{i}\ \text{Vm}^{-1}$

$\Rightarrow \quad \vec{\tau} = \left(4 \times 10^{-6}\right)\left[\left(\hat{i} - \hat{j} + \hat{k}\right) \times 20\hat{i}\right] = 8 \times 10^{-5}\left(\hat{k} + \hat{j}\right)$

Magnitude of torque is

$\tau = 8\sqrt{2} \times 10^{-5}\ \text{Nm}$

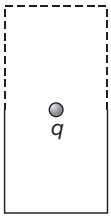
Hence, the correct answer is (C).

160. $\dfrac{\Sigma q}{\varepsilon_0} = \phi_{net} = -8 \times 10^3 + 4 \times 10^3 = -4 \times 10^3$

$\Rightarrow \quad \Sigma q_{enc} = -4000\varepsilon_0$ coulomb

Hence, the correct answer is (D).

161.

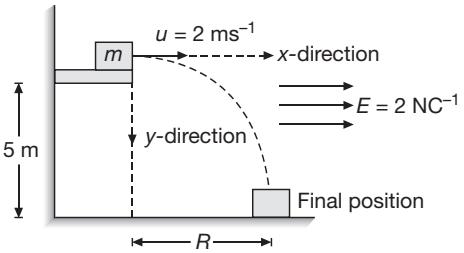


$\phi_{Gaussian} = \dfrac{q}{\varepsilon_0}$

$\Rightarrow \quad \phi_{vessel} = \dfrac{1}{2}\phi_{Gaussian} = \dfrac{q}{2\varepsilon_0}$

Hence, the correct answer is (A).

162.



Along y-direction

$S_y = u_y t + \dfrac{1}{2} a_y t^2 \qquad \ldots(1)$

$\Rightarrow \quad 5 = 0 + \dfrac{1}{2} \times g \times t^2$

$t = \sqrt{\dfrac{10}{g}} = 1\ s$

Along x-direction

$S_x = u_x t + \dfrac{1}{2} a_x t^2 \qquad \ldots(2)$

$$\Rightarrow \quad R = 2\times t + \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$\Rightarrow \quad R = 2\times 1 + \frac{1}{2}\times\frac{2\times10^{-9}\times 2}{2\times10^{-9}}\times 1$$

$$\Rightarrow \quad R = 2+1 = 3\text{ m}$$

Hence, the correct answer is (D).

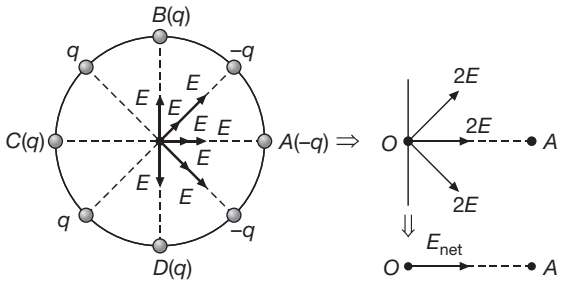
163. $E_{arc} = \dfrac{2\lambda}{4\pi\varepsilon_0 R}\sin\left(\dfrac{\theta}{2}\right)$

Field due to a single arc subtending an angle of $60°$ at centre is $E_1 = \dfrac{2\lambda}{4\pi\varepsilon_0 R}\left(\dfrac{1}{2}\right) = \dfrac{\lambda}{4\pi\varepsilon_0 R}$, where $\lambda = \dfrac{6Q}{2\pi R}$

$$\Rightarrow \quad E_{total} = \frac{6\lambda}{4\pi\varepsilon_0 R}, \text{ where } \lambda = \frac{3Q}{\pi R}$$

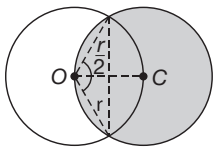
Hence, the correct answer is (D).

164. Since magnitude of each charges are same and situated at equal distance from centre O, so all charges will produce same magnitude of electric field at centre.



Hence, the correct answer is (A).

165.



$$\cos\theta = \frac{\left(\frac{r}{2}\right)}{r} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

Length of ring in sphere is

$$\ell = r\times 2\theta = \frac{2\pi r}{3}$$

So, charge enclosed in the sphere is

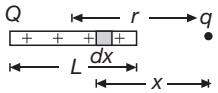
$$q_{enc} = \frac{Q}{2\pi r}\times\frac{2\pi r}{3} = \frac{Q}{3}$$

Hence, flux by Gauss Law is

$$\phi = \frac{Q}{3\varepsilon_0}$$

Hence, the correct answer is (C).

166.



$$dF = \frac{q\left(\frac{Q}{L}\right)dx}{4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad F = \frac{Qq}{4\pi\varepsilon_0 L}\int_{r-\frac{L}{2}}^{r+\frac{L}{2}}\frac{dx}{x^2} = -\frac{Qq}{4\pi\varepsilon_0 L}\left(\frac{1}{r+\left(\frac{L}{2}\right)} - \frac{1}{r-\left(\frac{L}{2}\right)}\right)$$

$$\Rightarrow \quad F = \frac{Qq}{4\pi\varepsilon_0\left(r^2 - \frac{L^2}{4}\right)} = \frac{Qq}{\pi\varepsilon_0\left(4r^2 - L^2\right)}$$

Hence, the correct answer is (B).

167. Effective distance in vacuum is given by

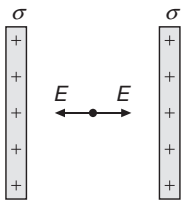
$$r = \sqrt{k_1}t_1 + \sqrt{k_2}t_2 - \left(t_1 + t_2\right)$$

So, $F = \dfrac{Q_1Q_2}{4\pi\varepsilon_0\left(t_1\sqrt{k_1} + t_2\sqrt{k_2} - t_1 - t_2\right)^2}$

$$\Rightarrow \quad F = \frac{Q_1Q_2}{4\pi\varepsilon_0\left[\left(\sqrt{k_1}-1\right)t_1 + \left(\sqrt{k_2}-1\right)t_2\right]^2}$$

Hence, the correct answer is (D).

168.



Electric field due to infinite sheet is independent of distance and hence electric field due to both sheets will be equal and opposite.

Hence, the correct answer is (D).

170. $f_\ell \geq F_e$, where $F_e = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$

Since $f = \mu N = \mu mg$

$$\Rightarrow \quad \mu mg \geq \frac{q^2}{4\pi\varepsilon_0 r^2}$$

CHAPTER 1

$$\Rightarrow \quad \mu \geq \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$$

Hence, the correct answer is (B).

171. Using Gauss Theorem, we have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ for } r \leq R \text{ and}$$

$$E = 0 \text{ for } r \geq R$$

So, correct graph is (A)

Hence, the correct answer is (A).

172. (–10 °C ice) (0 °C water)

$$q_1 \frac{k_1}{r_1 = 25 \text{ cm}} q_2 \qquad q_1 \frac{k_2 = 80}{r_2 = 5 \text{ cm}} q_2$$

Given $F_{\text{ice}} = F_{\text{water}}$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{k_1 r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{k_2 r_2^2}$$

$$\Rightarrow \quad k_1 r_1^2 = k_2 r_2^2$$

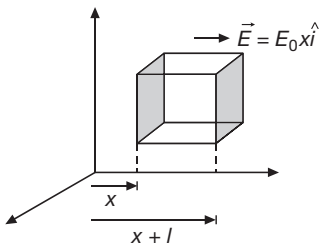
$$\Rightarrow \quad k_1 = k_2\left(\frac{r_2}{r_1}\right)^2 = 80 \times \left(\frac{5}{25}\right)^2 = 3.2$$

Hence, the correct answer is (B).

173. $\phi_{\text{net}} = E_0(x+\ell)\ell^2 - E_0 x\ell^2 = E_0\ell^3$

Since, $\sum q = (\phi_{\text{net}})\varepsilon_0$

$$\Rightarrow \quad q_{\text{enc}} = \varepsilon_0 E_0 \ell^3$$



Hence, the correct answer is (B).

175. Since $\phi_A + \phi_C + \phi_B = \frac{q}{\varepsilon_0}$

and $\phi_A = \phi_C$

$$\Rightarrow \quad \phi_A + \phi_A + \phi = \frac{q}{\varepsilon_0}$$

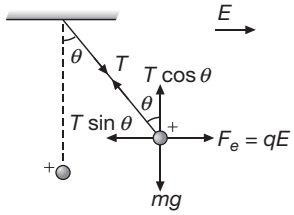
$$\Rightarrow \quad \phi_A = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$$

Hence, the correct answer is (D).

176. θ is inversely proportional to L

Hence, the correct answer is (A).

177.



For equilibrium

$$T\cos\theta = mg \text{ and}$$

$$T\sin\theta = qE$$

$$\Rightarrow \quad T = \sqrt{(mg)^2 + (qE)^2}$$

$$\Rightarrow \quad T = \sqrt{(80\times10\times10^{-6})^2 + (2\times10^{-8}\times2\times10^4)^2}$$

$$\Rightarrow \quad T = \sqrt{64\times10^{-8} + 16\times10^{-8}}$$

$$\Rightarrow \quad T = \sqrt{80\times10^{-8}} = 8.8\times10^{-4} \text{ N}$$

Hence, the correct answer is (C).

178. $\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0} = \frac{1 \text{ C/cm} \times 100 \text{ cm}}{\varepsilon_0} = \frac{100 \text{ C}}{\varepsilon_0}$

Hence, the correct answer is (D).

179. Since, positive charge diverts in the direction of field, so field must be upwards

So, particle (2) deflects down, (3) up and particle (4) deflects down

Hence, the correct answer is (A).

180. $\vec{A} = 100\hat{k}$

$$\Rightarrow \quad \phi = \vec{E}\cdot\vec{A} = 300 \text{ units}$$

Hence, the correct answer is (B).

181. $E_P = \frac{q}{4\pi\varepsilon_0 r^2}$

$$E_Q = \frac{q}{4\pi\varepsilon_0(2r)^2} + \frac{3q}{4\pi\varepsilon_0(2r)^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \quad E_P : E_Q = 1:1$$

Hence, the correct answer is (C).

182. For downward motion with constant velocity, we have

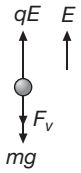
$$q = 2e \text{ and } m = 3.2\times10^{-17} \text{ kg}$$

Since motion is downwards, so viscous force is upwards.

$$6\pi\eta rv = mg \qquad \ldots(1)$$

For upwards motion with same constant speed, we have

$$6\pi\eta rv + mg = qE \qquad \ldots(2)$$



Substitute (1) in (2), we get

$$qE = 2mg$$

$$\Rightarrow \quad E = \frac{2mg}{q} = \frac{2\times 3.2\times 10^{-17}\times 10}{2\times 1.6\times 10^{-19}} = 2\times 10^{3}\ \text{Vm}^{-1}$$

Hence, the correct answer is (A).

183. $\phi_{\text{cube}} = \dfrac{q}{\varepsilon_0}$

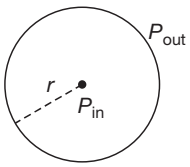
$$\Rightarrow \quad \phi_{\text{face}} = \frac{1}{6}\phi_{\text{cube}} = \frac{q}{6\varepsilon_0} = \frac{4\pi q}{24\pi\varepsilon_0}$$

Hence, the correct answer is (A).

184. The electric field due to Q at any point of square plate will be along the plane of square, therefore angle between electric field and area vector of plate is $90°$ and hence $\phi = 0$

Hence, the correct answer is (D).

185. Inside pressure must be $\dfrac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble. So, we have



$$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$$

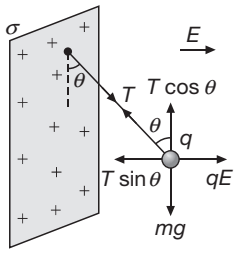
Since $\sigma = \dfrac{Q}{4\pi r^2}$

$$\Rightarrow \quad \frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0}$$

$$\Rightarrow \quad Q = 8\pi r\sqrt{2rT\varepsilon_0}$$

Hence, the correct answer is (B).

186.



$$T\sin\theta = qE \qquad \ldots(1)$$

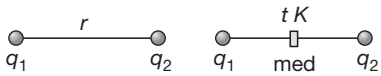
$$T\cos\theta = mg \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2\varepsilon_0 mg}$$

$$\Rightarrow \quad \sigma = \frac{2\varepsilon_0 mg\tan\theta}{q}$$

Hence, the correct answer is (B).

187.



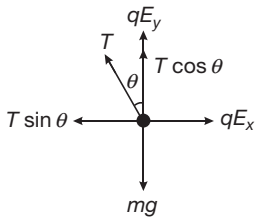
$$F_1 = \frac{Kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \text{ and } F_2 = \frac{q_1q_2}{4\pi\varepsilon_0(r - t + t\sqrt{k})^2}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $T\cos\theta + qE_y = mg \qquad \ldots(1)$

$$T\sin\theta = qE_x \qquad \ldots(2)$$



$$\Rightarrow \quad T(0.6) = 3\times 10^{5} q$$

$$\Rightarrow \quad q = \frac{6}{30}\times 10^{-5}\ T$$

$$\Rightarrow \quad q = 2\times 10^{-6} T$$

CHAPTER 1

$\Rightarrow \quad T(0.8)+(2\times10^{-6}T)(5\times10^{5})=\dfrac{1}{1000}(10)$

$\Rightarrow \quad T(0.8)+T=\dfrac{10}{1000}$

$\Rightarrow \quad T=\dfrac{1}{(100)(1.8)}$

$\Rightarrow \quad T=5.55\times10^{-3}\ \text{N}$

$\Rightarrow \quad q=11.1\times10^{-9}\ \text{C}$

$\Rightarrow \quad q\simeq11\ \text{nC}$

Hence, (A) and (C) are correct.

2. $\vec{E}_A$ is along $\overrightarrow{OA}$ and $\vec{E}_B$ is along $\overrightarrow{OB}$, where

$\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}$

Since, we observe that $\overrightarrow{OA}\cdot\overrightarrow{OB}=0$

$\Rightarrow \quad \vec{E}_A\perp\vec{E}_B$ (dot product of perpendicular vectors is zero)

Also, $|\vec{E}|\propto\dfrac{1}{r^2}$

and $|\overrightarrow{OC}|=2|\overrightarrow{OB}|$

$\Rightarrow \quad |\vec{E}_B|=4|\vec{E}_C|$

Hence, (A) and (C) are correct.

5. Charge density at A < charge density at B. Since field inside cavity is zero, hence
Potential at A = potential at B = a constant value.
By Gauss Theorem,

$$\begin{pmatrix}\text{Total electric}\\ \text{flux}\end{pmatrix}=\frac{1}{\varepsilon_0}\begin{pmatrix}\text{charge}\\ \text{enclosed}\end{pmatrix}=\left(\frac{q}{\varepsilon_0}\right)$$

Hence, (C) and (D) are correct.

7. Field lines must enter a surface or leave a surface normally.

Hence, option (D) is correct.

8. $t=\sqrt{\dfrac{2d}{a}}$ when $a=\dfrac{qE}{m}=\dfrac{qV}{md}$

$\Rightarrow \quad t=\sqrt{\dfrac{2d}{\frac{qV}{md}}}=\sqrt{\dfrac{2md^2}{qV}}$

$\Rightarrow \quad \dfrac{t}{T}=\sqrt{\dfrac{(2d)^2}{d^2}}=2$

$\Rightarrow \quad t=2T$

Hence, option (D) is correct.

10. $\dfrac{mv^2}{r}=qE=q\left(\dfrac{\lambda}{2\pi\varepsilon_0 r}\right)$

$\Rightarrow \quad v=\sqrt{\dfrac{q\lambda}{2\pi\varepsilon_0 m}}$

Hence, (A), (B) and (C) are correct.

14. $E=E_0t$

Since $a=\dfrac{qE}{m}=\dfrac{qE_0t}{m}=E_0st$

$\Rightarrow \quad \dfrac{dv}{dt}=E_0st \quad \Rightarrow \quad dv=E_0st\,dt$

$\Rightarrow \quad v=E_0s\displaystyle\int_0^t t\,dt=\frac{1}{2}E_0st^2$

$\Rightarrow \quad \dfrac{dx}{dt}=\dfrac{1}{2}E_0st^2$

$\Rightarrow \quad dx=\dfrac{1}{2}E_0st^2dt$

$\Rightarrow \quad \displaystyle\int_0^x dx=\frac{1}{2}E_0s\int_0^t t^2dt$

$\Rightarrow \quad x=\dfrac{1}{6}E_0st^3$

Hence, (B) and (D) are correct.

16. $\dfrac{1}{2}mv^2=QV$

$\Rightarrow \quad \dfrac{p^2}{2m}=QV$

$\Rightarrow \quad p=\sqrt{2mQV}$

$\Rightarrow \quad t=\dfrac{v}{a}=\dfrac{\sqrt{\frac{2QV}{m}}}{\frac{Q}{m}\left(\frac{V}{d}\right)}$ $\{\because\ v=0+at\}$

$\Rightarrow \quad t=\sqrt{\dfrac{2md^2}{QV}}$

Hence, (B) and (D) are correct.

18. $a=\dfrac{qE}{m}=sE$

Since, $L=ut$

$\Rightarrow \quad t=\dfrac{L}{u}$

Also, $\Delta = \frac{1}{2}at^2$

$\Rightarrow \quad \Delta = \frac{1}{2}(sE)\frac{L^2}{u^2}$

$\Rightarrow \quad \Delta = \frac{EsL^2}{2u^2}$

$\tan\theta = \frac{v_y}{v_x} = \frac{at}{u}$

$\Rightarrow \quad \tan\theta = \frac{Est}{u} = \frac{EsL}{u^2}$

Hence, (A), (B), (C) and (D) are correct.

20. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{R} + \frac{q_B}{2R}\right) = 2V$ …(1)

and, $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{2R} + \frac{q_B}{2R}\right) = \frac{3}{2}V$ …(2)

Solving equations (1) and (2), we get

$\frac{q_A}{q_B} = \frac{1}{2}$

When B is earthed, potential of B becomes zero. So,

$q_B' = -q_A' = -q_A$ {charge on A remains same}

$\Rightarrow \quad \frac{q_A'}{q_B'} = -1$

Also after earthing,

$V_A - V_B = \frac{q_A}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{q_A}{8\pi\varepsilon_0 R}$

Substituting $q_B = 2q_A$ in equation (1), we get

$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{2R} = \frac{V}{2}$

$\Rightarrow \quad V_A - V_B = \frac{V}{2}$

Since B is earthed, so $V_B = 0$ and hence $V_A = \frac{V}{2}$

Hence, (A) and (D) are correct.

22. Potential of innermost shell is zero, so

$\frac{Q_1}{r} + \frac{Q_2}{2r} + \frac{Q_3}{3r} = 0$

$\Rightarrow \quad 6Q_1 + 3Q_2 + 2Q_3 = 0$ …(1)

Similarly, potential of the outermost shell is also zero. So,

$\Rightarrow \quad \frac{Q_1}{3r} + \frac{Q_2}{3r} + \frac{Q_3}{3r} = 0$

$\Rightarrow \quad Q_1 + Q_3 = -Q_2$ …(2)

Solving equations (1) and (2), we get

$Q_1 = -\frac{Q_2}{4}$, $\frac{Q_3}{Q_1} = 3$ and $\frac{Q_3}{Q_2} = -\frac{3}{4}$

Hence, (A), (B) and (C) are correct.

24. $a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x)$ …(1)

$\Rightarrow \quad a = \frac{q\alpha}{m} - \frac{q\beta}{m}x$

$\Rightarrow \quad \ddot{x} = \frac{q\alpha}{m} - \frac{q\beta}{m}x$

Let $\frac{q\alpha}{m} - \frac{q\beta}{m}x = X$

$\Rightarrow \quad -\frac{q\beta}{m}\ddot{x} = \ddot{X}$

$\Rightarrow \quad \ddot{x} = -\frac{m}{q\beta}\ddot{X}$

$\Rightarrow \quad -\frac{m}{q\beta}\ddot{X} = X$

$\Rightarrow \quad \ddot{X} + \frac{q\beta}{m}X = 0$

$\Rightarrow \quad \ddot{X} + \omega^2 X = 0$

So, the motion of the particle is oscillatory and we observe that $a = 0$ at $x = \frac{\alpha}{\beta}$

At the mean position, no force acts on the particle

$\Rightarrow \quad F = 0$ at $x = x_0 = \frac{\alpha}{\beta}$

Equation (1) can be written as,

$v \cdot \frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$

$\Rightarrow \quad \int_0^v v\,dv = \frac{q}{m}\int_0^x (\alpha - \beta x)dx$

$\Rightarrow \quad v = \sqrt{\frac{2qx}{m}\left(\alpha - \frac{\beta}{2}x\right)}$

$v = 0$ at $x = 0$ and $x = \frac{2\alpha}{\beta}$

So, the particle reverses its direction of motion at $x = \frac{2\alpha}{\beta}$ (extreme position) and hence the particle will oscillate between $x = 0$ to $x = \frac{2\alpha}{\beta}$ with mean position

CHAPTER 1

at $x=\frac{\alpha}{\beta}$. So, amplitude of particle is $\frac{\alpha}{\beta}$ with maximum acceleration of particle at extreme positions (at $x=0$ and $x=\frac{2\alpha}{\beta}$ and $a_{\max}=\frac{q\alpha}{m}$ {from equation (1)}.

Hence, (A), (B), (C) and (D) are correct.

26. The electron will experience a downward acceleration a, given by

$$a=\frac{eE}{m}=\frac{1.6\times10^{-19}\times2000}{9\times10^{-31}}$$

$$\Rightarrow\quad a=\frac{eE}{m}=3.6\times10^{14}\ \text{ms}^{-2}$$

So, we can consider the electron to be a projectile launched with some initial velocity between the plates under influence of downward acceleration a.

From the point of launch let us consider the rightward horizontal as the x-axis and upward vertical as y-axis, let us calculate the height to which the electron will rise and its horizontal range. Considering upward vertical motion, we have

$$0=6\times10^{6}\sin45°t+\frac{1}{2}\times\left(-3.5\times10^{14}\right)t^{2}$$

$$\Rightarrow\quad t=2.4\times10^{-8}\ \text{s}$$

Considering motion along x-axis, we get

$$x(\text{range})=6\times10^{6}\cos45°\times2.4\times10^{-8}$$

$$\Rightarrow\quad x=0.102\ \text{m}=10.2\ \text{cm}$$

The vertical displacement in half the total time i.e., at maximum height, we have

$$t=\frac{T}{2}$$

The vertical height is given by

$$y=\left(6\times10^{6}\sin45°\right)\left(1.2\times10^{-8}\right)-\frac{1}{2}\left(3.5\times10^{14}\right)\left(1.2\times10^{-8}\right)^{2}$$

$$\Rightarrow\quad y=0.0509-0.0252$$

$$\Rightarrow\quad y=0.0257=2.57\ \text{cm}$$

So, the electron will not touch the upper plate and will strike the lower plate at the other edge.

Hence, (B) and (C) are correct.

27. $$\frac{1}{2}mv^{2}=\frac{Q^{2}}{4\pi\varepsilon_{0}d}$$

$$\Rightarrow\quad d=\frac{Q^{2}}{2\pi\varepsilon_{0}mv^{2}}$$

Hence, (A), (B) and (D) are correct.

28. Retardation $a=\frac{\text{force}}{\text{mass}}=\frac{eE}{m}$

$$\Rightarrow\quad a=\frac{1.6\times10^{-19}\times1000}{9\times10^{-31}}=1.78\times10^{14}\ \text{ms}^{-2}$$

Since, $v^{2}-v_{0}^{2}=2as$

$$\Rightarrow\quad 0^{2}-\left(5\times10^{6}\right)^{2}=2\times\left(-1.78\times10^{14}\right)s$$

$$\Rightarrow\quad s=0.07\ \text{m}$$

Now, $v=v_{0}+at$

$$\Rightarrow\quad 0=5\times10^{6}-\left(1.78\times10^{14}\right)t$$

$$\Rightarrow\quad t=2.8\times10^{-8}\ \text{s}=0.03\ \mu\text{s}$$

Loss of energy = work done $W=Fd=(eE)d$

$$\Rightarrow\quad \text{Fraction}\ f=\frac{eEd}{\frac{1}{2}mv^{2}}=\frac{2eEd}{mv^{2}}$$

$$\Rightarrow\quad \%\text{age Fraction is}$$

$$f=\frac{2\times1.6\times10^{-19}\times1000\times0.8\times10^{-2}}{9\times10^{-31}\times\left(5\times10^{6}\right)^{2}}=11\%$$

Hence, (B), (C) and (D) are correct.

30. The potential due to a pair of rings each of radius R and carrying charges $+q$ and $-q$ at a distance x from the mid point along the axis is given by

$$V=\frac{q\ell x}{4\pi\varepsilon_{0}\left(R^{2}+x^{2}\right)^{3/2}}$$

Now, consider a pair of rings of radius z and thickness dz. If dq be the infinitesimal charge on each ring, then

$$dq=2\pi z\sigma dz$$

and if dV be the potential due to this infinitesimal pair of elements at P

$$\Rightarrow\quad dV=\frac{(2\pi z\sigma dz)\ell x}{4\pi\varepsilon_{0}\left(z^{2}+x^{2}\right)^{3/2}}$$

$$\Rightarrow\quad dV=\frac{\sigma\ell x}{2\varepsilon_{0}}\frac{zdz}{\left(z^{2}+x^{2}\right)^{3/2}}$$

$$\Rightarrow\quad V=\frac{\sigma\ell x}{2\varepsilon_{0}}\int_{R}^{\infty}\frac{zdz}{\left(z^{2}+x^{2}\right)^{3/2}}$$

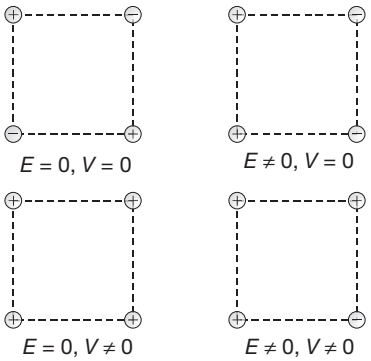
$$\Rightarrow\quad V=\frac{\sigma\ell x}{2\varepsilon_{0}\left(R^{2}+x^{2}\right)^{1/2}}$$

Since, $E=-\frac{\partial V}{\partial x}$

$$\Rightarrow \quad E = -\frac{\sigma \ell R^2}{2\varepsilon_0 \left(R^2 + x^2\right)^{3/2}}$$

Hence, (A) and (B) are correct.

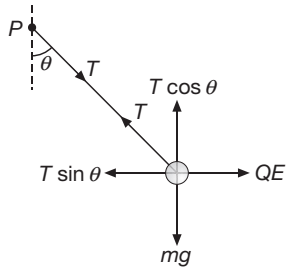
31.



Hence, (A), (B), (C) and (D) are correct.

32. Equating the forces in the vertical and horizontal directions

$$T\cos\theta = mg \text{ and } T\sin\theta = F = QE$$



Dividing, $\tan\theta = \frac{QE}{mg} = \frac{2\times10^{-8}\times2\times10^{4}}{71\times10^{-6}\times9.8}$

$$\Rightarrow \quad \tan\theta = 0.57 \cong \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30^\circ$$

Now, $T = \frac{71\times10^{-6}\times9.8}{\cos(30^\circ)} = 803\ \mu\text{N}$

Hence, (C) and (D) are correct.

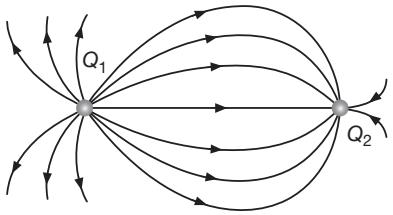
34. The given graph is of charged conducting sphere of radius. The entire charge distributes on the surface of the sphere.

Hence, (A), (B), (C) and (D) are correct.

36. Torque about Q of charge $-q$ is zero, so angular momentum charge $-q$ is constant, but distance between charges is changing, so force is charging, so speed and velocity are changing.

Hence, option (A) is correct.

38.



From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

Number of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude and opposite sign to that of Q_1.

Hence, (A) and (D) are correct.

40. Using Work Energy Theorem, we get

$$\frac{1}{2}mv^2 = qE\left(\ell - \ell\cos(60^\circ)\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{qE\ell}{m}}$$

So, OPTION (B) is correct.

At point B, we have

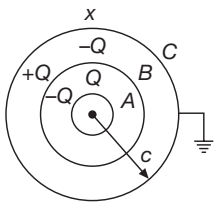
$$T - qE = \frac{mv^2}{\ell}$$

$$\Rightarrow \quad T = qE + \frac{m}{\ell}\left(\sqrt{\frac{qE\ell}{m}}\right)^2 = 2qE$$

So, OPTION (D) is also correct.

Hence, (B) and (D) are correct.

41. The distribution of charges is shown in figure. We see that the inner surface of B and C possess the same charge $-Q$. Also if x is charge on outer surface of C, then



$$V_C = 0$$

$$\Rightarrow \quad \frac{Q}{4\pi\varepsilon_0 c} + \left(\frac{-Q+Q}{4\pi\varepsilon_0 c}\right) + \frac{-Q+x}{4\pi\varepsilon_0 c} = 0$$

$$\Rightarrow \quad x = 0$$

Hence, (A) and (D) are correct.

CHAPTER 1

42. $\vec{E}=\dfrac{Q_1}{4\pi\varepsilon_0\left(2\dfrac{\sqrt{5}}{2}\right)^3}\left(\dfrac{a}{2}\hat{i}+a\hat{j}\right)+\dfrac{Q_2}{4\pi\varepsilon_0\left(a\dfrac{\sqrt{5}}{2}\right)^3}\left(-\dfrac{a}{2}\hat{i}+a\hat{j}\right)$

Since, x component is zero, so

$$Q_1=Q_2$$

Hence, (A) and (C) are correct.

43. Now electric field outside the sphere is due to charge on outer surface of sphere as E due Q and $-Q$ (induced charge) gets cancelled. Also E due to conducting sphere for from it will be zero

Hence, (B) and (D) are correct.

44. If $q_1=q_2=Q$, then

$$F_i\propto q_1q_2$$

$$\Rightarrow\quad F_i\propto Q^2$$

After touching, new charges on each ball is

$$\frac{Q_1+Q_2}{2}=\frac{2Q}{2}=Q$$

$$F_{\text{new}}\propto Q^2$$

$$\Rightarrow\quad F_i=F_{\text{new}}$$

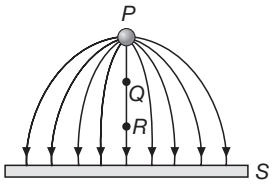
If $q_1\neq q_2$, then

$$F_{\text{new}}=\left(\frac{q_1+q_2}{2}\right)^2=\frac{q_1^2}{4}+\frac{q_2^2}{4}+\frac{2q_1q_2}{4}$$

$$\Rightarrow\quad F_{\text{new}}>F_i$$

Hence, (A) and (B) are correct.

46. As we are moving away from P towards sheet S spacing between electric lines of force is increasing.



$$\Rightarrow\quad E_R<E_Q$$

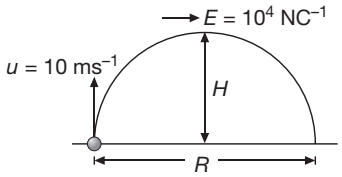
Also, in the direction of electric field potential decreases.

$$\Rightarrow\quad V_R<V_Q$$

Hence, (A) and (C) are correct.

47. Time of flight $(t)=\dfrac{2u}{g}=\dfrac{2\times10}{10}=2$ sec.

$$H=\frac{u^2}{2g}=\frac{10\times10}{2\times10}=5\text{ m}$$



$$R=0+\frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

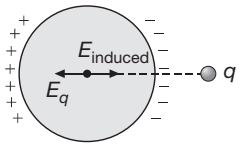
$$\Rightarrow\quad R=\frac{1}{2}\left(\frac{10^{-3}\times10^4\times2\times2}{2}\right)=10\text{ m}$$

Hence, (A), (B) and (C) are correct.

50. Charge is distributed over the surface of conductor in such a way that net field due to the charges inside and outside the conductor is zero inside. Field only due to the charge q inside the conductor is non-zero

Hence, (A), (B) and (C) are correct.

51.



Inside the conducting sphere, we have $E_{\text{net}}=0$

Since, $E_{\text{net}}=E_{\text{induced}}-E_{\text{due to q}}$

$$\Rightarrow\quad E_{\text{due to }q}=\frac{kq}{d^2}=E_{\text{induced}}$$

Hence, (A) and (D) are correct.

52. At a distance h above the sheet

$$E=E_{\text{sheet}}+E_{\text{slab}}$$

$$\Rightarrow\quad E=\frac{-\sigma}{2\varepsilon_0}+\frac{\rho D}{2\varepsilon_0}=\frac{\rho D-\sigma}{2\varepsilon_0}$$

At a distance h below the top surface of slab, we have

$$E_{\text{slab}}=\frac{\rho(D-2h)}{2\varepsilon_0}$$

$$\Rightarrow\quad E=E_{\text{sheet}}+E_{\text{slab}}=\frac{\sigma}{2\varepsilon_0}+\frac{\rho(D-2h)}{2\varepsilon_0}$$

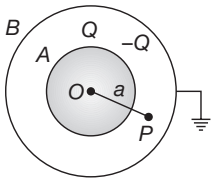
$$\Rightarrow\quad E=\frac{\sigma+\rho(D-2h)}{2\varepsilon_0}$$

At a distance h below the bottom surface of the slab

$$E=\frac{-\sigma}{2\varepsilon_0}+\frac{\rho D}{2\varepsilon_0}=\frac{\rho D-\sigma}{2\varepsilon_0}$$

Hence, (A), (B) and (D) are correct.

53.



$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, when $a \le r \le b$

$V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q}{r} - \dfrac{Q}{b}\right)$, where $a \le r \le b$

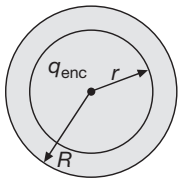
Potential of B is $V_B = 0$

Potential of A is $V_A = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q}{a} - \dfrac{Q}{b}\right)$

So, potential difference is $V_A - V_B = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

Hence, (A), (C) and (D) are correct.

54.

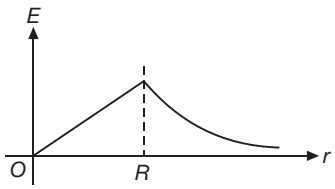


$E(4\pi r^2) = \dfrac{q_{enc}}{\varepsilon_0} = \dfrac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$

$\Rightarrow \quad E = \dfrac{\rho r}{3\varepsilon_0}$

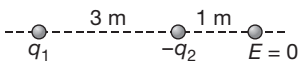
Slope of E-r graph (inside) i.e. for $r < R$ is $\dfrac{\rho}{3\varepsilon_0}$.

$\Rightarrow$ Slope $\propto \rho$



Hence, (B) and (C) are correct.

56.

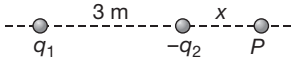


Since $\dfrac{kq_2}{1^2} = \dfrac{kq_1}{16}$

$\Rightarrow \quad q_1 = 16q_2$

Now, potential can be zero both inside and outside the charges on the line joining then

For a point P lying outside the two charges



$V_{\text{at } P} = 0$

$\Rightarrow \quad -\dfrac{kq_2}{x} + \dfrac{kq_1}{3+x} = 0$

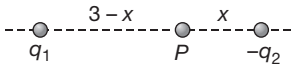
$\Rightarrow \quad +\dfrac{q_2}{x} = \dfrac{16q_2}{3+x}$

$\Rightarrow \quad 3 + x = 16x$

$\Rightarrow \quad 3 = 15x$

$\Rightarrow \quad x = \dfrac{1}{5} = 0.2 \text{ m}$

For a point P lying between the charges $V_{\text{at } P} = 0$



$\Rightarrow \quad \dfrac{kq_1}{3-x} = \dfrac{kq_2}{x}$

$\Rightarrow \quad 16x = 3 - x$

$\Rightarrow \quad x = \dfrac{3}{17}$ meter

Hence, (B) and (C) are correct.

58. $qEx_{\max} = \dfrac{1}{2}kx_{\max}^2$

$\Rightarrow \quad x_{\max} = \dfrac{2qE}{k}$

At Equilibrium, we have

$qE = kx_{eq}$

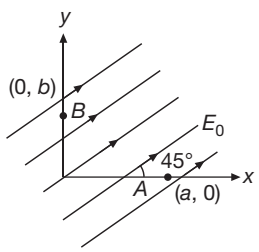
$\Rightarrow \quad x_{eq} = \dfrac{qE}{k}$

Amplitude of oscillation is equal to the displacement of block from the mean position, So, amplitude is

$A = \dfrac{2qE}{k} - \dfrac{qE}{k} = \dfrac{qE}{k}$

Hence, (A), (B) and (C) are correct.

59. When E_0 makes an angle of $45°$ with x-axis, then



CHAPTER 1

$$\vec{E} = \frac{E_0\hat{i}}{\sqrt{2}} + \frac{E_0\hat{j}}{\sqrt{2}}$$

Now, $V_A - V_B = -\vec{E}\cdot\overrightarrow{BA}$

$$\Rightarrow \quad V_A - V_B = -\left(\frac{E_0}{\sqrt{2}}\hat{i} + \frac{E_0}{\sqrt{2}}\hat{j}\right)\cdot\left(+a\hat{i} - b\hat{j}\right)$$

$$\Rightarrow \quad V_A - V_B = -\frac{E_0 a}{\sqrt{2}} + \frac{E_0 b}{\sqrt{2}} = \frac{E_0}{\sqrt{2}}(b-a)$$

If $a = b$

$\Rightarrow \quad V_A - V_B = 0$

If $a < b$

$\Rightarrow \quad V_A - V_B > 0$

$\Rightarrow \quad V_A > V_B$

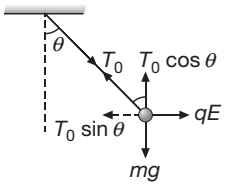
If $a > b$

$\Rightarrow \quad V_A - V_B < 0$

$\Rightarrow \quad V_A < V_B$

Hence, (B), (C) and (D) are correct.

60.



$$E = \frac{\sigma}{2\varepsilon_0}$$

Since, $T_0\cos\theta = mg$

$$T_0\sin\theta = qE$$

$$\Rightarrow \quad \tan\theta = \frac{q}{mg}\left(\frac{\sigma}{2\varepsilon_0}\right)$$

$$\Rightarrow \quad T_0 = \sqrt{(mg)^2 + (qE)^2}$$

$$\Rightarrow \quad T_0 > mg$$

i.e. the effective value of g is increased, hence time period T of oscillation decreases.

Hence, (A) and (D) are correct.

61. Near positive charge net potential is positive and near negative charge potential is negative. Q_1 is positive & Q_2 is negative, at 1 potential is zero and this point is near to Q_2. So, magnitude of Q_1 is more than magnitude of Q_2. Also, slope of $V - r$ graph gives E, which is zero at 3.

Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

1. Coulombic attraction exists even when one body is charged and the other is uncharged.

Hence, the correct answer is (C).

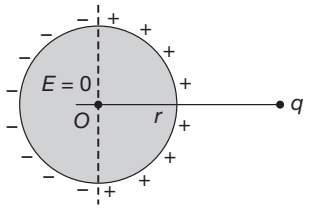
2. Force of interaction between two charge is independent of presence of other charge.

Hence, the correct answer is (B).

3. The rate of decrease of electric field is different in the two cases. In case of a point charge, it decreases as $\frac{1}{r^2}$ but in the case of electric dipole it decreases more rapidly, as $E = \frac{1}{r^2}$.

Hence, the correct answer is (D).

4. $$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\sum \text{Induced charge}}{r}\right)$$



$$\Rightarrow \quad V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + 0$$

Hence, potential is constant

Hence, the correct answer is (C).

6. Electric field due to line charge $E = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\alpha + \sin\beta)$

Hence, the correct answer is (D).

7. Electron being a negative charge, will move from lower to higher potential.

Hence, the correct answer is (D).

10. Since field is zero for a charged conductor on the surface and inside it also, so the surface of a charged conductor is always equipotential. Also, $\Delta V = 0$ for equipotential surface. Hence $\int \vec{E}\cdot d\vec{\ell} = 0$

$\Rightarrow \quad \vec{E} \perp d\vec{\ell}$. Hence both are true and Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

12. Path of charged particle is parabolic.

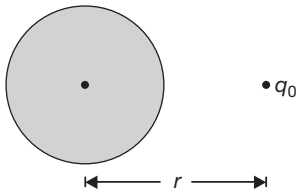
Hence, the correct answer is (C).

13. Inside the conductor, charged atoms are also present.
Hence, the correct answer is (B).

14. See theory part in detail to get the answer.
Hence, the correct answer is (B).

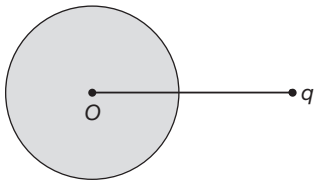
15. Statement-1 is also practical experience based; so it is true.
Statement-2 is also true but is not the correct explanation of Statement-1. Correct explanation is "there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled".
Hence, the correct answer is (B).

16. Electric field due to q_0 is towards left and is $\frac{q_0}{4\pi\varepsilon_0 r^2}$ but electric field due to induced charge is towards right and will have same magnitude $\frac{q_0}{4\pi\varepsilon_0 r^2}$ so that electric field inside the sphere is zero.



Hence, the correct answer is (D).

17.



At point O

$$\vec{E}_{\text{net}} = \vec{E}_{\text{in}} + \vec{E}_q$$

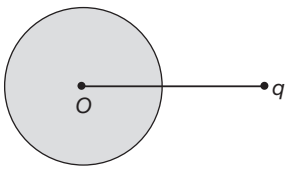
$$\vec{E}_{\text{net}} = 0$$

$$\Rightarrow \quad E_{in} + E_q$$

$$\Rightarrow \quad E_q = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \neq 0$$

Hence, the correct answer is (D).

18.



$$V_0 = \frac{q}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\int dq_{\text{induced}}}{R}\right)$$

$$\Rightarrow \quad V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \left\{\because \int dq_{\text{inside}} = 0\right\}$$

Hence, the correct answer is (A).

19. In a hollow spherical shield, the charge is present only on its surface but charge is zero at every point inside the hollow sphere. Hence, the metallic shield in form of hollow shell may be built to block an electric field.
Hence, the correct answer is (A).

20. Apply the concept of electric image.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. The magnitude of the force is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Now we can substitute our numerical values and find that the magnitude of the force between the proton and the electron in the hydrogen atom is

$$F_e = \frac{(9.0\times10^9\ \text{Nm}^2\text{C}^{-2})(1.6\times10^{-19}\ \text{C})^2}{(5.3\times10^{-11}\ \text{m})^2}$$

$$F_e = 8.2\times10^{-8}\ \text{N}$$

Hence, the correct answer is (C).

2. The magnitude of the electric field due to the proton is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{(90\times10^9\ \text{Nm}^2\text{C}^{-2})(1.6\times10^{-19}\ \text{C})}{(0.5\times10^{-10}\ \text{m})^2}$$

$$E = 5.76\times10^{11}\ \text{NC}^{-1}$$

Hence, the correct answer is (B).

3. The mass of the electron is $m_e = 9.1\times10^{-31}$ kg and the mass of the proton is $m_p = 1.7\times10^{-27}$ kg. Thus, the ratio of the magnitudes of the electric and gravitational force is given by

$$\text{Ratio} = \frac{\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)}{\left(G\frac{m_p m_e}{r^2}\right)} = \frac{\frac{1}{4\pi\varepsilon_0}e^2}{Gm_p m_e}$$

CHAPTER 1

$$\text{Ratio} = \frac{(9\times10^{9})(1.6\times10^{-19})^{2}}{(6.67\times10^{-11})(1.7\times10^{-27})(9.1\times10^{-31})}$$

$$\text{Ratio} = 2.2\times10^{39}$$

which is independent of r, the distance between the proton and the electron.

Hence, the correct answer is (C).

4. $M = \rho_{\text{oil}} V = \rho_{\text{oil}}\left(\frac{4}{3}\pi r^{3}\right)$

where the oil drop is assumed to be a sphere of radius r with volume $V = \frac{4\pi r^{3}}{3}$

$$\Rightarrow \quad M = \rho_{\text{oil}}\left(\frac{4}{3}\pi r^{3}\right)$$

$$\Rightarrow \quad M = \left(8.51\times10^{2}\ \text{kgm}^{-3}\right)\left(\frac{4\pi}{3}\right)\left(1.64\times10^{-6}\ \text{m}\right)^{3}$$

$$\Rightarrow \quad M = 1.57\times10^{-14}\ \text{kg}$$

Hence, the correct answer is (A).

5. The oil drop will be in static equilibrium when the gravitational force exactly balances the electrical force

$$\Rightarrow \quad \vec{F}_{g} + \vec{F}_{e} = 0$$

Since the gravitational force points downward so, the electric force on the oil must be acting upwards. So, we get

$$0 = m\vec{g} + q\vec{E}$$

$$\Rightarrow \quad mg = -qE$$

With the electrical field pointing downwards, we conclude that the charge on the oil drop must be negative.

$$q = -\frac{mg}{E_{y}}$$

$$q = -\frac{\left(1.57\times10^{-14}\ \text{kg}\right)\left(9.80\ \text{ms}^{-2}\right)}{1.92\times10^{5}\ \text{NC}^{-1}}$$

$$q = -8.03\times10^{-19}\ \text{C}$$

Since the electron has charge $e = 1.6\times10^{-19}$ C, the charge of the oil drop in units of e is

$$N = \frac{q}{e} = \frac{8.02\times10^{-19}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 5$$

Hence, the correct answer is (D).

6. Since the electron has a negative charge, $q = -e$, the force on the electron is

$$\vec{F}_{e} = q\vec{E} = -e\vec{E} = (-e)(-E)\hat{j} = eE\hat{j}$$

where the electric field is written as $\vec{E} = -E\hat{j}$, with $F_{y} > 0$. The force on the electron is upward. Note that the motion of the electron is analogous to the motion of a mass that is thrown horizontally in a constant gravitational field. The mass follows a parabolic trajectory downward. Since the electron is negatively charged, the constant force on the electron is upward and the electron will be deflected upwards on a parabolic path.

Hence, the correct answer is (D).

7. The acceleration of the electron is

$$\vec{a} = \frac{q\vec{E}}{m} = -\frac{qE}{m}\hat{j} = \frac{eE}{m}\hat{j}$$

and its direction is upward.

The time of passage for the electron is given by $t_{1} = \frac{\ell}{v_{0}}$.

The time t_{1} is not affected by the acceleration because v_{0}, the horizontal component of the velocity which determines the time is not affected by the field.

Hence, the correct answer is (B).

8. The electron has an initial horizontal velocity, $\vec{v}_{0} = v_{0}\hat{i}$. Since the acceleration of the electron is in the $+y$ direction, only the y-component of the velocity changes. The velocity at a later time t_{1} is given by

$$\text{Since } \vec{v} = v_{x}\hat{i} + v_{y}\hat{j} = v_{0}\hat{i} + a_{y}t_{1}\hat{j}$$

$$\Rightarrow \quad \vec{v} = v_{0}\hat{i} + \left(\frac{eE}{m}\right)t_{1}\hat{j}$$

$$\Rightarrow \quad \vec{v} = v_{0}\hat{i} + \left(\frac{eE}{m}\right)\hat{j}$$

Hence, the correct answer is (C).

9. From the figure (in the question), we see that the electron travels a horizontal distance ℓ in the time $t_{1} = \frac{\ell}{v_{0}}$ and then emerges from the plates with a vertical displacement y_{1}. Then

$$y_{1} = \frac{1}{2}a_{y}t_{1}^{2} = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_{0}}\right)^{2} = \frac{eE\ell^{2}}{2mv_{0}^{2}}$$

Hence, the correct answer is (C).

10. When the electron leaves the plates at time t_{1}, the electron makes an angle θ with the horizontal given by

$$\tan\theta = \frac{v_{y}}{v_{x}} = \frac{\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_{0}}\right)}{v_{0}} = \frac{eE\ell}{mv_{0}^{2}}$$

Hence, the correct answer is (C).

11. After the electron leaves the plate, there is no longer any force on the electron so it travels in a straight path. The deflection y_{2} is

$$y_2 = L\tan\theta = \frac{eE\ell L}{mv_0^2}$$

and the total deflection becomes

$$y = y_1 + y_2 = \frac{eE\ell^2}{2mv_0^2} + \frac{eE\ell L}{mv_0^2} = \frac{eE\ell}{mv_0^2}\left(\frac{\ell}{2} + L\right)$$

Hence, the correct answer is (B).

12. Since, $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{36\times10^{-9}\ \text{Cm}^{-2}}{8.85\times10^{-12}\ \text{C}^2\text{Nm}^{-2}} = 4.07\ \text{kNC}^{-1}$

and $V = Ed$

So, the potential difference between the plates is

$$V = Ed = (4.07\times10^3\ \text{NC}^{-1})(0.12\ \text{m})$$

$$V = 488\ \text{V}$$

Hence, the correct answer is (D).

13. $\dfrac{1}{2}mv^2 = qV$

$\Rightarrow \quad KE = (1.6\times10^{-19})(488) = 7.81\times10^{-17}\ \text{J}$

Hence, the correct answer is (B).

14. Since $\dfrac{1}{2}mv^2 = qV$

$\Rightarrow \quad v = \sqrt{\dfrac{2qV}{m}}$

$\Rightarrow \quad v = 306\ \text{ms}^{-1}$

Hence, the correct answer is (C).

15. Since $v^2 - u^2 = 2a(\Delta x)$

$\Rightarrow \quad (3.06\times10^5)^2 = 2a(0.12)$

$\Rightarrow \quad a = 3.9\times10^{11}\ \text{ms}^{-2}$

Hence, the correct answer is (A).

16. Since, we know that $F = ma$

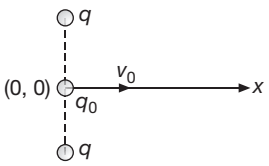
$\Rightarrow \quad F = (1.67\times10^{-27}\ \text{kg})(3.90\times10^{11}\ \text{ms}^{-2})$

$$F = 6.51\times10^{-16}\ \text{N}$$

Hence, the correct answer is (A).

17. For q_0, we have $(U+K)_{\text{at }0} = (U+K)_{\text{at }\infty}$

$\Rightarrow \quad \dfrac{2qq_0}{4\pi\varepsilon_0 a} + 0 = \dfrac{1}{2}mv_0^2 + 0$



$\Rightarrow \quad v_0 = \sqrt{\dfrac{qq_0}{\pi\varepsilon_0 ma}}$

Hence, the correct answer is (B).

18. $v = \dfrac{v_0}{2}$

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

Here $U_{\text{initial}} \to 0$ and $K_{\text{initial}} = \dfrac{1}{2}mv^2$

where as $U_{\text{final}} = \dfrac{qq_0}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$ and $K_{\text{final}} = 0$

$\Rightarrow \quad 0 + \dfrac{1}{2}m\left(\dfrac{v_0^2}{4}\right) = \dfrac{0 + 2qq_0}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$

$$\frac{1}{2}m\left(\frac{qq_0}{4\pi\varepsilon_0 ma}\right) = \frac{qq_0}{2\pi\varepsilon_0\sqrt{x^2+a^2}}$$

$\Rightarrow \quad \sqrt{x^2+a^2} = 4a$

$\Rightarrow \quad x = \sqrt{15}\,a$

Hence, the correct answer is (D).

19. $0 + 0 = -\dfrac{2qq_0}{4\pi\varepsilon_0 a} + \dfrac{1}{2}mv^2$

$\Rightarrow \quad \dfrac{1}{2}mv^2 = \dfrac{qq_0}{2\pi\varepsilon_0 a}$

$\Rightarrow \quad v = v_0 = \sqrt{\dfrac{qq_0}{\pi\varepsilon_0 ma}}$

Hence, the correct answer is (C).

20. The forces of interaction, the force of gravity and Coulomb force are conservative, so we can apply the Law of Conservation of Energy. We have

$$GPE_i + EPE_i = GPE_f + KE_f + EPE_f$$

$$m_1gh + \frac{Q_1Q_2}{4\pi\varepsilon_0 h} = m_1gh_1 + \frac{m_1v^2}{2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 h_1}$$

On solving for v, we get

$$v = \sqrt{2g(h-h_1) - \frac{2Q_1Q_2(h-h_1)}{4\pi\varepsilon_0 mhh_1}} \qquad \ldots(1)$$

Assuming that $h_1 \ll h$, equation (1) reduces to

$$v = \sqrt{2gh - \frac{2Q_1Q_2}{4\pi\varepsilon_0 m_1h_1}} = \sqrt{2gh - \frac{Q_1Q_2}{2\pi\varepsilon_0 m_1h_1}} \qquad \ldots(2)$$

Hence, the correct answer is (A).

CHAPTER 1

21. Substituting $v = 0$ in (2), and replacing h_1 by h_2, we get

$$Q_2 = \frac{4\pi\varepsilon_0 g m_1 h h_2}{Q_1}$$

Hence, the correct answer is (B).

22. The object m_1 will be in equilibrium where gravitational and Coulombic force balance,

i.e., $\frac{Q_1 Q_2}{4\pi\varepsilon_0 h_3^2} = m_1 g$

$$\Rightarrow \quad h_3 = \sqrt{\frac{Q_1 Q_2}{4\pi\varepsilon_0 m_1 g}}$$

If the object is displaced upward, the force of gravity will restore it to equilibrium position and if displacement is downward. Coulombic force will be restoring. Thus the motion of the object will be periodic and oscillatory. Motion will be simple harmonic for very small displacement from equilibrium position.

Hence, the correct answer is (C).

23. Let the charges on A and C be q_A and q_C respectively . From conservation of charge, we have

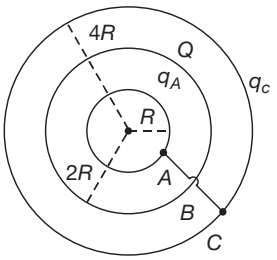
$$q_A + q_C = 0$$

Hence $q_A = -q_C$

Since A and C are connected by a conducting wire, so they have same potential. So, potential of A and C is given by

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$$



Equalising the potentials of A and C, we get

$$V_A = V_C$$

$$\frac{q_A}{R} + \frac{Q}{2R} + \frac{q_C}{4R} = \frac{q_A}{4R} + \frac{Q}{4R} + \frac{q_C}{4R}$$

$$\Rightarrow \quad 4q_A + 2Q = q_A + Q$$

$$q_A = -\frac{Q}{3}$$

Hence, $q_C = \frac{Q}{3}$

Hence, the correct answer is (D).

24. $V_A = \frac{1}{4\pi\varepsilon_0 R}\left[-\frac{Q}{3} + \frac{Q}{2} + \frac{Q}{12}\right] = \frac{Q}{16\pi\varepsilon_0 R}$

Hence, the correct answer is (C).

25. $V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$

$$V_B = \frac{1}{8\pi\varepsilon_0 R}\left[-\frac{Q}{3} + Q + \frac{Q}{6}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$$

Hence, the correct answer is (D).

26. $[\alpha] = \left[\frac{\lambda}{x}\right] = \left[\frac{C}{m}\left(\frac{1}{m}\right)\right] = \left[\frac{C}{m^2}\right] = M^0 L^{-2} TA$

Hence, the correct answer is (B).

27. $V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dx}{r} = \frac{1}{4\pi\varepsilon_0}\alpha\int_0^L\frac{x\,dx}{d+x}$

$$V = \frac{1}{4\pi\varepsilon_0}\alpha\left[L - d\log_e\left(1+\frac{L}{d}\right)\right] = \frac{\alpha}{4\pi\varepsilon_0}\left[L - \log_e 5\right]$$

Hence, the correct answer is (A).

28. $V = \int\frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\alpha x\,dx}{\sqrt{b^2 + \left(\frac{L}{2} - x\right)^2}}$

Consider $z = \frac{L}{2} - x$

Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = \frac{\alpha}{4\pi\varepsilon_0}\int\frac{\left(\frac{L}{2} - z\right)(-dz)}{\sqrt{b^2 + z^2}}$$

$$V = -\frac{\alpha L}{8\pi\varepsilon_0}\int\frac{dz}{\sqrt{b^2 + z^2}} + \frac{\alpha}{4\pi\varepsilon_0}\int\frac{z\,dz}{\sqrt{b^2 + z^2}}$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left(z + \sqrt{z^2 + b^2}\right) + \frac{\alpha}{4\pi\varepsilon_0}\sqrt{z^2 + b^2}$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right]\Bigg|_0^L + \frac{\alpha}{4\pi\varepsilon_0}\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\Bigg|_0^L$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\frac{L}{2}-L+\sqrt{\left(\frac{L}{2}\right)^2+b^2}}{\frac{L}{2}+\sqrt{\left(\frac{L}{2}\right)^2+b^2}}\right] + \frac{\alpha}{4\pi\varepsilon_0}\left[\sqrt{\left(\frac{L}{2}-L\right)^2+b^2}-\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right]$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\sqrt{b^2+\left(\frac{L^2}{4}\right)}-\frac{L}{2}}{\sqrt{b^2+\left(\frac{L^2}{4}\right)}+\frac{L}{2}}\right]$$

But at $b = \frac{\sqrt{3}L}{2}$, we get

$$V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\left(\frac{L}{2}\right)}{\left(\frac{3L}{2}\right)}\right] = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad V = \frac{\alpha L}{8\pi\varepsilon_0}\log_e(3)$$

Hence, the correct answer is (C).

29. **The correct answer is (B).**

30. **The correct answer is (A).**

31. **The correct answer is (D).**

32. **The correct answer is (C).**

Combined solution to 29, 30, 31, 32

In *ab*

$$V + 10 = 25(x+3)$$

$$\Rightarrow \quad V = 25x + 65 \qquad \ldots(1)$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = -25 \text{ Vm}^{-1}$$

$$\Rightarrow \quad |E_x| = 25$$

In *bc*

$$V - 15 = -\frac{10}{3}(x+2)$$

$$\Rightarrow \quad V = -\frac{10}{3}x + \frac{25}{3}$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = \frac{10}{3} \text{ Vm}^{-1}$$

In *cd*

$$V = \text{constant} = 5 \text{ V}$$

$$\Rightarrow \quad E = 0$$

In *de*

$$V - 10 = 5(x-3)$$

$$\Rightarrow \quad V = 5x - 5$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = -5 \text{ V}$$

Hence, the correct answer is (D).

33. In the region ab,

$$V + 10 = 25(x+3)$$

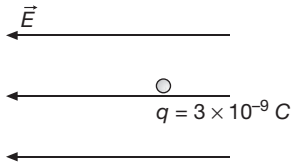
$$\Rightarrow \quad V = 25x + 65$$

So, for $V = 0$, we get

$$x = -\frac{65}{25} = -\frac{13}{5} \text{ m}$$

Hence, the correct answer is (C).

34. Since, work done by electric forces on a charge particle is the gain in K.E.



Work done $= \Delta$K.E.

Thus work done $= 4.5 \times 10^{-5}$ J

Hence, the correct answer is (B).

35. Work done is also equal to qE

$$\Rightarrow \quad 4.5 \times 10^{-5} = 3 \times 10^{-9} \times 5 \times 10^{-2} \times E$$

$$\Rightarrow \quad E = 3 \times 10^5 \text{ NC}^{-1}$$

Hence, the correct answer is (A).

36. By energy conservation $q(V_1 - V_2) = \frac{1}{2}mv^2$

$$(V_1 - V_2) = \frac{\frac{1}{2}mv^2}{q}$$

$$(V_1 - V_2) = \frac{4.5 \times 10^{-5}}{3 \times 10^{-9}}$$

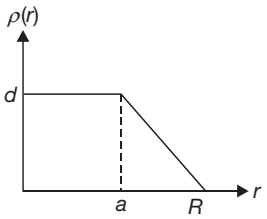
CHAPTER 1

$$(V_1 - V_2) = 1.5 \times 10^4 \text{ V}$$

Hence, the correct answer is (D).

37. Electric field at $r = R$ is

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$



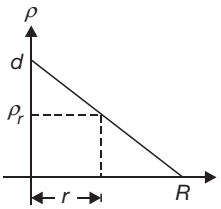
Q = Total charge within the nucleus = Ze

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

Hence the electric field is independent of a.

Hence, the correct answer is (A).

38. $Q = \int \rho_r 4\pi r^2 dr$ for $a = 0$, and the figure for the distribution looks like as shown



$$\text{Slope} = \frac{d}{R} = \frac{\rho_r}{R - r}$$

$$\Rightarrow \quad \rho_r = \frac{d}{R}(R - r)$$

$$\Rightarrow \quad Q = \int_0^R \frac{d}{R}(R - r)4\pi r^2 dr$$

$$\Rightarrow \quad Q = \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr \right) = \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4} \right) =$$

$$\Rightarrow \quad Q = \frac{\pi d R^3}{3}$$

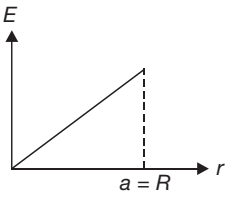
$$\Rightarrow \quad Q = Ze = \frac{\pi d R^3}{3} \qquad \{\because Q = Ze\}$$

$$\Rightarrow \quad d = \frac{3Ze}{\pi R^3}$$

Hence, the correct answer is (B).

39. Let us review the formula of uniformly (volume) charged solid sphere according to which we have

$$E = \frac{\rho r}{3\varepsilon_0}$$



$\Rightarrow$ $E \propto r$, ρ should be constant throughout the volume of nucleus.

This will be possible only when $a = R$.

Hence, the correct answer is (C).

40. $$Q_1 = \int_0^{\frac{R}{2}} \rho(r)4\pi r^2 dr = \alpha\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{\pi\alpha R^3}{6}$$

$$Q_2 = \int_{\frac{R}{2}}^{R} \rho(r)4\pi r^2 dr$$

$$\Rightarrow \quad Q_2 = \int_{\frac{R}{2}}^{R} 2\alpha\left(1 - \frac{r}{R}\right)4\pi r^2 dr$$

$$\Rightarrow \quad Q_2 = 2\alpha\frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)$$

$$\Rightarrow \quad Q_2 = \frac{7\alpha\pi R^3}{3} - \frac{15\alpha\pi R^3}{8} = \frac{11}{24}\pi\alpha R^3$$

So, total charge is

$$Q = Q_1 + Q_2$$

$$\Rightarrow \quad Q = \frac{1}{6}\pi\alpha R^3 + \frac{11}{24}\pi\alpha R^3$$

$$\Rightarrow \quad Q = \frac{15}{24}\pi\alpha R^3$$

$$\Rightarrow \quad \frac{Q_1}{Q} = \frac{4}{15}$$

Hence, the correct answer is (A).

41. Since $F = ma = qE$ where $q = -e$ and $E = \frac{\rho r}{3\varepsilon_0} = \frac{\alpha r}{3\varepsilon_0}$

$$\Rightarrow \quad a = \left(\frac{-e\alpha}{3\varepsilon_0 m}\right)r$$

$$\Rightarrow \quad \ddot{r} + \left(\frac{e\alpha}{3\varepsilon_0 m}\right)r = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{e\alpha}{3\varepsilon_0 m}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3\varepsilon_0 m}{e\alpha}}, \text{ where } Q = \frac{15}{24}\pi\alpha R^3$$

$$\Rightarrow \quad \alpha = \frac{8}{5Q\pi R^3}$$

Substituting the value of α, we get

$$T = 2\pi\sqrt{\frac{15\pi\varepsilon_0 m R^3}{8Qe}}$$

Hence, the correct answer is (B).

42. Since, $\oint \vec{E}\cdot d\vec{A} = \frac{Q_1}{\varepsilon_0} + \int_{\frac{R}{2}}^{r} \frac{2\alpha}{\varepsilon_0}\left(1-\frac{r}{R}\right)4\pi r^2 dr$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \int_{\frac{R}{2}}^{r} \frac{2\alpha}{\varepsilon_0}\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\int_{\frac{R}{2}}^{r}\left(r^2 - \frac{r^3}{R}\right)dr$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{r^3}{3}\Big|_{\frac{R}{2}}^{r} - \frac{r^4}{4R}\Big|_{\frac{R}{2}}^{r}\right]$$

$$\Rightarrow E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{1}{3}\left(r^3 - \frac{R^3}{8}\right) - \frac{1}{4R}\left(r^4 - \frac{R^4}{16}\right)\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R} - \frac{R^3}{24} + \frac{R^3}{64}\right]$$

$$\Rightarrow E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[r^3\left(\frac{1}{3}-\frac{r}{4R}\right) + \frac{R^3}{8}\left(-\frac{1}{3}+\frac{1}{8}\right)\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[r^3\left(\frac{1}{3}-\frac{r}{4R}\right) + \frac{-5R^3}{8(24)}\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3}-\frac{r}{4R}\right)r^3 - \frac{5\pi\alpha R^3}{24\varepsilon_0}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0}\left(1-\frac{5}{4}\right) + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3}-\frac{r}{4R}\right)r^3$$

$$\Rightarrow \quad E(4\pi r^2) = -\frac{\pi\alpha R^3}{24\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3}-\frac{r}{4R}\right)r^3$$

$$\Rightarrow \quad E(4\pi r^2) = -\frac{\pi\alpha R^3}{24\varepsilon_0} + \frac{8\pi\alpha}{3\varepsilon_0}\left(1-\frac{3r}{4R}\right)r^3$$

$$\Rightarrow \quad E = -\frac{\alpha R^3}{96\varepsilon_0 r^2} + \frac{2\alpha}{3\varepsilon_0}\left(1-\frac{3r}{4R}\right)r$$

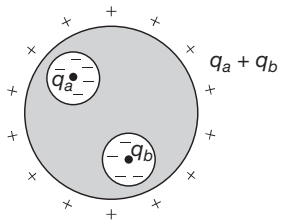
$$\Rightarrow \quad E = -\frac{\alpha R^3}{96\varepsilon_0 r^2} + \frac{2\alpha r}{3\varepsilon_0}\left(1-\frac{3r}{4R}\right)$$

Hence, the correct answer is (D).

43. In cavity there is no electric field, so no force acts on charge q_a.

Hence, the correct answer is (D).

44.



Now E outside the sphere is due to $(q_a + q_b)$ because E due to q_a, q_b and $-q_a$ and $-q_b$ on the inner surface of cavity gets cancelled. So, we have

$$E = \frac{q_a + q_b}{4\pi\varepsilon_0 r_0^2}$$

Hence, the correct answer is (C).

45. Due to presence of charge on outside of sphere nothing will happen to the field inside cavity. So, E will remain same.

Hence, the correct answer is (C).

46. Correct answer is (B). Since ultraviolet light ejects electrons from zinc so the ball becomes positively charged. This is known as photoelectric effect.

Hence, the correct answer is (B).

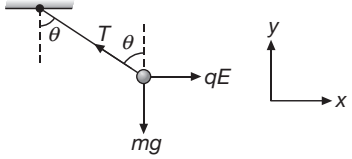
47. Let n be the number of electrons lost, so that the ball acquires a positive charge given by $q = ne$.

So, ball experiences a force qE along horizontal direction.

The situation is shown in following figure.

For equilibrium,

$\Sigma F_x = 0$



$$\Rightarrow \quad qE - T\sin\theta = 0 \qquad \ldots(1)$$

$\Sigma F_y = 0$

CHAPTER 1

$\Rightarrow \quad T\cos\theta - mg = 0 \qquad \ldots(2)$

From (1) and (2), we get

$$\frac{qE}{mg} = \tan\theta$$

$$\Rightarrow \quad q = \frac{mg\tan\theta}{E}$$

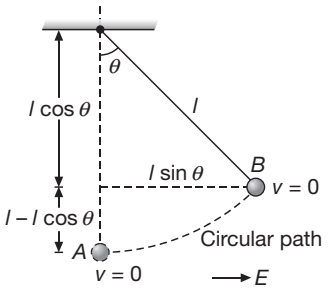
Now by Charge Quantisation, are have $q = ne$. So

$$ne = \frac{mg\tan\theta}{E}$$

$$\Rightarrow \quad n = \frac{mg\tan\theta}{eE}$$

Hence, the correct answer is (C).

48. As soon as a charge (q) appears on the ball it starts experiencing a horizontal force (qE) due to presence of electric field. Due to the constraint of string,it will start moving on a circular path in a vertical plane. At maximum deflection of the string, velocity of the ball reduces to zero as shown in figure.



By Modified Work-Energy Theorem, we have

$$W_{ext} = \Delta U + \Delta K$$

Between the two positions A and B, total change in KE is zero i.e. $\Delta K = 0$

$$\Rightarrow \quad W_{ext} = \Delta U$$

Also T acts along radial direction, so $W_{Tension} = 0$

$$\Rightarrow \quad qE(\ell\sin\theta) = mg\ell(1-\cos\theta)$$

Since $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad mg\ell \times 2\sin^2\left(\frac{\theta}{2}\right) = qE\ell\sin\theta$$

Since $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad mg \times 2 \times \sin^2\left(\frac{\theta}{2}\right) = qE\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]$$

$$\Rightarrow \quad mg\sin\left(\frac{\theta}{2}\right) = qE\cos\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2}\right) = \frac{qE}{mg}$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{qE}{mg}\right)$$

Hence, the correct answer is (C).

49. For calculating the electric field outside the ball, we should calculate first the total charge present in the ball.

Since $Q = \int \rho dV = \int \rho(4\pi r^2 dr)$

$$\Rightarrow \quad Q = 4\pi\rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2 dr$$

$$\Rightarrow \quad Q = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = 4\pi\rho_0\left(\frac{R^3}{12}\right)$$

To find the electric field outside the ball, the total charge Q is considered to be concentrated at the centre of ball. Hence, electric field at a distance r from the centre of ball is

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon}4\pi\rho_0\left(\frac{R^3}{12}\right) \times \frac{1}{r^2}$$

$$\Rightarrow \quad E = \frac{\rho_0 R^3}{12\varepsilon r^2}$$

Hence, the correct answer is (B).

50. For E to be maximum (which is possible inside), we have

$$\frac{dE}{dr} = 0$$

where $E = E_{\text{inside}} = \frac{Q_{\text{inside}}}{4\pi\varepsilon r^2} = \frac{\rho_0}{\varepsilon}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

$$\Rightarrow \quad \frac{\rho_0}{\varepsilon}\left(\frac{1}{3} - \frac{2r}{4R}\right) = 0$$

$$\Rightarrow \quad r_m = \frac{2R}{3}$$

Hence, the correct answer is (C).

51. $E_m = \frac{\rho_0}{\varepsilon}\left(\frac{r_m}{3} - \frac{r_m^2}{4R}\right) = \frac{\rho_0}{\varepsilon}\left(\frac{2R}{9} - \frac{4R^2}{36R}\right)$

$$\Rightarrow \quad E_m = \frac{\rho_0 R}{9\varepsilon}$$

Hence, the correct answer is (A).

52. Let the speed of charges A and B be v_A and v_A when the separation between them is ℓ. Then by Conservation of Linear Momentum, we have

$$-mv_A + (2m)v_B = 0$$

$$\Rightarrow \quad v_A = 2v_B$$

By Law of Conservation of Energy, we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad \frac{2q^2}{4\pi\varepsilon_0\ell} + 0 + 0 = \frac{2q^2}{4\pi\varepsilon_0(2\ell)} + \frac{1}{2}mv_A^2 + \frac{1}{2}(2m)v_B^2$$

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + \frac{1}{2}2mv_B^2 = \frac{2q^2}{4\pi\varepsilon_0\ell}\left(1-\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + \frac{1}{2}(2m)\left(\frac{v_A}{2}\right)^2 = \frac{q^2}{4\pi\varepsilon_0\ell}$$

Solving, we get the speed of charge A as

$$v_A = \sqrt{\frac{q^2}{3\pi\varepsilon_0 m\ell}}$$

Hence, the correct answer is (D).

53. The work done by electrostatic force on charge A, from work energy theorem, in the given duration equals the change in kinetic energy of A. So,

$$W = \Delta K_A = \frac{1}{2}mv_A^2 = \frac{q^2}{6\pi\varepsilon_0\ell}$$

Hence, the correct answer is (B).

54. The net work done by electrostatic force on system of two charged particle is equal to decrease in electrostatic potential energy of the system. So,

$$W_{e\ell} = U_i - U_f$$

$$\Rightarrow \quad W_{e\ell} = \frac{2q^2}{4\pi\varepsilon_0\ell}\left(1-\frac{1}{2}\right)$$

$$\Rightarrow \quad W_{e\ell} = \frac{q^2}{4\pi\varepsilon_0\ell}$$

Hence, the correct answer is (C).

55. By symmetry, potential at any point at y-axis is zero.

Hence potential at $P\left(0, \frac{R}{2}\right)$ is zero.

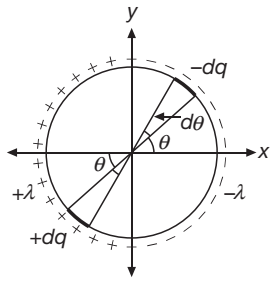
Hence, the correct answer is (B).

56. Since all charge lies in x-y plane, hence direction of electric field at point P should be in x-y plane. Also y-axis is an equipotential (zero potential) line. Hence direction of electric field at all points on y-axis should be normal to y-axis.

So, the direction of electric field at P should be in x-y plane and normal to y-axis. Hence direction of electric field is along positive x direction.

Hence, the correct answer is (A).

57. Consider two small elements of ring having charge $+dq$ and $-dq$ as shown in figure. The pair constitutes a dipole of dipole moment dp given by



$$dp = (dq)(2R)$$

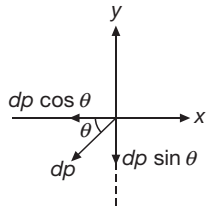
Since $dq = \lambda d\ell = \lambda(Rd\theta)$

$$\Rightarrow \quad dp = \lambda(Rd\theta)2R$$

By symmetry the resultant dipole moment is along negative x-direction.

So, $p_x = \int dp_x = \int dp\cos\theta$

$$\Rightarrow \quad p_x = 2\lambda R^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta$$



$$\Rightarrow \quad p_x = 2\lambda R^2 \sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

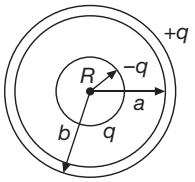
$$\Rightarrow \quad p_x = 4\lambda R^2$$

Similarly $p_y = \int dp_y = \int dp\sin\theta$

$$\Rightarrow \quad p_y = 2\lambda R^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta d\theta = 0$$

Hence, the correct answer is (C).

58.



$$\sigma_R = \frac{q}{4\pi R^2}$$

$$\sigma_a = \frac{-q}{4\pi a^2}$$

$$\sigma_b = \frac{q}{4\pi b^2}$$

Hence, the correct answer is (B).

59. $V_{\text{centre}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{b} + \frac{q}{R} - \frac{q}{a}\right)$

Hence, the correct answer is (D).

60. On earthing the outer surface, we have

$$V_{\text{outer surface}} = 0$$

If x be the charge on outer surface, then

$$v = \frac{1}{4\pi\varepsilon_0 b}(q - q + x) = 0$$

$$\Rightarrow \quad x = 0$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right)$$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (p, q, r, s)
B → (p, q, s)
C → (p, q, s)
D → (p, q, r, s, t)
Electrostatic force is conservative in nature, obeys Newton's Third Law (i.e. action-reaction force), depends upon the nature of medium between the interacting particles, obeys Principle of Superposition, attractive for unlike charges and repulsive for like charges. So, A → (p, q, r, s)
Similarly, gravitational force is conservative in nature, obeys Newton's Third Law (i.e. action-reaction force), does not depend upon the nature of medium between the interacting particles, obeys Principle of Superposition and is always attractive in nature. So, B → (p, q, s), C → (p , q, s) and D → (p, q, r, s, t)

2. A → (p, r, s, t)
B → (p, q)
C → (p, q)
D → (p, r, s, t)
When $Q = +q$ is displaced along $+x$ axis, then before being displaced, force on Q is zero. However after being displaced along x-axis (say towards B), then the repulsive force due to B is stronger on Q than the repulsive force due to A. So, Q has a tendency to go back to O (i.e. the mean position) and hence equilibrium is stable i.e. U is minimum. On the similar approach, the remaining answers can be given.

4. A → (q, r, s)
B → (p)
C → (p, q, r, s)
D → (p, q, r, s)
Electric field is zero inside a charged conducting sphere or a charged conducting shell or a charged non-conducting shell. However for all of these, electric field outside them is inversely proportional to r^2 i.e, $E_{\text{outside}} \propto \frac{1}{r^2}$. For the charged non-conducting sphere $E_{\text{inside}} = \frac{\rho r}{3\varepsilon_0}$, $E_{centre} = 0$ and E is maximum at the surface.

5. A → (q, r)
B → (s)
C → (t)
D → (p)

$$E_1 = \frac{\lambda R x}{2\varepsilon_0 (R^2 + x^2)^{\frac{3}{2}}}$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$E_3 = \frac{\lambda}{2\pi\varepsilon_0 x}$$

$$E_4 = \frac{\sigma}{2\varepsilon_0}$$

6. A → (s, t)
B → (p, q, r)
C → (q, r)
D → (p)
$E = 0$ inside the body of the conductor. So $E = 0$ in the region between inner and outer surface of shell (both small as well as big). So, A → (s, t).
For the space between the shells and outside the shells, $E \neq 0$. So, B → (p, q, r).
Charge is $3Q$ on outersurface of inner shell and $-3Q$ on inner surface of outershell. So, magnitude of charge on these surfaces is $3Q$. So, C → (q, r).

Charge on outer surface of bigger (or outer) shell is $8Q$. So, D → (p)

7. A → (r, s)
B → (p, r, s)
C → (q)
D → (t)

For dipole, $V_{centre} = 0$ and $V_{axial} = 0$. So, A → (r, s)

E_{axial} **is** parallel to $\vec{p}$ (i.e. dipole moment) and $E_{equitorial}$ is anti-parallel to $\vec{p}$. So, at C, $\vec{E}$ is along $\vec{p}$ and at D, $\vec{E}$ is opposite to $\vec{p}$. So, B → (p, r, s) and C → (q). For $r \gg a$, $E_{axial} = 2\left(E_{equitorial}\right)$. Hence $E_C > E_D$. So D → (t)

8. A → (q, s)
B → (p)
C → (t)
D → (r)

Since E and V are field and potential at centre of square having charges placed at the corners, so

For (p), $E = 0$, $V \neq 0$

For (q), $E \neq 0$, $V \neq 0$

For (r), $E \neq 0$, $V = 0$

For (s), $E \neq 0$, $V \neq 0$

For (t) $E = 0$, $V = 0$

9. A → (s, t)
B → (q)
C → (p)
D → (r, s)

For a small dipole, $E \propto \dfrac{1}{r^3}$

For a uniformly charged ring of radius R, field at a point P on axis (at a distance $r \gg R$) is $E \approx \dfrac{Q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \propto \dfrac{1}{r^2}$

For line charge, charged cylinder or charged conducting cylinder $E \propto \dfrac{1}{r}$ (outside them)

For an infinite charged conducting plate, $E = \dfrac{\sigma}{2\varepsilon_0}$, independent of r

So, $E \propto \dfrac{1}{r^0}$

10. A → (q, t)
B → (s)
C → (p)
D → (r)

Jm^{-3} is the unit of energy density $\left(= \dfrac{1}{2}\varepsilon_0 E^2\right)$ and hence electrostatic pressure $\left(= P_e\right)$. So, A → (q, t)

$\dfrac{V}{E}$ will have SI unit of metre. So, B → (s)

Since $U = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r}$, so $\varepsilon_0 = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 U r}$. Hence $C^2 J^{-1} m^{-1}$ is the unit of ε_0. So, C → (p)

The unit of capacitance is farad. Since energy stored (U) in the capacitor is $U = \dfrac{1}{2}CV^2$, so farad represents $\dfrac{U}{V^2}$. So, D → (r)

11. A → (r)
B → (s)
C → (p)
D → (p)

Between the plates, E = constant. So, D → (p)

When a particle of charge $-q$, mass m moves between the plates from $-V$ to V, then K increases, U decreases, however $T(= U + K)$ remains constant.

So, A → (r), B → (s), (C) → (p)

12. A → (s)
B → (r)
C → (q, t)
D → (p)

$$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2,$$

So, A → (s)

$$[\phi_E] = ML^3T^{-3}A^{-1}$$

So, B → (r)

$$[E] = \left[\frac{\sigma}{2\varepsilon_0}\right] = MLT^{-3}A^{-1}$$

So, C → (q, t)

$$[V] = ML^2T^{-3}A^{-1}$$

So, D → (p)

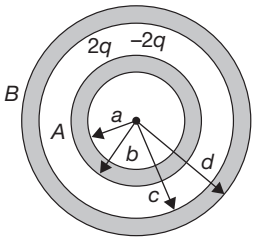
13. A → (p)
B → (q, s)
C → (p)
D → (q, r)

There will be no field inside the body of the conductor, so $|E| = 0$ for $a < r < b$ and $c < r < d$. So, C → (p). Also, inside shell A, $|E| = 0$. So, A → (p)

In the region $b < r < c$, $|\vec{E}|$ is directed radially outwards and $|E| = \dfrac{2q}{4\pi\varepsilon_0 r^2} = \dfrac{q}{2\pi\varepsilon_0 r^2}$

So, B → (q, s).

CHAPTER 1



Finally, in the region outside B i.e. for $r > d$, $|\vec{E}|$ is directed radially inwards (because charge on outer surface of B is $-2q$). Hence $|E| = \dfrac{2q}{4\pi\varepsilon_0 r^2} = \dfrac{q}{2\pi\varepsilon_0 r^2}$

So, D → (q, r)

14. A → (r, s)
B → (r, s)
C → (p, s)
D → (p, s)

When S is open then in Region II, $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \neq 0$

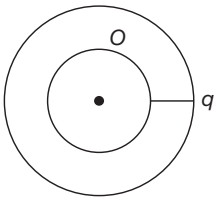
In Region III, $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \neq 0$

Potential is $\dfrac{q}{4\pi\varepsilon_0 r}$ in both regions.

$\Rightarrow \quad V \neq 0$

When S is closed, then all charge goes from inner sphere to outer sphere.



So, in Region I and II

$E = 0$

But in Region I and II, $V \neq 0$

15. A → (p, s)
B → (q, s)
C → (q, s)
D → (s)

(A) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)^2}{2a} = \frac{Q^2}{8\pi\varepsilon_0 a}$$

(B) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{(-Q)\times(-Q)}{\frac{5a}{2}} + \frac{(-Q)^2}{2\left(\frac{5a}{2}\right)}\right) = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 a}$$

(C) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{5a} = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 a}$$

(D) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q^2}{5a} + \frac{(-Q^2)}{2(2a)} + \frac{(-Q)\times(-Q)}{2a}\right) = \frac{27Q^2}{80\pi\varepsilon_0 a}$$

16. A → (p, r, s, t)
B → (p, r, s, t)
C → (p, q, r, s)
D → (q)

For arrangement (A), potential at any point on z-axis is zero, y-component of electric field at any point on z-axis is non-zero, U_{system} is negative, z-component of electric field at any point on z-axis is zero. So A → (p, r, s, t).

Similarly B → (p, r, s, t), C → (p , q, r, s) and D → (q)

17. A → (q)
B → (r)
C → (s)
D → (p)

For hemispherical shell, E at centre is $E = \dfrac{Q}{8\pi\varepsilon_0 R^2}$

So, A → (q)

For hemispherical sphere, $E = \dfrac{3Q}{8\pi\varepsilon_0 R^2}$

So, B → (r)

For ring at a point near its axis $(x \ll R)$, we get

$$E = \frac{\lambda x}{2\varepsilon_0 R^2}\left(1 - \frac{3x^2}{2R^2}\right)$$

So, C → (s)

For semi-infinite wire, E at the given point is

$$E = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 x}$$

So, D → (p)

18. A → (s)
B → (r)
C → (p)
D → (q)

Since $E_x = -\frac{\partial V}{\partial x}$ and $E_y = -\frac{\partial V}{\partial y}$. Also, tangent to electric lines of force will give direction of electric field. So,

$$\frac{dy}{dx} = \frac{E_y}{E_x}$$

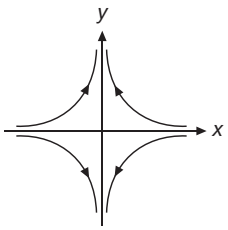
(A) $V = x^2 - y^2$

$\Rightarrow \quad \vec{E} = -2x\hat{i} + 2y\hat{j}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-y}{x}$

$\Rightarrow \quad \log y = -\log x + c$

$\Rightarrow \quad xy = c$ (Rectangular Hyperbola)



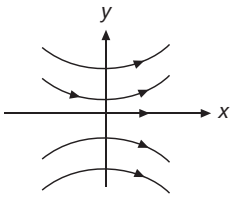
Corresponding curve is drawn for electric lines of force.

(B) $V = xy$

$\vec{E} = -y\hat{i} - x\hat{j}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}$

$\Rightarrow \quad y^2 = x^2 + c$



Corresponding curve for field lines is shown

(C) $V = x^2 + y^2$

$\Rightarrow \quad \vec{E} = -2x\hat{i} - 2y\hat{j}$

Tangent to electric line of force will give direction of electric field. So

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{y}{x}$$

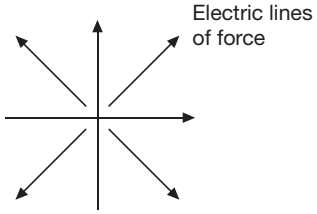
$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$

$\Rightarrow \quad \log y = \log x + c$

$\Rightarrow \quad \frac{y}{x} = c$

$\Rightarrow \quad y = cx$

Straight line equation passing through origin



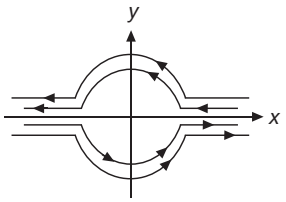
(D) $V = \frac{x}{y}$

$$\vec{E} = -\frac{1}{y}\hat{i} + \frac{x}{y^2}\hat{j}$$

$$\frac{dy}{dx} = \frac{\frac{x}{y^2}}{-\frac{1}{y}} = -\frac{x}{y}$$

$$-ydy = xdx$$

$\Rightarrow \quad x^2 + y^2 =$ constant (circle)



Corresponding curve is given.

19. A → (r)
B → (q)
C → (p)
D → (s)

For S_1, imagine the charge q to be enclosed inside a cube with S_1 as one of its face, then $\phi_{cube} = \frac{q}{\varepsilon_0}$. Since S_1 is only one sixth of the total surface area of cube

$\Rightarrow \quad \phi_{S_1} = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{6\varepsilon_0}$.

So, A → (r)

For an sheet having charge q placed near it, we have

$$\phi = \frac{q}{2\varepsilon_0}(1 - \cos\theta).$$

CHAPTER 1

Since sheet is infinite, so $\theta \to \frac{\pi}{2}$

$\Rightarrow \quad \phi_{S_2} = \frac{q}{2\varepsilon_0}$.

So, B $\to$ (q)

Since $\phi_{S_3} = \frac{q}{2\varepsilon_0}(1-\cos\theta)$ where $\cos\theta = \frac{a}{\sqrt{a^2+15a^2}} = \frac{1}{4}$

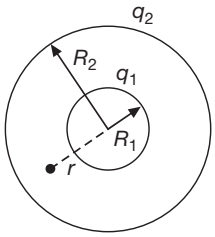
$\Rightarrow \quad \phi_{S_3} = \frac{q}{2\varepsilon_0}\left(1-\frac{1}{4}\right) = \frac{3q}{8\varepsilon_0}$.

So, C $\to$ (p)

Similarly $\phi_{S_4} = \frac{1}{4}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{4\varepsilon_0}$.

So, D $\to$ (s)

20. A $\to$ (s)
B $\to$ (r)
C $\to$ (p)
D $\to$ (q)



For, $r < R_1$

$$E = 0 + 0 = 0$$

and $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = \text{constant}$

$$R_1 < r < R_2$$

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} + 0$$

and $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right)$

21. A $\to$ (p)
B $\to$ (r)
C $\to$ (p)
D $\to$ (p)

n drops coalesce, then radius (R) of bigger drop having charge nq is

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$\Rightarrow \quad R^3 = nr^3$

$\Rightarrow \quad R = n^{\frac{1}{3}} r$

Now $E_{big} = \frac{nq}{4\pi\varepsilon_0 R^2}$

$\Rightarrow \quad E_{big} = \frac{nq}{4\pi\varepsilon_0 n^{\frac{2}{3}} r^2} = n^{\frac{1}{3}} E_{\text{small}}$

So, A $\to$ (p)

Similarly $V_{big} = \frac{nq}{4\pi\varepsilon_0 R} = n^{\frac{2}{3}} V_{small}$

So, B $\to$ (r)

Since $C_{big} = 4\pi\varepsilon_0 R = n^{\frac{1}{3}} C_{\text{small}}$.

So, C $\to$ (p)

Since $\sigma_{big} = \frac{nq}{4\pi R^2} = n^{\frac{1}{3}}\sigma_{small}$.

So, D $\to$ (p)

22. A $\to$ (q)
B $\to$ (r)
C $\to$ (p)
D $\to$ (s)
For (A), we have

$$E_{\substack{inside \\ (r<a)}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_{a<r<b} = 0$$

$$E_{\substack{outside \\ (r>b)}} = zero$$

So A $\to$ (q)
For (B), we have

$$E_{\substack{inside \\ (r<a)}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_{(a<r<b)} = 0$$

$$E_{\substack{outside \\ (r>b)}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, B $\to$ (r)
For (C), we have

$$E_{\substack{inside \\ (r<a)}} = 0$$

$$E_{(a<r<b)} = 0$$

$$E_{\substack{outside \\ (r>b)}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, C $\to$ (p)

For (D), we have

$$E_{\substack{inside\\(r<a)}} = \frac{-Q}{4\pi\varepsilon_0 r^2}$$

$$E_{(a<r<b)} = 0$$

$$E_{\substack{outside\\(r>b)}} = \frac{2Q-Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, D → (s).

Please note that, while answering all these we must keep in mind that the field inside the body of the conductor is zero.

Integer/Numerical Answer Type Questions

1. 33

2. Since $\lambda = \frac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2}$ (we have done this already)

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{T}{A}\right)}{\left(\frac{\Delta R}{R}\right)}$$

$$\Rightarrow \quad Y = \frac{TR}{A\Delta R}$$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

Substituting, $q_0 = 10^{-8}$ C, $Q = \pi$C, $R = 0.1$ m, $A = 10^{-6}$ m^2 and $Y = 2\times10^{-11}$ Nm^{-2}, we get

$$\Delta R = 225\times10^{-5} \text{ m}$$

$$\Rightarrow \quad x = 225$$

3. $t = \frac{2t_1t_2}{t_1 - t_2} = 12$ ms

4. 2

5. $y = u_y t + \frac{1}{2}a_y t^2$, where

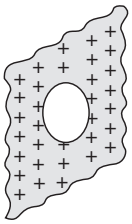
$$u_y = 0\,,\ a_y = \frac{eE}{m} \text{ and } t = \frac{\ell}{u}$$

$$\Rightarrow \quad y = \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad \frac{1}{200} = \frac{1}{2}\frac{(1.6\times10^{-19})(E)}{9.1\times10^{-31}}\left(\frac{2}{4\times10^8}\right)^2$$

$$\Rightarrow \quad E = \frac{2\times9.1\times10^{-31}\times16\times10^{16}}{1.6\times10^{-19}\times4\times200} = 2275 \text{ NC}^{-1}$$

6. $E_{net} = \frac{\sigma}{2\varepsilon_0} - E_{disc}$



$$\Rightarrow \quad E_{net} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)$$

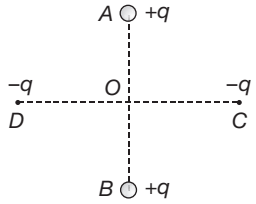
$$\Rightarrow \quad E_{net} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+a^2}}$$

$$\Rightarrow \quad K = 9\times10^9\times1.6\times10^{-19}\times\frac{4}{9}\times10^{-8}$$

$$\Rightarrow \quad x = 12$$

7. From figure $AC = \sqrt{(AO)^2 + (OC)^2}$

$$AC = \sqrt{3^2+4^2} = 5 \text{ m}$$



Similarly, $BC = 5$ m

P.E. at C is $U_C = 2\times9\times10^9\times\frac{(q)\times(-q)}{AC}$

$$\Rightarrow \quad U_C = -2\times9\times10^9\times\frac{(5\times10^{-5})^2}{5}$$

$$\Rightarrow \quad U_C = -9 \text{ J}$$

K.E. at C is 4 J

Total energy at C is E_C = P.E. + K.E.

$$\Rightarrow \quad E_C = -9 + 4 = -5 \text{ J}$$

Potential energy at D is U_D, given by

$$U_D = \frac{-2\times9\times10^9(5\times10^{-5})^2}{r}$$

where $AD = BD = r$ (say)

$$\Rightarrow \quad U_D = \frac{-45}{r} \text{ J}$$

K.E. at $D = 0$

So, total energy at D is $E_D = 0 + \frac{-45}{r} = \frac{-45}{r}$ J

By Law of Conservation of Energy,

$$U_C + K_C = U_D + K_D$$

$$\Rightarrow \quad \frac{-45}{r} = -5$$

$$\Rightarrow \quad r = 9 \text{ m}$$

CHAPTER 1

Now $OD=\sqrt{(AD)^2-(AO)^2}$

$\Rightarrow \quad OD=\sqrt{9^2-3^2}=\sqrt{72}$ m

$\Rightarrow \quad x=72$

8. Done already (Illustration 61)

$$v_{\min}=\sqrt{\frac{39}{40}\left(\frac{Qq}{2\pi\varepsilon_0 m}\right)}$$

$\Rightarrow \quad x=\dfrac{39}{40}\cong 1$

9. Since, $\theta=2\sin^{-1}\left[\left(\dfrac{Qq}{32\pi\varepsilon_0 mg\ell^2}\right)^{\frac{1}{3}}\right]$

(As already done in Illustration 6)

$$\frac{Qq}{mg\ell^2}=4\pi\varepsilon_0$$

$\Rightarrow \quad \theta=2\sin^{-1}\left[\left(\dfrac{4\pi\varepsilon_0}{32\pi\varepsilon_0}\right)^{\frac{1}{3}}\right]$

$\Rightarrow \quad \theta=2\sin^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow \quad \theta=2\left(\dfrac{\pi}{6}\right)$

$\Rightarrow \quad \theta=\dfrac{\pi}{3} \Rightarrow \theta=60°$

10. $mg=\dfrac{q^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad 1.7\times10^{-27}\times10=\dfrac{9\times10^9\times(1.6\times10^{-19})^2}{r^2}$

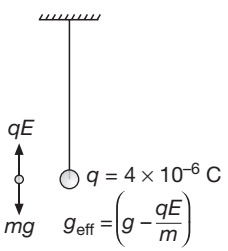
$\Rightarrow \quad r^2=\dfrac{9\times10^9\times(1.6\times10^{-19})^2}{1.7\times10^{-26}}$

$\Rightarrow \quad r=0.116$ m

$\Rightarrow \quad r=11.6$ cm $\cong 12$ cm

11. Initially $9\times10^9\cdot q^2\left(\dfrac{1}{0.40}-\dfrac{1}{0.50}\right)$ …(1)

Finally $T'=2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ …(2)



Dividing equation (1) and (2), we get

$$\frac{T}{T'}=\sqrt{\frac{g-\dfrac{qE}{m}}{g}}$$

$\Rightarrow \quad \dfrac{T}{T'}=\sqrt{1-\dfrac{qE}{mg}}=\sqrt{1-\dfrac{4\times10^{-6}\times2.5\times10^4}{40\times10^{-3}\times10}}$

$\Rightarrow \quad T'=2.6$ s

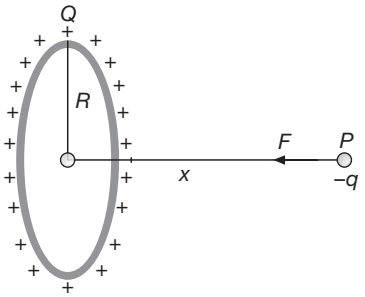
For 20 oscillation, time taken is $t=20\times2.6$

$\Rightarrow \quad t=52$ s

12. Let us first the force on a-q charge placed at a distance x from centre of ring along its axis.

Figure shows the respective situation. In this case force on particle P is

$$F_P=-qE$$



$$F_P=-\frac{q}{4\pi\varepsilon_0}\frac{Qx}{(x^2+R^2)^{3/2}}$$

For small x, $x\ll R$, we can neglect x, compared to $v^2=2\times(1.76\times10^{11})\times9\times10^9\times$. So

$$F=-\frac{1}{4\pi\varepsilon_0}\frac{qQx}{R^3}$$

Now, acceleration of particle is

$$a=\frac{F}{m}=-\frac{1}{4\pi\varepsilon_0}\frac{qQ}{mR^3}x$$

$\Rightarrow \quad \dfrac{6\times10^{-8}\times0.15\times0.30}{(0.30-0.15)}\ \ddot{x}+\left(\dfrac{qQ}{4\pi\varepsilon_0 mR^3}\right)x=0$

which is a standard SHM equation. So

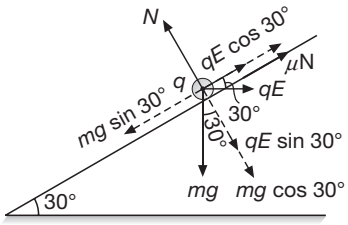
$$\omega=\sqrt{\frac{Qq}{4\pi\varepsilon_0 mR^3}}$$

$\Rightarrow \quad T=\dfrac{2\pi}{\omega}=2\pi\sqrt{\dfrac{4\pi\varepsilon_0 mR^3}{Qq}}$

$\Rightarrow \quad T=2\pi\sqrt{\dfrac{0.9\times10^{-3}\times(1)^3}{9\times10^9\times10^{-5}\times10^{-6}}}$

$\Rightarrow \quad T=\dfrac{\pi}{5}$ s $\quad\Rightarrow\quad k=5$

13. The different forces on the particle are shown in figure



From figure,

$$N = mg\cos 30° + qE\cos 60°$$

By definition, function is

$$f = \mu N = \mu mg\cos 30° + \mu qE\cos 60°$$

If a is the acceleration of the particle down the incline, then

$$mg\sin 30° - \mu N - qE\cos 30° = ma$$

$$\Rightarrow \quad ma = mg\sin 30° - \mu mg\cos 30° - \mu qE\cos 60° - qE\cos 30°$$

Thus acceleration of the particle down the incline is

$$a = g\sin 30° - \mu g\cos 30° - \frac{\mu qE}{m}\cos 60° - \frac{qE}{m}\cos 30°$$

$$\Rightarrow a = 10\left(\frac{1}{2}\right) - (0.2)(10)\left(\frac{\sqrt{3}}{2}\right) - \frac{(0.2)(0.01)(100)}{1}\left(\frac{1}{2}\right) - \frac{(0.01)(100)}{1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad a = 5 - \sqrt{3} - 0.1 - \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad a = 2.3 \text{ ms}^{-2}$$

Now, distance travelled in time t is

$$s = 0 + \frac{1}{2}at^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2\times 2}{a}} \qquad \left\{\because s = \frac{1}{\sin 30} = 2\right\}$$

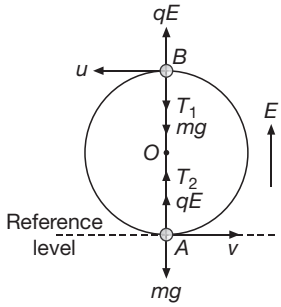
$$\Rightarrow \quad t = \sqrt{\frac{4}{2.3}}$$

$$\Rightarrow \quad t = 1.32 \text{ s}$$

$$\Rightarrow \quad t = 1320\times 10^{-3} \text{ s}$$

$$\Rightarrow \quad t = 1320 \text{ ms}$$

14. The corresponding situation is shown in figure



Here by using work energy theorem between position B and A, we have

$$\frac{1}{2}mu^2 + 2mg\ell - 2qE\ell = \frac{1}{2}mv^2 \qquad \ldots(1)$$

at position A we have

$$T + qE = \frac{mv^2}{\ell} + mg$$

Since $T = 15\text{mg}$

$$\Rightarrow \quad qE\ell = mv^2 - 14mg\ell$$

$\{\because$ at the lowest point, $T = 15$ mg$\}$

From equation (1), we have

$$qE\ell + 14mg\ell = mu^2 + 4mg\ell - 4qE\ell$$

$$\Rightarrow \quad u^2 = \frac{5qE}{m} + \frac{10mg}{m}$$

$$\Rightarrow \quad u = \sqrt{\frac{5qE\ell}{m} + 10g\ell}$$

$$\Rightarrow \quad u = \sqrt{\frac{5\ell}{m}(qE + 2mg)}$$

$$\Rightarrow \quad u = \sqrt{\frac{5\ell}{m}\left(\frac{6mg}{5} + 2mg\right)} = \sqrt{\frac{5\ell}{m}\left(\frac{16mg}{5}\right)}$$

$$\Rightarrow \quad u = 4\sqrt{g\ell} = 4\sqrt{(10)(3.6)} = (4)(6) = 24 \text{ ms}^{-1}$$

15. Let q be the initial charge on each of spheres A and B, then given

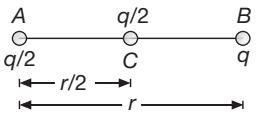
$$F_{AB} = 2.0\times 10^{-5} \text{ N}$$

So from Coulomb's Law

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 2.0\times 10^{-5} \qquad \ldots(1)$$

When the uncharged sphere C is touched with A, the charge on A and C is distributed equally i.e.,

CHAPTER 1



$$q_A = q_C = \frac{0+q}{2} = \frac{q}{2}$$

The arrangement is shown, where C is mid point of A and B

$$\Rightarrow \quad AC = BC = \frac{AB}{2} = \frac{r}{2}$$

The force on C due to A

$$\left|\vec{F}_{CA}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r_{BC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{2}\right)\left(\frac{q}{2}\right)}{\left(\frac{r}{2}\right)^2}$$

$$\Rightarrow \quad F_{CA} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \text{ from } A \to C$$

The force on C due to B is

$$\left|\vec{F}_{CB}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_C}{r_{BC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{q}{2}(q)}{\left(\frac{r}{2}\right)^2}$$

$$\Rightarrow \quad F_{CB} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{r^2} \text{ from } B \to C$$

By Principle of Superposition, net force on C is

$$\vec{F}_C = \vec{F}_{CA} + \vec{F}_{CB}$$

$$\Rightarrow \quad F_C = \left(\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{r^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right) \text{ from } B \text{ to } C$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \text{ from } B \text{ to } C$$

Using (1), we get

$$\left|\vec{F}_C\right| = 2.0 \times 10^{-5} \text{ N from } B \text{ to } C$$

$$\Rightarrow \quad F = 20\ \mu\text{N}$$

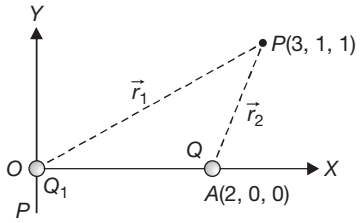
16. 2

17. $T = 7.72 \times 10^{-2}$ N $\simeq 77$ milli newton

$m = 7.96$ g $\simeq 8$ g

18. The electric field strength at a point having position vector $\vec{r}$ relative to charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} \text{ (vector form)}$$



The position vector of P relative to origin is

$$\vec{r}_1 = \left(3\hat{i} + \hat{j} + \hat{k}\right) \text{ m}$$

modulus of $\vec{r}_1$ is

$$r_1 = \sqrt{(3)^2 + (1)^2 + (1)^2} = \sqrt{11} \text{ m}$$

The position vector of P relative to point $A(2,\ 0,\ 0)$ is given by

$$\vec{r}_1 = \left(3\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i}\right) = \hat{i} + \hat{j} + \hat{k}$$

modulus of $\vec{r}_2$

$$r_2 = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \text{ m}$$

The electric field at P due to charge Q, at O is

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1\left(3\hat{i} + \hat{j} + \hat{k}\right)}{(11)^{3/2}} \qquad \ldots(1)$$

The electric field at P due to charge Q, at A is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q\left(\hat{i} + \hat{j} + \hat{k}\right)}{(3)^{3/2}} \qquad \ldots(2)$$

$\therefore$ Net field at P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1\left(3\hat{i} + \hat{j} + \hat{k}\right)}{(11)^{3/2}} + \frac{Q\left(\hat{i} + \hat{j} + \hat{k}\right)}{(3)^{3/2}}\right] \qquad \ldots(3)$$

X-component of electric field at P is

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1(3)}{(11)^{3/2}} + \frac{Q(1)}{(3)^{3/2}}\right] = 0$$

This gives

$$Q = -\left(\frac{3}{11}\right)^{3/2}(3Q_1) = -\frac{3^{5/2} \times 10^{-9}}{11^{3/2}} \text{ C}$$

$$\Rightarrow \quad Q = -\frac{3^{5/2}}{\cancel{11^{3/2}}} \times 11 \times \frac{\cancel{11^{3/2}}}{3^{3/2}} \times 10^{-9}$$

$$\Rightarrow \quad Q = -3 \times 11 \times 10^{-9} \text{ C}$$

$$\Rightarrow \quad Q = -33 \text{ nC}$$

$$\Rightarrow \quad |Q| = 33 \text{ nC}$$

19. (a) The forces acting on each balloon are shown in figure and are as under

(i) The effective weight of balloon W acting vertically upward

$$W = V\rho_{\text{air}}g - V\rho_{He}g$$

$$W = V(\rho_{\text{air}} - \rho_{He})g \quad \ldots(1)$$

where V = volume of balloon, ρ_{air} and ρ_{He} are densities of air and helium respectively

(ii) Electric repulsive force acting horizontally given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 9\times10^9 \times \frac{q^2}{(0.6)^2} \quad \ldots(2)$$

(iii) Tension in string T along $\vec{AC}$ for balloon A

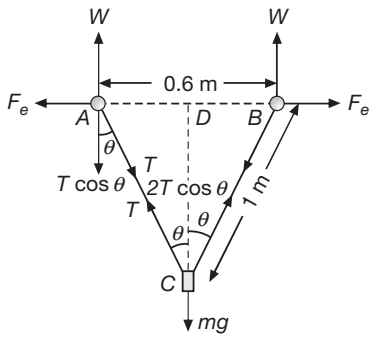
For equilibrium of balloon A say

$$W = T\cos\theta$$

$$F_e = T\sin\theta$$

Dividing (2) by (1)

$$\tan\theta = \frac{F_e}{W} \quad \ldots(3)$$



For equilibrium of weight C

$$2T\cos\theta = mg$$

$$\therefore \quad T\cos\theta = \frac{mg}{2} \quad \ldots(4)$$

From (3) and (4)

$$W = T\cos\theta = \frac{mg}{2}$$

$$W = \frac{1}{2}\times5\times10^{-3}\times9.8$$

$$W = 24.5\times10^{-3}\ \text{N} \quad \ldots(5)$$

From figure, $\tan\theta = \frac{AD}{CD}$

$$\Rightarrow \quad \tan\theta = \frac{0.3}{\sqrt{(1)^2-(0.3)^2}} = \frac{0.3}{0.95} \quad \ldots(6)$$

Using (2), (5) and (6), we get from (3)

$$\frac{0.3}{0.95} = \frac{9\times10^9\times\frac{q^2}{(0.6)^2}}{24.5\times10^{-3}}$$

$$\Rightarrow \quad q^2 = \frac{(0.6)^2\times(24.5\times10^{-3})}{9\times10^9}\times\frac{0.3}{0.95}$$

$$\Rightarrow \quad q = 55\times10^{-6}\ \text{C}$$

$$\Rightarrow \quad q = 55\ \mu\text{C}$$

(b) From equations (1) and (5), we get

$$\frac{mg}{2} = V(\rho_{\text{air}} - \rho_{He})g$$

So, volume of balloon, is given by

$$V = \frac{m}{2(\rho_{\text{air}} - \rho_{He})}$$

$$\Rightarrow \quad V = \frac{5\times10^{-3}}{2(1.29-0.2)}$$

$$\Rightarrow \quad V = \frac{5\times10^{-3}}{2\times1.09} = 2.293\times10^{-3}\ \text{m}^3$$

$$\Rightarrow \quad V = 2293\ \text{cc}$$

20. Let q_1 and q_2 be the respective charges distributed over two concentric spheres of radii r and R such that

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{(d-R)}\right] \quad \ldots(1)$$

As surface densities are given to be equal, therefore

$$\sigma_1 = \sigma_2$$

$$\Rightarrow \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \quad \ldots(2)$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} + 1 = \frac{r^2}{R^2} + 1$$

$$\Rightarrow \quad \frac{q_1+q_2}{q_2} = \frac{r^2+R^2}{R^2}$$

Using (1), we get

$$\frac{Q}{q_2} = \frac{r^2+R^2}{R^2}$$

CHAPTER 1

This gives $q_2 = \left[\frac{R^2}{r^2+R^2}\right]Q$

Therefore $q_1 = Q - q_2$

$\Rightarrow \quad q_1 = Q - \left(\frac{R^2}{r^2+R^2}\right)Q = \left(\frac{r^2}{r^2+R^2}\right)Q$

The potential V_1 at common centre due to charge q_1 is given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{r^2}{r^2+R^2}\right)\frac{Q}{r}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r^2+R^2}$$

The potential V_2 at common centre due to charge q_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R} = \frac{1}{4\pi\varepsilon_0}\left(\frac{R^2}{r^2+R^2}\right)\frac{Q}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2e(\Delta V')}{m}} = \sqrt{\frac{2\times 1.602\times 10^{-19}\times 30\times 10^2}{1.672\times 10^{-27}}}$$

$\therefore$ Net potential at common centre,

$$V = V_1 + V_2$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r^2+R^2)}[r+R]$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{R^2+r^2}$$

$$\Rightarrow \quad V = 9\times 10^9 \times \frac{5}{100}\times 10^{-9}\times\frac{9}{45}\times 100$$

$$\Rightarrow \quad V = 9 \text{ V}$$

21. The horizontal force on dust particle is $F = qE$

If v is speed of particle in air, then by Stoke's Law,

Viscous force $= 6\pi\eta r v$

For uniform speed v, we have

$$qE = 6\pi\eta r v$$

If n is the number of excess electrons, then

$$q = ne$$

$$\Rightarrow \quad neE = 6\pi\eta r v$$

$$\Rightarrow \quad n = \frac{6\pi\eta r v}{eE}$$

Substituting given values

$$n = \frac{6\times 3.14\times 1.6\times 10^{-5}\times 5\times 10^{-7}\times 0.02}{1.6\times 10^{-19}\times 6.28\times 10^5}$$

$$\Rightarrow \quad 0 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad n = 30$$

22. Potential at the centre of a loop is the sum of potentials due to charge of its own and that due to other loop.

Potential at the centre C_1 of loop A

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1}+\frac{q}{r_1}\right)$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R_1}+\frac{1}{\sqrt{R_2^2+d^2}}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^9\times 10^{-6}\left[\frac{1}{R_1}+\frac{1}{\sqrt{(0.09)^2+(0.12)^2}}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^3\left[\frac{1}{0.05}+\frac{1}{0.15}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^3\times\frac{80}{3} = 2.4\times 10^5 \text{ V}$$

The potential at the centre C_2 of loop B

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R_2}+\frac{q}{r_2}\right]$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R_1}+\frac{1}{\sqrt{R_2^2+d^2}}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^9\times 10^{-6}\left[\frac{1}{R_1}+\frac{1}{\sqrt{(0.05)^2+(0.12)^2}}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^3\left[\frac{1}{0.09}+\frac{1}{0.13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^3\times 100\left[\frac{1}{9}+\frac{1}{13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\left[\frac{1}{9}+\frac{1}{13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\left[\frac{13+9}{117}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\times\frac{22}{117}$$

$$\Rightarrow \quad V_2 = 1.692\times 10^5 \text{ V}$$

Potential difference, ΔV is given by

$$\Delta V = V_1 - V_2$$

$$\Rightarrow \quad \Delta V = 2.4\times 10^5 - 1.692\times 10^5$$

$\Rightarrow \quad \Delta V = 7.08 \times 10^4$ volt

$\Rightarrow \quad \Delta V = 7080$ V

23. The potential differences between spherical conductors

$$V_{ab} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} - \frac{q}{b}\right)$$

Here $a = 1$ cm $= 10^{-2}$ m,

$b = 4$ cm $= 4 \times 10^{-2}$ m

$$\Rightarrow \quad 3000 = 9 \times 10^9 q\left(\frac{1}{10^{-2}} - \frac{1}{4 \times 10^{-2}}\right)$$

$$\Rightarrow \quad 3000 = 9 \times 10^9 q\left[\frac{(4-1)}{4 \times 10^{-2}}\right]$$

$$\Rightarrow \quad q = \frac{3000 \times 4 \times 10^{-2}}{9 \times 10^9 \times 3} = \frac{4}{9} \times 10^{-8} \text{ C}$$

The potential at point distant $r_1 = 3$ cm from centre

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{b}\right)$$

The potential at point distant $r_2 = 2$ cm from centre,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_2} - \frac{q}{b}\right)$$

The gain in kinetic energy of electron

$$K = e(V_2 - V_1)$$

$$\Rightarrow \quad K = e\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_2} - \frac{q}{b} - \frac{q}{r_1} + \frac{q}{b}\right]$$

$$\Rightarrow \quad K = \frac{1}{4\pi\varepsilon_0} eq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Rightarrow \quad K = 9 \times 10^9 \times 1.6 \times 10^{-19} \times \frac{4}{9} \times 10^{-8} \times \left[\frac{1}{2 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}}\right]$$

$$\Rightarrow \quad K = \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad v^2 = \frac{2}{m} \times \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad v^2 = \left(\frac{2}{9 \times 10^{-31}}\right) \times \frac{32}{3} \times 10^{-17}$$

$$\Rightarrow \quad v^2 = \frac{64}{27} \times 10^{14}$$

$$\Rightarrow \quad v = 1.54 \times 10^7 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 154 \times 10^5 \text{ ms}^{-1}$$

$$\Rightarrow \quad x = 154$$

24. Initial potential energy of system

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{13}}\right]$$

$$\Rightarrow \quad U_1 = q \times 10^9\left[\frac{2 \times 3}{1} + \frac{3 \times 1}{1} + \frac{2 \times 1}{1}\right]$$

$$\Rightarrow \quad U_1 = 9 \times 10^9 \times 11$$

$$\Rightarrow \quad U_1 = 99 \times 10^9 \text{ J}$$

Final potential energy of system

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r'_{12}} + \frac{q_2q_3}{r'_{23}} + \frac{q_3q_1}{r'_{13}}\right]$$

$$\Rightarrow \quad U_2 = 9 \times 10^9\left[\frac{2 \times 3}{0.5} + \frac{3 \times 1}{0.5} + \frac{2 \times 1}{0.5}\right]$$

$$\Rightarrow \quad U_2 = 9 \times 10^9 \times 22 = 198 \times 10^9 \text{ J}$$

So, work done $W = U_2 - U_1$

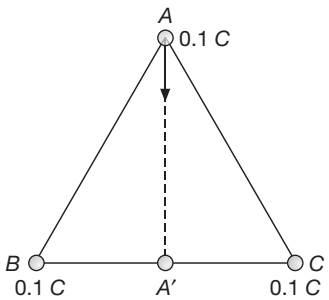
$$\Rightarrow \quad W = 198 \times 10^9 - 99 \times 10^9$$

$$\Rightarrow \quad W = 99 \times 10^9 \text{ J}$$

$$\Rightarrow \quad x = 99$$

25. Initial electrostatic energy of system of three charges,

$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{13}}\right)$$



Here $q_1 = q_2 = q_3 = q$ (say) $= 0.1$ C

$r_{12} = r_{23} = r_{13} = r$ (say) $= 1$ m

$$U_i = 3\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}\right)$$

$$\Rightarrow \quad U_i = 3\times 9\times 10^9 \times \frac{(0.1)^2}{1}$$

$$\Rightarrow \quad U_i = 2.7\times 10^8 \text{ J}$$

When charge A is moved to new position A' which is mid point of side BC, then final electrostatic energy,

$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{13}}\right)$$

$$\Rightarrow \quad U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{(A'B)} + \frac{q^2}{[BC]} + \frac{q^2}{(A'C)}\right]$$

But $A'B = A'C = \frac{r}{2} = 0.5$ m, so we get

$$U_f = 9\times 10^9\left[\frac{(0.1)^2}{0.5} + \frac{(0.1)^2}{1} + \frac{(0.1)^2}{0.5}\right]$$

$$\Rightarrow \quad U_f = 4.5\times 10^8 \text{ J}$$

Work done

$$W = U_f - U_i$$

$$\Rightarrow \quad W = 4.5\times 10^8 - 2.7\times 10^8$$

$$\Rightarrow \quad W = 1.8\times 10^8 \text{ J}$$

Since, by definition, $\text{Power} = \frac{\text{Work done}}{\text{Time}}$

$$\Rightarrow \quad P = \frac{W}{t}$$

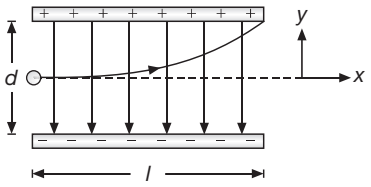
$$\Rightarrow \quad t = \frac{W}{P} = \frac{1.8\times 10^8}{10^3} \text{ second}$$

$$\Rightarrow \quad t = 1.8\times 10^5 \text{ s}$$

$$\Rightarrow \quad t = \frac{1.8\times 10^5}{3600} \text{ h}$$

$$\Rightarrow \quad t = 50 \text{ h}$$

26. The electron on entering the electric field, will experience a force $F = eE$ opposite to direction of electric field. On account of this force, it will be deflected in transverse direction. The path of electron is shown in figure.



Assuming initial velocity along X-axis, the transverse deflection will be along Y-direction,

$$a_y = \frac{F}{m} = \frac{eE}{m} = \text{constant}$$

$\therefore$ Initial velocity along Y-direction is zero and initial velocity along X-direction is u (say) therefore transverse deflection

$$y = \frac{1}{2}a_yt^2$$

$$\Rightarrow \quad y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{u}\right)^2 \qquad \left\{\because\ t = \frac{\ell}{u}\right\}$$

So, $y_{\text{MIN}} = \frac{1}{2}\frac{eE}{m}\left(\frac{\ell^2}{u_{\text{MAX}}^2}\right)$

Here $y_{\text{MIN}} = \frac{d}{2}$

and $E = \frac{V}{d}$

$$\Rightarrow \quad \frac{d}{2} = \frac{\ell}{2}\frac{e}{m}\left(\frac{V}{d}\right)\frac{\ell^2}{u_{\text{MAX}}^2}$$

$$\Rightarrow \quad u_{\text{MAX}}^2 = \frac{e}{m}\frac{V}{d}\left(\frac{\ell^2}{d}\right)$$

$$\Rightarrow \quad u_{\text{MAX}} = \frac{\ell}{d}\sqrt{\frac{eV}{m}}$$

Substituting given values

$$u_{MAX} = \frac{10^{-1}}{2\times 10^{-2}}\sqrt{\left(\frac{1.6\times 10^{-19}\times 300}{9\times 10^{-31}}\right)}$$

$$\Rightarrow \quad u_{\text{MAX}} = 3.65\times 10^7 \text{ ms}^{-1}$$

$$\Rightarrow \quad u_{\text{MAX}} = 365\times 10^5 \text{ ms}^{-1}$$

$$\Rightarrow \quad x = 365$$

27. (a) For equilibrium of oil drop between the plates the weight (mg) of drop must be balanced by upward electric force qE where E is electric field strength and q the charge on drop.

$$qE = mg$$

$$\Rightarrow \quad q = \frac{mg}{E}$$

where $E = \frac{V}{d} = \frac{1.5 \text{ kV}}{1.5\times 10^{-2} \text{ m}}$

$$\Rightarrow \quad E = \frac{1.5\times 10^3}{1.5\times 10^{-2}} \text{ Vm}^{-1} = 10^5 \text{ Vm}^{-1}$$

and $m = 4.9\times 10^{-15}$ kg

$$\Rightarrow \quad q = \frac{4.9\times10^{-15}\times9.8}{10^5}$$

$$\Rightarrow \quad q = 4.8\times10^{-19}\ \text{C}$$

If n is the number of electrons attached to oil drop and e is elementary charge, then by charge quantisation

$$q = ne$$

$$\Rightarrow \quad n = \frac{q}{e} = \frac{4.8\times10^{-19}}{1.6\times10^{-19}} = 3$$

(b) If the polarity of plates is reversed, then both weight of drop and electric force act downward therefore net downward force,

$$F = mg + qE = 2mg \qquad \{\text{since } mg = qE\}$$

Initial Acceleration of drop,

$$a = \frac{F}{m} = \frac{2mg}{q} = 2g \text{ downward}$$

$$\Rightarrow \quad a = 2\times10 = 20\ \text{ms}^{-2} \text{ downward}$$

(c) If the downward force F is balanced by upward viscous force, then drop attains terminal velocity v_T given by Stoke's Law

$$F = 6\pi\eta r v_T$$

$$\Rightarrow \quad 2mg = 6\pi\eta r v_T$$

$$\Rightarrow \quad v_T = \frac{2mg}{6\pi\eta r} = \frac{mg}{3\pi\eta r}$$

$$\Rightarrow \quad v_T = \frac{4.9\times10^{-15}\times10}{3\times3.14\times1.8\times10^{-5}\times5\times10^{-6}}$$

$$\Rightarrow \quad v_T = 5.8\times10^{-5}\ \text{ms}^{-1}$$

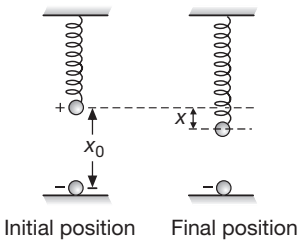
$$\Rightarrow \quad v_T = 58\times10^{-6}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_T = 58\ \mu\text{ms}^{-1}$$

28. When the thread is burnt, the positively charged sphere moves downward due to two forces

(i) Weight mg, acting vertically downward

(ii) Coulomb force of attraction $\frac{1}{4\pi\varepsilon_0}\frac{qq}{x^2}$, acting downward



At the position of maximum elongation of spring, the sphere moving downward comes to rest.

Let maximum elongation of spring be x, then by Law of Conservation of Energy

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{x_0} + mgx_0 + 0 = \frac{1}{4\pi\varepsilon_0}\frac{q(-q)}{(x_0-x)} + mg(x_0-x) + \frac{1}{2}K(x^2)$$

Total energy in initial position = Total energy in final position

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}q^2\left(\frac{1}{x_0-x} - \frac{1}{x_0}\right) = \frac{1}{2}Kx^2 - mgx \qquad \ldots(1)$$

Given $x = 0.1$ m, $x_0 = 0.5$ m,

$$K = 10^4\ \text{Nm}^{-1},\ g = 10\ \text{ms}^{-2}$$

$$\Rightarrow \quad \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{0.4} - \frac{1}{0.5}\right) = \frac{1}{2}(10^4)(0.1)^2 - (5)(10)(0.1)$$

$$\Rightarrow \quad 4.5\times10^9 q^2 = 45$$

$$\Rightarrow \quad q^2 = \frac{45}{4.5\times10^9} = 10^8$$

$$\Rightarrow \quad q = 10^4\ \text{C} = 100\ \mu\text{C}$$

29. Electric field due to a straight conductor,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

$$\therefore \quad \text{Force, } F = qE = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda q}{r}$$

$$\Rightarrow \quad F = 9\times10^9\cdot\frac{2\times6\times10^4\times1.6\times10^{-19}}{0.5}$$

$$\Rightarrow \quad F = 3.45\times10^{-4}\ \text{N}$$

$$\Rightarrow \quad F = 345\times10^{-6}\ \text{N} = 345\ \mu\text{N}$$

Since, by definition, we have

$$W = q(V_2 - V_1)$$

$$\Rightarrow \quad W = q\cdot\frac{1}{4\pi\varepsilon_0}2\lambda\log_e\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}(2\lambda q)\log_e\frac{r_2}{r_1}$$

$$\Rightarrow \quad W = 9\times10^9\times(2\times6\times10^4\times1.6\times10^{-19})\times\log_e\left(\frac{0.5}{0.2}\right)$$

$\Rightarrow \quad W = 1.58 \times 10^{-4}$ J

$\Rightarrow \quad W \cong 16$ mJ

30. Gain in KE $= e(V_2 - V_1)$

$$\Rightarrow \quad \frac{1}{2}mv^2 = e\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow \quad v^2 = \frac{2e}{m}\left[\frac{1}{4\pi\varepsilon_0}\left(q\frac{ab}{b-a}\right)\right]$$

$\Rightarrow \quad v = 8 \times 10^6$ ms^{-1}

So, the part not readable is 8.

31. The electric potential at P is due to two spheres

(i) A large solid sphere of charge density (ρ)

(ii) A small solid sphere of charge density $(-\rho)$

The electric potential due to whole solid sphere of radius R at a distance r from centre

$$V_1 = \frac{1}{4\pi\varepsilon_0}q\frac{(3R^2 - r^2)}{2R^3}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi R^3\rho\right)(3R^2 - r^2)}{2R^3}$$

The electric potential at external point at a distance

$$r' = \sqrt{x^2 + y^2} = 5 \text{ cm}$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r'} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi r'^3(-\rho)}{r'}$$

So, net potential

$$V = V_1 + V_2$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2}{3}\pi R\rho(3R^2 - r^2) - \frac{4}{3}\pi r^2\rho\right]$$

where $R = 5$ cm $= 5 \times 10^{-2}$ m,

$r = 4$ cm $= 4 \times 10^{-2}$ m,

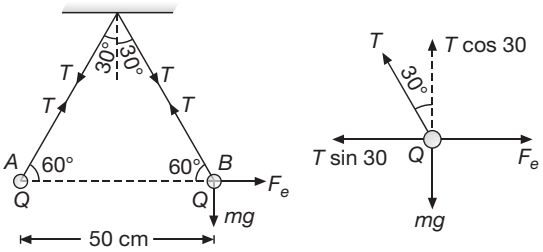
$r' = 5$ cm $= 5 \times 10^{-2}$ m,

$$\rho = \frac{10^{-6}}{\pi}$$

Substituting given values, $V = 35.16$ V

So, $V \cong 35$ V

32. Considering the equilibrium of ball P.



$$T\sin(30°) = F = F_e \qquad \ldots(1)$$

$$T\cos(30°) = mg \qquad \ldots(2)$$

$$\Rightarrow \quad T = \frac{mg}{\cos 30} = \frac{(0.866 \times 10^{-3}) \times 10}{0.866}$$

$\Rightarrow \quad T = 10^{-2}$ N

From equation (1)

$$T\sin 30° = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\Rightarrow \quad 10^{-2} \times \frac{1}{2} = \frac{9 \times 10^9 \times q^2}{(0.5)^2}$$

Solving we get,

$q = 3.726 \times 10^{-7}$ C

$q = 373$ nC

33. $\phi_E = \dfrac{q}{2\varepsilon_0}\left(1 - \dfrac{a}{\sqrt{R^2 + a^2}}\right)$ {as derived earlier}

$$\Rightarrow \quad \phi_E = \frac{8.85 \times 10^{-6}}{2(8.85 \times 10^{-12})}\left(1 - \frac{40}{50}\right)$$

$$\Rightarrow \quad \phi_E = \frac{10^6}{2}\left(1 - \frac{4}{5}\right)$$

$$\Rightarrow \quad \phi_E = \frac{10^6}{2} \times \frac{1}{5} = 10^5 \text{ NC}^{-1}$$

$\Rightarrow \quad \phi_E = 100$ kNC^{-1}

34. By Law of Conservation of Energy

$$(U + K)_{\text{initial}} = (U + K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{Qq}{4\pi\varepsilon_0 r_1} = \frac{1}{2}m(0)^2 + \frac{Qq}{4\pi\varepsilon_0 r_2}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Rightarrow \frac{1}{2}(2\times10^{-6})(1)^2=(10^{-9})(10^{-8})(9\times10^{9})\left(\frac{1}{r_2}-\frac{1}{\left(\frac{10}{100}\right)}\right)$$

$$\Rightarrow \quad \frac{100}{9}=\frac{1}{r_2}-10$$

$$\Rightarrow \quad \frac{1}{r_2}=\frac{190}{9}\ \text{m}$$

$$\Rightarrow \quad r_2=\frac{9}{190}\ \text{m}$$

$$\Rightarrow \quad r_2=4.7\times10^{-2}\ \text{m}$$

$$\Rightarrow \quad r_2=47\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad r_2=47\ \text{mm}$$

35. 63.5 g copper contains $N_A=6\times10^{23}$ copper atoms.

Therefore number of copper atoms in 10 g copper is

$$N=\frac{6\times10^{23}}{63.5}\times10$$

Since only one electron is transferred for every 1000 atoms, therefore the number of electrons transferred,

$$n=\frac{6\times10^{23}\times10}{63.5\times1000}$$

Magnitude of charge q is given by

$$q=ne=\frac{6\times10^{23}\times10}{63.5\times1000}\times1.6\times10^{-19}\ \text{C}$$

$$\Rightarrow \quad q=\frac{1920}{127}\ \text{C}$$

Separation between pieces is $r=1\ \text{cm}=10^{-2}\ \text{m}$

One piece of copper has positive charge and the other negative charge, so force of attraction between the pieces is

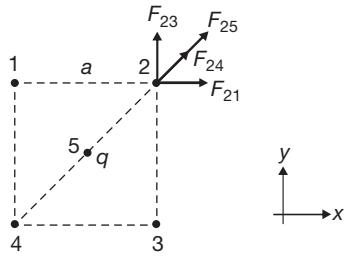
$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \quad F=\frac{9\times10^{9}\left(\frac{1920}{127}\right)\times\left(\frac{1920}{127}\right)}{(10^{-2})^2}\ \text{N}$$

$$\Rightarrow \quad F=2.057\times10^{6}\ \text{N}$$

So, $x\cong2$

36. Let us consider the equilibrium of charge Q placed at 2. Then for equilibrium



$$\vec{F}_2=\vec{0}$$

$$\Rightarrow \quad \vec{F}_{2x}\hat{i}+F_{2y}\hat{j}=\vec{0}$$

$$\Rightarrow \quad F_{2x}=0$$

$$\Rightarrow \quad F_{21}+F_{24}\cos45+F_{25}\cos45+F_{23}\cos90=0$$

$$\Rightarrow \quad \frac{Q^2}{4\pi\varepsilon_0a^2}+\frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2}\frac{1}{\sqrt{2}}+\frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}\frac{1}{\sqrt{2}}+0=0$$

$$\Rightarrow \quad Q+\frac{Q}{2\sqrt{2}}+\sqrt{2}q=0$$

$$\Rightarrow \quad Q(2\sqrt{2}+1)+4q=0$$

$$\Rightarrow \quad q=-\frac{Q}{4}(2\sqrt{2}+1)$$

Since $Q=\frac{8}{7}(1-2\sqrt{2})$

$$\Rightarrow \quad q=-\frac{1}{4}\left[\frac{8}{7}(1-2\sqrt{2})(1+2\sqrt{2})\right]$$

$$\Rightarrow \quad q=-\frac{1}{4}\left(\frac{8}{7}\right)(-7)=2\ \text{C}$$

37. Beta particles (i.e., electrons) emitted in t seconds

$$N=5\times10^{10}\ \text{t}$$

$\therefore$ The deficiency of electrons from a conductor results in positive charge

As 40% beta-particles escape from the source, positive charge gained by source (metal sphere),

$$q=\frac{40}{100}Ne=-\frac{40}{100}\times5\times10^{10}et$$

$$\Rightarrow \quad q=2\times10^{10}t\times1.6\times10^{-19}=3.2\times10^{-9}t\ \text{C}$$

Due to this charge the potential acquired by sphere

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

CHAPTER 1

Substituting given values

$$2 = 9\times10^{9}\times\frac{3.2\times10^{-9}t}{10^{-2}}$$

$$\Rightarrow\quad t = \frac{2\times10^{-2}}{9\times10^{9}\times3.2\times10^{-9}}$$

$$\Rightarrow\quad t = 7\times10^{-4}\ \text{s} = 700\ \mu\text{s}$$

38. For the ball just to complete the circle, the tension must vanish at the topmost point, i.e., $T_2 = 0$
From Newton's Second Law,

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0\ell^2} = \frac{mv^2}{\ell} \qquad \ldots(1)$$

At the topmost point $T_2 = 0$

$$mg - \frac{q^2}{4\pi\varepsilon_0\ell^2} = \frac{mv^2}{\ell} \qquad \ldots(2)$$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Energy at the}\\ \text{lowest point L}\end{pmatrix} = \begin{pmatrix}\text{Energy at the}\\ \text{highest point H}\end{pmatrix}$$

$$\Rightarrow\quad \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2\ell) \qquad \ldots(3)$$

$$\Rightarrow\quad v^2 = u^2 - 4g\ell \qquad \ldots(4)$$

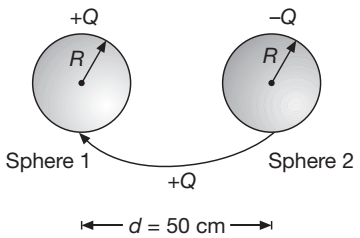
From equation (2), $v^2 = g\ell - \dfrac{q^2}{4\pi\varepsilon_0 m\ell}$ $\qquad \ldots(5)$

Equating (4) and (5), $u = \sqrt{5g\ell - \dfrac{q^2}{4\pi\varepsilon_0 m\ell}}$

$$\Rightarrow\quad u = \left(\frac{275}{8}\right)^{1/2} = 5.86\ \text{ms}^{-1}$$

$$\Rightarrow\quad u = 586\ \text{cms}^{-1}$$

39. Since the spheres are initially neutral, so when a charge Q is transferred from one sphere to the other, the charges on the sphere become $+Q$ and $-Q$ respectively. The potential difference between them is



$$\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \left(-\frac{Q}{4\pi\varepsilon_0 R}\right)$$

$$\Rightarrow\quad Q = 2\pi\varepsilon_0 R\,\Delta V$$

$$\Rightarrow\quad Q = \frac{2}{9\times10^{9}}\times100$$

Potential at the surface of first sphere,

V_1 = Potential due to charge on 1 + Potential due to charge on 2

$$\Rightarrow\quad V_1 = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0(d-R)} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{(d-R)}\right]$$

Potential at the surface of second sphere,

V_2 = Potential due to charge on 1 + Potential due to charge on 2

$$\Rightarrow\quad V_2 = \frac{+Q}{4\pi\varepsilon_0(d-R)} - \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(d-R)} - \frac{1}{R}\right]$$

Potential difference between the surfaces of the two spheres is $\Delta V'$ given by

$$\Delta V' = V_1 - V_2$$

$$\Rightarrow\quad \Delta V' = \frac{200}{10^{-2}}\left[\frac{1}{10} - \frac{1}{40}\right] - \frac{200}{10^{-2}}\left[\frac{1}{40} - \frac{1}{10}\right]$$

$$\Rightarrow\quad \Delta V' = 30\times10^{2}\ \text{V}$$

Kinetic energy gained by proton $= +e\Delta V' = \dfrac{1}{2}mv^2$

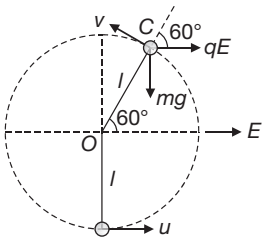
$$\Rightarrow\quad v = \sqrt{\frac{2e(\Delta V')}{m}}$$

$$\Rightarrow\quad v = \sqrt{\frac{2\times1.602\times10^{-19}\times30\times10^{2}}{1.672\times10^{-27}}}$$

$$\Rightarrow\quad v = 7.58\times10^{5}\ \text{ms}^{-1}$$

$$\Rightarrow\quad v = 758\times10^{3}\ \text{ms}^{-1} = 758\ \text{kms}^{-1}$$

40. Using the concept of Work-Energy Theorem,



we get

$$\frac{1}{2}m\left(v^2 - u^2\right) = -mg\ell(1+\sin60°) + qE\ell\cos60°$$

Substituting the values, we get

$$u^2 - v^2 = 32.32 \qquad \ldots(1)$$

Since, tension in the string at C is zero.

$$\Rightarrow \quad \frac{mv^2}{\ell} = mg\sin 60° - qE\cos 60°$$

$$\Rightarrow \quad v^2 = 3.66 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$u = 6 \text{ ms}^{-1}$$

41. (a) Consider the two spheres as a system

By Law of Conservation of Momentum

$$0 = m_1 v_1 \hat{i} + m_2 v_2 (-\hat{i})$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{m_2}$$

By Law of Conservation of Energy

$$0 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{r_1 + r_2}$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_1 + r_2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2m_2 q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

Substituting values, we get

$$v_1 = \sqrt{\frac{2(0.7)(9\times 10^9)(2\times 10^{-6})(3\times 10^{-6})}{(0.1)(0.8)}\left(\frac{1}{8\times 10^{-3}} - \frac{1}{1}\right)}$$

$$\Rightarrow \quad v_1 = 10.8 \text{ ms}^{-1}$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{m_2} = \frac{0.1 \text{ kg}(10.8 \text{ ms}^{-1})}{0.7 \text{ kg}}$$

$$\Rightarrow \quad v_2 = 1.55 \text{ ms}^{-1}$$

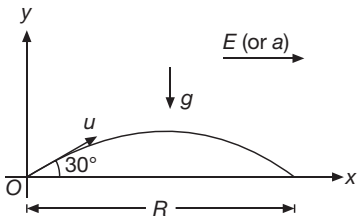
So, $v_1 = 1080 \text{ cms}^{-1}$ and $v_2 = 155 \text{ cms}^{-1}$

(b) If the spheres are conductors/metal, then electrons will move around them such that the centres of the excess charges can be placed inside the spheres without any loss in energy. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving faster than that calculated in (a).

42. $u = 20 \text{ ms}^{-1}$, $\alpha = 45°$

$$a_x = \frac{qE}{m} = 40 \text{ ms}^{-2}$$

$$a_y = g = -10 \text{ ms}^{-2}$$



Since at maximum height $v_y = 0$, so

$$v_y = u_y + a_y t$$

$$\Rightarrow \quad 0 = u\sin\alpha + (-g)T$$

$$\Rightarrow \quad T = \frac{2u_y}{|a_y|} = \frac{2u\sin\alpha}{g} = \frac{2\times 20}{10}\times\frac{1}{\sqrt{2}} = 2\sqrt{2} \text{ s}$$

In time $t = T$, the projectile touches the ground at a distance R from the launch point. So,

Range is given by

$$R = u_x T + \frac{1}{2}a_x T^2$$

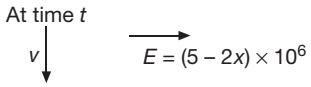
$$\Rightarrow \quad R = \left(\frac{20}{\sqrt{2}}\right)(2\sqrt{2}) + \left(\frac{1}{2}\right)(40)(2\sqrt{2})^2$$

$$\Rightarrow \quad R = 40 + 160$$

$$\Rightarrow \quad R = 200 \text{ m}$$

43. $a_x = \frac{qE}{m}$

$$\Rightarrow \quad \frac{dv_x}{dt} = \frac{q}{m}(5 - 2x)$$



$$\Rightarrow \quad v\frac{dv}{dx} = \frac{q}{m}(5 - 2x)$$

$$\Rightarrow \quad \frac{v_x^2}{2} = \frac{q}{m}(5x - x^2)$$

$$\Rightarrow \quad v_x^2 = \frac{2q}{m}(5x - x^2)$$

$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{2q}{m}(5x - x^2)}$$

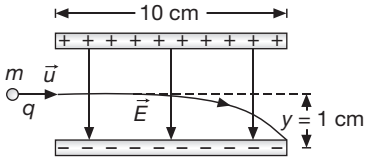
For x to be maximum $\frac{dx}{dt} = 0$

$$\Rightarrow \quad 5x - x^2 = 0$$

$$\Rightarrow \quad x = 5 \text{ m}$$

CHAPTER 1

44. The situation is visualised in the figure



Since, $E = \frac{V}{d}$

$\Rightarrow \quad E = \frac{300}{2/100} = 15000 \text{ Vm}^{-1}$

Since the particle does not come out hence its maximum deflection in vertical direction has to be

$$y = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}\left(\frac{\ell}{u}\right)^2 \qquad \left\{\text{As } a = \frac{qE}{m} \text{ and } t = \frac{\ell}{u}\right\}$$

$$\Rightarrow \quad u^2 = \frac{1}{2}\left(\frac{qE}{my}\right)x^2$$

$$\Rightarrow \quad u^2 = \frac{1}{2}\frac{(1.6\times10^{-19})(15000)}{(12\times10^{-24})(10^{-2})}\left(\frac{1}{10}\right)^2 = 10^8$$

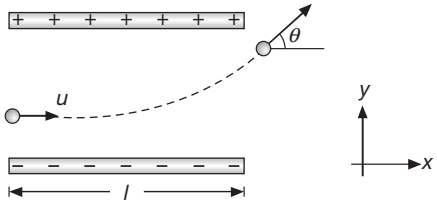
$$\Rightarrow \quad u = 10^4 \text{ ms}^{-1}$$

$$\Rightarrow \quad u = 10 \text{ kms}^{-1}$$

45. Along the x-direction speed of electron remains uniform; because of no component of field along horizontal. So,

$$v_x = u \text{ and } x = ut$$

$$\Rightarrow \quad \ell = ut \qquad \ldots(1)$$



Along y direction

$$u_y = 0$$

Acceleration along y-direction of motion of electron is given by

$$a_y = a = \frac{eE}{m}$$

Since, $v_y = u_y + a_y t$

$$\Rightarrow \quad v_y = 0 + \left(\frac{eE}{m}\right)t$$

From (1), we get $t = \frac{\ell}{u}$

$$v_{y_{\text{final}}} = v_{y_{\text{initial}}} + at$$

$$\Rightarrow \quad v_y = \left(\frac{eE}{m}\right)\left(\frac{\ell}{u}\right)$$

If θ be the angle of deviation, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(\frac{eE\ell}{mu}\right)}{u}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{eE\ell}{mu^2}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{eE\ell}{m\left(\sqrt{3}\frac{eE\ell}{m}\right)}\right] = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

46. Lets consider $V = 0$ at point P. Then the potential at the original position of the charge is

$$V_i = -Ex = -EL\cos\theta$$

At the final point A, $V_f = -EL$. Since the table is frictionless, by Law of Conservation of Energy

$$(K+U)_i = (K+U)_f$$

$$\Rightarrow \quad 0 - qEL\cos\theta = \frac{1}{2}mv^2 - qEL$$

$$\Rightarrow \quad v = \sqrt{\frac{2qEL(1-\cos\theta)}{m}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(2\times10^{-6} \text{ C})(300 \text{ NC}^{-1})(1.5 \text{ m})(1-\cos60^\circ)}{0.01 \text{ kg}}}$$

$$\Rightarrow \quad v = 0.3 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 30 \text{ cms}^{-1}$$

47. Total charge

$$Q = \int \rho dV = \int_{r=0}^{r=R} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}(R^{a+3})$$

$$Q' = \int \rho dV = \int_{r=0}^{r=\frac{R}{2}} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$$

According to question

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'}{\left(\frac{R}{2}\right)^2} = \frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)$$

Putting the value of Q and Q' get

$$a = 2$$

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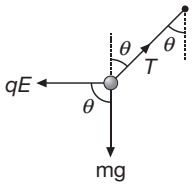
1. $T\cos\theta = mg$

$T\sin\theta = qE$



$\Rightarrow \quad \tan\theta = \dfrac{qE}{mg}$

$\Rightarrow \quad \tan\theta = \dfrac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \dfrac{1}{2}$

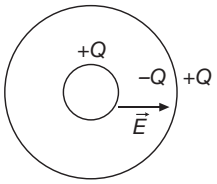
$\Rightarrow \quad \tan^{-1}\left(\dfrac{1}{2}\right)$

Hence, the correct answer is (D).

2. **For CASE-1**

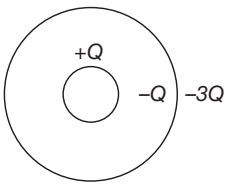
Electric field between spherical surface

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$



For CASE-2

Electric field between the surfaces remains unchanged.



So, potential difference between them also remains unchanged.

Hence, the correct answer is (B).

3. Since $dV = -\vec{E}\vec{dr} = -(Ax+B)dx$

$$\Rightarrow \quad \int_{v_2}^{v_1} dV = \int_{-5}^{1} -(Ax+B)dx$$

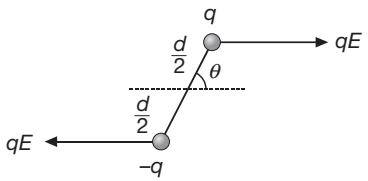
$$\Rightarrow \quad V_1 - V_2 = \left(-A\frac{x^2}{2} - Bx\right)_{-5}^{1}$$

$$\Rightarrow \quad V_1 - V_2 = \left(-\frac{A}{2} - B\right) + \left(\frac{A}{2}25 + B(-5)\right)$$

$$\Rightarrow \quad V_1 - V_2 = 12A - 6B = 240 - 60 = 180 \text{ V}$$

Hence, the correct answer is (A).

4. Since $\tau = \mathrm{I}\alpha$



$$\Rightarrow \quad -(qE)d\theta = \mathrm{I}\frac{d^2\theta}{dt^2} = 2\left(\frac{md^2}{4}\right)\left(\frac{d^2\theta}{dt^2}\right)$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} - \frac{2qE\theta}{md}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2qE}{md}}$$

Hence, the correct answer is (A).

5. $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$\int_{V_G}^{V_P} dV = \int -\frac{\lambda}{2\pi\varepsilon_0 r}dr$$

$$\Rightarrow \quad V_P - V_G = \frac{\lambda}{2\pi\varepsilon_0}\ell\mathrm{n}\left(\frac{r}{r_0}\right)$$

Since, $\dfrac{1}{2}mv^2 = q(V_P - V_G)$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r}{r_0}\right)$$

$$\Rightarrow \quad v \propto \left[\ell n\left(\frac{r}{r_0}\right)\right]^{\frac{1}{2}}$$

Hence, the correct answer is (A).

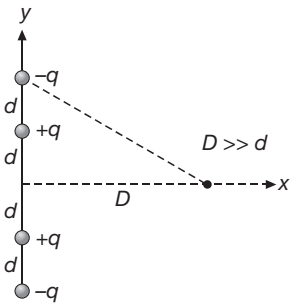
CHAPTER 1

6. $$U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}+\frac{Qq}{\left(\frac{d}{2}+D\right)}-\frac{Qq}{\left(D-\frac{d}{2}\right)}\right)$$

$$\Rightarrow\quad U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}-\frac{qQd}{D^2}\right)$$

Hence, the correct answer is (A).

7.



$$|\vec{E}|=\frac{2qD}{4\pi\varepsilon_0\left(D^2+4d^2\right)^{\frac{3}{2}}}-\frac{2qD}{4\pi\varepsilon_0\left(D^2+d^2\right)^{\frac{3}{2}}}$$

(along -ive x direction)

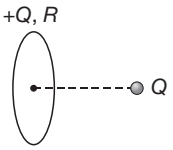
$$\Rightarrow\quad |\vec{E}|=\frac{2qD}{4\pi\varepsilon_0 D^3}\left[\frac{1}{\left(1+\left(\frac{2d}{D}\right)^2\right)^{\frac{3}{2}}}-\frac{1}{\left(1+\left(\frac{d}{D}\right)^2\right)^{\frac{3}{2}}}\right]$$

$$\Rightarrow\quad |\vec{E}|=\frac{2q}{4\pi\varepsilon_0 D^2}\left(1-\frac{3}{2}\frac{4d^2}{D^2}-1+\frac{3}{2}\frac{d^2}{D^2}\right)$$

$$\Rightarrow\quad |\vec{E}|=\frac{9qd^2}{4\pi\varepsilon_0 D^4}$$

Hence, the correct answer is (C).

8.



Potential at any point of the charged ring

$$V_P=\frac{Q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}$$

If v_0 is the minimum velocity for the point charge to reach the centre, then

$$\frac{1}{2}mv_0^2=Q\left(V_C-V_P\right)$$

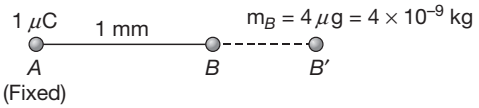
$$\Rightarrow\quad \frac{1}{2}mv_0^2=\frac{Q}{4\pi\varepsilon_0}\left(\frac{Q}{3a}-\frac{Q}{5a}\right)$$

$$\Rightarrow\quad v_0^2=\frac{4}{15}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q^2}{ma}\right)$$

$$\Rightarrow\quad v_0=\sqrt{\frac{2}{m}}\left[\frac{2}{15}\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)\right]^{\frac{1}{2}}$$

Hence, the correct answer is (D).

9.



Since, $U_1=\frac{q_1q_2}{4\pi\varepsilon_0 r_i}$ $\qquad U_f=\frac{q_1q_2}{4\pi\varepsilon_0 r_2}$

By Law of Conservation of Energy, we have

$$(U+K)_i=(U+K)_f$$

$$\frac{q_1q_2}{4\pi\varepsilon_0 r_1}=\frac{q_1q_2}{4\pi\varepsilon_0 r_2}+\frac{1}{2}mv^2$$

$$v^2=\frac{2q_1q_2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\Rightarrow\quad v^2=\frac{2\times9\times10^9\times10^{-12}}{4\times10^{-9}\times10^{-3}}\left(1-\frac{1}{9}\right)=4\times10^9$$

$$\Rightarrow\quad v=\sqrt{40}\times10^4\ \text{ms}^{-1}=6.32\times10^4\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

10. $$T=2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

where, $g_{\text{eff}}=\sqrt{g^2+\left(\frac{gE}{m}\right)^2}$

$$\Rightarrow\quad T=2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

Hence, the correct answer is (C).

11. $V=0$ and

$$\vec{E}=\frac{\vec{p}}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta+1}$$

For $\theta=\frac{\pi}{2}$ and $r=d$, we get

$$\vec{E}=\frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$$

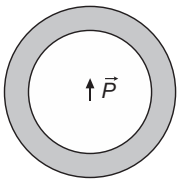
Hence, the correct answer is (C).

12. Since net charge on the dipole is zero, so the inside surface shall have non-zero and non-uniform charge distribution. Net field outside the region would be same as that would have been for point charge at surface.

Hence, the correct answer is (C).

13. According to Gauss Law, we have

$$E(4\pi a^2) = \frac{q_{enclosed}}{\varepsilon_0} \qquad \ldots(1)$$



$$\Rightarrow \quad q_{enc} = \int_0^a (kr) \times 4\pi r^2 dr$$

$$\Rightarrow \quad q_{enc} = 4\pi k\left(\frac{a^4}{4}\right) = 2Q \qquad \ldots(2)$$

From (1), we get

$$E(4\pi a^2) = \frac{4\pi k a^4}{4\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{ka^2}{4\varepsilon_0}$$

Since both charges do not experience any force, so

$$(F_{\text{net}})_A = 0$$

$$\Rightarrow$$

$$\Rightarrow \quad \frac{ka^2}{4\varepsilon_0} \times Q = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

Since, $2Q = \dfrac{4\pi k R^2}{4}$

$$\Rightarrow \quad k = \frac{2Q}{\pi R^4}$$

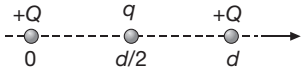
$$\left(\frac{2Q}{\pi R^4}\right)\left(\frac{a^2 Q}{4\varepsilon_0}\right) = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

$$\Rightarrow \quad 8a^4 = R^4$$

$$\Rightarrow \quad a = \frac{R}{8^{\frac{1}{4}}} = 8^{-\frac{1}{4}} R$$

Hence, the correct answer is (D).

14.



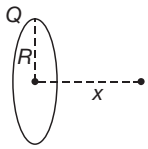
$$F_Q = \frac{Qq}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} + \frac{Q^2}{4\pi\varepsilon_0 d^2} = 0$$

$$\Rightarrow \quad 4Qq + Q^2 = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}$$

Hence, the correct answer is (C).

15. $E(x) = \dfrac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{\frac{3}{2}}}$



For maximum, $\dfrac{dE}{dx} = 0$

$$\Rightarrow \quad \frac{Q}{4\pi\varepsilon_0}\left[\frac{(R^2 + x^2)^{\frac{3}{2}} - (x)\frac{3}{2}(R^2 + x^2)^{\frac{1}{2}}(2x)}{(R^2 + x^2)^3}\right] = 0$$

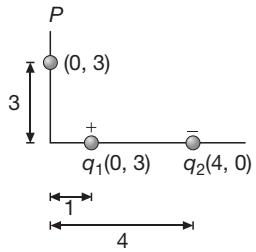
$$\Rightarrow \quad \frac{(R^2 + x^2)^{\frac{1}{2}} Q}{4\pi\varepsilon_0 (R^2 + x^2)^3}(x^2 + R^2 - 3x^2) = 0$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}$$

$$\Rightarrow \quad h = \frac{R}{\sqrt{2}}$$

Hence, the correct answer is (A).

16.



By Principle of Superposition, we have

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

where,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{\sqrt{10} \times 10^{-6}}{10}\left(-\frac{\hat{i} + 3\hat{j}}{\sqrt{10}}\right)$$

CHAPTER 1

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{(-25\times10^{-6})}{25}\left(-\frac{4\hat{i}+3\hat{j}}{5}\right)$$

$$\Rightarrow \quad \vec{E} = 9\times10^9\times10^{-6}\left[-\frac{\hat{i}+3\hat{j}}{10} - \left(-\frac{4\hat{i}+3\hat{j}}{5}\right)\right]$$

$$\Rightarrow \quad \vec{E} = \frac{9\times10^3}{10}\left(-\hat{i}+3\hat{j}+8\hat{i}-6\hat{j}\right)$$

$$\Rightarrow \quad \vec{E} = 9\times10^2\left(+7\hat{i}-3\hat{j}\right)$$

$$\Rightarrow \quad \vec{E} = \left(63\hat{i}-27\hat{j}\right)\times10^2 \text{ Vm}^{-1}$$

Hence, the correct answer is (A).

17. $$Q = \int_0^R 4\pi r^2\left(\frac{A}{r^2}e^{-\frac{2r}{a}}\right)dr$$

$$\Rightarrow \quad Q = \frac{4\pi Aa}{-2}e^{\frac{-2r}{a}}\Big|_0^R = 2\pi aA\left(1-e^{-\frac{2R}{a}}\right)$$

$$\Rightarrow \quad Q = e^{-\frac{2R}{a}} = 1-\frac{Q}{2\pi aA}$$

$$\Rightarrow \quad e^{\frac{2R}{a}} = \frac{1}{\left(1-\frac{Q}{2\pi aA}\right)}$$

$$\Rightarrow \quad R = \frac{a}{2}\ln\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

Hence, the correct answer is (A).

18. $$V \propto \frac{1}{d^2}$$

$$\Rightarrow \quad \frac{4qa}{x^2} = \frac{2qa}{(R-x)^2}$$

$$\Rightarrow \quad (R-x) = \frac{x}{\sqrt{2}}$$

$$\Rightarrow \quad x = \frac{\sqrt{2}R}{(\sqrt{2}+1)}$$

Hence, the correct answer is (B).

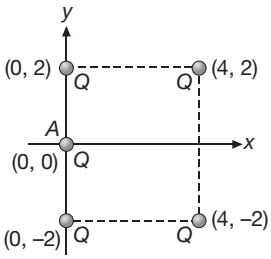
19. $$Q = 4\pi\sigma\left(a^2+b^2+c^2\right)$$

$$\Rightarrow \quad V = \frac{\sigma}{4\pi\varepsilon_0}\left(\frac{4\pi a^2}{a}+\frac{4\pi b^2}{b}+\frac{4\pi c^2}{c}\right)$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{a+b+c}{a^2+b^2+c^2}\right)$$

Hence, the correct answer is (A).

20.



$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2}+\frac{Q}{2}+\frac{q}{\sqrt{2^2+4^2}}+\frac{Q}{\sqrt{2^2+4^2}}\right)$$

$$\Rightarrow \quad V_A = \frac{1}{4\pi\varepsilon_0}\left(Q+\frac{2Q}{2\sqrt{5}}\right)$$

So, potential energy is given by

$$U = \frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$$

Hence, the correct answer is (D).

21. $F = \frac{p}{4\pi\varepsilon_0 y^3}Q$, for electric dipole

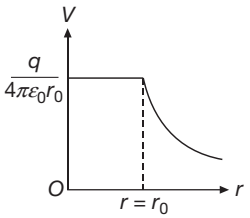
and $F' = \frac{p}{4\pi\varepsilon_0\left(\frac{y}{3}\right)^3}Q = \frac{27pQ}{4\pi\varepsilon_0 y^3}$

$$\Rightarrow \quad F' = 27F$$

Hence, the correct answer is (C).

22. For spherical shell $V = \frac{q}{4\pi\varepsilon_0 r_0}$ for $r \le r_0$

and $V = \frac{q}{4\pi\varepsilon_0 r}$ for $r > r_0$



Hence, the correct answer is (A).

23. $$0 = U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a}+\frac{Qq}{a}+\frac{Qq}{\sqrt{2}a}\right)$$

$$\Rightarrow \quad -q = Q\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad Q = \frac{-q\sqrt{2}}{\sqrt{2}+1}$$

Hence, the correct answer is (A).

24. $U = -\vec{p}\cdot\vec{E}$

$\Rightarrow \quad U = -pE\cos(45°)$

$\Rightarrow \quad U = -10^{-29}\times 10^{3}\times\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad U = -7\times 10^{-27}$ J

Hence, the correct answer is (C).

25. By Law of Conservation of Energy, we get

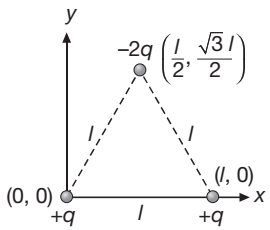
$$\frac{Q^2}{4\pi\varepsilon_0 R_0} = \frac{Q^2}{4\pi\varepsilon_0 R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{R_0}-\frac{1}{R}\right)}$$

Hence v increases and attains a finite value after a large time.

Hence, the correct answer is (D).

26.



Since $p_x = q_1x_1 + q_2x_2 + q_3x_3$

$\Rightarrow \quad p_x = (q)(0)+(q)(\ell)+(-2q)\left(\dfrac{\ell}{2}\right)$

$\Rightarrow \quad p_x = 0$

Similarly, $p_y = q_1y_1 + q_2y_2 + q_3y_3$

$\Rightarrow \quad p_y = (q)(0)+(q)(0)+(-2q)\left(\dfrac{\sqrt{3}\ell}{2}\right)$

$\Rightarrow \quad p_y = -\sqrt{3}q\ell$

$\Rightarrow \quad \vec{p} = p_x\hat{i} + p_y\hat{j}$

$\Rightarrow \quad \vec{p} = -\sqrt{3}q\ell\hat{j}$

Hence, the correct answer is (C).

27. Since $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$

$\Rightarrow \quad Q = \varepsilon_0 AE = (8.85\times 10^{-12})(1)(100)$

$\Rightarrow \quad Q = 8.85\times 10^{-10}$ C

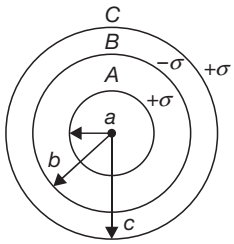
Hence, the correct answer is (A).

28. The potential of the shell B,

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{r_b}+\frac{q_B}{r_b}+\frac{q_C}{r_c}\right)$$

$$\Rightarrow \quad V_B = \frac{4\pi}{4\pi\varepsilon_0}\left(\frac{\sigma\times a^2}{b}-\frac{\sigma\times b^2}{b}+\frac{\sigma\times c^2}{c}\right)$$

$$\Rightarrow \quad v_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$$



Hence, the correct answer is (B).

29. Charged particle can be considered at the centre of a cube of side a, and the given surface represents its one side.

So, flux through each face $\phi = \dfrac{Q}{6\varepsilon_0}$

Hence, the correct answer is (B).

30. Charge density on given solid ball varies as

$$\rho = \rho_0\left(1-\frac{r}{R}\right), \text{ for } 0\le r\le R$$

Electric field outside the ball is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \ldots(1)$$

Now, $dq = \rho dV = \rho(4\pi r^2)dr$

$$\Rightarrow \quad q = \int dq = \int_0^R \rho_0\left(1-\frac{r}{R}\right)(4\pi r^2)dr$$

$$\Rightarrow \quad q = (4\pi\rho_0)\left[\frac{r^3}{3}-\frac{1}{R}\times\frac{r^4}{4}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3}-\frac{R^3}{4}\right)$$

$$\Rightarrow \quad q = 4\pi\rho_0\left(\frac{R^3}{12}\right) \qquad \ldots(2)$$

From (1) and (2), $E = \dfrac{\rho_0 R^3}{12\varepsilon_0 r^2}$

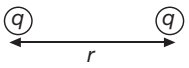
Hence, the correct answer is (A).

CHAPTER 1

31. Initially force between spheres A and B, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{r^2}$

When A and C are touched, charge on both will be $\dfrac{q}{2}$. Again C is touched with B the charge on B is given by

$$q_B = \frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$$



Required force between spheres A and B is given by

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{q}{2}\times\frac{3q}{4}}{r^2} = \frac{3}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right) = \frac{3}{8}F$$

Hence, the correct answer is (A).

32. Equilibrium position will shift to a point where resultant force is zero. So,

$$kx_{eq} = qE$$

$$\Rightarrow \quad x_{eq} = \frac{qE}{k}$$

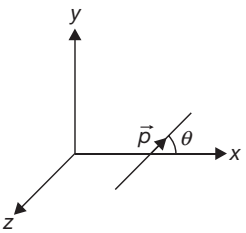
Total energy of the system is

$$E = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$$

Hence, the correct answer is (D).

33. $\vec{p} = p\cos\theta\hat{i} + p\sin\theta\hat{j}$

$$\vec{E}_1 = E\hat{i}$$



$$\Rightarrow \quad \vec{T}_1 = \vec{p}\times\vec{E}_1$$

$$\Rightarrow \quad \vec{T}_1 = \left(p\cos\theta\hat{i} + p\sin\theta\hat{j}\right)\times E\left(\hat{i}\right)$$

$$\Rightarrow \quad \tau\hat{k} = pE\sin\theta\left(-\hat{k}\right) \qquad \ldots(1)$$

$$\vec{E}_2 = \sqrt{3}E_1\hat{j}$$

$$\vec{T}_2 = \left(p\cos\theta\hat{i} + p\sin\theta\hat{j}\right)\times\sqrt{3}E_1\hat{j}$$

$$\Rightarrow \quad -\tau\hat{k} = \sqrt{3}pE_1\cos\theta\hat{k} \qquad \ldots(2)$$

From (1) and (2), we get

$$pE\sin\theta = \sqrt{3}pE\cos\theta$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Hence, the correct answer is (C).

35. $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

For S_1, $\phi_1 = \dfrac{2q}{\varepsilon_0}$

For S_2, $\phi_2 = \dfrac{3q-q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

For S_3, $\phi_3 = \dfrac{q+q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

For S_4, $\phi_4 = \dfrac{8q-6q}{\varepsilon_0} = \dfrac{2q}{\varepsilon_0}$

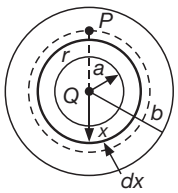
So, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \dfrac{2q}{\varepsilon_0}$

Hence, the correct answer is (A).

36. Applying Gauss's theorem for radius r, we get

$$\int\vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}(Q+q)$$

$$\Rightarrow \quad E\left(4\pi r^2\right) = \frac{1}{\varepsilon_0}(Q+q) \qquad \ldots(1)$$



where q is charge enclosed between $x = a$ and $x = r$.

$$q = \int_a^r \frac{A}{x}4\pi x^2dx = 4\pi A\int_a^r xdx = 4\pi A\left(\frac{x^2}{2}\right)_a^r$$

$$\Rightarrow \quad q = 2\pi A\left(r^2 - a^2\right)$$

Substituting the value of q in equation (1), we get

$$E\left(4\pi r^2\right) = \frac{1}{\varepsilon_0}\left[Q + 2\pi A\left(r^2 - a^2\right)\right]$$

$$\Rightarrow \quad E=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}+2\pi A-\frac{2\pi Aa^2}{r^2}\right)$$

E will be constant if it is independent of r

$$\Rightarrow \quad \frac{Q}{r^2}=\frac{2\pi Aa^2}{r^2}$$

$$\Rightarrow \quad A=\frac{Q}{2\pi a^2}$$

Hence, the correct answer is (A).

37. Since, $V(z)=\begin{cases}30-5z^2 \text{ for } |z|\le 1 \text{ m}\\ 35-10|z| \text{ for } |z|\ge 1 \text{ m}\end{cases}$

and $E(z)=-\dfrac{dV}{dz}=10z$ for $|z|\le 1$ m and

$E(z)=10$ for $|z|\ge 1$ m

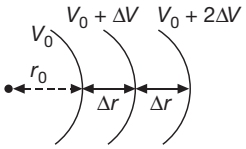
So, the source is an infinite non-conducting thick plate of thickness 2 m .

$$\Rightarrow \quad E=\frac{q}{2A\varepsilon_0} \text{ since } \rho_0=\frac{q}{At}=\frac{2E}{t}\varepsilon_0$$

$$\Rightarrow \quad \rho_0=\frac{2\times 10}{2}\varepsilon_0=10\varepsilon_0$$

Hence, the correct answer is (B).

38. Since, $E=-\left(\dfrac{\Delta V}{\Delta r}\right)$



where, ΔV and Δr are same for any pair of surfaces and hence, $E=$ constant

Since, electric field inside the spherical charge distribution is given by

$$E=\frac{\rho}{3\varepsilon_0}r=\text{constant}$$

$$\Rightarrow \quad \rho r=\text{constant}$$

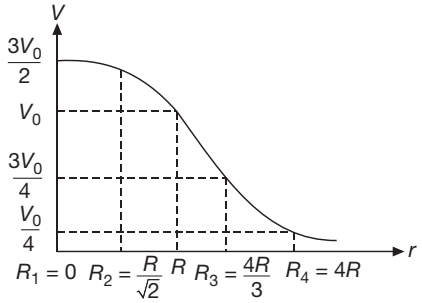
$$\Rightarrow \quad \rho(r)\propto\frac{1}{r}$$

Hence, the correct answer is (C).

39. $V=\begin{cases}\dfrac{Q}{8\pi\varepsilon_0 R^3}(3R^2-r^2), & \text{for} \quad r<R\\ \dfrac{Q}{4\pi\varepsilon_0 r}, & \text{for} \quad r\ge R\end{cases}$

So, $V_{\text{surface}}=V_0=\dfrac{Q}{4\pi\varepsilon_0 R}$,

$V_{\text{centre}}=\dfrac{3V_0}{2}$, so $R_1=0$ …(1)



Now, $V=\dfrac{5V_0}{4}=\dfrac{Q}{8\pi\varepsilon_0 R^3}\left(3R^2-R_2^2\right)$

$$\Rightarrow \quad \frac{5}{2}=3-\frac{R_2^2}{R^2}$$

$$\Rightarrow \quad R_2=\frac{R}{\sqrt{2}} \qquad \ldots(2)$$

Similarly, at R_3, $V=\dfrac{3V_0}{4}=\dfrac{Q}{4\pi\varepsilon_0 R_3}$

$$\Rightarrow \quad R_3=\frac{4R}{3} \qquad \ldots(3)$$

Again, at R_4, $V=\dfrac{V_0}{4}=\dfrac{Q}{4\pi\varepsilon_0 R_4}$

$$\Rightarrow \quad R_4=4R \qquad \ldots(4)$$

So, we observe from (1), (2), (3) and (4), that

$$R_1=0,\ R_4>2R \text{ and } R_4-R_3=4R-\frac{4R}{3}=\frac{8R}{3}>R_2$$

Hence, the correct answer is (C) and (D).

40. Electric field lines must originate from the positive charge and end at the negative charge. Also they must exit and enter a surface normally, must be smooth and continuous.

41. Electric field due to complete disc $(R=2a)$,

$$E_1=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{\sqrt{4a^2+h^2}}\right)$$

$$\Rightarrow \quad E_1=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{2a}\right) \qquad \{\because h\ll a\}$$

Electric field due to disc of radius $R=a$ is

$$E_2=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{a}\right)$$

CHAPTER 1

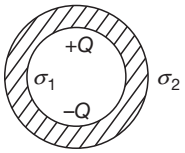
Hence, electric field due to given annular disc is

$$\Rightarrow \quad E = E_1 - E_2 = \frac{\sigma h}{4\varepsilon_0 a}$$

$$\Rightarrow \quad C = \frac{\sigma}{4a\varepsilon_0}$$

Hence, the correct answer is (C).

42.



On outer surface there will be no charge

So $Q_2 = \sigma_2 = 0$ on inner surface total charge will be zero, however, charge distribution will be there, so

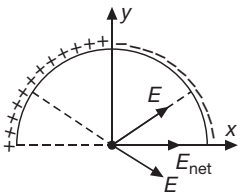
$$Q_1 = 0 \text{ and } \sigma_1 \neq 0$$

Hence, the correct answer is (C).

43. Due to quarter ring electric field intensity is

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\left(\frac{\theta}{2}\right)$$

So, due to each quarter section, $\theta = \frac{\pi}{2}$, so field intensity is



$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \vec{E}_{net} = \sqrt{2}E\,\hat{i} = \frac{\sqrt{2}\sqrt{2}\lambda}{4\pi\varepsilon_0 R}\hat{i} = \frac{2\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

Since $\lambda = \frac{Q}{\left(\frac{\pi R}{2}\right)} = \frac{2Q}{\pi R}$

$$\Rightarrow \quad \vec{E}_{net} = \frac{2(2Q)}{4\pi\varepsilon_0 \pi R^2}\hat{i} = \left(\frac{4Q}{4\pi^2\varepsilon_0 R^2}\right)\hat{i}$$

where $Q = 10^3\varepsilon_0 C$

and $\pi R = L = 20$ cm

$$\Rightarrow \quad \vec{E}_{net} = \left(\frac{4\times10^3\varepsilon_0}{4\pi^2\varepsilon_0 R^2}\right)\hat{i} = \left(\frac{4\times10^3}{4L^2}\right)\hat{i}$$

$$\Rightarrow \quad \vec{E}_{net} = \left(\frac{4\times10^3}{4\times(0.2)^2}\right)\hat{i} = \left(\frac{4\times10^3}{4\times0.04}\right)\hat{i}$$

$$\Rightarrow \quad \vec{E}_{net} = (25\times10^3\ \text{NC}^{-1})\hat{i}$$

Hence, the correct answer is (B).

44. Since $\Delta V = -\int \vec{E}\cdot d\vec{r}$

$$\Rightarrow \quad V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{r}$$

where, $d\vec{r} = dx\hat{i} + dy\hat{j}$ and

$$\vec{E} = \left(25\hat{i} + 30\hat{j}\right)\text{NC}^{-1}$$

$$\Rightarrow \quad \Delta V = V - 0 = -\int_{(0,0)}^{(2,2)} \left(25\hat{i} + 30\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j}\right)$$

$$\Rightarrow \quad V = -\left(\int_0^2 25dx + \int_0^2 30dy\right)$$

$$\Rightarrow \quad V = -\left(25(x)_0^2 + 30(y)_0^2\right)$$

$$\Rightarrow \quad V = -(25\times2 + 30\times2) \quad V = -110\text{ V} = -110\text{ J/C}$$

Hence, the correct answer is (D).

45. $V_A - V_O = -\int_O^A E_x dx$

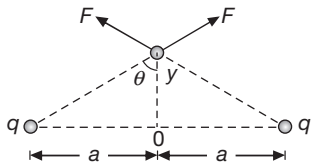
$$\Rightarrow \quad V_A - V_O = -30\int_0^2 x^2 dx$$

$$\Rightarrow \quad V_A - V_O = -30\left(\frac{x^3}{3}\Big|_0^2\right) = -10(8)$$

$$\Rightarrow \quad V_A - V_O = -80\text{ V}$$

Hence, the correct answer is (C).

46.



$$F_{net} = 2F\cos\theta$$

$$\Rightarrow \quad F_{net} = \frac{2\times kq^2}{2\left(a^2+y^2\right)}\times\frac{y}{\left(a^2+y^2\right)^{1/2}}$$

So, $F \approx \frac{kq^2y}{a^3}$ (for $y \ll a$)

$\Rightarrow \quad F \propto y$

Hence, the correct answer is (A).

47. $V = \int \frac{kdq}{L+x}$ where $k = \frac{1}{4\pi\varepsilon_0}$

$\Rightarrow \quad V = \frac{kQ}{L}\int_O^L \frac{dL}{L+x} = \frac{kQ}{L}\ln 2$

$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0 L}\ln 2$

Hence, the correct answer is (D).

48. Potential at the centre of the sphere, $V_C = \frac{R^2\rho}{2\varepsilon_0}$

Potential at the surface of the sphere, $V_S = \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}$

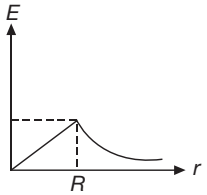
When a charge q is taken from the centre to the surface, the change in potential energy is.

$$\Delta U = (V_C - V_S)q = \left(\frac{R^2\rho}{2\varepsilon_0} - \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}\right)q = \frac{1}{6}\frac{R^2\rho q}{\varepsilon_0}$$

Statement 1 is false. Statement 2 is true.

Hence, the correct answer is (B).

49. For uniformly charged sphere



$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ (For $r < R$)

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (For $r = R$)

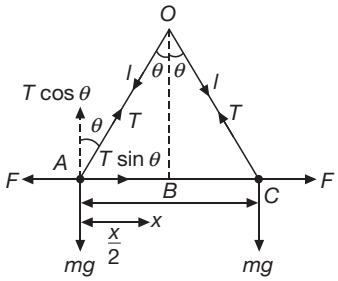
$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (For $r > R$)

The variation of E with distance r from the centre is as shown in figure.

Hence, the correct answer is (B).

50. **Method-I**

Figure shows equilibrium positions of the two spheres, so $T\cos\theta = mg$ and



$T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}$

$\Rightarrow \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$

$\Rightarrow \quad \frac{x}{2\ell} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg}$ $\left\{\because \tan\theta = \frac{x}{2\ell}\right\}$

$\Rightarrow \quad \frac{x}{2\ell} \propto \frac{q^2}{x^2}$

$\Rightarrow \quad q^2 \propto x^3$

$\Rightarrow \quad q \propto x^{\frac{3}{2}}$

when charge begins to leak from both the spheres at a constant rate, then

$\frac{dq}{dt} \propto \frac{3}{2}x^{\frac{1}{2}}\frac{dx}{dt}$

$\Rightarrow \quad v \propto x^{-\frac{1}{2}}$ $\left\{\because \frac{dq}{dt} = \text{constant}\right\}$

Method-II

At any instant, loss in electrostatic potential energy is equal to gain in kinetic energy.

So, $\frac{q^2}{4\pi\varepsilon_0 x} = 2\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad v \propto x^{-\frac{1}{2}}$

Hence, the correct answer is (A).

51. $\phi = ar^2 + b$

Electric field, $E = -\frac{d\phi}{dr} = -2ar$ …(1)

According to Gauss's theorem, $\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{inside}}}{\varepsilon_0}$

$\Rightarrow \quad (-2ar)4\pi r^2 = \frac{q_{\text{inside}}}{\varepsilon_0}$ {using (1)}

$\Rightarrow \quad q_{\text{inside}} = -8\varepsilon_0 a\pi r^3$

CHAPTER 1

Charge density inside the ball is

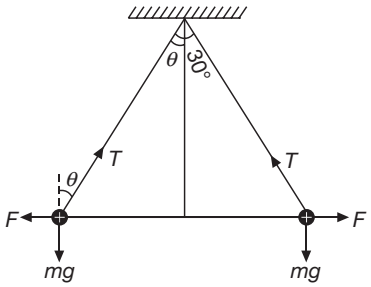
$$\rho_{\text{inside}} = \frac{q_{\text{inside}}}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho_{\text{inside}} = \frac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho_{\text{inside}} = -6a\varepsilon_0$$

Hence, the correct answer is (D).

52.



Initially, the forces acting on each ball are

(i) Tension T

(ii) Weight mg

(iii) Electrostatic force of repulsion F

For its equilibrium along vertical

$$T\cos\theta = mg \qquad \ldots(1)$$

and along horizontal

$$T\sin\theta = F \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\tan\theta = \frac{F}{mg} \qquad \ldots(3)$$

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, i.e. $F' = (F/K)$, whereas weight

$$mg' = mg - \text{upthrust}$$

$$\Rightarrow \quad mg' = mg - V\sigma g \qquad \{\because \text{Upthrust} = V\sigma g\}$$

$$\Rightarrow \quad mg' = mg\left(1 - \frac{\sigma}{\rho}\right) \qquad \left\{\because V = \frac{m}{\rho}\right\}$$

For equilibrium of balls, we have

$$\tan\theta' = \frac{F'}{mg'} = \frac{F}{Kmg\left(1-(\sigma/\rho)\right)} \qquad \ldots(4)$$

According to given problem, we have $\theta' = \theta$

From equations (3) and (4), we get $K = \dfrac{1}{\left(1-\dfrac{\sigma}{\rho}\right)}$

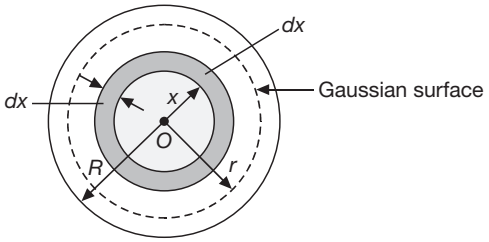
$$\Rightarrow \quad K = \frac{\rho}{(\rho-\sigma)} = \frac{1.6}{(1.6-0.8)} = 2$$

Hence, the correct answer is (D).

53. Consider a thin spherical shell of radius x and thickness dx as shown in the figure.

Volume of the shell,

$$dV = 4\pi x^2 dx$$



Let us draw a Gaussian surface of radius $r(r < R)$ as shown in the figure above.

Total charge enclosed (Q_{enc}) inside the Gaussian surface is

$$Q_{\text{enc}} = \int_0^r \rho dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right)4\pi x^2 dx$$

$$\Rightarrow \quad Q_{\text{enc}} = 4\pi\rho_0 \int_0^r \left(\frac{5}{4}x^2 - \frac{x^3}{R}\right)dx$$

$$\Rightarrow \quad Q_{\text{enc}} = 4\pi\rho_0\left(\frac{5}{12}x^3 - \frac{x^4}{4R}\right)_0^r = 4\pi\rho_0\left(\frac{5}{12}r^3 - \frac{r^4}{4R}\right)$$

$$\Rightarrow \quad Q_{\text{enc}} = \frac{4\pi\rho_0}{4}\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right) = \pi\rho_0\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right)$$

According to Gauss's Law, $E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0}$

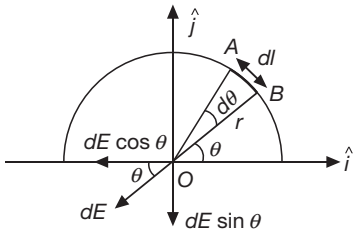
$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\rho_0}{\varepsilon_0}\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right)$$

$$\Rightarrow \quad E = \frac{\pi\rho_0 r^3}{4\pi r^2\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

Hence, the correct answer is (C).

54. Linear charge density, $\lambda = \frac{q}{\pi r}$

Consider a small element AB of length $d\ell$ subtending an angle $d\theta$ at the centre O as shown in the figure.



Charge dq on the element is $dq = \lambda d\ell$

$$\Rightarrow \quad dq = \lambda(rd\theta) \qquad \left\{\because d\theta = \frac{d\ell}{r}\right\}$$

the electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolving dE into two rectangular components, we get by symmetry,

$$\int dE\cos\theta = 0$$

So, the net electric field at O is

$$\vec{E} = \int_0^{\pi} dE\sin\theta\left(-\hat{j}\right) = \int_0^{\pi}\frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{E} = -\int_0^{\pi}\frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left\{\because \lambda = \frac{q}{\pi r}\right\}$$

$$\Rightarrow \quad \vec{E} = -\int_0^{\pi}\frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}(-\cos\theta)\Big|_0^{\pi}\hat{j}$$

$$\Rightarrow \quad \vec{E} = -\left(\frac{q}{2\pi^2\varepsilon_0 r^2}\right)\hat{j}$$

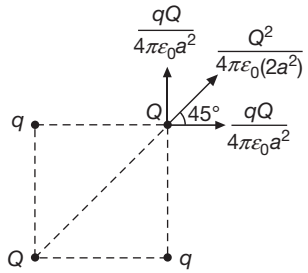
Hence, the correct answer is (D).

55. For net force on Q to be zero

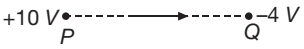
$$\frac{qQ}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)}\cos 45 + \frac{qQ}{4\pi\varepsilon_0 a^2}\cos 90 = 0$$

$$\Rightarrow \quad Q = -2\sqrt{2}q$$

$$\Rightarrow \quad \frac{Q}{q} = -2\sqrt{2}$$



Hence, the correct answer is (A).

56. 

Work done in moving 100 negative charges i.e. electrons from the positive to the negative potential i.e. from P to Q is

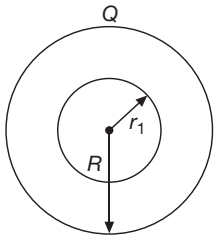
$$W = (100e^-)(V_Q - V_P) = (-100\times 1.6\times 10^{-19})(-14\text{ V})$$

$$\Rightarrow \quad W = 2.24\times 10^{-16}\text{ J}$$

Hence, the correct answer is (D).

57. If the charge density, $\rho = \frac{Q}{\pi R^4}r$,

The electric field at the point P distant r_1 from the centre, according to Gauss's theorem is



$$E\left(4\pi r_1^2\right) = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad E\left(4\pi r_1^2\right) = \frac{1}{\varepsilon_0}\int \rho dV \qquad \left\{\because q_{enc} = \int \rho dV\right\}$$

$$\Rightarrow \quad E\left(4\pi r_1^2\right) = \frac{1}{\varepsilon_0}\int_0^{r_1}\frac{Qr}{\pi R^4}\left(4\pi r^2 dr\right)$$

$$\Rightarrow \quad E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

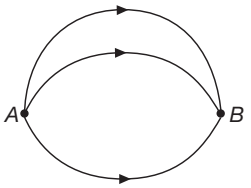
Hence, the correct answer is (C).

58. Work done = Potential difference × charge

$$\Rightarrow \quad W = (V_B - V_A)\times q,$$

CHAPTER 1

V_A and V_B only depend on the initial and final positions and not on the path. Electrostatic force is a conservative force. if the loop is completed, $V_A - V_A = 0$. No network is done as the initial and final potentials are the same.
Both the statements are true but Statement-2 is not the reason for Statement-1.



Hence, the correct answer is (C).

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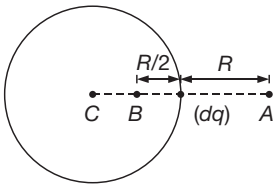
Single Correct Choice Type Problems

1. For uniformly distributed charged shell, the surface charge density (σ) is

$$\sigma = \frac{Q}{4\pi R^2}$$

So, charge of small area $dA = \alpha 4\pi R^2$ is

$$dq = \sigma dA = \alpha Q$$



Since potential at the surface before removing the charge dq is V_0, so

$$V_0 = \frac{Q}{4\pi\varepsilon_0 R}$$

When dq is removed, then potential at the centre is

$$V_{\text{centre}} = V_0 - V_{(dq)}$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{Q}{4\pi\varepsilon_0 R} - \frac{\alpha Q}{4\pi\varepsilon_0 R} = V_0(1-\alpha)$$

Similarly, potential at the point B (after removing dq) is

$$V_B = V_0 - \frac{\alpha Q}{4\pi\varepsilon_0\left(\frac{R}{2}\right)} = V_0(1-2\alpha)$$

$$\Rightarrow \quad \frac{V_{\text{centre}}}{V_B} = \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$$

According to principle of superposition, we get the electric field at A as

$$E_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(2R)^2} - \frac{\alpha Q}{(R)^2}\right) = E_{\text{shell}} - \frac{\alpha V_0}{R}$$

$$\Rightarrow \quad \Delta E_A = -\frac{\alpha V_0}{R}$$

So, at A, magnitude of electric field decreases by $\frac{\alpha V_0}{R}$

Similarly, $(E_C)_{\text{initial}} = 0$ and $(E_C)_{\text{final}} = \frac{\alpha Q}{4\pi\varepsilon_0 R^2}$

$$\Rightarrow \quad \Delta E_C = \frac{\alpha Q}{4\pi\varepsilon_0 R^2} = \frac{\alpha V_0}{R}$$

Hence, field at C increases by $\frac{\alpha V_0}{R}$

Hence, the correct answer is (D).

2. The sphere with cavity can be assumed as a complete sphere with positive charge of radius R_1 + another complete sphere with negative charge and radius R_2.

$E_+ \to \vec{E}$ due to total positive charge

$E_- \to \vec{E}$ due to total negative charge.

$E = E_+ + E_-$

If we calculate it at P, then E_- comes out to be zero.

So, $E = E_+$

and $E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1^3}(OP)$, in the direction of OP.

Here, q is total positive charge on whole sphere.

It is in the direction of OP or a.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

Hence, the correct answer is (D).

3. At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

Hence, the correct answer is (C).

4. $E_1 = \dfrac{kQ}{R^2}$, where $k = \dfrac{1}{4\pi\varepsilon_0}$

$$E_2 = \frac{k(2Q)}{R^2}$$

$$\Rightarrow \quad E_2 = \frac{2kQ}{R^2}$$

$$E_3 = \frac{k(4Q)R}{(2R)^3}$$

$$\Rightarrow \quad E_3 = \frac{kQ}{2R^2}$$

Hence, the correct answer is (C).

5. For inside points $(r \le R)$

$$E = 0 \;\Rightarrow\; V = \text{constant} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

For outside points $(r \ge R)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ or } E \propto \frac{1}{r^2}$$

and $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$ or $V \propto \dfrac{1}{r}$

On the surface $(r = R)$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

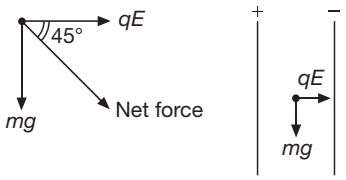
$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$$

where, $\sigma = \dfrac{q}{4\pi R^2}$ = surface charge density

corresponding to above equations the correct graphs are shown in option (D).

Hence, the correct answer is (D).

6.



Net force is at $45°$ from vertical.

$$\Rightarrow \quad qE = mg$$

$$\Rightarrow \quad \frac{qX}{d} = mg \qquad \left\{\because E = \frac{X}{d}\right\}$$

$$\Rightarrow \quad X = \frac{mgd}{q}$$

$$\Rightarrow \quad X = \frac{(1.67\times10^{-27})(9.8)(10^{-2})}{(1.6\times10^{-19})}$$

$$\Rightarrow \quad X \approx 1\times10^{-9} \text{ V}$$

Hence, the correct answer is (C).

7. Electric flux, $\phi = \vec{E}\cdot\vec{A}$

$$\Rightarrow \quad \phi = EA\cos\theta$$

Here, θ is the angle between $\vec{E}$ and $\vec{A}$

In this question $\theta = 45°$, because $\vec{A}$ is perpendicular to surface

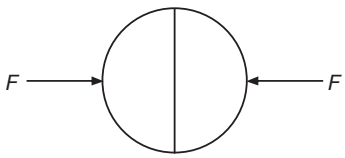
$$E = E_0 \text{ and}$$

$$A = (\sqrt{2}a)(a) = \sqrt{2}a^2$$

$$\Rightarrow \quad \phi = (E_0)(\sqrt{2}a^2)\cos(45°) = E_0a^2$$

Hence, the correct answer is (C).

8.



Electrostatics repulsive force :

$$F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$$

$$F = F_{ele} = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

Hence, the correct answer is (A).

9. In equilibrium, $mg = qE$

In absence of electric field, $mg = 6\pi\eta rv$

$$\Rightarrow \quad qE = 6\pi\eta rv$$

$$m = \frac{4}{3}\pi Rr^3d = \frac{qE}{g}$$

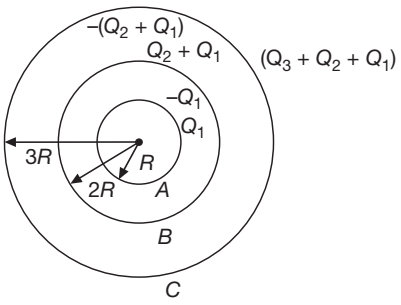
$$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$$

After substituting value we get,

$$q = 8\times10^{-19} \text{ C}$$

Hence, the correct answer is (D).

10. $Q_1 = \sigma(4\pi R^2) = 4\pi R^2\sigma$



$$Q_2 + Q_1 = 16\pi R^2\sigma$$

$$\Rightarrow \quad Q_2 = 16\pi R^2\sigma - Q_1 = 12\pi R^2\sigma$$

$$Q_3 + Q_2 + Q_1 = 36\pi R^2\sigma$$

$$\Rightarrow \quad Q_3 = 36\pi R^2\sigma - 16\pi R^2\sigma = 20\pi R^2\sigma$$

$$\Rightarrow \quad Q_1 : Q_2 : Q_3 = 1 : 3 : 5$$

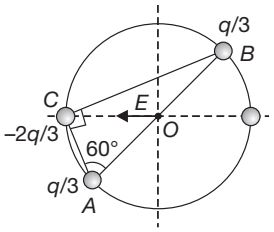
Hence, the correct answer is (B).

11. From Gauss's Law, we have flux equal to

$$\phi = \frac{\Sigma Q_{enc}}{\varepsilon_0} = \frac{\left(\frac{8C}{4}\right) - 7C + \left(\frac{6C}{2}\right)}{\varepsilon_0} = -\frac{2C}{\varepsilon_0}$$

Hence, the correct answer is (A).

12. Net electric field due to both charges $\frac{q}{3}$, will get cancelled. Electric field due to $\left(-\frac{2q}{3}\right)$ will be directed in –ve axis



$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)}{R^2}$$

$$\Rightarrow \quad E = \frac{q}{6\pi\varepsilon_0 R^2}$$

$$\text{P.E. of system} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\left(\frac{q}{3}\right)^2}{2R} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60°} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60°}\right)$$

P.E. of system $\neq 0$

Force between B and C

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60°)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\times 2q^2}{9\times 4\times 3R^2}$$

$$\Rightarrow \quad F = \frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2} \text{ (attractive)}$$

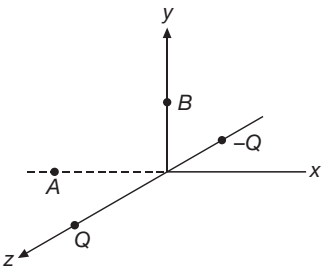
$$\Rightarrow \quad F = \frac{1}{54}\frac{q^2}{\pi\varepsilon_0 R^2}$$

Potential at O is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3} + \frac{q}{3} - \frac{2q}{3}\right)}{R} = 0$$

Hence, the correct answer is (C).

13. $A \equiv (-a,\ 0,\ 0)$, $B \equiv (0,\ a,\ 0)$



Since point charge is moved from A to B and

$$V_A = V_B = 0$$

$$\Rightarrow \quad W = 0$$

Hence, the correct answer is (C).

14. Charge will be induced in the conducting sphere, but net charge on it will be zero.

Hence, the correct answer is (D).

15. Inside the cavity, field at any point is uniform and non-zero.

Hence, the correct answer is (B).

16. There will be an electric field between two cylinders (using Gauss theorem). This electric field will produce a potential difference.

Hence, the correct answer is (A).

17. All the three plates will produce electric field at P along negative z-axis. Hence ,

$$\vec{E}_P = \left[\frac{\sigma}{2\varepsilon_o} + \frac{2\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_o}\hat{k}$$

The correct answer is (B).

Hence, the correct answer is (B).

18. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2.

Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

The correct answer is (C).

Hence, the correct answer is (C).

19. According to option (D), the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence, the correct answer is (D).

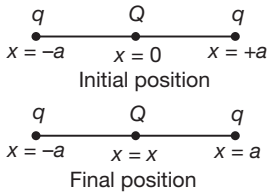
Hence, the correct answer is (D).

20. Electric field is zero everywhere inside a metal or conductor i.e. field lines do not enter a metal. Also, field lines must enter or leave the conductor surface at right angles because it is an equipotential surface.

Hence, the correct answer is (C).

21. $U_i = \dfrac{2Qq}{4\pi\varepsilon_0 a}$

and $U_f = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{a+x}+\dfrac{1}{a-x}\right]$



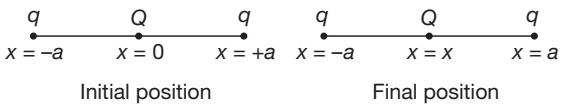
$\Delta U = U_f - U_i$

$\Rightarrow \quad |\Delta U| = \dfrac{2Qqx^2}{4\pi\varepsilon_0 a^3}$ for $x \ll a$

$\Rightarrow \quad \Delta U \propto x^2$

Hence, the correct answer is (B).

22.



$U_i = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{2Qq}{a}+\dfrac{q^2}{2a}\right)$

and $U_f = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{a+x}+\dfrac{1}{a-x}\right]+\dfrac{q^2}{4\pi\varepsilon_0(2a)}$

$\Rightarrow \quad \Delta U = U_f - U_i$

$\Rightarrow \quad |\Delta U| = \dfrac{2KQqx^2}{a^3}$

For $x \ll a$, we have

$\Delta U \propto x^2$

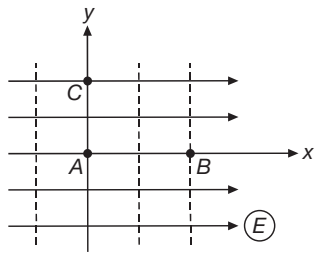
Hence, the correct answer is (B).

23. Electrostatic lines of force never form a closed loop. Therefore, options (B) and (D) are wrong, Electrostatic lines of force emanate from positive charge and terminate on negative charge, therefore, option (A) is also wrong.

Hence, the correct answer is (C).

24. Potential decreases in the direction of electric field. Dotted lines are equipotential lines. So

$V_A = V_C$ and $V_A > V_B$



Hence, the correct answer is (B).

25. Net electrostatic energy of the configuration will be

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a}+\frac{Qq}{\sqrt{2}\,a}+\frac{Qq}{a}\right]=0$$

$\Rightarrow \quad Q = \dfrac{-2q}{2+\sqrt{2}}$

Hence, the correct answer is (B).

26. $V = \dfrac{q}{4\pi\varepsilon_0 x_0}\left(1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}.....\right)$

$\Rightarrow \quad V = \dfrac{q}{4\pi\varepsilon_0 x_0}\log_e 2$

Hence, the correct answer is (D).

27. $s = \dfrac{1}{2}at^2$

where $a = \dfrac{qE}{m}$

$\Rightarrow \quad \dfrac{1}{2}\left(\dfrac{qE}{m_e}\right)t_1^2 = \dfrac{1}{2}\left(\dfrac{qE}{m_p}\right)t_2^2$

CHAPTER 1

$$\Rightarrow \quad \frac{t_2}{t_1} = \left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}$$

Hence, the correct answer is (B).

28. $-\int_{\ell=\infty}^{\ell=0} \vec{E}\cdot d\vec{\ell}$ is the line integral of electric field which is also called electrostatic potential. Since, charge is distributed non-uniformly, so $E \neq 0$ at the centre.

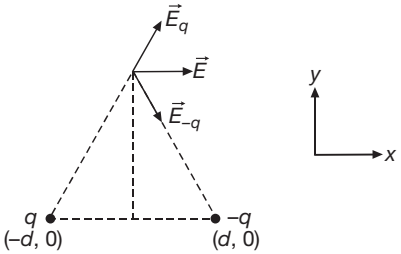
$$\Rightarrow \quad -\int_{\ell=\infty}^{\ell=0} \vec{E}\cdot d\vec{\ell} = \frac{q}{4\pi\varepsilon_0 R} = 2$$

Hence, the correct answer is (A).

29. Field lines always enter or leave a surface normal to it.

Hence, the correct answer is (D).

30. The electrical field $\vec{E}$ at all points on the x-axis will not have the same direction.



For $-d \le x \le d$, electric field is along positive x-axis, while for all other points it is along negative x-axis. The electric field $\vec{E}$ at all points on the y-axis will be parallel to the x-axis (i.e. along $\hat{i}$) (OPTION (C)).

The electrical potential at the origin due to both the charges is zero, hence, no work is done in bringing a test charge from infinity to the origin.

Dipole moment is directed from the $-q$ charge to the $+q$ charge (i.e. along $-\hat{i}$ direction).

Hence, the correct answer is (C).

31. $$V_1 = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$V_2 = \frac{Q_1}{4\pi\varepsilon_0\sqrt{2}a} + \frac{Q_2}{4\pi\varepsilon_0 a}$$

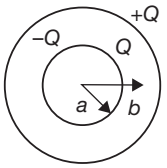
$$\Rightarrow \quad W_{1\to 2} = q\left(V_2 - V_1\right)$$

$$\Rightarrow \quad W_{1\to 2} = \frac{q\left(Q_1 - Q_2\right)}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad W = \frac{q\left(Q_1 - Q_2\right)\left(\sqrt{2}-1\right)}{4\sqrt{2}\pi\varepsilon_0 a}$$

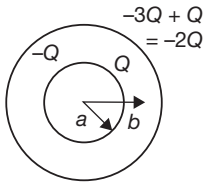
Hence, the correct answer is (B).

32.



$$V = V_{\substack{\text{surface}\\\text{sphere}}} - V_{\substack{\text{outer}\\\text{shell}}}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$



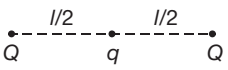
Potential difference $= V' = V_{\substack{\text{surface}\\\text{sphere}}} - V_{\substack{\text{outer}\\\text{shell}}}$

$$\Rightarrow \quad V' = \left(\frac{Q}{4\pi\varepsilon_0 a} + \frac{-2Q}{4\pi\varepsilon_0 b}\right) - \left(\frac{Q}{4\pi\varepsilon_0 b} + \frac{-2Q}{4\pi\varepsilon_0 b}\right)$$

$$\Rightarrow \quad V' = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = V$$

Hence, the correct answer is (A).

33. For equilibrium of Q



$$\frac{Qq}{4\pi\varepsilon_0\left(\frac{\ell}{2}\right)^2} + \frac{Q^2}{4\pi\varepsilon_0\ell^2} = 0$$

$$\Rightarrow \quad 4q + Q = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}$$

Here, we are asked to calculate equilibrium for system. In this system q is already in equilibrium. So, we are calculating equilibrium condition for Q.

Hence, the correct answer is (B).

34. By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will

be along CO. Under the action of this force charge Q will move towards O. If at any time charge Q is at a distance x from O.

$$F = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)}\times\frac{x}{(a^2+x^2)^{\frac{1}{2}}}$$

i.e., $F = \frac{1}{4\pi\varepsilon_0}\frac{2qQx}{(a^2+x^2)^{\frac{3}{2}}}$

As the restoring force F is not linear, motion will be oscillatory (with amplitude $2a$) but not simple harmonic.

Hence, the correct answer is (D).

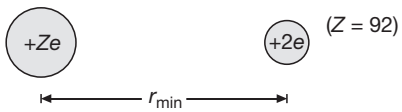
35. The potential at the centre of a hollow sphere is equal to the potential at its surface.

Hence, the correct answer is (B).

36. From conservation of mechanical energy

Decrease in kinetic energy = increase in potential energy

or $\frac{1}{4\pi\varepsilon_o}\frac{(Ze)(2e)}{r_{\min}} = 5 \text{ MeV} = 5\times1.6\times10^{-13}\text{ J}$



$$\Rightarrow \quad r_{\min} = \frac{1}{4\pi\varepsilon_o}\frac{2Ze^2}{5\times1.6\times10^{-13}} = \frac{\left(9\times10^9\right)(2)(92)\left(1.6\times10^{-19}\right)^2}{5\times1.6\times10^{-13}}$$

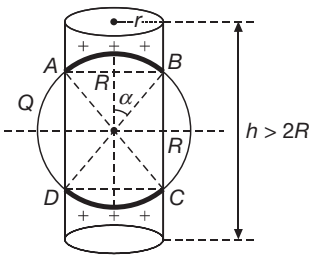
$$\Rightarrow \quad r_{\min} = 5.3\times10^{-14}\text{ m} = 5.3\times10^{-12}\text{ cm}$$

So, $r_{\min}$ is of the order of 10^{-12} cm.

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. **For OPTION (A)**



Since $h > 2R$ and $r = \frac{3R}{5}$, so we get

$$\sin\alpha = \frac{r}{R} = \frac{3}{5}$$

$$\Rightarrow \quad \cos\alpha = \frac{4}{5}$$

Since $\phi = 2\left[\frac{Q}{2\varepsilon_0}(1-\cos\alpha)\right]$ …(1)

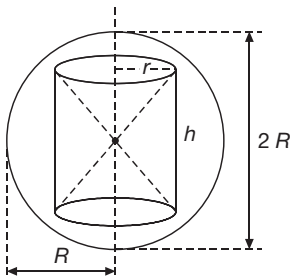
$$\Rightarrow \quad \phi = \frac{Q}{5\varepsilon_0}$$

Please note that a factor of 2 is used in equation (1), because we have two surfaces i.e., AB and CD of the shell that are enclosed by the cylinder.

For OPTION (B)

$$h < \frac{8R}{5} \text{ and } r = \frac{3R}{5}$$

$$h < 2R \text{ and } r < R$$

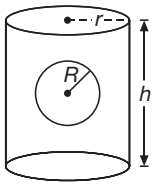


$$\Rightarrow \quad Q_{\text{enclosed}} = 0$$

$$\Rightarrow \quad \phi = 0$$

For OPTION (C)

$$h > 2R \text{ and } r > R$$

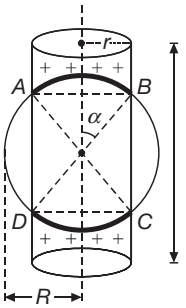


$$\Rightarrow \quad Q_{\text{enclosed}} = Q$$

$$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}$$

For OPTION (D)

$$h > 2R \text{ and } r = \frac{4R}{5} \text{ i.e., } r < R$$



$$Q_{\text{enclosed}} = Q[1-\cos\alpha]$$

Since, $\sin\alpha = \frac{r}{R} = \frac{3}{5}$

$$\Rightarrow \quad \cos\alpha = \frac{3}{5}$$

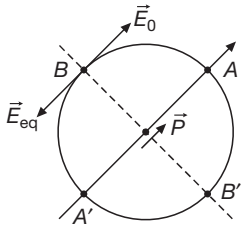
$$\Rightarrow \quad \phi = \frac{2Q}{2\varepsilon_0}(1-\cos\alpha)$$

$$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}\left(1-\frac{3}{5}\right)$$

$$\Rightarrow \quad \phi = \frac{2Q}{5\varepsilon_0}$$

Hence, (A), (B) and (C) are correct.

2. E_0 is the external field in the direction of $\vec{p} = \frac{p_0}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$



For an equipotential surface of radius r, the point B is the point at the equitorial line of the dipole. Since, on an equipotential surface, no tangential field exists, so we have

$$E_0 = E_{eq} \text{ (in magnitude)}$$

but both in opposite direction (as shown)

$$\Rightarrow \quad \frac{p_0}{4\pi\varepsilon_0 R^3} = E_0$$

$$\Rightarrow \quad R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{\frac{1}{3}}$$

Also, A lies at axial line of dipole, so

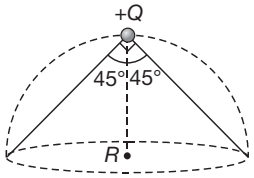
$$\vec{E}_A = \frac{2\vec{p}}{4\pi\varepsilon_0 R^3} + \vec{E}_0 = 2\vec{E}_0 + \vec{E}_0 = 3\vec{E}_0$$

Since $E_{eq} = E_0$

$$\Rightarrow \quad \left|\vec{E}_B\right| = E_{eq} - E_0 = 0$$

Hence, (A) and (C) are correct.

3. (a) $\Omega = 2\pi(1-\cos\theta)$, where $\theta = 45°$



$$\Rightarrow \quad \phi = -\frac{\Omega}{4\pi}\times\frac{Q}{\varepsilon_0}$$

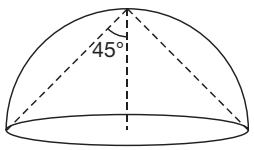
$$\Rightarrow \quad \phi = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

The negative sign signifies that the flux is due to field lines entering a surface.

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

(c) Total flux ϕ due to charge Q is $\frac{Q}{\varepsilon_0}$.



So, ϕ through the curved and flat surface will be less than $\frac{Q}{\varepsilon_0}$.

(d) Since, the circumference is equidistant from Q it will be equipotential $V = \frac{Q}{4\pi\varepsilon_0\sqrt{2}R}$.

Hence, (A) and (D) are correct.

4. $$\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow \quad Q = 2\pi\sigma r_0^2$$

(A) is incorrect, $r_0 = \dfrac{\lambda}{\pi\sigma}$

(B) is incorrect, $E_1\left(\dfrac{r_0}{2}\right) = 4E_1(r_0)$

As $E_1 \propto \dfrac{1}{r^2}$

$\Rightarrow \quad E_2\left(\dfrac{r_0}{2}\right) = 2E_2(r_0)$ as $E_2 \propto \dfrac{1}{r}$

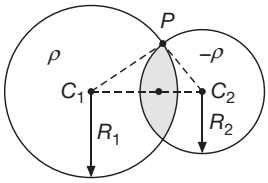
(C) is correct

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$$

as $E_3 \propto r^0$

Hence, option (C) is correct.

5.



For electrostatic field,

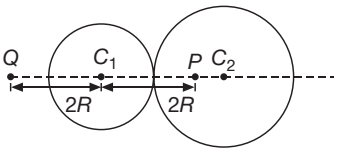
$$E_P = E_1 + E_2 = \frac{\rho}{3\varepsilon_0}C_1P + \frac{(-\rho)}{3\varepsilon_0}C_1P$$

$$\Rightarrow \quad E_P = \frac{\rho}{3\varepsilon_0}(C_1P + PC_2)$$

$$\Rightarrow \quad E_P = \frac{\rho}{3\varepsilon_0}C_1C_2$$

Hence, (C) and (D) are correct.

6. **At point P**



If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3}R$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$$

At point Q

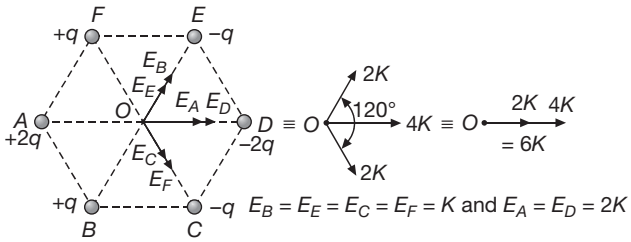
If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25} \qquad (\rho_1 \text{ must be negative})$$

Hence, (B) and (D) are correct.

7. (A)



Resultant of $2K$ and $2K$ (at $120°$) is also $2K$ towards $4K$. Therefore, net electric field is $6K$.

(B) $$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{L} + \frac{q_B}{L} + \frac{q_C}{L} + \frac{q_D}{L} + \frac{q_E}{L} + \frac{q_F}{L}\right]$$

$$V_0 = \frac{1}{4\pi\varepsilon_0 L}(q_A + \ldots + q_F) = 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(C) Only line PR, potential is same $(=0)$.

Hence, (A), (B) and (C) are correct.

8. OPTION (A) is correct due to symmetry.

OPTION (B) is wrong again due to symmetry.

OPTION (C) is correct because as per Gauss's theorem, net electric flux passing through any closed surface $= \dfrac{q_{in}}{\varepsilon_0}$

Here, $q_{in} = 3q - q - q = q$

So, net electric flux $= \dfrac{q}{\varepsilon_0}$

OPTION (D) is incorrect because there is no symmetry in two given planes.

Hence, (A) and (C) are correct.

9. If charges are of opposite signs, then the two fields are along the same direction. So, they cannot be zero. Hence, the charges should be of same sign.
Therefore, OPTION (C) is correct.

Further, $\begin{pmatrix}\text{Work done by}\\ \text{external force}\end{pmatrix} = \begin{pmatrix}\text{Change in electrostatic}\\ \text{potential energy}\end{pmatrix}$

$$\Rightarrow \quad W_{A\to B} = q(\Delta V) = (+1)(V_B - V_A)$$

$$\Rightarrow \quad W_{A\to B} = V_B - V_A$$

So, OPTION (D) is also correct.
Hence the correct OPTIONS are (C) and (D).

Hence, (C) and (D) are correct.

10. Inside a conducting shell electric field is always zero. Therefore, OPTION (A) is correct. When the two are connected, their potentials become the same.

$$\Rightarrow \quad V_A = V_B$$

$$\Rightarrow \quad \frac{Q_A}{R_A} = \frac{Q_B}{R_B} \qquad \left\{\because\ V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right\}$$

Since, $R_A > R_B$

$$\Rightarrow \quad Q_A > Q_B$$

So, OPTION (B) is correct.

Potential is also equal to, $V = \dfrac{\sigma R}{\varepsilon_0}$

Since $V_A = V_B$

$$\Rightarrow \quad \sigma_A R_A = \sigma_B R_B$$

$$\Rightarrow \quad \frac{\sigma_A}{\alpha_B} = \frac{R_B}{R_A}$$

$$\Rightarrow \quad \sigma_A < \sigma_B$$

So, OPTION (C) is correct.

Electric field on surface, $E = \dfrac{\sigma}{\varepsilon_0}$

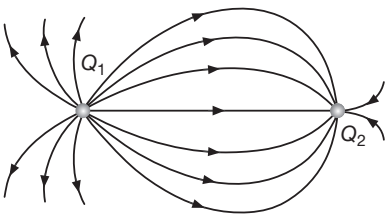
$$\Rightarrow \quad E \propto \sigma$$

Since, $\sigma_A < \sigma_B \ \Rightarrow \ E_A < E_B$

So, OPTION (D) is also correct.
Hence the correct OPTIONS are (A), (B), (C) and (D).

Hence, (A), (B), (C) and (D) are correct.

11.



From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

Number of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude and opposite sign to that of Q_1.

Hence, (A) and (D) are correct.

12. Net torque on $(-q)$ about a point (say P) lying over $+Q$ is zero. Therefore, angular momentum of $(-q)$ about point P should remain constant.

Hence, option (A) is correct.

13. The given graph is of charged conducting sphere of radius. The entire charge distributes on the surface of the sphere.

Hence, (A), (B), (C) and (D) are correct.

14. Charge density at $A <$ charge density at B. Since field inside cavity is zero, hence
Potential at A = potential at B = a constant value.
By Gauss Theorem,

$$\begin{pmatrix}\text{Total electric}\\ \text{flux}\end{pmatrix} = \frac{1}{\varepsilon_0}\begin{pmatrix}\text{charge}\\ \text{enclosed}\end{pmatrix} = \left(\frac{q}{\varepsilon_0}\right)$$

Hence, (C) and (D) are correct.

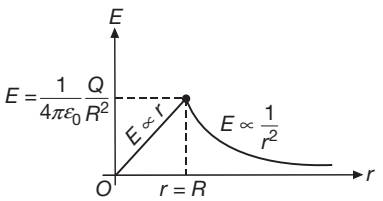
15. Inside the sphere i.e. for $r < R$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^3}r$$

$$\Rightarrow \quad E \propto r$$

i.e., E at centre is $E_{\text{centre}} = E|_{r=0} = 0\,(r = 0)$

and E at surface is $E_{\text{surface}} = E|_{r=R} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{R^2}$



Outside the sphere, i.e. for $r > R$

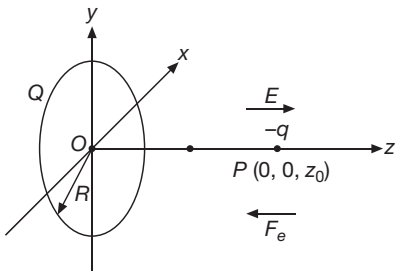
$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

$$\Rightarrow \quad E \propto \frac{1}{r^2}$$

Thus, variation of electric field (E) with distance (r) from the centre will be as follows.

Hence, (A) and (C) are correct.

16. Let Q be the charge on the ring, the negative charge $-q$ is released from point $P(0,\ 0,\ z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$$E=\frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{\left(R^2+z_0^2\right)^{\frac{3}{2}}}$$

$E=0$ at centre of the ring because $z_0=0$

Force on charge at P will be towards centre as shown, and its magnitude is

$$F_e=qE=\frac{1}{4\pi\varepsilon_0}\frac{Qqz_0}{\left(R^2+z_0^2\right)^{\frac{3}{2}}} \qquad \ldots(1)$$

Similarly, when it crosses the origin, the force is again towards centre O.

Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 \ll R$, then $\left(R^2+z_0^2\right)^{\frac{3}{2}} \approx R^3$

$$\Rightarrow \quad F_e \approx \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^3}z_0 \qquad \{\text{From equation (1)}\}$$

i.e. the restoring force $F_e \propto -z_0$. So, the motion of the particle will be simple harmonic. (The negative sign implies that the force is towards the mean position).

Reasoning Based Questions

1. Statement-1 is also practical experience based; so it is true.

Statement-2 is also true but is not the correct explanation of Statement-1. Correct explanation is "there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled".

Hence, the correct answer is (B).

Linked Comprehension Type Questions

1. After hitting the top plate, the balls will get negatively charged and will now get attracted to the bottom plate which is positively charged. The motion of the balls will be periodic but not SHM.

Hence, the correct answer is (A).

2. As the balls keep on carrying charge from one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q, then

$$\frac{kq}{r}=V_0$$

$$\Rightarrow \quad q=\frac{V_0 r}{k}$$

Inside the cylinder, electric field is

$$E=\left[V_0-\left(-V_0\right)\right]h$$

$$\Rightarrow \quad E=2V_0 h$$

$$\Rightarrow \quad \text{Acceleration of each ball, } a=\frac{qE}{m}=\left(\frac{2hr}{km}\right)V_0^2$$

$$\Rightarrow \quad \text{Time taken by balls to reach other plate is}$$

$$t=\sqrt{\frac{2h}{a}}=\sqrt{\frac{2hmk}{2hrV_0^2}}=\frac{1}{V_0}\sqrt{\frac{mk}{r}}$$

If there are n balls, then

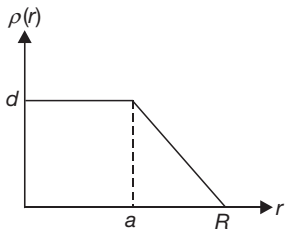
$$\text{Average current, } i_{av}=\frac{nq}{t}=n\times\frac{V_0 r}{k}\times V_0\sqrt{\frac{r}{mk}}$$

$$\Rightarrow \quad i_{av}\propto V_0^2$$

Hence, the correct answer is (B).

3. Electric field at $r=R$ is

$$E=\frac{Q}{4\pi\varepsilon_0 R^2}$$



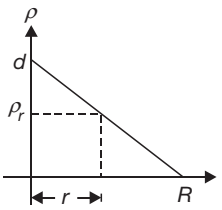
Q = Total charge within the nucleus = Ze

$$\Rightarrow \quad E=\frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

Hence the electric field is independent of a.

Hence, the correct answer is (A).

4. $Q=\int \rho_r 4\pi r^2 dr$ for $a=0$, and the figure for the distribution looks like as shown



$$\text{Slope} = \frac{d}{R} = \frac{\rho_r}{R-r}$$

$$\Rightarrow \quad \rho_r = \frac{d}{R}(R-r)$$

$$\Rightarrow \quad Q = \int_0^R \frac{d}{R}(R-r)4\pi r^2 dr$$

$$\Rightarrow \quad Q = \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr\right) = \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4}\right)$$

$$Q = \frac{\pi d R^3}{3}$$

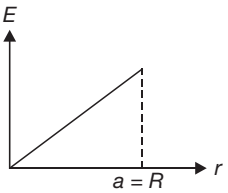
$$\Rightarrow \quad Q = Ze = \frac{\pi d R^3}{3} \qquad \{\because Q = Ze\}$$

$$\Rightarrow \quad d = \frac{3Ze}{\pi R^3}$$

Hence, the correct answer is (B).

5. Let us review the formula of uniformly (volume) charged solid sphere according to which we have

$$E = \frac{\rho r}{3\varepsilon_0}$$



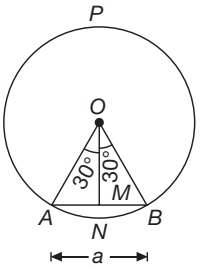
$\Rightarrow$ $E \propto r$, ρ should be constant throughout the volume of nucleus.

This will be possible only when $a = R$.

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. ANBP is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$$AM = \frac{a}{2}, \quad MO = \frac{\sqrt{3}a}{2}$$

$$\Rightarrow \quad \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$\Rightarrow \quad \angle AOM = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

Electric flux passing through $ABCD$ plane surface (shown only AB) = Electric flux passing through cylindrical surface ANB. So

$$\phi = \left(\frac{60°}{360°}\right)(\phi_1)$$

$$\Rightarrow \quad \phi = \frac{\lambda L}{6\varepsilon_0}$$

$$\Rightarrow \quad n = 6$$

2. Volume of cylinder per unit length $(\ell = 1)$ is

$$V = \pi R^2 \ell = (\pi R^2)$$

So, charge per unit length is

$$\lambda = \begin{pmatrix}\text{Volume per}\\ \text{unit length}\end{pmatrix} \times \begin{pmatrix}\text{Volume charge}\\ \text{density}\end{pmatrix} = (\pi R^2 \rho)$$

Now at P, $E_{\text{total}} = E_{\text{remainder}} + E_{\text{cavity}}$

$$\Rightarrow \quad E_R = E_T - E_C$$

Where R = Remainder portion

T = Total portion and

C = cavity portion

$$\Rightarrow \quad E_R = \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

where Q is charge on sphere given by

$$Q = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi R^3 \rho}{6}$$

Substituting the values, we get

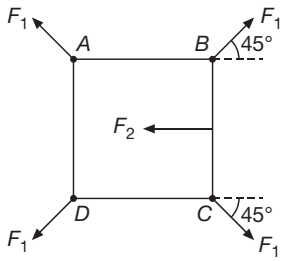
$$E_R = \frac{(\pi R^2 \rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}\frac{\left(\pi R^3 \frac{\rho}{6}\right)}{4R^2}$$

$$\Rightarrow \quad E_R = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)(6)\varepsilon_0}$$

$$\Rightarrow \quad k = 6$$

3. $F_1 =$ Net electrostatic force on any one charge due to rest of three charges. So

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$



$F_2 =$ Force due to surface tension $= \gamma a$

If we see the equilibrium of line BC, then

$$2F_1 \cos(45°) = F_2$$

$$\Rightarrow \quad \sqrt{2}F_1 = F_2$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(2 + \frac{1}{\sqrt{2}}\right) = \gamma a$$

$$\Rightarrow \quad a^3 = \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{\gamma}$$

$$\Rightarrow \quad a = \left(\frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\right)^{\frac{1}{3}}\left(\frac{q^2}{\gamma}\right)^{\frac{1}{3}} = k\left(\frac{q^2}{\gamma}\right)^{\frac{1}{3}}$$

where, $k = \left(\frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\right)^{\frac{1}{3}}$

$$\Rightarrow \quad N = 3$$

4. Total charge

$$Q = \int \rho dV = \int_{r=0}^{r=R} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}(R^{a+3})$$

$$Q' = \int \rho dV = \int_{r=0}^{r=\frac{R}{2}} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$$

According to question

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'}{\left(\frac{R}{2}\right)^2} = \frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)$$

Putting the value of Q and Q' get

$$a = 2$$

CHAPTER 1

CHAPTER 2: CAPACITANCE AND APPLICATIONS

Test Your Concepts-I (Based on General Capacitance)

1. The common potential,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Here $C_1 = 20\ \mu\text{F} = 20 \times 10^{-6}$ F, $V_1 = 500$ V

$C_2 = 10\ \mu\text{F} = 10 \times 10^{-6}$ F, $V_2 = 200$ V

$$\Rightarrow \quad V = \frac{20 \times 10^{-6} \times 500 + 10 \times 10^{6} \times 200}{20 \times 10^{-6} + 10 \times 10^{-6}}$$

$$\Rightarrow \quad V = 400 \text{ V}$$

2. The energy stored in the condenser is given by

$$U = \frac{1}{2}CV^2$$

where $C = 100\ \mu\text{F} = 100 \times 10^{-6}$ F, $V = 200$ V

$$U = \frac{1}{2} \times 100 \times 10^{-6} \times (200)^2 = 2 \text{ J}$$

When discharged through a resistor $(2\ \Omega)$, the whole energy is dissipated as heat. So, heat produced is given by

$$Q = U = 2 \text{ J}$$

3. When the dial is at $0°$, the capacitance of the capacitor is given by

$$C_1 = 50 \text{ pF} = 50 \times 10^{-12} \text{ F}$$

When dial is at $180°$, the capacitance is given by

$$C_2 = 950 \text{ pF} = 950 \times 10^{-12} \text{ F}$$

The potential difference across capacitor C_2, is given by

$$V_2 = 400 \text{ V}$$

Charge on capacitor C_2 is

$$q = C_2V_2 = 950 \times 10^{-12} \times 400$$

$$\Rightarrow \quad q = 380 \times 10^{-9} \text{ C}$$

(a) When battery is disconnected the charge remains the same and so, $q =$ constant. Let V_1 be the potential difference across capacitor when dial reads $0°$. Then

$$q = C_1V_1$$

$$\Rightarrow \quad 380 \times 10^{-9} = 50 \times 10^{-12} \times V_1$$

$$\Rightarrow \quad V_1 = \frac{380 \times 10^{-9}}{50 \times 10^{-12}} = 7600 \text{ V}$$

(b) Work required to turn the dial from $180°$ to $0°$ is $W =$ Gain in energy of capacitor

$$\Rightarrow \quad W = \frac{q^2}{2C_1} - \frac{q^2}{2C_2} = \frac{q^2}{2}\left(\frac{1}{C_1} - \frac{1}{C_2}\right) = \frac{q^2(C_2 - C_1)}{2C_1C_2}$$

$$\Rightarrow \quad W = \frac{(380 \times 10^{-9})^2 (950 - 50) \times 10^{-12}}{2 \times 50 \times 10^{-12} \times 950 \times 10^{-12}}$$

$$\Rightarrow \quad W = 1.368 \times 10^{-3} \text{ J}$$

4. Initial energy stored in the capacitors

$$U_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$\Rightarrow \quad U_1 = \frac{1}{2} \times 0.1 \times 10^2 + \frac{1}{2}C_2 \times 0 = 5 \text{ J}$$

When charging battery is removed, the charge remains constant. This charge is collected by first capacitor $(q_1 = C_1V_1)$ then redistributed equally in such a way that their potentials are equal i.e.,

$$V_1' = V_2'$$

$$\Rightarrow \quad \frac{q_1'}{C_1} = \frac{q_2'}{C_2}$$

Now since, $q_1' = q_2'$ {given}

$$\Rightarrow \quad C_1 = C_2 = C(\text{say})$$

$\therefore$ Common potential

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{CV_1 + 0}{C + C} = \frac{V_1}{2} = 5 \text{ V}$$

So, final energy stored is

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$\Rightarrow \quad U_2 = \frac{1}{2}(2C)V^2 = CV^2 = 0.1 \times (5)^2 = 2.5 \text{ J}$$

$$\Rightarrow \quad \frac{U_2}{U_1} = \frac{2.5}{5} = \frac{1}{2}$$

5. (a) Let the charge on sphere of radius R_1 be $q_1(=q)$ and that on the sphere of radius R_2 be $q_2(=Q-q)$, then the total energy is

$$U = U_1 + U_2 = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$

$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{(Q-q)^2}{8\pi\varepsilon_0 R_2}$$

For U to be minimum we must have

$$\frac{dU}{dq} = 0$$

$$\Rightarrow \quad \frac{2q}{8\pi\varepsilon_0 R_1} + \frac{2(Q-q)}{8\pi\varepsilon_0 R_2}(-1) = 0$$

$$\Rightarrow \quad \frac{q}{R_1} = \frac{Q-q}{R_2}$$

$$\Rightarrow \quad q = \left(\frac{R_1}{R_1+R_2}\right)Q = q_1$$

and $Q - q = \left(\frac{R_2}{R_1+R_2}\right)Q = q_2$

(b) $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{R_1 Q}{R_1(R_1+R_2)}$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1+R_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2} = \frac{1}{4\pi\varepsilon_0}\frac{R_2 Q}{R_2(R_1+R_2)}$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1+R_2}$$

$$\Rightarrow \quad V_1 - V_2 = 0$$

6. Let V_1 be the potential of C_0 after first charging, then

$$(C + C_0)V_1 = C_0 V_0$$

$$\Rightarrow \quad V_1 = \frac{C_0 V_0}{C + C_0}$$

Let V_2 be the potential of C_0 after second charging then

$$C_0 V_1 = (C + C_0)V_2$$

$$\Rightarrow \quad V_2 = \left(\frac{C_0}{C+C_0}\right)\left(\frac{C_0}{C+C_0}\right)V_0 = \left(\frac{C_0}{C+C_0}\right)^2 V_0$$

and so on

$$\Rightarrow \quad V_{10} = V = \left(\frac{C_0}{C+C_0}\right)^{10} V_0$$

$$\Rightarrow \quad C = \left[\left(\frac{V_0}{V}\right)^{\frac{1}{10}} - 1\right]C_0$$

Test Your Concepts-II (Based on Series and Parallel Combination of Capacitors)

1. (a) $U = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = C(\Delta V)^2$

(b) The altered capacitor has capacitance $C' = \frac{C}{2}$. The total charge is the same as before.

So, $q_{\text{initial}} = q_{\text{final}}$

$$\Rightarrow \quad C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V')$$

$$\Rightarrow \quad \Delta V' = \frac{4\Delta V}{3}$$

(c) $U' = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\left(\frac{C}{2}\right)\left(\frac{4\Delta V}{3}\right)^2$

$$\Rightarrow \quad U' = 4C\frac{(\Delta V)^2}{3}$$

(d) The extra energy comes from work put into the system by the agent pulling the capacitor plates apart.

2. (a) Capacitors connected in series.

Initially, capacitance C is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{\varepsilon_0 A} + \frac{d}{\varepsilon_0 A}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{2d}$$

When the plates are moved in the given manner, net capacity C' of the system is given by

$$\frac{1}{C'} = \frac{d-\Delta d}{\varepsilon_0 A} + \frac{d+\Delta d}{\varepsilon_0 A}$$

$$\left\{\because C_1' = \frac{\varepsilon_0 A}{d-\Delta d} \text{ and } C_2' = \frac{\varepsilon_0 A}{d+\Delta d}\right\}$$

$$\Rightarrow \quad C' = \frac{\varepsilon_0 A}{2d}$$

Net capacitance remains same as before.

(b) Two identical capacitors C_1 and C_2 connected in parallel. Initially the net capacity of the system is

$$C = C_1 + C_2 = 2\frac{\varepsilon_0 A}{d}$$

where, A is the area of the plates, and d the distance between the plates, for both the capacitors. When the plates are brought closer by Δd in one capacitor, its capacity increases to $\frac{\varepsilon_0 A}{d-\Delta d}$; whereas the capacity of the second capacitor,

CHAPTER 2

whose plates are moved away by Δd, decreases to $\frac{\varepsilon_0 A}{d+\Delta d}$. Net capacity of the system becomes

$$C' = \varepsilon_0 A\left(\frac{1}{d-\Delta d}+\frac{1}{d+\Delta d}\right)=\frac{2\varepsilon_0 Ad}{d^2-(\Delta d)^2}$$

$$\Rightarrow \quad C' = \frac{2\varepsilon_0 A}{d-\left(\frac{\Delta d^2}{d}\right)}$$

We find that $C' > C$ and hence net capacitance increases.

3. The initial charge on the larger capacitor is

$$Q = C\Delta V$$

$$\Rightarrow \quad Q = (10\ \mu\text{F})(15\ \text{V}) = 150\ \mu\text{C}$$

An additional charge q is pushed through the 50 V battery, giving the smaller capacitor charge q and the larger charge $150+q$ (in μC).

$$\Rightarrow \quad 50\ \text{V} = \frac{q}{5\ \mu\text{F}}+\frac{150\ \mu\text{C}+q}{10\ \mu\text{F}}$$

$$\Rightarrow \quad 500\ \mu\text{C} = 2q+150\ \mu\text{C}+q$$

$$\Rightarrow \quad q = 117\ \mu\text{C}$$

Across the 5 μF capacitor $\Delta V = \frac{q}{C}=\frac{117\ \mu\text{C}}{5\ \mu\text{F}} = 23.3\ \text{V}$

Across the 10 μF capacitor

$$\Delta V = \frac{150\ \mu\text{C}+117\ \mu\text{C}}{10\ \mu\text{F}} = 26.7\ \text{V}$$

4. (a) $U_i = \frac{q^2}{2C_i}=\left(\frac{q^2}{2\varepsilon_0 A}\right)x_1$ and

$$U_f = \frac{q^2}{2C_f}=\left(\frac{q^2}{2\varepsilon_0 A}\right)x_2$$

$$\Rightarrow \quad W_{\text{ext}} = W = U_f - U_i = \frac{q^2}{2\varepsilon_0 A}(x_2-x_1)$$

(b) $U_i = \frac{1}{2}C_i V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{x_1}\right)V^2$

$$U_f = \frac{1}{2}C_f V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{x_2}\right)V^2$$

$$\Rightarrow \quad W_{\text{external}} = U_f - U_i = \frac{1}{2}\varepsilon_0 AV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)$$

5. Initially $C_1(60) = C_2(40)$

$$3C_1 = 2C_2$$

Now, $q_2 = q_1 + q_{\text{across } 2\ \mu\text{F}}$

$$\Rightarrow \quad C_2(90) = C_1(10)+(2)(10)$$

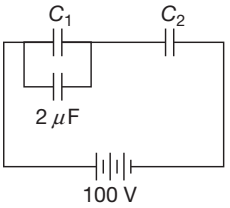
$$\Rightarrow \quad 9C_2 = C_1 + 2$$

$$\Rightarrow \quad 9C_2 = \frac{2C_2}{3}+2$$

$$\Rightarrow \quad 25C_2 = 6$$

$$\Rightarrow \quad C_2 = \frac{6}{25} = 0.24\ \mu\text{F}$$

and $C_1 = 0.16\ \mu\text{F}$



6. $U = \frac{1}{2}C_P V^2 = \frac{1}{2}(C_1+C_2)V^2$

$$\Rightarrow \quad \frac{1}{2}(C_1+C_2)(2)^2 = 0.1$$

$$\Rightarrow \quad (C_1+C_2) = 5\times10^{-2} \qquad \ldots(1)$$

Similarly $\frac{1}{2}C_S V^2 = 1.6\times10^{-2}$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(2)^2 = 1.6\times10^{-2}$$

$$\Rightarrow \quad C_1C_2 = 4\times10^{-4} \qquad \ldots(2)$$

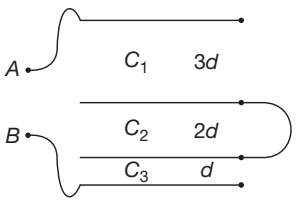
Solving equations (1) and (2), we get

$$C_1 = 40\ \text{mF} \ \text{ and } \ C_2 = 10\ \text{mF}$$

7. In the figure, $C_1 = \frac{\varepsilon_0 A}{3d}$

$$C_2 = \frac{\varepsilon_0 A}{2d}$$

and $C_3 = \frac{\varepsilon_0 A}{d}$



Now, C_2 is short circuited so that C_1 and C_3 are in series.

$$\Rightarrow \quad C_{AB} = \frac{C_1C_3}{C_1+C_3}$$

$$\Rightarrow \quad C_{AB} = \frac{\varepsilon_0 A}{4d}$$

8. In series the charge on each capacitor is same. Therefore, potential difference given by $V = \frac{q}{C}$ across the capacitors are in inverse ratio of their capacitors. Hence

$$V_1 = \frac{q}{C_1} = 130 \text{ V} \quad \text{and}$$

$$V_2 = \frac{q}{C_2} = 100 \text{ V}$$

$$\Rightarrow \quad \frac{C_2}{C_1} = \frac{V_1}{V_2} = \frac{130}{100}$$

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{10}{13} \qquad \ldots(1)$$

and so the smaller capacitance is C_1.

As capacitance of a capacitor is proportional to the dielectric constant (K), so

New capacitance C_1' of the first capacitor is

$$C_1' = \frac{5}{2}C_1 \qquad \left\{\because \frac{C_1'}{C_1} = \frac{K_1'}{K_1} = \frac{5}{2}\right\}$$

New capacitance of second capacitor C_2' does not change, so

$$C_2' = C_2$$

$$\Rightarrow \quad \frac{C_1'}{C_2'} = \frac{\frac{5}{2}C_1}{C_2} = \frac{5}{2} \times \frac{10}{13} = \frac{25}{13} \qquad \{\text{using (1)}\}$$

If V_1' and V_2' are the respective potential differences, then

$$\frac{V_1'}{V_2'} = \frac{C_2'}{C_1'} = \frac{13}{25} \qquad \ldots(2)$$

Also, $V_1' + V_2' = 230 \text{ V}$ $\qquad \ldots(3)$

Solving (2) and (3), we get

$$V_1' = 78.7 \text{ V} \quad \text{and} \quad V_2' = 151.3 \text{ V}.$$

9. Let d be the separation between the plates of first capacitor, the separation between the plates of second capacitor will be $[a-(b+d)]$.

The capacitances of these capacitors are

$$C_1 = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A}{a-(b+d)}$$

Since the capacitors are in series, so the equivalent capacitance of the combination C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)} + \frac{1}{\left(\frac{\varepsilon_0 A}{[a-(b+d)]}\right)}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{(\varepsilon_0 A)}[d + a - b - d] = \frac{a-b}{\varepsilon_0 A}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{a-b}$$

This is the required expression for the net capacitance and is independent of d and hence of the position of the central H-shaped part.

10. The given capacitor may be supposed to be formed of a large number of differential capacitors each connected in parallel. Consider one such capacitor of width dx at a distance x from O. The area of each plate of this small capacitor, $dA(=adx)$.

Separation between these plates, $EF = d + x\tan\theta$

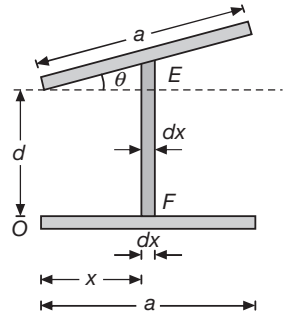
If dC be the capacitance of this small capacitor, then

$$dC = \frac{\varepsilon_0 dA}{(EF)} = \frac{\varepsilon_0 (adx)}{d + x\tan\theta} = \frac{\varepsilon_0 (adx)}{d + x\theta}$$

{since for small θ, $\tan\theta = \theta$}

$$\Rightarrow \quad dC = \frac{\varepsilon_0 (adx)}{d\left(1 + \frac{x\theta}{d}\right)} = \frac{\varepsilon_0 (adx)}{d}\left(1 + \frac{x\theta}{d}\right)^{-1}$$

$$\Rightarrow \quad dC = \frac{\varepsilon_0 (adx)}{d}\left(1 - \frac{\theta}{d}x\right) \{\text{using Binomial Theorem}\}$$

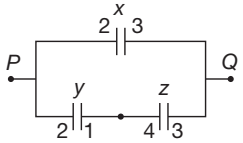


The net capacitance of capacitor is obtained by integrating the expression with respect to x between 0 to a i.e.,

$$C = \frac{\varepsilon_0 a}{d}\int_0^a \left(dx - \frac{\theta}{d}xdx\right) = \frac{\varepsilon_0 a}{d}\left[x - \frac{\theta}{d}\frac{x^2}{2}\right]_0^a$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a}{d}\left[a - \frac{a\theta^2}{2d}\right] = \frac{\varepsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right]$$

11. (a) The equivalent of Arrangement 1 is shown. Let C be capacitance of each capacitor.



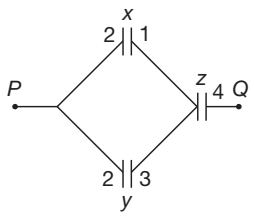
The net capacitance of y and z connected in series is

$$C_{yz} = \frac{C}{2}$$

Now C_{yz} and capacitor x are connected in parallel. Therefore net capacitance between P and Q,

$$C_{PQ} = C + \frac{C}{2} = \frac{3}{2}C = \frac{3}{2}\left(\frac{\varepsilon_0 A}{d}\right)$$

(b) The equivalent of Arrangement 2 is shown.



The net capacitance of x and y connected in parallel is

$$C_{xy} = 2C$$

Now C_{xy} and z are connected in series. Therefore net capacitance of arrangement between P and Q is

$$C_{PQ} = \frac{CC_{xy}}{C + C_{xy}} = \frac{C(2C)}{C + 2C} = \frac{2}{3}C = \frac{2}{3}\left(\frac{\varepsilon_0 A}{d}\right)$$

12. The capacitance is maximum when the plates of one group are parallel to the plates of other group, so that the effective area of each plate is A.
Since the alternate plates are connected together the potential difference across any two consecutive plates is same. The n plates of given arrangement form $(n-1)$ capacitors connected in parallel. Therefore the maximum capacitance of radio capacitor is

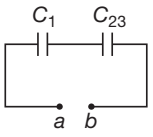
$$C = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} + \ldots (n-1) \text{ times}$$

$$\Rightarrow \quad C = (n-1)\frac{\varepsilon_0 A}{d}$$

13. (a) Capacitors C_2 and C_3 are in parallel, so their equivalent capacitance is

$$C_{23} = C_2 + C_3 = 4 + 8$$

$$\Rightarrow \quad C_{23} = 12\ \mu\text{F}$$



Capacitors C_1 and C_{23} are in series; so the equivalent capacitance C of the entire combination is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12}$$

$$C_{eq} = 4\ \mu\text{F}$$

(b) The total charge that flows from the battery is

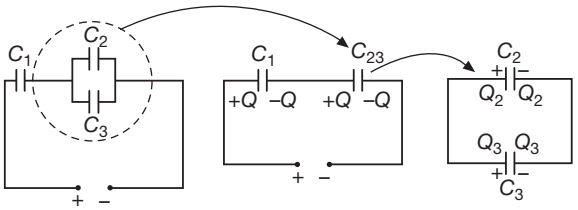
$$Q = C_{eq}V = (4 \times 10^{-6})(12) = 4.8 \times 10^{-5}\ \text{C}$$

Capacitors C_1 and C_{23} are in series, hence they carry the same charge $Q = 4.8 \times 10^{-5}$ C.
The voltage across C_1,

$$V_1 = \frac{Q}{C_1} = \frac{4.8 \times 10^{-5}}{6 \times 10^{-6}} = 8\ \text{V}$$

The voltage across the combination C_{23} is

$$V_{23} = \frac{Q}{C_{23}} = \frac{4.8 \times 10^{-5}}{12 \times 10^{-6}} = 4\ \text{V}$$



C_{23} is parallel combination of C_2 and C_3 and hence they have the same potential

$$V_2 = V_3 = 4\ \text{V}$$

The charges on C_2 and C_3 are

$$Q_2 = C_2V_2 = (4 \times 10^{-6})(4) = 1.6 \times 10^{-5}\ \text{C}$$

$$Q_3 = C_3V_3 = (8 \times 10^{-6})(4) = 3.2 \times 10^{-5}\ \text{C}$$

So we have $V_1 = 8$ V, $Q_1 = 48\ \mu$C

$V_2 = 4$ V, $Q_2 = 16\ \mu$C

$V_3 = 4$ V, $Q_3 = 32\ \mu$C

Also, we note that $Q_2 + Q_3 = Q_1 = Q$

14. (a) The two condensers in the middle are connected together and hence act as a single conductor. Thus effectively there are three plates which form two capacitors in parallel.
Capacitance of each capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

So, net capacitance of combination is

$$C_{PQ} = 2C = \frac{2\varepsilon_0 A}{d}$$

(b) The four plates are alternatively connected and form three capacitors in parallel. So, the net capacitance of combination is given by

$$C_{PQ} = 3C = \frac{3\varepsilon_0 A}{d}$$

15. (a) The capacitance of capacitor (A,B) is

$$C_1 = \frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 \pi R_1^2}{d_1}$$

$$\Rightarrow \quad C_1 = \frac{\left(\frac{1}{36\pi \times 10^9}\right)\pi\left(\frac{0.1}{2}\right)^2}{2\times 10^{-3}} = \frac{1}{288\times 10^8} \text{ F}$$

The capacitance of capacitor (C,D) is

$$C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{\varepsilon_0 \pi R_2^2}{d_2}$$

$$\Rightarrow \quad C_2 = \frac{\left(\frac{1}{36\pi \times 10^9}\right)\pi\left(\frac{0.12}{2}\right)^2}{3\times 10^{-3}} = \frac{1}{300\times 10^8} \text{ F}$$

The capacitors C_1 and C_2 are in series. Therefore, equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = 288\times 10^8 + 300\times 10^8$$

$$\Rightarrow \quad \frac{1}{C} = 588\times 10^8$$

$$\Rightarrow \quad C = \frac{1}{588\times 10^8} = 17\times 10^{-12} \text{ F} = 17 \text{ pF}$$

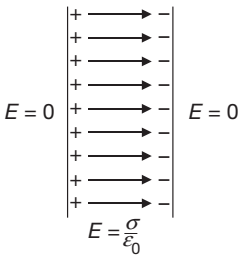
(b) Energy stored by capacitors

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times(17\times 10^{-12})\times(120)^2$$

$$\Rightarrow \quad U = 1.224\times 10^{-7} \text{ J}$$

16. (a) Electric field is uniform between the plates of the capacitor. The magnitude of this field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$



Therefore, the energy density (u) should also be constant.

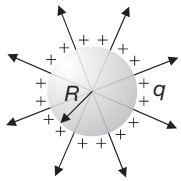
$$U = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{2A^2\varepsilon_0}$$

So, total stored energy, $U = (u)$ (total volume)

$$U = \left(\frac{q^2}{2A^2\varepsilon_0}\right)(A.d) = \frac{q^2}{2\left(\frac{A\varepsilon_0}{d}\right)}$$

$$\Rightarrow \quad U = \frac{q^2}{2C} \qquad \left\{\because C = \frac{A\varepsilon_0}{d}\right\}$$

(b) In case of a spherical conductor (of radius R) the excess charge resides on the outer surface of the conductor. The field inside the conductor is zero. It extends from surface to infinity. And since the potential energy is stored in the field only, it will be stored in the region extending from surface to infinity. Also the field is non-uniform, the energy density u is also non-uniform. Hence, the total energy can be calculated using the concept of integration. Electric field at a distance r from the centre is,



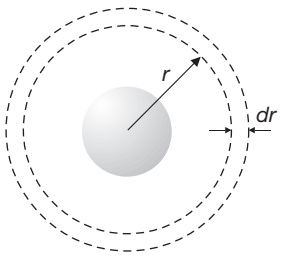
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$u(r) = \frac{1}{2}\varepsilon_0 E^2$$

$$\Rightarrow \quad u(r) = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2$$

Energy stored in a volume $dV = (4\pi r^2)dr$ is

$$dU = u\,dV$$



$$\Rightarrow \quad \text{Total energy stored is, } U = \int_{r=R}^{r=\infty} dU$$

Substituting the values and integrating, we get

$$U = \frac{q^2}{2(4\pi\varepsilon_0 R)}$$

$$\Rightarrow \quad U = \frac{q^2}{2C} \qquad \{\because C = 4\pi\varepsilon_0 R\}$$

Test Your Concepts-III (Based on Dielectrics and Breakdown)

1. Arrangement A is equivalent to a combination of two capacitors each of area A, separation $\frac{d}{2}$ and connected in series. So,

$$\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{(d/2)}{\varepsilon_0 K_1 A}+\frac{d/2}{\varepsilon_0 K_2 A}$$

$$\frac{1}{C_S}=\frac{\varepsilon_0 A}{d}\left[\frac{K_1+K_2}{K_1K_2}\right]$$

where $K_S=\frac{K_1K_2}{K_1+K_2}$

Arrangement B is equivalent to combination of two capacitors each of area $\frac{A}{2}$, separation d and connected in parallel. So,

$$C_p=C_3+C_4=\frac{\varepsilon_0 K_1\left(\frac{A}{2}\right)}{d}+\frac{\varepsilon_0 K_2\left(\frac{A}{2}\right)}{d}$$

$$C_p=\frac{\varepsilon_0 A}{d}\left[\frac{K_1+K_2}{2}\right]$$

where $K_P=\frac{K_1+K_2}{2}$

Now $\frac{C_S}{C_P}=\frac{2K_1K_2}{K_1+K_2}\times\frac{2}{(K_1+K_2)}=\frac{4K_1K_2}{(K_1+K_2)^2}$

$$\Rightarrow\quad \frac{C_S}{C_P}=\frac{4\times2\times3}{(2+3)^2}=\frac{24}{25}$$

2. (a) $C_0=\frac{\varepsilon_0 A}{d}=\frac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage, then

$$C=\kappa C_0=\frac{Q}{\Delta V_0}$$

Since, $U_0=\frac{1}{2}C_0(\Delta V_0)^2$

and $U=\frac{1}{2}C(\Delta V)^2=\frac{\kappa C_0(\Delta V_0^2)}{2}$

$$\Rightarrow\quad \frac{U}{U_0}=\kappa$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0=C_0\Delta V_0$

and $Q=C\Delta V_0=\kappa C_0\Delta V_0$

$$\Rightarrow\quad \frac{Q}{Q_0}=\kappa$$

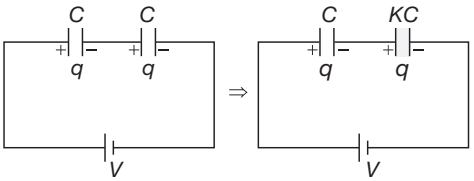
Please note that this situation is not the same as the situation where the battery is removed from the circuit before inserting the dielectric.

3. (a) Net capacitance without inserting the slab is, $\frac{C}{2}$

$$\Rightarrow\quad q=\left(\frac{C}{2}\right)V$$

Since, electric field

$$E=\frac{\text{Potential Difference}}{\text{separation between the plates}}=\frac{\left(\frac{V}{2}\right)}{d}=\frac{V}{2d}$$



Net capacitance after inserting the slab is

$$C'=\left(\frac{K}{K+1}\right)C$$

$$\Rightarrow\quad q'=CV\left(\frac{K}{K+1}\right)$$

Electric field

$$E'=\frac{\text{Potential Difference}}{\text{Separation between the plates}}$$

$$E'=\frac{V\left(\frac{1}{K+1}\right)}{d}\frac{V}{(K+1)d}$$

Electric field decreases by a factor

$$\frac{E}{E'}=\frac{\left(\frac{V}{2d}\right)}{\left(\frac{V}{(K+1)d}\right)}=\frac{K+1}{2}$$

(b) Charge that flows through the battery is $q'-q$

$$\text{Charge Flowing}=\frac{CV(K-1)}{2(K+1)}$$

4. $C=\frac{\varepsilon_0 A}{d-t+\frac{t}{k}}=\frac{C_0}{1-\frac{t}{d}+\frac{t}{kd}}$ $\quad\left\{\because C_0=\frac{\varepsilon_0 A}{d}\right\}$

where $t=2(0.1\text{ mm})=0.2\text{ mm}$, $d=4\text{ mm}$ and $K=3$

Putting values, we get $C_0=44.25\text{ pF}$

Now, $C = \dfrac{44.25}{1 - \dfrac{0.2}{4} + \dfrac{0.2}{(3)(4)}}$

$\Rightarrow \quad C = \dfrac{44.25}{1 - \dfrac{1}{20} + \dfrac{1}{60}} pF$

$\Rightarrow \quad C = \dfrac{44.25 \times 60}{60 - 3 + 1} = \dfrac{2655}{58}$

$\Rightarrow \quad C = 45.77\%$

So, %age change is

$$\frac{\Delta C}{C_0} \times 100 = \left(\frac{45.77 - 44.25}{44.25} \right) \times 100 = 3.4\%$$

$\Rightarrow$ Percentage increase in the value of capacitance is 3.4%

5. (a) $C = \dfrac{\varepsilon_0 A}{d}$

(b) $C = \dfrac{\varepsilon_0 A}{d - t}$

Since, $t \to 0$

(c) $C = \dfrac{\varepsilon_0 A}{d/2} = \dfrac{2\varepsilon_0 A}{d}$

6. (a) Since both are connected to the same potential, hence they are in parallel. So,

$$C_{net} = K_1 \left(\frac{\varepsilon_0 A}{d_1} \right) + K_2 \left(\frac{\varepsilon_0 A}{d_2} \right)$$

$$\Rightarrow \quad C_{net} = \varepsilon_0 A \left(\frac{K_1}{d_1} + \frac{K_2}{d_2} \right)$$

(b) $\sigma = \dfrac{Q}{A}$

$$\Rightarrow \quad \sigma = \frac{CV}{A}$$

$$\Rightarrow \quad \sigma = \frac{\varepsilon_0 AV}{A} \left(\frac{K_1}{d_1} + \frac{K_2}{d_2} \right)$$

$$\Rightarrow \quad \sigma = \varepsilon_0 V \left(\frac{K_1}{d_1} + \frac{K_2}{d_2} \right)$$

(c) $U_1 = \dfrac{1}{2} \varepsilon E_1^2$

Since $\varepsilon = K_1 \varepsilon_0$ and $E_1 = \dfrac{V_1}{d}$, so

$$U_1 = \frac{1}{2} (K_1 \varepsilon_0) \left(\frac{V}{d_1} \right)^2$$

$$\Rightarrow \quad U_1 = \frac{1}{2} (K_1 \varepsilon_0) \left(\frac{V^2}{d_1^2} \right)$$

$$\Rightarrow \quad U_1 = \frac{1}{2} \left(\frac{\varepsilon_0 K_1 V^2}{d_1^2} \right)$$

7. The system of three parallel plates form two parallel plate capacitors connected in series.
The capacitance of first capacitor

$$C_1 = \frac{K_1 \varepsilon_0 A}{d_1}$$

and capacitance of second capacitor

$$C_2 = \frac{K_2 \varepsilon_0 A}{d_2}$$

The net capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\left(\dfrac{K_1 \varepsilon_0 A}{d_1} \right)} + \frac{1}{\left(\dfrac{K_2 \varepsilon_0 A}{d_2} \right)}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{\varepsilon_0 A} \left[\frac{d_1}{K_1} + \frac{d_2}{K_2} \right] = \frac{1}{\varepsilon_0 A} \frac{d_1 K_2 + d_2 K_1}{K_1 K_2}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 K_1 K_2 A}{d_1 K_2 + d_2 K_1}$$

8. The capacitances C_2 and C_3 are in series, each having area $\dfrac{A}{2}$ and separation $\dfrac{d}{2}$.

$$C_2 = K_2 \left(\frac{\varepsilon_0 \dfrac{A}{2}}{\dfrac{d}{2}} \right) = K_2 \left(\frac{\varepsilon_0 A}{d} \right)$$

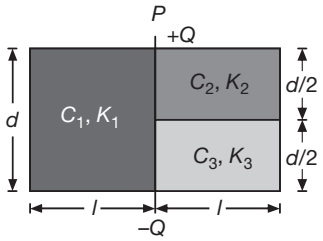
$$\Rightarrow \quad C_3 = K_3 \left(\frac{\varepsilon_0 \left(\dfrac{A}{2} \right)}{\dfrac{d}{2}} \right) = K_3 \left(\frac{\varepsilon_0 A}{d} \right)$$

Equivalent capacitance of C_2 and C_3 is

$$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{K_2 K_3 \left(\dfrac{\varepsilon_0 A}{d} \right)^2}{(K_2 + K_3) \left(\dfrac{\varepsilon_0 A}{d} \right)} = \left(\frac{K_2 K_3}{K_2 + K_3} \right) \frac{\varepsilon_0 A}{d}$$

Also $C_1 = K_1 \left(\dfrac{\varepsilon_0 \left(\dfrac{A}{2} \right)}{d} \right) = K_1 \left(\dfrac{\varepsilon_0 A}{2d} \right)$

CHAPTER 2



From figure it is clear that C_1 is in parallel with C' (combination of C_2 and C_3). Hence net capacitance between P and Q.

$$C = C_1 + C' = K_1\left(\frac{\varepsilon_0 A}{2d}\right) + \left(\frac{K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2K_3}{(K_2+K_3)}\right)$$

Substituting given values

$$C = \frac{8.8\times10^{-12}\times(1\times10^{-4})}{2\times10^{-3}}\left(\frac{4}{2}+\frac{6\times2}{6+2}\right)$$

$$\Rightarrow \quad C = 0.44\times10^{-12}\left(\frac{7}{2}\right)$$

$$\Rightarrow \quad C = 1.54\times10^{-12}\ \text{F} = 1.54\ \text{pF}$$

9. With the switch S closed, the potential difference across capacitors A and B is same. So,

$$V = \frac{Q_A}{C} = \frac{Q_B}{C}$$

The initial charges on the capacitors are given by

$$Q_A = Q_B = CV$$

When dielectric is introduced, the new capacitance of either capacitor.

$$C' = KC = 3C$$

Now, when the switch S is opened, let the potential difference across capacitor A be V volt and that across the capacitor B be V' volt.

When dielectric is introduced with the switch open (i.e., battery disconnected) then, the charge on capacitor B remains unchanged, so

$$Q_B = CV = C'V'$$

$$\Rightarrow \quad V' = \frac{C}{C'}V = \frac{V}{3}\ \text{volt}$$

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{5}{3}CV^2$$

$$\Rightarrow \quad \frac{U_i}{U_f} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

10. (a) $C = \frac{\varepsilon_0}{d}[(\ell - x)\ell + \kappa\ell x] = \frac{\varepsilon_0}{d}[\ell^2 + \ell x(\kappa - 1)]$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(\frac{\varepsilon_0(\Delta V)^2}{d}\right)[\ell^2 + \ell x(\kappa-1)]$

(c) $F = -\left(\frac{dU}{dx}\right)\hat{i} = \frac{\varepsilon_0(\Delta V)^2}{2d}\ell(\kappa-1)$ to the left (out of the capacitor)

(d) $F = \frac{(2000)^2(8.85\times10^{-12})(0.0500)(4.50-1)}{2(2.00\times10^{-3})}$

$F = 1.55\times10^{-3}$ N

11. The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_P = \frac{\varepsilon_0\left(\frac{A}{2}\right)}{d} + \frac{\kappa\varepsilon_0\left(\frac{A}{2}\right)}{d} = \left(\frac{\kappa+1}{2}\right)\frac{\varepsilon_0 A}{d}$$

where A is the area of either plate and d is the separation of the plates. The horizontal orientation produces two capacitors in series. If f is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa\varepsilon_0 A} + \frac{(1-f)d}{\varepsilon_0 A} = \left[\frac{f+\kappa(1-f)}{\kappa}\right]\frac{d}{\varepsilon_0 A}$$

$$\Rightarrow \quad C_s = \left[\frac{\kappa}{f+\kappa(1-f)}\right]\frac{\varepsilon_0 A}{d}$$

For having $C_p = C_s$ we get $\frac{\kappa+1}{2} = \frac{\kappa}{f+\kappa(1-f)}$

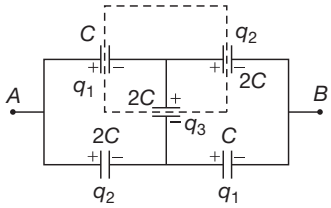
$$\Rightarrow \quad (\kappa+1)[f+\kappa(1-f)] = 2\kappa$$

For $\kappa = 2$, we get $3[2-(1)f] = 4$

$$\Rightarrow \quad f = \frac{2}{3}$$

Test Your Concepts-IV (Based on Capacitor Circuits, Kirchhoff's Laws, Charge Flown and Generation of Heat)

1. The given circuit forms an unbalanced Wheatstone bridge. Let us suppose that the point A is connected to the positive terminal of a hypothetical battery and B to the negative terminal of the same battery. Hence, a total charge q is stored in the capacitors. Seeing the symmetry about the input and output points, we can say that charges will be distributed as shown.



$$q_1 + q_2 = q \qquad \ldots(1)$$

Applying Second Law, we have

$$-\frac{q_1}{C} - \frac{q_3}{2C} + \frac{q_2}{2C} = 0$$

$$\Rightarrow \quad q_2 - q_3 - 2q_1 = 0 \qquad \ldots(2)$$

Plates inside the dotted line form an isolated system. Hence,

$$q_2 + q_3 - q_1 = 0 \qquad \ldots(3)$$

Solving these three equations, we have

$$q_1 = \frac{2}{5}q\,,\ q_2 = \frac{3}{5}q \text{ and } q_3 = -\frac{q}{5}$$

Now, let C_{eq} be the equivalent capacitance between A and B. Then,

$$V_A - V_B = \frac{q}{C_{eq}} = \frac{q_1}{C} + \frac{q_2}{2C}$$

$$\Rightarrow \quad \frac{q}{C_{eq}} = \frac{2q}{5C} + \frac{3q}{10C} = \frac{7q}{10C}$$

$$\Rightarrow \quad C_{eq} = \frac{10}{7}C$$

2. $$V_A - \frac{q_1}{C_1} + E - \frac{q_2}{C_2} - V_B = 0$$

$$5 - \frac{q_1}{1} + 10 - \frac{q_2}{2} = 0$$

$$\Rightarrow \quad q_1 + \frac{q_2}{2} = 15 \qquad \ldots(1)$$

Also, we observe that

$$-q_1 + q_2 = 0$$

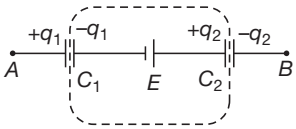
$$\Rightarrow \quad q_1 = q_2 \qquad \ldots(2)$$

From (1) and (2), we get

$$q_1 + \frac{q_1}{2} = 15$$

$$\Rightarrow \quad q_1 = 10\ \mu\text{C}$$

$$q_2 = 10\ \mu\text{C}$$



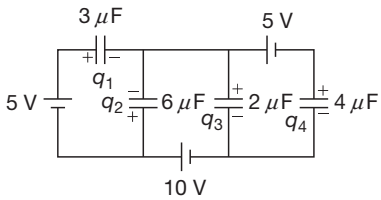
So, voltage across the first capacitor is

$$V_1 = \frac{q_1}{C_1} = 10\ \text{V}$$

and across the second is

$$V_2 = \frac{q_2}{C_2} = 5\ \text{V}$$

3. Here, we observe that q_1, q_2, q_3 and q_4 are in μC (microcoulomb) and any battery supplies same magnitude of charge from its both terminals.



Select the sign of the charges on the capacitors arbitrarily, we get

$$-q_1 - q_2 + q_3 + q_4 = 0 \qquad \ldots(1)$$

Applying Loop Law in three loops, one by one, we get

$$5 - \frac{q_1}{3} + \frac{q_2}{6} = 0 \qquad \ldots(2)$$

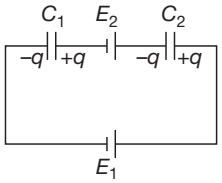
$$10 - \frac{q_2}{6} - \frac{q_3}{2} = 0 \qquad \ldots(3)$$

and $$5 - \frac{q_3}{2} + \frac{q_4}{4} = 0 \qquad \ldots(4)$$

Solving these four equations, we get

$$q_2 = q_3 = 10\ \mu\text{C} \text{ and } q_1 = q_4 = \frac{40}{3}\ \mu\text{C}$$

4. The charge distribution is shown in figure. The circuit is equivalent to two capacitors connected in series with a battery in between them. Let the charge on C_1 is q. The same charge will also appear on C_2. The reason is as follows : Whatever charge appears on plate of C_1 (say- q) then equal but opposite charge goes to C_2 through E_1. The potential drops across C_1 and C_2 will be $\frac{q}{C_1}$ and $\frac{q}{C_2}$ respectively. Which plate would be at higher potential will be decided by relative strength of E_1 and E_2. If we move along the complete loop, the total potential difference must be zero. Let $E_2 > E_1$.



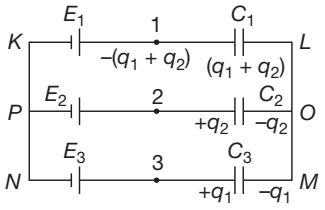
Thus, $\frac{q}{C_1}+E_1+\frac{q}{C_2}-E_2=0$

$$\Rightarrow \quad q=\frac{(E_2-E_1)C_1C_2}{C_1+C_2}$$

$$|q|=\frac{|E_2-E_1|C_1C_2}{C_1+C_2}$$

It is obvious from the figure that the addition of a battery in between two capacitors is equivalent to charging the potential applied to the two capacitors in series.

5. Charge distribution is shown in figure. Consider the loop $NMLKN$. Applying $-\Delta V=0$, we have



$$-E_3+\frac{q_1}{C_3}+\left(\frac{q_1+q_2}{C_1}\right)+E_1=0 \qquad \ldots(1)$$

Similarly, for loop $OLKPO$, we get

$$\left(\frac{q_1+q_2}{C_1}\right)+E_1-E_2+\frac{q_2}{C_2}=0 \qquad \ldots(2)$$

Solving these equations, we get

$$q_1+q_2=\frac{E_2C_2-E_1C_2-E_1C_3+E_3C_3}{\frac{C_3}{C_1}+\frac{C_2}{C_1}+1} \qquad \ldots(3)$$

Now $V_1-V_0=V_1=-\frac{(q_1+q_2)}{C_1} \qquad \{\because V_0=0\}$

$$\Rightarrow \quad V_1=\frac{E_1(C_2+C_3)-E_2C_2-E_3C_3}{C_1+C_2+C_3}$$

Similarly, $V_2=\frac{E_2(C_1+C_3)-E_1C_1-E_3C_3}{C_1+C_2+C_3}$

$$V_3=\frac{E_3(C_1+C_2)-E_1C_1-E_2C_2}{C_1+C_2+C_3}$$

6. Capacitors C_1 and C_2 are in series; their effective capacitance is

$$C_{12}=\frac{C_1C_2}{C_1+C_2}$$

Effective capacitance of C_3 and C_4 is

$$C_{34}=\frac{C_3C_4}{C_3+C_4}$$

C_{12} and C_{34} are in parallel; equivalent capacitance of the circuit is

$$C_{eq}=C_{12}+C_{34}$$

total charge on the capacitors is

$$Q=C_{eq}V$$

Charge Q divides into charge Q_{12} in C_1 or C_2 and Q_{34} in C_3 or C_4. We have

$$Q_{12}=C_{12}V \text{ and } Q_{34}=C_{34}V$$

Potential across capacitor C_1 is

$$V_1=\frac{Q_{12}}{C_1}=\frac{C_2}{C_1+C_2}V$$

Potential drop at capacitor C_3

$$V_3=\frac{Q_{34}}{C_3}=\frac{C_4}{C_3+C_4}V$$

Potential of A and B are

$$V_A=V-V_1=\left(1-\frac{C_2}{C_1+C_2}\right)V$$

and $V_B=V-V_3=\left(1-\frac{C_4}{C_3+C_4}\right)V$

Thus, potential differences between A and B are equal to

$$V_A-V_B=\left(\frac{C_4}{C_3+C_4}-\frac{C_2}{C_1+C_2}\right)V=\frac{C_1C_4-C_2C_3}{(C_1+C_2)(C_3+C_4)}V$$

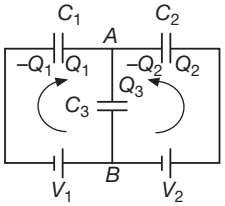
This will be zero, if $C_1C_4-C_2C_3=0$

$$\Rightarrow \quad \frac{C_1}{C_2}=\frac{C_3}{C_4}$$

This is the condition for a balanced Wheatstone bridge.

7. Let the charges on capacitors C_1, C_2, and C_3 be Q_1, Q_2 and Q_3. Potential differences are therefore $\frac{Q_1}{C_1}, \frac{Q_2}{C_2}$ and $\frac{Q_3}{C_3}$ respectively. Look at the inner plates of the three capacitors connected to point A. From conservation of charge, we have

$$Q_1-Q_2+(\pm Q_3)=0 \qquad \ldots(1)$$



Because all the inner three plates were neutral initially, therefore, whatever charge is induced on one is obviously drawn from others. Q_3, the charge on the upper plate of C_3, may be positive or negative depending upon the value of $(Q_1 - Q_2)$.

As we move along the closed loops in the sense shown, we have

$$-V_1 + \frac{Q_1}{C_1} \mp \frac{Q_3}{C_3} = 0 \qquad \ldots(2)$$

and $$V_2 - \frac{Q_2}{C_2} \mp \frac{Q_3}{C_3} = 0 \qquad \ldots(3)$$

If Q_3 is positive, then potential falls as we go from A to B. Hence $\mp Q_3$ in (1) implies $\mp\left(\frac{Q_3}{C_3}\right)$ in (2) and (3).

From (2) and (3), we get

$$Q_1 = \left(V_1 \mp \frac{Q_3}{C_3}\right)C_1 \text{ and } Q_2 = \left(V_2 \mp \frac{Q_3}{C_3}\right)C_2$$

Substituting for Q_1 and Q_2 in (1),we get

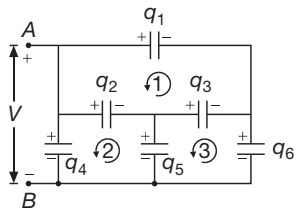
$$\frac{V_1C_1 - V_2C_2 \pm Q_3(C_1 + C_2 + C_3)}{C_3} = 0$$

$$\Rightarrow \quad \frac{Q_3}{C_3} = (\pm)\frac{V_2C_2 - V_1C_1}{C_1 + C_2 + C_3}$$

Thus potential difference between A and B

$$V_{AB} = \frac{V_2C_2 - V_1C_1}{C_1 + C_2 + C_3}$$

8. Let the potential difference across the battery terminals be V and the charge of the battery Q. To find the capacitance of the battery means to find the capacitance of a capacitor which would have the same charge Q on its plates as the battery at voltage V. Hence,



$$C_{AB} = \frac{Q}{V} \qquad \ldots(1)$$

where from conservation of charge we must have

$$Q = q_1 + q_2 + q_3 = q_4 + q_5 + q_6 \qquad \ldots(2)$$

and $$V = V_4 = \frac{q_4}{C}$$

In a closed loop the net potential drop must be zero, from KVL. Therefore, for loop 1, loop 2 and loop 3, we have

For Loop 1

$$\frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

For Loop 2

$$\frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0$$

For Loop 3

$$\frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} = 0 \qquad \ldots(3)$$

The conductor that connects the second, third and fifth capacitors is electrically neutral. Hence,

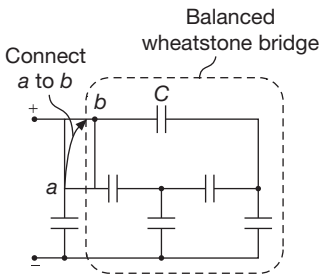
$$q_3 + q_5 - q_2 = 0 \qquad \ldots(4)$$

On solving, we get

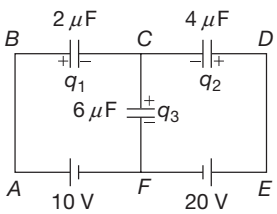
$$q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2} \text{ and } q_3 = 0$$

Therefore, $C_{AB} = 2C$

Method 2. Connect the connection of point a to b and note the indicated Wheatstone bridge. Now simplify the circuit to obtain the desired result.



9. Let the charges in three capacitors be as shown in figure



Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2. Then,

$$q_1 + q_2 = q_3$$

This relation can also be obtained by using the Junction Law. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries. Thus,

$$q_3 - q_1 - q_2 = 0$$

$$\Rightarrow \quad q_3 = q_1 + q_2 \qquad \ldots(1)$$

Applying Kirchhoff's Second Law to loops $BCFAB$ and $CDEFC$, we get

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

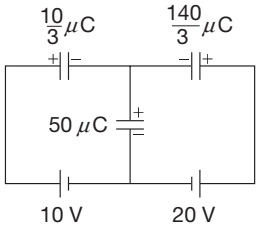
$$\Rightarrow \quad q_3 + 3q_1 = 60 \qquad \ldots(2)$$

and $$\frac{q_2}{4} - 20 + \frac{q_3}{6} = 0$$

$$\Rightarrow \quad 3q_2 + 2q_3 = 240 \qquad \ldots(3)$$

Solving equations (1), (2) and(3), we have

$$q_1 = \frac{10}{3}\ \mu\text{C},\ q_2 = \frac{140}{3}\ \mu\text{C} \text{ and } q_3 = 50\ \mu\text{C}$$



10. Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries V_1 and V_2.

From KVL we have

$$\frac{Q_i}{C} - V_2 + V_1 = 0$$

$$\Rightarrow \quad Q_i = (V_2 - V_1)C \qquad \ldots(1)$$

depending upon the sign of $(V_2 - V_1)$, charge Q_i on the left plate may be positive (if $V_2 > V_1$), or negative $(\text{if } V_2 < V_1)$. However, charge on the right plate would be equal and opposite.

When the switch is moved to position 2, the left plate earlier having charge $+Q_i$, will have charge

$$Q_f = -V_1 C \qquad \ldots(2)$$

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = \left[-V_1 - (V_2 - V_1)\right]C = -V_2 C$$

We can say that a net positive charge equal to V_2C is pulled by the battery of emf V_1 from the left plate of the capacitor, which flows through battery V_1 and is transferred to the right plate of the capacitor. Work done by battery V_1 in the process of charge transfer is given by

$$W_{\text{battery}} = V_1|\Delta Q| = V_1 V_2 C \qquad \ldots(3)$$

A part of this work changes the energy of the capacitor

$$\Delta U_C = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C}$$

$$\Delta U_C = \frac{1}{2}V_1^2 C - \frac{1}{2}(V_2 - V_1)^2 C$$

$$\Delta U_C = \frac{1}{2}(2V_1V_2 - V_2^2)C$$

and the remaining part is lost as Joule heating. Hence the heat generated H is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = \frac{1}{2}V_2^2 C$$

11. When the switch is open, then $C_{\text{eq}} = 4\ \mu\text{F}$

So, $q = C_{eq}V$

$$\Rightarrow \quad q = 800\ \mu\text{C}$$

This charge divides equally amongst the upper branch and lower branch. So,

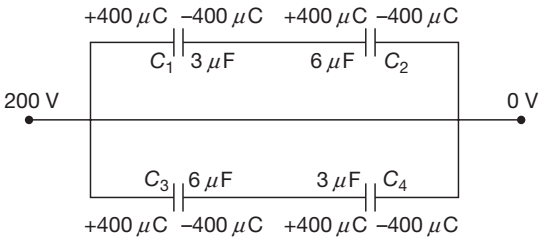
$$V_1 - V_b = \frac{400}{3}\ \text{V}$$

$$V_1 - V_a = \frac{400}{6}\ \text{V} = \frac{200}{3}\ \text{V}$$

$$\Rightarrow \quad V_a - V_b = \frac{200}{3}\ \text{V}$$

When the switch is closed, then $V_a - V_b = 0$

The charges on the capacitors before the switch S is closed are shown.



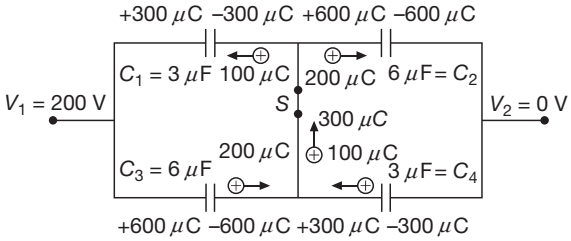
Now, when the switch is closed, then $C'_{eq} = \frac{q}{2}\ \mu\text{F}$

$$\Rightarrow \quad q' = C'_{eq}V = 900\ \mu\text{C}$$

This $900\ \mu\text{C}$ is shared amongst $3\ \mu\text{F}$ and $6\ \mu\text{F}$ in direct ratio of their capacitances. So

$$q_{3\ \mu\text{F}} = \left(\frac{3}{3+6}\right)900 = 300\ \mu\text{C} \text{ and}$$

$$q_{6\,\mu F} = \left(\frac{6}{3+6}\right)900 = 600\ \mu C$$

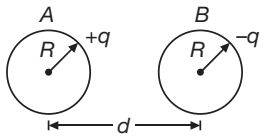


	Initial Charge	Final Charge	Charge Flown $\lvert\Delta q\rvert$
C_1	400 μC	300 μC	$\lvert\Delta q_1\rvert = 100$ μC
C_2	400 μC	600 μC	$\lvert\Delta q_2\rvert = 200$ μC
C_3	400 μC	600 μC	$\lvert\Delta q_3\rvert = 200$ μC
C_4	400 μC	300 μC	$\lvert\Delta q_4\rvert = 100$ μC

Test Your Concepts-V (Based on Spherical and Cylindrical Capacitors)

1. Let us assume a positive charge $+q$ on A and a negative charge $-q$ on B. Then their potentials are

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{d} \text{ and } V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{d} - \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$



The potential difference is

$$V = V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{2q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{2q}{d}$$

$$\Rightarrow\quad V = 2\frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R} - \frac{1}{d}\right] = \frac{q}{2\pi\varepsilon_0}\left(\frac{d-R}{Rd}\right)$$

Since $d \gg R$

$$\Rightarrow\quad V = \frac{q}{2\pi\varepsilon_0 R}$$

Since, $C = \frac{q}{V}$

$$\Rightarrow\quad C = 2\pi\varepsilon_0 R$$

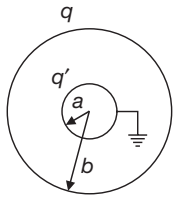
$$\Rightarrow\quad C = \frac{2}{9\times10^9}\times\frac{9}{1000}$$

$$\Rightarrow\quad C = 2\times10^{-12}\ \text{F}$$

$$\Rightarrow\quad C = 2\ \text{pF}$$

2. (a) Given, $a = 2\times10^{-2}$ m, $b = 4\times10^{-2}$ m and $q = 2\times10^{-6}$ C

Let q' charge comes on the inner sphere. Then,



$$V_{\text{inner}} = 0$$

$$\Rightarrow\quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{a} + \frac{q}{b}\right] = 0$$

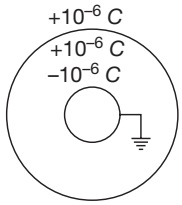
$$\Rightarrow\quad q' = -\frac{a}{b}\cdot q = -10^{-6}\ \text{C}$$

Now, $V_{\text{inner}} - V_{\text{outer}} = \frac{q'}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

$$\Rightarrow\quad 0 - V_{\text{outer}} = -10^{-6}\times10^9\times10^2\times\frac{1}{4}$$

$$\Rightarrow\quad V_{\text{outer}} = 2.5\times10^4\ \text{V}$$

(b) Charges appearing on different surfaces are as shown in figure. Hence the charge retained on the outer surface of the outer sphere is $+10^{-6}$ C or $+1\ \mu$C.



3. Let q, v and c be the charge, potential and capacitance of individual (small) drop. Also let Q, V, C be the corresponding quantities for bigger drop.

As total charge is conserved, therefore the charge of bigger drop

$$Q = N\times\text{charge on small drop}$$

$$\Rightarrow\quad Q = Nq$$

$$\Rightarrow\quad \frac{Q}{q} = N \qquad \ldots(1)$$

Since the capacitance of a spherical drop is proportional to its radius $(C = 4\pi\varepsilon_0 r)$, therefore if R and r are radii of big drop and small drop respectively, then

$$\frac{C}{c} = \frac{R}{r} \qquad \ldots(2)$$

Now, since total mass of the drops is conserved, so, we have

Mass of bigger drop $= N\times$ Mass of small drop

CHAPTER 2

$$\Rightarrow \quad \frac{4}{3}\pi R^3\rho = N\left(\frac{4}{3}\pi r^3\rho\right) \quad \{\rho \text{ is density of mercury}\}$$

$$\Rightarrow \quad R = N^{1/3}r$$

$$\Rightarrow \quad \frac{C}{c} = \frac{R}{r} = N^{1/3} \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{C}{c} = N^{\frac{1}{3}}$$

Again, by definition

$$V = \frac{Q}{C} \text{ and } v = \frac{q}{c}$$

$$\Rightarrow \quad \frac{V}{v} = \frac{Q}{C}\cdot\frac{c}{q} = \left(\frac{Q}{q}\right)\left(\frac{c}{C}\right) = \frac{N}{N^{1/3}} = N^{2/3} \qquad \ldots(4)$$

$$\Rightarrow \quad \frac{V}{v} = N^{\frac{2}{3}}$$

Also, $U = \dfrac{Q^2}{2C}$ and $u = \dfrac{q^2}{2c}$

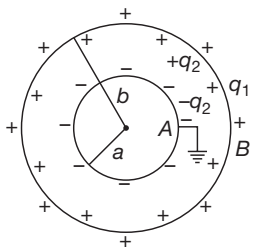
$$\Rightarrow \quad \frac{U}{u} = \frac{\frac{Q^2}{2C}}{\frac{q^2}{2c}} = \left(\frac{Q}{q}\right)^2\left(\frac{c}{C}\right)$$

$$\Rightarrow \quad \frac{U}{u} = N^2 \times N^{-\frac{1}{3}} = N^{\frac{5}{3}}$$

$$\Rightarrow \quad \frac{U}{u} = N^{\frac{5}{3}}$$

4. Let Q be the charge given to outer sphere B. This charge will be partly on outer surface and partly on inner surface. Let q_1 be charge on outer surface and q_2 on inner surface. Then

$$q_1 + q_2 = Q \qquad \ldots(1)$$



The charge induced on inner sphere $= -q_2$
The potential of B,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 + q_2 - q_2}{b} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b} \qquad \ldots(2)$$

The potential of A,

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q_2}{a} + \frac{q_1 + q_2}{b}\right)$$

As sphere A is earthed $V_A = 0$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{-q_2}{a} + \frac{q_1 + q_2}{b}\right] = 0 \qquad \ldots(3)$$

Equation (3) gives

$$\frac{-q_2}{a} + \frac{q_1 + q_2}{b} = 0 \text{ i.e., } \frac{-q_2}{a} + \frac{Q}{b} = 0$$

$$\Rightarrow \quad \frac{q_2}{Q} = +\frac{a}{b} \qquad \ldots(4)$$

Using this equation (1) gives

$$q_1 + \frac{a}{b}Q = Q$$

$$\Rightarrow \quad q_1 = Q\left(1 - \frac{a}{b}\right) = \frac{b-a}{b}Q$$

i.e., $\dfrac{Q}{q_1} = \dfrac{b}{b-a}$ $\qquad \ldots(5)$

Potential difference between A and B

$$V_{ab} = V_B - V_A = V_B - 0 = V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b}$$

$\therefore$ Capacitance

$$C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{b}} = 4\pi\varepsilon_0 b\frac{Q}{q_1}$$

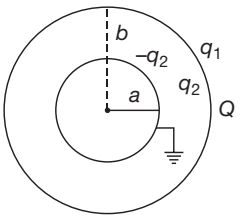
Using (5), we get

$$C = 4\pi\varepsilon_0 b\left(\frac{b}{b-a}\right) = \frac{4\pi\varepsilon_0 b^2}{b-a} \qquad \{\text{done already}\}$$

5. Charge on inner sphere is given by

$$-q_2 = -\frac{a}{b}Q$$

$$\Rightarrow \quad -q_2 = -\frac{8 \text{ cm}}{10 \text{ cm}} \times 30 \text{ nC} = -24 \text{ nC}$$



Charge on inner surface of outer sphere is

$$q_2 = +24 \text{ nC}$$

The charge Q is distributed into q_1 and q_2 so that the electrostatic potential of inner sphere is zero.

6. The given arrangement is a parallel combination of two capacitors.
The capacity of a cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_1 = \frac{2\pi\varepsilon_0 K_1\left(\frac{L}{2}\right)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_2 = \frac{2\pi\varepsilon_0 K_2\left(\frac{L}{2}\right)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_{eq} = C_1 + C_2 = \frac{\pi L}{\log_e\left(\frac{b}{a}\right)}(K_1 + K_2)$$

Single Correct Choice Type Questions

1. When the switch is closed, the inner plates of the two capacitors get connected whereas the outer plates still are not connected and hence the circuit is not complete.
Hence, the correct answer is (A).

3. $C = k\varepsilon_0 \frac{A}{d}$

$$\Rightarrow \quad 1.77\times10^{-6} = 200\left(8.85\times10^{-12}\right)\frac{A}{d}$$

$$\Rightarrow \quad \frac{A}{d} = 10^3$$

This ratio is satisfied by both **OPTIONS (A)** and **(B)**. So to arrive at a conclusion we take help of the break through strength.

FOR OPTION (A)

$$d = 10^{-6} m$$

$$\Rightarrow \quad E = \frac{V}{d}$$

$$\Rightarrow \quad E = \frac{20}{10^{-6}}$$

$$\Rightarrow \quad E = 2\times10^7 \ Vm^{-1} > 3\times10^6 \ Vm^{-1}$$

which is impossible

FOR OPTION (B)

$$d = 10^{-5}\text{m}$$

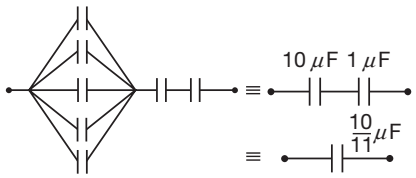
$$\Rightarrow \quad E = \frac{V}{d}$$

$$\Rightarrow \quad E = \frac{20}{10^{-5}}$$

$$\Rightarrow \quad E = 2\times10^6 \ \text{Vm}^{-1} < 3\times10^6 \ \text{Vm}^{-1}$$

Hence **OPTION (B)** is right to give as an answer.
Hence, the correct answer is (B).

4.



Hence, the correct answer is (A).

5. The circuit show is a balanced Wheat Stone Bridge.
Hence, the correct answer is (D).

6. The given system is equivalent to a spherical capacitor of inner radius b and outer radius c. So, the capacitance of the system will be $4\pi\varepsilon_0\left(\frac{bc}{c-b}\right)$.

Hence, the correct answer is (D).

7. $C = 4\pi\varepsilon_0 R$

$$\Rightarrow \quad C = 711 \ \mu\text{F}$$

$$\Rightarrow \quad C \cong 1000 \ \mu\text{F}$$

$$\Rightarrow \quad C \cong 10^{-3} \ \text{F} = 1 \ \text{mF}$$

Hence, the correct answer is (B).

8. $Q_2 = -Q_3$ {By induction}

Further,

$$\Delta V = \frac{Q_2}{C} = \frac{2Q_2}{2C} = \frac{Q_2 - (-Q_2)}{2C}$$

$$\Rightarrow \quad \Delta V = \frac{Q_2 - Q_3}{2C}$$

Hence, the correct answer is (C).

10. On connecting both with a thin wire, the common potential V is given by

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$

$$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

CHAPTER 2

Total initial energy $= U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

Total final energy $= U_f = \frac{1}{2}(C_1 + C_2)V^2$

Loss = Total Initial Energy – Total Final Energy

$\Rightarrow$ Loss $= U_i - U_f$

$\Rightarrow$ Loss $= \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$

Hence, the correct answer is (C).

12. $C' = \frac{C}{1 - \frac{1}{2}}$ $\left\{\because C' = \frac{C}{1 - \frac{t}{d}}\right\}$

$C' = 2C$

Hence, the correct answer is (B).

14. The capacitance of the capacitor increases initially and then decreases and hence the positive charge on plate A first increases and then decreases. Due to this the current in the outer circuit first flows from B to A and then from A to B.

Hence, the correct answer is (C).

15. $C_S = 2\ pF$

$q = C_SV$

$\Rightarrow$ $q = 10\ \mu C$

when connected in parallel

$C_P = C_1 + C_2$

$\Rightarrow$ $C_P = 9\ pF$

$\Rightarrow$ $V = \frac{q}{C_P}$

$\Rightarrow$ $V = \frac{10}{9}\ kV$

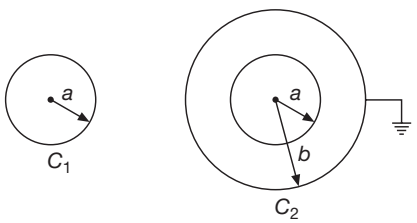
$\Rightarrow$ $V = 1111\ V$

Hence, the correct answer is (D).

16. $C_1 = 4\pi\varepsilon_0 a$

and $C_2 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

Given that $C_2 = nC_1$



$\Rightarrow$ $\frac{ab}{b-a} = na$

$\Rightarrow$ $\frac{b}{b-a} = n$

$\Rightarrow$ $b = nb - na$

$\Rightarrow$ $na = (n-1)b$

$\Rightarrow$ $\frac{b}{a} = \frac{n}{n-1}$

Hence, the correct answer is (B).

17. $\frac{1}{C} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$

$\Rightarrow$ $\frac{1}{C} = \frac{1/2}{1 - 1/2}$

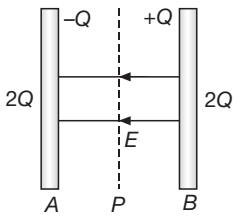
$\Rightarrow$ $C = 1$

Hence, the correct answer is (D).

18. Since, the charges on the facing surfaces are $-Q$ and Q (used Gauss's Law to find them)

So, $E = \frac{Q}{A\varepsilon_0}$

$\Rightarrow$ $V_P - V_A = +\frac{Ed}{2}$



$\Rightarrow$ $V_P - V_A = \frac{Qd}{2A\varepsilon_0} = \frac{Q}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{Q}{2C_0}$

Hence, the correct answer is (B).

19. For series combination

$C_S = \frac{C_1C_2}{C_1 + C_2}$

$\Rightarrow$ $C_S = \frac{2}{3}\mu F$

When connected in series the maximum charge that can flow through the combination equals the lower value of charge accommodated by the first capacitor i.e., 6000 μC

$\therefore$ $Q_1 = 6000\ \mu C$ and $Q_2 = 8000\ \mu C$

$$\Rightarrow \quad V_S = \frac{Q_1}{C_S}$$

$$\Rightarrow \quad V_S = \frac{6000\ \mu C}{\frac{2}{3}\mu C}$$

$$\Rightarrow \quad V_S = 9\ kV$$

Hence, the correct answer is (C).

20. Let C be the capacitance of the capacitor without slab. Then after removing the slab i.e., finally the net capacitance is $\frac{C}{2}$

$$q_1 = q_2 = \frac{CE}{2}$$

Before the slab is removed

$$C_1 = C \text{ and } C_2 = KC$$

$$\Rightarrow \quad C_{net} = \left(\frac{K}{K+1}\right)C$$

$$\Rightarrow \quad Q_1 = Q_2 = \left(\frac{KCE}{K+1}\right)$$

$$\Rightarrow \quad \frac{q_2}{Q_2} = \frac{K+1}{2K}$$

Hence, the correct answer is (A).

21. $U_i = \frac{1}{2}C_0E^2$ …(1)

On inserting a dielectric new capacitance is

$$C = kC_0 \qquad \ldots(2)$$

and new potential difference is $\{V = Ed\}$

$E' = f_e'd$ where f_e is the electrostatic field. But on inserting the dielectric, the new field f_e' becomes $\frac{1}{k}$ times the original field.

So $f_e' = \frac{f_e}{k}$

$$\Rightarrow \quad E' = \frac{E}{k} \qquad \ldots(3)$$

So $U_f = \frac{1}{2}CE'^2$

$$\Rightarrow \quad U_f = \frac{1}{2}(kC_0)\left(\frac{E}{k}\right)^2$$

$$\Rightarrow \quad U_f = \left(\frac{1}{2}C_0E^2\right)\frac{1}{k}$$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C_0E^2\left(\frac{1}{k} - 1\right)$$

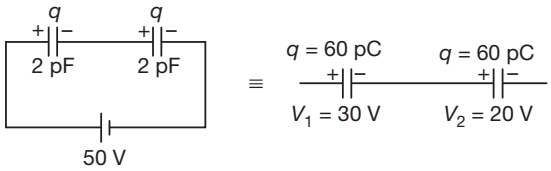
Hence, the correct answer is (D).

22. k_1 in series with half of k_3 and hence equivalent dielectric constant is $\frac{k_1k_3}{k_1+k_3}$

k_2 in series with half of k_3 and hence equivalent dielectric constant is $\frac{k_2k_3}{k_2+k_3}$ and then both in parallel to give $k = \frac{k_1k_3}{k_1+k_3} + \frac{k_2k_3}{k_2+k_3}$

Hence, the correct answer is (D).

24. Charges on the capacitors are



$$q_1 = (30)(2) = 60\ pC$$

$$\Rightarrow \quad q_2 = (20)(3) = 60\ pC$$

$$\Rightarrow \quad q_1 = q_2 = q \quad \text{(say)}$$

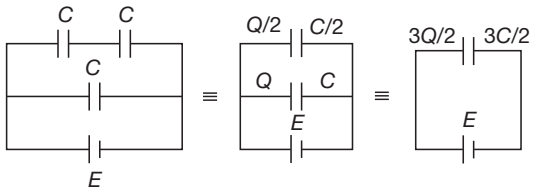
The situation is similar to the two capacitors in series which are first charged with a battery of emf 50 V and then disconnected.

So, when S_3 is closed, $V_1 = 30$ V and $V_2 = 20$ V

Other way of looking at the thing is, when S_1 and S_3 both are closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

Hence, the correct answer is (D).

26.



$\{\because Q = CE\}$

Hence, the correct answer is (B).

27. $W = \frac{1}{2}C(100 - 25)$ $\left\{\because W = \frac{1}{2}C\left(V_2^2 - V_1^2\right)\right\}$

$$\Rightarrow \quad W = \frac{75}{2}C \qquad \ldots(1)$$

$$\Rightarrow \quad W' = \frac{1}{2}C(225 - 100)$$

$$\Rightarrow \quad W' = \frac{125}{2}C \qquad \ldots(2)$$

CHAPTER 2

From (1) and (2)

$$W' = \frac{5}{3}W$$

Hence, the correct answer is (C).

28. $V = 4\pi\sigma\left(\frac{t}{k_1} + \frac{d-t}{k_2}\right)$

$$\Rightarrow \quad V = \frac{4\pi Q}{A}\left(\frac{t}{k_1} + \frac{d-t}{k_2}\right)$$

Hence, the correct answer is (B).

29. $V_{\text{combined}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$\Rightarrow \quad V_{\text{combined}} = \frac{(1)(200) + 2(100)}{1+2}$$

$$V_{\text{combined}} = \frac{400}{3} \simeq 133\ V$$

Hence, the correct answer is (C).

30. $\text{Loss} = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}\left(\frac{C}{2}\right)(V_1 - V_2)^2 = \frac{1}{4}C(V_1 - V_2)^2$$

Hence, the correct answer is (C).

31. During electrostatic equilibrium.

Electrostatic attraction between the plates = Spring force

$$\Rightarrow \quad \frac{q^2}{2\varepsilon_0 A} = kx$$

$$\Rightarrow \quad \frac{(CV)^2}{2\varepsilon_0 A} = k\left(d - \frac{4d}{5}\right) \qquad \{\because q = CV\}$$

$$\Rightarrow \quad \frac{\left(\frac{\varepsilon_0 A}{\left(\frac{4d}{5}\right)}\right)^2 V^2}{2\varepsilon_0 A} = 0.2kd$$

$$\Rightarrow \quad k = \frac{\varepsilon_0 AV^2}{0.256d^3} \approx \frac{4\varepsilon_0 AV^2}{d^3}$$

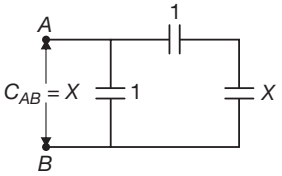
Hence, the correct answer is (C).

32. $\frac{X(1)}{X+1} + 1 = X$

$$\Rightarrow \quad X = X^2 - 1$$

$$\Rightarrow \quad X^2 - X - 1 = 0$$

$$\Rightarrow \quad X = \frac{1+\sqrt{5}}{2}$$



$$\Rightarrow \quad X = \frac{1 + 2.236}{2}$$

$$\Rightarrow \quad X \simeq \frac{3.2}{2}$$

$$\Rightarrow \quad X \simeq 1.6\ \mu F$$

Hence, the correct answer is (C).

33. Let d be the thickness of the slab, then

$$t = \frac{d}{2}$$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow \quad \frac{1}{\frac{d}{2}\left(1 + \frac{1}{K}\right)} = \frac{4}{3d}$$

$$\Rightarrow \quad 1 + \frac{1}{K} = \frac{3}{2}$$

$$\Rightarrow \quad K = 2$$

Hence, the correct answer is (A).

34. $C_{AB} = \frac{3C}{2}$

Total potential across the series combination of 2C, C and 2C is 90 V. If Q is the total charge then

$$Q = C_{AB}(90)$$

$$\Rightarrow \quad Q = 135C$$

Charge in SR branch $= Q_1 = 90C$

Charge in SPQR branch $= Q_2 = 45C$

$$\Rightarrow \quad V_{MN} = \text{ Potential across C}$$

$$\Rightarrow \quad V_{MN} = \frac{Q_2}{C} = 45\ V$$

Hence, the correct answer is (C).

35. The spring force $\vec{F}_S$ acting on plate a is given by

$$\vec{F}_S = -kx\,\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate b is

$$\vec{F}_e = QE\hat{i} = Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i} = \frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where A is the area of the plate. Notice that charges on plate a cannot exert a force on itself, as required by Newton's Third Law. Thus, only the electric field due to plate b is considered. At equilibrium the two forces cancel and we have

$$kx = Q\left(\frac{Q}{2A\varepsilon_0}\right)$$

which gives

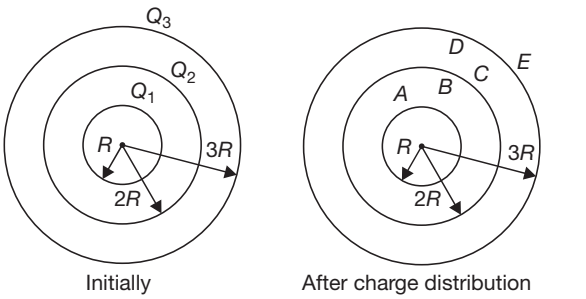
$$x = \frac{Q^2}{2kA\varepsilon_0}$$

Hence, the correct answer is (C).

36. The charge distribution on the surfaces of the shells are given. As per the given condition, the surface charge densities of the outer surfaces are equal. So,

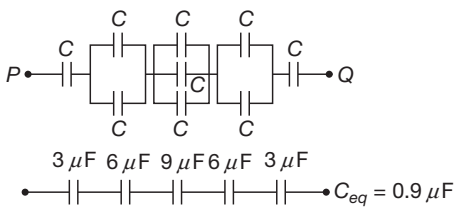
$$\frac{Q_1}{4\pi R^2} = \frac{Q_1+Q_2}{4\pi(2R)^2} = \frac{Q_1+Q_2+Q_3}{4\pi(3R)^2}$$

$$\Rightarrow \quad \frac{Q_1}{1} = \frac{Q_2}{3} = \frac{Q_3}{5}$$



Hence, the correct answer is (B).

37.



Hence, the correct answer is (C).

38. $V = \frac{q}{C}$

Since $C \propto \frac{1}{d}$

So, as d is halved, C becomes double

$$\Rightarrow \quad V' = \frac{q}{C'}$$

$$\Rightarrow \quad \frac{V}{V'} = \frac{C'}{C} = 2$$

$$\Rightarrow \quad V' = \frac{V}{2} = 250\ V$$

$$U_i = \frac{1}{2}CV^2$$

$$U_f = \frac{1}{2}C'V'^2 = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$$

$$U_f = \frac{1}{4}CV^2$$

$$\Rightarrow \quad Change = \frac{1}{4}CV^2$$

$$\Rightarrow \quad \text{Change} = \frac{1}{4}(100\times10^{-6})(500)^2$$

$$\Rightarrow \quad \Delta U = 6.25\ J$$

Hence, the correct answer is (B).

39. $C = \frac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad C = \frac{1}{4\pi\times9\times10^9}\ \frac{\pi(0.08)^2}{10^{-3}}$$

$$\Rightarrow \quad C = 1.8\times10^{-10}\ \text{F}$$

Since $Q = CV$

$$\Rightarrow \quad Q = 1.8\times10^{-8}\ \text{C}$$

Hence, the correct answer is (B).

40. Total Initial Charge

$$Q_i = (2C)(2V) - CV = 3CV$$

Total Final Charge

$$Q_f = 2CV' + CV'$$

where V' is common potential.

By Law of Conservation of Charge

$$Q_i = Q_f$$

$$\Rightarrow \quad 3CV = 3CV'$$

$$\Rightarrow \quad V' = V$$

So, final energy of combination is

$$U_f = \frac{1}{2}(C+2C)V^2 = \frac{3}{2}CV^2$$

Hence, the correct answer is (B).

41. $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

$$C = \frac{1}{9\times10^9}\ \frac{(0.5)(0.6)}{0.1}$$

$$\Rightarrow \quad C = \frac{0.3}{9\times10^8}$$

CHAPTER 2

$\Rightarrow \quad C' = kC$

$\Rightarrow \quad C' = 2 \times 10^{-9}$ F

Hence, the correct answer is (B).

42. The 3 μF capacitor has energy 600 μJ

$\Rightarrow \quad 600 \times 10^{-6} = \frac{1}{2}(3 \times 10^{-6}) V_0^2$

$\Rightarrow \quad V_0 = 20$ V

$\Rightarrow \quad V_{AB} = 3V_0 = 60$ V

This 60 V has to divide in between 2 μF and 6 μF in the inverse ratio of their capacitances i.e., 3 : 1 .

So, $V_{2\,\mu F} = \left(\frac{3}{3+1}\right) 60 \text{ V} = 45 \text{ V}$

Hence, the correct answer is (C).

45. The arrangement is a balanced Wheatstone Bridge. So, no charge will exist across the branch PQ. Hence

$C_{net} = 10\ \mu F$

Hence, the correct answer is (D).

47. On connecting by a wire, let V be the common potential. Then, by Law of Conservation of Charge

$Q_{initial} = Q_{final}$

$\Rightarrow \quad C_1V_1 + C_2V_2 = C_1V + C_2V$

$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

If E_i be total initial energy and E_f be the total final energy, then

$\text{Loss} = E_i - E_f$

$\Rightarrow \quad \text{Loss} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$

$\Rightarrow \quad \text{Loss} = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

We get above result by putting value of V and then solving.

Hence, the correct answer is (C).

48. $C_P = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \ldots$

$\Rightarrow \quad C_P = C\left(\frac{1}{1 - \frac{1}{2}}\right)$

$\Rightarrow \quad C_P = 2C = 2\ \mu F$

Hence, the correct answer is (C).

49. Total initial energy is

$E_i = 2\left(\frac{1}{2}CV^2\right) = CV^2$

Initial charge across A is CV

Initial charge across B is CV

On, opening the switch when a dielectric with $k = 3$ is introduced in both A and B, then

$C_A = 3C$ and $C_B = 3C$

The new potential across A is still V. Let V' be new potential across B. So, new charge across B is $(3C)V'$.

By Law of Conservation of Charge

$CV = (3C)V'$

$\Rightarrow \quad V' = \frac{V}{3}$

So, total final energy is

$E_f = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2$

$\Rightarrow \quad E_f = \frac{3}{2}CV^2\left(1 + \frac{1}{9}\right)$

$\Rightarrow \quad E_f = \frac{5}{3}CV^2$

$\Rightarrow \quad \frac{E_i}{E_f} = \frac{3}{5}$

Hence, the correct answer is (B).

50. $C_s = \frac{4 \times 6}{4 + 6}\ \mu F$

$\Rightarrow \quad C_s = 2.4\ \mu F$

$q = C_sV = 2.4 \times 500 \times 10^{-6}$

$q = 1200\ \mu C$

Hence, the correct answer is (C).

52. $Q_1 = C_0V_0$

$Q_2 = (kC_0)V_0$

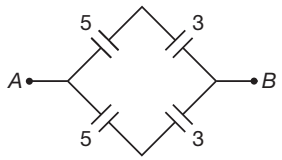
$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{1}{k}$

$\Rightarrow \quad \frac{40}{100} = \frac{1}{k}$

$\Rightarrow \quad k = 2.5$

Hence, the correct answer is (B).

53. It is a balanced Wheatstone Bridge.



$$\Rightarrow \quad C_{AB} = \frac{5\times 3}{5+3} + \frac{5\times 3}{5+3}$$

$$\Rightarrow \quad C_{AB} = 2\times\frac{15}{8}$$

$$\Rightarrow \quad C_{AB} = 3.75\ \mu F$$

Hence, the correct answer is (A).

54. Initial charge on first capacitor is $CV = Q_1$
Initial charge on second capacitor is $2CV = Q_2$
Final capacitance of first capacitor is KC
If V' is the common potential then

$$V' = \frac{Q_1 + Q_2}{C_1' + C_2}$$

$$\Rightarrow \quad V' = \frac{CV + 2CV}{KC + 2C}$$

$$\Rightarrow \quad V' = \frac{3V}{2+K}$$

Hence, the correct answer is (A).

55. $U_i = \frac{q^2}{2C_i} = \frac{q^2}{2(3C_0)}$

$$U_f = \frac{q^2}{2C_f} = \frac{q^2}{2(2C_0)}$$

$$\Rightarrow \quad \frac{U_f}{U_i} = \frac{3}{2}$$

Hence, the correct answer is (B).

56. By Law of Conservation of Charge

$$C_1V = C_1V' + C_2V'$$

$$\Rightarrow \quad V' = \left(\frac{C_1}{C_1 + C_2}\right)V$$

Hence, the correct answer is (A).

57. $U_i = U = \frac{1}{2}C_1V^2$

$$U_f = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2$$

$$\Rightarrow \quad \frac{U_f}{U} = \left(\frac{C_1 + C_2}{C_1}\right)\frac{V'^2}{V^2}$$

$$\Rightarrow \quad U_f = \left[\frac{C_1 + C_2}{C_1}\left(\frac{C_1}{C_1 + C_2}\right)^2\right]U$$

$$\Rightarrow \quad U_f = \left(\frac{C_1}{C_1 + C_2}\right)U$$

Hence, the correct answer is (A).

58. While drawing the dielectric plate outside, the capacitance decreases till the entire plate comes out and then becomes constant.
So, V increases and then becomes constant.
Hence, the correct answer is (B).

59. C_2 and C_3 in parallel

$$\Rightarrow \quad V_2 = V_3\ .\ \text{Also,}$$

$$V = V_1 + V_2 = V_1 + V_3$$

$$Q_2 + Q_3 = Q_1$$

Hence, the correct answer is (C).

60. When we move from left to right the voltage increases and this increase is just within the capacitors and remains constant in the conducting wires. Also for series combination $V \propto \frac{1}{C}$ and according to graph $V_2 > V_1$

$$\Rightarrow \quad C_1 > C_2$$

Hence, the correct answer is (C).

61. By Law of Conservation of Charge

$$Q_{\text{initial}} = Q_{\text{final}}$$

$$Q_{\text{initial}} = 5\times 12 = 60\ \mu C$$

$$Q_{\text{final}} = (C+5)3\ \mu C$$

$$\Rightarrow \quad (C+5)3 = 60$$

$$\Rightarrow \quad C = 15\ \mu F$$

Hence, the correct answer is (B).

62. Potential difference across the branch de is 6 V.
Net capacitance of de branch is $2.1\ \mu F$.

So, $q = CV$

$$\Rightarrow \quad q = 2.1\times 6\ \mu C$$

$$\Rightarrow \quad q = 12.6\ \mu C$$

Potential across $3\ \mu F$ capacitance is

$$V = \frac{12.6}{3} = 4.2\ volt$$

Potential across 2 and 5 combination in parallel is $6 - 4.2 = 1.8\ V$

CHAPTER 2

So,

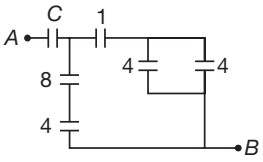
$$q' = (1.8)(5) = 9\ \mu C$$

Hence, the correct answer is (B).

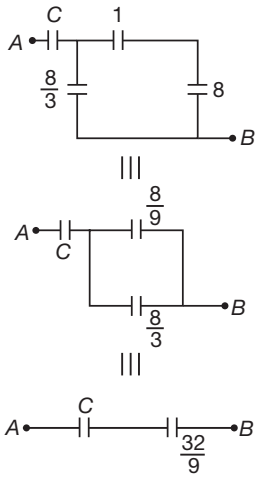
63. Since all are in series.

Hence, the correct answer is (D).

64.



Next equivalent circuit diagram is



$$\Rightarrow \quad \frac{\frac{32}{9}C}{\frac{32}{9}+C} = 1$$

$$\Rightarrow \quad C = \frac{32}{23}\ \mu F$$

Hence, the correct answer is (A).

65. $\frac{1}{C_s} = 1 + \frac{1}{2} + \frac{1}{4} + \ldots$

$$\frac{1}{C_s} = \frac{1}{1-\frac{1}{2}} = 2$$

$$\Rightarrow \quad C_s = 0.5\ \mu F$$

Hence, the correct answer is (C).

66. $C = \dfrac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Hence, the correct answer is (C).

68. $V_0 = \dfrac{q}{C_0}$

$$V = \frac{q}{C}$$

$$\Rightarrow \quad \frac{V}{V_0} = \frac{C_0}{C}$$

$$\Rightarrow \quad \frac{C}{C_0} = \frac{500}{75} = \frac{20}{3}$$

By definition

$$C = kC_0$$

$$\Rightarrow \quad k = \frac{20}{3}$$

Hence, the correct answer is (C).

69. $\dfrac{\sigma}{\varepsilon_0} d = \dfrac{\sigma}{\varepsilon_0}\left(d + 2.4 - 3 + \dfrac{3}{k}\right)$

$$\Rightarrow \quad d = d - 3 + 2.4 + \frac{3}{k}$$

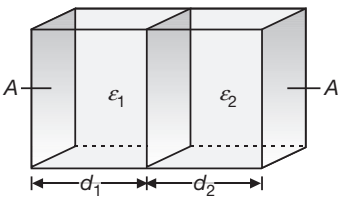
$$\Rightarrow \quad 3 - 2.4 = \frac{3}{k}$$

$$\Rightarrow \quad 0.6 = \frac{3}{k}$$

$$\Rightarrow \quad k = 5$$

Hence, the correct answer is (B).

70.



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

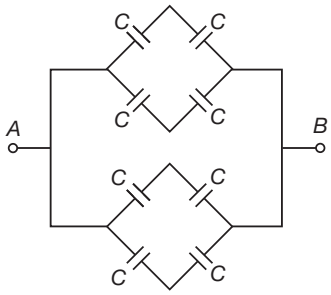
$$\Rightarrow \quad \frac{1}{C_s} = \frac{1}{\frac{\varepsilon_1 A}{d_1} + \frac{\varepsilon_2 A}{d_2}}$$

$$\Rightarrow \quad \frac{1}{C_s} = \frac{1}{A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$\Rightarrow \quad C_s = \frac{A\varepsilon_1\varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$$

Hence, the correct answer is (A).

71.

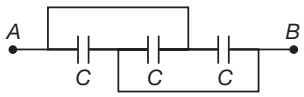


The figure shows two independent balanced Wheatstone Bridges connected in parallel each having a capacitance C. So,

$$C_{net} = C_{AB} = 2C$$

Hence, the correct answer is (B).

72.



$$C_{AB} = 3C$$

The circuit contains such two combinations in series. Hence

$$C_{\text{total}} = \frac{(3C)(3C)}{3C+3C}$$

$$\Rightarrow \quad C_{\text{total}} = \frac{3C}{2}$$

Hence, the correct answer is (B).

73. (A), (B) and (C) are different forms of wheatstone bridge.
Hence, the correct answer is (D).

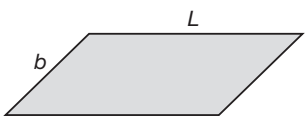
74. When disconnected from the source, q = constant.
When immersed in dielectric, we have $C' = KC$. So, we get

$$V' = \frac{q}{C'} = \frac{q}{KC} = \frac{V}{K}$$

Hence $E' = Vd = \frac{Vd}{K} = \frac{Vd}{K} = \frac{E}{K}$

Hence, the correct answer is (A).

75. Let area of plate be A, then $A = bL$, where b is width of plate



On partial insertion of dielectric, the net capacitance of capacitor is

$$C = \frac{\varepsilon_0 b(L-x)}{d} + \frac{K\varepsilon_0 bx}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[L+(K-1)x]$$

Now $U = \frac{q^2}{2C}$ $\qquad \{\because\ q = \text{constant}\}$

Since, $F = -\frac{dU}{dx}$, so we get

$$dU = \frac{q^2}{2}(-1)C^{-2}\frac{dC}{dx}$$

$$\Rightarrow \quad dU = -\frac{q^2}{2C^2}\frac{d}{dx}\left\{\frac{\varepsilon_0 b}{d}(L+(K-1)x)\right\}$$

$$\Rightarrow \quad dU = -\frac{q^2}{2\left(\frac{\varepsilon_0 b}{d}\right)^2}\frac{\frac{\varepsilon_0 b}{d}(K-1)dx}{[L+(K-1)x]^2}$$

$$\Rightarrow \quad F = -\frac{dU}{dx} = \frac{q^2(K-1)d}{2\varepsilon_0 b[L+(K-1)x]^2}$$

$$\Rightarrow \quad F = \frac{q^2(K-1)}{2\left(\frac{\varepsilon_0(bL)}{d}\right)L\left(1+\left(\frac{K-1}{L}\right)x\right)^2}$$

But $\frac{\varepsilon_0(bL)}{d} = C_0$

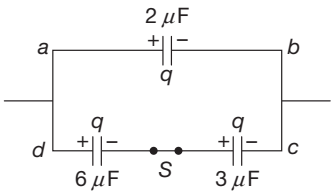
$$\Rightarrow \quad F = \frac{q^2(K-1)}{2C_0 L\left(1+\left(\frac{K-1}{L}\right)x\right)^2}, \text{ attractive}$$

Hence, the correct answer is (C).

76. Potential across the first capacitor is V, equal to emf of the battery.
Thus no charge will further flow through the circuit.
Hence, the correct answer is (D).

77. As soon as the switch is closed, for the charge to flow in the circuit (through S), there must exist a potential difference in the circuit. Here, we observe, that for the closed loop $abcd$, we have

$$\Delta V = -\frac{q}{2} + \frac{q}{3} + \frac{q}{6} = 0$$



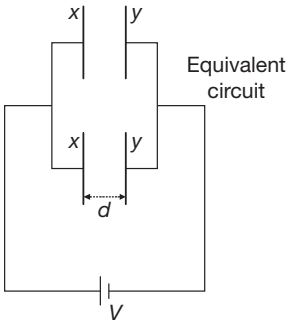
CHAPTER 2

Hence, no charge will flow through S, when closed.

Hence, the correct answer is (A).

78. Equivalent capacitance of the circuit

$$C = 2\left(\frac{\varepsilon_0 A}{d}\right)$$



Energy stored, $U = \frac{1}{2}CV^2$

$$\Rightarrow \quad U = \frac{1}{2}\left(\frac{2(\varepsilon_0 A)}{d}\right)V^2 = \frac{\varepsilon_0 A V^2}{d}$$

Hence, the correct answer is (B).

79. Starting from positive plate, at higher potential (say V_0) and going to negative plate at lower potential, we have potential after covering a

(a) distance d in air, is $V_1 = V_0 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

(b) further d in conductor, is still V_1, because field inside conductor is zero and hence potential stays constant.

(c) further d in air is $V_2 = V_1 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

(d) further d in dielectric is $V_3 = V_2 - Ed$ where $E = \frac{\sigma}{K\varepsilon_0}(< E_0)$

(e) last d in air is $V_4 = V_3 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

In all the cases except (b), potential decreases linearly with field and in (d) the field has lesser value than cases (a), (c) and (e). So OPTION (C) is correct.

Hence, the correct answer is (C).

80. $\frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$, where $t = \frac{d}{2}$

$$\Rightarrow \quad \frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}}$$

$$\Rightarrow \quad \frac{4}{3d} = \frac{1}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

$$\Rightarrow \quad 1 + \frac{1}{K} = \frac{3}{2}$$

$$\Rightarrow \quad K = 2$$

Hence, the correct answer is (A).

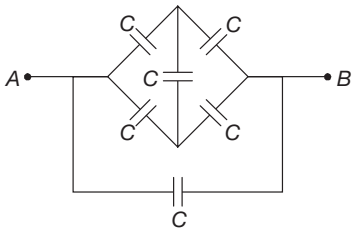
81. $F = \frac{q^2}{2\varepsilon_0 A}$

$$\Rightarrow \quad F \propto q^2$$

Since capacitor stays connected to the battery, so V = constant. Hence when separation is halved, C becomes twice. So, $q(= CV)$ becomes twice. Hence F becomes four times.

Hence, the correct answer is (C).

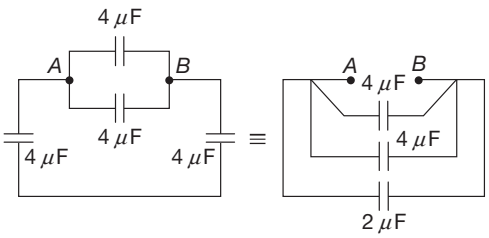
82. This circuit is a balanced Wheatstone Bridge with one capacitor connected in parallel to the bridge. So, the equivalent circuit is shown here. The net capacitance of the circuit between the points A and B is $2C$.



Hence, the correct answer is (D).

83. Since both free terminals are earthed, hence they being at same potential can be connected to give the following equivalent circuit.

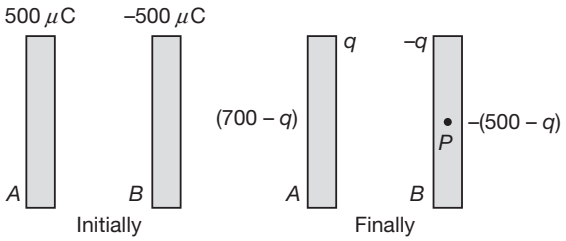
Now, all three being in parallel between A and B, will given net capacitance as $10\ \mu F$.



Hence, the correct answer is (D).

84. Charge attained by the plates of capacitor is

$$q_0 = CV_0 = (10\ \mu F)(50\ V) = 500\ \mu C$$



Now, when a charge $200\ \mu C$ is given to the positive plate of capacitor, then the net positive charge becomes $700\ \mu C$, while charge on negative plate is stil $-500\ \mu C$.

Let a charge q appear on inner surface of A, then surface of B facing A gets a charge $-q$. Such that a charge $(700-q)$ and $-(500-q)$ appears on outer surfaces of A and B respectively. Consider a point P inside plate B. Then net field at P, inside the conductor is zero. So

$$E_P = 0$$

$$\Rightarrow \quad \frac{700-q}{2\varepsilon_0 A}+\frac{q}{2\varepsilon_0 A}-\frac{q}{2\varepsilon_0 A}+\frac{500-q}{2\varepsilon_0 A}=0$$

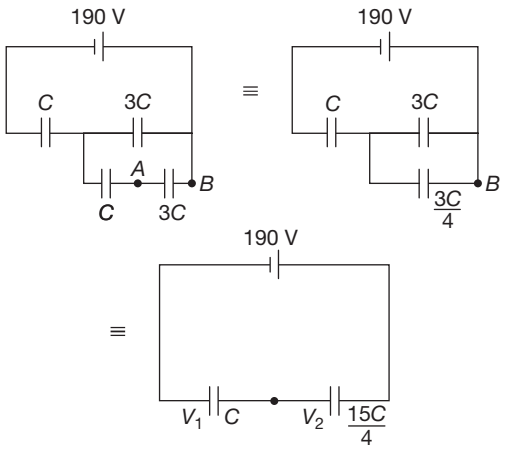
$$\Rightarrow \quad q = 600\ \mu C$$

So, potential difference between the plates is

$$\Delta V = \frac{q}{C}=\frac{600\ \mu C}{10\ \mu F}=60\ V$$

Hence, the correct answer is (B).

85. $C_{net}=\dfrac{15C}{19}$



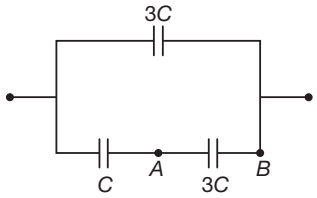
$$\Rightarrow \quad q = C_{net}V$$

$$\Rightarrow \quad q=\left(\frac{15C}{19}\right)(190)=150C$$

Now, potential across $\dfrac{15C}{4}$ is

$$V_2=\frac{q}{\left(\frac{15C}{4}\right)}=\frac{150C}{\frac{15C}{4}}=40\ V$$

So, potential across $3C$ and combination of $3C$ and C is also $40\ V$.

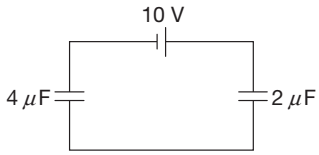


This $40\ V$ is distributed between C and $3C$ (in series), in the inverse ratio of their capacitance i.e., $3:1$. Hence

$$V_{AB} = 10\ V$$

Hence, the correct answer is (A).

86. The equivalent circuit is shown here for convenience



Potential across each capacitor is $5\ V$ (equally divided)

$$\Rightarrow \quad q = CV = (4\ \mu F)(5\ V) = 20\ \mu C$$

Hence, the correct answer is (B).

87. Using Kirchhoffs' Law, we get

$$8-\frac{q_1}{C_1}-2-\frac{q_3}{C_3}-3=0$$

Since, $C_1 = C_3 = 2\ \mu F$

$$\Rightarrow \quad q_1+q_2=6 \qquad \ldots(1)$$

Also

$$8-\frac{q_1}{C_1}+\frac{q_2}{C_2}-2=0$$

Since $C_1 = C_2 = 2\ \mu F$

$$\Rightarrow \quad q_1-q_2=12 \qquad \ldots(2)$$

Also, we observe that

$$-q_1-q_2+q_3=0$$

$$\Rightarrow \quad q_1+q_2=q_3 \qquad \ldots(3)$$

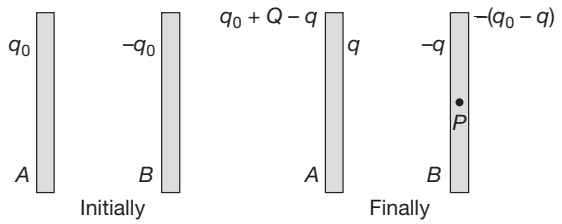
CHAPTER 2

Solving equations (1), (2) and (3) we get

$$q_1 = 6\ \mu C\ ,\ q_2 = -6\ \mu C \text{ and } q_3 = 0$$

Hence, the correct answer is (C).

88. Initial charge on plates of capacitor will be $q_0(say)$. Then $q_0 = CV$



When charge Q is given to the positive plate of capacitor, then the net positive charge becomes $q_0 + Q$ while charge on negative plate is still $-q_0$. Finally the charge distribution on the plates is also shown such that both inner faces have charge q and $-q$ respectively. When a point P is considered inside plate B, then net field at P, inside the conductor, is always zero. So

$$E_P = 0$$

$$\Rightarrow \quad \frac{1}{2\varepsilon_0 A}\left[(q_0 + Q - q) + q - q + (q_0 - q)\right] = 0$$

$$\Rightarrow \quad q_0 + Q + q_0 - 2q = 0$$

$$\Rightarrow \quad 2q = 2q_0 + Q$$

Since $q_0 = CV$, so we get

$$q = CV + \frac{Q}{2}$$

So, potential difference between the plates is

$$\Delta V = \frac{q}{C} = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

Hence, the correct answer is (C).

Problem Solving Technique(s)

We can also arrive at the result by using the concept that

(a) Charges on the surfaces facing each other are equal and opposite.
(b) Charges on the outer surfaces are equal.
(c) Capacitance of a parallel plate capacitor depends upon its shape i.e., A and d, but not on charge distribution.

So, we have

$$q_0 + Q - q = -(q_0 - q)\text{, where } q_0 = CV$$

$$CV + Q - q = -CV + q$$

$$\Rightarrow \quad 2q = 2CV + Q$$

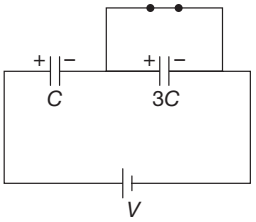
$$\Rightarrow \quad q = CV + \frac{Q}{2}$$

$$\Rightarrow \quad \Delta V = \frac{q}{C} = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

89. In the circuit $C_{net} = \frac{3C}{4}$

When key K was open, charge stored in the capacitors was $q_{\text{initial}} = \frac{3CV}{4}$

When key K is closed, capacitor $3C$ becomes short circuited and hence the new charge on C is $q_{\text{final}} = CV$



So, charge flowing is

$$\Delta q = q_{\text{final}} - q_{\text{initial}}$$

$$\Rightarrow \quad \Delta q = CV - \frac{3CV}{4} = \frac{CV}{4}$$

Hence, the correct answer is (C).

90. Energy stored in capacitor is, $\frac{1}{2}CV^2 = 3\text{ J}$

On connecting this capacitor to an uncharged capacitor, since charge distributes equally, hence both capacitors must be same capacitance.

Now common potential, $V' = \frac{CV + C(0)}{C + C} = \frac{V}{2}$

Total energy stored in two capacitors is

$$U' = \frac{1}{2}CV'^2 + \frac{1}{2}CV'^2 = C\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$$

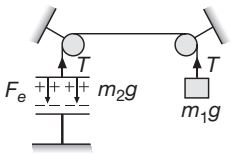
$$\Rightarrow \quad U' = \frac{3}{2} = 1.5\text{ J}$$

Hence, the correct answer is (A).

91. $C_1 + C_2 = \frac{25}{6}\left(\frac{C_1C_2}{C_1+C_2}\right)$

Hence, the correct answer is (A).

92.



For upper plate, we have

$$T = F_e + m_2g \qquad \ldots(1)$$

For m_1, we have

$$T = m_1g \qquad \ldots(2)$$

Equating (1) and (2), we get

$$m_1g = \frac{q^2}{2A\varepsilon_0} + m_2g$$

$$\Rightarrow \quad m_1 = \frac{q^2}{2A\varepsilon_0 g} + m_2$$

Hence, the correct answer is (C).

93. $V_{C_1} = 10 - 4 = 6$ V

$$V_{C_2} = 4 - 0 = 4 \text{ V}$$

Since, in series, we have

$$\frac{V_{C_1}}{V_{C_2}} = \frac{C_2}{C_1}$$

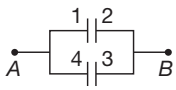
$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{V_{C_2}}{V_{C_1}} = \frac{4}{6} = \frac{2}{3}$$

Hence, the correct answer is (B).

94. It is a Wheatstone bridge

Hence, the correct answer is (A).

95.



$$C_{eq} = \frac{2\varepsilon_0 A}{d}$$

Hence, the correct answer is (B).

96. The capacitor is charged by a battery of 25 V. Let the magnitude of surface charge density on each plate be σ. Before inserting the dielectric slab, electric field strength between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{V}{d}$$

$$\Rightarrow \quad E_0 = \frac{\sigma}{\varepsilon_0} = \frac{25}{5\times10^{-3}} = 5000 \text{ NC}^{-1}$$

The capacitor is disconnected from the battery but charge on it will not change so that σ has the same value. When a dielectric slab of thickness 3 mm is placed between the plates, the thickness of air between the plates will be

$$t = 5 - 3 = 2 \text{ mm}$$

Electric field strength in air will have the same value (5000 NC^{-1}) but inside the dielectric, it will be

$$E = \frac{5000}{K} = \frac{5000}{10} = 500 \text{ NC}^{-1}$$

So potential difference is $V = E_{\text{air}}d_{\text{air}} + E_{\text{med}}d_{\text{med}}$

$$\Rightarrow \quad V = 5000\times(2\times10^{-3}) + 500\times(3\times10^{-3})$$

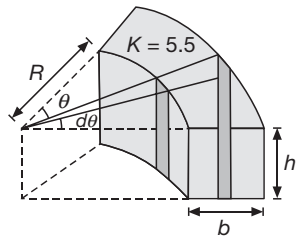
$$\Rightarrow \quad V = 11.5 \text{ V}$$

Hence, the correct answer is (C).

97. $dC = \frac{\varepsilon_0 KR d\theta \times h}{b}$

Since all small dC are in parallel, so

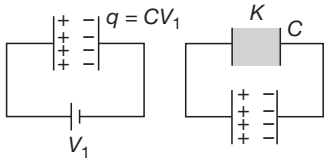
$$C_{eq} = \Sigma dC = \int_{\theta=0}^{\frac{\pi}{6}} dC$$



$$\Rightarrow \quad C_{eq} = \frac{6KhR}{b}\int_{\theta=0}^{\frac{\pi}{6}} d\theta = \frac{\pi}{6}\left(\frac{\varepsilon_0 KhR}{b}\right)$$

Hence, the correct answer is (A).

99.



$$V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$\Rightarrow \quad V = \frac{CV_1}{C + KC}$$

CHAPTER 2

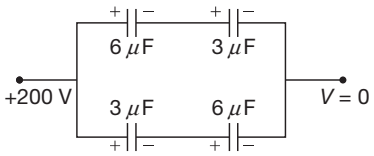
$$\Rightarrow \quad CV + KCV = CV_1$$

$$\Rightarrow \quad K = \frac{CV_1 - CV}{CV}$$

$$\Rightarrow \quad K = \frac{V_1 - V}{V}$$

Hence, the correct answer is (D).

100. **When switch is open**

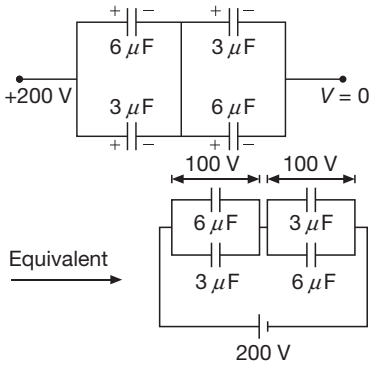


$q = C_{eq}V$ (for both branches)

$$\Rightarrow \quad q = 2 \times 200 = 400$$

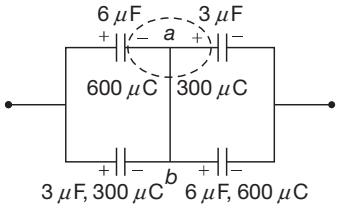
Hence charge on each capacitor is $400\ \mu C$

When switch is closed

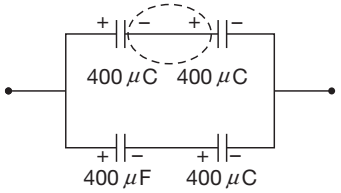


$q_6 = 600\ \mu C$ and $q_3 = 300\ \mu C$

Final charge on both plates is $300\ \mu C$



Initial charge on both plates is zero



Hence charge flown is 300 μC.

Hence, the correct answer is (A).

101. Since $F = kx$

$$\frac{q^2}{2A\varepsilon_0} = kx$$

$$\Rightarrow \quad x = \frac{q^2}{2A\varepsilon_0 k}$$

Hence, the correct answer is (A).

102. $C_{eq} = C_1 + C_2 = 4\pi\varepsilon_0(r_1 + r_2)$

Hence, the correct answer is (B).

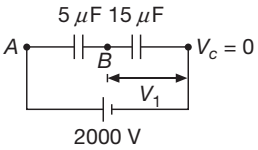
104. $\text{Loss} = -\Delta U = \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Since $V_1 = V$ and $V_2 = -V$

$$\Rightarrow \quad -\Delta U = \frac{C^2}{4C}(2V)^2 = CV^2$$

Hence, the correct answer is (B).

105.



$$V_1 = \frac{2000 \times 5}{20} = 500$$

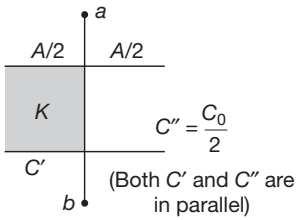
$$\Rightarrow \quad V_B - V_C = 500$$

Since $V_C = 0$

$$\Rightarrow \quad V_B = 500$$

Hence, the correct answer is (D).

106. Equivalent capacitance of Arrangement-1



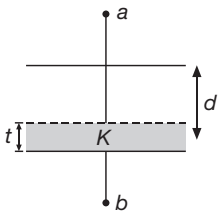
$$C' = \frac{\varepsilon_0 A}{2d}K = \frac{C_0 K}{2}$$

$$\Rightarrow \quad C_1 = C' + C''$$

$$\Rightarrow \quad C_1 = \frac{C_0}{2} + \frac{C_0 K}{2} = \frac{C_0}{2}(1 + K)$$

Equivalent capacitance of Arrangement-2

$$C_2 = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$



Both are same, so

$$\frac{C_0}{2}(K+1) = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{2d}(2+1) = \frac{\varepsilon_0 A}{d - t + \frac{t}{2}}$$

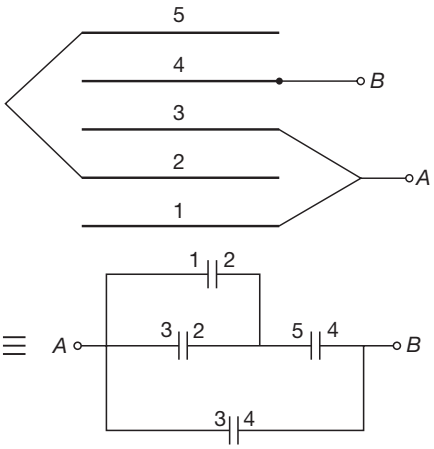
$$\Rightarrow \quad 3d - \frac{3t}{2} = 2d$$

$$\Rightarrow \quad d = \frac{3t}{2}$$

$$\Rightarrow \quad t = \frac{2d}{3}$$

Hence, the correct answer is (A).

107.



The capacity of each capacitor is, $C_0 = \frac{\varepsilon_0 A}{d}$

From figure it is clear that $C_{eq} = \frac{5}{3}C_0 = \frac{5}{3}\left(\frac{\varepsilon_0 A}{d}\right)$

Hence, the correct answer is (C).

108. Before the switch S is closed, then

$$q_i = C_{eq} V$$

$$\Rightarrow \quad q_i = 5 \times 4 = 20\ \mu\text{C}$$

When the switch S is closed, then

$$q_f = CV$$

$$\Rightarrow \quad q_f = 10 \times 4 = 40\ \mu\text{C}$$

$$\Rightarrow \quad W_{\text{battery}} = (\Delta q) V_{\text{battery}} = (q_f - q_i) V_{\text{battery}}$$

$$\Rightarrow \quad W_{\text{battery}} = 20 \times 4 = 80\ \mu\text{J}$$

Hence, the correct answer is (C).

109. Since between the plates, we have

$$E = \frac{2Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{3Q}{2A\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{3}{2}\frac{Q}{Cd} \qquad \left\{\because\ C = \frac{\varepsilon_0 A}{d}\right\}$$

$$\Rightarrow \quad Ed = \frac{3Q}{2C} = V$$

Also, force on one plate due to other is

$$F = Q_2 E_1$$

$$\Rightarrow \quad F = (-Q)\left(\frac{2Q}{2\varepsilon_0 A}\right)$$

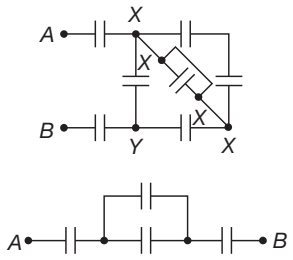
$$\Rightarrow \quad F = -\frac{Q^2}{A\varepsilon_0} = -\frac{Q^2}{Cd}$$

$$\text{Energy} = \frac{1}{2}\varepsilon_0 E^2 Ad$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0\left(\frac{3Q}{2Cd}\right)^2 Ad = \frac{9}{8}\frac{Q^2}{C}$$

Hence, the correct answer is (D).

110. When no capacitor, resistor or battery exist between any two points in a branch, then those two points are taken at same potential. Using this concept to solve the circuit, we get



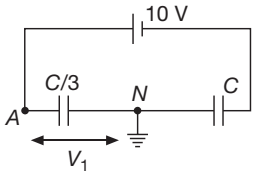
$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{2C}{5}$$

Hence, the correct answer is (B).

112. Earthing a single point in a circuit does not change the charge or current flowing in the circuit, So,

CHAPTER 2



$$V_A - V_N = \frac{10 \times C}{C + \frac{C}{3}} = \frac{30C}{4C} = 7.5$$

$\Rightarrow \quad V_A - 0 = 7.5$

$\Rightarrow \quad V_A = 7.5 \text{ V}$

Hence, the correct answer is (B).

113. $W = \Delta U = U_f - U_i = \frac{Q^2}{2C_f} - \frac{Q^2}{2C_i} = \frac{Q^2}{2}\left(\frac{2d}{\varepsilon_0 A} - \frac{d}{\varepsilon_0 A}\right)$

$\Rightarrow \quad W = \frac{Q^2}{2}\frac{d}{\varepsilon_0 A} = \frac{C^2V^2}{2C} = \frac{CV^2}{2} = \frac{\varepsilon_0 A V^2}{2d}$

Hence, the correct answer is (A).

114. Energy density $= \frac{1}{2}\varepsilon_0 E^2$ and $E = \frac{V}{d}$

Hence, the correct answer is (C).

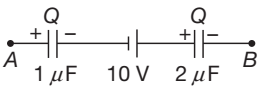
115. $Q = CV = 120\ \mu\text{C}$ and

$Q' = C'V' = 50 \times 12 = 600\ \mu\text{C}$

$\Rightarrow$ Charge Flown $= \Delta Q = 600 - 120 = 480\ \mu\text{C}$

Hence, the correct answer is (C).

116.



Applying Kirchhoff's Voltage Law (KVL) between A and B, we get

$$V_A - \frac{Q}{1\times10^{-6}} + 10 - \frac{Q}{2\times10^{-6}} - V_B = 0$$

$\Rightarrow \quad (V_A - V_B) + 10 = \frac{3Q}{2\times10^{-6}}$

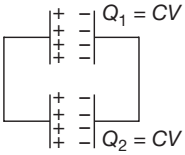
$\Rightarrow \quad Q = 10\ \mu\text{C}$

$\Rightarrow \quad V_{1\,\mu\text{C}} = \frac{Q}{C} = \frac{10\ \mu\text{C}}{1\ \mu\text{F}} = 10 \text{ V}$ and

$V_{2\,\mu\text{C}} = \frac{Q}{C} = \frac{10\ \mu\text{C}}{2\ \mu\text{F}} = 5 \text{ V}$

Hence, the correct answer is (C).

117.



Since loss in energy is given by

$$-\Delta U = \frac{C_1 C_2}{2(1+C_2)}(V_1 - V_2)^2$$

$\Rightarrow \quad -\Delta U = \frac{C^2}{2\times 2C}[V - (-V)]^2$

$\Rightarrow \quad -\Delta U = \frac{C}{4} \times 4V^2 = CV^2$

Hence, the correct answer is (B).

119. $i = \frac{dq}{dt}$

$\Rightarrow \quad q = it + a$, where $a =$ constant

Since $V = \frac{q}{C}$

$\Rightarrow \quad V = \frac{it + a}{C}$

So, V is proportional to time

Hence, the correct answer is (D).

120. $C_{eff} = \frac{\varepsilon_0 A}{d}$

Since effective capacitance between plates A and E is zero, so

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{2d}V^2$$

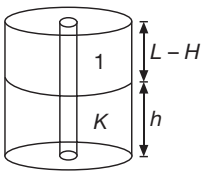
Hence, the correct answer is (C).

121. Gravitational force $= mg = \rho\pi(b^2 - a^2)gh$

Net upward force $F = \frac{dU}{dh} = \frac{1}{2}V^2\frac{dC}{dh}$

where h is the height of liquid.

Now let us calculate C as a function of h.



Since, $C_{eq} = C_{Air} + C_K = \frac{2\pi\varepsilon_0(L-h)}{\log_e\left(\frac{b}{a}\right)} + \frac{2\pi\varepsilon_0 Kh}{\log_e\left(\frac{b}{a}\right)}$

$$\Rightarrow \quad C = \frac{2\pi\varepsilon_0 (L + (K-1)h)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad F = \frac{1}{2}V^2 \times \frac{dC}{dh} = \frac{1}{2}V^2 \times \frac{2\pi\varepsilon_0 (K-1)}{\log_e\left(\frac{b}{a}\right)}$$

Since $F = mg$, so we get

$$\frac{\frac{1}{2}V^2 \times 2\pi\varepsilon_0 (K-1)}{\log_e\left(\frac{b}{a}\right)} = \rho\pi\left(b^2 - a^2\right)gh$$

$$\Rightarrow \quad h = \frac{\pi\varepsilon_0 V^2 (K-1)}{\rho\pi\left(b^2 - a^2\right)g \log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad h = \frac{\varepsilon_0 V^2 (K-1)}{\rho\left(b^2 - a^2\right)g \log_e\left(\frac{b}{a}\right)}$$

Hence, the correct answer is (B).

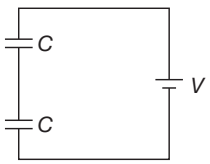
122. $U = \frac{1}{2}qV$

Work done, $W = qV$

So, Loss of energy $= qV - \frac{1}{2}qV = \frac{qV}{2}$

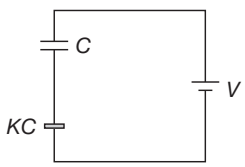
Hence, the correct answer is (D).

123.



Initially, energy stored is

$$U_i = \frac{1}{2}\left(\frac{C}{2}\right)V^2 = \frac{CV^2}{4}$$



Finally, energy stored is

$$U_f = \frac{1}{2}\left(\frac{C \times KC}{C + KC}\right)V^2 = \frac{KCV^2}{2(1+K)}$$

Since, $W = \Delta U = U_f - U_i$

$$\Rightarrow \quad W = \frac{KCV^2}{2(1+K)} - \frac{CV^2}{4} = \frac{CV^2}{2}\left(\frac{K}{1+K} - \frac{1}{2}\right)$$

$$\Rightarrow \quad W = \frac{CV^2}{2}\left(\frac{2K - 1 - K}{2(1+K)}\right) = \frac{CV^2}{4}\left(\frac{K-1}{K+1}\right)$$

Hence, the correct answer is (B).

124. Total charge $= 1.25\ \mu C + 0.75\ \mu C = 2\ \mu C$

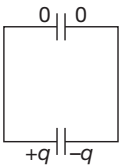
Since, $q_1' : q_2' = R_1 : R_2 = 1 : 2$

$$\Rightarrow \quad q_1' = \frac{1}{3} \times 2 = \frac{2}{3}\ \mu C$$

and $q_2' = \frac{2}{3} \times 2 = \frac{4}{3}\ \mu C$

Hence, the correct answer is (A).

125.



Energy loss $= -\Delta U = \frac{1}{2}\left(\frac{C^2}{C+C}\right)V^2 = \frac{1}{2}\frac{C}{2} \times \frac{q^2}{C^2}$

$$\Rightarrow \quad -\Delta U = \frac{q^2}{4C} = \frac{q^2 d}{4\varepsilon_0 A}$$

Hence, the correct answer is (A).

126. $U_i = \frac{Q^2}{2C_i}$

$$U_f = \frac{Q^2}{2C_f}$$

As surface was deformed in such a way that charge on original surface are coming closer or moving perpendicular to electric force acting on them, so total energy of foil increases.

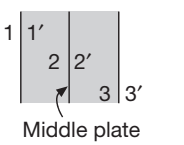
$$\Rightarrow \quad U_f > U_i$$

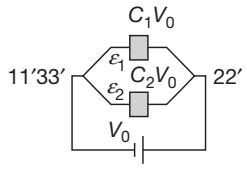
$$\Rightarrow \quad \frac{Q^2}{2C_f} > \frac{Q^2}{2C_i}$$

$$\Rightarrow \quad C_i > C_f$$

Hence, the correct answer is (B).

127.





CHAPTER 2

Total charge on 2 and 2′ plate is

$$q = C_{eq}V = \left(\frac{\varepsilon_1\varepsilon_0 A}{d_1} + \frac{\varepsilon_2\varepsilon_0 A}{d_2}\right)V$$

$$\Rightarrow \quad \sigma = \frac{q}{A} = \varepsilon_0 V\left(\frac{\varepsilon_1}{d_1} + \frac{\varepsilon_2}{d_2}\right)$$

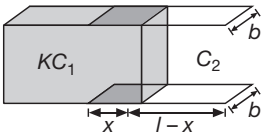
Hence, the correct answer is (A).

128. $V_{\text{common}} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{C_1 \times V + C_2 \times 0}{C_1 + C_2}$

$$\Rightarrow \quad V_{\text{common}} = \frac{C_1V}{C_1 + C_2}$$

Hence, the correct answer is (B).

129.



$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 bxK}{d} + \frac{\varepsilon_0(\ell - x)b}{d}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 b}{d}(\ell b + x(K-1))$$

$$\Rightarrow \quad \frac{dC}{dt} = \frac{\varepsilon_0 b}{d}(K-1)\frac{dx}{dt} = \frac{\varepsilon_0 b}{d}(K-1)v$$

Hence, the correct answer is (B).

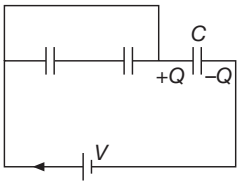
130. Charge in the circuit flows only when potential difference across C_1 is either greater or less than that across C_2

$$\Rightarrow \quad \frac{q_1}{C_1} \neq \frac{q_2}{C_2}$$

$$\Rightarrow \quad q_1C_2 \neq q_2C_1$$

Hence, the correct answer is (D).

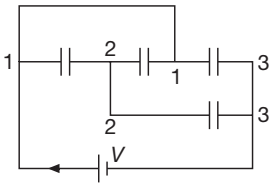
131. When K_1 is closed



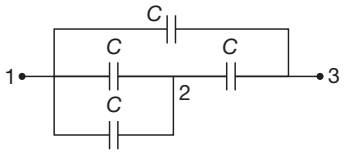
$$Q = CV$$

So, initial Energy is $U_i = \dfrac{1}{2}CV^2$

When K_2 is also closed



Equivalent circuit is shown here.



$$C_{eq} = C + \frac{2C}{3} = \frac{5C}{3}$$

So, final Energy is $U_f = \dfrac{1}{2} \times \dfrac{5C}{3}V^2 = \dfrac{5}{6}CV^2$

Charge supplied by battery after closing K_2 is

$$\Delta q = \frac{5}{3}CV - CV = \frac{2}{3}CV$$

Energy supplied by battery is $(\Delta q)V_{battery}$

Also energy supplied by battery $= U_f - U_i + \Delta H$

$$\Rightarrow \quad \frac{2}{3}CV^2 = \frac{5}{6}CV^2 - \frac{1}{2}CV^2 + \Delta H$$

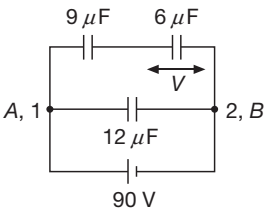
$$\Rightarrow \quad \Delta H = \frac{1}{3}CV^2$$

Hence, the correct answer is (C).

132. $E = \dfrac{V}{d}$

Hence, the correct answer is (A).

133.



$$V' = \frac{90 \times 9}{15} = 54$$

$$\Rightarrow \quad q = CV' = 6 \times 54 = 324\ \mu\text{C}$$

Hence, the correct answer is (C).

134. $C = \dfrac{q}{V}$. If potential difference between plates is zero then capacitance will be infinite.

Hence, the correct answer is (D).

135. $C_K = \dfrac{\varepsilon_0 A}{d - b + \dfrac{b}{K}}$

If we set $b = 0$, we get

$$C_K = \frac{\varepsilon_0 A}{d} = C$$

If $C_K = 2C$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d - b + \dfrac{b}{K}} = \frac{2\varepsilon_0 A}{d}$$

$$\Rightarrow \quad K = \frac{2b}{2b - d}$$

Since, we know that $K > 0$ and $b \le d$

$$\Rightarrow \quad K = \frac{2b}{2b - d} \text{ and}$$

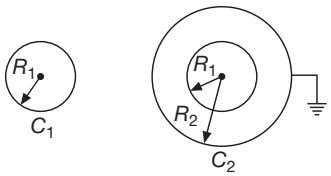
$$2b - d > 0$$

$$\Rightarrow \quad b > \frac{d}{2}$$

$$\Rightarrow \quad \frac{d}{2} < b \le d$$

Hence, the correct answer is (B).

136.



$$C_1 = 4\pi\varepsilon_0 R_1$$

and $C_2 = 4\pi\varepsilon_0 \left(\dfrac{R_1 R_2}{R_2 - R_1} \right)$

Given that $C_2 = nC_1$

$$\Rightarrow \quad \frac{R_2 R_1}{R_2 - R_1} = nR_1$$

$$\Rightarrow \quad \frac{\dfrac{R_2}{R_1}}{\dfrac{R_2}{R_1} - 1} = n$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \frac{n}{n-1}$$

Hence, the correct answer is (B).

137. For Capacitor 1, we have $\begin{cases} C = 1\ \mu\text{F} \\ V_{max} = 16\ \text{KV} \\ Q_{max} = 16\ \text{mC} \end{cases}$

For Capacitor 2, we have $\begin{cases} C = 2\ \mu\text{F} \\ V_{max} = 4\ \text{KV} \\ Q_{max} = 8\ \text{mC} \end{cases}$

For series combination of capacitors, we have

$$\begin{cases} C_{eq} = \dfrac{1 \times 2}{1 + 2} = \dfrac{2}{3}\ \mu\text{F} \\ Q_{max} = 8\ \text{mC} \\ \text{and } V_{max} = \dfrac{Q_{max}}{C_{eq}} = \dfrac{8\ \text{mC}}{\dfrac{2}{3}\ \mu\text{F}} = 12\ \text{KV} \end{cases}$$

Hence, the correct answer is (B).

138. All the $3C$ capacitors are in series, so

$$\frac{1}{3C} + \frac{1}{3C} + \frac{1}{3C} = \frac{1}{C_{eq}}$$

$$\Rightarrow \quad C_{eq} = C$$

Since C and C are in parallel

$$\Rightarrow \quad C_{eq} = 2C$$

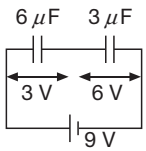
Hence, the correct answer is (B).

139. Initially

$$q_{6\ \mu\text{F}} = 18\ \mu\text{C}$$

$$q_{1\ \mu\text{F}} = 6\ \mu\text{C}$$

$$q_{2\ \mu\text{F}} = 12\ \mu\text{C}$$

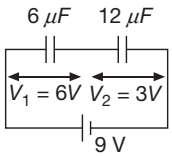


Finally

$$q_{6\ \mu\text{F}} = 36\ \mu\text{C}$$

$$q_{2\ \mu\text{F}} = 6\ \mu\text{C}$$

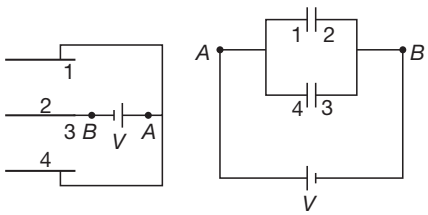
$$q_{1\ \mu\text{F}} = 3\ \mu\text{C}$$



Hence charge on C_1 increases but on C_2 decreases.

Hence, the correct answer is (C).

140.

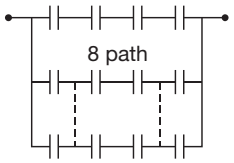


$$U = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 = \frac{\varepsilon_0 A V^2}{d}$$

Hence, the correct answer is (B).

CHAPTER 2

141.



So, $N = 4 \times 8 = 32$

Hence, the correct answer is (D).

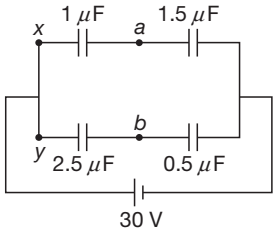
142. Since, $Q = CV$

So, for $Q = 5C$, we get $V = 2.5$ V

Since the capacitor is charged at a steady rate, hence it should be a straight line graph.

Hence, the correct answer is (A).

143.



$$V_x - V_a = \frac{30 \times 1.5}{2.5} = 18 \quad \ldots(1)$$

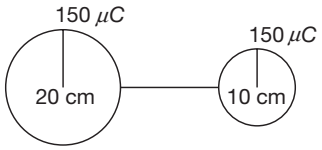
$$V_y - V_b = \frac{30 \times 0.5}{3} = 5 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$V_a - V_b = -13 \text{ V}$$

Hence, the correct answer is (C).

144.

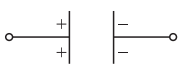


Since $V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{150 \times 2 \times 10^{-6}}{4\pi\varepsilon_0 (r_1 + r_2)}$

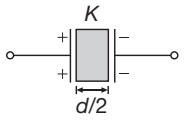
$$\Rightarrow \quad V = \frac{300 \times 10^{-6} \times 9 \times 10^9}{30 \times 10^{-2}} = 9 \times 10^6 \text{ V}$$

Hence, the correct answer is (A).

146.



$$V = \frac{q}{C} = \frac{qd}{\varepsilon_0 A}$$



$$V' = \frac{q}{C} = \frac{q\left(\frac{d - t + t}{K}\right)}{\varepsilon_0 A}$$

$$\Rightarrow \quad V' = \frac{q}{\varepsilon_0 A}\left(d - \frac{d}{2} + \frac{d}{2 \times 2}\right)$$

$$\Rightarrow \quad V' = \frac{qd}{\varepsilon_0 A}\left(\frac{1}{2} + \frac{1}{4}\right)$$

$$\Rightarrow \quad V' = \frac{qd}{\varepsilon_0 A}\frac{3}{4}$$

$$\Rightarrow \quad V' = \frac{3}{4}V$$

Hence, the correct answer is (D).

147. Let, $Q_0 = C_0 V_0 =$ Initial charge on capacitor

Battery connected means V remains unchanged

As separation is doubled, so $C = \frac{C_0}{2}$

$$\Rightarrow \quad U_1 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{C_0 V_0^2}{4}$$

When battery disconnected, charge remains unchanged

As separation is doubled, so $C = \frac{C_0}{2}$

$$\Rightarrow \quad U_2 = \frac{Q_0^2}{2C} = \frac{C_0^2 V_0^2}{2\left(\frac{C_0}{2}\right)} = C_0 V_0^2$$

$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{1}{4}$$

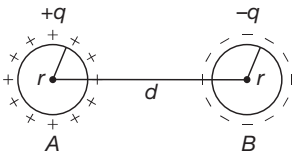
Hence, the correct answer is (A).

148. $C_{eq} = C_1 + C_2 + C_3$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d \times 2} = \frac{\varepsilon_0 A}{3 \times 3d} = \frac{11\varepsilon_0 A}{18d}$$

Hence, the correct answer is (C).

150.

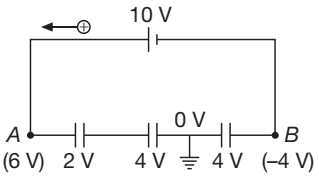


Since $V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \left(-\frac{Kq}{r}\right)\right) = \frac{2q}{4\pi\varepsilon_0 r}$

$$\Rightarrow \quad C = \frac{q}{V_A - V_B} = 2\pi\varepsilon_0 r$$

Hence, the correct answer is (B).

151.



Hence, the correct answer is (A).

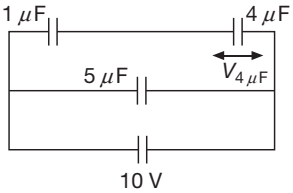
153. $Q_1 = C_1 V$ and $Q_2 = C_2 V$

$$\Rightarrow \quad \frac{50}{150} = \frac{\varepsilon_0}{K\varepsilon_0}$$

$$\Rightarrow \quad K = 3$$

Hence, the correct answer is (D).

156.



$2\ \mu F$ and $2\ \mu F$ are reduced to 1 capacitor of $1\ \mu F$ and $3\ \mu F$ and $1\ \mu F$ are reduced to $4\ \mu F$.

$$\Rightarrow \quad V_{4\,\mu F} = \frac{10 \times 1}{5} = 2$$

(potential difference across $4\ \mu F$ capacitor is calculated by voltage division Rule)

Since, $V_{5\,\mu F} = 10$ V

(potential difference across $5\ \mu F$ is same as Battery)

$$\Rightarrow \quad \frac{V_{1\,\mu F}}{V_{5\,\mu F}} = \frac{2}{10} = \frac{1}{5}$$

Hence, the correct answer is (C).

157. When switch is open then

$$Q_1 = \frac{8 \times 4}{12} \times 40 = \frac{320}{3}\ \mu C$$

When switch is closed, then

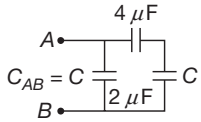
$$Q_2 = 8 \times 40 = 320\ \mu C$$

Charge flown through AB is

$$\Delta q = q_2 - q_1 = 320 - \frac{320}{3} = \frac{640}{3}\ \mu C$$

Hence, the correct answer is (B).

158.



$$\frac{4C}{4+C} + 2 = C$$

$$\Rightarrow \quad C = 4\ \mu F$$

Hence, the correct answer is (A).

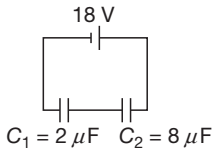
160. $U = \frac{1}{2} C V_0^2 = \frac{1}{2} \frac{\varepsilon_0 A}{x} V_0^2$

$$\frac{dU}{dt} = -\frac{V_0^2 \varepsilon_0 A}{2} \frac{1}{x^2} \left(\frac{dx}{dt} \right) = -\frac{V_0^2 \varepsilon_0 A v}{2x^2}$$

Hence, the correct answer is (A).

161. $V_1 = \frac{18 \times C_2}{C_1 + C_2} = \frac{18 \times 8}{10} = 14.6$ V

and $V_2 = \frac{18 \times 1}{10} = 1.8$ V



Hence, the correct answer is (D).

162. $C_{eq} = \left(\frac{\varepsilon r_1 + \varepsilon r_2}{2} \right) C$

$$\Rightarrow \quad C_{eq} = \left(\frac{4+6}{2} \right) (1\ \mu F) = 5\ \mu F$$

Hence, the correct answer is (A).

164. $C_1 = 10\ \mu F$, $C_2 = ?$

$V_1 = 100$ V, $V_2 = 0$

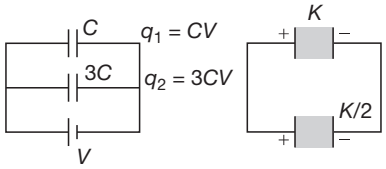
and $V_{common} = 40$ V

Since $V_{common} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$\Rightarrow \quad C_2 = 15\ \mu F$$

Hence, the correct answer is (A).

165.



CHAPTER 2

Common potential is given by

$$V_{\text{common}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{CV + 3CV}{KC + \frac{K}{2}3C}$$

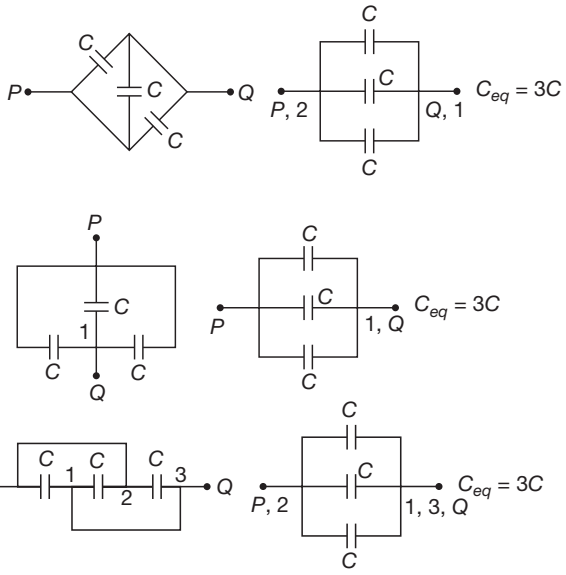
$$\Rightarrow \quad V_{\text{common}} = \frac{4CV}{\frac{5KC}{2}} = \frac{8V}{5K}$$

Hence, the correct answer is (C).

166. $\text{Loss} = -\Delta U = \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

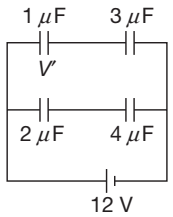
Hence, the correct answer is (C).

167.



Hence, the correct answer is (C).

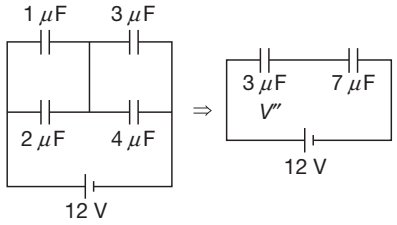
168. **When only S_1 is closed**



$$V' = \frac{12 \times 3}{4} = 9V$$

$$\Rightarrow \quad q_{1\,\mu F} = 1 \times 9 = 9\ \mu C$$

When both S_1 and S_2 are closed



$$V'' = \frac{12 \times 7}{10} = \frac{42}{5}\ \text{V}$$

$$\Rightarrow \quad q'_{1\,\mu F} = 1 \times \frac{42}{5} = \frac{42}{5}\ \mu C$$

$$\Rightarrow \quad \frac{q_1}{q'_1} = \frac{9}{\frac{42}{5}} = \frac{45}{42} = \frac{15}{14}$$

Hence, the correct answer is (C).

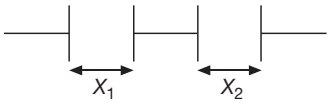
169. $I_0 = \dfrac{E_0}{R}$ Since both capacitors are charged to same potential difference hence current at $t = 0$ will be same in both circuits.

Time taken in 50% discharging is $t = 0.693RC$

So, $1\ \mu F$ will take less time compared to $2\ \mu F$

Hence, the correct answer is (C).

171. When two capacitors are in series



$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{x_1}{\varepsilon_0 A} + \frac{x_2}{\varepsilon_0 A} = \frac{x_1 + x_2}{\varepsilon_0 A}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{x_1 + x_2}$$

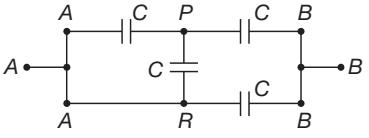
$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{\text{Sum of separations between plates}}$$

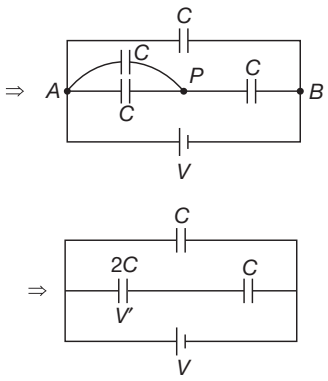
Now in given arrangement, the capacitors are in series and sum of separations $= a - b$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{a - b}$$

Hence, the correct answer is (B).

172.





Potential across $2C$ be V', then

$$V' = \frac{V \times C}{3C} = \frac{V}{3}$$

$$\Rightarrow \quad q = \frac{CV}{3}$$

Hence, the correct answer is (B).

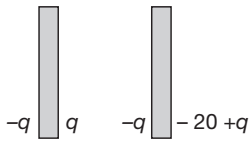
173. Battery disconnected

$\Rightarrow$ Q = constant

$\Rightarrow$ As d increases, C decreases. So, U increases, V decreases

Hence, the correct answer is (A).

174. Since outer surfaces both have equal charge, so

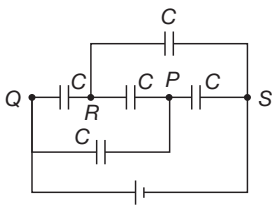


$$-q = -20 + q$$

$$\Rightarrow \quad q = \frac{20}{2} = 10\ \mu C$$

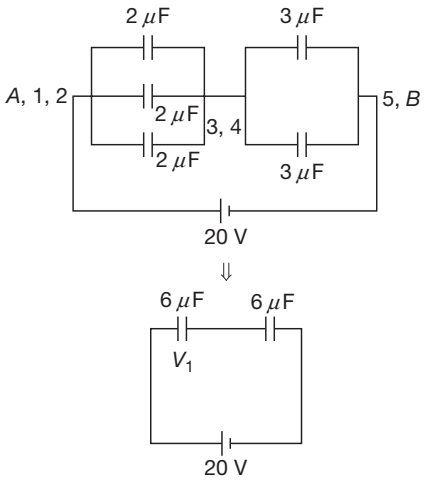
Hence, the correct answer is (A).

175. Capacitor between P and R will not acquire any charge.



Hence, the correct answer is (A).

178.



$$V_1 = 10\ V$$

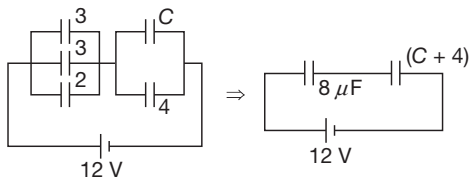
$$\Rightarrow \quad q_{C_0} = 10 \times 2 = 20\ \mu C$$

Hence, the correct answer is (B).

179. $U_i = \frac{1}{2} C_1 V_0^2$ and $U_f = \frac{C_1^2 V_0^2}{2(C_1 + C_2)}$

Hence, the correct answer is (A).

180. Using point potential method, the circuit can be redrawn as



Since, $q = C_{eq} V$

$$\Rightarrow \quad 48 = \frac{8 \times (C+4)}{8 + 4 + C} \times 12$$

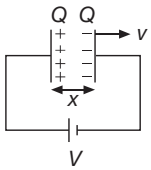
$$\Rightarrow \quad 48 + 4C = 8C + 32$$

$$\Rightarrow \quad 16 = 4C$$

$$\Rightarrow \quad C = 4$$

Hence, the correct answer is (A).

181.



CHAPTER 2

Since $U = \frac{1}{2}\frac{\varepsilon_0 A}{x}V^2$

$$\Rightarrow \quad \frac{dU}{dt} = \frac{1}{2}\varepsilon_0 A V^2 \frac{d}{dt}^{(x^{-1})}$$

$$\Rightarrow \quad \frac{dU}{dt} = \frac{1}{2}\left(\frac{\varepsilon_0 A}{x^2}\right)\left(\frac{dx}{dt}\right)V^2$$

$$\Rightarrow \quad \frac{dU}{dt} \propto \frac{1}{x^2} \propto x^{-2}$$

Hence, the correct answer is (A).

182. $C_{eq} = C_1 + C_2 = 2 + 2 = 4\ \mu F$

$$\Rightarrow \quad q = C_{eq}V = 4\ \mu F \times 8 = 32\ \mu C$$

Hence, the correct answer is (D).

183. $q_1 = C_{eq}V$

$$\Rightarrow \quad q_1 = \frac{10 \times 20 \times 10^{-6}}{30} \times 3 \times 10^3 = 2 \times 10^{-2}$$

$$\Rightarrow \quad q_1 = 20000 \times 10^{-6} = 20000\ \mu C$$

Hence, the correct answer is (B).

Multiple Correct Choice Type Questions

1. The capacitances

$$C_1 = \frac{K_1\varepsilon_0 A}{\left(\frac{d}{2}\right)} \text{ and } C_2 = \frac{K_2\varepsilon_0 A}{\left(\frac{d}{2}\right)}$$

The capacitors are in series. So, the effective capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2K_1K_2}{K_1 + K_2}\left(\frac{\varepsilon_0 A}{d}\right)$$

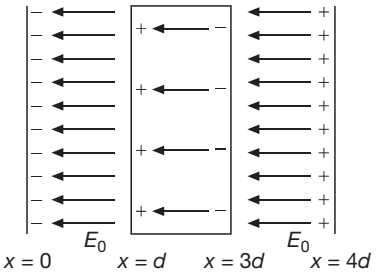
So, the **OPTIONS (A), (B), (C)** and **(D)** all are correct

Hence, (A), (B), (C) and (D) are correct.

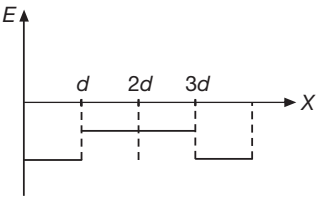
3. Since P and R are earthed and so they can be assumed to be connected to each other. Hence, $'P+Q'$ and $'Q+R'$ are two capacitors with the same potential difference. If Q is closer to P than to R then the capacitance C_{PQ} is $> C_{QR}$. The upper surface of Q will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2 so, there will be a net upward force on Q which can balance its weight.

Hence, (B) and (D) are correct.

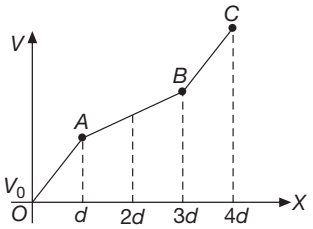
4. The magnitude and direction of electric field in different regions are shown in figure. The direction of the electric field remains the same. Hence, **OPTION (B)** is correct.



Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 4d$. Therefore, **OPTION (C)** is also correct.



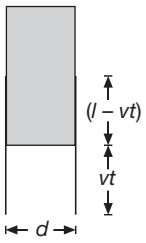
The variation of electric field (E) and potential (V) with x will be as follows :



8. Let the length, width, thickness and dielectric constant of the slab be ℓ, b, d and K respectively. The initial capacitance of the capacitor is given by

$$C_0 = \frac{\varepsilon_0 K \ell b}{d}$$

When the dielectric is pulled out of the capacitor with a speed v then, at time t, the length of the slab that has been pulled out is vt. At this instant, the capacitor may be considered as a parallel combination of two capacitors, an air capacitor having area $b(vt)$ and a dielectric capacitor having area $b(\ell - vt)$.



Hence, the net capacitance as a function of time will be equal to

$$C = \frac{\varepsilon_0 (vt) b}{d} + \frac{\varepsilon_0 K b (\ell - vt)}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}\left(vt + K(\ell - vt)\right)$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}\left(K\ell - (K-1)vt\right)$$

It is observed that the net capacitance C varies with time t linearly and slope of this line is negative and has positive intercept on C-axis. Hence, the **OPTION (D)** is correct.

When the battery is disconnected, the charge on the capacitor remains constant and at time t, the potential difference across the capacitor will be $V = \frac{q}{C}$ and hence the curve between C and V is a rectangular hyperbola as shown in **OPTION (C)**. Now since we observe that C decreases with time as a result of which V also increases from initial value $V_0 = \frac{q}{C_0}$. So, **OPTION (A)** is also correct.

Let the potential difference between the plates of the capacitor be V, then the energy stored in it is equal to $U = \frac{1}{2}qV$, where q remains constant. Hence, energy $U \propto V$. So, the curve between U and V will be a straight line passing through origin but since the minimum possible non-zero value of C is $C_{MIN} = \frac{\varepsilon_0 \ell b}{d}$, therefore, the curve must start from this minimum value of C. Hence, the **OPTION (B)** is also correct.

Hence, (A), (B), (C) and (D) are correct.

9. Since the given circuit is a series combination of two identical dielectric capacitors, therefore, charges on these two capacitors are equal. So, when the dielectric slab of the capacitor B is pulled out, its capacitance decreases, as a result of which the equivalent capacitance of the series combination also decreases. Hence the charge on series combination begins to decrease as a result of which an anticlockwise current starts flowing through the circuit or a charge flows from a to b. Hence, the **OPTION (A)** is correct.

Also the capacitors are in series with each other, so the charge on two capacitors remains same or both the capacitors possess a common charge at all instants and hence, the **OPTION (B)** is incorrect.

As studied and discussed earlier, we know that a charged capacitor always attracts the dielectric in the space between the plates. So, capacitor B exerts a force of attraction on the dielectric slab. On pulling the dielectric slab out of the capacitor, a work has to be done by the external force F. But if the slab is pulled out slowly then a very small current will flow through the circuit. In that case, heat generated in the circuit will be negligible. Hence, it is not necessary that significant amount of heat is produced in the circuit. Therefore, the **OPTION (C)** is incorrect.

It is also observed that an anticlockwise current flows in the circuit during the process i.e., the battery is being charged which further means that the internal energy of the battery increases. Therefore, the **OPTION (D)** is also correct.

Hence, (A) and (D) are correct.

10. Let C_0 be the capacitance initially and C be the capacitance finally . Then $C_0 = \frac{\varepsilon_0 A}{d}$

Since, $Q = C_0 V$

$$\Rightarrow \quad Q = \frac{\varepsilon_0 A V}{d}$$

Further $E_0 = \frac{V}{d}$ and $E = \frac{E_0}{K}$

$$\Rightarrow \quad E = \frac{V}{Kd}$$

Also, if U_i is the initial energy, then $U_i = \frac{1}{2}C_0 V^2$

After the introduction of slab if U_f be the final energy, then

$$U_f = \frac{1}{2}CV_{slab}^2 = \frac{1}{2}(KC_0)\left(\frac{V}{K}\right)^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}\frac{C_0 V^2}{K}$$

$$\Rightarrow \quad \Delta U = U_2 - U_1$$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C_0 V^2\left(\frac{1}{K} - 1\right)$$

Since work done = Decrease in Potential Energy

$$\Rightarrow \quad W = -\Delta U$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{\varepsilon_0 A V^2}{d}\left(1 - \frac{1}{K}\right)$$

Hence, (A), (C) and (D) are correct.

11. Capacitance without water is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 0.01}{0.01} = \varepsilon_0$$

$$\Rightarrow \quad C_0 = 8.85 \times 10^{-12} \text{ F}$$

When water rises by 1 cm

$$C = \frac{\varepsilon_0 \times 81 \times 0.001}{0.01} + \frac{\varepsilon_0 (0.01 - 0.001)}{0.01}$$

$$\Rightarrow \quad C = 8.1\varepsilon_0 + 0.9\varepsilon_0 = 9\varepsilon_0$$

$$\Rightarrow \quad C = 9\left(8.85 \times 10^{-12}\ \text{F}\right) = 80\ \text{F}$$

Let m be the mass of water that rises to a height of 100 cm, then

$$m = (0.001 \times 0.01) \times 1000 = 0.01\ \text{kg}$$

By Law of Conservation of Energy, we have

$$\frac{q^2}{2C_0} = \frac{q^2}{2C} + \frac{mgh}{2}$$

$$\Rightarrow \quad \frac{q^2}{C_0} = \frac{q^2}{C} + mgh$$

Now, $q = C_0 V$

$$\Rightarrow \quad \frac{V^2 C_0^2}{C_0} = \frac{V^2 C_0^2}{C} + mgh$$

$$\Rightarrow \quad V^2 C_0 \left(1 - \frac{C_0}{C}\right) = mgh$$

$$\Rightarrow \quad V^2 \varepsilon_0 \left(1 - \frac{\varepsilon_0}{9\varepsilon_0}\right) = 0.01 \times 9.8 \times 0.01$$

$$\Rightarrow \quad V = \sqrt{\frac{9 \times 0.01 \times 9.8 \times 0.01}{8.85 \times 10^{-12} \times 8}}$$

$$\Rightarrow \quad V = 1110\ \text{V}$$

Hence, (A), (C) and (D) are correct.

12. On connecting the conducting sphere having charge Q with a conducting shell having no charge, the entire charge must flow out to the uncharged shell.
{OPTION (C)}
The capacitance of the outer shell of radius $2R$ is $C = 4\pi\varepsilon_0 (2R)$.

So, energy liberated is $U = \frac{Q^2}{2C} = \frac{Q^2}{16\pi\varepsilon_0 R}$.

Hence, (C) and (D) are correct.

13. The spring force $\vec{F}_S$ acting on plate a is given by

$$\vec{F}_S = -kx\,\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate b is

$$\vec{F}_e = QE\hat{i} = Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i} = \frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where A is the area of the plate. Notice that charges on plate a cannot exert a force on itself, as required by Newton's Third Law. Thus, only the electric field due to plate b is considered. At equilibrium the two forces cancel and we have

$$kx = Q\left(\frac{Q}{2A\varepsilon_0}\right) = QE$$

which gives

$$x = \frac{Q^2}{2kA\varepsilon_0} = \frac{QE}{k}$$

Hence, (A), (B) and (C) are correct.

16. The effective capacitance

$$C = \frac{4 \times 5}{9} = \frac{20}{9}\ \mu\text{F}$$

The charge on C_1 is given by

$$Q_1 = CV = \frac{20}{9} \times 9 = 20\ \mu\text{C}$$

The charge on C_2 is given by

$$Q_2 = \left(\frac{C_2}{C_2 + C_3}\right) Q_1 = \frac{2}{5} \times 20 = 8\ \mu\text{C}$$

The charge on C_3 is given by

$$Q_3 = \left(\frac{C_3}{C_2 + C_3}\right) Q_1 = \frac{3}{5} \times 20 = 12\ \mu\text{C}$$

Since, $\frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

$$\Rightarrow \quad Q_1 = Q_2 + Q_3 = Q_2 + \frac{C_3}{C_2} Q_2 = Q_2 \left(\frac{C_2 + C_3}{C_2}\right)$$

$$\Rightarrow \quad Q_2 = \left(\frac{C_2}{C_2 + C_3}\right) Q_1 = \left(\frac{2}{2+3}\right) 20 = 8\ \mu\text{C}$$

and $Q_3 = \left(\frac{C_3}{C_2 + C_3}\right) Q_1 = \left(\frac{3}{2+3}\right) 20 = 12\ \mu\text{C}$

Hence, (A), (C) and (D) are correct.

17. When either A or C is earthed (both not earthed together), a parallel plate capacitor is formed with B, with $\pm Q$ charges on the inner surfaces. The other plate, which is not earthed, plays no role, hence charge of amount $+Q$ flows to the earth.
When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio $1:2$, and hence share charge Q in the same ratio.
Hence, (A), (B) and (C) are correct.

18. Condition for a balanced Wheat Stone Bridge.

Hence, (A) and (C) are correct.

19. Since, $F = \dfrac{q^2}{2\varepsilon_0 A}$

Hence, (A) and (D) are correct.

21. The distribution of charges is shown in figure. Applying Kirchhoff's loop rules for loops ①, ②, and ③, we get

LOOP ①

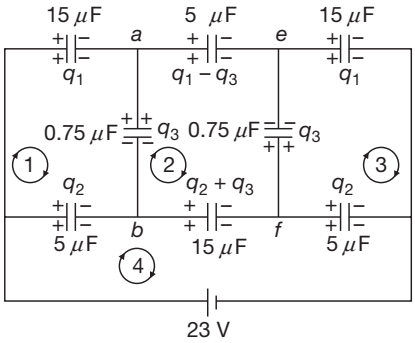
$$-\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\Rightarrow \quad q_1 - 3q_2 + 20q_3 = 0 \qquad \ldots(1)$$

LOOP ②

$$-\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\Rightarrow \quad 3q_1 - q_2 - 44q_3 = 0 \qquad \ldots(2)$$



LOOP ③

$$23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$\Rightarrow \quad 345 = 7q_2 + q_3 \qquad \ldots(3)$$

Solving for q_1, q_2, q_3, we get

$$q_1 = \frac{19 \times 345}{92}, \; q_2 = \frac{13 \times 345}{92}, \; q_3 = \frac{345}{92}$$

Potential Difference between a and b is

$$V_{ab} = \frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$$

Potential Difference between e and f is

$$V_{ef} = -5 \text{ V}$$

Hence, (A) and (B) are correct.

27. The equivalent capacitance of the arrangement here is C where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C = \frac{20}{3} \; \mu\text{F}$$

The potential difference across the combination is 90 V and hence the total charge is

$$Q = CV = \frac{20}{3} \times 10^{-6} \times 90 = 6 \times 10^{-4} \text{ C} = 600 \; \mu\text{C}$$

Since all the capacitors are in series and hence all capacitors will have the same charge given by

$$Q = CV$$

The potential difference across each capacitor will be in the inverse ratio of their capacitances.

So, the potential difference across C_1 is 30 V, the potential difference across C_2 is 20 V and the potential difference across C_3 is 40 V and hence all the **OPTIONS (A), (B), (C)** and **(D)** are correct.

Hence, all the options are correct.

30. $q = CV = C_0 V_0$

where $C_0 = \dfrac{\varepsilon_0 A}{d}$ and $q = \dfrac{\varepsilon_0 A V_0}{d} = \dfrac{\varepsilon_0 A V}{(d + \ell)}$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d} V_0 = \frac{\varepsilon_0 A}{(d + \ell)} V \quad \Rightarrow \quad V = \frac{(d + \ell)}{d} V_0 = \left(1 + \frac{\ell}{d}\right) V_0$$

and $C = \dfrac{\varepsilon_0 A}{d + \ell} = \dfrac{\varepsilon_0 A}{d\left(1 + \dfrac{\ell}{d}\right)} = \dfrac{C_0}{\left(1 + \dfrac{\ell}{d}\right)}$

Hence, (B) and (C) are correct.

31. $C_s = \dfrac{2 \times 8}{2 + 8} = 1.6 \; \mu F$

Since $Q = C_s V = 1.6 \times 10^{-6} \times 300$

$$Q = 4.8 \times 10^{-4} \; C$$

$$V_1 = \frac{4.8 \times 10^{-4}}{2 \times 10^{-6}} = 240 \; V$$

$$V_2 = \frac{4.8 \times 10^{-4}}{8 \times 10^{-6}} = 60 \; V$$

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}$$

$$\Rightarrow \quad U = \frac{(4.8 \times 10^{-4})^2}{2} \left(\frac{1}{1.6 \times 10^{-6}}\right)$$

$$\Rightarrow \quad U = 3 \times 2.4 \times 10^{-2} \; J$$

$$\Rightarrow \quad U = 2.4 \times 10^{-2} \; J$$

Hence, (A) and (D) are correct.

32. Since battery is still in connection. So,

$$V = V_0$$

CHAPTER 2

$\Rightarrow \quad Q_0 = C_0 V_0$ and

$Q = kC_0 V_0$

$\Rightarrow \quad Q = kQ_0$

Since $k > 1$

$\Rightarrow \quad Q > Q_0$

Also $U_0 = \frac{1}{2} Q_0 V_0$ and

$$U = \frac{1}{2} QV = kU_0 \qquad \{\because Q = kQ_0 \text{ and } V = V_0\}$$

Hence $U > U_0$

Hence, (A) and (D) are correct.

33. Since same charge flows through $7\ \mu F$ and $3\ \mu F$ (as both are in series), so

$$Q_7 = Q_3 = 42\ \mu C$$

$\Rightarrow \quad 7 \times 6 = 3V_3$

$\Rightarrow \quad V_3 = 14\ V$

So, $V_{3.9} = 20\ V$

Charge on $12\ \mu F$ capacitor is

$$Q_{12} = Q_{3.9} + Q_7$$

$$\Rightarrow \quad Q_{12} = 3.9 \times 20 + \frac{7 \times 3}{7 + 30} \times 20$$

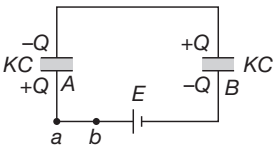
$\Rightarrow \quad Q_{12} = 120\ \mu C =$ Total charge in the circuit.

$$\Rightarrow \quad V_{12} = \frac{120}{12} = 10\ V$$

Voltage across battery is $14 + 6 + 10 = 30\ V$

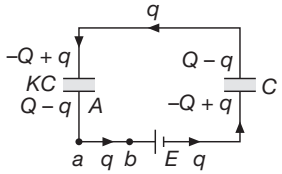
Hence, (A), (C) and (D) are correct.

34.



$$Q = \frac{KCE}{2}$$

On removal of the dielectric from B, let a charge q flow from a to b. Then by applying Kirchhoff's Loop Law, we have



$$\frac{Q-q}{KC} + \frac{Q-q}{C} = E$$

$$\Rightarrow \quad Q - x = \left(\frac{K}{K+1}\right) CE$$

$$\Rightarrow \quad \frac{KCE}{2} - q = \left(\frac{K}{K+1}\right) CE$$

$$\Rightarrow \quad q = KCE\left(\frac{1}{2} - \frac{1}{K+1}\right) = \frac{KCE}{2}\left(\frac{K-1}{K+1}\right)$$

$$\Rightarrow \quad q = Q\left(\frac{K-1}{K+1}\right)$$

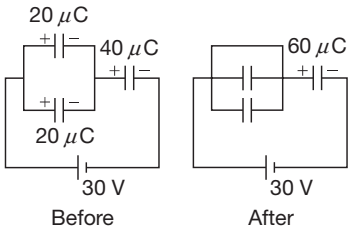
Since K is always greater than one, so q is positive and hence we have the charge flowing from a to b. So, option (A) is correct.

Final charge on B equals the final charge on A, so option (B) is incorrect.

During this process, energy received by the is battery is $(\Delta q) V_{\text{battery}} = qE$. So, option (D) is correct.

Hence, (A) and (D) are correct.

35. The charges stored in different capacitors before and after the switch S is closed are shown in figure(s).



The amount of charge flown through the battery is

$$\Delta q = q_f - q_i = 20\ \mu C$$

So, energy supplied by the battery is

$$W_{\text{battery}} = (\Delta q) V_{\text{battery}}$$

$$\Rightarrow \quad W_{\text{battery}} = (20 \times 10^{-6})(30) = 0.6\ \text{mJ}$$

Energy stored in all the capacitors before closing the switch S is

$$U_i = \frac{1}{2} C_{\text{net}} V^2 = \frac{1}{2}\left(\frac{4}{3} \times 10^{-6}\right)(30)^2 = 0.6\ \text{mJ}$$

and after closing the switch is

$$U_f = \frac{1}{2} C_{\text{net}} V^2 = \frac{1}{2}\left(2 \times 10^{-6}\right)(30)^2 = 0.9\ \text{mJ}$$

Since we know that $W_{\text{battery}} = \Delta U_C + \Delta H$

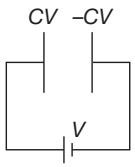
So, heat generated ΔH is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = 0.3\ \text{mJ}$$

and charge flown through the switch is $60\ \mu C$.

Hence, (A), (C) and (D) are correct.

37.

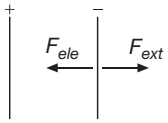


When the battery is disconnected, then the charge will remain constant. Now when the plates are pulled apart, then we have

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

So, E remains constant and hence option (A) is correct

Work is done against attractive force by the external force F_{ext}



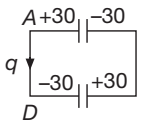
So, option (B) is also correct.

Since, $U = \frac{1}{2}CV^2$

When the battery stays connected to the capacitor, then V = constant . When the separation between the plates is increased, then capacitance given by $C = \frac{\varepsilon_0 A}{d}$ decreases and hence U also decreases.
So option (C) is also correct.

Hence, (A), (B) and (C) are correct.

38.



The common potential is given by

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{30 - 30}{C_1 + C_2} = 0$$

Final charge on the first capacitor is

$$Q_1' = C_1V = 0$$

Final charge on the second capacitor is

$$Q_2' = C_2V = 0$$

So, final energy stored in the arrangement is zero.
Let a charge q flow from A to D, then

$$30 - q = 0$$

$$\Rightarrow \quad q = 30\ \mu C$$

Hence, (A) and (C) are correct.

39. $\tau_{C_1} = \tau_{C_2} = 2RC = \tau_C$ (say)

$$q_1 = (CE)\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

and $q_2 = (2CE)\left(1 - e^{-\frac{t}{\tau_C}}\right)$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{1}{2} \text{ (at any time)}$$

So, the ratio of charges in steady state is also $1:2$

Now, $\frac{dq_1}{dt} = \frac{CE}{\tau_C}e^{-\frac{t}{\tau_C}}$

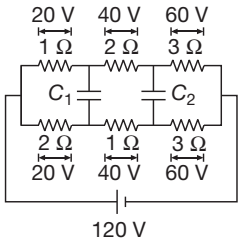
and $\frac{dq_2}{dt} = \frac{2CE}{\tau_C}e^{-\frac{t}{\tau_C}}$

$$\Rightarrow \quad \left(\frac{dq_1}{dt}\right) \neq \left(\frac{dq_2}{dt}\right)$$

The option (A) is wrong.

Hence, (B), (C) and (D) are correct.

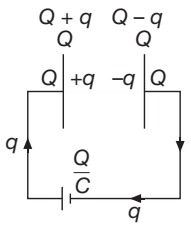
40. In steady state, the potential difference across C_1 is 20 V and potential difference across C_2 is 0 V. Then, charge stored on C_1 is $40\ \mu C$ and charge stored on C_2 is $0\ \mu C$.



Hence, (B) and (D) are correct.

41. Let the charge flowing from the positive terminal to the negative terminal of the battery be q, then

$$q = CV = C\left(\frac{Q}{C}\right) = Q$$



Total charge on plate x is

$$Q + q = Q + Q = 2Q$$

Total charge on plate y is

$$Q - q = Q - Q = 0$$

The energy supplied by the cell is

$$W_{\text{battery}} = \left(\frac{Q}{C}\right)q = \frac{Q^2}{C} = CV^2$$

Hence, all the options are correct.

42. (A) $U = \frac{1}{2}CV^2$

As source between the plates is connected, so potential difference remains constant. But capacitance C becomes KC hence energy stored is increased by factor K.

(B) Electric field $\frac{V}{d}$ is not changed.

(C) Charge on each plate is increased by factor K hence force between them increases by a factor of K^2.

(D) Since $Q = CV$

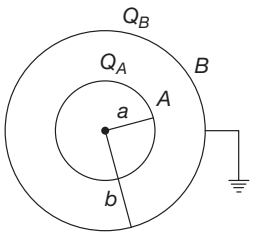
Hence charge becomes KQ as C becomes KC and V remains unchanged.

Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

6. $V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{a} + \frac{Q_B}{b}\right)$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{b} + \frac{Q_B}{b}\right)$$



$$\Rightarrow \quad V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{a} - \frac{Q_A}{b}\right) = \frac{Q_A}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Hence, the correct answer is (A).

7. Equivalent capacitance of parallel combination is

$$C_{\text{parallel}} = C_1 + C_2 + C_3$$

Hence, the correct answer is (C).

8. When the battery is disconnected from the capacitor, then $Q = \text{constant}$

$$\text{Energy} = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$$\Rightarrow \quad \text{Energy} \propto d$$

Hence, the correct answer is (D).

9. Since $V = \frac{Q}{C}$

If sizes of two capacitors are different then potentials will also be different. Thus potential difference may exist between them although they carry same amount of positive charge.

Hence, the correct answer is (D).

10. The sum of charges on both the plates should be zero.

Hence, the correct answer is (A).

11. The capacitance depends upon the geometrical parameters only. And if Q is increased then V increase.

Hence, the correct answer is (C).

12. In the given case $V = V_0$ (constant)

Energy stored in the capacitor $= \frac{1}{2}CV^2$

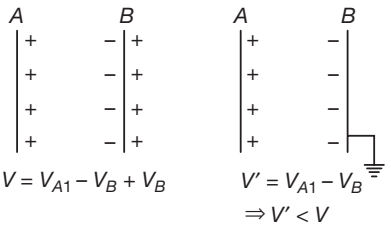
$C' = CK$, so energy stored will become A times

$Q = CV$, so Q will become K times

$$\therefore \quad \text{surface charge density } \sigma' = \frac{Kq}{A} = K\sigma_0$$

Hence, the correct answer is (C).

13.



Hence, the correct answer is (C).

14. $C_1 = \frac{\varepsilon_0 KA}{d}$

$$\frac{C_1}{C_2} = \left(\frac{K_1}{d_1}\right)\left(\frac{d_2}{K_2}\right) = \left(\frac{K_1}{K_2}\right)\left(\frac{d_2}{d_1}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$$

Hence, the correct answer is (B).

Linked Comprehension Type Questions

1. Since,

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - \frac{d}{3} + \frac{d}{6}}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - \frac{d}{6}}$$

$$\Rightarrow \quad C = \frac{6}{5}\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow \quad C = \frac{6}{5}C_0$$

Hence, the correct answer is (D).

2. According to question

$$\frac{\varepsilon_0 A}{d - \frac{t}{2}} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

Solving we get $\frac{t}{d} = \frac{2}{3}$

Hence, the correct answer is (A).

3. For dielectric $U_1 = \frac{q^2}{2C_1} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d - \frac{t}{2}}\right)} = \frac{q^2}{2\left(\frac{3\varepsilon_0 A}{2d}\right)}$

For conductor with air filling the complete space, we have

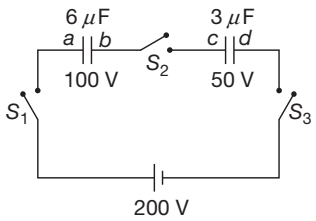
$$U_2 = \frac{q^2}{2C_2} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} \quad \Rightarrow \quad \frac{U_2}{U_1} = \frac{3}{2}$$

Hence, the correct answer is (B).

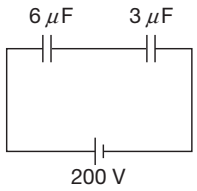
4. Plate b and c are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they together form an isolated system.

Hence, the correct answer is (D).

5. The charges, before the switches are closed are shown here



In steady state, after closing the switch, let q charge goes from battery.



Then charge on each will increase by q

Applying Kirchhoff's Voltage Law, we get

$$200 - \left(\frac{600 + q}{6}\right) - \left(\frac{150 + q}{3}\right) = 0 \quad \Rightarrow \quad q = 100\ \mu C$$

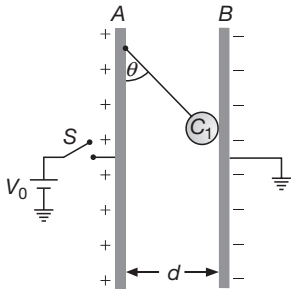
Hence, charge on $6\ \mu F$ capacitor will be $700\ \mu C$ and that on the $3\ \mu F$ capacitor will be $250\ \mu C$.

Hence, the correct answer is (B).

6. From all the switches $100\ \mu C$ of charge will flow.

Hence, the correct answer is (D).

7. Let d be the separation between the plates and ℓ be the length of the silk thread, then in the final position, we have



$$\sin\theta = \frac{d}{\ell} = \frac{50}{100} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 30°$ {with plate A}

Hence, the correct answer is (A).

8. When the ball again touches the plate A, the charge on plate A, i.e., $Q_0 = C_0 V_0$, redistributes on A and ball. Let the charge on plate be Q_0' and on ball be Q after redistribution such that the common potential is V. Then

$$Q_0' + Q = C_0 V_0$$

$$\frac{Q_0'}{Q} = \frac{C_0}{C}$$

$$\Rightarrow \quad 1 + \frac{Q_0'}{Q} = 1 + \frac{C_0}{C}$$

$$\Rightarrow \quad Q=\left(\frac{C}{C+C_0}\right)(Q+Q_0')$$

$$\Rightarrow \quad Q=\left(\frac{C}{C+C_0}\right)C_0V_0$$

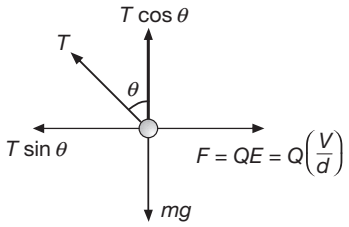
$$\Rightarrow \quad V=\frac{Q}{C}=\frac{C_0V_0}{C+C_0}$$

Hence, the correct answer is (B).

9. In the final position of the ball, the free body diagram shows that

$$T\sin\theta=QE=Q\left(\frac{V}{d}\right)$$

and $T\cos\theta=mg$



$$\Rightarrow \quad \tan\theta=\frac{QV}{mgd}$$

$$\Rightarrow \quad mg\tan\theta=\frac{QV}{d}=\frac{(CV)V}{d}=\frac{CV^2}{d}$$

Since $V=\dfrac{C_0V_0}{C+C_0}$

$$\Rightarrow \quad mg\tan\theta=\frac{C}{d}\left(\frac{C_0V_0}{C_0+C}\right)^2$$

$$\Rightarrow \quad \frac{C_0^2V_0^2}{(C_0+C)^2}=\frac{mgd\tan\theta}{C}$$

$$\Rightarrow \quad V_0=\frac{C_0+C}{C_0}\sqrt{\frac{mgd\tan\theta}{C}}$$

$$\Rightarrow \quad V_0=\left(1+\frac{C}{C_0}\right)\sqrt{m(10)\left(\frac{50}{1000}\right)\frac{\tan 30}{C}}$$

$$\Rightarrow \quad V_0=\left(1+\frac{C}{C_0}\right)\sqrt{\frac{m}{2\sqrt{3}C}}$$

Hence, the correct answer is (C).

10. $\dfrac{C_A}{C_B}=\dfrac{1}{K}$

Hence, the correct answer is (B).

15. $C\propto r$

$$\Rightarrow \quad \frac{C_A}{C_B}=\frac{r_A}{r_B}=\frac{2}{3}$$

Hence, the correct answer is (A).

16. The charge will be shared till both attain a common potential V (say).
Then

$$\frac{Q_2}{Q_1}=\frac{C_2V}{C_1V}=\frac{C_2}{C_1}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{Q_2}{Q_1}+1=\frac{3}{2}+1$$

$$\Rightarrow \quad \frac{Q_2+Q_1}{Q_1}=\frac{3+2}{2}$$

$$\Rightarrow \quad \frac{Q}{Q_1}=\frac{5}{2}$$

$$\Rightarrow \quad Q_1=\frac{2Q}{5}$$

Hence, the correct answer is (C).

17. $\dfrac{(U_A)_i}{(U_A)_f}=\dfrac{q_i^2}{q_f^2}=\left(\dfrac{2}{5}\right)^2=\dfrac{4}{25}$

Hence, the correct answer is (C).

18. $\dfrac{U_i}{U_f}=\dfrac{\frac{1}{2}C_1V^2}{\frac{1}{2}(C_1+C_2)V^2}=\dfrac{C_1}{C_1+C_2}=\dfrac{2}{2+3}=\dfrac{2}{5}$

Hence, the correct answer is (B).

22. Capacitor A is a combination of two capacitors C_K and C_O in parallel. Hence,

$$C_A=C_K+C_O=\frac{K\varepsilon_oA}{d}+\frac{\varepsilon_oA}{d}=(K+1)\frac{\varepsilon_oA}{d}$$

Here, $A=0.02\ \text{m}^2$ Substituting the values, we have

$$C_A=(9+1)\frac{8.85\times10^{-12}(0.02)}{\left(8.85\times10^{-4}\right)}\ \text{F}$$

$$\Rightarrow \quad C_A=2\times10^{-9}\ \text{F}=2\ \text{nF}$$

Hence, the correct answer is (B).

23. Energy stored in capacitor A, when connected with a 110 V battery is

$$U_A=\frac{1}{2}C_AV^2=\frac{1}{2}\left(2\times10^{-9}\right)(110)^2$$

$$\Rightarrow \quad U_A=1.21\times10^{-5}\ \text{J}\cong12\ \mu\text{J}$$

Hence, the correct answer is (C).

24. Charge stored in the capacitor

$$q_A = C_A V = \left(2\times10^{-9}\right)(110)$$

$$\Rightarrow \quad q_A = 2.2\times10^{-7}\ \text{C}$$

Now, this charge remains constant even after battery is disconnected. But when the slab is removed, capacitance of A will get reduced. Let the new capacitance be C'_A. So

$$C'_A = \frac{\varepsilon_o(2A)}{d} = \frac{\left(8.85\times10^{-12}\right)(0.04)}{8.85\times10^{-4}}\ \text{F}$$

$$\Rightarrow \quad C'_A = 0.4\times10^{-9}\ \text{F}$$

Energy stored in this case would be

$$U'_A = \frac{1}{2}\frac{(q_A)^2}{C'_A} = \frac{1}{2}\frac{\left(2.2\times10^{-7}\right)^2}{\left(0.4\times10^{-9}\right)}\ \text{J}$$

$$\Rightarrow \quad U'_A = 6.05\times10^{-5}\ \text{J} > U_A$$

So, work done to remove the slab would be

$$W = U'_A - U_A = (6.05-1.21)\times10^{-5}\ \text{J}$$

$$\Rightarrow \quad W = 4.84\times10^{-5}\ \text{J} = 48.4\ \mu\text{J}$$

Hence, the correct answer is (A).

25. Capacitance of B when filled with dielectric is

$$C_B = \frac{K\varepsilon_o A}{d} = \frac{(9)\left(8.85\times10^{-12}\right)(0.02)}{\left(8.85\times10^{-4}\right)}\ \text{F}$$

$$\Rightarrow \quad C_B = 1.8\times10^{-9}\ \text{F}$$

These two capacitors are in parallel. Therefore, net capacitance of the system is

$$C = C'_A + C_B = (0.4+1.8)\times10^{-9}\ \text{F}$$

$$\Rightarrow \quad C = 2.2\times10^{-9}\ \text{F}$$

Charge stored in the system is $q = q_A = 2.2\times10^{-7}\ \text{C}$

So, energy stored, $U = \frac{1}{2}\frac{q^2}{C} = \frac{q_A^2}{2C}$

$$\Rightarrow \quad U = \frac{1}{2}\frac{\left(2.2\times10^{-7}\right)^2}{\left(2.2\times10^{-9}\right)}$$

$$\Rightarrow \quad U = 1.1\times10^{-5}\ \text{J} = 11\ \mu\text{J}$$

Hence, the correct answer is (B).

26. Capacitors 2 and 3 are in parallel and give equivalent capacitance 6C, Which further is in series with capacitor 1, so the battery sees capacitance $\frac{1}{C_{\text{net}}} = \frac{1}{3C} + \frac{1}{6C}$

$$\Rightarrow \quad C_{\text{net}} = 2C$$

Hence, the correct answer is (C).

27. If they were initially uncharged, C_1 stores the same charge as C_2 and C_3 together. With greater capacitance, C_3 stores more charge than C_2. Then $Q_1 > Q_2 > Q_3$

Hence $C_1 > C_2 > C_3$ (in terms of charges)

Hence, the correct answer is (A).

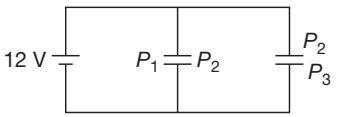
28. The $(C_2 \| C_3)$ equivalent capacitor stores the same charge as C_1. Since it has greater capacitance, so $\Delta V = \frac{Q}{C}$ implies that it has smaller potential difference across it than C_1. In parallel with each other, C_2 and C_3 have equal voltages. $\Delta V_1 > \Delta V_2 = \Delta V_3$. Hence $C_1 > C_2 = C_3$ (in terms of potentials)

Hence, the correct answer is (C).

29. If C_3 is increased, the overall equivalent capacitance increases. More charge moves through the battery and Q increases. As ΔV_1 increases, ΔV_2 must decrease so Q_2 decreases. Then Q_3 must increase even more: Q_3 and Q_1 increase; Q_2 decreases. Hence charge on C_1 and C_3 increase but that on C_2 decreases.

Hence, the correct answer is (B).

30. Each face of P_2 carries charge, so the three-plate system is equivalent to



Each capacitor by itself has capacitance

$$C = \frac{K\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\right)\left(7.5\times10^{-4}\right)}{\left(1.19\times10^{-3}\right)} = 5.58\ \text{pF}$$

So, equivalent capacitance = $5.58 + 5.58 = 11.2$ pF

Hence, the correct answer is (B).

31. $Q = C\Delta V + C\Delta V$

$$\Rightarrow \quad Q = \left(11.2\times10^{-12}\right)(12\ \text{V}) = 134\ \text{pC}$$

Hence, the correct answer is (C).

32. Now P_3 has charge on two surfaces and in effect three capacitors are in parallel

$$C = 3(5.58\ \text{pF}) = 16.7\ \text{pF}$$

Hence, the correct answer is (D).

33. Only one face of P_4 carries charge

$$Q = C\Delta V = 5.58\times10^{-12}\ \text{F}(12\ \text{V}) = 67\ \text{pC}$$

Hence, the correct answer is (B).

34. Let V_1 and V_2 be the voltage across 1 and 2 μF capacitors respectively. Then

$$V_1 + V_2 = 300 \text{ V}$$

In series the charge Q on each capacitor is same, therefore we have

$$\frac{Q}{C_1} + \frac{Q}{C_2} = 300$$

$$\Rightarrow \quad \left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q = 300$$

Here $C_1 = 1\ \mu\text{F}$, $C_2 = 2\ \mu\text{F}$

$$\Rightarrow \quad Q = \frac{300C_1C_2}{C_1 + C_2} = \frac{300(1\times10^{-6})(2\times10^{-6})}{(1\times10^{-6} + 2\times10^{-6})}$$

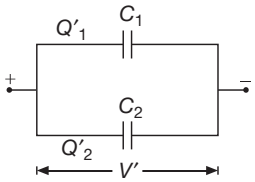
$$\Rightarrow \quad Q = 200\times10^{-6}\ \text{C} = 200\ \mu\text{C}$$

$$\Rightarrow \quad V_1 = \frac{Q}{C_1} = \frac{200\ \mu\text{C}}{1\ \mu\text{F}} = 200 \text{ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{200\ \mu\text{C}}{2\ \mu\text{F}} = 100 \text{ V}$$

Hence, the correct answer is (D).

35. When the plates of same polarity are connected together, the capacitors are in usual parallel combination. In parallel the potential difference across each capacitor is the same.



$\therefore$ The common potential difference

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{2Q}{C_1 + C_2}$$

$$\Rightarrow \quad V = \frac{2\times(200\ \mu\text{C})}{(1\ \mu\text{F} + 2\ \mu\text{F})} = \frac{400\ \mu\text{C}}{3\ \mu\text{F}} = \frac{400}{3} \text{ V}$$

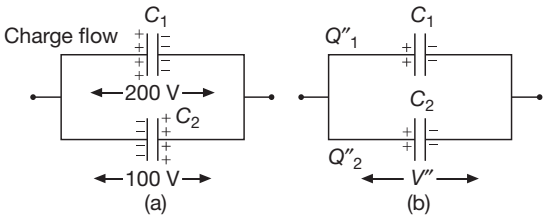
If Q_1' and Q_2' are new charges on the respective capacitors, then

$$Q_1' = C_1V = (1\ \mu\text{F})\times\left(\frac{400}{3}\text{ V}\right) = \frac{400}{3}\ \mu\text{C}$$

$$Q_2' = C_2V = (2\ \mu\text{F})\times\left(\frac{400}{3}\text{ V}\right) = \frac{800}{3}\ \mu\text{C}$$

Hence, the correct answer is (C).

36. When the opposite plates of **SITUATION (I)** are connected together, the charge will flow from the capacitor of higher potential to that of lower potential until their potentials are same. The net charge is then $(Q_1 - Q_2)$.



If V'' is common potential difference across the capacitors, then

$$V'' = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{(200\ \mu\text{F} - 200\ \mu\text{C})}{1\ \mu\text{F} + 2\ \mu\text{F}} = 0$$

The charges after sharing

$$Q_1'' = C_1V'' = C_1(0) = 0$$

$$Q_2'' = C_2V'' = C_2(0) = 0$$

Hence, the correct answer is (A).

37. When the opposite plates of **SITUATION (II)** are connected together, the charge will flow from one having larger quantity of charge of that of smaller quantity of charge i.e., from C_2 to C_1, to make common potential V''' (say) given by

$$V''' = \frac{(Q_2' - Q_1')}{C_1 + C_2} = \frac{\left(\frac{800}{3} - \frac{400}{3}\right)\mu\text{C}}{(1+2)\mu\text{F}} = \frac{400}{9} \text{ V}$$

$\Rightarrow$ Charges after sharing are

$$Q_1'' = C_1V''' = \left(1\ \mu\text{F}\times\frac{400}{9}\text{ V}\right) = \frac{400}{9}\ \mu\text{C}$$

and $Q_2'' = C_2V''' = \dfrac{800}{9}\ \mu\text{C}$

Hence, the correct answer is (B).

38. Initial charges on first and second capacitors are $q_1 = C_1V_0 = 100\ \mu\text{C}$ and $q_2 = -C_2V_0 = -300\ \mu\text{C}$ finally, let the common potential be V, then $q_{\text{final}} = (C_1 + C_2)V$. So, by Law of Conservation of Charge, we get

$$(C_1 + C_2)V = C_1V_0 + C_2(-V_0)$$

$$\Rightarrow \quad V = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)V_0$$

$$\Rightarrow \quad V = \left(\frac{3-1}{3+1}\right)100 = 50 \text{ V}$$

Hence, the correct answer is (B).

39. $(q_1)_f = C_1V = (1\ \mu\text{F})(50\ \text{V}) = 50\ \mu\text{C}$

Hence, the correct answer is (A).

40. $(q_2)_f = C_2V = (3\ \mu\text{F})(50\ \text{V}) = 150\ \mu\text{C}$

Hence, the correct answer is (C).

41. $U_f = \frac{1}{2}(C_1 + C_2)V^2$

$\Rightarrow \quad U_f = \frac{1}{2}(4\times10^{-6})(2500)$

$\Rightarrow \quad U_f = 2\times10^{-6}\times2500$

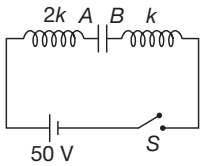
$\Rightarrow \quad U_f = 5\times10^{-3}\ \text{J} = 5\ \text{mJ}$

and $U_i = \frac{1}{2}(C_1 + C_2)V_0^2 = 200\ \text{mJ}$

$\Rightarrow$ Loss $= U_i - U_f = 15\ \text{mJ}$

Hence, the correct answer is (C).

42.



Final energy of capacitor is 10×10^{-3} J

$\Rightarrow \quad 10\times10^{-3} = \frac{1}{2}C_{\text{final}}(50)^2$

$\Rightarrow \quad C_{\text{final}} = C_2 = 8\ \mu\text{F}$

Since, $C = \frac{\varepsilon_0 A}{d}$

$\Rightarrow \quad \frac{C_1}{C_2} = \frac{d_2}{d_1}$

$\Rightarrow \quad C_1 = \left(\frac{d_2}{d_1}\right)C_2 = \left(\frac{2}{8}\right)(8) = 2\ \mu\text{F}$

So, the capacitance of the capacitor when the switch S is not closed is $2\ \mu$F.

Hence, the correct answer is (C).

43. If x_1 and x_2 be the extensions in springs connected to plates A and B respectively, then we observe that

$F = (2k)x_1 = kx_2$

$\Rightarrow \quad 2x_1 = x_2$

Also, $x_1 + x_2 = d_1 - d_2 = 8 - 2 = 6$ mm

$\Rightarrow \quad \frac{x_2}{2} + x_2 = 6$

$\Rightarrow \quad \frac{3x_2}{2} = 6$

$\Rightarrow \quad x_2 = 4$ mm

So, $x_1 = \frac{x_2}{2} = 2$ mm

Hence, the correct answer is (B).

44. Since force between plates of the capacitor connected to the battery is given by

$$F = \frac{1}{2}\frac{C_2V^2}{d_2} = \frac{10\times10^{-3}}{2\times10^{-3}} = 5\ \text{N}$$

$\Rightarrow \quad (2k)x_1 = 5$

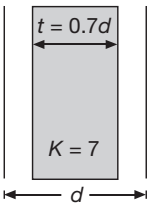
$\Rightarrow \quad k_A = 2k = \frac{5}{x_1} = \frac{5}{2\times10^{-3}} = 2500\ \text{Nm}^{-1}$

Hence, the correct answer is (C).

45. Since, we know that

$$C_{\text{eq}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{\frac{0.3d}{1} + \frac{0.7d}{7}} = \frac{\varepsilon_0 A}{0.4d} \qquad \ldots(1)$$

When plate was absent, then



$$C_0 = \frac{\varepsilon_0 A}{d} = 10\ \mu\text{F}$$

So from equation (1), we get

$$C_{\text{eq}} = \frac{10}{0.4} = 25\ \mu\text{F}$$

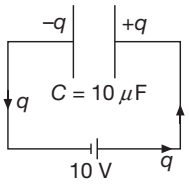
Charge stored $= C_{\text{eq}}V = 25\times10 = 250\ \mu\text{C}$

Energy stored $= \frac{1}{2}(25)(10)^2 = 1250\ \mu\text{J}$

Hence, the correct answer is (B).

46. After disconnecting the battery, when the slab is removed, then let new charge on the capacitor plate be q_f. So, we have

CHAPTER 2



$$-10+\frac{q_f}{10}=0$$

$$\Rightarrow \quad q_f = 100\ \mu\text{C}$$

$$\text{New energy} = \frac{(100)^2}{2\times 10} = 500\ \mu\text{J}$$

Hence, the correct answer is (A).

47. Since $W_{\text{battery}} = \Delta U_C + \Delta H$

where, $W_{\text{battery}} = (\Delta q)V_{\text{battery}}$

Since the positive plate of the capacitor is connected to the negative of the battery, so $q_{\text{final}} = -100\ \mu\text{C}$, whereas we observe that $q_{\text{initial}} = 250\ \mu\text{C}$. Hence the charge flown through the battery is

$$|\Delta q| = |q_{\text{final}} - q_{\text{initial}}| = 350\ \mu\text{C}$$

Also, $\Delta U_C = U_{\text{final}} - U_{\text{initial}} = 500 - 1250 = -750\ \mu\text{J}$

$$\Rightarrow \quad 3500 = -750 + \Delta H$$

$$\Rightarrow \quad \Delta H = 4250\ \mu\text{J}$$

Hence, the correct answer is (D).

48. Method-1

$$E_1 = \frac{Q}{2A\varepsilon_0} \text{ and } E_2 = \frac{2Q}{2A\varepsilon_0}$$

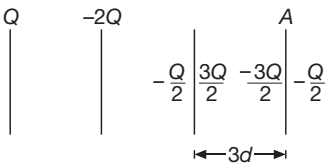
$$\Rightarrow \quad E = E_1 + E_2$$

$$\Rightarrow \quad E = \frac{3Q}{2A\varepsilon_0}$$

Potential difference between the plates is

$$\Delta V = E(3d) = \left(\frac{3Q}{2A\varepsilon_0}\right)3d = \frac{9Q}{2C}$$

Method-2



$$C_{\text{eq}} = \frac{\varepsilon_0 A}{3d}$$

$$V = \frac{Q_{\text{inside}}}{C_{eq}} = \frac{\left(\frac{3Q}{2}\right)}{\left(\frac{\varepsilon_0 A}{3d}\right)} = \frac{9Q}{2C}$$

where, $C = \frac{\varepsilon_0 A}{d}$ as said in the question.

$$\Rightarrow \quad V = \frac{9Q}{2C}$$

Hence, the correct answer is (B).

49. Method-1

Since field between the plates is

$$E = \frac{3Q}{2A\varepsilon_0}$$

Since electric energy density is given by

$$u_{\text{electric}} = \frac{1}{2}\varepsilon_0 E^2$$

$$\Rightarrow \quad u_{\text{electric}} = \frac{1}{2}\varepsilon_0\left(\frac{3Q}{2A\varepsilon_0}\right)^2 = \frac{9Q^2}{8\varepsilon_0 A^2}$$

So, the electric energy U is obtained by taking the product of the volume of the capacitor with the energy density. Hence

$$U = A(3d)\left(\frac{1}{2}\varepsilon_0 \mathrm{E}^2\right)$$

$$U = A(3d)\left(\frac{1}{2}\varepsilon_0 \mathrm{E}^2\right) = 3Ad\left(\frac{9Q^2}{8\varepsilon_0 A^2}\right) = \frac{27Q^2}{8C}$$

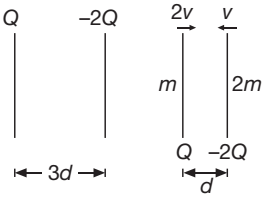
Method-2

$$U = \frac{(Q_{inside})^2}{2C_{eq}}$$

$$\Rightarrow \quad U = \frac{\left(\frac{3Q}{2}\right)^2}{2\left(\frac{\varepsilon_0 A}{3d}\right)} = \frac{27Q^2}{8C}$$

Hence, the correct answer is (C).

50.



By Law of conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Electrostatic}\\ \text{Energy of system}\end{pmatrix} = \begin{pmatrix}\text{Gain in Kinetic}\\ \text{Energy of system}\end{pmatrix}$$

$$\Rightarrow \quad U_{\text{initial}} - U_{\text{final}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2$$

$$\Rightarrow \quad \frac{27Q^2}{8C} - \frac{9Q^2}{8C} = 3mv^2$$

$$\Rightarrow \quad \frac{9Q^2}{4C} = 3mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{3Q^2}{4mC}} = \frac{3Q}{2}\sqrt{\frac{1}{3mC}}$$

So the relative velocity of approach of the plates is

$$v_{\text{rel}} = 3v = \frac{9Q}{2}\sqrt{\frac{1}{3mC}}$$

Hence, the correct answer is (D).

52. Since both the capacitors are in parallel, so we have

$$C = C_1 + C_2$$

$$\Rightarrow \quad C = \frac{K\varepsilon_0 Wh}{d} + \frac{\varepsilon_0 W(L-h)}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 W}{d}[(K-1)h + L]$$

$$\Rightarrow \quad K_{\text{eff}} = \left[\frac{(K-1)h}{L} + 1\right]$$

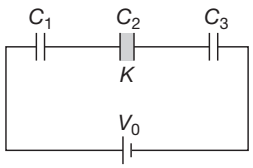
Hence, the correct answer is (A).

53. When the tank is one-fourth empty then it is three-fourth full, so $h = \frac{3L}{4}$ and hence

$$K_{\text{eff}} = \frac{K+1}{4}$$

Hence, the correct answer is (D).

54. It is observed that potential difference across C_3 is 12 V, so we have

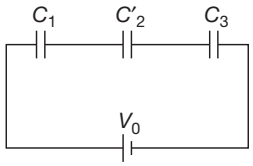


$$12 = \left(\frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right)V_0 \qquad \ldots(1)$$

Now, when the dielectric material $(K=3)$ between the plates of C_2 is removed then the new capacitance becomes C_2', so that

$$C_2' = \frac{C_2}{3}$$

In this situation, the new potential difference across C_3 is found to be 9 V, so



$$9 = \left(\frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2'}+\frac{1}{C_3}}\right)V_0 \qquad \ldots(2)$$

It is also observed that the potential differences across C_1 and C_2' are equal, so we have

$$\Rightarrow \quad C_1V_1 = C_2'V_1$$

$$\Rightarrow \quad C_1 = C_2' = \frac{C_2}{3}$$

$$\Rightarrow \quad C_2 = 3C_1 \qquad \ldots(3)$$

Substituting (3) in (1) and (2), we get

$$12 = \left(\frac{\frac{1}{C_3}}{\frac{4}{3C_1}+\frac{1}{C_3}}\right)V_0 = \frac{V_0}{\frac{4}{3}\frac{C_3}{C_1}+1} \qquad \ldots(4)$$

$$9 = \left(\frac{\frac{1}{C_3}}{\frac{2}{C_2'}+\frac{1}{C_3}}\right)V_0 = \frac{V_0}{\frac{2C_3}{C_1}+1} \qquad \ldots(5)$$

Now assuming $\frac{C_3}{C_1} = x$ and dividing equation (4) by equation (5), we get

$$\frac{12}{9} = \frac{2x+1}{4x+3}$$

$$\Rightarrow \quad 16x + 12 = 18x + 9$$

$$\Rightarrow \quad x = \frac{3}{2}$$

$$\Rightarrow \quad x = \frac{C_3}{C_1} = \frac{3}{2}$$

Substituting the value of x in equation (4), we get

$$12 = \frac{V_0 \times 3}{4x+3}$$

CHAPTER 2

$$\Rightarrow \quad 4 = \frac{V_0}{6+3}$$

$$\Rightarrow \quad V_0 = 36 \text{ V}$$

Hence, the correct answer is (D).

55. If C_2 is $6\ \mu F$, then $C_2 = 3C_1$

$$\Rightarrow \quad C_1 = \frac{C_2}{3} = \frac{6}{3} = 2\ \mu\text{F}$$

Hence, the correct answer is (D).

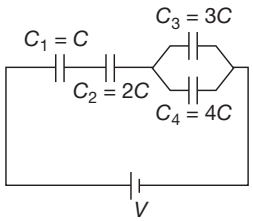
56. $C_3 = xC_1 = \frac{3}{2} \times 2 = 3\ \mu\text{F}$

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A → (q, s)
B → (p)
C → (q, s)
D → (r)

2. A → (q)
B → (p)
C → (s)
D → (r)



Effective capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{7C}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{14+7+2}{14C} = \frac{23}{14C}$$

$$\Rightarrow \quad C_{eq} = \frac{14C}{23}$$

So, charge across C_1 and C_2 is $q = \frac{14CV}{23}$

Charge will get distributed across C_3 and C_4 in direct ratio of their capacitances and hence charge across C_3 is

$$q_{C_3} = \frac{3q}{7} = \frac{3}{7} \times \frac{14CV}{23} = \frac{6CV}{23}$$

Hence, (A) → (q)

Potential differences across C_3 and C_4 would be the same, because they are in parallel so,

$$V_{C_3} = V_{C_4} = \frac{q}{7C}$$

$$\Rightarrow \quad V_{C_3} = V_{C_4} = \frac{1}{7C}\left(\frac{14CV}{23}\right) = \frac{2V}{23}$$

Hence (B) → (p)

Energy stored across C_3 is

$$U_3 = \frac{1}{2}C_3 V_{C_3}{}^2 = \frac{1}{2}(3C)\left(\frac{q}{7C}\right)^2$$

$$\Rightarrow \quad U_3 = \frac{1}{2}(3C)\frac{q^2}{49C^2} = \frac{3q^2}{98C}$$

$$\Rightarrow \quad U_3 = \frac{3q^2}{98C} = \frac{3}{98C}\left(\frac{14CV}{23}\right)^2$$

$$\Rightarrow \quad U_3 = \frac{3 \times 196CV^2}{98 \times 23 \times 23} \simeq \frac{CV^2}{88}$$

Hence (C) → (s)

Energy stored across C_4 is

$$U_4 = \frac{1}{2}C_4 V_{C_4}^2 = \frac{1}{2}4C\left(\frac{q}{7C}\right)^2$$

$$\Rightarrow \quad U_4 = \frac{1}{2}(4C)\left(\frac{q^2}{49C^2}\right)$$

$$\Rightarrow \quad U_4 = \frac{4q^2}{98C} = \frac{4 \times 196CV^2}{98 \times 23 \times 23} \simeq \frac{CV^2}{66}$$

Hence (D) → (r)

3. A → (p, r, s)
B → (p, r)
C → (p, r, s)
D → (p, r)

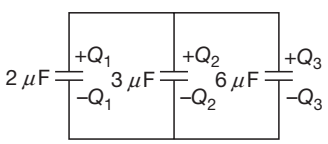
4. A → (s)
B → (q)
C → (r)
D → (p)

After reconnection, charges are redistributed as shown

$$Q_1 = (2\ \mu\text{F})V$$

$$Q_2 = (3\ \mu\text{F})V$$

$$Q_3 = (6\ \mu\text{F})V$$



Since, initially the capacitors were in series, so, 12 μC charge appears on each of the positive plates and -12 μC charge appears on each of negative plates. So a total of 36 μC charge on the three positive plates now redistribute as Q_1, Q_2 and Q_3.

$$Q_1 + Q_2 + Q_3 = 36\ \mu C$$

$$(11\ \mu F)V = 36\ \mu C$$

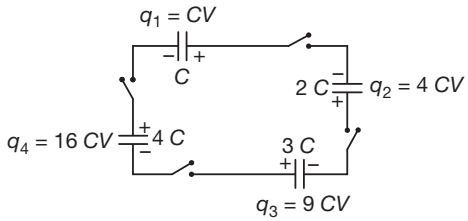
$$\Rightarrow \quad V = \frac{36}{11}\ V$$

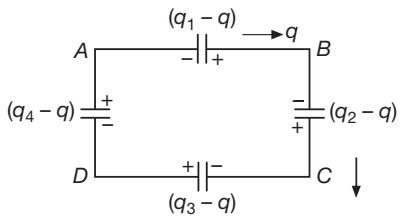
$$\Rightarrow \quad Q_1 = \frac{72}{11}\ \mu C$$

$$\Rightarrow \quad Q_2 = \frac{108}{11}\ \mu C$$

$$\Rightarrow \quad Q_3 = \frac{216}{11}\ \mu C$$

5. A → (r)
B → (p)
C → (q)
D → (s)





When circuit is closed, let charge q flows in the circuit; then applying Loop Law in ABCDA

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting the value of q_1, q_2, q_3 and q_4 we get

$$q = \left(\frac{24}{5}\right)(CV)$$

Hence $V_1 = \dfrac{q_1 - q}{C} = \dfrac{19V}{5}$

$$V_2 = \frac{q_2 - q}{2C} = \frac{2V}{5}$$

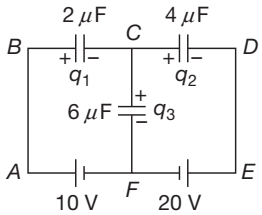
$$V_3 = \frac{q_3 - q}{3C} = \frac{7V}{5}$$

$$V_4 = \frac{q_4 - q}{4C} = \frac{14V}{5}$$

6. A → (q)
B → (r)
C → (p)
D → (s)

The charges on three plates which are in contact add to zero, because these plates taken together form an isolated system which cannot receive charges from the batteries

Thus, $q_3 - q_1 - q_2 = 0$ …(1)



Applying Kirchhoff's Law in loop ABCFA and CDEFC

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

$$\Rightarrow \quad q_3 + 3q_1 = 60 \qquad \ldots(2)$$

and $\quad 20 - \dfrac{q_2}{4} - \dfrac{q_3}{6} = 0$

$$\Rightarrow \quad 3q_2 + 2q_3 = 240 \qquad \ldots(3)$$

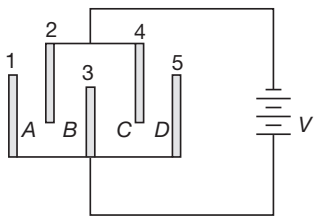
Solving the above three equations, we have

$$q_1 = \frac{10}{3}\ \mu C,\ q_2 = \frac{140}{3}\ \mu C,\ q_3 = 50\ \mu C$$

Potential difference across 6 μC is

$$V_{6\ \mu C} = \frac{q_3}{6} = \frac{50\ \mu C}{6\ \mu F} = \frac{25}{3}\ V$$

7. A → (q)
B → (p)
C → (s)
D → (r)



These five plates constitute four identical capacitors in parallel, each of capacity $\frac{\varepsilon_0 A}{d}$. Now as plate 1 is connected to positive terminal of battery and is a part of one capacitor only, so charge on it

$$q_1 = +\left(\frac{\varepsilon_0 AV}{d}\right) = 1\left(\frac{\varepsilon_0 AV}{d}\right)$$

So (A) → (q)

However the plate 4 is connected to negative terminal of battery and in common to two identical capacitors in parallel.

So $q_4 = -\frac{2\varepsilon_0 AV}{d} = -2\left(\frac{\varepsilon_0 AV}{d}\right)$

Hence (B) → (p)

From circuit, it is obvious that between the plates 2 and 3, battery is connected so potential difference will be $V\left(=\frac{1}{2}(2V)\right)$. Between 1 and 5. Plates 1 and 5 gets connected through connecting wire, so potential difference is zero $(=0(2\text{ V}))$.

Hence (C) → (s), (D) → (r)

8. A → (q)
B → (r)
C → (s)
D → (p)
Done Already

9. A → (p, s)
B → (q, r, s)
C → (q, r, s)
D → (p)

10. A → (p, q)
B → (r, q)
C → (p, q, s)
D → (r, q)

(A) For an isolated capacitor

Q = constant

So, if distance between plate decreases, then capacitance will increase.

Since $Q = CV$

$$\Rightarrow \quad V \propto \frac{1}{C}$$

So as C increases V decreases.

Since Energy $U = \frac{Q^2}{2C}$

So as C increases energy decreases.

Since Force $F = \frac{Q^2}{2A\varepsilon_0}$

So, when Q = constant, then F = constant.

(B) As battery is connected to capacitor, so V is constant

when slab is inserted, C increases

So, energy $U = \frac{1}{2}CV^2$ also increases

Since $F = \frac{CV^2}{2d}$

So, as U increases, F also increases.

(C) If area of plates of capacitor increases, then C increases.

As capacitor is isolated, so

Q = constant

Since, $V = \frac{Q}{C}$

As C increases, V decreases.

Since, $U = \frac{Q^2}{2C}$

As C increases, U decreases.

Since, $F = \frac{q^2}{2\varepsilon_0 A} = \frac{q^2}{2Cd}$

As U decreases i.e. C increases, F decreases.

(D) Capacitor is connected with battery

V = constant

As d decreases, C increases

Since, $U = \frac{1}{2}CV^2$

As C increases, U increases.

Since, $F = \frac{Q^2}{2A\varepsilon_0} = \frac{C^2V^2}{2A\varepsilon_0}$

As C increases, so F increases.

11. A → (q, s)
B → (r, s)
C → (p, r, s)
D → (p, r, s)

For A: If $V = E$, i.e. potential difference across the capacitor is same as that of e.m.f. of battery, no charge will be supplied by battery and hence work done by battery $= 0$.

$$U_i = U_f \text{ so } \Delta H = 0$$

In all cases, zero charge appears on outer surfaces of the plates of capacitor.

For B: If $V > E$, so capacitor charges the battery and hence thermal energy will be developed in circuit.

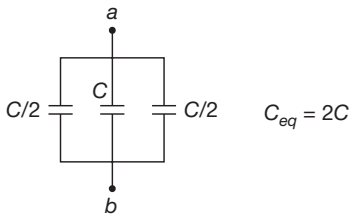
For C: If $V < E$ so battery performs some work on the capacitor.

For D: Combination of B and C.

12. A → (q)
B → (p)
C → (r)
D → (s)

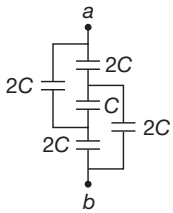
13. A → (p)
B → (p)
C → (r)
D → (q)

(A)



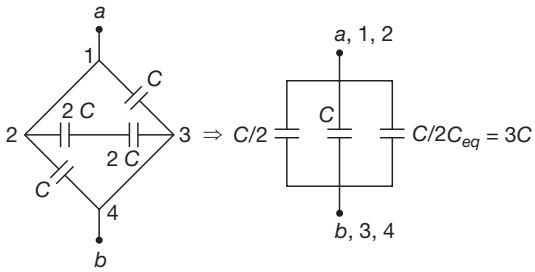
$C_{eq} = 2C$

(B)



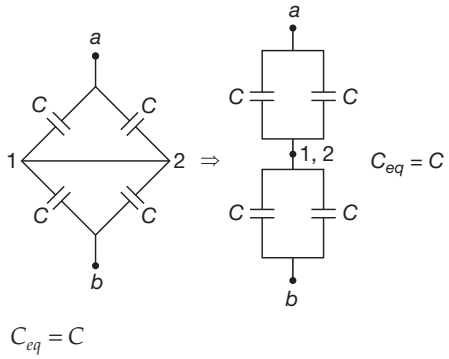
$C_{eq} = 2C$

(C)



$C_{eq} = 3C$

(D)



$C_{eq} = C$

$C_{eq} = C$

14. A → (p, s)
B → (r, s)
C → (p, q)
D → (q)

For A: Charge is constant, C increases, so energy stored decreases. Since system is isolated, so

$$\Delta U < 0 \text{ and } W_{ext} = \Delta U < 0$$

For B: Since potential is constant, so C increases, so energy stored increases

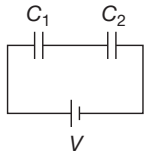
$$\Rightarrow \quad \Delta U > 0 \text{ and } W_{e\ell} = -\Delta U < 0$$

Since charge of capacitor increases, which means that work done by battery is greater then zero.

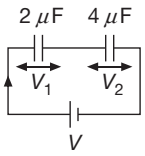
For C: Potential is constant, C decreases, Q decreases and U also decreases. So, $W_{battery} < 0$, $\Delta U < 0$, $W_{ext} > 0$

For D: Charge is constant, C decreases, U increases So, $\Delta U > 0$, $W_{ext} > 0$

15. A → (p, r)
B → (q)
C → (s, q)
D → (q)



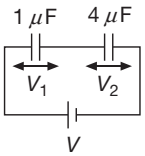
$$C = \frac{\varepsilon_0 A}{d}$$



So, $V_1 = \frac{4}{6} \times V = \frac{4}{6} V = \frac{2}{3} V = 0.6 \text{ V}$ and

$$V_2 = \frac{2}{6} \times V = \frac{V}{3} = 0.3 \text{ V}$$

When d is double d, then C becomes $C' = \frac{2}{2} = 1\ \mu\text{F}$



$$V_1' = \frac{4V}{5} = 0.8 \text{ V}$$

$$V_2' = \frac{1V}{5} = 0.2 \text{ V}$$

So, potential difference across C_1 increases and that across C_2 decreases.

Initial energy stored in

$$C_1 \text{ is } (U_1)_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 2\times\frac{16}{36}V^2 = \frac{16}{36}V^2$$

$$C_2 \text{ is } (U_2)_i = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 4\times\frac{V^2}{9} = \frac{8}{9}V^2$$

Final energy stored in

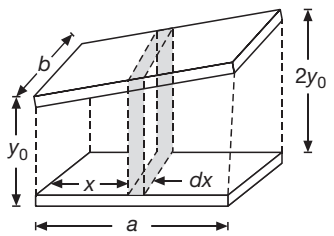
$$C_1 \text{ is } (U_1)_f = \frac{1}{2}\times 1\times\frac{16}{25}V^2 = \frac{8}{25}V^2$$

$$C_2 \text{ is } (U_2)_f = \frac{1}{2}\times 4\times\frac{V^2}{25} = \frac{2}{25}V^2$$

So, potential energy of C_1 decreases by a factor of $\frac{18}{25}$ and potential energy of C_2 decreases.

Integer/Numerical Answer Type Questions

1. Let us consider an infinitesimal strip of width dx and length b to approximate a differential capacitor of area bdx and separation $d = y_0 + \left(\frac{y_0}{a}\right)x$. All such differential capacitors are in parallel arrangement.



$$dC = \frac{\varepsilon_0(bdx)}{y_0 + \left(\frac{y_0}{a}\right)x}$$

$$C = \varepsilon_0 b\int_0^a \frac{dx}{\left(y_0 + \frac{y_0}{a}x\right)}$$

$$C = \frac{\varepsilon_0 b}{\left(\frac{y_0}{a}\right)}\left[\log_e\left(\frac{y_0 + \frac{y_0}{a}\times a}{y_0}\right)\right]$$

$$C = \frac{\varepsilon_0 ab}{y_0}\log_e 2$$

$$\Rightarrow \quad \alpha = 2$$

2. The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2\times 10^{-16}\text{ kg}\right)\left(2\times 10^{6}\text{ ms}^{-1}\right)^2$$

$$\Rightarrow \quad K = 4\times 10^{-4}\text{ J}$$

The potential difference across the capacitor is

$$\Delta V = \frac{Q}{C} = \frac{1000}{10}\frac{\mu\text{C}}{\mu\text{F}} = 100\text{ V}$$

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = \left(-3\times 10^{6}\text{ C}\right)(-100\text{ V})$$

$$\Rightarrow \quad U = 3\times 10^{-4}\text{ J}$$

Since its original kinetic energy is greater than this, the particle will reach the negative plate

As the particle moves, the system keeps constant total energy

$$(K+U)_{\text{at + plate}} = (K+U)_{\text{at-plate}}$$

$$4\times 10^{-4}\text{ J} + \left(-3\times 10^{-6}\text{ C}\right)(+100\text{ V}) = \frac{1}{2}\left(2\times 10^{-16}\right)v_f^2 + 0$$

$$\Rightarrow \quad v_f = \sqrt{\frac{2\left(1\times 10^{-4}\text{ J}\right)}{2\times 10^{-16}\text{ kg}}} = 10^{6}\text{ ms}^{-1} = 1000\text{ kms}^{-1}$$

3. Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6\ \mu\text{F})(250\text{ V}) = 1500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2\ \mu\text{F})(250\text{ V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1000\ \mu\text{C}$$

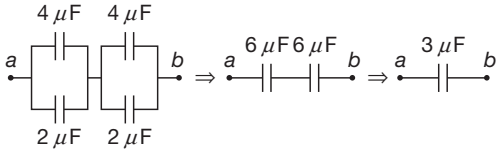
and $\Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1000\ \mu\text{C}}{8\ \mu\text{F}} = 125\text{ V}$

Therefore,

$$q'_1 = C_1(\Delta V') = (6\ \mu\text{F})(125\text{ V}) = 750\ \mu\text{C}$$

$$q'_2 = C_2(\Delta V') = (2\ \mu\text{F})(125\text{ V}) = 250\ \mu\text{C}$$

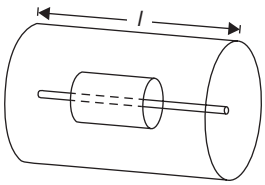
4. Assume a potential difference across a and b, and notice that the potential difference across the $8\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit.



$$C_{ab} = 3\ \mu\text{F}$$

5. $2\pi r\ell E = \frac{q_{in}}{\varepsilon_0}$

So $E = \frac{\lambda}{2\pi r\varepsilon_0}$



$$\Delta V = -\int_{r_1}^{r_2} E \cdot dr = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r\varepsilon_0} dr = \frac{\lambda}{2\pi\varepsilon_0} \log_e\left(\frac{r_1}{r_2}\right)$$

$$\frac{\lambda_{max}}{2\pi\varepsilon_0} = E_{max} r_{inner}$$

$$\Delta V = (1.2\times10^6 \text{ Vm}^{-1})(0.1\times10^{-3} \text{ m})\log_e\left(\frac{25}{0.2}\right)$$

$$\Delta V_{max} = 579 \text{ V}$$

6. Capacitance of metal surface of radius

$r = 0.5\times10^{-3}$ m

$$C = 4\pi\varepsilon_0 r = \frac{0.5\times10^{-3}}{9\times10^9} = \frac{1}{18}\times10^{-12} \text{ farad}$$

Rate of escape of charge from surface is R is

$$R = \frac{80}{100}\times ne$$

$$\Rightarrow \quad R = \frac{80}{100}\times6.25\times10^{10}\times1.6\times10^{-19} \text{ Cs}^{-1}$$

$$\Rightarrow \quad R = 8\times10^{-9} \text{ Cs}^{-1}$$

$\therefore$ If t is the required time, then charge escaped

$$q = Rt = (8\times10^{-9})t$$

From relation $q = CV$, we have

$$8\times10^{-9}t = \frac{1}{18}\times10^{-12}\times1 \qquad \{\because \text{ V} = 1 \text{ V}\}$$

$$\Rightarrow \quad t = \frac{10^{-12}}{8\times10^{-9}\times18} = \frac{10^{-3}}{144}$$

$$\Rightarrow \quad t = \frac{1000\times10^{-6}}{144} s = 6.95\ \mu s = 6950 \text{ ns}$$

7. If d is the separation between the plates (each of area A_0) of a parallel plate condenser in air, then its capacitance $C = \frac{\varepsilon_0 A}{d}$ …(1)

If a slab of thickness t is introduced between the plates with new separation d', then its new capacitance,

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1-\left(\frac{1}{K}\right)\right)} \qquad \ldots(2)$$

As $Q = CV$, the charge on the capacitor is same in both cases, therefore to maintain the same potential difference the capacitance C and C' must be same i.e., from (1) and (2).

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left(1-\left(\frac{1}{K}\right)\right)}$$

$$\Rightarrow \quad d = d' - t\left(1-\left(\frac{1}{K}\right)\right)$$

Here, $d' = d + 2.4 \text{ mm} = d + 2.4\times10^{-3}$ m

$t = 3 \text{ mm} = 3\times10^{-3}$ m

$$\Rightarrow \quad d = d + 2.4\times10^{-3} - 3\times10^{-3}\left\{1-\left(\frac{1}{K}\right)\right\}$$

$$\Rightarrow \quad 3\left\{1-\left(\frac{1}{K}\right)\right\} = 2.4$$

$$\Rightarrow \quad K = 5$$

8. (a) The capacitors of 3, 5 and 4 μF are in parallel, therefore their equivalent capacitance

$$C_1 = 3+5+4 = 12\ \mu\text{F}$$

Again the capacitor of 4 and 2 μF are equivalent to a single capacitor of capacitance

$$C_3 = 4+2 = 6\ \mu\text{F}$$

For equivalent capacitance between x and y, C_1 and C_2 are in series. If C' is the equivalent capacitance of C_1 and C_2, then

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{12\times4}{12+4} = \frac{48}{16} = 3\ \mu\text{F}$$

Similarly, C_3 and C_4 are in series and their equivalent capacitance C'' is given by

$$C'' = \frac{C_3C_4}{C_3+C_4} = \frac{6\times3}{6+3} = \frac{18}{9} = 2\ \mu\text{F}$$

For equivalent capacitance between x and y, C' and C'' are in parallel, therefore the equivalent capacitance between x and y

$$C = C' + C'' = 3+2 = 5\ \mu\text{F}$$

(b) The charge of 5 μF capacitor

$$= 120\ \mu\text{C} = 120\times10^{-6} \text{ C}$$

CHAPTER 2

The p.d. across $5\ \mu F$ capacitor

$$= \frac{120\times10^{-6}}{5\times10^{-6}} = 24\ V$$

As the capacitor 3, 5 and $4\ \mu F$ are in parallel, the p.d. across each of these capacitors is 24 V.

$\therefore$ The charge on $3\ \mu F$ capacitor

$$= 3\times10^{-6}\times24\ C$$

$$= 72\times10^{-6}\ C$$

and the charge on $4\ \mu F$ capacitor

$$= 4\times10^{-6}\times24 = 96\times10^{-6}\ C$$

Therefore, the total charge flowing through C_1 and C_2 is

$$= (72+120+96)\times10^{-6}\ C$$

$$= 288\times10^{-6}\ C$$

The potential difference across C_2

$$= \frac{288\times10^{-6}}{4\times10^{-6}} = 72\ V$$

$\therefore$ Net potential difference (p.d.) between x and y

$$= 24+72 = 96\ V$$

As equivalent capacitance of C_3 and C_4 is $2\ \mu F$, the charge flowing through C_3 and C_4

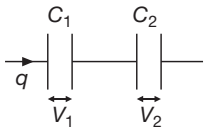
$$= 96\times2\times10^{-6}\ C$$

$$= 192\times10^{-6}\ C$$

The potential difference between x and z is

$$V_{xz} = \frac{192\times10^{-6}}{3\times10^{-6}} = 64\ V$$

9. Given $V_1 = 6\ kV$, $V_2 = 4\ kV$



Charge on first capacitor

$$q_1 = C_1V_1 = 1\ \mu F\times(6\times10^3\ V)$$

$$\Rightarrow\quad q_1 = 6000\ \mu C$$

Charge on second capacitor

$$q_2 = C_2V_2$$

$$\Rightarrow\quad q_2 = (2\ \mu F)\times(4\times10^3\ V) = 8000\ \mu C$$

In series the charge on each capacitor remains same, but maximum charge on first capacitor will be $6000\ \mu C$.

Therefore charge on second capacitor must also be $6000\ \mu C$

The potential across second capacitor

$$V_2 = \frac{q}{C_2} = \frac{6000\ \mu C}{2\ \mu F} = 3000\ V = 3\ kV$$

So, maximum voltage across system will be

$$= V_1 + V_2 = 6\ kV + 3\ kV = 9\ kV$$

10. Capacitance of parallel plate capacitor.

$$C = \frac{\varepsilon_0 A}{d}$$

Charge on capacitor $q = CV = \dfrac{\varepsilon_0 A}{d}V$

If the separation is doubled, the capacitance becomes half, i.e., here capacitance $C' = \dfrac{C}{2}$. If battery is disconnected the charge remains same i.e., q = constant.

$\therefore$ Net potential V' is given by $CV = C'V'$

$$V' = \frac{CV}{C'}$$

As $\quad C' = \dfrac{C}{2}$

$$V' = \frac{CV}{\left(\frac{C}{2}\right)} = 2V = 2\times10^3\ V$$

Work required, $W = U_2 - U_1$

$$= \frac{q^2}{2C'} - \frac{q^2}{2C} = \frac{q^2}{2}\left[\frac{1}{\frac{C}{2}} - \frac{1}{C}\right] = \frac{q^2}{2C}$$

$$= \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

Here $A = 0.2\ m^2$, $d = 10^{-2}\ m$

$$\Rightarrow\quad W = \frac{1}{2}\left(\frac{1}{36\pi\times10^7}\right)\times\frac{0.2}{10^{-2}}\times(10^3)^2$$

$$\Rightarrow\quad W = \frac{1}{36\pi}\times10^{-2}\ J = 8.8\times10^{-5}\ J$$

Final voltage (V') across capacitor is given by

$$q = CV = C'V'$$

i.e., $V' = \dfrac{CV}{C'} = \dfrac{C}{\frac{C}{2}} V = 2V = 2 \times 10^3$ V

11. First of all we shall calculate the equivalent capacitance of this arrangement. The condensers (1) and (2) are in series. Their equivalent capacitance is give by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\Rightarrow \quad C' = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow \quad C' = \frac{C_1 \times \eta C_1}{C_1 + \eta C_1} = \frac{\eta C_1}{1 + \eta} \qquad \ldots(1)$$

Now C' and capacitor (4) are in parallel. Hence

$$C'' = C' + C_2 = \frac{\eta C_1}{1 + \eta} + \eta C_1$$

$$\Rightarrow \quad C'' = \frac{(\eta^2 + 2\eta) C_1}{1 + \eta} \qquad \ldots(2)$$

Further C'' and capacitor (3) are in series. So the equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{(1+\eta)}{(\eta^2 + 2\eta) C_1} = \frac{(\eta^2 + 2\eta) + (1 + \eta)}{(\eta^2 + 2\eta) C_1}$$

$$\Rightarrow \quad C = \frac{(\eta^2 + 2\eta) C_1}{\eta^2 + 3\eta + 1} \qquad \ldots(3)$$

$$\Rightarrow \quad q = CE = \frac{(\eta^2 + 2\eta) C_1 E}{\eta^2 + 3\eta + 1} \qquad \ldots(4)$$

The voltage V across equivalent capacitor is given by

$$V = \frac{q}{C''} = \frac{(\eta^2 + 2\eta) C_1 E}{\eta^2 + 3\eta + 1} \times \frac{(1+\eta)}{(\eta^2 + 2\eta) C_1}$$

$$\Rightarrow \quad V = \frac{(1+\eta) E}{\eta^2 + 3\eta + 1} \qquad \ldots(5)$$

($\because$ The charge q will be across q_1 and equivalent capacitor C'')

The voltage V will be across C_2 and equivalent capacitor C'

$$q' = C'V = \left(\frac{\eta C_1}{1 + \eta} \right) \left[\frac{(1+\eta) E}{\eta^2 + 3\eta + 1} \right]$$

$$\Rightarrow \quad q' = \frac{\eta C_1 E}{\eta^2 + 3\eta + 1} \qquad \ldots(6)$$

The charge will be distributed across C_1 and C_2.

Voltage across C_2 is given by

$$V = \frac{q'}{C_2} = \frac{q'}{\eta C_1} = \frac{1}{\eta C_1} \left[\frac{\eta C_1 E}{\eta^2 + 3\eta + 1} \right]$$

$$\Rightarrow \quad V = \frac{E}{\eta^2 + 3\eta + 1} \qquad \ldots(7)$$

Substituting the values, we get

$$V = \frac{110}{4 + 6 + 1} = \frac{110}{11} = 10 \text{ V}.$$

12. Charge on 10 μF condenser

$$q_1 = 10 \times 10^{-6} \times 150 \text{ C} = 2.5 \times 10^{-3} \text{ C}$$

Charge on 20 μF condenser

$$q_2 = 20 \times 10^{-6} \times 300 \text{ C} = 6 \times 10^{-3} \text{ C}$$

When the two condensers are connected in parallel, the total charge

$$q = (1.5 \times 10^{-3} + 6 \times 10^{-3}) = 7.5 \times 10^{-3} \text{ C} = 7500 \ \mu\text{C}$$

The equivalent capacitance $= 30 \ \mu$F

If V be the potential, then

$$30 \times 10^{-6} V = 7.5 \times 10^{-3}$$

$$\Rightarrow \quad V = \frac{7.5 \times 10^{-3}}{30 \times 10^{-6}} = 250 \text{ V}$$

Energy of first condenser before connection

$$U_1 = \frac{1}{2} \times (1.5 \times 10^{-3}) 150$$

Energy of second condenser before connection

$$U_2 = \frac{1}{2} \times (6 \times 10^{-3}) 300$$

Total energy of condensers before connection

$$U_{\text{initial}} = \frac{1}{2} \times (1.5 \times 10^{-3}) 150 + \frac{1}{2} (6 \times 10^{-3}) 300$$

$$\Rightarrow \quad U_i = 1.0125 \text{ J}$$

Energy after connection

$$U_f = \frac{1}{2} \times (7.5 \times 10^{-3}) 250 = 0.9375 \text{ J}$$

Energy lost

$$-\Delta U = 1.0125 - 0.9375 = 0.075 \text{ J} = 75 \text{ mJ}$$

13. The capacitance C_0 before the slab is introduced

$$V_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.9 \times 10^{-12})(10^{-2})}{10^{-2}} = 8.9 \times 10^{-12} \text{ farad}$$

CHAPTER 2

$\therefore$ Free charge

$$q = C_0V_0 = (8.9\times10^{-12})\times100 = 890\times10^{-12}\ \text{C} = 890\ \text{pC}$$

Now, Electric field intensity

$$E_0 = \frac{V_0}{d} = \frac{100}{10^{-2}} = 1\times10^4\ \text{Vm}^{-1} = 10\ \text{kVm}^{-1}$$

Electric field intensity in dielectric

$$E = \frac{E_0}{K} = \frac{1\times10^4}{7} = 1.43\times10^3\ \text{Vm}^{-1} = 1430\ \text{Vm}^{-1}$$

Potential difference between the plates with dielectric present is given by

$$V = E_0(d-b)+Eb$$

$$\Rightarrow\quad V = (1\times10^4)(10^{-2}-0.6\times10^{-2})+(1.43\times10^3)(0.5\times10^{-2})$$

$$\Rightarrow\quad V = 57\ \text{V}$$

The free charge on the plate is the same as before. The capacitance with dielectric present is

$$C = \frac{q}{V} = \frac{8.9\times10^{-10}\ \text{C}}{57\ \text{V}} = 16\times10^{-12}\ \text{farad} = 16\ p\text{F}$$

14. Potential difference without introducing the slab is given by

$$V_0 = E\times d = 300\times5 = 1500\ \text{V}$$

The capacitance C_0 is given by

$$C_0 = \frac{\varepsilon_0A}{d} = \frac{\varepsilon_0A}{0.05}\ \text{farad}$$

$$\Rightarrow\quad \varepsilon_0A = 0.05C_0 \qquad \ldots(1)$$

When the slab is inserted, the capacitance C is given by

$$C = \frac{\varepsilon_0A}{d-t+\frac{t}{K}}\quad \text{where } t = \text{thickness of slab}$$

$$\Rightarrow\quad C = \frac{0.05C_0}{0.05+\frac{0.01}{5}} = \frac{25C_0}{21} \qquad \ldots(2)$$

Now, $C_0V_0 = CV$

$$C_0\times1500 = \frac{25C_0}{21}\times V$$

$$\Rightarrow\quad V = \frac{1500\times21}{25} = 1260\ \text{V}$$

Let x be the thickness of the metal plate. Now the effective distance in air between the plates is $(5-x)$ cm.

Here it should be remembered that the metal sheet splits the single capacitor into two in series.

$$\therefore\quad C_f = \frac{\varepsilon_0A}{\frac{(5-x)}{100}}\ \text{farad}$$

Again $CV = C_0V_0 = C_fV_f$

$$\Rightarrow\quad V_f = V_0\frac{C_0}{C_f} = 1500\times\frac{\varepsilon_0A}{0.05}\times\frac{(5-x)}{\varepsilon_0A\times100}$$

According to the problem $V_f = 1260$ V

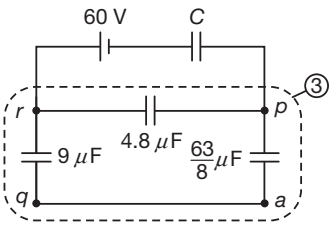
$$\Rightarrow\quad 1500\times\frac{\varepsilon_0A}{0.05}\times\frac{(5-x)}{\varepsilon_0A\times100} = 1260$$

$$\Rightarrow\quad (5-x) = 4.2$$

$$\Rightarrow\quad x = 0.8\ \text{cm} = 8\ \text{mm}$$

15. The 3 μF and 5 μF capacitors are in series, the potential difference across them will be in inverse ratio of capacitance.

Thus P.D. across 5 μF capacitor $= 10\times\frac{3}{5} = 6$ V

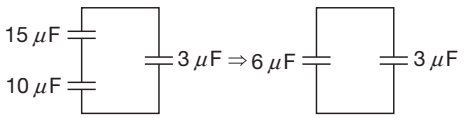


P.D. across p and b $= (10+6)V = 16$ V

Charge on 6 μF capacitor $= (16\times6)\mu\text{C} = 96\ \mu C$

Total charge in enclosed portion 1 of the circuit

$$= (96+30)\ \mu\text{C} = 126\ \mu\text{C}$$



The two enclosed portions (1) and (2) are in series combination as shown in figure. Charge on 9 μF capacitor $= 126\ \mu$C .

If Q_1 and Q_2 are charges on 3 μF and 6 μF branches respectively.

$$\frac{Q_1}{3}=\frac{Q_2}{6} \qquad \ldots(1)$$

and $Q_1+Q_2=126\ \mu C$ $\ldots(2)$

Now solving equations (1) and (2) simultaneously, we get

$Q_1=42\ \mu C$,

$Q_2=84\ \mu C$

The P.D. across $3\ \mu F$ capacitor (that is between points q and r)

$$=\frac{Q_1}{C}=\frac{42}{3}=14\ V$$

Thus the P.D. between p and $r=(16+14)$ V=30 V

The total charge on $4.8\ \mu F$ capacitor

$=(4.8\times 30)\mu C=144\ \mu C$

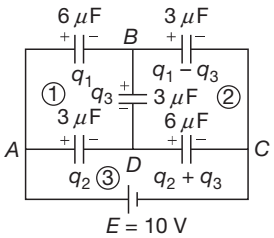
Total charge in the enclosed region 3

$=144\ \mu C+126\ \mu C=270\ \mu C$

The capacitor C is in series with enclosed portion 3, so charge on it is $270\ \mu C$ and potential $(60-30)$ V

Hence $C=\left(\frac{270}{60-30}\right)\mu F=9\ \mu F$

16. Let the charge on each capacitor be as shown in figure. Applying *KVL* in the loop (1), traversing clock-wise,



$$-\frac{q_1}{6}-\frac{q_3}{3}+\frac{q_2}{3}=0$$

$\Rightarrow \quad -q_1-2q_3+2q_2=0$ $\ldots(1)$

For loop (2), traversing clockwise, we have

$$-\left(\frac{q_1-q_3}{3}\right)+\frac{q_2+q_3}{6}+\frac{q_3}{3}=0$$

$$-2q_1+5q_3+q_2=0 \qquad \ldots(2)$$

For loop $ABCEA$, traversing clockwise, we have

$$-\frac{q_1}{6}-\frac{(q_1-q_3)}{3}+10=0$$

$3q_1-2q_3=60$ $\ldots(3)$

Solving equations (1), (2) and (3), we get

$q_1=24\ \mu C$;

$q_2=18\ \mu C$;

$q_3=6\ \mu C$

Charge on $6\ \mu F$ capacitor is $24\ \mu C$

Charge on $3\ \mu F$ capacitor is $18\ \mu C$

17. (a) $C=\frac{Q}{\Delta V}$

$\Rightarrow \quad 6\times 10^{-6}=\frac{Q}{20}$ and $Q=120\ \mu C$

(b) Since $Q=120\ \mu C$

$\Rightarrow \quad Q_1+Q_2=120\ \mu C$

$\Rightarrow \quad Q_1=120-Q_2$ $\ldots(1)$

Also, $\frac{Q_1}{C_1}=\frac{Q_2}{C_2}$

$\Rightarrow \quad \frac{120-Q_2}{6}=\frac{Q_2}{3}$

$\Rightarrow \quad Q_2=40\ \mu C$ and $Q_1=80\ \mu C$

18. With switch closed, distance $d'=\frac{d}{2}$ and capacitance

$$C'=\frac{\varepsilon_0 A}{d'}=\frac{2\varepsilon_0 A}{d}=2C$$

(a) $Q=C'(\Delta V)=2C(\Delta V)=2(2\times 10^{-6}\ F)(100\ V)$

$\Rightarrow \quad Q=400\ \mu C$

(b) The force stretching out one spring is

$$F=\frac{Q^2}{2\varepsilon_0 A}=\frac{4C^2(\Delta V)^2}{2\varepsilon_0 A}=\frac{2C^2(\Delta V)^2}{\left(\frac{\varepsilon_0 A}{d}\right)d}$$

$$F=\frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x=\frac{d}{4}$

CHAPTER 2

$$\Rightarrow \quad k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2}$$

$$\Rightarrow \quad k = \frac{8(2\times10^{-6}\text{ F})(100\text{ V})^2}{(8\times10^{-3}\text{ m})^2}$$

$$\Rightarrow \quad k = 2500\text{ Nm}^{-1}$$

19. The energy transferred is

$$H_{ET} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50\text{ C})(1\times10^8\ V)$$

$$H_{ET} = 2.5\times10^9\text{ J}$$

and 1% of this is absorbed by the tree
So, energy absorbed is given by

$$\Delta E_{\text{absorbed}} = 2.5\times10^7\text{ J}$$

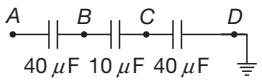
If m is the amount of water boiled away, then

$$\Delta E_{\text{int}} = m(4186\text{ Jkg}^{-1}\text{ °C})(100\text{ °C} - 30\text{ °C}) + m(2.26\times10^6\text{ Jkg}^{-1}) = 2.5\times10^7\text{ J}$$

$$\Rightarrow \quad m = 9.79\text{ kg}$$

So, $m \cong 10$ kg

20. The given combination of capacitors is equivalent to the circuit diagram shown in the figure



Assume the charge on point A to be Q. Then,

$$Q = (40\ \mu\text{F})\Delta V_{AB} = (10\ \mu\text{F})\Delta V_{BC} = (40\ \mu\text{F})\Delta V_{CD}$$

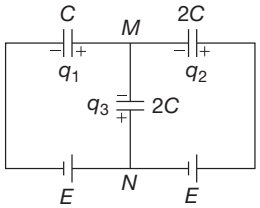
$$\Rightarrow \quad \Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$$

So, the centre capacitor will break down first, at $\Delta V_{BC} = 15$ V and when this occurs, we have

$$\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 4\text{ V}$$

$$\Rightarrow \quad V_{AD} = V_{AB} + V_{BC} + V_{CD} = 4\text{ V} + 16\text{ V} + 4\text{ V} = 24\text{ V}$$

21. Let $C_1 = C$ and $C_2 = 2C$ and the charges on the different capacitors be as shown in figure.



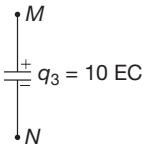
Net charge on isolated system should be zero.

Hence, $q_1 - q_2 - q_3 = 0$ …(1)

Applying Loop's Law in two loops we have,

$$E - \frac{q_3}{2C} - \frac{q_1}{C} = 0 \qquad \ldots(2)$$

$$E - \frac{q_2}{2C} + \frac{q_3}{2C} = 0 \qquad \ldots(3)$$



Solving these three equations, we get

$$q_3 = -10EC$$

$$\Rightarrow \quad V_{MN} = \frac{q_3}{2C} = 5E$$

$$\Rightarrow \quad V_{MN} = 550\text{ V}$$

22. $$W_{\text{ext}} = \frac{q^2}{2\varepsilon_0 A}(x_f - x_i) = \frac{q^2}{2\varepsilon_0 A}(2d - d) = \frac{q^2}{2\varepsilon_0 A}d$$

$$\Rightarrow \quad W_{\text{ext}} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{q^2}{2C} = \frac{(20\times10^{-12}\times200)^2}{2\times(20\times10^{-12})}$$

$$\Rightarrow \quad W_{\text{ext}} = 0.4\ \mu\text{J} = 400\text{ nJ}$$

23. $$W = U_f - U_i = \frac{\varepsilon_0 A V^2}{2}\left(\frac{1}{x_f} - \frac{1}{x_i}\right) = \frac{V^2}{2}(C_f - C_i)$$

$$\Rightarrow \quad W = 160\text{ mJ}$$

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1. $C = \dfrac{k\varepsilon_0 A}{d}$

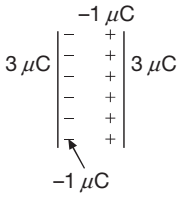
Since, $E = \dfrac{V}{d}$

$\Rightarrow \quad 15\times10^{-12} = \dfrac{k\times8.86\times10^{-12}\times10^{-4}\times10^{6}}{500}$

$k = 8.5$

Hence, the correct answer is (C).

2.



Since $V = \dfrac{q}{C} = \dfrac{1\ \mu C}{1\ \mu F}$

$\Rightarrow \quad V = 1\ V$

Hence, the correct answer is (B).

3. $U_i = \dfrac{Q^2}{2C_1}$ and $U_f = \dfrac{Q^2}{2C_2}$

Since $W = U_f - U_i$

$\Rightarrow \quad W = \dfrac{Q^2}{2}\left(\dfrac{1}{C_2} - \dfrac{1}{C_1}\right)$

$\Rightarrow \quad W = \dfrac{(5)^2}{2}\left[\dfrac{1}{2} - \dfrac{1}{5}\right]$

$\Rightarrow \quad W = 3.75\ \mu J$

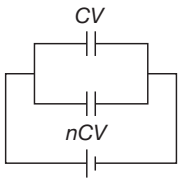
Hence, the correct answer is (C).

4. Initially, we have $C_{eq} = C + nC = (1+n)C$

$\Rightarrow \quad Q = CV = CV(1+n)$ …(1)

Finally, $C'_{eq} = KC + nC = (K+n)C$

$\Rightarrow \quad Q' = C'V' = (K+n)CV'$ …(2)



Since $Q =$ constant because system is disconnected from battery, so

$Q = Q'$

$\Rightarrow \quad CV(1+n) = (K+n)CV'$

$\Rightarrow \quad V' = \dfrac{V(n+1)}{K+n}$

$\Rightarrow \quad V = \dfrac{CV(1+n)}{(K+n)C} = \dfrac{V(1+n)}{(K+n)}$

Hence, the correct answer is (A).

5. Equivalent capacitance for series combination is

$C' = \dfrac{C_1C_2}{C_1+C_2}$

For parallel combination, $C'' = C_1 + C_2$

Since, $C'' > C'$

$\Rightarrow \quad C_1 + C_2 = \dfrac{500}{10} = 50\ \mu F$ …(1)

and $\dfrac{C_1C_2}{C_1+C_2} = \dfrac{80}{10} = 8\ \mu F$

$\Rightarrow \quad C_1C_2 = 400\ \mu F$ …(2)

Solving equations (1) and (2), we get

$C_1 = 40\ \mu F \quad C_2 = 10\ \mu F$

Hence, the correct answer is (A).

6. $\dfrac{1}{C_1} = \dfrac{d}{3A\varepsilon_0}\left(\dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{K_3}\right)$

$\Rightarrow \quad C_1 = \dfrac{3A\varepsilon_0(K_1K_2K_3)}{d(K_1K_2 + K_2K_3 + K_3K_1)}$

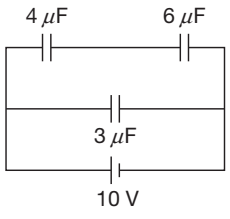
$\Rightarrow \quad C_2 = \dfrac{A\varepsilon_0}{3d}(K_1 + K_2 + K_3)$

$\Rightarrow \quad \dfrac{E_1}{E_2} = \dfrac{C_1}{C_2} = \dfrac{3K_1K_2K_3}{(K_1K_2 + K_2K_3 + K_3K_1)} \times \dfrac{3}{(K_1 + K_2 + K_3)}$

$\Rightarrow \quad \dfrac{E_1}{E_2} = \dfrac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}$

Hence, the correct answer is (C).

7.



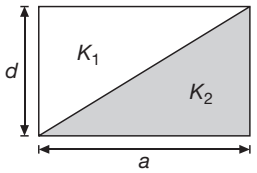
CHAPTER 2

$$Q = Q_{(4\ \mu F)} = \left(\frac{4\times 6}{4+6}\right)\times 10\ \mu C$$

$$\Rightarrow \quad Q = 24\ \mu C$$

Hence, the correct answer is (D).

8. Since $C_{eq} = \dfrac{\varepsilon_0 K_1 K_2 a^2 \ln\dfrac{K_1}{K_2}}{(K_1 - K_2)d}$ where $K_2 = K$ and $K_1 = 1$



$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 K a^2 \ln\left(\dfrac{1}{K}\right)}{d(1-K)}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 K a^2}{d(K-1)}\ln K$$

Hence, the correct answer is (A).

9. $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} + \dfrac{C_3 C_4}{C_3 + C_4}$

where $C_1 = K_1 C_0$ and $C_0 = \dfrac{\varepsilon_0 \dfrac{A}{2}}{\dfrac{d}{2}}$

Similarly

$$C_2 = K_2 C_0$$

$$C_3 = K_3 C_0$$

$$C_4 = K_4 C_0$$

$$\Rightarrow \quad K_{eq}\left(\frac{\varepsilon_0 A}{d}\right) = \left(\frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 \dfrac{A}{2}}{\dfrac{d}{2}}$$

$$\Rightarrow \quad K_{eq} = \frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}$$

*No given option is correct.

10. $C = C_1 + C_2 + C_3$

$$\frac{\varepsilon_0 K A}{d} = \frac{\varepsilon_0 K_1 A}{3d} + \frac{\varepsilon_0 K_2 A}{3d} + \frac{\varepsilon_0 K_3 A}{3d}$$

$$\Rightarrow \quad K = \frac{K_1 + K_2 + K_3}{3} = \frac{10 + 12 + 14}{3} = 12$$

Hence, the correct answer is (C).

11. $U_{in} = \dfrac{1}{2}\times 12\times 10^{-12}\times 100$

$$\Rightarrow \quad U_{in} = 600\ \text{pJ}$$

Since $Q = CV$

$$\Rightarrow \quad Q = 120\ \text{pC}$$

$$U_{final} = \frac{1}{2}\times\frac{120\times 120\times 10^{-24}\times 2}{12\times 10^{-12}\times 13}$$

$$\Rightarrow \quad U_{final} = 92.3\ \text{pJ}$$

$$\Rightarrow \quad W_{cap} = U_{initial} - U_{final} \approx 508\ \text{pJ}$$

Hence, the correct answer is (C).

12. Let the charges be Q_1 and Q_2, then

$$\frac{Q_1}{6} = \frac{Q_2}{4}$$

Also, $Q_1 + Q_2 = 30$

$$\Rightarrow \quad Q_1 = 18\ \mu C,\ Q_2 = 12\ \mu C$$

Hence, the correct answer is (A).

13. Since for combination in OPTION (A), we have

$$C_{eq} = \frac{6\times\left(\dfrac{2}{4}\right)}{6 + \dfrac{2}{4}}$$

$$\Rightarrow \quad C_{eq} = \frac{3}{6 + \dfrac{1}{2}} = \frac{6}{13}\ \mu F$$

Hence, the correct answer is (A).

14. Heat produced is given by

$$\Delta H = \Delta U$$

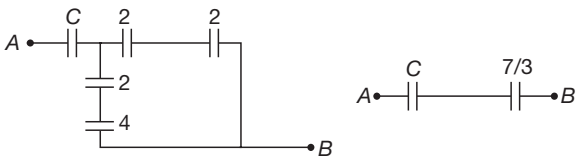
$$\Rightarrow \quad \Delta H = \frac{1}{2}\times\frac{C\times 3C}{C + 3C}(V^2)$$

$$\Rightarrow \quad \Delta H = \frac{1}{2}\times\frac{3}{4}CV^2 = \frac{3}{8}CV^2$$

$$\Rightarrow \quad \Delta H = \frac{3}{8}\frac{Q^2}{C}$$

Hence, the correct answer is (A).

15.



$$\Rightarrow \quad \frac{\frac{7}{3}C}{C+\frac{7}{3}} = \frac{1}{2}$$

$$\Rightarrow \quad 14C = 3C + 7$$

$$\Rightarrow \quad C = \frac{7}{11}\ \mu\text{F}$$

Hence, the correct answer is (A).

16. $\text{I} = \dfrac{dq}{dt} = \text{Slope of } q-t \text{ graph.}$

$$\Rightarrow \quad I = 0$$

Hence, the correct answer is (B).

17. Induced charge on dielectric, $Q_{ind} = Q\left(1-\dfrac{1}{K}\right)$

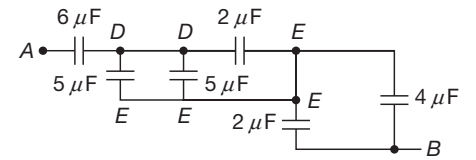
Final charge on capacitor, $Q = K(C_0V)$

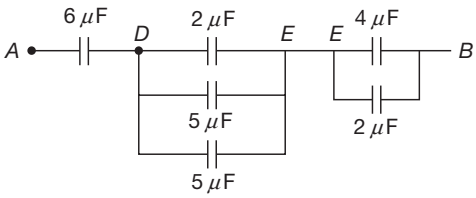
$$\Rightarrow \quad Q = \frac{5}{3}\times 90\times 10^{-12}\times 20 = 3\times 10^{-9}C = 3\text{ nC}$$

$$\Rightarrow \quad Q_{ind} = 3\left(1-\frac{3}{5}\right) = 3\times\frac{2}{5} = 1.2\text{ nC}$$

Hence, the correct answer is (A).

18.





$$\frac{1}{C_{eq}} = \frac{1}{6}+\frac{1}{12}+\frac{1}{6} = \frac{5}{12}$$

$$\Rightarrow \quad C_{eq} = \frac{12}{5} = 2.4\ \mu\text{F}$$

Hence, the correct answer is (A).

19. Given $A = 200\text{ cm}^2 = 2\times 10^{-2}\text{ m}^2$

$d = 1.5\text{ cm} = 1.5\times 10^{-2}\text{ m}$, $F = 25\times 10^{-6}\text{ N}$, $V = ?$

Since, $F = \dfrac{1}{2}(\varepsilon_0 A)\dfrac{V^2}{d^2}$

$$\Rightarrow \quad V = d\sqrt{\frac{2F}{\varepsilon_0 A}}$$

$$\Rightarrow \quad V = 1.5\times 10^{-2}\sqrt{\frac{2\times 25\times 10^{-6}}{8.85\times 10^{-12}\times 2\times 10^{-2}}}$$

$$\Rightarrow \quad V = 1.5\times 10^{2}\sqrt{\frac{25}{8.85}}$$

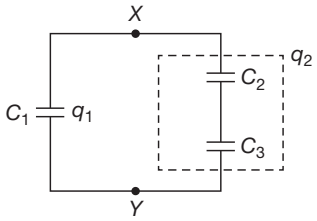
$$\Rightarrow \quad V \approx 250\text{ V}$$

Hence, the correct answer is (C).

20. Initially, potential on C_1 is $V_0 = 60$ V

$$\Rightarrow \quad q_0 = C_1V_0 = 1(\mu F)(60\text{ V}) = 60\ \mu\text{C}$$

Finally, circuit can be redrawn as shown.



Charge starts flowing from C_1 till the potential difference across C_1 is equal to potential difference across series combination of C_2 and C_3.

$$\Rightarrow \quad \frac{q_1}{C_1} = \frac{q_2}{\frac{C_2C_3}{C_2+C_3}}$$

Since, $C_1 = 1\ \mu\text{F}$, $C_2 = 3\ \mu\text{F}$, $C_3 = 6\ \mu\text{F}$

$$\Rightarrow \quad q_1 = \frac{q_2}{2}$$

$$\Rightarrow \quad q_2 = 2q_1 \qquad \ldots(1)$$

Also, $q_1 + q_2 = 60$ $\qquad \ldots(2)$

From (1) and (2), we get

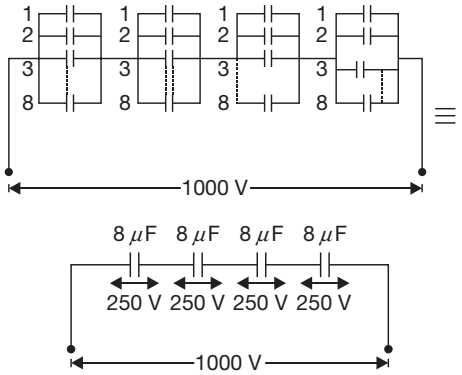
$q_1 = 20\ \mu\text{C}$ and $q_2 = 40\ \mu\text{C}$

Hence, the correct answer is (C).

21. Following arrangement will meet the required condition.

Eight capacitors of $1\ \mu\text{F}$ in parallel with four such branches in series.

CHAPTER 2



Hence, the correct answer is (D).

22. The energy stored in the electric field produced by a metal sphere is 4.5 J

$$\Rightarrow \quad \frac{Q^2}{2C} = 4.5$$

$$\Rightarrow \quad C = \frac{Q^2}{2\times 4.5} \qquad \ldots(1)$$

Capacitance of spherical conductor is $C = 4\pi\varepsilon_0 R$

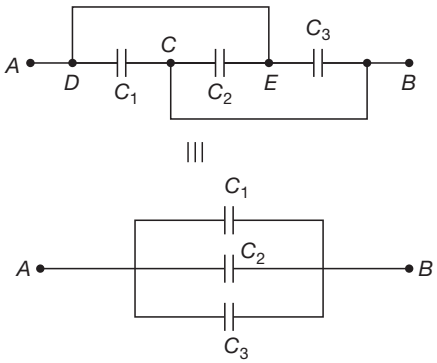
$$\Rightarrow \quad C = 4\pi\varepsilon_0 R = \frac{Q^2}{2\times 4.5} \qquad \{\text{From (1)}\}$$

$$\Rightarrow \quad R = \frac{1}{4\pi\varepsilon_0}\times\frac{(4\times 10^{-6})^2}{2\times 4.5} = 9\times 10^9 \times \frac{16}{9}\times 10^{-12}$$

$$\Rightarrow \quad R = 16\times 10^{-3}\ \text{m} = 16\ \text{mm}$$

Hence, the correct answer is (C).

23.



As the capacitors are in parallel combination so they have equal potential differences.

$$C_{\text{before}} = \frac{\varepsilon_0 A}{3} \qquad \ldots(1)$$

$$C_{\text{after}} = \frac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{\frac{k\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}} \qquad \ldots(2)$$

From (1) and (2), $\frac{\varepsilon_0 A}{3} = \frac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{k\frac{\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$

$$\Rightarrow \quad 3k = 2.4\,k + 3$$

$$\Rightarrow \quad 0.6k = 3$$

$$\Rightarrow \quad k = \frac{3}{0.6}$$

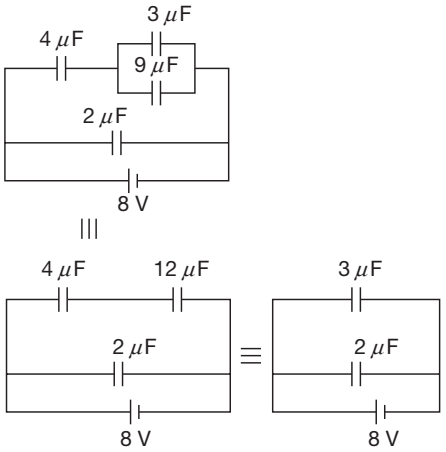
$$\Rightarrow \quad k = 5$$

Hence, the correct answer is (B).

24. 3 μF and 9 μF are in parallel combination, so the equivalent capacitance is $C_{eq} = (3+9) = 12\ \mu\text{F}$

Now, 4 μF and 12 μF are in series, so their equivalent capacitance is $C'_{eq} = \frac{4\times 12}{16} = 3\ \mu\text{F}$

Charge on 3 μF is $q_1 = (3\ \mu\text{F})\times(8\ \text{V}) = 24\ \mu\text{C}$



So charge on 4 μF and 12 μF are same (i.e. 24 μC) as they are in series.

Charge on 9 μF is $q_2 = \left(\frac{9}{9+3}\right)\times 24\ \mu C = 18\ \mu\text{C}$

So, required charge Q = Charge on 4 μF + Charge on 9 μF

$\Rightarrow \quad Q = (24+18)\ \mu C = 42\ \mu C$ and hence the required electric field is

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

$$\Rightarrow \quad E = (9\times10^9)\frac{(42\times10^{-6})}{(30)^2} = 420\ \text{NC}^{-1}$$

Hence, the correct answer is (C).

25. For (A), $\frac{1}{C_a} = \frac{1}{4}+\frac{1}{4}+\frac{1}{4} = \frac{3}{4} \Rightarrow C_a = \frac{4}{3}\mu F$

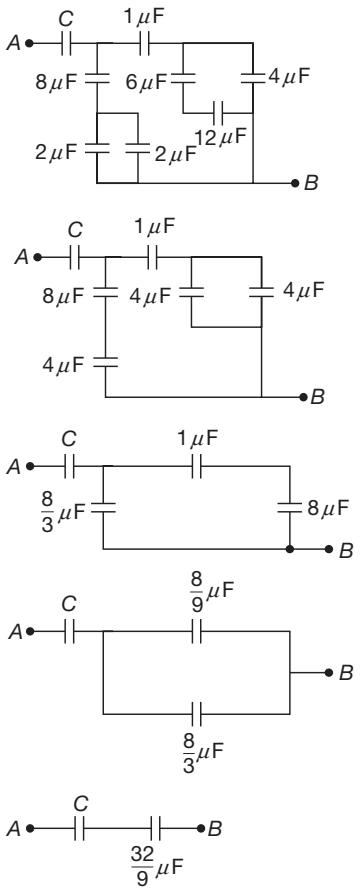
For (B), $C_b = 4+4+4 = 12\ \mu F$

For (C), $C_c = \frac{(4+4)\times4}{(4+4)+4} = \frac{8}{3}\mu F$

For (D), $C_d = \frac{4\times4}{4+4}+4 = 6\ \mu F$

Hence, the correct answer is (D).

26.



Here $C_{AB} = 1\ \mu F = \dfrac{C\times\frac{32}{9}}{C+\frac{32}{9}}$

$$\Rightarrow \quad C+\frac{32}{9} = \frac{32C}{9}$$

$$\Rightarrow \quad \frac{23}{9}C = \frac{32}{9}$$

$$\Rightarrow \quad C = \frac{32}{23}\mu F$$

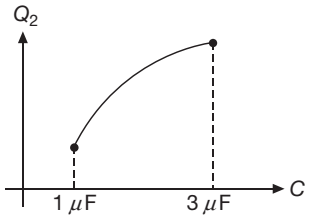
Hence, the correct answer is (A).

27. $Q_2 = \frac{2Q}{2+1} = \frac{2Q}{3}$

Now, $Q = C_{net}E$

$$\Rightarrow \quad Q = \left(\frac{3C}{3+C}\right)E$$

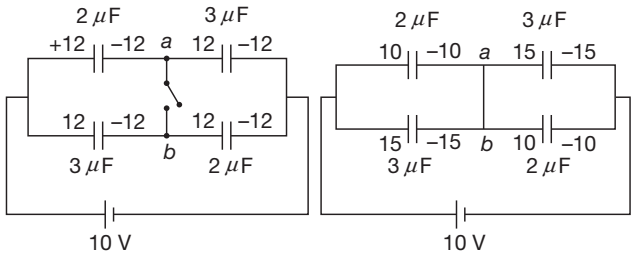
$$\Rightarrow \quad Q_2 = \frac{2}{3}\left(\frac{3CE}{3+C}\right) = \frac{2CE}{3+C}$$



Hence, the correct answer is (B).

28. For upper and lower links, $C_{eq} = \frac{6}{5}\mu F$

$$\Rightarrow \quad Q_{\text{upper}} = Q_{\text{lower}} = 12\ \mu C$$



On closing the switch, charge on $2\ \mu F$ is $10\ \mu C$ and that on $3\ \mu F$ is $15\ \mu C$

At a, we have $q_i = -12+12 = 0$ and

$$q_f = 15 - 10 = 5\ \mu C$$

So, charge flowing from b to a is $5\ \mu C$.

Hence, the correct answer is (A).

29. Inside the dielectric,

$$E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0} = 3\times 10^4$$

$$\Rightarrow \quad \sigma = K\varepsilon_0 (3\times 10^4) = 6\times 10^{-7}\ Cm^{-2}$$

Hence, the correct answer is (A).

30. For this, charge must be same $Q = C_1V_1 = C_2V_2$

$$\Rightarrow \quad 120C_1 = 200C_2$$

$$\Rightarrow \quad 3C_1 = 5C_2$$

Hence, the correct answer is (B).

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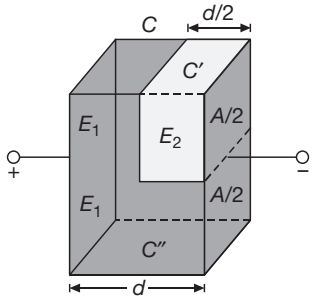
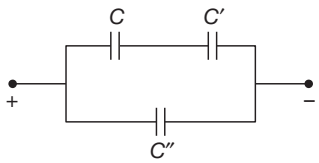
Single Correct Choice Type problems

1.





$$C_1 = \frac{\varepsilon_0 A}{d},\ C = \frac{2\varepsilon_0\left(\frac{A}{2}\right)}{\frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$$

$$C' = \frac{4\varepsilon_0\left(\frac{A}{2}\right)}{\frac{d}{2}} = \frac{4\varepsilon_0 A}{d}$$

and $C'' = \dfrac{2\varepsilon_0\left(\frac{A}{2}\right)}{d} = \dfrac{\varepsilon_0 A}{d}$

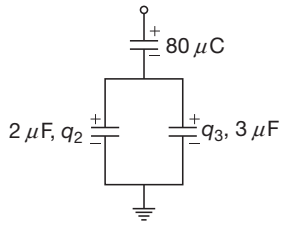
Since, $C_2 = \dfrac{CC'}{C+C'} + C'' = \dfrac{4}{3}\dfrac{\varepsilon_0 A}{d} + \dfrac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad C_2 = \frac{7}{3}\frac{\varepsilon_0 A}{d}\frac{C_2}{C_1} = \frac{7}{3}$$

Hence, the correct answer is (D).

2. Total charge of $80\ \mu C$ will distribute between $2\ \mu F$ and $2\ \mu F$ capacitors (in parallel) in direct ratio of capacitances. So

$$\frac{q_3}{q_2} = \frac{3}{2}$$



$$\Rightarrow \quad q_3 = \left(\frac{3}{3+2}\right)(80) = 48\ \mu C$$

Hence, the correct answer is (C).

3. $q_i = C_1V = 2V = q$

This charge remains constant after the switch is shifted from position 1 to position 2. So,

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2\times 2} = \frac{q^2}{4} \text{ and}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2\times 10} = \frac{q^2}{20}$$

So, energy dissipated is

$$-\Delta U = U_i - U_f = \frac{q^2}{5}$$

This energy dissipated $\left(=\dfrac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(=\dfrac{q^2}{4}\right)$

Hence, the correct answer is (D).

4. $\text{Loss} = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1-V_2)^2$

$\Rightarrow \quad \text{Loss} = \frac{1}{2}\left(\frac{C}{2}\right)(V_1-V_2)^2 = \frac{1}{4}C(V_1-V_2)^2$

Hence, the correct answer is (C).

5. When the switch is closed, the inner plates of the two capacitors get connected whereas the outer plates still are not connected and hence the circuit is not complete.

Hence, the correct answer is (A).

6. k_1 in series with half of k_3 and hence equivalent dielectric constant is $\frac{k_1k_3}{k_1+k_3}$

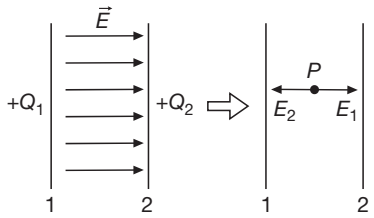
k_2 in series with half of k_3 and hence equivalent dielectric constant is $\frac{k_2k_3}{k_2+k_3}$ and then both in parallel to give $k = \frac{k_1k_3}{k_1+k_3} + \frac{k_2k_3}{k_2+k_3}$

Hence, the correct answer is (D).

7. Electric field at point P within the plates is $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$

$$E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_o} - \frac{Q_2}{2A\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Q_1 - Q_2}{2A\varepsilon_o}$$

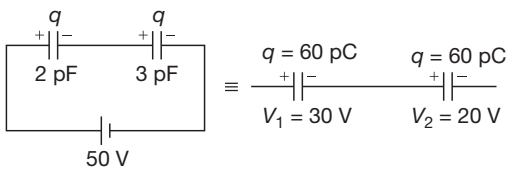


So, potential difference between the plates is

$$V_A - V_B = Ed = \left(\frac{Q_1-Q_2}{2A\varepsilon_o}\right)d = \frac{Q_1-Q_2}{2\left(\frac{A\varepsilon_o}{d}\right)} = \frac{Q_1-Q_2}{2C}$$

Hence, the correct answer is (D).

8. Charges on the capacitors are



$q_1 = (30)(2) = 60$ pC

$\Rightarrow \quad q_2 = (20)(3) = 60$ pC

$\Rightarrow \quad q_1 = q_2 = q$ (say)

The situation is similar to the two capacitors in series which are first charged with a battery of emf 50 V and then disconnected.

So, when S_3 is closed, $V_1 = 30$ V and $V_2 = 20$ V

Other way of looking at the thing is, when S_1 and S_3 both are closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

Hence, the correct answer is (D).

9. $\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}q$

$\Rightarrow \quad E(2\pi r\ell) = \frac{1}{\varepsilon_0}(\lambda\ell)$

$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Hence, the correct answer is (C).

10. Total Initial Charge

$$Q_i = (2C)(2V) - CV = 3CV$$

Total Final Charge

$$Q_f = 2CV' + CV'$$

where V' is common potential.

By Law of Conservation of Charge

$$Q_i = Q_f$$

$\Rightarrow \quad 3CV = 3CV'$

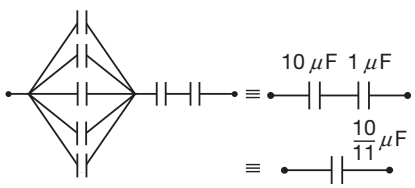
$\Rightarrow \quad V' = V$

So, final energy of combination is

$$U_f = \frac{1}{2}(C+2C)V^2 = \frac{3}{2}CV^2$$

Hence, the correct answer is (B).

11.



Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. $C = C_1 + C_2$

$$C_1 = \frac{k\varepsilon_0 \frac{A}{3}}{d}$$

$$C_2 = \frac{\frac{\varepsilon_0 2A}{3}}{d}$$

$$\Rightarrow \quad C = \frac{(K+2)\varepsilon_0 A}{3d}$$

$$\Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \frac{V}{d}$, where V is potential difference between the plates.

Hence, (A) and (D) are correct.

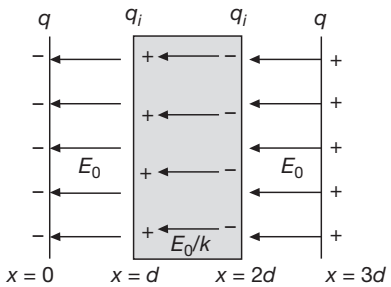
2. After pressing S_1 charge on upper plate of C_1 is $+2CV_0$. After pressing S_2 this charge equally distributes in two capacitors. Therefore charge an upper plates of both capacitors will be $+CV_0$.

When S_2 is released and S_3 is pressed, charge on upper plate of C_1 remains unchanged $(=+CV_0)$ but charge on upper plate of C_2 is according to new battery $(=-CV_0)$.

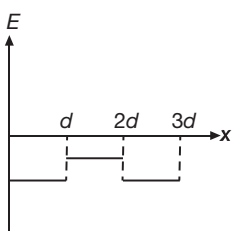
Hence, (B) and (D) are correct.

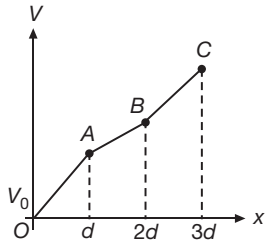
3. The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence, answer (B) is correct.

Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.



Therefore, answer (C) is also correct. The variation of electric field (E) and potential (V) with x is shown.





$OA \parallel BC$ and $(\text{Slope})_{OA} > (\text{Slope})_{AB}$

So, $E_{0\to d} = E_{2d\to 3d}$ and $E_{0\to d} > E_{d\to 2d}$

Hence, (B) and (C) are correct.

4. Let C_0 be the capacitance initially and C be the capacitance finally. Then $C_0 = \frac{\varepsilon_0 A}{d}$

Since, $Q = C_0 V$

$$\Rightarrow \quad Q = \frac{\varepsilon_0 AV}{d}$$

Further $E_0 = \frac{V}{d}$ and $E = \frac{E_0}{K}$

$$\Rightarrow \quad E = \frac{V}{Kd}$$

Also, if U_i is the initial energy, then $U_i = \frac{1}{2}C_0 V^2$

After the introduction of slab if U_f be the final energy, then

$$U_f = \frac{1}{2}CV_{slab}^2 = \frac{1}{2}(KC_0)\left(\frac{V}{K}\right)^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}\frac{C_0 V^2}{K}$$

$$\Rightarrow \quad \Delta U = U_2 - U_1$$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C_0 V^2\left(\frac{1}{K} - 1\right)$$

Since work done = Decrease in Potential Energy

$$\Rightarrow \quad W = -\Delta U$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1 - \frac{1}{K}\right)$$

Hence, (A), (C) and (D) are correct.

5. Charging battery is removed. Therefore, q = constant.

Distance between the plates is increased. Therefore, C decreases.

Now, $V = \frac{q}{C}$, q is constant and C is decreasing.

Therefore, V should increase.

$U = \frac{1}{2}\frac{q^2}{C}$ again q is constant and C is decreasing.

Therefore U should increase.

Hence, (B) and (D) are correct.

6. Since battery is still in connection. So,

$V = V_0$

$\Rightarrow \quad Q_0 = C_0 V_0$ and

$Q = kC_0 V_0$

$\Rightarrow \quad Q = kQ_0$

Since $k > 1$

$\Rightarrow \quad Q > Q_0$

Also $U_0 = \frac{1}{2} Q_0 V_0$ and

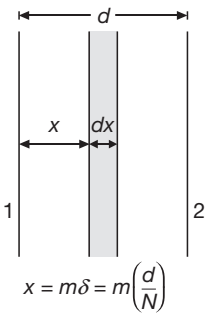
$U = \frac{1}{2} QV = kU_0 \qquad \{\because Q = kQ_0 \text{ and } V = V_0\}$

Hence $U > U_0$

Hence, (A) and (D) are correct.

Integer/Numerical Answer Type Questions

1. $dC = \frac{K\varepsilon_0 A}{dx}$



$x = m\delta = m\left(\frac{d}{N}\right)$

Since all capacitors connected in series, so

$\frac{1}{C} = \int \frac{1}{dC}$

$\Rightarrow \quad \frac{1}{C} = \int \frac{dx}{K\varepsilon_0 A\left(1 + \frac{m}{N}\right)}$

Distance of m^{th} layer from plate 1 is

$x = m\delta = m\left(\frac{d}{N}\right)$

$\Rightarrow \quad \frac{1}{C} = \int \frac{dx}{K\varepsilon_0 A\left(1 + \frac{x}{d}\right)}$

$\Rightarrow \quad \frac{1}{C} = \frac{d}{K\varepsilon_0 A}\int_0^x \frac{dx}{d + x}$

$\Rightarrow \quad \frac{1}{C} = \frac{d\ln(2)}{K\varepsilon_0 A}$

$\Rightarrow \quad C = \frac{K\varepsilon_0 A}{d\ln(2)}$

$\Rightarrow \quad \alpha = 1$

CHAPTER 2

CHAPTER 3: ELECTRIC CURRENT AND CIRCUITS

Test Your Concepts-I (Based on Current Definition)

1. (a) Using $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}=\frac{mv^2}{r}$, we get

$$v=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}}=2.19\times10^6\ \text{ms}^{-1}$$

(b) The time for the electron to revolve around the proton once is:

$$t=\frac{2\pi r}{v}=\frac{2\pi(5.29\times10^{-11}\ \text{m})}{2.19\times10^6\ \text{ms}^{-1}}=1.52\times10^{-16}\ \text{s}.$$

The total charge flow in this time is 1.60×10^{-19} C, so the current is

$$I=\frac{1.60\times10^{-19}\ \text{C}}{1.52\times10^{-16}\ \text{s}}=1.05\times10^{-3}\ \text{A}=1.05\ \text{mA}$$

2. $q=4t^3+5t+6$

$$A=(2\ \text{cm}^2)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2=2\times10^{-4}\ \text{m}^2$$

(a) $I(1\ \text{s})=\left.\frac{dq}{dt}\right|_{t=1\ \text{s}}=(12t^2+5)|_{t=1\ \text{s}}=17\ \text{A}$

(b) $J=\frac{I}{A}=\frac{17\ \text{A}}{2\times10^{-4}\ \text{m}^2}=85\ \text{kAm}^{-2}$

3. $I=\frac{dq}{dt}$

$$q=\int dq=\int Idt=\int_0^{\frac{1}{240}\text{s}}(100\ \text{A})\sin\left(\frac{120\pi t}{s}\right)dt$$

$$q=\frac{-100\ \text{C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right)-\cos\ 0\right]=\frac{+100\ \text{C}}{120\pi}=0.265\ \text{C}$$

4. Let us write the condition of the zero potential of the sphere, and hence, of any point inside it (in particular, its centre), at any instant of time t. We shall find out three times intervals:

$$(1)\ t<\frac{a}{v},\ (2)\ \frac{a}{v}\le t<\frac{b}{v},\ (3)\ t\ge\frac{b}{v}$$

Denoting the charge of the sphere by $q(t)$, we obtain the following expression for an instant t from the first time interval:

$$\frac{q_1}{a}+\frac{q_2}{b}+\frac{q(t)}{vt}=0,$$

Hence, $q(t)=-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right)t,$

$$I_1(t)=-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right)$$

For an instant t from the second time interval, we find that the fields inside and outside the sphere are independent, and hence

$$\frac{q(t)+q_1}{vt}=-\frac{q_2}{b},$$

$$I_2(t)=-v\frac{q_2}{b}$$

Finally, as soon as the sphere absorbs the two point charges q_1 and q_2, the current will stop flowing through the earthing conductor, and we can write $I_3(t)=0$.

Thus,

$$I(t)=\begin{cases}-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right), & t<\frac{a}{v},\\ -v\frac{q_2}{b}, & \frac{a}{v}\le t<\frac{b}{v},\\ 0, & t\ge\frac{b}{v}.\end{cases}$$

5. (a) $I=\frac{dq}{dt}$

$$\Rightarrow\quad\int_0^q dq=\int_0^5 Idt$$

$$\Rightarrow\quad q=\int_0^5(3+2t)dt$$

$$\Rightarrow\quad q=\left[3t+t^2\right]_0^5=[15+25]$$

$$\Rightarrow\quad q=40\ \text{C}$$

(b) $I=\frac{q}{t}=\frac{40}{4}$

$$\Rightarrow\quad I=10\ \text{A}$$

6. Let n be the number of electrons at distance x. Now,

$$\frac{dn}{dx}=\text{increase in } n \text{ per unit length}$$

$$\Rightarrow\quad\frac{dn}{dx}=\alpha n$$

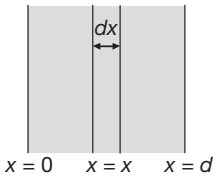
$$\Rightarrow \quad \frac{dn}{n} = \alpha dx$$

$$\Rightarrow \quad \int_{n_0}^{n} \frac{dn}{n} = \alpha \int_{0}^{d} dx$$

$$\Rightarrow \quad \log_e\left(\frac{n}{n_0}\right) = \alpha d$$

$$\Rightarrow \quad \frac{n}{n_0} = e^{\alpha d}$$

$$\Rightarrow \quad n = n_0 e^{\alpha d}$$



where, n is number of electrons reaching at other plate per unit time.

$$\begin{pmatrix}\text{The electronic current}\\ \text{at opposite plate}\end{pmatrix} = \begin{pmatrix}\text{charge reaching}\\ \text{per unit time}\end{pmatrix}$$

$$\Rightarrow \quad I = en$$

$$\Rightarrow \quad I = en_0 e^{\alpha d}$$

7. Convectional current is the current which is developed due to the transportation of charge

$$I = \frac{q_{\text{transported}}}{t} = 2\lambda v$$

Test Your Concepts-II (Based on Resistance, Resistivity and Ohm's Law)

1. $\Delta V = -E\ell$

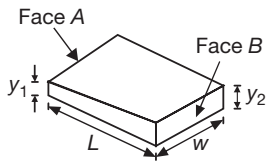
$$\Rightarrow \quad dV = -Edx$$

$$\Delta V = -IR = -E\ell$$

$$\Rightarrow \quad I = \frac{dq}{dt} = \frac{E\ell}{R} = \frac{A}{\rho\ell}E\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \sigma A\left|\frac{dV}{dx}\right|$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

2. $R = \int \frac{\rho dx}{A} = \int \frac{\rho dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L}x$



$$\Rightarrow \quad R = \frac{\rho}{w}\int_0^L \frac{dx}{y_1 + \left[\frac{(y_2 - y_1)}{L}\right]x}$$

$$\Rightarrow \quad R = \frac{\rho L}{w(y_2 - y_1)}\log_e\left(y_1 + \frac{y_2 - y_1}{L}x\right)_0^L$$

$$\Rightarrow \quad R = \frac{\rho L}{w(y_2 - y_1)}\log_e\left(\frac{y_2}{y_1}\right)$$

3. Resistance of original wire,

$$R_1 = \frac{\rho\ell_1}{A_1} = 8\ \Omega c$$

Resistance of modified wire,

$$R_2 = \frac{\rho\ell_2}{A_2} = \frac{\rho(2\ell_1)}{4A_1} = \frac{1}{2}\frac{\rho\ell_1}{A} = \frac{1}{2}(8\ \Omega) = 4\ \Omega$$

4. (a) The resistance of a wire is given by

$$R = \rho\frac{\ell}{A}$$

As volume is constant, $V = A\ell =$ constant

$$R = \rho\left(\frac{\ell^2}{V}\right)$$

The fractional charge, $\frac{dR}{R} = 2\frac{d\ell}{\ell} = 2(0.1\%) = 0.2\%$

The percentage change in resistance is 0.2%

(b) Similarly $R = \rho\frac{V}{A^2} \Rightarrow \frac{dR}{R} = -2\frac{dA}{A}$

Since cross-sectional area $A = \pi r^2$

$$\Rightarrow \quad \frac{dA}{A} = 2\frac{dr}{r} \Rightarrow \frac{dR}{R} = -2\frac{dA}{A} = -4\frac{dr}{r}$$

If radius is increased by 1%, the resistance is decreased by 4%. If area is increased by 1%, the resistance is decreased by 2%. The significance of negative sign is that change in resistance is opposite to that of radius and area.

Here please note that calculus can be employed for small percentage change only.

5. In electrostatic condition, electric field inside a conductor is zero. But when a current flows through the conductor electric field is non-zero.

6. (a) $R = \int_a^b \frac{\rho dr}{4\pi r^2}$ $\left\{\text{Resistance} = \frac{\rho\ell}{A}\right\}$

$$\Rightarrow \quad R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b) $I = \frac{V_{ab}}{R}$ and $j = \frac{I}{A} = \frac{I}{4\pi r^2}$

$$\Rightarrow \quad j = \frac{V_{ab}}{4\pi R r^2} = \frac{V_{ab}}{\rho\left(\frac{1}{a} - \frac{1}{b}\right)r^2}$$

$$\Rightarrow \quad j = \frac{abV_{ab}}{\rho(b-a)r^2}$$

CHAPTER 3

Test Your Concepts-III (Based on Variation of Resistance with Temperature)

1. Resistance at temperature T is given by

$$R_T = R_0(1+\alpha_0 T) = R(t)$$

From the given data,

$$R_0 = R_1[1+\alpha_1(0-20^\circ)]$$

$$\Rightarrow \quad R_0 = 2(1-20\times 0.00575)$$

$$\Rightarrow \quad R_0 = 1.77\ \Omega$$

Since, $T = 20+10t$

$$\Rightarrow \quad R(t) = 1.77[1+0.0065(20+10t)]$$

$$\Rightarrow \quad R(t) = (2+0.115t)\ \Omega$$

2. Since $R_T = R_0(1+\alpha_0 T)$ …(1)

and $R_0 = R_T(1-\alpha_T T)$ …(2)

From equation (2), we have

$$\alpha_T = \frac{R_T - R_0}{TR_T} \qquad \ldots(3)$$

Substituting for R_T in equation (3) , we have

$$\alpha_T = \frac{R_0(1+\alpha_0 T)-R_0}{TR_0(1+\alpha_0 T)} = \frac{\alpha_0}{1+\alpha_0 T}$$

3. Since, we have

$$\frac{1}{\alpha_T} = \frac{1+\alpha_0 T}{\alpha_0} = \frac{1}{\alpha_0} + T$$

So we have $\frac{1}{\alpha_1} = \frac{1}{\alpha_0} + T_1$

and $\frac{1}{\alpha_2} = \frac{1}{\alpha_0} + T_2$

Subtracting, we get

$$\frac{1}{\alpha_1} - \frac{1}{\alpha_2} = T_1 - T_2$$

$$\Rightarrow \quad \alpha_2 = \frac{1}{\left(\frac{1}{\alpha_1}\right)+(T_2-T_1)} = \frac{\alpha_1}{1+\alpha_1(T_2-T_1)}$$

4. Let R_0 be the resistance at 0 °C . Then at the temperatures T_1 and T_2 , we have

$$R_1 = R_0(1+\alpha_0 T_1) \qquad \ldots(1)$$

and $R_2 = R_0(1+\alpha_0 T_2)$ …(2)

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{1+\alpha_0 T_1}{1+\alpha_0 T_2} \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{R_1}{R_2} = (1+\alpha_0 T_1)(1+\alpha_0 T_2)^{-1}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = (1+\alpha_0 T_1)\left[1-\alpha_0 T_2+(\alpha_0 T_2)^2-\ldots\right]$$

$$\Rightarrow \quad \frac{R_1}{R_2} \simeq 1+\alpha_0(T_1-T_2)$$

{We have neglected the higher powers of $\alpha_0 T_2$ }

5. From figure, the slope of the straight line is

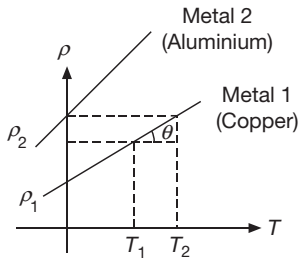
$$\tan\theta = m = \frac{\rho_2-\rho_1}{T_2-T_1}$$

which on rearranging yields

$$\rho_2 = \rho_1 + m(T_2-T_1)$$

$$\Rightarrow \quad \rho_2 = \rho_1\left[1+\frac{m}{\rho_1}(T_2-T_1)\right]$$

where m is the slope from the $\rho - T$ graph.



6. If coefficient of linear expansion is α, then the percentage change in length is

$$\left(\frac{d\ell}{\ell}\right)(100) = (\alpha dT)(100)$$

Coefficient of area ℓ expansion is 2α, so percentage change in area is

$$\left(\frac{dA}{A}\right)(100) = (2\alpha dT)(100)$$

percentage change in resistivity is

$$\left(\frac{d\rho}{\rho}\right)(100) = (\alpha_R dT)(100)$$

where α_R denotes thermal coefficient of resistance.
All the variables ℓ, A and ρ are functions of T. Consequently, R is also a function of T. We find, therefore,

$$\frac{dR}{dT} = \frac{d}{dT}\left(\frac{\rho\ell}{A}\right)$$

$$\Rightarrow \quad \frac{dR}{dt} = \frac{\ell}{A}\frac{d\rho}{dT} + \frac{\rho}{A}\frac{d\ell}{dT} + \rho\ell\frac{d}{dT}\left(\frac{\ell}{A}\right)$$

$$\Rightarrow \quad \frac{dR}{dt} = \alpha_R\rho\frac{\ell}{A} + \alpha\ell\frac{\rho}{A} - \frac{1}{A^2}2\,\alpha A\,\rho\ell$$

$$\Rightarrow \quad \frac{dR}{dt} = (\alpha_R+\alpha-2\alpha)\frac{\rho\ell}{A}$$

$$\Rightarrow \quad \frac{dR}{dt} = (\alpha_R-\alpha)\frac{\rho\ell}{A}$$

where we have written $\frac{d\rho}{dT} = \alpha_R \rho$ from equation $d\rho = \alpha_R \rho dT$. Similarly $\frac{d\ell}{dT} = \alpha \ell$ and $\frac{dA}{dT} = 2\alpha A$

Thus change in resistance due to temperature change is given by

$$dR = (\alpha_R - \alpha) R dT$$

We find that since α is quite small as compared to α_R, we have approximately

$$dR = \alpha_R R dT$$

and, therefore, percentage change in R is the same as that in resistivity.

So, percentage change in Length $= 0.0017\%$

percentage change in Area $= 0.0034\%$

percentage change in Resistance $= 0.39\%$

7. $R_{50\,^\circ C} = (R_1)_{50\,^\circ C} + (R_2)_{50\,^\circ C}$

$$R_{50\,^\circ C} = (R_1)_{20\,^\circ C}(1 + \alpha_1 \Delta\theta) + (R_2)_{20\,^\circ C}(1 + \alpha_2 \Delta\theta)$$

$$R_{50\,^\circ C} = 600(1 + 0.001 \times 30) + 300(1 + 0.004 \times 30)$$

$$R_{50\,^\circ C} = 954\ \Omega$$

Further, $R_{50\,^\circ C} = R_{20\,^\circ C}(1 + \alpha \Delta\theta)$

$$\Rightarrow \quad \alpha = \frac{\left(\frac{R_{50\,^\circ C}}{R_{20\,^\circ C}} - 1\right)}{\Delta\theta} = \frac{\left(\frac{954}{900} - 1\right)}{30}$$

$$\Rightarrow \quad \alpha = 0.002\ (^\circ C)^{-1}$$

Test Your Concepts-IV (Based on Series and Parallel Combination of Resistances)

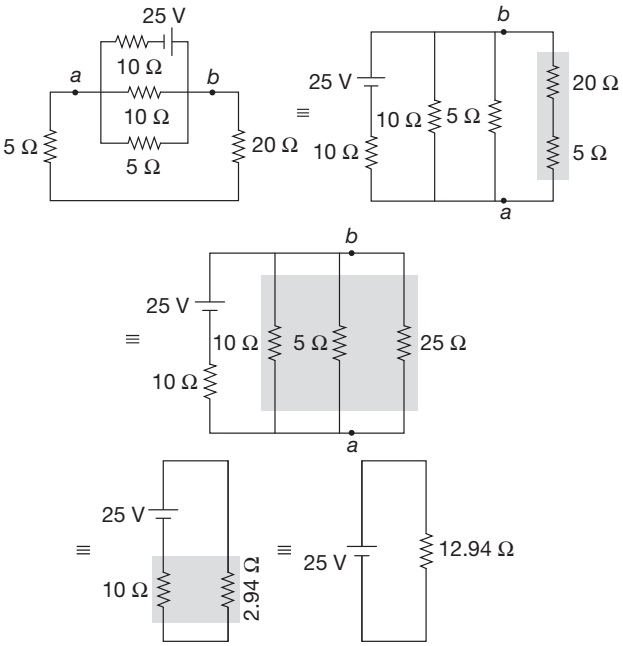
1. $120\ V = IR_{eq} = I\left(\frac{\rho\ell}{A_1} + \frac{\rho\ell}{A_2} + \frac{\rho\ell}{A_3} + \frac{\rho\ell}{A_4}\right)$

$$\Rightarrow \quad I\rho\ell = \frac{(120\ V)}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)}$$

$$\Rightarrow \quad \Delta V_2 = \frac{I\rho\ell}{A_2} = \frac{(120\ V)}{A_2\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)} = 29.5\ V$$

2. The equivalent circuit is shown for the convenience of evaluating.

$$R_{eq} = \frac{1}{\left(\frac{1}{10\ \Omega} + \frac{1}{5\ \Omega} + \frac{1}{25\ \Omega}\right)} = 2.94\ \Omega$$



Applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The $12.94\ \Omega$ resistor is connected across $25\ V$, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25\ V}{12.94\ \Omega} = 1.93\ A$$

This $1.93\ A$ goes through the $2.94\ \Omega$ equivalent resistor to give a potential difference of

$$\Delta V = IR = (1.93\ A)(2.94\ \Omega) = 5.68\ V$$

We see that this potential difference is the same across ab, the $10\ \Omega$ resistor, and the $5\ \Omega$ resistor.

(a) Therefore, $\Delta V_{ab} = 5.68\ V$

(b) Since the current through the $20\ \Omega$ resistor is also the current through the $25\ \Omega$ line ab,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ V}{25\ \Omega} = 0.227\ A = 227\ mA$$

3. When S is open, R_1, R_2 and R_3 are in series with the battery. So,

$$R_1 + R_2 + R_3 = \frac{6\ V}{10^{-3}\ A} = 6\ k\Omega \qquad \ldots(1)$$

When S is closed in position 1, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. So,

CHAPTER 3

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times10^{-3}\text{A}} = 5\text{ k}\Omega \qquad \ldots(2)$$

When S is closed in position 2, R_1 and R_2 are in series with the battery. R_3 is shorted. Hence,

$$R_1 + R_2 = \frac{6\text{ V}}{2\times10^{-3}A} = 3\text{ k}\Omega \qquad \ldots(3)$$

From (1) and (3), we get $R_3 = 3\text{ k}\Omega$

Subtract (2) from (1), we get $R_2 = 2\text{ k}\Omega$ and from (3), we get $R_1 = 1\text{ k}\Omega$

$\Rightarrow \quad R_1 = 1\text{ k}\Omega, R_2 = 2\text{ k}\Omega, R_3 = 3\text{ k}\Omega$

4. Denoting the two resistors as x and y, we get

$$x + y = 690 \text{ and } \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

$$\Rightarrow \quad \frac{1}{150} = \frac{1}{x} + \frac{1}{690-x} = \frac{(690-x)+x}{x(690-x)}$$

$$\Rightarrow \quad x^2 - 690x + 103\ 500 = 0$$

$$\Rightarrow \quad x = \frac{690 \pm \sqrt{(690)^2 - 414000}}{2}$$

$$\Rightarrow \quad x = 470\ \Omega \text{ and } y = 220\ \Omega$$

5. $I = \dfrac{200}{5+10+25} = 5\text{ A}$

(i) $0 - (10+5)\times5 = V_1$

$\Rightarrow \quad V_1 = -75\text{ V}$

(ii) $0 - 10\times5 = V_2$

$\Rightarrow \quad V_2 = -50\text{ V}$

(iii) $0 + 25\times5 = V_3$

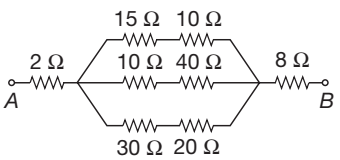
$\Rightarrow \quad V_3 = 125\text{ V}$

(iv) $V_{32} = V_3 - V_2 = 175\text{ V}$

(v) $V_{12} = V_1 - V_2 = -25\text{ V}$ and

(vi) $V_{13} = V_1 - V_3 = -200\text{ V}$

6. The simple circuit is as shown in figure which gives $R_{AB} = 22.5\ \Omega$



7. Two resistances are in parallel. Hence, net resistance across the battery,

$$R = \frac{12\times8}{12+8} = 4.8\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{24}{4.8}\text{ A} = 5\text{ A}$$

8. All the four resistances are in parallel. Hence the equivalent resistance is,

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{15}{24}\ \Omega$$

$$\Rightarrow \quad R = \frac{8}{5}\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{24}{8/5} = 15\text{ A}$$

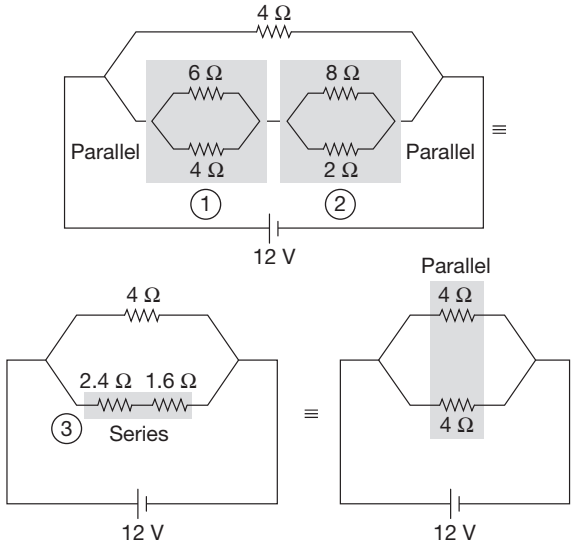
9. All the three resistances are in parallel. Hence, the equivalent resistance will be given by,

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{6} + \frac{1}{12} = \frac{9}{24}$$

$$\Rightarrow \quad R = \frac{8}{3}\ \Omega$$

$$\Rightarrow \quad \text{I} = \frac{V}{R} = \frac{24}{8/3} = 9\text{ A}$$

10. The equivalent circuit is as shown in figure.



For ①: $\dfrac{6\times4}{6+4} = 2.4\ \Omega$

For ②: $\dfrac{8\times2}{8+2} = 1.6\ \Omega$

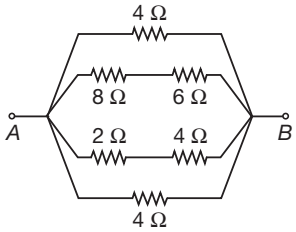
$$\Rightarrow \quad R = \frac{4(2.4+1.6)}{4+2.4+1.6} = 2\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{12}{2} = 6\text{ A}$$

11. (a) The simplified circuit is as shown in figure,

$$\frac{1}{R_{AB}}=\frac{1}{4}+\frac{1}{8+6}+\frac{1}{2+4}+\frac{1}{4}$$

$$\Rightarrow \quad R_{AB}=\frac{42}{31}\ \Omega$$

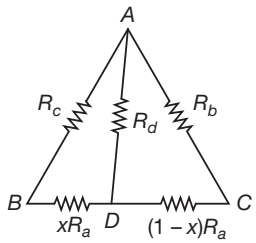


(b) The $6\ \Omega$ and $4\ \Omega$ resistances are in parallel. Similarly $8\ \Omega$ and $2\ \Omega$ resistances are also in parallel. These two are then in series. Hence,

$$R_{AB}=\left(\frac{6\times 4}{6+4}\right)+\left(\frac{8\times 2}{8+2}\right)=4\ \Omega$$

12. First find the value of R_{AD} as a function of x and then for obtaining its maximum value we perform $\frac{dR_{AD}}{dx}=0$, we get

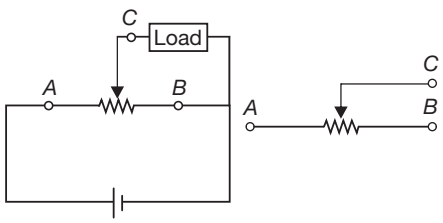
$$(R_{AD})_{MAX}=\frac{R_d(R_d+2R_b)(R_a+2R_c)}{4R_d(R_a+R_b+R_c)+(R_a+2R_b)(R_a+2R_c)}$$



13. $R_{CD}=\frac{R_2 r}{R_2+r}=\frac{60\times 120}{60+120}=40\ \Omega$

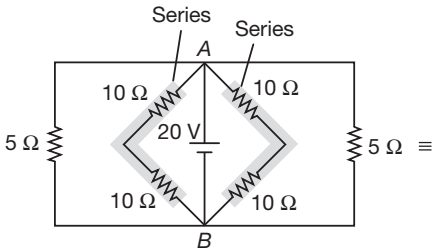
$$\Rightarrow \quad V_{CD}=\left(\frac{R_{CD}}{R_{CD}+R_1}\right)V_{AB}=\left(\frac{40}{40+60}\right)120=48\ \text{V}$$

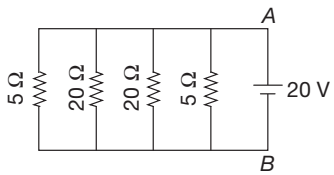
14. The rheostat is as shown in figure. Battery should be connected between A and B and the load between C and B.



15. Note that both the $5\ \Omega$ resistors are connected to same points A and B, across which battery is connected. Therefore each of $5\ \Omega$ resistors is in parallel arrangement with battery, therefore the current through it,

$$I=\left(\frac{20}{5}\right)\text{A}=4\ \text{A}$$





The equivalent resistance between A and B is

$$\frac{1}{R_{eq}}=\frac{1}{20}+\frac{1}{20}+\frac{1}{5}+\frac{1}{5}=\frac{10}{20}$$

$$R_{eq}=2\ \Omega$$

Thus the current supplied by battery $=\left(\frac{20}{2}\right)\text{A}=10\ \text{A}$

16. It is zero, because the current will follow the path of least resistance. In fact a path of zero resistance is not available. Hence $I=0$.

Test Your Concepts-V (Based on Series and Parallel Combination of Resistances)

1. (a) Let us split the medium into infinitesimal differential cylindrical shell elements, of width dr, in series. The current flow is cylindrically symmetric $(L\gg b)$. The area through which the current flows across a shell of radius r is $A(r)=2\pi rL$. The length the current flows, passing through a shell of radius r is dr. So, the resistance of the shell of radius r is

$$dR=\frac{1}{\sigma}\frac{dr}{2\pi rL} \qquad \ldots(1)$$

Since the shells are connected in a series, we get

$$R_{ab}=\int_a^b dR=\frac{\log_e\left(\frac{b}{a}\right)}{2\pi\sigma L}=\frac{1}{2\pi\sigma L}\log_e\left(\frac{b}{a}\right) \qquad \ldots(2)$$

CHAPTER 3

(b) Using $\sigma(r)=\frac{\sigma_0}{r}$, we have from equation (1),

$$dR=\frac{dr}{2\pi rL\sigma(r)}=\frac{dr}{2\pi rL\left(\frac{\sigma_0}{r}\right)}=\frac{dr}{2\pi L\sigma_0} \quad \ldots(3)$$

$$\Rightarrow \quad R_{ab}=\int_a^b dR=\frac{b-a}{2\pi L\sigma_0} \quad \ldots(4)$$

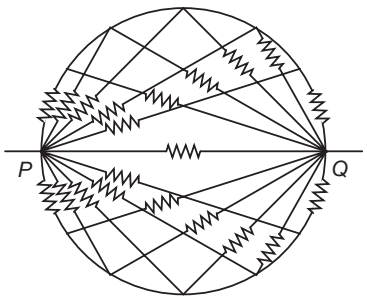
2. (a) Since each of the terminals is connected by an insulated wire to each of the remaining 11 terminals by a resistor R, the symmetry shows all the twelve terminals to be symmetrically equivalent before any voltage is applied.

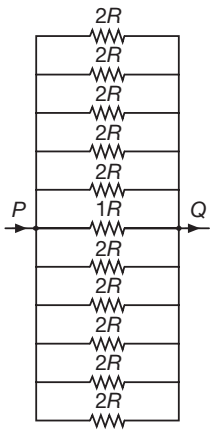
However, asymmetry is introduced just at the point where current enters and also at the point where the current leaves the circuit. All other ten points are symmetry points, all at the same potential. Hence the given network reduces to the following network

Thus if X be the equivalent resistance between P and Q

$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots. \text{ to 10 terms } +\frac{1}{R}$$

$$\frac{1}{X}=\frac{10}{2R}+\frac{1}{R}=\frac{6}{R}$$





Hence $X=\frac{R}{6}$

The equivalent resistance $=\frac{R}{6}\ \Omega=1\ \Omega$

(b) Proceeding in the same manner as in (a) we find that the equivalent resistor is given by X where

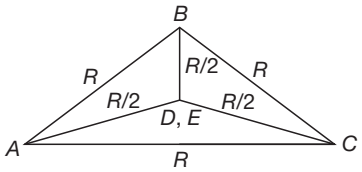
$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots \text{ to } (n-2) \text{ terms } +\frac{1}{R}$$

$$\frac{1}{X}=\frac{n-2}{2R}+\frac{1}{R}=\frac{n}{2R}$$

Hence equivalent resistance $=\frac{2R}{n}$

The above symmetry simplification can be done only if n is even.

3. (a) Points D and E are symmetrically located with respect to points A and C. The circuit can be redrawn as shown in figure.

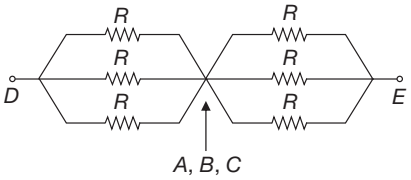


This is a combination of a balanced Wheatstone bridge which is in parallel with a resistance R. So, the resistance between B and D (or E) can be removed to get

$$\frac{1}{R_{AC}}=\frac{1}{R}+\frac{1}{\frac{R}{2}+\frac{R}{2}}+\frac{1}{R}$$

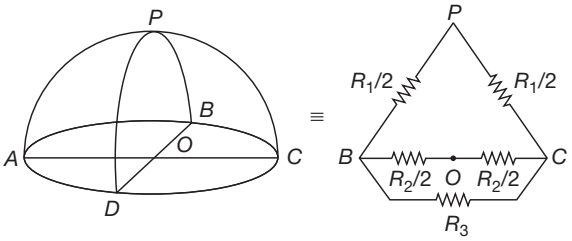
$$\Rightarrow \quad R_{AC}=\frac{2R}{5}$$

(b) With respect to D and E the points A, B and C all are symmetrically located. Hence, the simplified equivalent circuit is shown here.



$$\Rightarrow \quad R_{DE}=\frac{R}{3}+\frac{R}{3}=\frac{2R}{3}$$

4. Points (A and C) and (D and B) are symmetrically located with respect to points O and P. Hence, the circuit can be drawn as shown in figure.



This is a balanced Wheatstone bridge between P and O

Hence, R_3 can be removed. And,

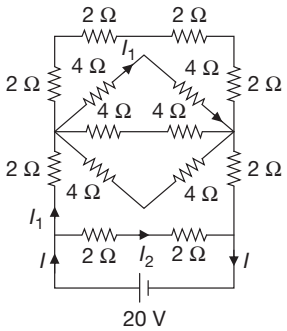
$$R_{PO} = \frac{R_1 + R_2}{4}$$

where $R_1 = R_{PB} = R_{PD} = \frac{(\pi a)\lambda}{2}$

and $R_2 = R_{OB} = (a)\lambda$

$\Rightarrow \quad R_{PO} = \frac{(2+\pi)a\lambda}{8}$

5. The simplified circuit is as shown in figure



Total resistance of circuit

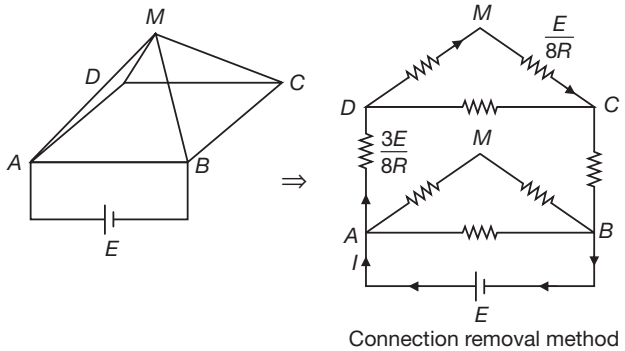
$R = 2.4\ \Omega$

$\Rightarrow \quad I = \frac{20}{2.4} = 8.33\ \text{A}$

So, $I_1 = \left(\frac{4}{10}\right)(I) = 3.33\ \text{A}$

The desired current is $I' = \frac{I_1}{4} = 0.833\ \text{A}$

6. (a) $R_{\text{net}} = \frac{8R}{15}$

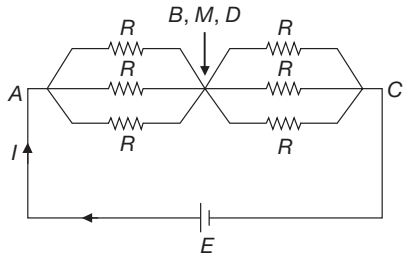


Connection removal method

$\Rightarrow \quad I = \frac{E}{8R/15} = \frac{15E}{8R}$

Hence current through MC is $I = \frac{E}{8R}$

(b) Points B, D and M are symmetrical with respect to points A and C both. Hence, the simple equivalent circuit can be drawn as shown

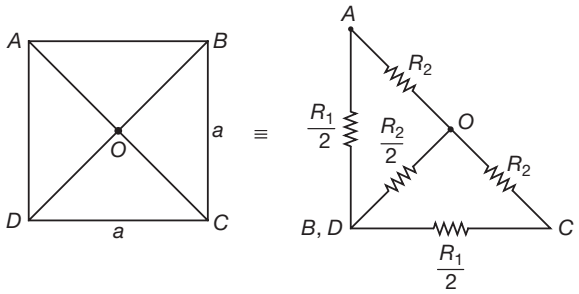


$R_{\text{net}} = \frac{2R}{3}$

$\Rightarrow \quad I = \frac{E}{R_{\text{net}}} = \frac{3E}{2R}$

Hence, current through MA (or AM) $= \frac{I}{3} = \frac{E}{2R}$

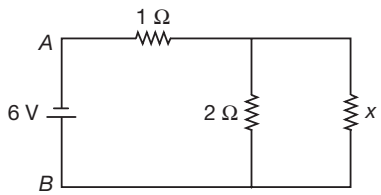
7. Since points B and D are symmetrically located with respect to points A and O. So they are at same potential. The simple circuit can be drawn as shown.



Here, $R_1 = \rho a$ and $R_2 = \frac{\rho a}{\sqrt{2}}$

$\Rightarrow \quad R_{AO} = \left(\frac{\sqrt{2}}{2\sqrt{2}+1}\right)\lambda a$

8. (a) Let $R_{AB} = x$. Then, we can break one chain and connect a resistance of magnitude x that replaces the remaining circuit.



Thus, the equivalent circuit remains as shown in the figure.

CHAPTER 3

Now, $2\ \Omega$ and x are in parallel. So, their combined is $\dfrac{2x}{2+x}$

$$\Rightarrow \quad R_{AB} = 1 + \frac{2x}{2+x}$$

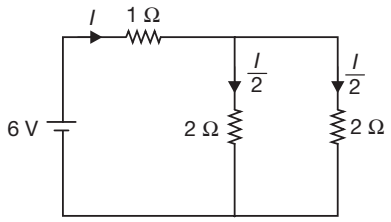
But R_{AB} is assumed to x. Therefore,

$$x = 1 + \frac{2x}{2+x}$$

Solving this equation, we get

$x = 2\Omega$ Hence proved.

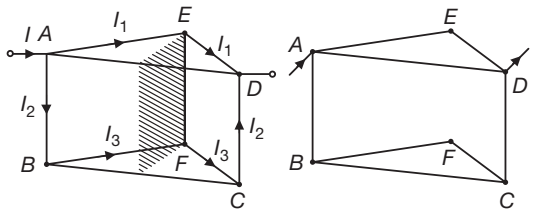
(b) Net resistance of circuit $R = 1 + \dfrac{2\times 2}{2+2} = 2\ \Omega$



$\therefore$ Current through battery $I = \dfrac{6}{2} = 3$ A

This current is equally distributed in $2\ \Omega$ and $2\ \Omega$ resistances. Therefore, the desired current is $\dfrac{I}{2}$ or 1.5 A.

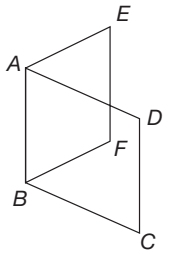
9. (a) Due to symmetry about the shaded plane, current distribution on either side of the plane will be identical and points E and F will be at same potential and no current will flow through it,



$$\Rightarrow \quad R_{AD} = \frac{\frac{2}{3}r \times \frac{8}{3}r}{\frac{2}{3}r + \frac{8}{3}r} = \frac{8}{15}r$$

(b) Redrawing the given arrangement for resistance across AB. Potentials $V_D = V_E$

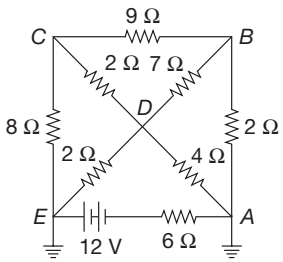
$V_C = V_F$



Therefore no current flows through DE and CF.

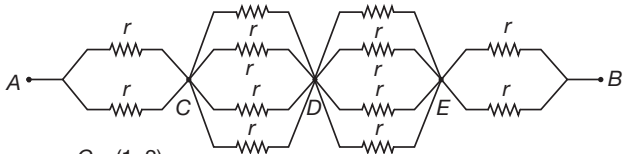
$$\Rightarrow \quad R_{AB} = \frac{\frac{3}{2}r \times r}{\frac{3}{2}r + r} = \frac{3}{5}r$$

10. Both the ends of the bottom branch are grounded, so the net potential difference between E and A is zero. If we traverse from E to A there is a potential drop of 12 V across battery, so there must be a potential gain of 12 V in the resistor. Therefore



the current in $6\ \Omega$ resistor is $I = \left(\dfrac{12}{6}\right)$A $= 2$ A and to the left. The current in the $9\ \Omega$ resistor is zero because there is no potential difference across E and A. The entire current of the battery goes from E into the zero resistance path back to A via ground. Please remember that when ground connections are shown, it is assumed that all such points are wired to a common line even if not shown.

11. In figure points $C(1, 2)$, $D(3, 4, 5)$ and $E(6, 7)$ are at same potential. Equivalent circuit can be redrawn as in figure.

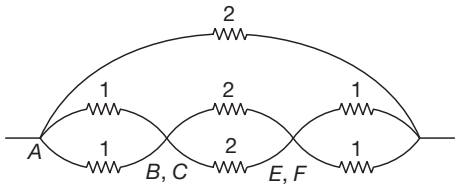


$C \equiv (1, 2)$
$D \equiv (3, 4, 5)$
$E \equiv (6, 7)$

The equivalent resistance of this series combination is

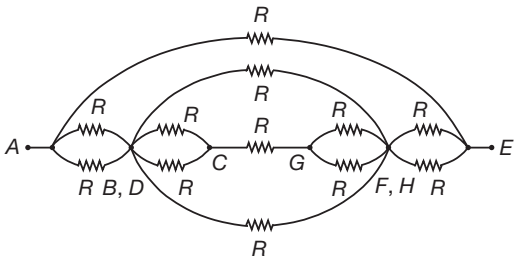
$$R_{eq} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$

12. Points B and C, E and F are at the same potential, so the circuit can be redrawn as in figure.



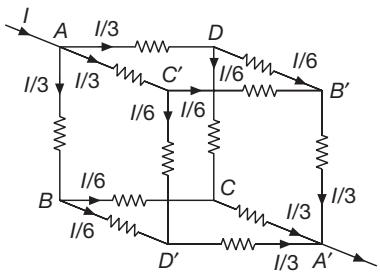
Thus the equivalent resistance is $1\ \Omega$.

13. Points B and D have same potential, similarly F and H have same potential. The equivalent circuit is shown in figure.



The equivalent resistance of network is $\frac{7R}{2}$.

14. (a) The potential difference between any two points is same no matter what path we take to arrive at the second beginning from first.
The circuit is symmetrical as shown in figure, the entry point and exit point are identical. Therefore at A, a current $\frac{I}{3}$ flows in each branch, similarly at exit point A' each branch must have current $\frac{I}{3}$.
At point B the current divides equally to $\frac{I}{6}$ and $\frac{I}{6}$ in branches BC and BD'.
Similarly current divides equally to $\frac{I}{6}$ and $\frac{I}{6}$ in branches $C'$and D' respectively.



Let equivalent resistance between A and A' be R_{eq} and the potential drop across it IR_{eq}
From given circuit the potential difference between A and A' can be determined as follows

$$V_A - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = V_B$$

$$\Rightarrow \quad V_A - V_B = \frac{I}{3}R + \frac{I}{3}R + \frac{I}{3}R$$

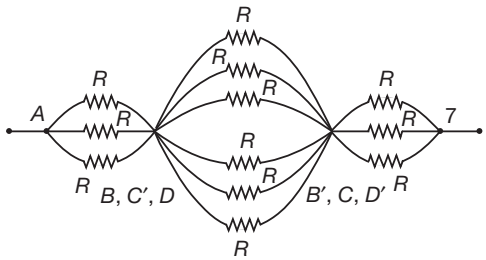
From equivalent circuit $V_A - V_B = IR_{eq}$

Thus we have, $IR_{eq} = \frac{5}{6}IR$

$$R_{eq} = \frac{5}{6}R$$

Method 2. In between A and A', symmetry of the circuit indicates that B, C', D are at equal potential and similarly B', C, D'. So the cube may be redrawn as

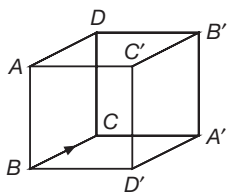
$$R_{eq} = \frac{R}{3} \text{ '+' } \frac{R}{6} \text{ '+' } \frac{R}{3}$$



where '+' stands for the series

$$= \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}R$$

(b) Once again the circuit is symmetrical. Figure shows the current distribution at junctions A and B. The incoming current at D must be equal to outgoing current, similar situation exists at C'. Therefore the current in branches CD and $C'D'$ are zero or we can say that the points C and D are equipotential, similarly C' and D' have same potential.
The equivalent circuit is reduced to figure.



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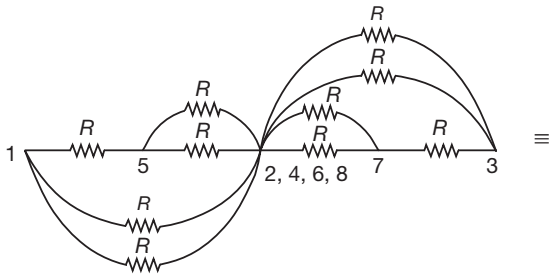
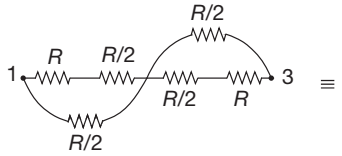
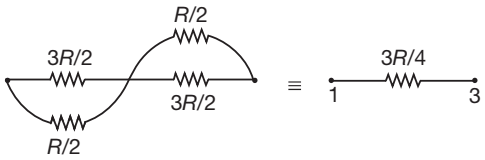
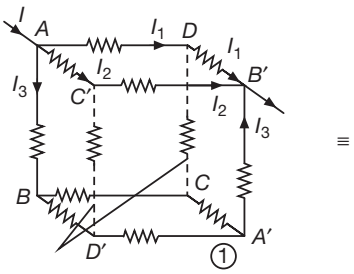
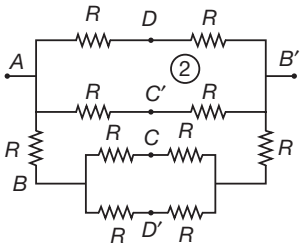


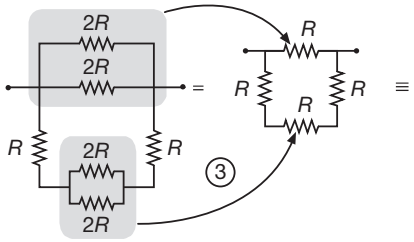


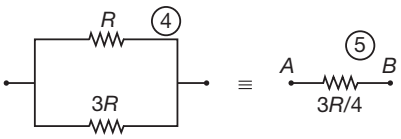




Resistors irrelevant from point of view of current







$$\Rightarrow \quad R_{AB'} = \frac{3R}{4}$$

Method 2. In between points 1 and 3, 2 and 4 are at the same potential. Current in the wires $2-6$ and $4-8$ is zero. Potential difference between $2-6$ and $4-8$ is zero.

(c) Let a current I enter the point A and leave the point B. Figure shows distribution of current in the circuit. the circuit is symmetrical, note the currents at entry and exit points. The wires AC' and AD have equal resistance and located symmetrically, so they have same current; similar situation exists with DC and $C'D'$. From KCL we have

$$I = I_1 + 2I_2 \qquad \ldots(1)$$

For the loop $AC'D'BA$, applying KVL, we have

$$-I_2R - I_3R - I_2R + I_1R = 0$$

$$\Rightarrow \quad I_1 = 2I_2 + I_3 \qquad \ldots(2)$$

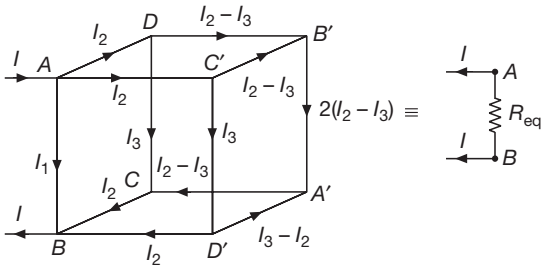
For the loop $C'D'A'B'$, applying KVL we have

$$-I_3R + (I_2 - I_3)R + 2(I_2 - I_3)R + (I_2 - I_3)R = 0$$

$$\Rightarrow \quad 4I_2 - 5I_3 = 0 \qquad \ldots(3)$$

On solving equations $(1), (2)$ and (3), we get

$$I_1 = \frac{7}{12}I, \; I_2 = \frac{5}{24}I \text{ and } I_3 = \frac{1}{6}I$$



The potential difference across $AB = I_1R = \frac{7}{12}IR$

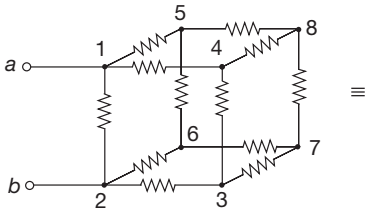
The potential difference across equivalent resistance is $V = IR_{eq}$

$$\Rightarrow \quad IR_{eq} = \frac{7}{12}IR$$

$$\Rightarrow \quad R_{eq} = \frac{7}{12}R$$

Method 2: From considerations of symmetry alone, points 3 and 6 must be at the same potential, and so must points 4 and 5.

If two points in a circuit have the same potential, the currents in the circuit do not change if they are connected at these points by a wire. There is no current in the wire because there is no potential difference between its ends. Points 3 and 6 may therefore be connected by a wire, and similarly points 4 and 5 may be connected.



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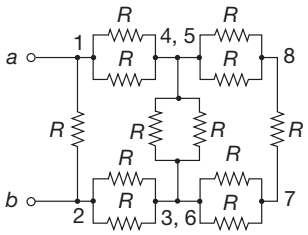

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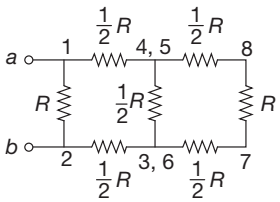

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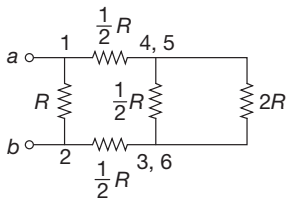

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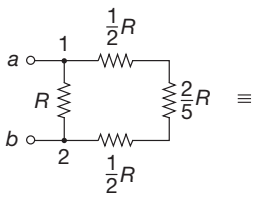

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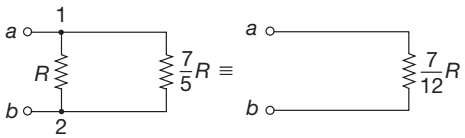


Thus the equivalent resistance between a and b is $\frac{7}{12}R$

15. Before connecting B with D and C with E,

$$R_{AF} = 5R$$

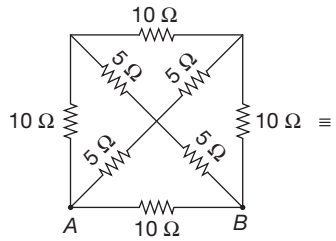
After connecting B with D and C with E, a balanced Wheatstone bridge is formed between B and E.

So, $R_{BE} = R$

and $R'_{AF} = 3R = 0.6R_{AF}$

i.e., the new equivalent resistance in the circuit has become 0.6 times.

16. The simplified circuit is as shown in figure.



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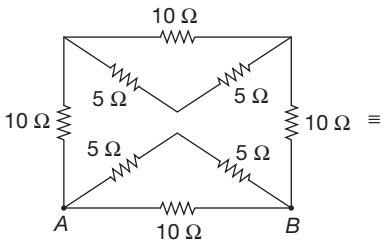

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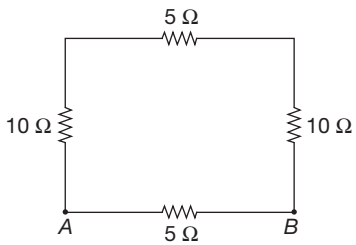
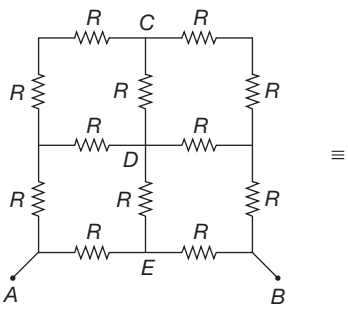


$$R_{AB} = \frac{(25)(5)}{(25)+(5)} = \frac{25}{6}\ \Omega$$

17.



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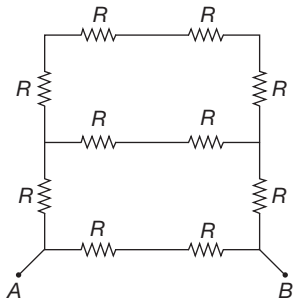
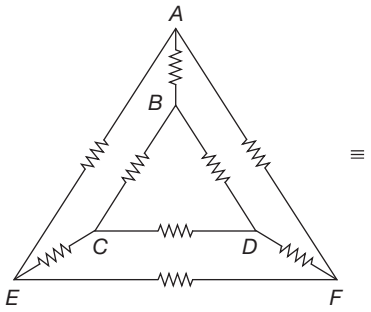
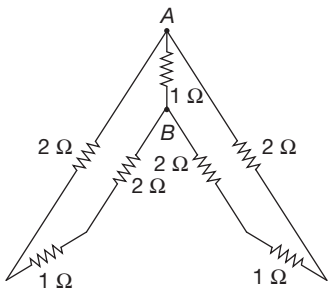


Points C, D and E are at same potential. The circuit can now be redrawn as,

The equivalent resistance between A and B can now be easily determined as $\frac{5R}{4}$.

18. The extended line AB is a line of symmetry. Hence, points C and D and also points E and F are at the same potentials. So, the resistance between them can be removed from the circuit.





The equivalent resistance between A and B can now be worked out to be $\frac{5}{7}\ \Omega$.

19. (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$dR = \frac{\rho d\ell}{A} = \frac{\rho dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

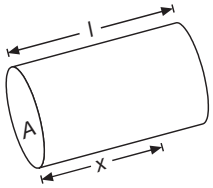
$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\rho}{2\pi L}\log_e\left(\frac{r_b}{r_a}\right)$$

(b) In this equation $\frac{\Delta V}{I} = \frac{\rho}{2\pi L}\log_e\left(\frac{r_b}{r_a}\right)$

$$\Rightarrow \quad \rho = \frac{2\pi L\Delta V}{I\log_e\left(\frac{r_b}{r_a}\right)}$$

20. (a) Consider the cylinder as composed of thin discs of width dx connected in series. The resistance of a disc at a distance x away from the cylinder end is given by

$$dR = \frac{1}{\sigma(x)}\frac{dx}{A} = \frac{\sqrt{x}\,dx}{A\ell\sigma_0} \qquad \ldots(1)$$



where A is the cross-section area of the disc and dx is its width. Since the discs are connected in series, the total resistance is

$$R = \int_0^{\ell} dR = \frac{1}{A\ell\sigma_0}\int_0^{\ell}\sqrt{x}dx = \frac{2\sqrt{\ell}}{3A\sigma_0} \qquad \ldots(2)$$

(b) From Ohm's Law, we deduce that the current flowing across the cylinder is given by

$$I = \frac{V_0}{R} = \frac{3A\sigma_0 V_0}{2\sqrt{\ell}} \qquad \ldots(3)$$

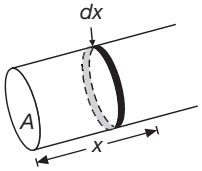
The current density is, therefore

$$J = \frac{I}{A} = \frac{3\sigma_0 V_0}{2\sqrt{\ell}} \qquad \ldots(4)$$

The electric field in the cylinder may be found by using Ohm's Law, according to which

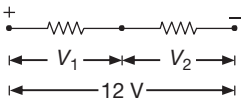
$$J = \sigma(x)E(x)$$

$$\Rightarrow \quad E(x) = \frac{J}{\sigma(x)} = \frac{J}{\sigma_0\left(\frac{1}{\sqrt{x}}\right)} = \frac{3V_0\sqrt{x}}{2\ell^{\frac{3}{2}}} \qquad \ldots(5)$$



21. Refer to solution of ILLUSTRATION 24, we get on solving $R_{AB} = \left(\frac{\sqrt{7}-1}{3}\right)R_0$

22. Note that points a, h, g and f have same potential, they are connected by conducting wires without any circuit elements between them. Similarly points b, c, d and e have same potential. Hence the potential drop across branch e and f and a and b is same. The two resistors ($6\ \Omega$ and $4\ \Omega$ in series) are directly connected across the terminals of 12 V battery.
The complex circuitry in the middle has no effect on the potential drop across the upper $10\ \Omega$ branch. If the current through it is I



Potential drop across R_1, $V_1 = IR_1$
Potential drop across R_2, $V_2 = IR_2$
Potential drop across branch,

$$V = V_1 + V_2 = I(R_1 + R_2)$$

The current $I = \frac{V}{R_1 + R_2} = \frac{12}{10} = 1.2$ A

Hence $V_1 = (1.2)(6) = 7.2$ V

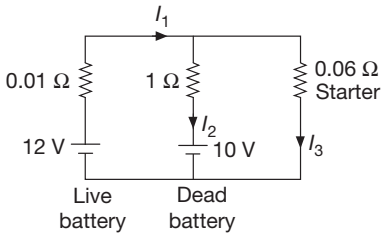
$V_2 = (1.2)(4) = 4.8$ V

23. Since the polarity of batteries in the lowest horizontal branch and in the right vertical branch oppose each other, so the net emf in these branches is zero. So there is no current anywhere in the circuit.

Test Your Concepts-VI (Based on Kirchhoff's Laws and Nodal Analysis)

1. Using Kirchhoff's Rules,

$$12 - (0.01)I_1 - (0.06)I_3 = 0$$
$$10 + (1)I_2 - (0.06)I_3 = 0$$



and $I_1 = I_2 + I_3$

$$12 - (0.01)I_2 - (0.07)I_3 = 0$$
$$10 + (1)I_2 - (0.06)I_3 = 0$$

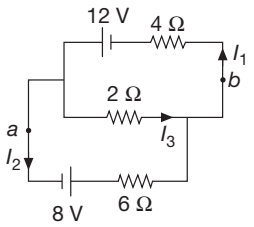
Solving simultaneously,

$I_2 = 0.283$ A, downwards in the dead battery

and $I_3 = 171$ A, downwards in the starter.

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

2. Let the currents be I_1, I_2, and I_3 as shown.



(a) $I_1 = I_2 + I_3$ …(1)

Traversing the top loop counter clockwise, we get

$$12\ \text{V} - (2\ \Omega)I_3 - (4\ \Omega)I_1 = 0$$

$$\Rightarrow \quad I_1 = 3 - \frac{1}{2}I_3 \qquad \ldots(2)$$

Traversing the bottom loop, we get

$$8\ \text{V} - (6\ \Omega)I_2 + (2\ \Omega)I_3 = 0$$

$$\Rightarrow \quad I_2 = \frac{4}{3} + \frac{1}{3}I_3 \qquad \ldots(3)$$

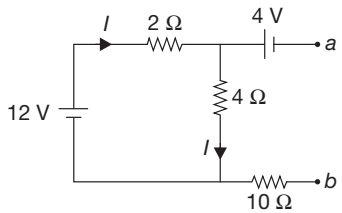
From (1), (2) and (3), we get

$I_3 = 909$ mA

(b) $V_a - (0.909\ \text{A})(2\ \Omega) = V_b \Rightarrow V_b - V_a = -1.82$ V

3. Using Kirchhoff's Loop Rule for the closed loop, $+12 - 2I - 4I = 0$, so $I = 2$ A

$$V_b - V_a = +4\ \text{V} - (2\ \text{A})(4\ \Omega) - (0)(10\ \Omega) = -4\ \text{V}$$

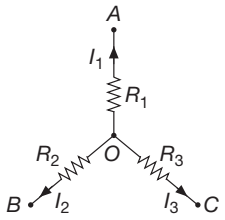


Thus, $|\Delta V_{ab}| = 4$ V and point a is at the higher potential.

4. Let the current through each branch be I_1, I_2, I_3 as shown in figure. Applying KCL to O, we get

$$I_1 + I_2 + I_3 = 0 \qquad \ldots(1)$$

CHAPTER 3



Now if V_0 is the potential at junction point O, then

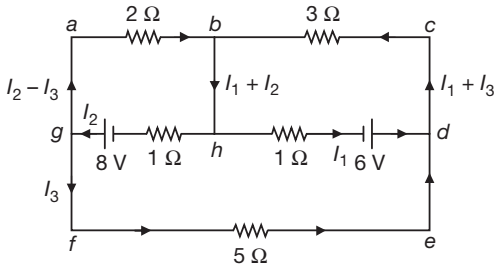
$$I_1 = \frac{(V_0 - V_1)}{R_1},\ I_2 = \frac{(V_0 - V_2)}{R_2} \text{ and } I_3 = \frac{(V_0 - V_3)}{R_3}$$

So substituting these values of I_1, I_2 and I_3 in equation (1), we have

$$V_0\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)=0$$

$$\Rightarrow\quad V_0 = \frac{\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$$

5. Applying Kirchhoff's Loop Law in three loops, we have,



For loop *abhga* $8 - 2(I_2 - I_3) - 1(I_2) = 0$ …(1)

For loop *dhbcd* $-6 + 1(I_1) + 3(I_1 + I_3) = 0$ …(2)

For loop hgfedh $-8 + 1(I_2) - 1(I_1) + 6 + 5I_3 = 0$ …(3)

Solving these three equations, we have

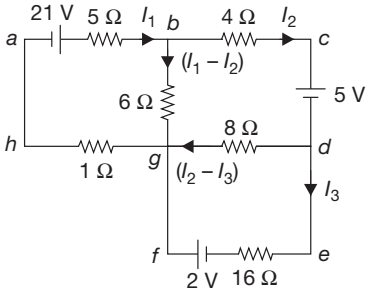
$$I_1 = \frac{108}{77}\text{ A},\ I_2 = \frac{212}{77}\text{ A and } I_3 = \frac{10}{77}\text{ A}$$

$V_A - V_B = (I_2 - I_3)(2\ \Omega)$ and $V_B - V_C = -(I_1 + I_3)(3\ \Omega)$

$$\Rightarrow\quad V_A - V_B = \left(\frac{212-10}{77}\right)(2) = \frac{404}{77}\text{ V}$$

and $V_B - V_C = -\frac{354}{77}$ V

6. Using the loop current method.
Three equations are as under



For loop abgha $21 - 5I_1 - 6(I_1 - I_2) - 1I_1 = 0$ …(1)

For loop cdgbc $-5 - 8(I_2 - I_3) + 6(I_1 - I_2) - 4I_2 = 0$ …(2)

For loop efgde $2 + 8(I_2 - I_3) - 16I_3 = 0$ …(3)

On solving, we get

$I_1 = 2$ A, $I_2 = 0.5$ A and $I_3 = 0.25$ A

So, current through 1 Ω and 5 Ω is $I_1 = 2$ A

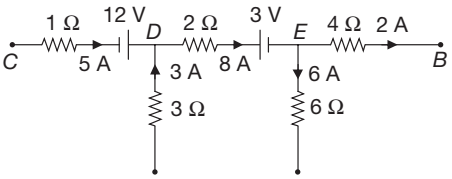
current through 6 Ω is $I_1 - I_2 = 1.5$ A

current through 8 Ω is $I_2 - I_3 = 0.25$ A

current through 4 Ω is $I_2 = 0.5$ A and

current through 16 Ω is $I_3 = 0.25$ A

7. Applying Kirchhoff's Junction Law at E current in wire DE is 8 A from D to E. Now further applying Junction Law at D. The current in 3 Ω resistance will be 3 A towards D.



Power dissipated in 3 Ω resistance

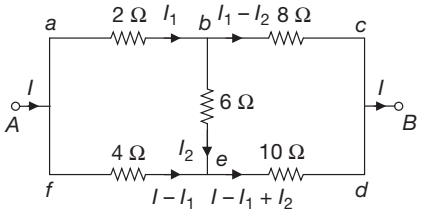
$$= I^2R = (3)^2(3) = 27\text{ W}$$

$V_C - V_B$: $V_C - 5 \times 1 + 12 - 8 \times 2 - 3 - 4 \times 2 = V_B$

$$\Rightarrow\quad V_C - V_B = 5 - 12 + 16 + 3 + 8$$

$$\Rightarrow\quad V_C - V_B = 20\text{ V}$$

8. This is an unbalanced Wheatstone bridge.



Applying Kirchhoff's Second Law in loops *abefa* and *bcdeb* we have,

$$-2I_1 - 6I_2 + 4(I - I_1) = 0 \quad \ldots(1)$$

$$-8(I_1 - I_2) + 10(I - I_1 + I_2) + 6I_2 = 0 \quad \ldots(2)$$

Solving these two equations we have,

$$I_1 = \frac{13}{21}I \text{ and } I_2 = \frac{I}{21}$$

For $AabcB$, we have

$$V_A - 2I_1 - 8(I_1 - I_2) - V_B = 0$$

$$\Rightarrow \quad V_{AB} = 2I_1 + 8(I_1 - I_2)$$

Substituting values of I_1 and I_2,

$$V_{AB} = \frac{122}{21}I \quad \ldots(3)$$

According to Thevenin's Theorem, "any complicated circuit can be reduced to an equivalent resistance" and hence the potential difference (across the terminals for which equivalent resistance has been calculated) equals the total current (entering the terminal) times the equivalent resistance. So,

$$V_{AB} = IR_{AB} \quad \ldots(4)$$

Equating equations (3) and (4), we get

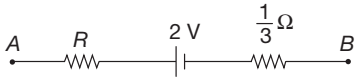
$$R_{AB} = \frac{122}{21}\ \Omega$$

9. (a) Equivalent emf of three batteries would be :

$$E_{eq} = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{(3/1 + 2/1 + 1/1)}{(1/1 + 1/1 + 1/1)} = 2\text{ V}$$

Further r_1, r_2 and r_3 each are of 1 Ω. Therefore, internal resistance of the equivalent battery will be $\frac{1}{3}$ Ω as all three are in parallel.

The equivalent circuit is therefore, shown in the figure.



Since, no current is taken from the battery.

$$V_{AB} = 2\,V \ (\text{From } V = E - Ir)$$

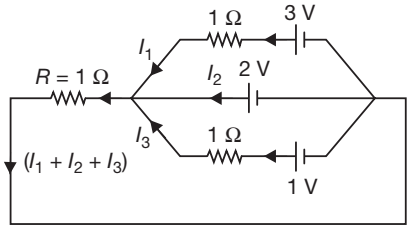
Further, $V_A + I_1 r_1 - E_1 = V_B$

$$\Rightarrow \quad I_1 = \frac{V_B - V_A + E_1}{r_1} = \frac{-2+3}{1} = 1\text{ A}$$

Similarly, $I_2 = \frac{V_B - V_A + E_2}{r_2} = \frac{-2+2}{1} = 0$

and $I_3 = \frac{V_B - V_A + E_3}{r_3} = \frac{-2+1}{1} = -1\text{ A}$

(b) r_2 is short circuited means resistance of this branch becomes zero. Making a closed circuit with a battery and resistance R, applying Kirchhoff's Second Law in three loops so formed.



$$3 - I_1 - (I_1 + I_2 + I_3) = 0 \quad \ldots(1)$$

$$2 - (I_1 + I_2 + I_3) = 0 \quad \ldots(2)$$

$$1 - I_3 - (I_1 + I_2 + I_3) = 0 \quad \ldots(3)$$

From equation (2),

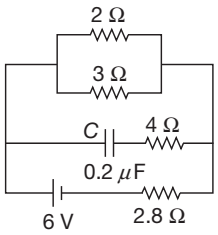
$$I_1 + I_2 + I_3 = 2\text{ A}$$

$\Rightarrow$ Substituting in (1) we get, $I_1 = 1$ A

Substituting in (3), we get $I_3 = -1$ A

$\Rightarrow \quad I_2 = 2$ A

10. In steady state situation no current will flow through the capacitor, 2 Ω and 3 Ω are in parallel.



Therefore, their combined resistance will be

$$R = \frac{2\times 3}{2+3} = 1.2\ \Omega$$

Net current through the battery

$$I = \frac{6}{1.2 + 2.8} = 1.5\text{ A}$$

This current will distribute in inverse ratio of their resistances 2 Ω and 3 Ω.

$$\Rightarrow \quad \frac{I_2}{I_3}=\frac{3}{2}$$

$$\Rightarrow \quad I_2=\left(\frac{3}{3+2}\right)(1.5)=0.9 \text{ A}$$

11. Applying Kirchhoff's Second Law in Loop $ADBA$:

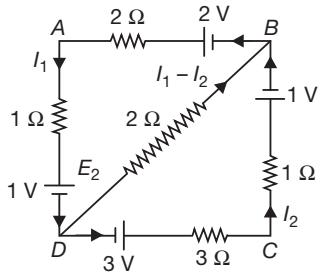
$$2-2I_1-I_1-1-2(I_1-I_2)=0 \qquad \ldots(1)$$

Similarly applying Kirchhoff's Second Law in Loop $BDCB$

$$2(I_1-I_2)+3-3I_2-I_2-1=0 \qquad \ldots(2)$$

Solving in equations (1) and (2) we get,

$$I_1=\frac{5}{13}, I_2=\frac{6}{13} \text{ and } I_1-I_2=-\frac{1}{13}$$



(a) Potential difference between B and D.

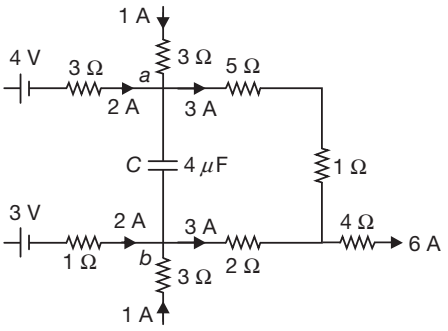
$$V_B+2(I_1-I_2)=V_D$$

$$\Rightarrow \quad V_B-V_D=-2(I_1-I_2)=\frac{2}{13} \text{ V}$$

(b) $V_G=E_G-I_2r_G=3-\frac{6}{13}\times 3=\frac{21}{13}$ V

$$V_H=E_H+I_2r_H=1+\frac{6}{13}\times 1=\frac{19}{13} \text{ V}$$

12. Using Kirchhoff's First Law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potential difference V_{ab} across it.

$$V_a-3\times 5-3\times 1+3\times 2=V_b$$

$$\Rightarrow \quad V_a-V_b=V_{ab}=12 \text{ V}$$

$$\Rightarrow \quad U=\frac{1}{2}CV_{ab}^2=\frac{1}{2}\left(4\times 10^{-6}\right)(12)^2 \text{ J}=0.288 \text{ mJ}$$

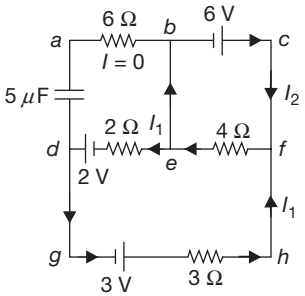
13. In steady state no current will flow through $R_1=6\ \Omega$.

Potential difference across R_3 or 4 Ω is E_1 or 6 V.

$\therefore$ Current through it will be $\frac{6}{4}=1.5$ A from right to left.

Because left hand side of this resistance is at higher potential.

Now, suppose this 1.5 A distributes in I_1 and I_2 as shown.



Applying Kirchhoff's Second Law in Loop $dghfed$:

$$3-3I_1-4\times 1.5-2I_1+2=0$$

$$\Rightarrow \quad I_1=-\frac{1}{5} \text{ A}=-0.2 \text{ A}$$

To find energy stored in capacitor we will have to find potential difference across it. Or V_{ad}.

Now, $V_a-2I_1+2=V_d$

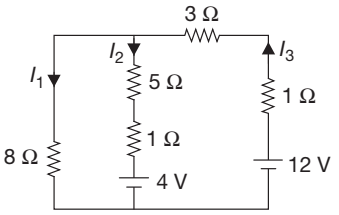
$$\Rightarrow \quad V_a-V_d=2I_1-2=-2.4\ V$$

$$\Rightarrow \quad V_d-V_a=2.4\ V=V_{da}$$

Energy stored in capacitor:

$$U=\frac{1}{2}CV_{da}^2=\frac{1}{2}\left(5\times 10^{-6}\right)(2.4)^2=1.44\times 10^{-5} \text{ J}$$

14. We name currents I_1, I_2 and I_3 as shown.



From Kirchhoff's Current Rule, $I_3=I_1+I_2$.

Applying Kirchhoff's Voltage Rule to the loop containing I_2 and I_3

$$12\text{ V} - (4)I_3 - (6)I_2 - 4\text{ V} = 0$$

$$8 = (4)I_3 + (6)I_2$$

Applying Kirchhoff's Voltage Rule to the loop containing I_1 and I_2,

$$-(6)I_2 - 4\text{ V} + (8)I_1 = 0$$

$$(8)I_1 = 4 + (6)I_2 .$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \text{ or } \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}$$

Then $I_2 = 1.33(0.846\text{ A}) - 0.667$

and $I_3 = I_1 + I_2$ give $I_1 = 846$ mA, $I_2 = 462$ mA, $I_3 = 1.31$ A

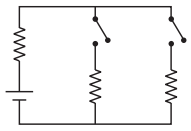
All currents are in the directions indicated by the arrows in the circuit diagram.

Test Your Concepts-VII (Based on emf, Internal Resistance and Combination of Cells)

1. (a) Here $E = I(R + r)$,

$$\text{so } I = \frac{E}{R+r} = \frac{12.6\text{ V}}{(5\ \Omega + 0.08\ \Omega)} = 2.48\text{ A}$$

Then, $\Delta V = IR = (2.48\text{ A})(5\ \Omega) = 12.4\text{ V}$



(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

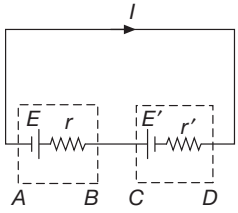
Then, $I_1 = I_2 + 35$ A, and $E - I_1 r - I_2 r = 0$

$$\Rightarrow \quad E = (I_2 + 35\text{ A})(0.08\ \Omega) + I_2(5\ \Omega) = 12.6\text{ V}$$

giving $I_2 = 1.93$ A

Thus, $\Delta V_2 = (1.93\text{ A})(5\ \Omega) = 9.65\text{ V}$

2. Two batteries AB and CD emf's E and $E'(E' > E)$ and internal resistances r and r' respectively are connected in series as shown in the figure.



If I is the current in the circuit, the total potential drop in the circuit must be equal to net emf:

$$I(r + r') = E' - E$$

$$\Rightarrow \quad I = \frac{E' - E}{r + r'}$$

The potential difference across terminals of battery AB is

$$V_B - V_A = Ir + E$$

Obviously $(V_B - V_A) > E$ by Ir, i.e.,

$$(V_B - V_A) - E = \frac{(E' - E)r}{r + r'}$$

when a current is flowing in any battery opposite to its emf, then terminal voltage is given by

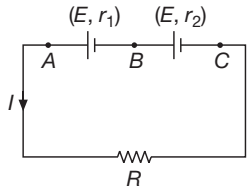
$$V = E + Ir$$

3. Two batteries are connected in series. The effective emf in the circuit is therefore $2E$ because both push the charge in the same direction. Hence emf's are added.

Net resistance in the circuit is $(r_1 + r_2 + R)$

Therefore, current in the circuit

$$I = \frac{2E}{(r_1 + r_2 + R)}$$



The potential difference between the terminals of first battery is $(V_A - V_B)$, terminal potential difference is given by

$$(V_A - V_B) = E - Ir_1$$

where E is the emf of the battery and r_1 is its internal resistance. Substituting the value of I, we get

$$V_A - V_B = E - \frac{2Er_1}{r_1 + r_2 + R} = E\frac{(R + r_2 - r_1)}{(R + r_2 + r_1)}$$

For $(V_A - V_B)$ to be zero, we must have

$$R = (r_1 - r_2)$$

This gives meaningful result only if $r_1 > r_2$. Otherwise, if $r_2 > r_1$, then $R = r_2 - r_1$ will produce terminal voltage across second cell to be zero $(V_{BC} = 0)$.

4. The voltage supplied by the charging plant is here constant which is equal to,

$$V = E_I + I_I r = (90) + (10)(2)$$

$$V = 110\text{ V}$$

Let I_f be the current at the end of charging

CHAPTER 3

Then, $V = E_f + I_f r$

$\Rightarrow \quad I_f = \dfrac{V - E_f}{r} = \dfrac{110 - 100}{2}$

$\Rightarrow \quad I_f = 5 \text{ A}$

5. In the first case current will be maximum when, Total external resistance = total internal resistance

$\Rightarrow \quad R = \dfrac{nr}{m} \Rightarrow r = \dfrac{mR}{n}$ …(1)

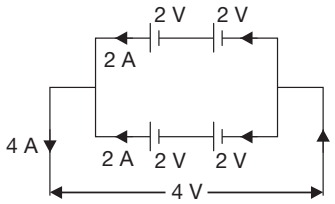
$\Rightarrow \quad I = \dfrac{nE}{2R}$ …(2)

In the second case,

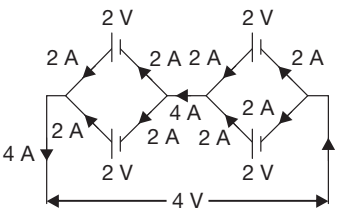
$$I' = \frac{mE}{R + \dfrac{mr}{n}} = \frac{mE}{R + \dfrac{m^2}{n^2}R}$$

$$\Rightarrow \quad I' = \frac{(mE)n^2}{R(n^2 + m^2)} = \frac{nE}{2R}\left(\frac{2mn}{n^2 + m^2}\right) = \left(\frac{2mn}{m^2 + n^2}\right)I$$

6. To get 4 V, two cells must be connected in series, and to get 4 A two cells must be connected in parallel. Following two arrangements of cells are possible, which can supply 4 A at 4 V.

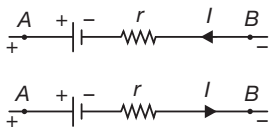


(A)



(B)

7. Let A, B represent the terminals of the cell.



$V_{AB} = E - IR$

$\Rightarrow \quad 8.5 = E - 3R$ …(1)

$V'_{AB} = E + IR$

$\Rightarrow \quad 11 = E + 2R$ …(2)

Solving (1) and (2), we get,

$E = 10 \text{ V}$ and $r = 0.5\ \Omega$

8. The last cell is a voltage divider that reduces the potential of the nth point k times as compared with the $(n-1)$ th point. Hence,

$$U_n = \frac{U_{n-1}}{R_1 + R_3}R_3 = \frac{U_{n-1}}{k}$$

$$\Rightarrow \quad \frac{R_1}{R_3} = k - 1$$

The relation $U_i = \dfrac{U_{i-1}}{k}$ should be true for any cell. For this reason the resistance of the last cell, of the last two, of the last three, etc., cells should also be R_3. Therefore,

$$\frac{1}{R_3} = \frac{1}{R_2} + \frac{1}{R_1 + R_3}$$

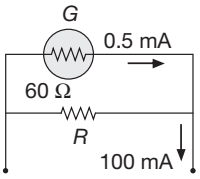
$$R_2 = \frac{R_3(R_1 + R_3)}{R_1} = R_3\frac{k}{k-1}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 = (k-1)^2 : k : (k-1)$$

Test Your Concepts-VIII (Based on Galvanometer, Voltmeter and Ammeter)

1. $\Delta V = I_g r_g = (I - I_g)R_p$

$$\Rightarrow \quad R_p = \frac{I_g r_g}{(I - I_g)} = \frac{I_g(60\ \Omega)}{(I - I_g)}$$

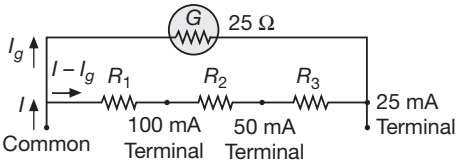


Therefore, to have $I = 0.1 \text{ A} = 100 \text{ mA}$ when $I_g = 0.5 \text{ mA}$:

$$R_p = \frac{(0.5 \text{ mA})(60\ \Omega)}{99.5 \text{ mA}} = 0.302\ \Omega$$

2. Consider the circuit diagram, realizing that $I_g = 1 \text{ mA}$.

For the 25 mA scale, we have $(I - I_g)S = I_g G$



$\Rightarrow \quad (24\text{ mA})(R_1+R_2+R_3)=(1\text{ mA})(25\ \Omega)$

$\Rightarrow \quad R_1+R_2+R_3=\left(\frac{25}{24}\right)\Omega \qquad \ldots(1)$

For the 50 mA scale:

$(49\text{ mA})(R_1+R_2)=(1\text{ mA})(25\ \Omega+R_3)$

$\Rightarrow \quad 49(R_1+R_2)=25\ \Omega+R_3 \qquad \ldots(2)$

For the 100 mA scale:

$(99\text{ mA})R_1=(1\text{ mA})(25\ \Omega+R_2+R_3)$

$\Rightarrow \quad 99R_1=25\ \Omega+R_2+R_3 \qquad \ldots(3)$

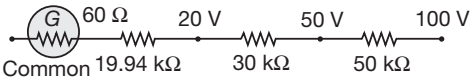
Solving (1), (2) and (3) simultaneously yields

$R_1=0.26\ \Omega,\ R_2=0.261\ \Omega,\ R_3=0.521\ \Omega$

3. $\Delta V=IR$ and for a voltmeter we have $V=I_g(G+R)$

(a) $20\text{ V}=(1\times10^{-3}\text{ A})(R_1+60\ \Omega)$

$R_1=1.994\times10^4\ \Omega=19.94\text{ k}\Omega$



(b) $50\text{ V}=(1\times10^{-3}\text{ A})(R_2+R_1+60\ \Omega)$

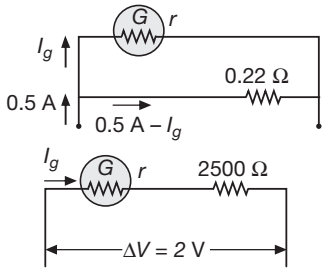
$R_2=30\text{ k}\Omega$

(c) $100\text{ V}=(1\times10^{-3}\text{ A})(R_3+R_1+60\ \Omega)$

$R_3=50\text{ k}\Omega$

4. For ammeter, we have $I_g r=(0.5\text{ A}-I_g)(0.22\ \Omega) \quad \ldots(1)$

$\Rightarrow \quad I_g(r+0.22\ \Omega)=0.11\text{ V}$



For voltmeter, we have $2\text{ V}=I_g(r+2500\ \Omega) \qquad \ldots(2)$

Solve (1) and (2) simultaneously, we get

$I_g=0.756\text{ mA}$ and $r=145\ \Omega$

5. (a) Potential difference across $R=(I_{\text{meas}}-I_1)R$; potential difference across voltmeter $=I_1R_v=V_{\text{meas}}$ and this is measured potential difference. The two branches are in parallel.

$(I_{\text{meas}}-I_1)R=V_{\text{meas}}$

$\Rightarrow \quad \left(I_{\text{meas}}-\frac{V_{\text{meas}}}{R_v}\right)R=V_{\text{meas}}$

$\Rightarrow \quad R=\frac{V_{\text{meas}}}{\left(I_{\text{meas}}-\frac{V_{\text{meas}}}{R_v}\right)}$

(b) From the expression for R, we get

$\frac{1}{R}=\frac{I_{\text{meas}}}{V_{\text{meas}}}-\frac{1}{R_v}$

If $\quad R_v\gg\frac{V_{\text{meas}}}{I_{\text{meas}}}$

$\frac{I}{R_v}\ll\frac{I_{\text{meas}}}{V_{\text{meas}}}$

$\Rightarrow \quad \frac{1}{R}\simeq\frac{I_{\text{meas}}}{V_{\text{meas}}}$

$\Rightarrow \quad R=\frac{V_{\text{meas}}}{I_{\text{meas}}}$

6. (a) The voltmeter and ammeter branches are in parallel arrangement. Hence,

$V_{\text{meas}}=(I-I_{\text{meas}})R_V=I_{\text{meas}}(R_A+R)$

Hence, $R_A+R=\frac{V_{\text{meas}}}{I_{\text{meas}}}$

$R=\left(\frac{V_{\text{meas}}}{I_{\text{meas}}}-R_A\right)$

(b) For $\frac{V_{\text{meas}}}{I_{\text{meas}}}\gg R_A$, $R\simeq\frac{V_{\text{meas}}}{I_{\text{meas}}}$

7. The given parameters are

$I_S=1\text{ mA}=1\times10^{-3}\text{ A}$

$G=20\ \Omega$

$I=I_{\max}=50\times10^{-3}\text{ A}$

Since, $S=\frac{I_G G}{(I-I_G)}$

$\Rightarrow \quad S=\frac{(1\times10^{-3})(20)}{50\times10^{-3}-1\times10^{-3}}$

$\Rightarrow \quad S=0.408\ \Omega$

The equivalent resistance of the instrument is

$\frac{1}{R_{\text{eq}}}=\frac{1}{G}+\frac{1}{S}=\frac{1}{20}+\frac{1}{0.408}$

$R_{\text{eq}}=0.4\ \Omega$

Note that shunt resistance is so small in comparison to the galvanometer resistance that the equivalent resistance is very nearly equal to the shunt resistance.

8. The maximum current through the galvanometer is

$I_G=\frac{V}{G}$

With a multiplier resistor in series with galvanometer the potential difference across the entire branch is

$$nV = I_G G + I_G R$$

$$\Rightarrow \quad nV = \left(\frac{V}{G}\right)G + \left(\frac{V}{G}\right)R$$

$$\Rightarrow \quad nV = V + \left(\frac{V}{G}\right)R$$

$$\Rightarrow \quad R = (n-1)G$$

Usually n is referred to as multiplication factor, e.g., a galvanometer with a range 0 to 1 mV is to be converted to a voltmeter of range 0 to 1 V.

We have $n = \dfrac{1\text{ V}}{1\text{ mV}} = 10^3$

The multiplier resistor $R = (10^3 - 1)G = 999G$.

So you can guess how large the multiplier resistor is in comparison to the galvanometer resistance G.

9. The currents in the ammeters are proportional to deflections produced, let α_1 and α_2 be the proportionality constants.

$$I_1 = \alpha_1 n_1,$$

$$I_2 = \alpha_2 n_2$$

The ammeters are in series, hence

$$I_1 = I_2$$

$$\Rightarrow \quad \alpha_1 n_1 = \alpha_2 n_2 \qquad \ldots(1)$$

In the second arrangement the potential drops across resistors are equal as they are in parallel arrangement. Resistances of ammeters have been ignored assuming them to be ideal.

Thus we have $I_1' R_1 = I_2' R_x$

Also $I_1' = \alpha_1 N_1$ and $I_2' = \alpha_2 N_2$

$$\Rightarrow \quad R_1 \alpha_1 N_1 = R_x \alpha_2 N_2 \qquad \ldots(2)$$

From equations (1) and (2), eliminating α_1 and α_2, we get

$$\frac{R_1 N_1}{n_1} = \frac{R_x N_2}{n_2}$$

$$\Rightarrow \quad R_x = \frac{R_1 n_2 N_1}{n_1 N_2}$$

10. Before the ammeter is connected, $I_0 = \dfrac{V}{R}$, and after it is connected, $I = \dfrac{V}{R + R_0}$, where R_0 is the resistance of the ammeter. The error is given by

$$\varepsilon = \frac{I_0 - I}{I_0} = \frac{1}{1 + \dfrac{R}{R_0}} = \frac{R_0}{R + R_0}$$

When $R_0 \ll R$, then the error may be neglected.

11. The sought error is $\varepsilon = \dfrac{V_0 - V}{V_0}$, where V_0 is the voltage across the resistance R before the voltmeter is switched on and V the voltage after it is switched on. According to Ohm's Law, $V_0 = IR$ and $V = I\left(\dfrac{RR_0}{R + R_0}\right)$, where R_0 is the resistance of the voltmeter. Hence, the error is

$$\varepsilon = \frac{\dfrac{R}{R_0}}{1 + \dfrac{R}{R_0}} = \frac{R}{R + R_0}$$

This is determined only by the ratio between the resistances of the section of the circuit and the voltmeter. When $R_0 \gg R$, the error may be neglected.

12. A reduction in the sensitivity n times means that the galvanometer carries a current I_1 which is n times smaller than the current in the remaining circuit before the branching off, Hence the current I_2 through the shunt is $\left(\dfrac{n-1}{n}\right)I$ in the remaining circuit. Hence,

$$\frac{r}{R} = \frac{I_1}{I_2} = \frac{1}{n-1}$$

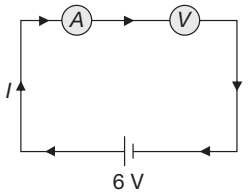
$$\Rightarrow \quad r = \frac{R}{n-1} \cong 204\ \Omega$$

13. Let R_1 = resistance of ammeter,

R_2 = combined resistance of ammeter and voltmeter

V = Voltage across voltmeter and

V_A = Voltage across ammeter



In the first case current in the circuit,

$$I = \frac{6}{R_2} \qquad \ldots(1)$$

and voltage across voltmeter $V = 6 -$ voltage across ammeter

$$\Rightarrow \quad V = 6 - V_A$$

$$\Rightarrow \quad V = 6 - IR_1$$

$$\Rightarrow \quad V = 6 - \frac{6R_1}{R_2} \qquad \ldots(2)$$

In the second case reading of ammeter becomes two times, i.e., the total resistance becomes half while the resistance of ammeter remains unchanged. Hence,

$$I' = \frac{6}{R_2/2} = \frac{12}{R_2} \quad \ldots(3)$$

and $V' = 6 - (I')R_1$

$$\Rightarrow \quad V' = 6 - \frac{12R_1}{R_2} \quad \ldots(4)$$

Further, it is given that $V' = \frac{V}{2}$

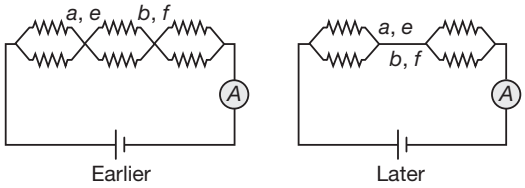
$$\Rightarrow \quad 6 - \frac{12R_1}{R_2} = 3 - \frac{3R_1}{R_2} \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{1}{3}$$

Substituting this value in equation (4), we have

$$V' = 6 - (12)\left(\frac{1}{3}\right)$$

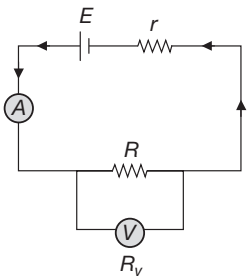
$$\Rightarrow \quad V' = 2 \text{ V}$$

14. The two conductors ace and bdf have different potentials, with the result that when the switch S is closed a current will flow from c to d, while the currents will flow from a to c, from e to c, from d to f and from d to b.



So, the closing of switch leads to an increase in the current flowing through the ammeter. Please note that the sections ab and ef of the resistors are shorted.

15. Net resistance $= \frac{E}{I} = \frac{3.4}{0.04}\ \Omega = 85\ \Omega$



$$\Rightarrow \quad R_A + r + \frac{RR_V}{R + R_V} = 85$$

$$\Rightarrow \quad 2 + 3 + \frac{100R_V}{100 + R_V} = 85$$

$$\Rightarrow \quad R_V = 400\ \Omega$$

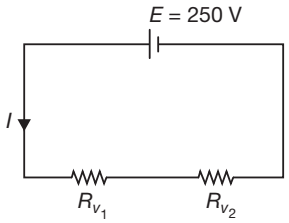
Further, $\frac{100R_V}{100 + R_V} = 80\ \Omega$

Hence, reading of voltmeter $= (0.04)(80) = 3.2$ V

Had it been an ideal voltmeter $(R_V \to \alpha)$, its reading would had been,

$$\left(\frac{3.4}{100 + 5}\right)(100) = 3.238 \text{ V}$$

16. (a) When the switch is open, no current flows through the branch that contains R_1 and R_2. Hence



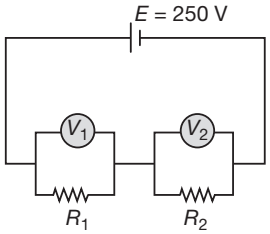
$$I = \frac{250}{10000} = \frac{1}{40} \text{ A}$$

According to Ohm's Law

$$V_1 = \frac{1}{40}(6000) = 150 \text{ V} \quad \text{and}$$

$$V_2 = (250 - 150) \text{ V} = 100 \text{ V}$$

(b) When the switch is closed the equivalent circuit is shown.



Since we observe that V_1 and R_1 are in parallel and V_2 and R_2 are in parallel and both in series and also simultaneously we have

$$\frac{R_1(R_{V_1})}{R_1 + R_{V_1}} = \frac{R_2(R_{V_2})}{R_2 + R_{V_2}} = 2.4 \text{ k}\Omega$$

Hence the potential of 250 V must be equally divided. So, reading of both the voltmeters is

$$V_1 = V_2 = \frac{E}{2} = 125 \text{ V}$$

17. (a) $\dfrac{RR_V}{R+R_V}=\dfrac{50\times200}{50+200}=40\ \Omega$

Current in the circuit, $I=\dfrac{4.3}{1+40+2}=0.1$ A

and voltage across voltmeter $=40I=4$ V

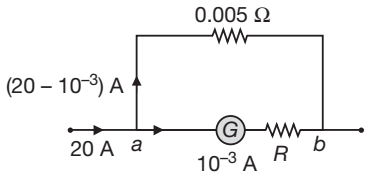
(b) $\dfrac{(R+R_A)R_V}{R+R_A+R_V}=\dfrac{52\times200}{52+200}=41.27\ \Omega$

$I=\dfrac{4.3}{1+41.27}=0.102$ A

Reading of voltmeter $=41.27I=4.2$ V

Reading of ammeter $=\left(\dfrac{R_V}{R+R_A+R_V}\right)(I)=0.08$ A

18. $V_{ab}=(0.005)(20-10^{-3})=10^{-3}(20+R)$



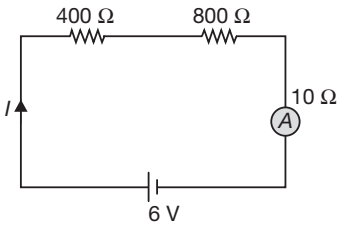
$\Rightarrow \quad R=79.995\ \Omega$

19. $\Delta R=G-\dfrac{GS}{G+S}=\dfrac{G^2}{G+S}$

20. $100=\left(\dfrac{2500\times r}{2500+r}\right)\times5$

$\Rightarrow \quad r=20.16\ \Omega$

21. Refer figure (a): Current through ammeter,



(a)

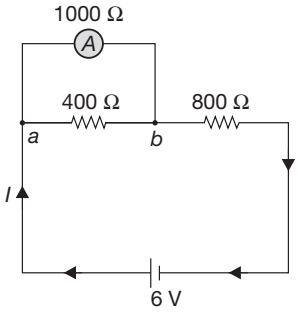
$I=\dfrac{\text{net emf}}{\text{net resistance}}=\dfrac{6}{400+800+10}=$

4.96×10^{-3} A $=4.96$ mA

Refer figure (b): Combined resistance of 1000 Ω voltmeter and 400 Ω resistance is,

$R=\dfrac{1000\times400}{1000+400}=285.71\ \Omega$

$\Rightarrow \quad I=\dfrac{6}{(285.71+800)}=5.53\times10^{-3}$ A

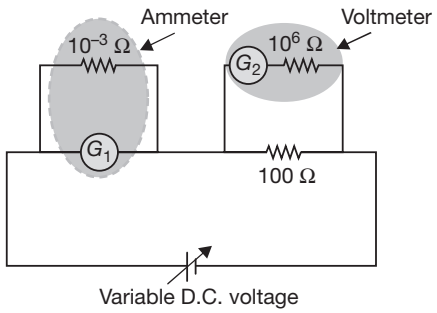


(b)

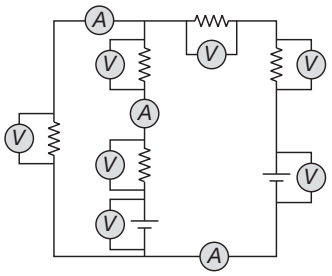
Reading of voltmeter

$=V_{ab}=I'R=\left(5.53\times10^{-3}\right)(285.71)=1.58$ V

22.



23. The usage of voltmeters and ammeters is shown in the figure.



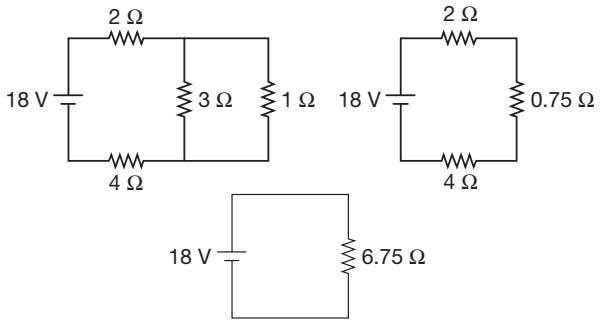
Test Your Concepts-IX (Based on Heating Effects and Power Consumption)

1. $\dfrac{1}{R_p}=\left(\dfrac{1}{3}+\dfrac{1}{1}\right)$

$\Rightarrow \quad R_p=0.75\ \Omega$

$R_s=(2+0.75+4)\Omega=6.75\ \Omega$

$I_{\text{battery}}=\dfrac{\Delta V}{R_S}=\dfrac{18\text{ V}}{6.75\ \Omega}=2.67$ A



Since, $P = I^2R \Rightarrow P_2 = (2.67\text{ A})^2(2\ \Omega)$

$\Rightarrow \quad P_2 = 14.2\text{ W in } 2\ \Omega$

$P_4 = (2.67\text{ A})^2(4\text{ A}) = 28.4\text{ W in } 4\ \Omega$

$\Delta V_2 = (2.67\text{ A})(2\ \Omega) = 5.33\text{ V}$

$\Delta V_4 = (2.67\text{ A})(4\ \Omega) = 10.67\text{ V}$

$\Delta V_p = 18\text{ V} - \Delta V_2 - \Delta V_4 = 2\text{ V}(= \Delta V_3 = \Delta V_1)$

So, $P_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2\text{ V})^2}{3\ \Omega} = 1.33\text{ W in } 3\ \Omega$

$P_1 = \dfrac{(\Delta V_1)}{R_1} = \dfrac{(2\text{ V})^2}{1\ \Omega} = 4\text{ W in } 1\ \Omega$

2. A certain quantity of energy $\Delta E_{\text{int}} = P(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $P = I\Delta V = \dfrac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus comparing coils 1 and 2, we have for the energy

$$H = \frac{(\Delta V)^2\,\Delta t}{R_1} = \frac{(\Delta V)^2\,2\Delta t}{R_2}$$

$\Rightarrow \quad R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{1}{\frac{1}{R_1}+\frac{1}{2R_1}} = \frac{2R_1}{3}$$

and $H = \dfrac{(\Delta V)^2\,\Delta t}{R_1} = \dfrac{(\Delta V)^2\,\Delta t_p}{\frac{2R_1}{3}}$

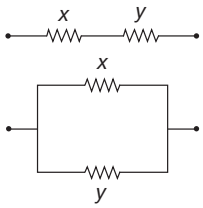
$\Rightarrow \quad \Delta t_p = \dfrac{2\Delta t}{3}$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and $\dfrac{(\Delta V)^2\,\Delta t}{R_1} = \dfrac{(\Delta V)^2\,\Delta t_s}{3R_1}$

$\Rightarrow \quad \Delta t_s = 3\Delta t$

3. Let the two resistances be x and y.

Then, $R_s = x + y = \dfrac{P_s}{I^2}$ and $R_p = \dfrac{xy}{x+y} = \dfrac{P_p}{I^2}$



From the first equation, $y = \dfrac{P_s}{I^2} - x$ and

the second becomes $\dfrac{x\left(\dfrac{P_s}{I^2 - x}\right)}{x + \left(\dfrac{P_s}{I^2 - x}\right)} = \dfrac{P_p}{I^2}$

$\Rightarrow \quad x^2 - \left(\dfrac{P_s}{I^2}\right)x + \dfrac{P_sP_p}{I^4} = 0$

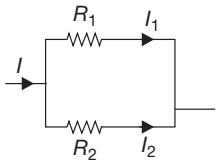
Using the quadratic formula, $x = \dfrac{P_s \pm \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

Then, $y = \dfrac{P_s}{I^2} - x$ gives $y = \dfrac{P_s \mp \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

The two resistances are $\dfrac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$ and $\dfrac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

4. (a) $\Delta V_1 = \Delta V_2$

$I_1R_1 = I_2R_2$



$\Rightarrow \quad I = I_1 + I_2 = I_1 + \dfrac{I_1R_1}{R_2} = I_1\dfrac{R_2 + R_1}{R_2}$

$\Rightarrow \quad I_1 = \dfrac{IR_2}{R_1 + R_2}$ and $I_2 = \dfrac{I_1R_1}{R_2} = \dfrac{IR_1}{R_1 + R_2} = I_2$

(b) The power delivered to the pair is $P = I_1^2R_1 + I_2^2R_2 = I_1^2R_1 + (I - I_1)^2R_2$. For minimum power let us find I_1 such that $\dfrac{dP}{dI_1} = 0$

$\Rightarrow \quad \dfrac{dP}{dI_1} = 2I_1R_1 + 2(I - I_1)(-1)R_2 = 0$

$$\Rightarrow \quad I_1R_1 - IR_2 + I_1R_2 = 0$$

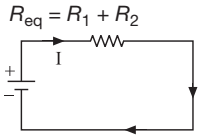
$$\Rightarrow \quad I_1 = \frac{IR_2}{R_1 + R_2}$$

This is the same condition as that derived in part (a).

5. For the resistors in series, from KVL we have

$$V_0 - I(R_1 + R_2) = 0$$

$$I = \frac{V_0}{(R_1 + R_2)}$$



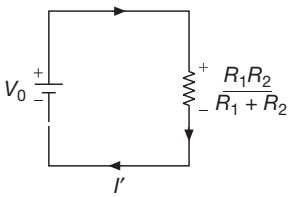
Equivalent series circuit

The power absorbed by equivalent resistor,

$$R_s = R_1 + R_2$$

$$\Rightarrow \quad I = \frac{V_0}{R_s} = \frac{V_0}{R_1 + R_2}$$

$$P = IV = \left(\frac{V_0}{R_1 + R_2}\right)V_0 = \frac{V_0^2}{R_1 + R_2}$$



Equivalent parallel circuit

For a parallel connection the equivalent resistance is

$$R_p = \frac{R_1R_2}{R_1 + R_2}$$

If I' be the current in the circuit , then

$$I' = \frac{V_0}{R_P} = \frac{V_0(R_1 + R_2)}{R_1R_2}$$

The power absorbed by equivalent resistor is,

$$P' = I'V = \left(V_0\frac{R_1 + R_2}{R_1R_2}\right)V_0 = V_0^2\left(\frac{R_1 + R_2}{R_1R_2}\right)$$

The ratio of the power absorbed in parallel and series circuit is

$$\frac{P'}{P} = \frac{V_0^2\left(\frac{(R_1 + R_2)}{R_1R_2}\right)}{\frac{V_0^2}{(R_1 + R_2)}} = \frac{(R_1 + R_2)^2}{R_1R_2}$$

$$\Rightarrow \quad \frac{P'}{P} = \frac{(R_1 + R_2)^2}{R_1R_2} = 2 + \frac{R_1}{R_2} + \frac{R_2}{R_1} > 1$$

The ratio is greater than unity. Therefore, power absorbed in a parallel connection of two resistors is greater than in a series connection of the same resistors for a given voltage.

6. (a) Power consumed in resistor 1 is

$$P_1 = \frac{V^2}{R_1} = \frac{V^2}{\left(\frac{\rho\ell}{A_1}\right)} = \frac{V^2A_1}{\rho\ell} = \frac{\pi}{4}\left(\frac{V^2D_1^2}{\rho\ell}\right)$$

Similarly power consumed in resistor 2 is

$$P_2 = \frac{V^2}{R_2} = \frac{\pi}{4}\frac{V^2D_2^2}{\rho\ell}$$

Since $D_1 = 2D_2$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{D_1^2}{D_2^2} = \frac{4D_2^2}{D_2^2} = 4$$

(b) Power consumed in resistor 1 is

$$P_1 = I^2R_1 = I^2\frac{\rho\ell}{A_1} = \frac{4}{\pi}\frac{I^2\rho\ell}{D_1^2}$$

Power consumed in resistor 2 is

$$P_2 = I^2R_2 = \frac{4}{\pi}\frac{I^2\rho\ell}{D_2^2}$$

If $D_1 = 2D_2$, then $\frac{P_1}{P_2} = \frac{\frac{1}{D_1^2}}{\frac{1}{D_2^2}} = \frac{D_2^2}{4D_2^2} = \frac{1}{4}$

7. The ratio of resistances at the two temperatures is given by

$$\frac{R_{20}}{R_{2000}} = \frac{1 + 20\ \alpha_0}{1 + 2000\ \alpha_0} = \frac{1 + 20 \times 0.0055}{1 + 2000 \times 0.0055}$$

$$\Rightarrow \quad \frac{R_{20}}{R_{2000}} = 9.25 \times 10^{-2}$$

At 2000 °C : $R_{2000} = \frac{V^2}{P} = \frac{(110)^2}{100} = 121\ \Omega$

At 20 °C : $R_{20} = 121 \times 9.25 \times 10^{-2} = 11.2\ \Omega$

Current at 20 °C, $I_{20} = \frac{P}{V} = \frac{110}{11.2} = 9.82$ A

Current at 2000 °C, $I_{2000} = \frac{P}{V} = \frac{100}{110} = 0.91$ A

Thus we can see that at the instant the bulb is turned on and when temperature of filament is 20 °C, the current is very large (9.82 A) as compared to current at 2000 °C ($I_{2000} = 0.91$ A)

8. (a) When a current is passed through a wire, its temperature rises with time due to heat produced. Since the fuse wire is exposed to the surroundings it also loses heat mainly due to radiation, heat loss depends upon temperature and it increases as temperature rises. The temperature attains a steady value when the rate at which heat is produced in the wire is equal to the rate at which heat is lost due to radiation. Let the steady temperature be equal to the melting point of the wire and I be the current at the melting point. The rate at which heat is produced in the wire is given by

$$P = I^2R$$

where R is the resistance of the wire given by

$$R = \frac{\rho\ell}{\pi r^2}$$

where ρ = resistivity, ℓ = length and r = radius of the wire. Thus

$$P = \frac{I^2\rho\ell}{\pi r^2} \qquad \ldots(1)$$

Now let h be the rate of loss of heat per unit surface area of the wire. The rate of loss of heat from the wire is

$$P' = 2\pi r\ell \times h \qquad \ldots(2)$$

In the steady state, $P = P'$. Therefore equating equations (1) and (2) we have

$$\frac{I^2\rho\ell}{\pi r^2} = 2\pi r\ell h$$

$$\Rightarrow \quad I = \sqrt{\frac{2\pi^2 r^3 h}{\rho}} \qquad \ldots(3)$$

Thus the current at which the melting point is reached is independent of the length, ℓ of the wire, it depends upon the values of r, ρ and h. If the two wires have the same value of r, they are made of the same material (ρ is the same for both wires), the rate of loss of heat per unit surface area (h) depends upon the material of the wire and the melting point which is same for both the wires, both fuses will melt at the same value of current, inspite of different lengths.

(b) From equation (3), we have

$$I = \sqrt{\frac{2\pi^2 r^3 h}{\rho}}$$

$$\Rightarrow \quad \frac{I^2}{r^3} = \frac{2\pi^2 h}{\rho} = \text{constant}$$

$$\Rightarrow \quad \frac{I_1^2}{r_1^3} = \frac{I_2^2}{r_2^3}$$

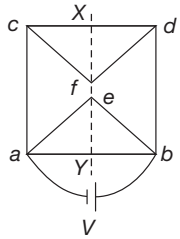
$$\Rightarrow \quad r_2 = r_1\left(\frac{I_2}{I_1}\right)^{\frac{2}{3}}$$

Substituting numerical values $r_1 = 0.1$ mm, $I_1 = 10$ A and $I_2 = 20$ A, we have

$$r_2 = (0.1)\left(\frac{20}{10}\right)^{\frac{2}{3}} = (0.1)(2)^{\frac{2}{3}}$$

$$\Rightarrow \quad r_2 = 0.159 \text{ mm} \approx 0.16 \text{ mm}$$

9. The circuit has asymmetry about line XY, i.e., the current in left and right are not mirror images. Just imagine the central junction of wires in the form of two junctions connected by wire ef as shown. Then it follows from symmetry consideration that there is no current in the wire ef, thus we can remove it from the circuit. The resistors of the wires will be proportional to their lengths.



$$R_{ab} = R_{ac} = R_{cd} = R_{bd} = r$$

$$R_{ae} = R_{be} = R_{cf} = R_{df} = \frac{r}{\sqrt{2}}$$

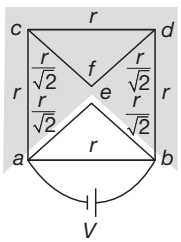
The power dissipated in resistor ab is

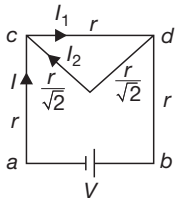
$$P_{ab} = \frac{V^2}{r}$$

By Applying Ohm's Law to the upper part, we can determine current through cd,

$$I_{cd} = \frac{V}{(\sqrt{2}+3)r} \qquad \{\because V_{ab} = V_{aeb} = V_{\text{shaded}} = V\}$$

The power dissipated in the conductor cd,





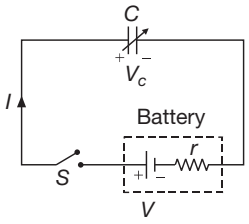
$$P_2 = I_{cd}^2 r = \frac{V^2}{(\sqrt{2}+3)^2 r}$$

Thus the required ratio is

$$\frac{P_1}{P_2} = \left(\sqrt{2}+3\right)^2 = 11+6\sqrt{2}$$

10. (a) The current in the circuit is related to the voltage across the capacitor as

$$I = C\frac{dV_c}{dt}$$



As the capacitor charges up, V_C rises toward its limiting value and its rate of change goes to zero. Therefore, in order to keep I constant, capacitance, C, should compensate for this drop in $\frac{dV_C}{dt}$ (observe that, as time $t \to \infty$, $\frac{dV_c}{dt} \to 0$ and we should have $C_\infty \to \infty$). Ignoring the practical difficulties of maintaining a constant I, asymptotically we have a continuous charge flow in the circuit

$$I = \frac{dq}{dt}$$

and $V_C = V$

The battery supplies a constant power since its emf is constant,

$$P_B = VI$$

(b) The capacitor is being continuously charged. Once the voltage across the capacitor stabilizes at $t \to \infty$, we have

$$V = Ir + V_{C,\infty}$$

$$\Rightarrow \quad V_{C,\infty} = V - Ir$$

The energy stored in the capacitor becomes

$$W = \frac{1}{2}q_\infty V_{C,\infty}$$

$$\Rightarrow \quad W = \frac{1}{2}q_\infty (V - Ir)$$

The rate of change of W with time is

$$P_c = \frac{dE}{dt} = \frac{1}{2}V_{c,\infty}\frac{dq_\infty}{dt} = \frac{1}{2}I(V - Ir)$$

The rate of dissipation of energy in the resistor r is $P_r = I^2 r$,

Therefore the power supplied directly to the capacitor is

$$P = P_B - P_r = IV - I^2 r = I(V - Ir),$$

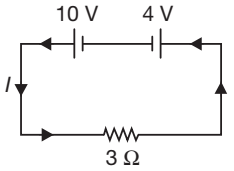
which is twice as large as that used in charging the capacitor. So,

$$P = 2P_c$$

The battery supplies twice as much energy to the capacitor as is stored in the capacitor. The difference is the work done by the capacitor against the external agent which is changing the capacitance.

11. Net emf of the circuit $= (10 - 4)$ V $= 6$ V

Total resistance of the circuit $= 3\ \Omega$



$$\Rightarrow \quad \text{Current in the circuit } I = \frac{\text{net emf}}{\text{total resistance}} = \frac{6}{3} = 2 \text{ A}$$

(a) Power supplied by 10 V battery

$= EI = (10)(2) = 20$ W

(b) Power consumed by 4 V battery

$= EI = (4)(2) = 8$ W

(c) Power consumed by $3\ \Omega$ resistance

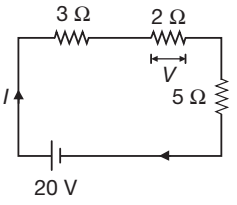
$= I^2R = (2)^2(3) = 12$ W

Here we can see that total power supplied by 10 V battery (i.e., 20 W) = power consumed by 4 V battery and $3\ \Omega$ resistance. Which proves that conservation of energy holds good in electric circuits also.

12. The $6\ \Omega$ and $3\ \Omega$ resistances are in parallel. So their combined resistance is,

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \text{ or } R = 2\ \Omega$$

The equivalent simple circuit can be drawn as shown. Current in the circuit,



$$I = \frac{\text{net emf}}{\text{total resistance}} = \frac{20}{3+2+5} = 2 \text{ A}$$

$V = IR = (2)(2) = 4$ V

i.e., potential difference across $6\ \Omega$ and $3\ \Omega$ resistances are 4 V. Now,

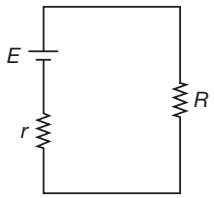
$$H_{3\,\Omega} = I^2Rt = (2)^2(3)(2) = 24 \text{ J}$$

$$H_{6\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2}{6}(2) = \frac{16}{3} \text{ J}$$

$$H_{3\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2(2)}{3} = \frac{32}{3} \text{ J}$$

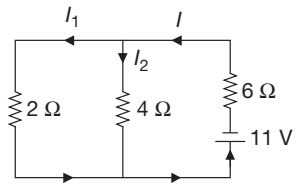
and $H_{5\,\Omega} = I^2Rt = (2)^2(5)(2) = 40 \text{ J}$

13. Suppose, we insert the battery with $2\,\Omega$ resistance. Then we can take $2\,\Omega$ as the internal resistance (r) of the battery and combined resistance of the other two as the external resistance (R). The circuit in that case shown in figure.



Now power, $P = \dfrac{E^2}{R+r}$

This power will be minimum where $R+r$ is maximum and we can see that $(R+r)$ will be maximum when the battery is inserted with $6\,\Omega$ resistance as shown in figure.



Net resistance in this case is

$$6 + \frac{2\times 4}{2+4} = \frac{22}{3}\ \Omega$$

$$\Rightarrow \quad I = \frac{11}{22/3} = 1.5 \text{ A}$$

This current will be distributed in $2\,\Omega$ and $4\,\Omega$ in the inverse ratio of their resistances.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{4}{2} = 2$$

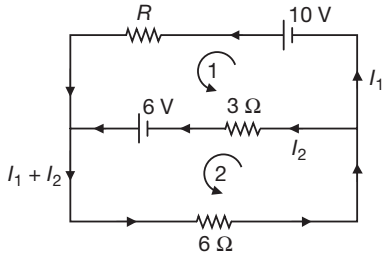
$$\Rightarrow \quad I_1 = \left(\frac{2}{2+1}\right)(1.5) = 1 \text{ A}$$

So, $P_6 = (1.5)^2 6 = 13.5 \text{ W}$

$$P_4 = (0.5)^2 4 = 1 \text{ W}$$

$$P_2 = (1)^2(2) = 2 \text{ W}$$

14. Applying Kirchhoff's Laws to the two loops we have,



$$10 - I_1R - 6 + 3I_2 = 0 \qquad \ldots(1)$$

$$6 - 6(I_1 + I_2) - 3I_2 = 0 \qquad \ldots(2)$$

Solving these two equations we get,

$$I_1 = \frac{6}{R+2}$$

Power developed in R,

$$P = I_1^2R = \frac{36}{(R+2)^2}R \qquad \ldots(3)$$

For power to be maximum,

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \quad (R+2)^2(36) - (36R)(2)(R+2) = 0$$

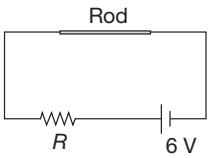
$$\Rightarrow \quad R + 2 - 2R = 0$$

$$\Rightarrow \quad R = 2\ \Omega$$

For maximum power from equation (3), we have

$$P_{\max} = 4.5 \text{ W}$$

15. (a) At $I = 0.4$ A, voltage across the rod,



$$V = \left(\frac{0.4}{0.2}\right)^2 = 4 \text{ V}$$

$$\Rightarrow \quad V_R = 6 - 4 = 2 \text{ V}$$

Now, $IR = V_R = 2$ V

$$\Rightarrow \quad R = \frac{2}{0.4} = 5\ \Omega$$

(b) $P_{\text{supplied}} = EI = 6I$

$$P_R = I^2R$$

$$\Rightarrow \quad P_{\text{rod}} = 6I - I^2R$$

Given that, $6I - I^2R = 2I^2R$

$$\Rightarrow \quad 6 - IR = 2IR$$

$$\Rightarrow \quad IR = 2 \qquad \ldots(1)$$

Further, $6 = V_R + V_{\text{rod}} = IR + \dfrac{I^2}{0.04}$ $\qquad \ldots(2)$

$$\left\{\because\ I = 0.2\ \text{V}^{\frac{1}{2}} \Rightarrow I^2 = 0.04 V_{\text{rod}}\right\}$$

Solving equations (1) and (2), we get

$R = 5\ \Omega$

16. $V = E - Ir$

$\Rightarrow \quad 2 = 2.6 - (1)(r)$

$\Rightarrow \quad r = 0.6\ \Omega$

Power generated by battery $= VI = 2$ W

and power developed in it by electric forces $= I^2 r = 0.6$ W

17. (a) By the 4 V battery

$$\Delta U = (\Delta V) I \Delta t = (4\ \text{V})(-0.462\ \text{A})120\ \text{s} = -222\ \text{J}$$

By the 12 V battery

$$(12\ \text{V})(1.31\ \text{A})120\ \text{s} = 1.88\ \text{kJ}$$

(b) By the 8 Ω resistor

$$I^2 R \Delta t = (0.846\ \text{A})^2 (8\ \Omega)120\ \text{s} = 687\ \text{J}$$

By the 5 Ω resistor

$$I^2 R \Delta t = (0.462\ \text{A})^2 (5\ \Omega)120\ \text{s} = 128\ \text{J}$$

By the 1 Ω resistor

$$I^2 R \Delta t = (0.462\ \text{A})^2 (1\ \Omega)120\ \text{s} = 25.6\ \text{J}$$

By the 3 Ω resistor

$$I^2 R \Delta t = (1.31\ \text{A})^2 (3\ \Omega)120\ \text{s} = 616\ \text{J}$$

By the 1 Ω resistor

$$I^2 R \Delta t = (1.31\ \text{A})^2 (1\ \Omega)120\ \text{s} = 205\ \text{J}$$

(c) $-222\ \text{J} + 1.88\ \text{kJ} = 1.66\ \text{kJ}$ from chemical to electrical

$687\ \text{J} + 128\ \text{J} + 25.6\ \text{J} + 616\ \text{J} + 205\ \text{J} = 1.66\ \text{kJ}$ from electrical to internal

Test Your Concepts-X (Based on Wheatstone Bridge and Potentiometer)

1. (a) The value of E is given by ($E_1 = 1.02$ V is the emf of the standard cell C_1)

$$E = \frac{1.02 \times 82.3}{67.3} = 1.25\ \text{V}$$

(b) The high resistance R keeps the current drawn from the standard cell within permissible limit and prevents a large current to flow through the galvanometer when far away from the balance point.

(c) At null point no current flows through R, hence no effect on null point.

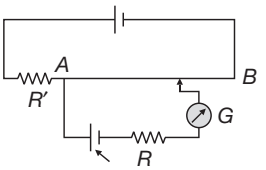
(d) The null point depends on the terminal voltage of C and the emf of C_1 only, hence no effect.

(e) The method would not work if the potential difference across AB due to the driver cell C became less than the emf of cell C_1, because there would be no null point on the wire.

(i) In this case the potential difference across AB due to cell C of emf 2 V will be less than 1 V. Since the emf of the standard cell is greater than this value, there will be no null point on the wire. So, it will not work.

(ii) Similarly, the method would not work if the emf of C were 1 V instead of 2 V.

(f) Suppose the emf E to be measured is, say, $1\ \text{mV} = 1 \times 10^{-3}$ V, the potential difference across AB due to C is 2 V, the length AB of the wire is 100 cm and its resistance per centimetre is λ (say). Then the balance length will be



1×10^{-3} V = P.D. across AJ due to C = current in the main circuit × resistance of AJ

$$\Rightarrow \quad 1 \times 10^{-3} = \frac{2}{100\lambda} \lambda AJ = \frac{2\ AJ}{100}\ \text{V}$$

$$\Rightarrow \quad AJ = 0.05\ \text{cm}$$

Thus the null point will be very close to A and there is an extremely large percentage error in its measurement. To have a large balance length, the circuit shown in figure is modified by putting a suitable resistor R'. The balance length will then be measurable and the percentage error will be much smaller.

2. Let E be the emf of the cell C' and r its internal resistance. Let $\ell = AJ$ be the balance length when switch S is open. When a resistance R is introduced by closing the switch, a current begins to flow through the cell C' and resistance R. The potential difference between the terminals of the cell falls and the balance length decreases to $\ell' = AJ'$. The internal resistance of the cell is given by

$$r = \frac{E - V}{I}$$

where V is the terminal voltage of C' and I is the current in the circuit involving C' and R. Also

$$I = \frac{V}{R}$$

$$\Rightarrow \quad r = \left(\frac{E}{V} - 1\right)R$$

But $\frac{E}{V} = \frac{\ell}{\ell'}$. Hence

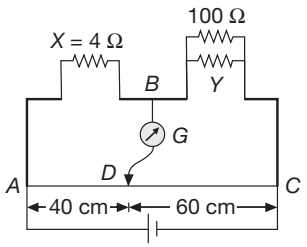
$$\Rightarrow \quad r = R\left(\frac{\ell - \ell'}{\ell'}\right) = 4\left(\frac{76.3 - 60}{60}\right) = 1.1\ \Omega$$

3. The resistances of the two segments of the wire AD and DC are in the ratio of their lengths. If R_0 is the resistance of Y in melting ice $(0\ °C)$, the balance condition of Wheatstone bridge gives

$$\frac{X}{R_0} = \frac{\lambda\ell}{\lambda(100 - \ell)} = \frac{\ell}{100 - \ell}$$

where λ is the resistance per centimetre of wire AC. Now, $\ell = 40$ cm and $X = 4\ \Omega$. Substituting these values, we get $R_0 = 6\ \Omega$. Let R_t be the resistance of Y when heated to a temperature $t = 100\ °C$. When it is connected in parallel with $100\ \Omega$ resistor as shown in figure, the net resistance becomes

$$R' = \frac{100R_t}{R_t + 100}$$



Since the null point remains unchanged, we have

$$\frac{X}{R'} = \frac{40}{60}$$

$$\Rightarrow \quad R' = 6\ \Omega$$

$$\Rightarrow \quad 6 = \frac{100R_t}{R_t + 100}$$

$$\Rightarrow \quad R_t = 6.38\ \Omega$$

Temperature coefficient of resistance of the coil Y is

$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{6.38 - 6}{6 \times 100} = 6.3 \times 10^{-4}\ K^{-1}$$

4. $$\frac{E}{V} = \frac{0.52}{0.4} = \frac{13}{10} = 1.3$$

$$\Rightarrow \quad \frac{E}{E - Ir} = 1.3$$

$$\Rightarrow \quad 1.3Ir = 0.3E$$

$$\Rightarrow \quad 1.3\left(\frac{Er}{R + r}\right) = 0.3E \qquad \left\{\because I = \frac{E}{R + r}\right\}$$

$$\Rightarrow \quad r = 0.3R = (0.3)(5) = 1.5\ \Omega$$

5. Emf of cell = (potential gradient) × (balancing length)

$$\Rightarrow \quad E_1 = \frac{V_{AB}}{L} \times x$$

$$\Rightarrow \quad 2.4 = \frac{V_{AB}}{10} \times 8$$

$$\Rightarrow \quad V_{AB} = 3\ V$$

Consider the loop containing E and applying potential divider concept, we get

$$V_{AB} = E\frac{R_{AB}}{R_{AB} + r}$$

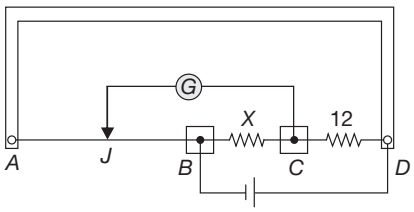
$$\Rightarrow \quad 3 = 4\frac{(1.6)(10)}{(1.6)(10) + r} \quad \left\{\because R_{AB} = \left(1.6\ \Omega m^{-1}\right)(10\ m)\right\}$$

$$\Rightarrow \quad r = \frac{16}{3}\ \Omega$$

Note that as there is no current through the cell and galvanometer, the battery E, the internal resistance r and the potentiometer wire AB are in series.

6. (a) There are no positive and negative terminals on the galvanometer because only zero deflection is needed.

(b)



(c) $AJ = 60$ cm

$$\Rightarrow \quad BJ = 40\ cm$$

If no deflection is taking place. Then, the Wheatstone bridge is said to be balanced.

Hence, $\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}}$

$$\Rightarrow \quad \frac{X}{12} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow \quad x = 8\ \Omega$$

7. Slide wire bridge is most sensitive when the resistance of all the four arms of bridge is same.
Hence, for the resistor B the value of X is the most accurate answer.

8. Resistance of x metre length of wire,

$$R_1 = \frac{R_0}{L}x$$

Resistance R and R_1 are in parallel, so their effective resistance,

$$R' = \frac{RR_1}{R+R_1}$$

This is in series with length $(L-x)$ of wire. Resistance of length $(L-x)$ of wire is

$$R_2 = \frac{R_0}{L}(L-x)$$

Hence, total resistance is given by

$$R_{\text{eff}} = R' + R_2 = \frac{RR_1}{R+R_2} + R_2$$

So, current in the main circuit is given by

$$I = \frac{V_0}{R_{\text{eff}}} = \frac{V_0}{\left(\frac{RR_1}{R+R_1}+R_2\right)}$$

Potential difference across the length x is

$$V = IR' = I\frac{RR_1}{R+R_1}$$

$$\Rightarrow \quad V = \frac{V_0}{\left(\frac{RR_1}{R+R_1}+R_2\right)}\left(\frac{RR_1}{R+R_1}\right)$$

$$\Rightarrow \quad V = \frac{V_0RR_1}{RR_1+RR_2+R_1R_2}$$

$$\Rightarrow \quad V = \frac{V_0RR_1}{R(R_1+R_2)+R_1R_2} = \frac{V_0RR_1}{R_0R+R_1R_2}$$

$$\Rightarrow \quad V = \frac{V_0R\left(\frac{R_0}{L}x\right)}{R_0R+\left(\frac{R_0}{L}x\right)\left(\frac{R_0}{L}(L-x)\right)}$$

$$\Rightarrow \quad V = \frac{V_0Rx}{RL+R_0x\left(1-\frac{x}{L}\right)}$$

For $R \gg R_0$, we get $V \cong \frac{V_0x}{L}$

9. Since $r = \left(\frac{\ell_{\text{key open}}}{\ell_{\text{key closed}}}-1\right)R$

$$\Rightarrow \quad r = \left(\frac{60}{40}-1\right)4$$

$$\Rightarrow \quad r = 2\ \Omega$$

10. Since $r = \left(\frac{\ell_{\text{key open}}}{\ell_{\text{key closed}}}-1\right)R$

$$\Rightarrow \quad r = \left(\frac{75}{60}-1\right)6$$

$$\Rightarrow \quad r = \left(\frac{5}{4}-1\right)6$$

$$\Rightarrow \quad r = 1.5\ \Omega$$

Test Your Concepts-XI (Based on RC Circuit)

1. $Q(t) = \int_0^t Idt = I_0\tau\left(1-e^{-\frac{t}{\tau}}\right)$

(a) $Q(\tau) = I_0\tau(1-e^{-1}) = (0.632)I_0\tau$

(b) $Q(10\tau) = I_0\tau(1-e^{-10}) = (0.99995)I_0\tau$

(c) $Q(\infty) = I_0\tau(1-e^{-\infty}) = I_0\tau$

2. (a) The resistance of the dielectric block is $R = \frac{\rho\ell}{A} = \frac{d}{\sigma A}$

The capacitance of the capacitor is $C = \frac{\kappa\varepsilon_0 A}{d}$

Then $RC = \frac{d}{\sigma A}\frac{\kappa\varepsilon_0 A}{d} = \frac{\kappa\varepsilon_0}{\sigma}$ is a characteristic of the material only.

(b) $R = \frac{\kappa\varepsilon_0}{\sigma C} = \frac{\rho\kappa\varepsilon_0}{C} = \frac{75\times10^{16}(3.78)8.85\times10^{-12}}{14\times10^{-9}}$

$R = 1.79\times10^{15}\ \Omega$

3. (a) $I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-\frac{t}{RC}}\right) = -I_0e^{-\frac{t}{RC}}$

where, $I_0 = \frac{Q}{RC} = \frac{5.1\times10^{-6}}{(1300)(2\times10^{-9})} = 1.96$ A

$$\Rightarrow \quad I(t) = -(1.96\text{ A})\exp\left[\frac{-9\times10^{-6}}{(1\,300)(2\times10^{-9})}\right]$$

$I(t) = -61.6$ mA

(b) $q(t) = Qe^{-\frac{t}{RC}} = (5.1\ \mu\text{C})\exp\left[\frac{-8\times10^{-6}}{(1\,300)(2\times10^{-9})}\right]$

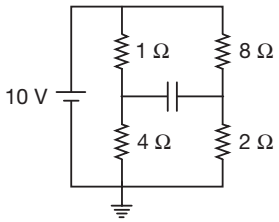
$= 0.235\ \mu$C

(c) The magnitude of the maximum current is $I_0 = 1.96$ A

4. $U = \frac{1}{2}C(\Delta V)^2$ and $\Delta V = \frac{Q}{C}$

Therefore, $U = \frac{Q^2}{2C}$ and when the charge decreases to half its original value, the stored energy is one-quarter its original value and so, $U_f = \frac{1}{4}U_0$.

5. (a) Let us denote the potential at the left junction V_L and at the right V_R. After a long time, the capacitor is fully charged.



$V_L = 8$ V because of voltage divider

$I_L = \frac{10\text{ V}}{5\ \Omega} = 2$ A

$V_L = 10\text{ V} - (2\text{ A})(1\ \Omega) = 8$ V

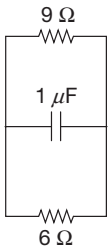
Similarly, $V_R = \left(\frac{2\ \Omega}{2\ \Omega + 8\ \Omega}\right)(10\text{ V}) = 2$ V

$\Rightarrow \quad I_R = \frac{10\text{ V}}{10\ \Omega} = 1$ A

$V_R = (10\text{ V}) - (8\ \Omega)(1\text{ A}) = 2$ V

Therefore, $\Delta V = V_L - V_R = 8 - 2 = 6$ V

(b) Redraw the circuit



$R = \frac{1}{\left(\frac{1}{9}\ \Omega\right) + \left(\frac{1}{6}\ \Omega\right)} = 3.6\ \Omega$

$RC = 3.6 \times 10^{-6}$ s

and $e^{-\frac{t}{RC}} = \frac{1}{10}$

$\Rightarrow \quad t = RC\log_e 10 = 8.29\ \mu$s

6. The total resistance between points b and c is

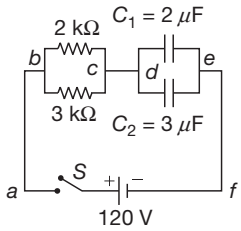
$R = \frac{(2)(3)}{2+3} = 1.2\ \text{k}\Omega$

The total capacitance between points d and e is

$C = 2\ \mu\text{F} + 3\ \mu\text{F} = 5\ \mu\text{F}$

The potential difference between point d and e in this series RC circuit at any time is

$\Delta V = E\left[1 - e^{-\frac{t}{RC}}\right] = (120\text{ V})\left[1 - e^{-\frac{1000t}{6}}\right]$



Therefore, the charge on each capacitor between points d and e is

$q_1 = C_1\Delta V = (2\ \mu\text{F})(120\text{ V})\left[1 - e^{-\frac{1000t}{6}}\right]$

$\Rightarrow \quad q_1 = (240\ \mu\text{C})\left[1 - e^{-\frac{1000t}{6}}\right]$

$q_2 = C_2(\Delta V) = (3\ \mu\text{F})(120\text{ V})\left[1 - e^{-\frac{1000t}{6}}\right]$

$\Rightarrow \quad q_2 = (360\ \mu\text{C})\left[1 - e^{-\frac{1000t}{6}}\right]$

7. The current in a charging series RC circuit is given by

$I = I_0 e^{-\frac{t}{RC}}$

when the current is halved,

$\frac{I_0}{2} = I_0 e^{-\frac{t}{RC}}$

$\Rightarrow \quad \frac{1}{2} = e^{-\frac{t}{RC}}$

Taking the natural logarithms on both sides,

$\log_e 1 - \log_e 2 = -\frac{t}{RC}$

On solving for t, we get

$t = RC\log_e 2 = \tau\log_e 2 = 0.693\tau$

8. The charge at time t in a discharging circuit is given by

$q = Q_0 e^{-\frac{t}{RC}} = Q_0 e^{-\frac{t}{\tau}}$

CHAPTER 3

When the charge is halved,

$$\frac{Q_0}{2} = Q_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-\frac{t}{\tau}}$$

Taking the natural logarithms on both sides, we get

$$-0.693 = -\frac{t}{\tau}$$

$$\Rightarrow \quad t = 0.693\tau$$

9. The electric potential energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}Q_0^2 e^{-\frac{2t}{\tau}}$$

Since $U_0 = \frac{Q_0^2}{2C}$ is initial potential energy, we have

$$U = U_0 e^{-\frac{2t}{\tau}}$$

When the potential energy is half its initial value, we have

$$\frac{1}{2} = e^{-\frac{2t}{\tau}}$$

Taking natural logarithms of both sides, we get

$$-0.693 = -\frac{2t}{\tau}$$

$$\Rightarrow \quad t = 0.34\tau$$

The potential energy decreases to half its value in half the time it takes the charge to decrease to half its value.

10. (a) Rate at which energy is dissipated in the resistor is

$$P(t) = I^2 R$$

$$\Rightarrow \quad P(t) = \left(\frac{V_0}{R} e^{-\frac{t}{RC}}\right)^2 R$$

$$\Rightarrow \quad P(t) = \frac{V^2}{R} e^{-\frac{2t}{RC}}$$

(b) We find the total joule heat by integrating from $t = 0$ to $t \to \infty$

$$W = \int P(t)\, dt = \int_0^\infty \frac{V_0^2}{R} e^{-\frac{2t}{RC}}\, dt$$

$$\Rightarrow \quad W = \frac{V_0^2 C}{2}\left(e^{-\frac{2t}{RC}}\right)_0^\infty$$

$$\Rightarrow \quad W = \frac{V_0^2 C}{2}(-0+1)$$

$$\Rightarrow \quad W = \frac{1}{2}\,\mathrm{CV}_0^2$$

Observation: The result is independent of R. Therefore, when a capacitor is charged by a battery with a constant emf, half the energy provided by the battery is stored in the capacitor and half goes into thermal energy, independent of resistance; the thermal energy includes the energy that goes into the internal resistance of the battery.

While charging, the charge flowing through battery = CV_0

Work done by battery $= (CV_0)V_0 = CV_0^2$

Power dissipated in the resistor $= \frac{1}{2}CV_0^2$

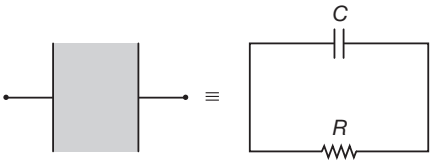
Energy stored in the capacitor $= \frac{1}{2}\,\mathrm{CV}_0^2$

11. The problem is basically of discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.

$$R = \frac{d}{\sigma A} \qquad \left\{\because\ R = \frac{\ell}{\sigma A}\right\}$$

and $C = \frac{K\varepsilon_o A}{d}$

$\therefore$ Time constant, $\tau_C = CR = \frac{K\varepsilon_o}{\sigma}$



Substituting the values, we have

$$\tau_1 = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98\ s$$

Charge at any time decrease exponentially as

$$q = q_0 e^{-t/\tau_C}$$

Here $q_o = 8.85 \times 10^{-6}\,C$ (Charge at time $t = 0$)

Therefore, discharging (leakage) current at time t will be given by

$$I = \left(-\frac{dq}{dt}\right) = \frac{q_o}{\tau_C} e^{-t/\tau_C}$$

or current at $t = 12\ s$ is

$$I = \frac{\left(8.85 \times 10^{-6}\right)}{5.98} e^{-12/5.98} = 0.198 \times 10^{-6}\,\mathrm{A} = 0.198\ \mu\mathrm{A}$$

12. In SITUATION 1, the capacitor and resistor form a series RC circuit. Charge at time t is

$$q = q_0\left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad CV = CV_0\left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad \frac{V}{V_0} = \left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad \frac{0.75}{1.5} = \left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad e^{-\frac{t}{CR}} = \frac{1}{2}$$

Taking natural logarithm on both sides, we have

$$t = CR\log_e 2$$

$$\Rightarrow \quad t = (10\times10^{-6})(0.1\times10^{6})\log_e 2$$

$$\Rightarrow \quad t = \log_e 2$$

$$\Rightarrow \quad t = 0.693 \text{ s}$$

In SITUATION 2, the capacitor is connected in parallel with the battery. Initially it acts like a short circuit and will be charged instantaneously. Now,

$$RC = \frac{8.846\times10^{-12}\times5}{7.4\times10^{-12}}$$

$$\Rightarrow \quad RC = 6$$

$$\Rightarrow \quad I = \frac{8.85\times10^{-6}}{6}e^{-\frac{12}{6}}$$

$$\Rightarrow \quad I = 0.24\ \mu\text{A} \qquad \{\text{where } e = 2.718 \text{ and } e^2 = 7.39\}$$

13. The charge on a capacitor at time t is

$$q = q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad CV = CV_0\left(1 - e^{-\frac{t}{RC}}\right)$$

So, the potential difference at time t is given by

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \qquad \ldots(1)$$

Let the potential difference V_e be attained at time t and V_f at time $t+T$, we have

$$V_e = V_0\left(1 - e^{-\frac{t}{RC}}\right) \qquad \ldots(2)$$

$$V_f = V_0\left(1 - e^{-\frac{(t+T)}{RC}}\right) \qquad \ldots(3)$$

From equation (2),

$$e^{-\frac{t}{RC}} = V_0 - V_e \qquad \ldots(4)$$

From equation (3),

$$e^{-\frac{(t+T)}{RC}} = V_0 - V_f \qquad \ldots(5)$$

Dividing equation (5) by (4), we get

$$\frac{V_0 - V_f}{V_0 - V_e} = e^{-\frac{(t+T)}{RC}+\frac{t}{RC}} = e^{-\frac{T}{RC}} \qquad \ldots(6)$$

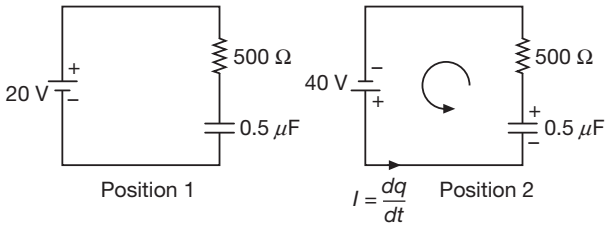
Now taking logarithm of both sides of equation (6), we get

$$T = RC\log_e\left(\frac{V_0 - V_e}{V_0 - V_f}\right)$$

14. When the switch is shifted to position 1, the capacitor gets charged and the instantaneous charge is

$$q = (0.5\times10^{-6})(20)(1 - e^{-4000t})$$

$$\Rightarrow \quad q = 10(1 - e^{-4000t})\ \mu\text{C} \qquad \ldots(1)$$

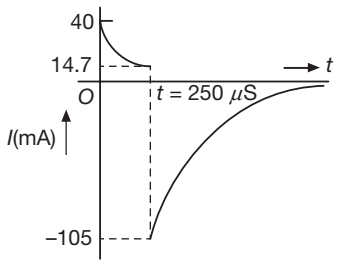


Since time constant, $\tau = RC$

$$\Rightarrow \quad \tau = (500)(0.5\times10^{-6}) = 250\ \mu\text{s}$$

$$\Rightarrow \quad \frac{1}{\tau} = \frac{10^6}{250} = 4000\ \text{s}^{-1}$$

At $t = 250\ \mu$s, we have $q = 10(1 - e^{-1}) = 6.3\ \mu$C



When the switch is shifted to position 2 at $t = 250\ \mu$s, applying KVL, we get

$$-R\frac{dq}{dt} - \frac{q}{C} - 40 = 0$$

Putting $R = 500\ \Omega$, C = 0.5 μF and $\frac{1}{\tau} = 4000\ \text{s}^{-1}$, we get

$$\int_{6.3\ \mu\text{C}}^{q}\frac{dq}{q+20} = -4000\int_{250\ \mu\text{s}}^{t}dt$$

$$\Rightarrow \quad \log_e\left|\frac{q+20}{26.3}\right| = -(4000t-1)$$

$$\Rightarrow \quad q = 26.3e^{-(4000t-1)} - 20 = \left(26.3\,e^{-4000t}\right)(2.718) - 20$$

$$\Rightarrow \quad q = 71.5e^{-4000t} - 20 \qquad \ldots(2)$$

Differentiating equations (1) and (2), we get

$$I(in\ \text{mA}) = \frac{dq}{dt} = \begin{bmatrix} 40e^{-4000t} & 0 < t < 250\ \mu\text{s} \\ -286e^{-4000t} & t > 250\ \mu\text{s} \end{bmatrix}$$

15. Time constant of the circuit,

$$\tau = CR = \left(10^{-4}\right)\left(\frac{20\times 40}{20+40}\right)\text{s} = \frac{40}{3}\times 10^{-4}\ \text{s}$$

The potential across capacitor will reach to 20 V (or half its steady state value) after a time,

$$t = \tau\log_e 2 = (0.693)\times\frac{40}{3}\times 10^{-4}\ \text{s} = 9.24\times 10^{-4}\ \text{s}$$

Current through R_2 at time t would be,

$$I = \frac{2}{3}I_0 e^{-t/\tau}$$

Here $I_0 = \dfrac{E}{R} = \dfrac{40}{\left(\frac{40}{3}\right)} = 3\ \text{A}$

$$\Rightarrow \quad I = 2e^{-\frac{3\times 10^4 t}{40}} = 2e^{-\frac{3\times 10^3 t}{4}}$$

$$\Rightarrow \quad H = \int_0^t I^2 R_2 dt = \int_0^{9.24\times 10^{-4}} \left(4e^{-1.5\times 10^3 t}\right)(20)dt$$

$$\Rightarrow \quad H = \frac{80}{1.5\times 10^3}\left(1 - e^{-1.5\times 9.24\times 10^{-1}}\right)\text{J}$$

$$\Rightarrow \quad H = 40\times 10^{-3}\ \text{J} = 40\ \text{mJ}$$

16. (a) Half of the energy supplied by the battery is stored in the capacitor and rest half is lost as a heat. So,

$$(\Delta H) = \frac{1}{2}(2E)^2 C = 2CE^2$$

Alternate Method:

$$(\Delta H) = \int_0^\infty I^2 R\,dt = \int_0^\infty \left(I_0 e^{-t/\tau_c}\right)^2 R\,dt = 2CE^2$$

Here $I_0 = \dfrac{2E}{R}$ and $\tau_c = CR$

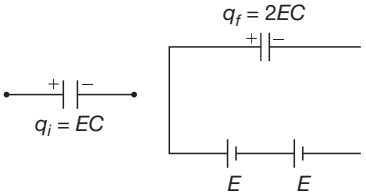
(b) $\Delta H = (\Delta H)_1 + (\Delta H)_2$

Here $(\Delta H)_1 = \dfrac{1}{2}CE^2$

Calculation of $(\Delta H)_2$:

Charge transferred from the battery =

$$\Delta q = q_f - q_i = EC$$



So, energy supplied by the battery $= (\Delta q)(2E) = 2CE^2$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C\left[4E^2 - E^2\right] = \frac{3}{2}CE^2$$

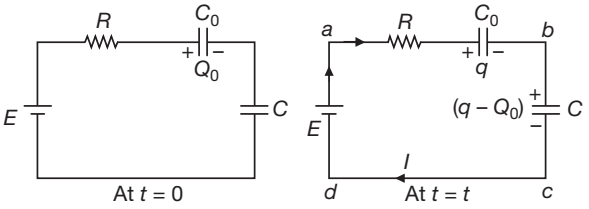
$$\Rightarrow \quad (\Delta H)_2 = 2CE^2 - \frac{3}{2}CE^2 = \frac{1}{2}CE^2$$

$$\Rightarrow \quad \Delta H = CE^2$$

17. Applying Loop Law to abcda, we get

$$-IR + E - \frac{q}{C_0} - \left(\frac{q-Q_0}{C}\right) = 0$$

Substituting $C_0 = C$, we get



$$E + \frac{Q_0}{C} - \frac{2q}{C} - IR = 0$$

$$\Rightarrow \quad \left(\frac{dq}{dt}\right)R = E + \frac{Q_0}{C} - \frac{2q}{C} \qquad \left\{\because I = \frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \int_{Q_0}^{q} \frac{dq}{E + \frac{Q_0}{C} - \frac{2q}{C}} = \frac{1}{R}\int_0^t dt$$

Solving this, we get

$$q = \frac{1}{2}\left[(EC + Q_0) - (EC - Q_0)e^{-\frac{2t}{CR}}\right]$$

and $q - Q_0 = \dfrac{1}{2}\left[(EC - Q_0) - (EC - Q_0)e^{-\frac{2t}{CR}}\right]$

So, $I = \dfrac{dq}{dt} = \dfrac{1}{CR}(EC - Q_0)e^{-\frac{2t}{CR}}$

$$\Rightarrow \quad I = \left(\frac{E}{R} - \frac{Q_0}{CR}\right)e^{-\frac{2t}{CR}}$$

Substituting the given values, we have

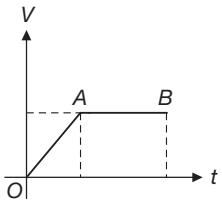
$$I=\left(\frac{4}{100}-\frac{2\times10^{-6}}{2\times10^{-6}\times100}\right)e^{-\frac{2t}{2\times10^{-6}\times100}}$$

or $I=\left(0.03e^{-10^4t}\right)$ A

In this case, though the capacitors are in series yet the charges on the capacitors are not equal (even in steady state).

18. **From O to A:** $V=at$, where a is a positive constant

$$\Rightarrow \quad V=at=\frac{q}{C}+IR$$



Differentiating w.r.t. time, we get

$$a=\frac{1}{C}\left(\frac{dq}{dt}\right)+\left(\frac{dI}{dt}\right)R$$

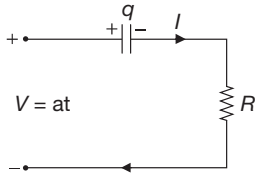
$$\Rightarrow \quad \left(\frac{dI}{dt}\right)R=a-\frac{I}{C} \qquad \left\{\because I=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \int_0^I\frac{dI}{a-\frac{I}{C}}=\int_0^t\frac{dt}{R}$$

$$\Rightarrow \quad I=aC\left(1-e^{-\frac{t}{CR}}\right)$$

i.e., current in the circuit increases exponentially and hence

$$V_{CD}=IR=aCR\left(1-e^{-\frac{t}{CR}}\right)$$



i.e., V_{CD} also increases exponentially.

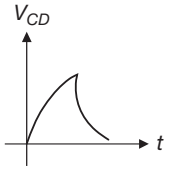
From A onwards: When $V=$ constant (say V_0), then

$$V_0=at$$

$$\Rightarrow \quad t=\frac{V_0}{a}$$

$$\Rightarrow \quad V_{CD}=aCR\left(1-e^{\frac{-V_0}{aCR}}\right)$$

After this V_{CD} will decrease exponentially. Hence, a rough graph is as shown in figure.

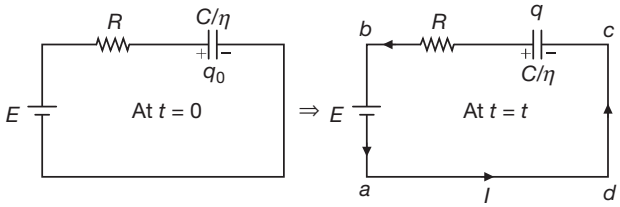


19. At $t=0$, charge stored in the capacitor is,

$$q_0=EC$$

Since, the capacity of the capacitor has been decreased η fold, the charge stored in the capacitor will decrease. Hence, the current in the circuit will flow anticlockwise, with

$$I=-\frac{dq}{dt}$$



Applying Loop Law to $abcda$, we get, $E+IR-\frac{q}{C/\eta}=0$

$$\Rightarrow \quad \left(\frac{dq}{dt}\right)R=E-\left(\frac{\eta}{C}\right)q$$

$$\Rightarrow \quad \int_{q_0}^{q}\frac{dq}{E-\frac{\eta}{C}q}=\frac{1}{R}\int_0^t dt$$

$$\Rightarrow \quad q=\frac{EC}{\eta}\left[1+(\eta-1)e^{-\frac{\eta t}{CR}}\right]$$

$$\Rightarrow \quad I=-\frac{dq}{dt}=\frac{E}{R}(\eta-1)e^{-\frac{\eta t}{CR}}$$

20. Charge on capacitor

$$q=CV \qquad \ldots(1)$$

But from Gauss Theorem

$$\Rightarrow \quad q=KE_0\int_S\vec{E}\cdot d\vec{A} \qquad \ldots(2)$$

Also, $I=\int\vec{J}\cdot d\vec{A}$

$$\Rightarrow \quad I=\int\sigma\vec{E}\cdot d\vec{A}=\sigma\int\vec{E}\cdot d\vec{A} \qquad \ldots(3)$$

$\{\because \vec{J}=\sigma\vec{E}$ where $\sigma=$ conductivity$\}$

CHAPTER 3

Now using (2) and (3), we get

$$I = \frac{\sigma q}{KE_0} = \frac{1}{\rho}\left(\frac{CV}{KE_0}\right) \qquad \left\{\because \sigma = \frac{1}{\rho}\right\}$$

$$\Rightarrow \quad I = \frac{(4\times10^{-9})(2\times10^{3})}{(10^{11})(6)(8.85\times10^{-12})}$$

$$\Rightarrow \quad I = 1.5\times10^{-6}\ \text{A} = 1.5\ \mu\text{A}$$

Single Correct Choice Type Questions

1. $j = \frac{I}{A} = nqv$

where v is the drift velocity of charge carriers each with charge q.

Hence, the correct answer is (D).

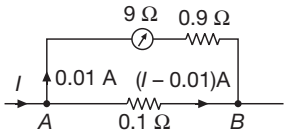
2. $\hbar = \frac{h}{2\pi}$ (Just for the sake of knowledge)

$$I = \frac{\text{Charge Circulating}}{\text{Time for one revolution}}$$

$$\Rightarrow \quad I = \frac{e}{\left(\frac{2\pi r}{v}\right)} = \frac{e\left(\frac{e^2}{\hbar}\right)}{2\pi\frac{\hbar^2}{me^2}} = \frac{me^5}{2\pi\hbar^3}$$

Hence, the correct answer is (A).

3. In parallel, current distributes in inverse ratio of resistance.



$$\Rightarrow \quad \frac{0.01}{I-0.01} = \frac{0.1}{9+0.9}$$

Solving we get, $I = 1$ A

Hence, the correct answer is (C).

4. $Q = Q_0 e^{-t/\tau} = \frac{Q_0}{\eta}$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{\eta}$$

$$\Rightarrow \quad e^{t/\tau} = \eta$$

$$\Rightarrow \quad \frac{t}{\tau} = \log_e \eta$$

Hence, the correct answer is (B).

5. Let us assume that charge flow between the rod and cylinder be radial. Then at point P at distance r from centre of rod the current density J is

$$J = \frac{I}{2\pi r\ell}$$

Further

$$E = \rho J = \frac{\rho I}{2\pi r\ell} \qquad \{\because J = \sigma E\}$$

with both J and E directed radially outwards.

Since

$$dV = -E\,dr$$

$$\Rightarrow \quad dV = -\frac{\rho I}{2\pi\ell}\frac{dr}{r}$$

$$\Rightarrow \quad \int_a^b dV = -\frac{\rho I}{2\pi\ell}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad -V = -\frac{\rho I}{2\pi\ell}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad I = \frac{2\pi\ell V}{\rho\log_e\left(\frac{b}{a}\right)}$$

Hence, the correct answer is (B).

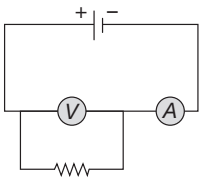
6. Let V be the voltage of the source and R be the resistance of each bulb. Then, we have

$R = \frac{V^2}{P}$ when N bulbs are joined in series across V, current in each bulb is given by $I = \frac{V}{NR}$. Power drawn by each bulb is $I^2R = \left(\frac{V^2}{N^2R^2}\right)R = \frac{V^2}{N^2R} = \frac{P}{N^2}$

Hence, total power drawn is $Np = N\left(\frac{P}{N^2}\right) = \frac{P}{N}$

Hence, the correct answer is (C).

7. Whenever a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases so that the current increases and hence ammeter reading also increases. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.



Hence, the correct answer is (A).

8. Since the resistivity of a current carrying conductor carrying charge carriers each of charge q, mass m is given by

$$\rho = \frac{m}{nq^2\tau}$$

where n is number density of charge carriers and τ is the average relaxation time.

Hence, the correct answer is (B).

9. Convectional current is the current which is developed due to the transportation of charge.

$$\Rightarrow \quad I = \frac{q_{\text{transported}}}{t} = 2\lambda v$$

Hence, the correct answer is (B).

10. This is basically a RC circuit, being charged from a battery. The resistance (R) of the voltmeter is the resistance in the circuit. The voltage across R is

$$V = (\text{Circuit Current})(\text{Resistance}) = \text{Reading of the voltmeter } (V)$$

Hence the nature of the V-t curve is the same as that of the I-t curve.

Hence, the correct answer is (C).

13. $R_1 + R_2 = \text{Constant}$

$$\Rightarrow \quad \Delta(R_1 + R_2) = 0$$

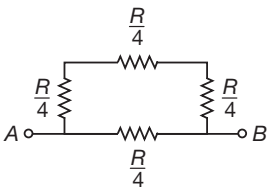
$$\Rightarrow \quad |\Delta R_1| = |\Delta R_2|$$

$$\Rightarrow \quad R_1\alpha\Delta T = R_2\beta\Delta T$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\beta}{\alpha}$$

Hence, the correct answer is (D).

14.



$$\Rightarrow \quad R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\ \Omega$$

Hence, the correct answer is (C).

15. $$I = \frac{2(1.5)}{R + 2r}$$

$$\Rightarrow \quad R + 2r = 3 \qquad \ldots(1)$$

$$0.6 = \frac{1.5}{R + (r/2)}$$

$$\Rightarrow \quad 2R + r = 5 \qquad \ldots(2)$$

From (1) and (2), we get

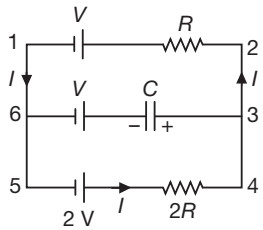
$$R = \frac{7}{3}\ \Omega,\ r = \frac{1}{3}\ \Omega$$

Hence, the correct answer is (C).

16. In steady state the branch containing the capacitor can be omitted and hence current in the circuit is

$$I = \frac{2V - V}{R + 2R}$$

$$\Rightarrow \quad I = \frac{V}{3R}$$



For loop 36543

$$-V_C - V + 2V - I(2R) = 0$$

$$\Rightarrow \quad V_C = -V + 2V - \frac{V}{3R}(2R)$$

$$\Rightarrow \quad V_C = V - \frac{2V}{3} = \frac{V}{3}$$

Hence, the correct answer is (C).

17. $G = 25\ \Omega$

$$I_g = (2 \times 10^{-4})(50)$$

$$\Rightarrow \quad I_g = 0.01\ \text{A}$$

Since

$$R = \frac{V}{I_g} - G$$

$$\Rightarrow \quad R = \frac{25}{0.01} - 25$$

$$\Rightarrow \quad R = 2500 - 25$$

$$\Rightarrow \quad R = 2475\ \Omega$$

Hence, the correct answer is (C).

18. On short circuiting

$$I = \frac{E}{R}$$

$$\Rightarrow \quad 3 = \frac{1.5}{R}$$

$$\Rightarrow \quad R = 0.5\ \Omega$$

Hence, the correct answer is (A).

CHAPTER 3

19. When K is open

$$R_{net} = \frac{3R}{2}$$

$$\Rightarrow \quad i_1 = \frac{E}{\left(\frac{3R}{2}\right)} = \frac{2E}{3R}$$

When K is closed

$$R_{net} = 2\left[\frac{R \times 2R}{R + 2R}\right] = \frac{4}{3}R$$

$$\Rightarrow \quad i_2 = \frac{E}{\left(\frac{4R}{3}\right)} = \frac{3E}{4R}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{8}{9}$$

Hence, the correct answer is (C).

20. 7 Ω and 3 Ω are in parallel; 6 Ω and 4 Ω are in parallel and both in series.
So

$$\Rightarrow \quad R_{eq} = \frac{7 \times 3}{7 + 3} + \frac{4 \times 6}{4 + 6}$$

$$\Rightarrow \quad R_{eq} = 2.1 + 2.4$$

$$\Rightarrow \quad R_{eq} = 4.5\ \Omega$$

Hence, the correct answer is (A).

21. Let E be the e.m.f. and r be the internal resistance of battery. Then

$$0.5 = \frac{E}{20 + r}$$

$$\Rightarrow \quad E = 10 + 0.5\, r \qquad \ldots(1)$$

and $\quad 0.8 = \frac{E}{10 + r}$

$$\Rightarrow \quad E = 8 + 0.8\, r \qquad \ldots(2)$$

From (1) and (2)

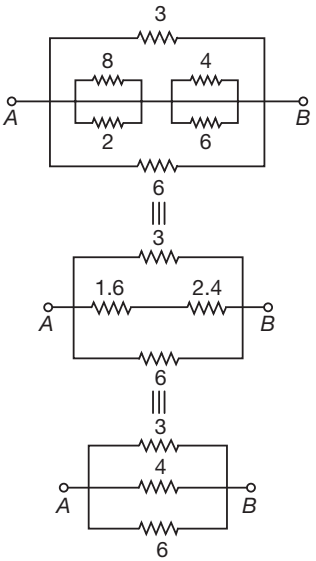
$$2 = 0.3\, r$$

$$\Rightarrow \quad r = \frac{20}{3}\ \Omega \text{ and}$$

$$E = \frac{40}{3}\ \text{V}$$

Hence, the correct answer is (D).

22.



3, 4 and 6 Ω all in parallel. Hence

$$\frac{1}{R_P} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{R_P} = \frac{4 + 3 + 2}{12}$$

$$\Rightarrow \quad R_P = \frac{12}{9}\ \Omega = \frac{4}{3}\ \Omega$$

Hence, the correct answer is (A).

24. $0.2 = \frac{1.5}{5 + r}$

$$\Rightarrow \quad 1 + 0.2\ \text{r} = 1.5$$

$$\Rightarrow \quad r = \frac{5}{2} = 2.5\ \Omega$$

Hence, the correct answer is (C).

25. In parallel, the potential across all the resistances is the same.
Hence, the correct answer is (D).

26. Since $I = \frac{E}{R_{eq}} = \frac{E}{R + 10}$

$$\Rightarrow \quad I = \frac{2}{10 + R}$$

100 cm has a resistance of 10 Ω

$$\Rightarrow \quad 40 \text{ cm has a resistance of } \frac{10}{100} \times 40$$

$$\Rightarrow \quad R' = 4\ \Omega$$

So, potential difference across $4\ \Omega$ is

$$V' = IR' = 10\ \text{mV}$$

$$\Rightarrow \quad V' = \left(\frac{2}{10+R}\right)4 = 10^{-2}$$

$$\Rightarrow \quad 800 = 10 + R$$

$$\Rightarrow \quad R = 790\ \Omega$$

Hence, the correct answer is (B).

27. If A is fused, then complete circuit is broken.
Hence, the correct answer is (C).

28. $R_1 + R_2 = 18$ and $\dfrac{R_1 R_2}{R_1 + R_2} = 4$

Hence, the correct answer is (C).

29. If a wire is stretched to n times its original length, then new resistance is n^2 times original resistance. Stretching to make the wire 0.1% longer implies its new length is

$$\frac{0.1}{100}\ell_0 + \ell_0 = \frac{1001}{1000}\ell_0$$

So, $\quad R_{\text{new}} = \left(\dfrac{1001}{1000}\right)^2 R_0$

$\Rightarrow$ %age increment is

$$\left[\left(\frac{1001}{1000}\right)^2 - 1\right] \times 100\%$$

$$= \left[\left(1 + \frac{1}{1000}\right)^2 - 1\right] \times 100\%$$

$$= \left[1 + \frac{2}{1000} - 1\right] \times 100 = 0.2\%$$

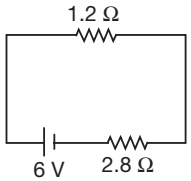
(Since according to Binomial Theorem

$$(1+x)^n \simeq 1 + nx$$

For $|x| << 1$ or $-1 < x < 1$)

Hence, the correct answer is (C).

30. Since capacitor is a dc blocking element, so no current flows through the branch containing the capacitor. However, a voltage drop will exist across this branch.
$R_{\text{net}} = 4\ \Omega$



$$\Rightarrow \quad I_{\text{total}} = \frac{V}{R_{\text{net}}} = 1.5\ \text{A}$$

Voltage drop across $2.8\ \Omega$ is $V_1 = (2.8)(1.5)$

$$\Rightarrow \quad V_1 = 4.2\ \text{V}$$

So, potential across parallel combination of $2\ \Omega$ and $3\ \Omega$ is

$$V_2 = 6 - 4.2 = 1.8\ \text{V}$$

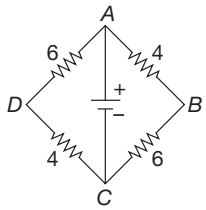
If I_2 is the current in $2\ \Omega$ resistor, then

$$I_2 = \frac{1.8}{2} = 0.9\ \text{A}$$

Hence, the correct answer is (B).

31. Let $E =$ emf of the cell

$$V_A - V_B = \frac{E}{4\ \Omega + 6\ \Omega} \times 4\ \Omega = \frac{4E}{10}$$



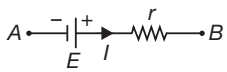
$$V_A - V_D = \frac{E}{6\ \Omega + 4\ \Omega} \times 6\ \Omega = \frac{6E}{10}$$

$$V_B - V_D = \frac{E}{5}$$

Hence, the correct answer is (C).

32. Current in the circuit is $I = \dfrac{NE}{Nr} = \dfrac{E}{r}$

The equivalent circuit of one cell is shown in the figure.



Potential difference across the cell equals

$$V_A - V_B = -E + Ir = -E + \left|\frac{E}{r}\right| r = 0$$

Hence, the correct answer is (D).

33. $I = \dfrac{2Q}{2\pi/\omega}$

$$\Rightarrow \quad I = \frac{Q\omega}{\pi}$$

Hence, the correct answer is (C).

34. No current will flow through voltmeter. As it is ideal (infinite resistance). Current through two batteries

CHAPTER 3

$$i = \frac{1.5 - 1.3}{r_1 + r_2} = \frac{0.2}{r_1 + r_2}$$

Now, $V = E_2 - ir_2$

$$\Rightarrow \quad 1.45 = 1.5 - \left(\frac{0.2}{r_1 + r_2}\right)(r_2)$$

Solving this equation, we get

$$r_1 = 3r_2$$

Hence, the correct answer is (B).

35. $F = \frac{dp}{dt} = \dot{p}$

Momentum of each charge carrier moving with a drift velocity v is mv.

Total number of charge carriers in the sample is $N = n(A\ell)$, where n is number of charge carriers per unit volume, A is area of cross-section of the conductor.

Total momentum = $p = N(mv) = nA\ell mv$

Further we have $v = \frac{I}{neA}$

$$\Rightarrow \quad p = nA\ell m\left(\frac{I}{neA}\right)$$

$$\Rightarrow \quad p = \ell\left(\frac{m}{e}\right)I$$

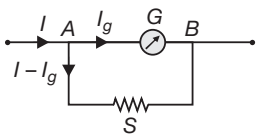
Since $F = \frac{dp}{dt}$

$$\Rightarrow \quad F = \frac{\ell}{s}\dot{I} \qquad \left\{\because \ s = \text{specific charge} = \frac{e}{m}\right\}$$

$$\Rightarrow \quad \frac{F}{\ell} = \frac{\dot{I}}{s}$$

Hence, the correct answer is (C).

36. $I_g = 10 \text{ mA} = 0.01 \text{ A}, \ G = 1\ \Omega, \ I = 1 \text{ A}$



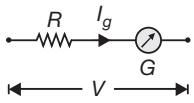
Since, $V_A - V_B = I_g G = (I - I_g)S$

$$S = \frac{I_g G}{I - I_g} = \frac{(0.01 \text{ A} \times 1\ \Omega)}{1 \text{ A} - 0.01 \text{ A}} = \frac{1}{99}\ \Omega$$

Hence, the correct answer is (C).

37. $I_g = 0.01 \text{ A}, \ G = 1\ \Omega, \ V = 10 \text{ V}$

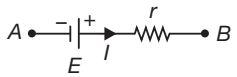
Since, $V = (G + R)I_g$



$$\Rightarrow \quad R = \frac{V}{I_g} - G = \frac{10 \text{ V}}{0.01 \text{ A}} - 1\ \Omega = 999\ \Omega$$

Hence, the correct answer is (C).

38. $I = \frac{(N-2)E}{Nr}$



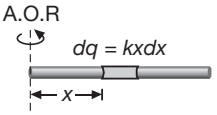
$$V_B - V_A = -Ir + E = E - \frac{(N-2)E}{Nr}r$$

$$\Rightarrow \quad V_B - V_A = E\left(1 - \frac{N-2}{N}\right) = \frac{2E}{N}$$

Hence, the correct answer is (A).

39. Consider an element of length dx at a distance x from the axis of rotation (A.O.R.)

$$\Rightarrow \quad q = \int_0^L dq = k\int_0^L x\, dx$$



$$\Rightarrow \quad q = \frac{1}{2}kL^2 = \frac{\lambda L}{2}$$

$$\Rightarrow \quad I = \frac{q}{T} = \frac{\left(\frac{\lambda L}{2}\right)}{\left(\frac{2\pi}{\omega}\right)}$$

$$\Rightarrow \quad I = \frac{\lambda L\omega}{4\pi} = \frac{kL^2\omega}{4\pi}$$

Hence, the correct answer is (C).

40. For every cell that is wrongly connected, the emf decreases by $2E_0$. However, internal resistance does not depend on direction and therefore remains the same for all cells.

Hence, the correct answer is (D).

41. Let x, y and $\frac{x}{3}$ be the dimensions of the block.

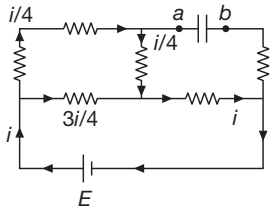
$$\Rightarrow \quad R_{\max} = \frac{\rho x}{y\left(\frac{x}{3}\right)}$$

$$\Rightarrow \quad R_{\min} = \frac{\rho\left(\frac{x}{3}\right)}{xy}$$

$$\Rightarrow \quad \frac{R_{\max}}{R_{\min}} = 9$$

Hence, the correct answer is (B).

42.



$$V_a - \frac{i}{4}R - iR = V_b$$

$$\Rightarrow \quad V_a - V_b = \frac{5}{4}iR = 10 \qquad \ldots(1)$$

$$R_{\text{net}} = R + \frac{(3R)(R)}{(3R+R)}$$

$$\Rightarrow \quad R_{\text{net}} = \frac{7}{4}R$$

$$\Rightarrow \quad i = \frac{E}{R_{\text{net}}}$$

$$\Rightarrow \quad i = \frac{E}{\left(\frac{7}{4}\right)R} = \left(\frac{4E}{7R}\right)$$

Substituting in Equation (1), we get

$$\left(\frac{5}{4}\right)\left(\frac{4E}{7R}\right) \times R = 10$$

$$\Rightarrow \quad E = 14 \text{ V}$$

Hence, the correct answer is (A).

43. Since $\text{Mass} = (\text{Density})(\text{Volume})$

$$\Rightarrow \quad m = (d)(A\ell)$$

$$\Rightarrow \quad A = \frac{m}{\ell d}$$

Further since

$$\Rightarrow \quad R = \frac{\rho\ell}{A}$$

$$\Rightarrow \quad R = \frac{\rho\ell}{\left(\frac{m}{\ell d}\right)}$$

$$\Rightarrow \quad R = \frac{\rho\ell^2 d}{m}$$

As all the three wires are made of Cu, so all have same d and ρ.

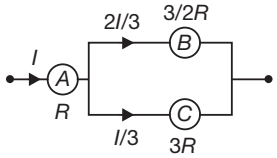
$$\Rightarrow \quad R \propto \frac{\ell^2}{m}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 :: 25 : 3 : \frac{1}{5}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 :: 125 : 15 : 1$$

Hence, the correct answer is (C).

46. $V_A = IR$, $V_B = \frac{2I}{3}\left(\frac{3}{2}R\right) = IR$



and $V_C = \left(\frac{I}{3}\right)(3R) = IR$

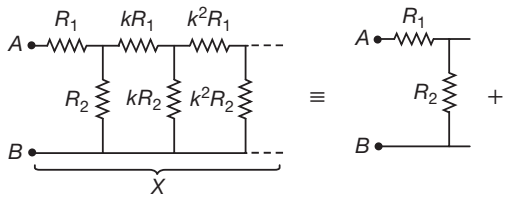
Hence, the correct answer is (D).

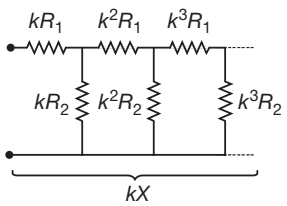
47. $r = R\left(\frac{E}{V} - 1\right) = 5\left(\frac{2.2}{1.8} - 1\right) = \frac{10}{9}\ \Omega$

Hence, the correct answer is (A).

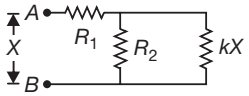
48. The filament of the heater attains steady resistance when the heater reaches its steady temperature, which is much higher than the room temperature. The resistance at room temperature is thus much lower than the resistance at its steady state. When the heater is switched on, it draws a larger current than its steady state current. As the filament heats up, its resistance increases and the current falls to its steady state value.
Hence, the correct answer is (D).

49. When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times.
Let the equivalent resistance between A and B be X.





So, equivalent circuit becomes



For $k = \frac{1}{2}$

$$\Rightarrow \quad X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6\,R_1R_2}}{2}$$

Hence, the correct answer is (C).

50. Since the capacitors are identical so they will finally attain a charge $\frac{Q}{2}$ each. Now,

Initial energy of the system $= E_i = \frac{Q^2}{2C}$

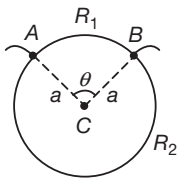
Final energy of the system $= E_f = 2\left[\frac{\left(\frac{Q}{2}\right)^2}{2C}\right] = \frac{Q^2}{4C}$

So, heat produced = Loss in energy $= E_i - E_f = \frac{Q^2}{4C}$

Hence, the correct answer is (B).

51. Since $R \propto \ell$

$$\Rightarrow \quad R_1 = \frac{R}{2\pi a}(a\theta)$$



$$\Rightarrow \quad R_1 = \frac{R\theta}{2\pi}$$

Similarly $R_2 = \frac{R}{2\pi a}(2\pi a - a\theta)$

$$\Rightarrow \quad R_2 = \frac{R}{2\pi}(2\pi - \theta)$$

Since, $R_{AB} = \frac{R_1R_2}{R_1 + R_2}$

$$\Rightarrow \quad R_{AB} = \frac{R\theta(2\pi - \theta)}{4\pi^2}$$

Hence, the correct answer is (A).

52. Both the resistances are in parallel.

So, $\frac{1}{R_p} = \frac{1}{R_{Cu}} + \frac{1}{R_{Ni}}$

where $R_{Cu} = \frac{\rho_C\ell}{\pi r^2}$

$$R_{Ni} = \frac{\rho_N\ell}{\pi(2R)^2 - \pi R^2} = \frac{\rho_N\ell}{3\pi R^2}$$

$$\Rightarrow \quad \frac{1}{R_P} = \frac{\pi r^2}{P_C\ell} + \frac{3\pi R^2}{\rho N\ell}$$

$$\Rightarrow \quad R_P = \frac{\ell}{\pi r^2\left(\frac{1}{\rho_C} + \frac{3}{\rho_N}\right)}$$

Hence, the correct answer is (C).

55. Since the materials are same, the resistivity is same. If R_1 and R_2 are their resistances, we have $R_1\pi r_1^2 / \ell_1 = R_2\pi r_2^2 / \ell_2$, where ℓ_1 and ℓ_2 are their respective lengths. Here

$$\ell_1 = \ell_2,\ r_1^2/r_2^2 = R_2/R_1\ .$$

If ℓ is the balancing length measured from the left of metre bridge, we have

$$R_1/R_2 = \ell/(100 - \ell) = 25/75\ .$$

This gives $r_1/r_2 = \sqrt{75/25} = \sqrt{3}$

Hence, the correct answer is (A).

56. $I = \frac{E}{R_2 + r} = \frac{5}{4+1}$

$$\Rightarrow \quad I = 1\ A$$

Voltage across R_2 is

$$V = IR_2 = 4\ \text{V}$$

So, voltage across each capacitor is 2 V.

$$\Rightarrow \quad Q = CV = 3(2)$$

$$\Rightarrow \quad Q = 6\ \mu C$$

Hence, the correct answer is (B).

57. For metre bridge to be balanced

$$\frac{P}{Q} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow \quad P = \frac{2}{3}Q$$

When Q is shunted i.e. a resistance of $10\ \Omega$ is connected in parallel across Q. So net resistance is $\frac{10Q}{10+Q}$.

Now the balance point shifts to 50 cm i.e.

$$\frac{P}{\left(\frac{10Q}{10+Q}\right)} = 1$$

$$\Rightarrow \quad \frac{2}{3}=\frac{10}{10+Q}$$

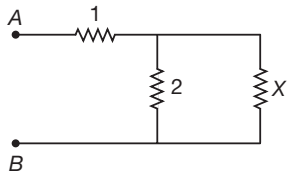
$$\Rightarrow \quad 20+2Q=30$$

$$\Rightarrow \quad Q=5\ \Omega \text{ and } P=\frac{10}{3}\ \Omega$$

Hence, the correct answer is (A).

58. Let the equivalent resistance between A and B be X. If we retain the basic repetitive unit then the remaining circuit is identical to the total circuit and hence will also possess a resistance X.

$$R_{AB}=X=\frac{2X}{2+X}+1$$



$$\Rightarrow \quad 2X=(2+X)(X-1)$$

$$\Rightarrow \quad 2X=2X+X^2-X-2$$

$$\Rightarrow \quad X^2-X-2=0$$

$$\Rightarrow \quad X^2-2X+X-2=0$$

$$\Rightarrow \quad (X+1)(X-2)=0$$

$$\Rightarrow \quad X=2\ \Omega$$

Hence, the correct answer is (B).

59. $I_{\text{total}}=\dfrac{V}{R_{AB}}=\dfrac{6}{2}$

$$\Rightarrow \quad I_{\text{total}}=3\ \text{A}$$

Voltage drop across $2\ \Omega$ resistor is 3 V.

So, $I=\dfrac{3}{2}=1.5\ \text{A}$

Hence, the correct answer is (A).

60. Let the potential of the battery be V

$$R_1=\frac{V}{1}$$

$$R_2=\frac{V}{2}$$

$$R_3=\frac{V}{4}$$

On connecting in series

$$R_S=R_1+R_2+R_3$$

$$\Rightarrow \quad R_S=V\left(1+\frac{1}{2}+\frac{1}{4}\right)$$

$$\Rightarrow \quad R_S=\frac{7}{4}\text{V}$$

Since, by definition

$$I=\frac{V}{R_S}$$

$$\Rightarrow \quad I=\frac{4}{7}A$$

Hence, the correct answer is (C).

62. Since A and B will remain connected in parallel across the capacitor so, at all the instants they will produce the same readings.

Hence, the correct answer is (D).

63. $2=1+(0.00125)(T-300)$

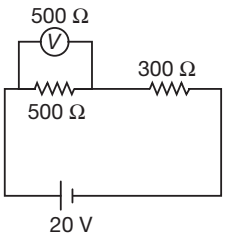
$$\Rightarrow \quad T-300=\frac{100000}{125}$$

$$\Rightarrow \quad T-300=800$$

$$\Rightarrow \quad T=1100\ K$$

Hence, the correct answer is (B).

64.



When we do not connect the voltmeter then potential across $500\ \Omega$ resistor is

$$V_1=\frac{500}{800}\times 20=12.5\ \text{V}$$

The measure this value of potential difference we connect a voltmeter in parallel across $500\ \Omega$ resistor. Net resistance of parallel combination is $250\ \Omega$ and also voltage drop across both $500\ \Omega$ and voltmeter is the same (as both are in parallel). So, voltage drop across combination is

$$V_2=\frac{250}{250+300}\times 20=9.1\ \text{V}$$

So, error in the reading is $12.5-9.1=3.4$ V

Hence, the correct answer is (C).

65. For potential difference between B and D to be zero

$$\frac{10}{5+x}=\frac{0.5}{1} \quad \Rightarrow \quad 20=5+x$$

$$\Rightarrow \quad x=15\ \Omega$$

Hence, the correct answer is (C).

66. Since net resistance is to be found between A and B. So let a current I enter at A and then exit at B.
When I enters at A then by symmetry a current $\frac{I}{6}$ must flow in the branch AB from A to B.
For current I to exit from B, a current $\frac{I}{6}$ must flow in the branch AB from A to B. Super-imposing the two, we conclude that a current $\left(\frac{I}{6}+\frac{I}{6}\right)$ must flow in the branch AB from A to B.
According to Thevenin's Theorem we have

$$I_{\text{total}}\, R_{\text{eq}} = V_{AB} = \left(\frac{I}{6}+\frac{I}{6}\right)R_0 = \frac{IR_0}{3}$$

$$\Rightarrow \quad IR_{\text{eq}} = \frac{IR_0}{3}$$

$$\Rightarrow \quad R_{\text{eq}} = \frac{R_0}{3}$$

Hence, the correct answer is (C).

67. Equivalent emf of two batteries E_1 and E_2 is

$$E = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\left(\frac{2}{2}\right)+\left(\frac{4}{6}\right)}{\left(\frac{1}{2}\right)+\left(\frac{1}{6}\right)}$$

$$\Rightarrow \quad E = 2.5 \text{ V}$$

Now, $V_{\text{AN}} = E$

$$\Rightarrow \quad (I_{\text{AN}})(R_{\text{AN}}) = E$$

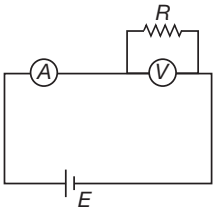
$$\Rightarrow \quad \left(\frac{12}{4+4\times 4}\right)(4)(\ell) = 2.5$$

Solving this equation, we get

$$\ell = \frac{25}{24} \text{ m}$$

Hence, the correct answer is (D).

68. When a resistance is joined in parallel to a voltmeter the net resistance of parallel combination decreases as a result of which total resistance of circuit decreases, hence net current increases. So reading of ammeter increases. Since net resistance of parallel combination decreases so reading of voltmeter decreases



Hence, the correct answer is (D).

69. $$I = \frac{\text{Charge Circulating}}{\text{Time to Complete One Revolution}}$$

$$\Rightarrow \quad I = \frac{Q}{\left(\frac{2\pi}{\omega}\right)}$$

$$\Rightarrow \quad I = \frac{Q\omega}{2\pi}$$

Hence, the correct answer is (C).

70. $$R \propto \frac{1}{A}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{A_2}{A_1}$$

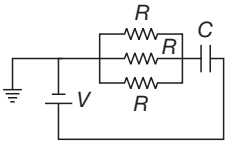
$$\Rightarrow \quad \frac{10}{R_2} = \frac{1}{3}$$

$$\Rightarrow \quad R_2 = 30\ \Omega$$

$$\Rightarrow \quad R_s = 30 + 10 = 40\ \Omega$$

Hence, the correct answer is (A).

71. The simple circuit is as shown below.



$$R_{net} = \frac{R}{3}$$

$$\Rightarrow \quad \tau_C = CR_{\text{net}} = \frac{CR}{3}$$

$$\Rightarrow \quad q = q_0\left(1 - e^{-\frac{t}{\tau_C}}\right),$$

where, $q_0 = CV$

Hence, the correct answer is (C).

72. Since, $P = \frac{V^2}{R}$, so the bulb with higher power rating has lower resistance, and vice versa. When the bulbs are joined in series, they draw the same current. Since $P = I^2R$, the bulb with the lower power rating draws more power.
Hence, the correct answer is (C).

73. Just after the switch is closed C_1 is short-circuited and current passes through R_1 and C_1 only.
Hence, the correct answer is (B).

74. Current can flow in a circuit or part of a circuit only when it has a return path, i.e., it can return to its starting point. Here, current flowing from PQ cannot return to P.
Hence, the correct answer is (C).

76. For series connection, $I_{\max} = \frac{NE}{Nr} = \frac{E}{r}$

For parallel connection, $I_{\max} = \frac{E}{r/N} = \frac{NE}{r}$

Hence, the correct answer is (D).

77. In the steady state no current will flow in the branch containing the capacitor. $\{\because R_2$ and C are in parallel$\}$

Hence $I = \dfrac{V}{R_1 + R_2}$

However potential across the capacitor will be there and equals the potential across the resistor R_2.

$$\Rightarrow \quad V_C = \left(\frac{R_2}{R_1 + R_2}\right)V$$

Hence, the correct answer is (D).

78. In a Wheatstone bridge, the deflection in the galvanometer does not change whenever the battery and galvanometer are interchanged.
Hence, the correct answer is (D).

79. $$\frac{E^2 r_1}{(r + r_1)^2} = \frac{E^2 r_2}{(r + r_2)^2}$$

$$\Rightarrow \quad r_1\left(r^2 + r_2^2 + 2\,rr_2\right) = r_2\left(r^2 + r_1^2 + 2\,rr_1\right)$$

$$\Rightarrow \quad r_1 r^2 + r_1 r_2^2 + 2rr_1 r_2 = r_2 r^2 + r_2 r_1^2 + 2rr_1 r_2$$

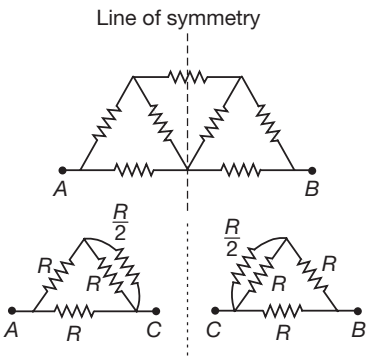
$$\Rightarrow \quad r^2(r_1 - r_2) = r_1 r_2 (r_1 - r_2)$$

$$\Rightarrow \quad r = \sqrt{r_1 r_2}$$

Hence, the correct answer is (B).

80. When we move in the direction of the current in a uniform conductor, the potential decreases linearly. When we pass through the cell, from its negative to its positive terminal, the potential increase by an amount equal to its potential difference.
This is less than its emf, as there is some potential drop across its internal resistance when the cell is driving current.
Hence, the correct answer is (D).

81. $R_{AB} = R_{AC} + R_{CB}$.



Hence, the correct answer is (D).

82. Current in the circuit $= I = \dfrac{E}{R + r}$

Potential difference across cell = potential difference across R is $IR = \dfrac{ER}{R + r}$

Hence, the correct answer is (C).

84. Since $q_g = q_0\left(1 - e^{-\frac{t}{RC}}\right)$

$$\Rightarrow \quad I_g = \frac{dq_g}{dt} = \frac{q_0}{RC}e^{-\frac{t}{RC}}$$

Also $q_d = q_0 e^{-\frac{t}{RC}}$

$$\Rightarrow \quad I_d = \frac{dq_d}{dt} = \frac{-q_0}{RC}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I_g = -I_d$$

Hence, the correct answer is (C).

85. By Kirchhoff's Laws

$$E - \frac{q_0}{C_0} - I_C R = 0$$

$$\Rightarrow \quad I_C = \frac{\left(E - \dfrac{q_0}{C_0}\right)}{R}$$

Hence, the correct answer is (C).

86. $$I = \frac{E}{R_{AB} + R} = \frac{4}{2 + 2.4}$$

$$\Rightarrow \quad I = \frac{40}{44} = \frac{10}{11}\ \text{A}$$

Now, potential difference across wire AB is

$$V_{AB} = IR_{AB} = \frac{20}{11}\ \text{V}$$

$$\Rightarrow \quad \frac{20}{11}\ \text{V} \xrightarrow[\text{length of}]{\text{across a}} 100\ \text{cm}$$

$$\Rightarrow \quad 1\ \text{V} \xrightarrow[\text{length of}]{\text{across a}} \frac{100}{\left(\frac{20}{11}\right)}$$

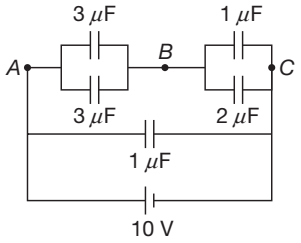
$$\Rightarrow \quad 1.5\ \text{V} \xrightarrow[\text{length of}]{\text{across a}} \frac{1100}{20} \times 1.5\ \text{cm} = 82.5\ \text{cm}$$

Hence, the correct answer is (A).

87. The equivalent resistance of the circuit is $\dfrac{7}{5}R$. Hence the time constant $\tau = \dfrac{7}{5}RC$.

Practically, the steady state value is attained after five time constants
i.e., $t_0 \simeq 5\tau = 7\,RC$
Hence, the correct answer is (B).

88.



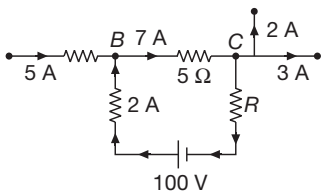
$$\frac{V_{AB}}{V_{BC}} = \frac{(C)_{BC}}{(C)_{AB}} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \quad V_{AB} = \left(\frac{1}{1+2}\right)(10) \text{ V}$$

$$\Rightarrow \quad V_{AB} = \frac{10}{3} \text{ V}$$

Hence, the correct answer is (D).

89.



Applying loop equation in closed loop we have,

$$+100 - 30 - 35 - 2R = 0$$

$$\Rightarrow \quad 2R = 35 \text{ V} = V_R$$

$$V_{5\,\Omega} = 7 \times 5 = 35 \text{ V}$$

$$\Rightarrow \quad \frac{V_{5\,\Omega}}{V_R} = 1$$

Hence, the correct answer is (D).

90. At $t = 0$ when capacitors are initially uncharged, their equivalent resistance is zero. Hence, whole current passes through these capacitors.

Hence, the correct answer is (D).

91. Since the balancing length is at the mid point, each wire has a resistance equal to the known resistance value R. When they are in series, if ℓ is the balancing length measured from the left, we have resistance R in the left gap and $2R$ in the right gap.

Thus $\frac{R}{2R} = \frac{\ell}{100 - \ell}$

$$\Rightarrow \quad \ell = 33.3 \text{ cm}$$

Hence, the correct answer is (B).

93. $$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R}$$

Hence, the correct answer is (B).

94. Slope $= \frac{V}{I} =$ Resistance

So, $R_1 > R_2$

$$\Rightarrow \quad T_1 > T_2$$

Hence, the correct answer is (A).

95. The circuit is a balanced Wheatstone Bridge with net resistance

$$R_{\text{total}} = \frac{(3R)(6R)}{3R + 6R} = 2R$$

For maximum power

$$R_{\text{internal}} = R_{\text{total external}}$$

$$\Rightarrow \quad 4 = 2R$$

$$\Rightarrow \quad R = 2\ \Omega$$

Hence, the correct answer is (B).

96. $$\frac{E_1 + E_2}{E_1 - E_2} = \frac{204}{36} = \frac{51}{9}$$

$$\Rightarrow \quad 9\,E_1 + 9\,E_2 = 51\,E_1 - 51\,E_2$$

$$\Rightarrow \quad 42\,E_1 = 60\,E_2$$

$$\Rightarrow \quad E_1 = \frac{60}{42}(1.4)$$

$$\Rightarrow \quad E_1 = 2\ V$$

Hence, the correct answer is (D).

97. According to Kirchhoff's Junction Law

$$I_{CO} + I_{BO} + I_{AO} = I_{OD}$$

$$\Rightarrow \quad \frac{V_C - V_O}{R} + \frac{V_B - V_O}{R} + \frac{V_A - V_O}{R} = \frac{V_O - V_D}{R}$$

$$\Rightarrow \quad 3\left(\frac{V_A - V_O}{R}\right) = \frac{V_O - V_A + V_A - V_D}{R}$$

$$\Rightarrow \quad 3\left(\frac{V_A - V_O}{R}\right) = -\left(\frac{V_A - V_O}{R}\right) + \left(\frac{V_A - V_D}{R}\right)$$

$$\Rightarrow \quad 4\left(\frac{V_A - V_O}{R}\right) = \frac{V_A - V_D}{R}$$

$$\Rightarrow \quad 4(V_A - V_O) = 40$$

$$\Rightarrow \quad V_A - V_O = 10 \text{ V}$$

Hence, the correct answer is (A).

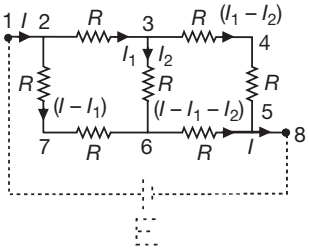
98. e.m.f. is the value when no current is withdrawn from the circuit. So

$$E = 2 \text{ V}$$

Also $r =$ slope $= \frac{2}{5} = 0.4\ \Omega$

Hence, the correct answer is (B).

99. Let a fictitious battery E be connected across the circuit.



Loop 23672

$$-I_1R - I_2R + 2(I - I_1)R = 0$$

$$\Rightarrow \quad 3I_1 + I_2 = 2I \qquad \ldots(1)$$

Loop 34563

$$-2(I_1 - I_2)R + (I - I_1 + I_2)R + I_2R = 0$$

$$\Rightarrow \quad -3I_1 + 4I_2 = -I \qquad \ldots(2)$$

Add (1) and (2), we get

$$5I_2 = I$$

$$\Rightarrow \quad I_2 = \frac{I}{5}$$

and $I_1 = \frac{3I}{5}$

For $127658EI$

$$-2R(I - I_1) - R(I - I_1 + I_2) + E = 0$$

$$\Rightarrow \quad E = 2R\left(I - \frac{3I}{5}\right) + R\left(I - \frac{3I}{5} + \frac{I}{5}\right)$$

$$\Rightarrow \quad E = \frac{4IR}{5} + \frac{3IR}{5}$$

$$\Rightarrow \quad E = \frac{7}{5}IR$$

Finally, we have

$$E = I_{\text{total}}\, R_{\text{net}}$$

$$\Rightarrow \quad IR_{\text{net}} = \frac{7}{5}IR$$

$$\Rightarrow \quad R_{\text{net}} = \frac{7}{5}R = 7\ \Omega$$

Hence, the correct answer is (B).

101. $R_{\max} = \dfrac{\rho \ell_{\max}}{A_{\min}}$

$$\Rightarrow \quad R_{\max} = \frac{\rho(3\ell_0)}{\ell_0^2}$$

$\Rightarrow \quad \ell_0 =$ Minimum Length

$$R_{\min} = \frac{\rho \ell_{\min}}{A_{\max}}$$

$$\Rightarrow \quad R_{\min} = \frac{\rho(\ell_0)}{3\ell_0^2}$$

$$\Rightarrow \quad \frac{R_{\max}}{R_{\min}} = 9$$

Hence, the correct answer is (C).

102. Since

$$r = \left(\frac{\ell}{\ell'} - 1\right) \times R$$

$$\Rightarrow \quad r = \left(\frac{75}{60} - 1\right) \times 10$$

$$\Rightarrow \quad R = \frac{15}{60} \times 10$$

$$\Rightarrow \quad R = \frac{5}{2} = 2.5\ \Omega$$

Hence, the correct answer is (B).

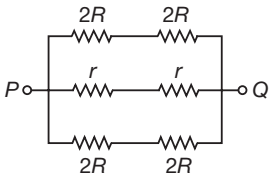
103. Current decreases $\frac{20}{30}$ times or $\frac{2}{3}$ times. Therefore, net resistance should become $\frac{3}{2}$ times.

$$\Rightarrow \quad R + 50 = \frac{3}{2}(2950 + 50)$$

Solving we get, $R = 4450\ \Omega$

Hence, the correct answer is (A).

104.



Hence, the correct answer is (A).

105. Out of 5 one is wrongly connected i.e. 4 are rightly connected

Net $EMF = 4E - E = 3E$

Net resistance $= 5r$

Hence, the correct answer is (B).

106. $I = I_1 + I_2 \qquad \ldots(1)$

$$r = \frac{r_1 r_2}{r_1 + r_2} \qquad \ldots(2)$$

Further

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

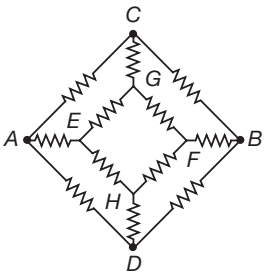
$$\Rightarrow \quad \frac{E}{r} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

$$\Rightarrow \quad E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Hence, the correct answer is (C).

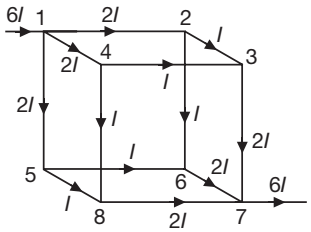
CHAPTER 3

107. The circuit reduces to the following



Hence, the correct answer is (D).

108. Resistance between diagonal corners of cube



$$\Rightarrow \quad V_{17} = (V_1 - V_5) + (V_5 - V_8) + (V_8 - V_7)$$

$$\Rightarrow \quad V_{17} = 2IR_0 + IR_0 + 2IR_0$$

$$\Rightarrow \quad V_{17} = 5IR_0$$

Also,

$$V_{17} = x(6I)$$

$$\Rightarrow \quad x(6I) = 5IR_0$$

$$\Rightarrow \quad x = \frac{5R_0}{6}$$

Hence, the correct answer is (D).

109. $R_{\text{total}} = \dfrac{5R}{11} + \dfrac{6R}{11} = R$

Since $I = \dfrac{V}{R_{\text{total}}}$

$$\Rightarrow \quad 2 = \frac{12}{R}$$

$$\Rightarrow \quad R = 6\,\Omega$$

Hence, the correct answer is (C).

110. In a galvanometer deflection is directly proportional to the current. So 20 divisions correspond to current through galvanometer and 30 divisions for current through shunt.

So, voltage drop a cross galvanometer $= 20\,G$

where G is resistance of galvanometer. Since a fall of 30 divisions is registered when a $12\,\Omega$ shunt (a resistance) is connected in parallel across a galvanometer. Hence voltage drop across shunt = (30)(12)

$$\Rightarrow \quad 20\,G = (30)(12)$$

$$\Rightarrow \quad G = 18\,\Omega$$

Hence, the correct answer is (A).

111. Since

$$I = neAv$$

If N is total number of electrons in the wire then

$$N = nA\ell$$

Momentum possessed by a single electron is mv.

Total Momentum = Nmv

$$\Rightarrow \quad p = (nA\ell)m\frac{I}{neA}$$

$$\Rightarrow \quad p = \frac{m\ell I}{e}$$

$$\Rightarrow \quad \frac{p}{\ell} = \frac{I}{e/m} = \frac{I}{s}$$

Hence, the correct answer is (B).

112. Since $E \propto \ell$. So, for $E_1 > E_2$ we have $\ell_1 > \ell_2$ and hence null point will be obtained at shorter length i.e. to left of C.

Hence, the correct answer is (A).

113. Voltage drop across $50\,\Omega$ and $30\,\Omega$ (n series) is

$$V' = (80)(0.1) = 8\text{ V}$$

Voltage across $20\,\Omega$ branch is also 8 V.

So, current in $20\,\Omega$ branch is

$$I_2 = \frac{8}{20} = 0.4\text{ A}$$

Hence total current in the circuit is 0.5 A.

Also voltage drop across R is 12 – 8 = 4 V.

$$\Rightarrow \quad 4 = I_{\text{total}}R$$

$$\Rightarrow \quad 4 = (0.5)R$$

$$\Rightarrow \quad R = 8\,\Omega$$

Hence, the correct answer is (B).

114. Net resistance of the balanced Wheatstone Bridge is R which is connected in parallel with another resistance of same value.

Hence

$$R_{AB} = R/2$$

Hence, the correct answer is (A).

115. $R_{\text{eq}} = 220\,\Omega$

$$\Rightarrow \quad I_{\text{total}} = 1\text{ A}$$

Reading of ammeter is the current which flows through 3 branches.

Hence reading $= \frac{3}{5}$ A

Hence, the correct answer is (C).

116. Shunt $= 0.1\ \Omega = S$

$G_{net} = 9.9\ \Omega$

$I_g = 10$ mA

$I = ?$

Since $S = \frac{I_g G}{I - I_g}$

$\Rightarrow \quad 0.1 = \frac{(10 \times 10^{-3})(9 \cdot 9)}{I - 10 \times 10^{-3}}$

$\Rightarrow \quad I - 10^{-2} = 99 \times 10^{-2}$

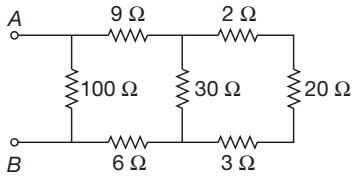
$\Rightarrow \quad I = 100 \times 10^{-2}$

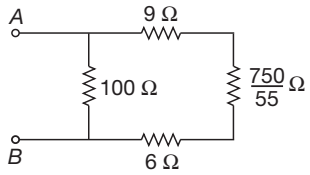
$\Rightarrow \quad I = 1$ A

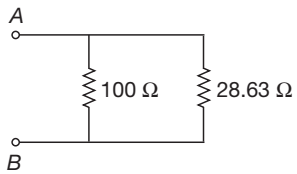
Hence, the correct answer is (C).

117. $\Rightarrow \quad R_{AB} = \frac{(28.63)(100)}{128.63}$

$\Rightarrow \quad R_{AB} \simeq 22 \cdot 26\ \Omega$







Hence, the correct answer is (C).

118. Taking the total current as $6I$, current through $4R$ is $2I$ and current through $2R$ is $4I$. Applying Kirchhoff's Second Law, we get current through $2R$ is $2E/7R$.

Hence, the correct answer is (C).

119. $R_{total} = R_1 + r$

(Since capacitor is a dc blocking element)

$V_C = V_{R_1}$

$V_C = \frac{ER_1}{R_1 + r} \qquad \left\{ \because\ I = \frac{E}{R_1 + r} \right\}$

Since

$Q = CV_C$

$\Rightarrow \quad Q = \frac{CER_1}{R_1 + r}$

Hence, the correct answer is (B).

120. $N = 24 = mn$ …(1)

For current to be MAXIMUM

$R_{internal} = R_{external}$

$\Rightarrow \quad \frac{mr}{n} = 3$

$\Rightarrow \quad \frac{m}{n}(0.5) = 3$

$\Rightarrow \quad \frac{m}{n} = 6$

$\Rightarrow \quad m = 6n$ …(2)

Substituting (2) in (1), we get

$24 = 6n^2$

$\Rightarrow \quad n = 2$

$\Rightarrow \quad m = 12$

Hence, the correct answer is (A).

121. Initial current, $i_1 = \frac{E_1 + E_2}{R + r_1 + r_2}$

Final current, when second battery is short circuited is

$i_2 = \frac{E_1}{R + r_1}$

$i_2 > i_1$ if $\frac{E_1}{R + r_1} > \frac{E_1 + E_2}{R + r_1 + r_2}$

$\Rightarrow \quad E_1 R + E_1 r_1 + E_1 r_2 > E_1 R + E_1 r_1 + E_2 R + E_2 r_1$

$\Rightarrow \quad E_1 r_2 > E_2 (R + r_1)$

Hence, the correct answer is (B).

122. $V_0 = i_0 R = (10)(10) = 100$ V

After 2 s, current becomes $\frac{1}{4}$th. Therefore, after 1 s, current will remain half also called half-life.

$t_{\frac{1}{2}} = (\ln 2)\tau_C = (\ln 2)CR$

CHAPTER 3

$$\Rightarrow \quad C=\frac{\left(t_{\frac{1}{2}}\right)}{(\ln 2)R}=\frac{1}{10\ln 2}F$$

Total heat $\Delta H=\frac{1}{2}CV_0^2$

$$\Rightarrow \quad \Delta H=\frac{1}{2}\times\frac{1}{10\ln 2}(100)^2$$

$$\Rightarrow \quad \Delta H=\frac{500}{\ln 2}J$$

Hence, the correct answer is (D).

123. $\frac{E_1+E_2}{E_1-E_2}=\frac{400}{200}$

$$\Rightarrow \quad E_1+E_2=2(E_1-E_2)$$

$$\Rightarrow \quad E_1=3E_2$$

$$\Rightarrow \quad \frac{E_1}{E_2}=\frac{3}{1}$$

Hence, the correct answer is (B).

124. $I=\frac{dQ}{dt}$

$$\Rightarrow \quad I=\alpha-2\beta t$$

Since at $t_0=t=\frac{\alpha}{2\beta}$, I becomes zero. Hence we shall calculate the heat produced from

$$t=0 \text{ to } t_0=\frac{\alpha}{2\beta}$$

$$\Rightarrow \quad H=\int_0^{t_0} I^2R\,dt$$

$$\Rightarrow \quad H=R\int_0^{\frac{\alpha}{2\beta}}(\alpha-2\beta t)^2\,dt$$

$$\Rightarrow \quad H=R\frac{(\alpha-2\beta t)^{2+1}}{-6\beta}\Bigg|_0^{\frac{\alpha}{2\beta}}$$

$$\Rightarrow \quad H=R\left(0-\frac{\alpha^3}{-6\beta}\right)$$

$$\Rightarrow \quad R=\frac{\alpha^3R}{6\beta}$$

Hence, the correct answer is (D).

125. In position-1, initial maximum current is

$$i_0=\frac{V}{R}=\frac{10}{5}=2 \text{ A}$$

At the given time, given current is 1 A of half of the above value. Hence, at this is instant capacitor is also charged to half of the final value of 5 V.
Now, it is shifted to position-2 wherein steady state it is again charged to 5 V but with opposite polarity.

$$U_i=U_f=\frac{1}{2}CV^2 \qquad \{\because V=5 \text{ V}\}$$

So, total energy supplied by the lower battery is converted into heat. But double charge transfer (from the normal) takes place from this battery.

$$\Rightarrow \quad \text{Heat produced} = \text{Energy supplied by the battery}$$

$$\Rightarrow \quad \Delta H=(\Delta q)V=(2CV)(V)=2CV^2$$

$$\Rightarrow \quad \Delta H=2\times 2\times 10^{-6}\times(5)^2$$

$$\Rightarrow \quad \Delta H=100\times 10^{-6} \text{ J}$$

$$\Rightarrow \quad \Delta H=100\ \mu\text{J}$$

Hence, the correct answer is (C).

126. $\frac{R_1+10}{R_2}=\frac{50}{50}$

$$\Rightarrow \quad R_1+10=R_2 \qquad \ldots(1)$$

Now, when 10 Ω is removed, then

$$\frac{R_1}{R_2}=\frac{40}{60}$$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{2}{3}$$

$$\Rightarrow \quad R_2=\frac{3}{2}R_1 \qquad \ldots(2)$$

Put (2) in (1), we get

$$R_1+10=\frac{3}{2}R_1$$

$$\Rightarrow \quad 2R_1+20=3R_1$$

$$\Rightarrow \quad R_1=20 \text{ Ω}$$

Hence, the correct answer is (A).

127. Resistance between A and B can be removed due to balanced Wheatstone bridge concept. Now, R_{DE} and R_{GH} are in series and they are connected in parallel with 10 V battery.

$$\Rightarrow \quad I_{DE}=\frac{10}{R_{DE}+R_{HG}}=\frac{10}{2+2}$$

$$\Rightarrow \quad I_{DE}=2.5 \text{ A}$$

Hence, the correct answer is (B).

128. $R_{AB} = 2$ [Net resistance of infinite series] +1
In parallel net resistance is always less than the smallest one. Hence, net resistance of infinite series is less than $1\ \Omega$.

$\Rightarrow \quad 1\ \Omega < R_{AB} < 3\ \Omega$

Hence, the correct answer is (C).

129. $V = iR$

$\Rightarrow \quad V \propto R \qquad (\text{as } i = \text{constant})$

$\Rightarrow \quad \dfrac{V_A}{V_B} = \left(\dfrac{\rho \ell_A}{\pi r_A^2}\right)\left(\dfrac{\pi r_B^2}{\rho \ell_B}\right)$

$\Rightarrow \quad \dfrac{r_B}{r_A} = \sqrt{\dfrac{V_A}{V_B} \times \dfrac{\ell_B}{\ell_A}}$

$= \sqrt{\dfrac{3}{2} \times \dfrac{1}{6}} = \dfrac{1}{2}$

Hence, the correct answer is (B).

130. $\tau_C = CR$

$\Rightarrow \quad \tau_C = \left(\dfrac{K\varepsilon_0 A}{d}\right)\left(\dfrac{d}{A\sigma}\right) \qquad \left\{\because\ R = \dfrac{\ell}{\sigma A}\right\}$

$\Rightarrow \quad \tau_C = \dfrac{K\varepsilon_0}{\sigma}$

$\Rightarrow \quad \tau_C = \dfrac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}}$

$\Rightarrow \quad \tau_C = 6\ \text{s}$

Hence, the correct answer is (D).

131. $r = R\left(\dfrac{\ell_1}{\ell_2} - 1\right) = 132.4\left(\dfrac{70}{60} - 1\right) \approx 22.1\ \Omega$

Hence, the correct answer is (A).

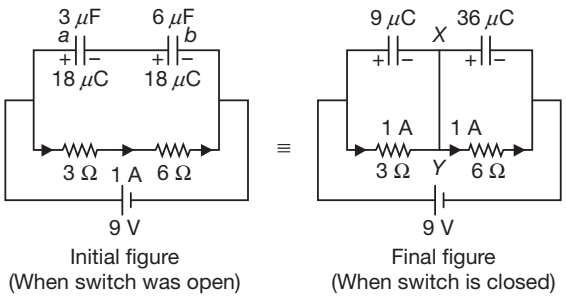
132. Since $\dfrac{X}{20} = \dfrac{1}{80}$

$\Rightarrow \quad X = 0.25\ \Omega$

Hence, the correct answer is (A).

133. From Y to X charge flows to plates a and b

$(q_a + q_b)_i = 0,\ (q_a + q_b)_f = 27\ \mu\text{C}$



Initial figure (When switch was open)

Final figure (When switch is closed)

$\therefore$ 27 μC charge flows from Y to X

Hence, the correct answer is (C).

134. Initially, the rate of charging is fast.

Hence, the correct answer is (B).

135. $\alpha(T) = \dfrac{1}{R_0}\dfrac{dR}{dT}$

$\Rightarrow \quad (3T^2 + 2T) = \dfrac{1}{R_0}\dfrac{dR}{dT}$

$\Rightarrow \quad dR = R_0(3T^2 + 2T)dT$

$\Rightarrow \quad \displaystyle\int_{R_0}^{R} dR = R_0\left[3\int_0^T T^2\,dT + 2\int_0^T T\,dT\right]$

$\Rightarrow \quad R = R_0[1 + T^2 + T^3]$

Hence, the correct answer is (C).

136. Let $R = at + b$

At $t = 10$ s, $R = 20\ \Omega$

$\Rightarrow \quad 20 = 10a + b \qquad \ldots(1)$

At $t = 30a + b \qquad \ldots(2)$

Solving these two equations, we get

$a = 1.0\ \Omega\text{s}^{-1}$

and $b = 10\ \Omega$

$\Rightarrow \quad R = (t + 10)$

Since, $i = \dfrac{E}{R} = \dfrac{10}{t + 10}$

and $\Delta q = \displaystyle\int_{10}^{30} i\,dt$

$\Rightarrow \quad \Delta q = \displaystyle\int_{10}^{30}\left(\dfrac{10}{t + 10}\right)dt$

$\Rightarrow \quad \Delta q = 10\log_e(2)$

Hence, the correct answer is (D).

137. During charging capacitor and resistance of its wire are independently connected with the battery.
Hence,

$\tau_C = CR$

During discharging capacitor is discharged through both resistors (in series). Hence,

$\tau_C = C(2R) = 2CR$

Hence, the correct answer is (C).

138. Mean free path is the average distance travelled by the charge carriers between two successive collisions

$\Rightarrow \quad \ell = v_d\tau = \dfrac{j\tau}{nq}$

Hence, the correct answer is (B).

CHAPTER 3

139. The given time is the half-life time of the exponentially decreasing equation. So,

$$t = t_{\frac{1}{2}} = (\ln 2)\tau_C = (\ln 2)CR_{net}$$

$$\Rightarrow \quad R_{net} = \frac{t}{(\ln 2)C}$$

$$\Rightarrow \quad R_{net} = \frac{2(\ln 2)\mu s}{(\ln 2)(0.5\ \mu F)} = 4\ \Omega$$

So, resistance of ammeter is $2\ \Omega$

Hence, the correct answer is (C).

140. 9, 9 and 9 are in parallel to give 3 Ω.
This 3 Ω is further in series with the other 3 Ω to give 6 Ω

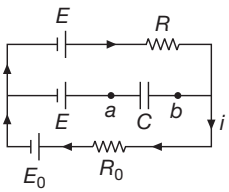
Hence, the correct answer is (B).

141. $V_A - 6 - 3\times 2 + \frac{9}{1} - 3\times 3 = V_B$

$$\Rightarrow \quad V_A - V_B = 12\ \text{V}$$

Hence, the correct answer is (A).

142.



$$i = \frac{[E - E_0]}{R + R_0}$$

Now, $V_a - E + E_0 + iR_0 = V_b$

$$\Rightarrow \quad V_a - V_b = (E - E_0) - iR_0$$

$$\Rightarrow \quad V_a - V_b = (E - E_0)\left[1 - \frac{R_0}{R + R_0}\right]$$

$$\Rightarrow \quad V_a - V_b = \frac{R(E - E)}{R + R_0}$$

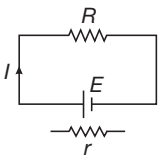
$$\Rightarrow \quad q = C(V_a - V_b) = \frac{CR(E - E_0)}{R + R_0}$$

Hence, the correct answer is (D).

143. $I = \frac{E}{R + r}$

$$\Rightarrow \quad P = I^2R \ = \text{Power consumed across external resistance}$$

$$\Rightarrow \quad P = \frac{E^2R}{(R + r)^2}$$



For P to be Maximum

$$\Rightarrow \quad \frac{dP}{dR} = 0$$

$$\Rightarrow \quad R = r$$

Hence, the correct answer is (A).

144. $i_1 = \left(\frac{V}{2R}\right)e^{-\frac{t}{6CR}}$

$$\Rightarrow \quad i_2 = \left(\frac{V}{R}\right)e^{-\frac{t}{CR}}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{e^{\frac{5t}{6CR}}}{2}$$

We can see that this ratio is increasing with time.

Hence, the correct answer is (B).

145. $mn = 24$ …(1)

For power consumption across the load to be maximum

$$\left(\begin{matrix}\text{External}\\ \text{Resistance}\end{matrix}\right) = \left(\begin{matrix}\text{Total Internal}\\ \text{Resistance}\end{matrix}\right)$$

$$\Rightarrow \quad R = \frac{nr}{m}$$

$$\Rightarrow \quad \frac{n}{m} = 10 \qquad \ldots(2)$$

$$\Rightarrow \quad n = 10\ m$$

$$\Rightarrow \quad 10\ m^2 = 24$$

$$\Rightarrow \quad m = \sqrt{2.4}$$

$$\Rightarrow \quad m \simeq 1.55 \qquad \textbf{Option-1}$$

$$m = 1\ ;\ n = 24$$

Since

$$\Rightarrow \quad I_1 = \frac{nE}{R + \frac{nr}{m}}$$

$$\Rightarrow \quad I_1 = \frac{24}{34}E \qquad \textbf{Option-2}$$

$$m = 2\ ;\ n = 12$$

Since

$$\Rightarrow \quad I_2 = \frac{nE}{R + \frac{nr}{m}} = \frac{12}{16}E$$

$$\Rightarrow \quad I_2 = \frac{24}{32}E$$

From above we observe that $I_2 > I_1$

Hence $n = 12, m = 2$ for the power consumption across the load to be maximum.

Hence, the correct answer is (B).

146. In steady state condition, current flows from outer-most loop.

$$i = \frac{12}{6+2} = 1.5 \text{ A}$$

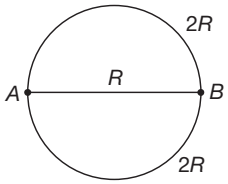
Now, $V_C = V_{6\,\Omega} = iR$

$\Rightarrow \quad V_C = 1.5 \times 6 = 9 \text{ V}$

$\Rightarrow \quad q = CV_C = 18\ \mu C$

Hence, the correct answer is (D).

147.



$$R = \left(\frac{4}{2\pi r}\right)(2r) = \frac{4}{\pi}$$

Now 2 Ω, 2 Ω and R are in parallel.

Hence, the correct answer is (A).

149. $E - ir = 0$

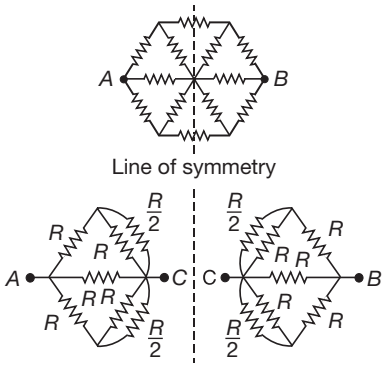
$$\Rightarrow \quad 3 - \left(\frac{3+15}{1+2+R}\right)(1) = 0$$

Solving this equation, we get

$R = 3\ \Omega$

Hence, the correct answer is (C).

150.



Let each resistance be having a value R

$\Rightarrow \quad R_{AB} = R_{AC} + R_{CB}$

Hence, the correct answer is (B).

151. At $t = 0$, when capacitor is under charged equivalent resistance of capacitor $= 0$

In this case, 6 Ω and 3 Ω are parallel (equivalent = 2 Ω)

$\Rightarrow \quad R_{\text{net}}(1+2)\Omega = 3\ \Omega$

$\Rightarrow \quad$ Current from battery $= \frac{12}{3} = 4$ A

$=$ Current through 1 Ω resistor

Hence, the correct answer is (B).

152. $\tau_C = CR = 6$ s

$q_0 = CV = 10\ \mu C$

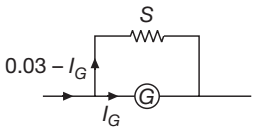
Now, $q = q_0 e^{-\frac{t}{\tau C}} = (10\ \mu C) e^{-\frac{12}{6}}$

$\Rightarrow \quad q = \left(\frac{1}{e}\right)^2 (10\ \mu C)$

$\Rightarrow \quad q = (0.37)^2 (10\ \mu C)$

Hence, the correct answer is (B).

154.



In parallel, current distributers in inverse ratio of resistance.

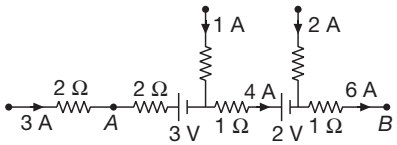
$$\frac{0.03 - I_G}{I_G} = \frac{G}{S} = \frac{r}{\left(\frac{r}{4}\right)} = 4$$

Solving this equation, we get

$I_G = 0.006$ A

Hence, the correct answer is (C).

156.

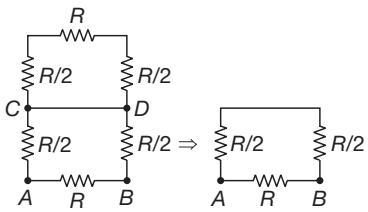


$$V_A - 3 \times 2 - 3 - 1 \times 4 + 2 - 1 \times 6 = V_B$$

$\Rightarrow \quad V_A - V_B = 17 \text{ V}$

Hence, the correct answer is (D).

157.



Hence, the correct answer is (D).

159. $V_{1\,\Omega} = 5 + 2 = 7$ V

$\Rightarrow \quad i_{1\,\Omega} = \dfrac{V}{R} = 7$ A

$V_{2\,\mu F} = 6$ V

$\Rightarrow \quad q_{2\,\mu F} = CV = 12\ \mu C$

Hence, the correct answer is (B).

161. $H_1 = H_2 = U_i - U_f$

The only change is by increasing the resistance τ_C increase. Hence, process of redistribution of charge slows down.

Hence, the correct answer is (A).

162. When K_1 and K_2 both are closed R_1 is short-circuited,

$R_{net} = (50 + r)\ \Omega$

When K_1 is open and K_2 is closed, current remains half.

Therefore, net resistance of the circuit becomes two times.

$\Rightarrow \quad (50 + r) + R_1 = 2(50 + r)$

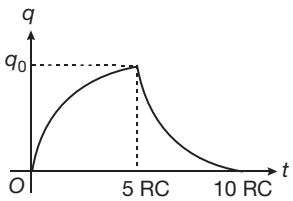
Of the given options, the above equation is satisfied if

$r = 0$ and $R_1 = 50\ \Omega$

Hence, the correct answer is (D).

163. Practically, the steady state is attained when five time constants have elapsed. As soon as the key is opened the peak value of charge also decays in five time constants. Hence pulse period is

$5RC + 5RC = 10RC$



Hence, the correct answer is (C).

164. Potential drop across potentiometer wire

$= (0.2 \times 10^{-3})(100) = 0.02$ V

Now given resistance and potentiometer wire are in series with given battery. So, potential will drop in direct ratio of resistance.

$\Rightarrow \quad \dfrac{0.02}{2 - 0.02} = \dfrac{R}{490}$

$\Rightarrow \quad R = 4.9\ \Omega$

Hence, the correct answer is (A).

165. $q_g + q_d = q_0 = 10\ \mu C$

$\Rightarrow \quad q_d = 7\ \mu C$

Hence, the correct answer is (B).

166. Total heat produced $\Delta H = H = \dfrac{1}{2}CV^2$

$\Rightarrow \quad \Delta H = \dfrac{1}{2}(2\ \mu F)(5)^2$

$\Rightarrow \quad \Delta H = 25\ \mu J$

Now, this should distribute in inverse ratio of resistors, as they are in parallel.

$\Rightarrow \quad \dfrac{H_{5\,\Omega}}{H_R} = \dfrac{R}{5}$

$\Rightarrow \quad H_{5\,\Omega} = \left(\dfrac{R}{R+5}\right)$ (Total heat)

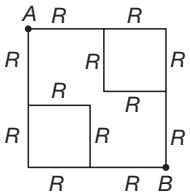
$\Rightarrow \quad 10 = \left(\dfrac{R}{R+5}\right)(25)$

Solving this equation, we get

$R = \left(\dfrac{10}{3}\right)\Omega$

Hence, the correct answer is (C).

167.



Connection can be removed from centre. $3R$ and $3R$ from two sides of AB are in parallel.

Hence, the correct answer is (B).

168. $\dfrac{1}{10} = 1 - e^{-t/\tau}$

$\Rightarrow \quad e^{-t/\tau} = \dfrac{9}{10}$

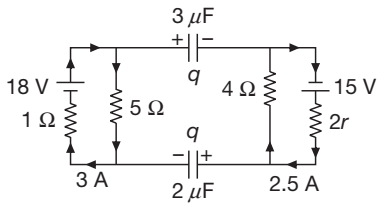
$\Rightarrow \quad e^{t/\tau} = \dfrac{10}{9}$

$\Rightarrow \quad \dfrac{t}{\tau} = \ell n\left(\dfrac{10}{9}\right)$

$\Rightarrow \quad t = \tau \ell n\left(\dfrac{10}{9}\right)$

Hence, the correct answer is (B).

169. Applying Kirchhoff's loop law in outermost loop, we have



$$-\frac{q}{3}+15-2\times2.5-\frac{q}{2}-3\times1+18=0$$

Solving this equation, we get

$$q=30\ \mu C$$

Hence, the correct answer is (A).

171. Suppose $n(<1)$ fraction of length is stretched to m times.

Then, $(1-n)\ell+(n\ell)m=1.5\ell$

$$\Rightarrow \quad nm-n=0.5 \qquad \ldots(1)$$

$$R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\left(\frac{V}{\ell}\right)} \qquad (V=\text{volume})$$

$$\Rightarrow \quad R=\frac{\rho\ell^2}{V}$$

$$\Rightarrow \quad R\propto\ell^2 \qquad (\text{if } V=\text{constant})$$

Now, the second condition is

$$(1-n)R+(nR)m^3=4R$$

$$\Rightarrow \quad nm^2-n=3 \qquad \ldots(2)$$

Solving these two equations, we get

$$n=\frac{1}{8}$$

Hence, the correct answer is (B).

172. Total potential of 10 V equally distributes between 50 Ω and other parallel combination of 100 Ω and voltmeter. Hence, their net resistance should be same.

$$\Rightarrow \quad \frac{100\times R}{100+R}=50$$

$$\Rightarrow \quad R=100\ \Omega=\text{resistance of voltmeter}$$

Hence, the correct answer is (B).

174. All these resistors are in parallel.

$$\Rightarrow \quad R_{net}=\frac{R}{3}+r=4\ \Omega$$

Hence. the main current

$$i=\frac{E}{R_{net}}=1\text{ A}$$

Current through either of the resistance is $\frac{i}{3}$

$$\Rightarrow \quad \frac{1}{3}\text{ A}$$

$$\Rightarrow \quad V=iR=\left(\frac{1}{3}\right)(9)=3\text{ V}$$

Hence, the correct answer is (A).

175. Since capacitors are identical, hence they share equal amount of charge $\frac{Q_0}{2}$.

$$E_i=\frac{Q_0^2}{2C}$$

$$\Rightarrow \quad E_f=2\left[\frac{\left(\frac{Q_0}{2}\right)^2}{2C}\right]=\frac{Q_0^2}{4C}$$

{Due to sharing the new capacitance is 2C}}

$$\text{Heat produced}=E_i-E_f=\frac{Q_0^2}{4C}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Let us assume that charge flow between the rod and cylinder be radial. Then at point P at distance r from centre of rod the current density J is

$$J=\frac{I}{2\pi r\ell}$$

Further

$$E=\rho J=\frac{\rho I}{2\pi r\ell} \qquad \{\because J=\sigma E\}$$

with both J and E directed radially outwards.
Since

$$dV=-E\,dr$$

$$\Rightarrow \quad dV=-\frac{\rho I}{2\pi\ell}\frac{dr}{r}$$

$$\Rightarrow \quad \int_a^b dV=-\frac{\rho I}{2\pi\ell}\int_a^b\frac{dr}{r}$$

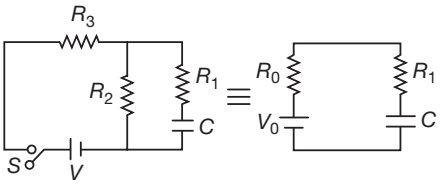
$$\Rightarrow \quad -V=-\frac{\rho I}{2\pi\ell}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad I=\frac{2\pi\ell V}{\rho\log_e\left(\frac{b}{a}\right)}$$

Hence, (A) and (B) are correct.

2. $R_0=\frac{R_2R_3}{R_2+R_3}$ and $V_0=\frac{V\,R_2}{R_2+R_3}$

CHAPTER 3



$$\Rightarrow \quad \tau = C(R_1 + R_0) = C\left(R_1 + \frac{R_2R_3}{R_2 + R_3}\right)$$

$$\Rightarrow \quad q = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad q = CV_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

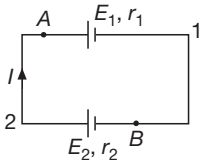
$$\Rightarrow \quad q = \frac{CVR_2}{R_2 + R_3}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Hence, (B) and (C) are correct.

3. Let $E_2 > E_1$, then

$$I = \frac{E_2 - E_1}{r_1 + r_2}$$

For potential difference across battery 1, apply *KVL* to $A1B$, we get



$$V_A - E_1 - Ir_1 - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = E_1 + Ir_1 \qquad \ldots(1)$$

$$\Rightarrow \quad V_A - V_B = E_1 + \left(\frac{E_2 - E_1}{r_1 + r_2}\right)r_1$$

$$\Rightarrow \quad V_A - V_B = \frac{\cancel{E_1r_1} + E_1r_2 + E_2r_1 - \cancel{E_1r_1}}{r_1 + r_2}$$

$$\Rightarrow \quad V_A - V_B = \frac{E_1r_2 + E_2r_1}{r_1 + r_2} \qquad \ldots(2)$$

For potential difference across battery 2, apply *KVL* to $A2B$, we get

$$V_A + Ir_2 - E_2 - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = E_2 - Ir_2 \qquad \ldots(3)$$

$$\Rightarrow \quad V_A - V_B = E_2 - \left(\frac{E_2 - E_1}{r_1 + r_2}\right)r_2$$

$$\Rightarrow \quad V_A - V_B = \frac{E_2r_1 + \cancel{E_2r_2} - \cancel{E_2r_2} + E_1r_2}{r_1 + r_2}$$

$$\Rightarrow \quad V_A - V_B = \frac{E_1r_2 + E_2r_1}{r_1 + r_2} \qquad \ldots(4)$$

Also, we could have thought of equating equation (1) and (2), to get

$I = \frac{E_2 - E_1}{r_1 + r_2}$, which also satisfies the same result obtained.

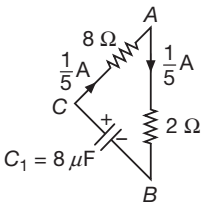
Also, from equation (1), we get that potential difference across 1 is greater than its emf E_1.

Now, since the current I flows from the positive plate to the negative plate inside the battery 2, hence energy is absorbed by it or in other words the battery will continuously get the energy supplied by the other battery. Hence all the statements A, B, C and D are correct.

Hence, (A), (B), (C) and (D) are correct.

4. In steady state, no current passes through the branches containing the capacitors. So,

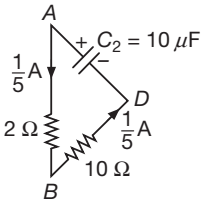
$$I = \frac{4}{8 + 2 + 10} = \frac{4}{20} = \frac{1}{5}\text{ A}$$



Loop ABCA

$$-\left(\frac{1}{5}\right)8 - \left(\frac{1}{5}\right)2 + \Delta V_1 = 0$$

$$\Rightarrow \quad \Delta V_1 = \frac{10}{5} = 2\text{ V}$$



Loop ABDA

$$-\left(\frac{1}{5}\right)2 - \left(\frac{1}{5}\right)(10) + \Delta V_2 = 0$$

$$\Rightarrow \quad \Delta V_2 = 2 + 0.4$$

$$\Rightarrow \quad \Delta V_2 = 2.4\text{ V}$$

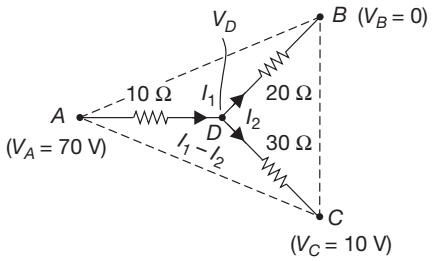
Hence, (B) and (D) are correct.

6. Let V_D be the potential at the point D, then

$$70 - V_D = I_1 \times 10\ \Omega$$

$$V_D - 0 = I_2 \times 20\ \Omega$$

$$V_D - 10 = (I_1 - I_2) \times 30\ \Omega$$



Solve for I_1, I_2 and V_D, we get

$$V_D = 40\ \text{V}$$

$$I_1 = 3\ \text{A},\ I_2 = 2\ \text{A} \text{ and } (I_1 - I_2) = 1\ \text{A}$$

$$P_{\text{total}} = (3)^2(10) + (2)^2(20) + (1)^2(30)$$

$$\Rightarrow \quad P_{\text{total}} = 90 + 80 + 30 = 200\ \text{W}$$

$$\Rightarrow \quad I_1 : I_2 : (I_1 - I_2) :: 3 : 2 : 1 \text{ and } P_{\text{total}} = 200\ \text{W}$$

Hence, (A), (B) and (D) are correct.

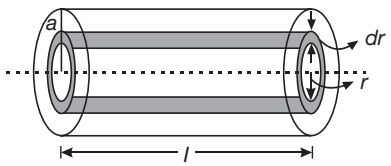
7. Consider a cylindrical element of radius r, thickness dr. If dR is the resistance of this element then

$$dR = \frac{\rho(r)\ell}{2\pi r\, dr}$$

Total resistance of the cylinder is given by

$$\frac{1}{R_{\text{total}}} = \int \frac{1}{dR} = \frac{2\pi}{\alpha\ell}\int_0^a r^3 dr$$

$$\Rightarrow \quad \frac{1}{R_{\text{total}}} = \frac{2\pi}{\alpha\ell}\left(\frac{a^4}{4}\right)$$



$$\Rightarrow \quad R_{\text{total}} = \frac{2\alpha\ell}{\pi a^4}$$

$$\Rightarrow \quad \frac{R_{\text{total}}}{\ell} = \frac{2\pi\alpha}{(\pi a^2)^2}$$

$$\Rightarrow \quad R = \frac{2\pi\alpha}{A^2}$$

Since $E = \frac{V}{\ell}$ (in magnitude)

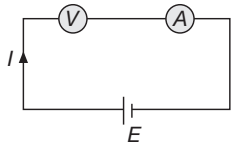
$$\Rightarrow \quad E = I\left(\frac{R_{\text{total}}}{\ell}\right) \qquad \{\text{By Ohm's Law}\}$$

$$\Rightarrow \quad E = \frac{2\pi\alpha I}{A^2}$$

Hence, (A) and (C) are correct.

8. $$I = \frac{E}{R_A + R_V}$$

$$\Rightarrow \quad R_A + R_V = \frac{E}{I} \qquad \ldots(1)$$



Since $E = V + IR_A$

$$\Rightarrow \quad V = E - IR_A < E \qquad \ldots(2)$$

Also, $V = IR_V$

$$\Rightarrow \quad R_V = \frac{V}{I} \qquad \ldots(3)$$

Potential difference across the ammeter is $\Delta V_A = IR_A$

$$\Rightarrow \quad \Delta V_A = IR_A = E - V \qquad \ldots(4)$$

So, from (1), (2), (3) and (4), we observe all the options to be correct.

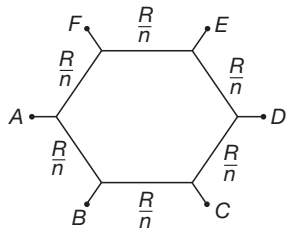
Hence, (A), (B), (C) and (D) are correct.

10. Consider the 'n' sided polygon, shown below

For the points that face each other like (A, D), (B, E) and (C, F), we have

$$R_{\text{MAX}} = R_{AD} = R_{BE} = R_{CF}$$

Resistance for the upper half between A and D is $\left(\frac{R}{n}\right)\left(\frac{n}{2}\right) = \frac{R}{2}$ and that for the lower half between A and D is also $\frac{R}{2}$.



These both are in parallel across A and D, hence

CHAPTER 3

$$R_{MAX} = R_{AD} = R_{BE} = R_{CF} = \frac{\left(\frac{R}{2}\right)\left(\frac{R}{2}\right)}{\frac{R}{2}+\frac{R}{2}} = \frac{R}{4}$$

We shall get the minimum value of resistance between the adjacent points of the polygon i.e., AB, BC, CD, DE, EF or FA. Between two adjacent points we have two resistors connected in parallel one having resistance $\frac{R}{n}$ and other having resistance $(n-1)\frac{R}{n}$

So, $R_{MIN} = R_{\substack{\text{ADJACENT}\\\text{POINTS}}} = \dfrac{\left(\frac{R}{n}\right)(n-1)\left(\frac{R}{n}\right)}{\frac{R}{n}+(n-1)\frac{R}{n}} = \left(\frac{n-1}{n^2}\right)R$

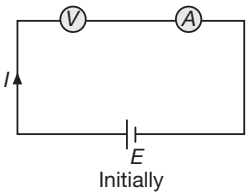
Hence, (A) and (C) are correct.

11. Let the respective resistance of the voltmeter and ammeter be R_V and R_A. Then, initially

$$I_{initial} = \frac{E}{R_V + R_A} \qquad \ldots(1)$$

Finally, when another resistance of value R_A is connected in parallel with the ammeter, then we have

$$I_{final} = \frac{E}{R_V + \frac{R_A}{2}}$$



New ammeter reading is

$$\frac{I_{final}}{2} = \frac{E}{2R_V + R_A}$$

Initial reading of ammeter was

$$I_{initial} = \frac{E}{R_V + R_A}$$

So, we observe that

$$\frac{E}{2R_V + R_A} > \frac{1}{2}\left(\frac{E}{R_V + R_A}\right)$$

$$\Rightarrow \quad I_{final} > \frac{1}{2}(I_{initial})$$

Initial reading of the voltmeter is

$$V_{initial} = \left(\frac{E}{R_V + R_A}\right)R_V$$

Final reading of the voltmeter is

$$V_{final} = \left(\frac{E}{R_V + \frac{R_A}{2}}\right)R_V > V_{initial}$$

Hence, (B) and (D) are correct.

12. In steady state,

$$q_C = EC \text{ and } q_{2C} = 2EC$$

$$\tau_C = 2CR \text{ of both circuits}$$

At time t,

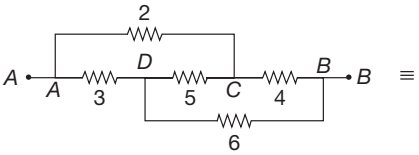
$$q_C = EC\left(1 - e^{\frac{t}{\tau_C}}\right)$$

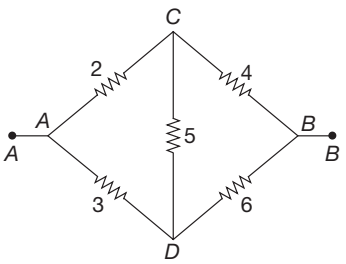
$$q_{2C} = 2EC\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

$$\Rightarrow \quad \frac{q_C}{q_{2C}} = \frac{1}{2}$$

Hence, (B), (C) and (D) are correct.

14.

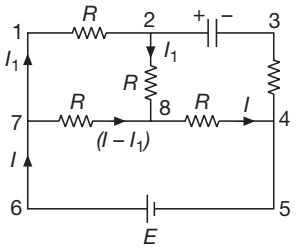




Rearrangement of the circuit as shown gives a balanced Wheatstone bridge, and no current flows through the $5\ \Omega$ resistor. It can thus be removed from the circuit to find the net resistance. When the current in the $5\ \Omega$ branch is zero, then $\Delta V = 0$.

Hence, (B) and (D) are correct.

15. In steady state, no current passes through the branch that contains a fully charged capacitor, because a fully charged capacitor is a dc blocking element. Hence the circuit becomes



For Loop 1284561

$$-2I_1R - IR + E = 0$$

$\Rightarrow \quad 6I_1 + 3I = 15 \qquad \ldots(1)$

For Loop 784567

$$-(I - I_1)R - IR + E = 0$$

$$-2IR + I_1R + E = 0$$

$\Rightarrow \quad 3I_1 - 6I = -15$

$\Rightarrow \quad \frac{3I_1}{2} - 3I = -\frac{15}{2} \qquad \ldots(2)$

Add (1) and (2), we get

$$\left(\frac{3}{2} + 6\right)I_1 = 15 - \frac{15}{2}$$

$\Rightarrow \quad \left(\frac{3+12}{2}\right)I_1 = \frac{30-15}{2}$

$\Rightarrow \quad 15I_1 = 15$

$\Rightarrow \quad I_1 = 1 \text{ A}$

$\Rightarrow \quad 6 + 3I = 15$

$\Rightarrow \quad 3I = 9$

$\Rightarrow \quad I = 3 \text{ A}$

For Loop 23482

$$-V_C + I(3) + I_1(3) = 0$$

$\Rightarrow \quad V_C = 9 + 3 = 12 \text{ V}$

Hence, (C) and (D) are correct.

Please note that though no current flows through the branch containing the fully charged capacitor, yet potential difference across the capacitor will be finite non zero value given by $\Delta V = \pm\frac{q}{C}$.

16. $V_1 = \left(\frac{R_1}{R_1 + R_2 + R_3 + ...}\right)V = \left(\frac{R_1}{R_s}\right)V$

$\Rightarrow \quad V_2 = \left(\frac{R_2}{R_1 + R_2 + R_3 + ...}\right)V = \frac{R_2}{R_s}V$

Hence, (A), (B) and (C) are correct.

17. Acceleration of each electron is

$$a = \frac{eE}{m} = \frac{eV}{md}$$

$\Rightarrow \quad v^2 - 0^2 = 2ax$

$\Rightarrow \quad v = \sqrt{\frac{2eVx}{md}}$

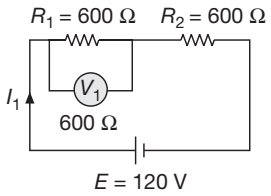
Since $I = neAv$

and $\rho = en = e\frac{I}{veA}$

$\Rightarrow \quad \rho = I\sqrt{\frac{md}{2eVxA^2}}$

Hence, (A) and (B) are correct.

18. **CASE-1:** Net resistance of the circuit is



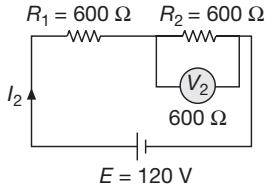
$$R = \frac{(600)(600)}{600 + 600} + 300$$

$$R = 600 \ \Omega$$

$\Rightarrow \quad I_1 = \frac{120}{600} = \frac{1}{5} \text{ A}$

$$V_1 = \frac{1}{5}(300) = 60 \text{ V}$$

CASE-2: Net resistance of circuit is



$$R' = 600 + \frac{(300)(600)}{300 + 600}$$

$\Rightarrow \quad R' = 600 + 200 = 800 \ \Omega$

$\Rightarrow \quad I_2 = \frac{E}{R'} = \frac{120}{800} = \frac{3}{20} \text{ A}$

Now $V_2 = \left(\frac{3}{20}\right)\left(\frac{300 \times 600}{300 + 600}\right) = 30 \text{ V}$

Hence, (C) and (D) are correct.

20. For conductors, the resistance decreases with the increase in temperature as random motion of the charge carriers decreases, so $\alpha_2 > 0$

For semiconductors, the resistance increases with the decrease in temperature and hence $\alpha_1 < 0$.

Hence, (B) and (C) are correct.

21. $\Rightarrow \quad I = \frac{9}{9} = 1 \text{ A}$

At A a current of 1 A divides into 0.5 A and 0.5 A.

At B the current of 0.5 A divides into 0.25 A and 0.25 A

Hence, (A) and (D) are correct.

CHAPTER 3

22. See Theory
Hence, (A) and (C) are correct.

23. The electrical paths AC_1D_2B and AD_1C_2B are in parallel. The resistance of each of them is $10\ \Omega$, and hence the current is 1 A.
When C_1C_2 and D_1D_2 are joined, the ammeters are in parallel combination. As they are of a same resistance, their readings will be equal.
Hence, (B), (C) and (D) are correct.

24. Since $R = \dfrac{\rho\ell}{A} = \dfrac{\ell}{\sigma A}$

and $C = \dfrac{K\varepsilon_0 A}{\ell}$

$\Rightarrow \quad \tau = RC = \dfrac{\varepsilon_0 K}{\sigma}$

Since $q = q_0 e^{-t/\tau}$

$\Rightarrow \quad q = q_0 e^{-\frac{\sigma t}{K\varepsilon_0}}$

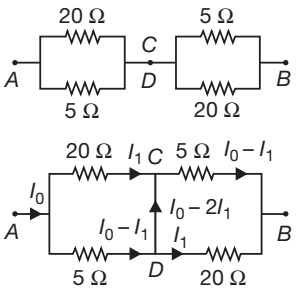
Leakage current $= I = \dfrac{dq}{dt}$

$\Rightarrow \quad I = -\dfrac{q_0\sigma}{\varepsilon_0 K} e^{-\frac{t\sigma}{\varepsilon_0 K}}$

$\Rightarrow \quad I = I_0 e^{-\frac{t\sigma}{\varepsilon_0 K}}$ where $I_0 = -\dfrac{q_0\sigma}{\varepsilon_0 K}$

Hence, (A), (C) and (D) are correct.

25. As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as $8\ \Omega$. When we distribute current in the network using symmetry,



$V_A - V_D = V_A - V_C$

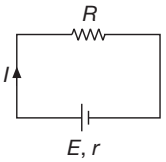
$\Rightarrow \quad 20I_1 = 5(I_0 - I_1)$

$\Rightarrow \quad I_1 = \dfrac{I_0}{5}$

$\Rightarrow \quad I_0 - 2I_1 = I_0 - \dfrac{2I_0}{5} = \dfrac{3I_0}{5}$ which is the current flowing from D to C.

Hence, (A), (B) and (C) are correct.

28. $I = \dfrac{E}{R+r}$



$P = EI$

$\Rightarrow \quad P = \dfrac{E^2}{R+r}$

So, heat produced per second across the external circuit is

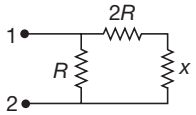
$$H = I^2R = I(IR) = \frac{E(IR)}{R+r}$$

and that across the internal circuit is

$$h = I^2r = \frac{E^2 r}{(R+r)^2}$$

Hence, (A), (B), (C) and (D) are correct.

29. **CIRCUIT A**



$x = \dfrac{R(2R+x)}{3R+x}$

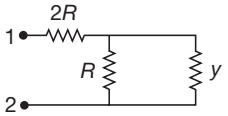
$\Rightarrow \quad 3Rx + x^2 = 2R^2 + Rx$

$\Rightarrow \quad x^2 + 2Rx - 2R^2 = 0$

$\Rightarrow \quad x = -\dfrac{2R \pm \sqrt{4R^2 + 8R^2}}{2}$

$\Rightarrow \quad x = \dfrac{-2R + 2\sqrt{3}R}{2}$

$\Rightarrow \quad x = (\sqrt{3} - 1)R$



CIRCUIT B

$$y = \frac{yR}{y+R} + 2R$$

$\Rightarrow \quad y^2 + Ry = yR + 2Ry + 2R^2$

$\Rightarrow \quad y^2 - 2yR - 2R^2 = 0$

$\Rightarrow \quad y = (\sqrt{3}+1)R$

Hence, (A), (B), (C) and (D) are correct.

31. Since voltmeter is a device connected in parallel across the circuit, hence

$$R_{\text{equivalent}} = \frac{R_V R_0}{R_V + R_0}$$

For $R_0 \ll R_V$

$\Rightarrow \quad R_{\text{equivalent}} \simeq R_0$

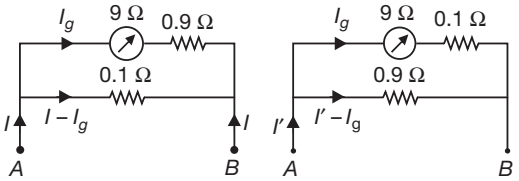
{i.e., resistance of the circuit remains unaltered when a voltmeter of extremely high resistance is applied across the circuit}

Hence, (A) and (B) are correct.

33. $I_g = 10 \text{ mA} = 0.01 \text{ A}$

$$V_A - V_B = (I - I_g)(0.1 \text{ A}) = I_g(9.9)$$

$\Rightarrow \quad I \times 0.1 = I_g \times 10$



$$\Rightarrow \quad I = \frac{0.01 \text{ A} \times 10}{0.1} = 1 \text{ A}$$

Similarly $(I' - I_g)(0.9) = I_g(9.1)$

$\Rightarrow \quad I'(0.9) = I_g(10)$

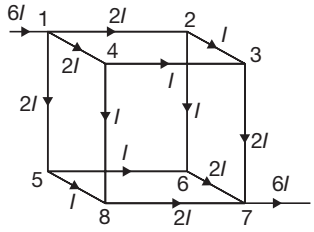
$$\Rightarrow \quad I' = \frac{1}{9} \text{ A} = 111 \text{ mA}$$

Hence, (A) and (C) are correct.

34. Resistance between diagonal corners of cube

$\Rightarrow \quad V_{17} = (V_1 - V_5) + (V_5 - V_8) + (V_8 - V_7)$

$\Rightarrow \quad V_{17} = 2IR_0 + IR_0 + 2IR_0$



$\Rightarrow \quad V_{17} = 5IR_0$

Also,

$V_{17} = x(6I)$

$\Rightarrow \quad x(6I) = 5IR_0$

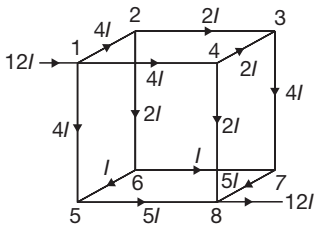
$$\Rightarrow \quad x = \frac{5R_0}{6}$$

$\Rightarrow \quad V_{18} = (V_1 - V_5) + (V_5 - V_8)$

$\Rightarrow \quad V_{18} = 4IR_0 + 5IR_0$

$\Rightarrow \quad V_{18} = 9IR_0$

Also $V_{18} = y(12I)$



$\Rightarrow \quad (12I)y = 9IR_0$

$$\Rightarrow \quad y = \frac{3}{4}R_0$$

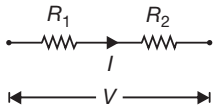
$$\Rightarrow \quad \frac{x}{y} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{3}{4}\right)} = \frac{20}{18} = \frac{10}{9}$$

$$\Rightarrow \quad x - y = \frac{5}{6}R_0 - \frac{3}{4}R_0$$

$$\Rightarrow \quad x - y = \frac{R_0}{12}$$

Hence, (A) and (B) are correct.

35. Let $V = 220$ V and R_1 and R_2 be the resistances of the 25 W and 100 W bulbs.



Since, $P_A = 25 \text{ W} = \frac{V^2}{R_1}$ and $P_B = 100 \text{ W} = \frac{V^2}{R_2}$

$$\Rightarrow \quad R_1 = \frac{V^2}{25 \text{ W}} \text{ and } R_2 = \frac{V^2}{100 \text{ W}}$$

When the bulbs are joined in series then the current is

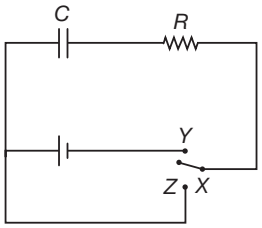
$$I = \frac{V}{R_1 + R_2}$$

Power in the 25 W bulb is R_1I^2 and that in the 100 W bulb is R_2I^2.

Hence, (A) and (C) are correct.

36. When X is joined to Y for a long time (charging), the energy stored in the capacitor is equal to the heat produced in R i.e., H_1.

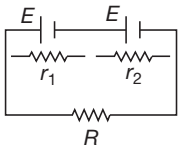
When X is joined to Z (discharging), the energy stored in $C(=H_1)$ reappears as heat (H_2) in R. So, $H_1 = H_2$. Also, we observe that energy supplied by the battery is $H = H_1 + H_2$.



Hence, (A), (C) and (D) are correct.

37. $I = \dfrac{2E}{R + r_1 + r_2}$

$V_1 = 0$, for terminal potential difference to be zero across source 1.



$$\Rightarrow \quad E - Ir_1 = 0$$

$$\Rightarrow \quad E - \frac{2Er_1}{R + r_1 + r_2} = 0$$

$$\Rightarrow \quad 2r_1 = R + r_1 + r_2$$

$$\Rightarrow \quad R = r_1 - r_2$$

$V_2 = 0$, for terminal potential difference to be zero across source 2.

$$\Rightarrow \quad E - Ir_2 = 0$$

$$\Rightarrow \quad E - \frac{2\,Er_2}{R + r_1 + r_2} = 0$$

$$\Rightarrow \quad 2r_2 = R + r_1 + r_2$$

$$\Rightarrow \quad R = r_2 - r_1$$

Hence, (A) and (C) are correct.

38. Let R_1 and R_2 be the resistances of the two heaters. Let, initially, the heat produced be H, then

$$H = \left(\frac{V^2}{R_1}\right)t_A = \left(\frac{V^2}{R_2}\right)t_B \qquad \ldots(1)$$

When used in series,

$$H = \left(\frac{V^2}{R_1 + R_2}\right)T \qquad \ldots(2)$$

When used in parallel

$$H = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right)t \qquad \ldots(3)$$

Using (1), (2) and (3), we get the desired results.

Hence, (B) and (D) are correct.

39. When steady state is reached no current will flow in the branch having the capacitor. So

$$R_{\text{total}} = 2.8 + \frac{(2)(3)}{2+3}$$

$$\Rightarrow \quad R_{\text{total}} = 4\ \Omega$$

$$\Rightarrow \quad I = \frac{6}{4} = 1.5\ \text{A}$$

Current in 2 Ω resistor = $I_2 = \left(\dfrac{1/2}{1/2 + 1/3}\right)1.5$

$$\Rightarrow \quad I_2 = \left(\frac{3}{5}\right)(1.5)$$

$$\Rightarrow \quad I_2 = 0.9\ \text{A}$$

Potential drop across 2.8 Ω is

$$V = (2.8)(1.5)$$

$$\Rightarrow \quad V = 4.2\ \text{V}$$

Hence, (A), (B) and (D) are correct.

40. $V_A - V_B = 9\ \text{V}$

15 V battery and 24 V battery form the dominating combination and hence I must flow from D to A through R.

$$\Rightarrow \quad I = \frac{24 + 15 - 6}{R + 1 + 2 + 1}$$

$$\Rightarrow \quad I = \frac{33}{R + 4}$$

Since inside the 6 V battery the current is going from positive terminal to the negative terminal so

$$V_A - V_B = 6 + Ir$$

$$\Rightarrow \quad 9 = 6 + \frac{33}{R + 4}$$

$$\Rightarrow \quad R + 4 = 11$$

$$\Rightarrow \quad R = 7\ \Omega$$

$$\Rightarrow \quad V_{BC} = 15 - 3(2) = 9\ \text{V} \qquad \left\{\because I = \frac{33}{11} = 3\ \text{A}\right\}$$

$\Rightarrow \quad V_{BD} = 39 - 3(3)$

$\Rightarrow \quad V_{BD} = 30\ V$

Hence, (B) and (C) are correct.

41. $V = V_0\left(1 - e^{-t/\tau}\right)$

$\tau = \dfrac{RC}{2}$

$V_0 = \dfrac{E}{2}$ (as both R and R are in series)

$\Rightarrow \quad V = \dfrac{E}{2}\left(1 - e^{-\frac{2t}{RC}}\right)$

Hence, (C) and (D) are correct.

42. $H = \dfrac{V^2}{R_1}t_1$

$\Rightarrow \quad R_1 = \dfrac{V^2 t_1}{H}$

Similarly, $R_2 = \dfrac{V^2 t_2}{H}$

In series, $H = \left(\dfrac{V^2}{R_1 + R_2}\right)t$

$t = \dfrac{H(R_1 + R_2)}{V^2}$

Substituting the values of R_1 and R_2, we get

$t = t_1 + t_2$

In parallel, $H = \dfrac{V^2}{R_{\text{net}}}t = V^2 t\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$= V^2 t\left(\dfrac{H}{V^2 t_1} + \dfrac{H}{V^2 t_2}\right)$

Solving we get, $t = \dfrac{t_1 t_2}{t_1 + t_2}$

Hence, (A) and (C) are correct.

43. $I = \dfrac{dQ}{dt} = a - 2bt$, which decreases linearly with t.

$I = 0$ for $t = \dfrac{a}{2b}$ and rate of charging is $\dfrac{dI}{dt}$. So,

$\dfrac{dI}{dt} = -2b$

Hence the correct options are **(A), (C)** and **(D)**
Hence, (A), (C) and (D) are correct.

45. The resistance in the middle plays no part in the charging process of C, as it does not alter either the potential difference across the RC combination or the current through it.

C discharges through $R + R$ in series.
Hence, (A) and (B) are correct.

46. $Q = Q_0 e^{-t_1/\pi}$ and potential difference across C is proportional to Q.

For the potential difference to fall by 10%, Q must fall by 10%. So,

$Q = 0.9\ Q_0 = Q_0 e^{-t_1/\tau}$

$\Rightarrow \quad e^{t_1/\tau} = \dfrac{10}{9}$

$\Rightarrow \quad \dfrac{t_1}{\tau} = \log_e\left(\dfrac{10}{9}\right)$

$\Rightarrow \quad t_1 = \tau \log_e\left(\dfrac{10}{9}\right)$

$Q = 0.1\ Q_0 = Q_0 e^{-t_2/\tau}$

$\Rightarrow \quad e^{t_2/\tau} = 10$

$\Rightarrow \quad \dfrac{t_2}{\tau} = \log_e 10 = 2.303$

$\Rightarrow \quad t_2 = 2.303\,\tau$

Hence, (B) and (C) are correct.

47. $Q = Q_0\left(1 - e^{-t_1/\tau}\right) = 0.1 Q_0$

$\Rightarrow \quad e^{-t_1/\tau} = 0.9$

$\Rightarrow \quad e^{t_1/\tau} = \dfrac{10}{9}$

$\Rightarrow \quad t_1 = \tau \log_e\left(\dfrac{10}{9}\right)$

$Q = Q_0\left(1 - e^{-t_2/\tau}\right) = 0.9 Q_0$

$\Rightarrow \quad e^{-t_2/\tau} = 0.1$

$\Rightarrow \quad e^{t_2/\tau} = 10$

$\Rightarrow \quad t_2 = \tau \log_e(10)$

$\Rightarrow \quad (t_1 + t_2) = \tau \log_e\left(\dfrac{100}{9}\right) = 2\tau \log_e\left(\dfrac{10}{3}\right)$

and $(t_2 - t_1) = 2\tau \log_e(3)$

Hence, (B) and (D) are correct.

49. By closing S_1, net external resistance will decrease. So, main current will increase.
By closing S_2, net emf will remain unchanged but net internal resistance will decrease. Hence, main current will increase.
Hence, (A) and (C) are correct.

50. $11.4 = I_1(9.5)$

$\Rightarrow \quad I_1 = 1.2\ \text{A}$

CHAPTER 3

Since, $E-11.4=I_1 r$

$\Rightarrow \quad E-11.4=1.2r \quad \ldots(1)$

Similarly $I_2=1$ A

$\Rightarrow \quad E-11.5=1r \quad \ldots(2)$

$0.1=0.2r$

$\Rightarrow \quad r=0.5\ \Omega$ and $E=12$ V

Hence, (A) and (B) are correct.

51. $C_s=\dfrac{10}{7}\ \mu F$

$Q=C_s V=\dfrac{10}{7}\times 14$

$\Rightarrow \quad Q=20\ \mu C$

Further $V_{C_1}=\dfrac{Q}{C_1}=\dfrac{20}{5}=4$ V

Similarly $V_{C_2}=\dfrac{Q}{C_2}=\dfrac{20}{2}=10$ V

Hence, (B) and (C) are correct.

52. At $t=0$, emf of the circuit $=PD$ across the capacitor $=6$ V.

$\Rightarrow \quad i=\dfrac{6}{1+2}=2$ A

Half-life of the circuit

$=(\ln 2)\tau_C=(\ln 2)CR=(6\ln 2)s.$

In half-life time, all values get halved. For example

$V_C=\dfrac{6}{2}=3$ V

$i=\dfrac{2}{1}=1$ A

$\Rightarrow \quad V_{1\,\Omega}=iR=1$ V and

$V_{2\,\Omega}=iR=2$ V

Hence, (A), (B), (C) and (D) are correct.

54. If switch S is open,

$i_1\lambda\ell=E_2$

where, $i_1=$ current in upper circuit and λ is resistance per unit length of potentiometer wire.

$\Rightarrow$ Null point length, $\ell=\dfrac{E_2}{i_1\lambda}$

(a) If jockey is shifted towards right, resistance in upper circuit will increase. So, current i_1 will decrease. Hence, ℓ will increase.

(b) If E_1 is increased, i_1 will also increase, So, ℓ will decrease.

(c) $\ell \propto E_2$

(d) If switch is closed, then null point will be obtained corresponding to

$V_2=E_2-i_2 r_2$

which is less than E_2. Hence, null point length will decrease.

Hence, (A), (B), (C) and (D) are correct.

55. Current through A is the main current passing through the battery. So, this current is more than the current passing through B. Hence, during charging more heat is produced in A.

In steady state,

$i_C=0$

and $i_A=i_B$

Hence, heat is produced at the same rate in A and B. Further, in steady state

$V_C=V_B=\dfrac{\varepsilon}{2}$

$\Rightarrow \quad U=\dfrac{1}{2}CV_C^2=\dfrac{1}{8}C\varepsilon^2$

Hence, (A), (B) and (D) are correct.

Reasoning Based Questions

2. The electrons are in motion which constitute electric current in a conductor but no. of positive and negative charges are same.

Hence, the correct answer is (D).

3. When current flows through a conductor, it always remains uncharged. Hence no electric field is produced outside it.

Hence, the correct answer is (A).

4. Direction of flow of current is from higher potential to lower potential.

Hence, the correct answer is (D).

5. Since current arises due to continuous flow of charged particles. There is no free charge in insulator. Hence no flow of charges is possible. Therefore current does not flow through insulators.

Hence, the correct answer is (A).

6. $V=IR$

Hence, the correct answer is (D).

8. Since, in the case of stretching the length n times, resistance becomes n^2 times the original value. So, Statement 1 is false, however Statement 2 is true.

Because $R=\rho\dfrac{\ell}{A}$

Hence, the correct answer is (D).

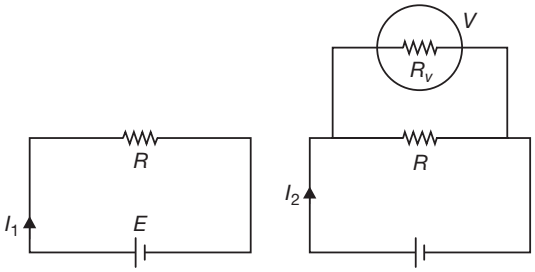
9. On increasing temperature of wire, the kinetic energy of the electrons increases and so they collide more rapidly with each other as a result of which their drift velocity decreases and resistivity increases which is inversely proportional to the conductivity of material.
Hence, the correct answer is (B).

10. Voltmeter gives terminal potential (V) thought it can give emf if internal resistance of the cell is zero.
Hence, the correct answer is (D).

12. $I_1 = \dfrac{E}{R}$

$$R_{eq} = \frac{RR_V}{R+R_V}$$

$$I_2 = \frac{E}{\left(\dfrac{RR_V}{R+R_V}\right)}$$



Hence, the correct answer is (A).

13. In a simple battery circuit the point at lowest potential is the negative terminal of battery. The current flows in the circuit from positive terminal to negative terminal.
Hence, the correct answer is (D).

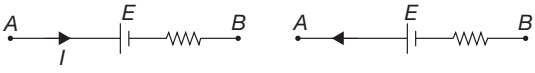
15. Internal resistance is always in series with the battery.
Hence, the correct answer is (D).

16. $P = \dfrac{V^2}{R}$

Hence, the correct answer is (D).

17. $V_A - E - Ir = V_B$

$\Rightarrow \quad V_A - V_B = E + Ir$



Similarly $V_A - V_B = E - Ir$

Hence, the correct answer is (A).

18. Power used $= I^2R$
Hence, power is consumed and not the current
Hence, the correct answer is (D).

19. $P = I^2R$

$$\Rightarrow \quad \left(\frac{dP}{P}\right)(100\%) = 2\left(\frac{dI}{I}\right)100\%$$

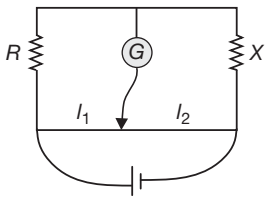
Hence, the correct answer is (B).

20. If either the e.m.f. of the driver cell or potential difference across whole potentiometer wire is lesser than the e.m.f. of the experimental cell, then balance point will not obtained.
Hence, the correct answer is (A).

21. $V = E + Ir$ when charging of cell take place, i.e., $V > E$
Hence, the correct answer is (D).

22. The resistance of galvanometer is fixed. In metre bridge experiments, to protect the galvanometer from a high current, high resistance is connected to the galvanometer in order to protect it from damage (because then less current can pass through the galvanometer).
Hence, the correct answer is (C).

23. $R\ell_2 = \ell_1 x$



To get null point at the same position, means ℓ_1 and ℓ_2 are still the same. As temperature increases, value of unknown resistance increases. To get the same null point, R must be increased. So Statement-1 is wrong. Statement-2 is True.
Hence, the correct answer is (D).

25. Charge on capacitor $q = CE\left(1 - e^{-t/CR_{eq}}\right)$

$$\Rightarrow \quad I = \frac{dq}{dt} = \frac{E}{R_{eq}} e^{-t/CR}$$

At $t = 0$

$$I = \frac{E}{R_{eq}} = \frac{E}{R+r}$$

$\Rightarrow$ Resistance offered by capacitor is zero.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. To find the charge per pulse let us integrate the expression $dQ = Idt$. Since the current to be constant for the entire duration of the pulse, so

$$Q_{pulse} = I\int dt = I\Delta t = (250\times10^{-3}\,\text{A})(200\times10^{-9}\,\text{s})$$

$\Rightarrow \quad Q_{\text{pulse}} = 5\times10^{-8}\ \text{C}$

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse. So, we have

$$\text{Electrons per pulse} = \frac{5\times10^{-8}\ \text{C/Pulse}}{1.6\times10^{-19}\ \text{C/electron}} = 3.13\times10^{11}\ \text{electrons/pulse}$$

Hence, the correct answer is (C).

2. Average current is given by Equation, $I_{av} = \dfrac{\Delta Q}{\Delta t}$. Because the time interval between pulses is 4 ms, and also we have already calculated the charge per pulse, so we obtain

$$I_{av} = \frac{Q_{\text{pulse}}}{\Delta t} = \frac{5\times10^{-8}\ \text{C}}{4\times10^{-3}\ \text{s}} = 12.5\ \mu\text{A}$$

This represents only 0.005% of the peak current, which is 250 mA.

Hence, the correct answer is (A).

3. Since, power is defined as the energy delivered per unit time interval. Thus, the peak power is equal to the energy delivered by a pulse divided by the pulse duration.

$$P_{\text{peak}} = \frac{\text{pulse energy}}{\text{pulse duration}} \qquad \ldots(1)$$

$$P_{\text{peak}} = \frac{\left(3.13\times10^{11}\ \text{electrons/pulse}\right)\left(40\ \text{MeV/electron}\right)}{2\times10^{-7}\ \text{s/pulse}}\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)$$

$\Rightarrow \quad P_{\text{peak}} = 1\times10^{7}\ \text{W} = 10\ \text{MW}$

Also we could have calculated the power directly by assuming that each electron has zero energy before being accelerated. Thus, by definition, each electron must go through a potential difference of 40 MV to acquire a final energy of 40 MeV. Hence, we have

$$P_{\text{peak}} = I_{\text{peak}}\Delta V \qquad \ldots(2)$$

$\Rightarrow \quad P_{\text{peak}} = \left(250\times10^{-3}\ \text{A}\right)\left(40\times10^{6}\ \text{V}\right)$

$\Rightarrow \quad P_{\text{peak}} = 10\ \text{MW}$

Hence, the correct answer is (B).

4. To calculate the average power, we shall make use of the time interval between pulses rather than the duration of a pulse. So, the average power is given by

$$P_{av} = \frac{\text{pulse energy}}{\text{time interval between pulses}}$$

$$P_{av} = \frac{\left(3.13\times10^{11}\ \text{electrons/pulse}\right)\left(40\ \text{MeV/electron}\right)}{4\times10^{-3}\ \text{s/pulse}}\times\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = 500\ \text{W}$$

Otherwise we could have calculated the average power using the similar formula where we have the average power given by the relation

$$P_{av} = I_{av}\Delta V = \left(12.5\times10^{-6}\ \text{A}\right)\left(40\times10^{6}\ \text{V}\right)$$

$\Rightarrow \quad P_{av} = 500\ \text{W}$

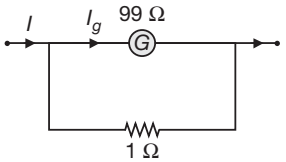
Notice that these two calculations agree with each other and that the average power is much lower than the peak power.

Hence, the correct answer is (D).

5. For ammeter $99I_g = \left(I - I_g\right)1$

$\Rightarrow \quad I = 100I_g \qquad \ldots(1)$

I_g is the full scale deflection current of the galvanometer I and the range of ammeter



For the circuit in figure, given in the question

$$\frac{12V}{2 + r + \dfrac{99\times1}{99+1}} = 3\ \text{A}$$

$\Rightarrow \quad r = 1.01\ \Omega$

Hence, the correct answer is (C).

6. **The correct answer is (C).**

7. **Hence, the correct answer is (B).**

Combined Solution to 6 & 7

For voltmeter, range

$$V = I_g\left(99 + 101\right)$$

$$V = 200\,I_g \qquad \ldots(2)$$

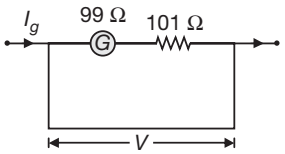
Also resistance of the voltmeter is R_V = 99 + 101 = 200 Ω. In figure resistance across the terminals of the battery is given by

$$R_1 = r + \frac{200\times2}{200} = 2.99\ \Omega$$

$\Rightarrow$ Current drawn from the battery,

$$I_1 = \frac{12}{2.99} = 4.01\ \text{A}$$

$\Rightarrow$ Voltmeter reading



$$\frac{4}{5}V = 12 - I_1 r$$

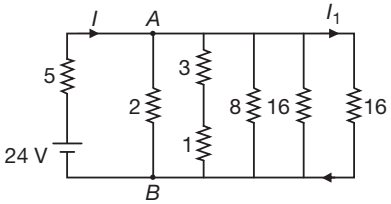
$$\Rightarrow \quad \frac{4}{5}V = 12 - 4.01 \times 1.01$$

$$\Rightarrow \quad V = 7.96 \times \frac{5}{4} = 9.95 \text{ V}$$

Using (2), $I_g = \dfrac{9.95}{200} = 0.05$ A

Using (1) range of the ammeter $I = 100 I_g = 5$ A

8. The equivalent circuit is as shown in figure.



Net resistance of circuit $R = 6\ \Omega$. Hence,

$$I = \frac{V}{R} = \frac{24}{6} = 4 \text{ A}$$

Hence, the correct answer is (B).

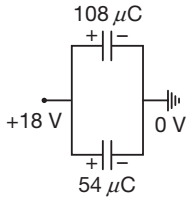
9. $I_1 = \dfrac{V_{AB}}{16} = \dfrac{4}{16} = \dfrac{1}{4}$ A

Hence, the correct answer is (C).

10. $V_{AB} = E - Ir = 24 - (4)(5) = 4$ V

Hence, the correct answer is (A).

11. When switch S is open circuit is as shown in figure.



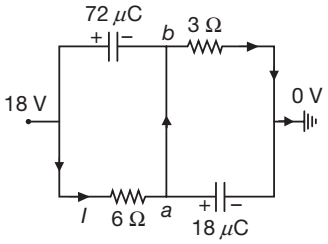
$V_a = +18$ V

and $V_b = 0$

$\Rightarrow \quad V_{ab} = 18$ V

Hence, the correct answer is (B).

12.



When switch S is closed, a current,

$I = \dfrac{18}{6+3} = 2$ A, flows in the circuit

$V_b - 0 = 3 \times 2 = 6$ V

$\Rightarrow \quad V_b = 6$ V

Hence, the correct answer is (C).

13. $\Delta q = (\Delta q)_{3\ \mu F} = (18 - 54)\ \mu C = -36\ \mu C$

and $\Delta q' = (\Delta q)_{6\ \mu F} = (72 - 108)\ \mu C = -36\ \mu C$

Hence, the correct answer is (A).

14. **The correct answer is (C).**

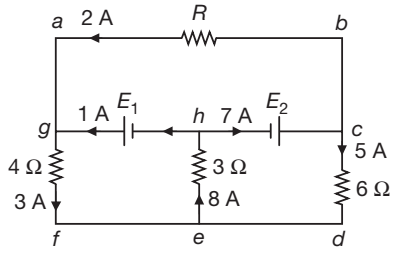
15. **The correct answer is (B).**

16. **Hence, the correct answer is (D).**

Combined Solution to 14, 15 & 16

Applying Junction Law, we get

$I_{3\ \Omega} = 5 + 3 = 8$ A



Applying Kirchhoff's Loop Law, we have for

Loop $ghefg$, $E_1 - 12 - 24 = 0$

$\Rightarrow \quad E_1 = 36$ V

Loop $chedc$, $-E_2 + 24 + 30 = 0$

$\Rightarrow \quad E_2 = 54$ V

Loop $cbagc$, $-2R - E_1 + E_2 = 0$

$\Rightarrow \quad R = \dfrac{E_2 - E_1}{2} = 9\ \Omega$

17. Time constant of circuit is

$$\tau = CR = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{\rho d}{A}\right) = K\varepsilon_0\rho$$

$$\Rightarrow \quad \tau = 18\left(\frac{1}{4\pi\times 9\times 10^9}\right)\left(4\pi\times 10^3\right)$$

$$\Rightarrow \quad \tau = 2\times 10^{-6}\ \text{s} = 2\ \mu\text{s}$$

Hence, the correct answer is (B).

18. Since, $q = q_0 e^{-\frac{t}{CR}}$

$$\Rightarrow \quad q = 2\times 10^{-6}\times e^{-\frac{2}{2}} = \frac{2\times 10^{-6}}{e}$$

$$\Rightarrow \quad q = 0.37\times 2\times 10^{-6} = 0.74\ \mu\text{C}$$

Hence, the correct answer is (D).

19. $q = q_0 e^{-\frac{t}{CR}}$

$$\Rightarrow \quad I = -\frac{dq}{dt} = -q_0 e^{-\frac{t}{CR}}\left(-\frac{1}{CR}\right)$$

$$\Rightarrow \quad I = \frac{q_0}{CR}e^{-\frac{t}{CR}}$$

$$\Rightarrow \quad I = \frac{2}{2}e^{-\frac{4}{2}} = \frac{1}{e^2} = \frac{1}{7.39} \quad (\text{As, } e = 2.718 \text{ and } e^2 = 7.39)$$

$$\Rightarrow \quad I = 0.13\ \mu\text{A}$$

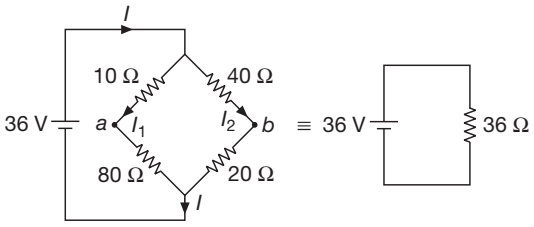
Hence, the correct answer is (D).

20. In steady state there is no current in the capacitor, the equivalent circuit is shown.

$$R_{\text{eq}} = 36\ \Omega$$

The current I is given by

$$I = \left(\frac{36}{36}\right)\text{A} = 1\ \text{A}$$



The current divides into two parallel branches, as I_1 and I_2. In parallel branches current divides in inverse ratio of resistance, thus

$$I_1 = \left(\frac{60}{90+60}\right)I = 0.4\ \text{A} \qquad \{\because\ I = 1\ \text{A}\}$$

$$I_2 = \left(\frac{90}{90+60}\right)I = 0.6\ \text{A} \qquad \{\because\ I = 1\ \text{A}\}$$

The capacitor is connected across points a and b. From KVL we have

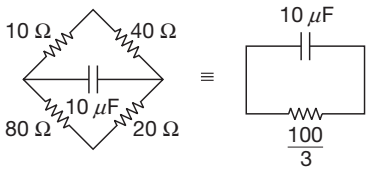
$$V_a + 10I_1 - 40I_2 = V_b$$

$$V_a - V_b = 40I_2 - 10I_1 = 20\ \text{V}$$

Thus, the voltage across capacitor is 20 V

Hence, the correct answer is (C).

21. When battery is disconnected, the equivalent circuit is shown here.



The initial charge on capacitor is

$$Q_0 = (10\times 20)\ \mu\text{C}$$

$$\Rightarrow \quad Q_0 = 200\ \mu\text{C}$$

The time constant of circuit is

$$\tau = RC = \left(\frac{100}{3}\times 10\right)\mu\text{s} = \left(\frac{1000}{3}\right)\mu\text{s}$$

The charge on a capacitor, Q as a function of time in a discharging circuit is given by the relation

$$Q = Q_0 e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad Q = 200e^{-\frac{t}{\left(\frac{1000}{3}\right)}}$$

$$\Rightarrow \quad Q = 200e^{-\frac{3t}{1000}} = 200\ \exp\left(-\frac{3t}{1000}\right)\mu\text{C}$$

Hence, the correct answer is (C).

22. Voltage, V across capacitor as a function of time is

$$CV = CV_0 e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad V = V_0 e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad V = 200e^{-\frac{t}{RC}}$$

$$I = 200e^{-\frac{t}{RC}}$$

Taking log both sides, we get

$$t = RC\log_e 200 = \frac{1000}{3}\log_e 200$$

$$\Rightarrow \quad t = 1766.1\ \mu\text{s}$$

$$\Rightarrow \quad t = 1.766\ \text{ms}$$

Hence, the correct answer is (B).

23. $\tau_1 = RC = (1.5\times10^5\ \Omega)(10\times10^{-6}\ \text{F}) = 1.5\ \text{s}$

Hence, the correct answer is (C).

24. $\tau_2 = (1\times10^5\ \Omega)(10\times10^{-6}\ \text{F}) = 1\ \text{s}$

Hence, the correct answer is (B).

25. The battery carries current

$$\frac{10\ \text{V}}{50\times10^3\ \Omega} = 200\ \mu\text{A}$$

The 100 kΩ carries current of magnitude

$$I = I_o e^{-\frac{t}{RC}} = \left(\frac{10\ \text{V}}{100\times10^3\ \Omega}\right)e^{-\frac{t}{1\ \text{s}}} = 100e^{-t}\ \mu\text{A}$$

So the switch carries downward current, I' given by

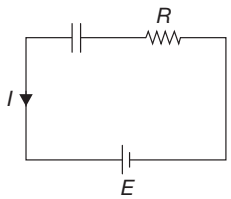
$$I' = 200\ \mu\text{A} + (100\ \mu\text{A})e^{-\frac{t}{1\ \text{s}}}$$

$$\Rightarrow\quad I' = 100(2+e^{-t})\ \mu\text{A}$$

Hence, the correct answer is (C).

26. Taking loop equation, we get

$$E - \frac{q}{C_f} + RI = 0 \quad [C_f \text{ is the capacitance at } t>0] \quad \ldots(1)$$



As charge on the capacitor cannot change abruptly, the current I_0, just after the capacitance has been changed, will be given by

$$E - \frac{q_0}{C_f} + RI_0 = 0$$

where q_0 is charge on capacitor at $t = 0$,

Hence, $I_0 = \dfrac{q_0 - EC_f}{RC_f}$

As $q_0 = C_i E$ $\quad \{C_i = \text{initial capacitance}\}$

putting the values, we get

$$I_0 = \frac{5\times10}{5\times50} = \frac{1}{5}\ \text{A}$$

Hence, the correct answer is (A).

27. Differentiating equation (1), we get

$$R\frac{dI}{dt} = \frac{I}{C_f}\frac{dq}{dt}$$

as $\quad I = \left|\dfrac{dq}{dt}\right|$

$$\Rightarrow\quad R\frac{dI}{dt} = -\frac{I}{C_f}$$

$$\Rightarrow\quad R\int_{I_0}^{I}\frac{dI}{I} = -\int_0^t\frac{dt}{C_f}$$

$$\Rightarrow\quad I = I_0 e^{-\frac{t}{RC_f}} \qquad \ldots(2)$$

Hence, $I = (0.2\ \text{A})e^{-\frac{t}{(2.5\times10^{-4})}} = 0.2\exp(-4000t)\ \text{A}$ where t is in seconds.

Hence, the correct answer is (B).

28. Substituting I in equation (1) we get

$$E - \frac{q}{C_f} + RI_0 e^{-\frac{t}{RC_f}} = 0$$

$$\Rightarrow\quad q = C_f\left(E + RI_0 e^{-\frac{t}{RC_f}}\right)$$

$$\Rightarrow\quad q = (5)\left(10 + 10e^{-\frac{t}{2.5\times10^{-4}}}\right)$$

$$\Rightarrow\quad q = (5\times10^{-5})\left(1 + e^{-\frac{t}{2.5\times10^{-4}}}\right)\text{C}$$

$$\Rightarrow\quad q = 5\times10^{-5}(1+e^{-4000t})$$

$$\Rightarrow\quad q = 50(1+e^{-4000t})\ \mu\text{C}$$

Hence, the correct answer is (C).

29. Power in resistor $= I^2R$

Hence, heat produced in the wire as function of time is H, given by

$$H = \int_0^t (I^2R)dt = I_0^2R\int_0^t e^{-\frac{2t}{RC_f}}dt = \frac{-I_0^2R^2C_f}{2}\left(e^{-\frac{2t}{RC_f}}\right)\Big|_0^t$$

$$\Rightarrow\quad H = +\frac{I_0^2R^2C_f}{2}\left(1 - e^{-\frac{2t}{RC_f}}\right)$$

$$\Rightarrow\quad H = \frac{(0.2\times50)^2\times(5\times10^{-6})}{2}[1-e^{-8000t}]\ \text{J}$$

$$\Rightarrow\quad H = (2.5\times10^{-4})[1-e^{-8000t}]\ \text{J}$$

$$\Rightarrow\quad H = 250(1-e^{-8000t})\ \mu\text{J}$$

Hence, the correct answer is (C).

30. Finally, the capacitors are in parallel and total charge $(=q_0)$ distributes between them in direct ratio of capacity.

$$\Rightarrow \quad q_{C_2} = \left(\frac{C_2}{C_1 + C_2}\right) q_0 \rightarrow \text{ in steady state.}$$

But this charge increases exponentially.

Hence, charge on C_2 at any time t is

$$q_{C_2} = \left(\frac{C_2 q_0}{C_1 + C_2}\right)\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

Initially, C_2 is uncharged so, whatever is the charge on C_2, it is charge flown through switches.

Hence, the correct answer is (B).

31. Common potential in steady state when they finally come in parallel is

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{q_0}{C_1 + C_2}$$

Total heat dissipated is $\Delta H = U_i - u_f$

$$\Rightarrow \quad \Delta H = \frac{q_0^2}{2C_1} - \frac{1}{2}(C_1 + C_2)\left(\frac{q_0}{C_1 + C_2}\right)^2$$

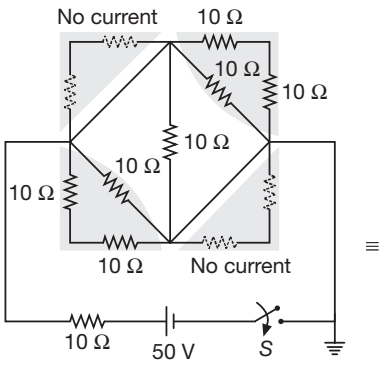
$$\Rightarrow \quad \Delta H = \left(\frac{q_0^2}{2C_1}\right)\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

Hence, the correct answer is (A).

32. Immediately after closing the switch a capacitor behaves as a short circuit. Figure shows the effective circuit from the point of view of current.

Figure shows the equivalent circuit, current I from battery is

$$I = \frac{50}{\frac{5}{2} + 10} = 4 \text{ A}$$



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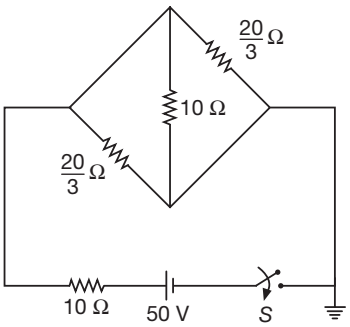
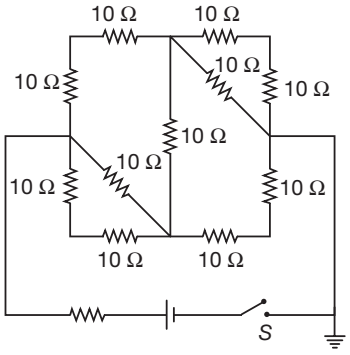


Hence, the correct answer is (B).

33. A long time after closing the switch a capacitor behaves as an open circuit. The equivalent circuits are shown. Using Kirchhoff's Laws, we get

$$I = \frac{70}{33} \text{ A} \quad \text{and} \quad I_1 = \frac{25}{33} \text{ A}$$



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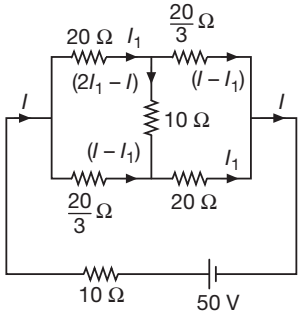


Hence, the correct answer is (C).

34. The $10\ \mu\text{F}$ capacitor is connected across the $20\ \Omega$ resistor of figure (see question)

$$\text{P.D. across } 20\ \Omega \text{ resistor} = 20 \times I_1 = \frac{500}{33} \text{ V}$$

So, charge on $10\ \mu\text{F}$ capacitor is given by

$$q_{10} = \left(10 \times \frac{500}{33}\right)\mu C = \frac{5000}{33}\ \mu C$$

The $5\ \mu F$ capacitor is connected in parallel to the $20\ \Omega$ resistor hence charge on the $5\ \mu F$ capacitor, is

$$q_5 = \left(\frac{5 \times 500}{33}\right)\mu C = \frac{2500}{33}\ \mu C$$

Hence, the correct answer is (C).

35. Immediately after closing switch S_1 the capacitor C_1 short-circuits the part of circuit to the right of it. So, the current will flow through the loop $ABGHA$ only, with a value

$$I_0 = I(t=0) = \frac{V}{R} = \frac{12}{100} = 0.12\ A$$

Hence, the correct answer is (C).

36. Long time after closing the switches the capacitors do not allow the current to pass through them. So, the current will now flow in the loop $ABCFGHA$ (with no capacitor in the branch BG). So, at $t \to \infty$, the net resistance offered to the flow of current is given by

$$R_{eq} = 100 + 50 + 150 = 300\ \Omega$$

$$\Rightarrow \quad I = I(t \to \infty) = \frac{V}{R_{eq}} = \frac{12}{300} = 0.04\ A$$

Hence, the correct answer is (B).

37. C_1 is connected across points B and G. We traverse the circuit along $BAHGB$ and apply KVL, we have

$$V_B + 100I - 12 - V_G = 0$$

$$V_B - V_G = 12 - 100I = 12 - (100)(0.04) = 8\ V$$

Final voltage across C_1 is 8 V

Hence, the correct answer is (D).

38. Capacitor C_2 is connected across the $150\ \Omega$ resistor, hence voltage across C_2 is

$$V_2 = (150)(0.04) = 6\ V$$

Hence, the correct answer is (C).

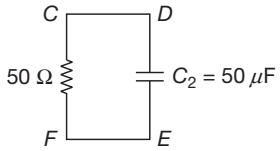
39. When switch S_2 is opened, the loop $CDEF$ is a discharging CR circuit. The charge as function of time is

$$Q = Q_0 e^{-\left(\frac{t}{RC_2}\right)}$$

$$\Rightarrow \quad Q = (C_2 \times 6)e^{-\left(\frac{t}{RC_2}\right)} \qquad \{\because\ Q_0 = C_2V_2 = 6C_2\}$$

Current $I = \dfrac{dQ}{dt}$

$$\Rightarrow \quad I = -\frac{6C_2}{RC_2}e^{-\left(\frac{t}{RC_2}\right)}$$



$$\Rightarrow \quad I = -\frac{6}{R}e^{-\left(\frac{t}{RC_2}\right)}$$

$$\Rightarrow \quad I = -\frac{6}{150}e^{-\frac{t}{(50\times10^{-6})(150)}}$$

$$\Rightarrow \quad I = -0.04e^{-\frac{400t}{3}}$$

$$\Rightarrow \quad I = -0.04\exp\left(-\frac{400t}{3}\right)$$

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (s)
C → (p)
D → (r)

2. A → (s)
B → (q)
C → (q)
D → (p)

Since $R = \dfrac{\rho\ell}{A}$ or $R \propto \dfrac{\ell}{A}$

$$\Rightarrow \quad R_1 : R_2 : R_3 = \left(\frac{\ell}{A}\right)_1 : \left(\frac{\ell}{A}\right)_2 : \left(\frac{\ell}{A}\right)_3$$

$$\Rightarrow \quad R_1 : R_2 : R_3 = \frac{1}{2} : \frac{1}{2} : \frac{1}{3} = 1 : 1 : 6$$

Hence (A) → (s)
As V is constant so

$$I \propto \frac{1}{R} \text{ and } P \propto \frac{1}{R}, \text{ so } P \propto I$$

Hence, $I_1 : I_2 : I_3 = P_1 : P_2 : P_3 = \left(\dfrac{2}{1}\right) : \left(\dfrac{2}{1}\right) : \left(\dfrac{1}{3}\right)$

$$\Rightarrow \quad I_1 : I_2 : I_3 = 6 : 6 : 1$$

So (B) → (q), (C) → (q)
Since wires are of same material, so

$$\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$$

Hence (D) → (p)

3. A → (q, r, s)
B → (q, s)
C → (p, s)
D → (p, s)

CHAPTER 3

Since,

$$q = q_0\left(1 - e^{-t/RC}\right)$$

So, the charging current is

$$I = \frac{dq}{dt} = \frac{V}{R}e^{-t/RC}$$

$$\Rightarrow \quad I = \frac{V}{R}e^{-t/RC} = I_0 e^{-t/RC}$$

At $t = 0$, we get

$$I = I_0 = \frac{V}{R}$$

Hence current is maximum and capacitor acts as a short circuit.
So, (A) → (q, r, s)
The decay charge is given by

$$q = q_0 e^{-t/RC}$$

Hence, the discharging current is

$$I = -\frac{dq}{dt} = I_0 e^{-t/RC}$$

$$\Rightarrow \quad I = I_0 e^{-t/RC}$$

at $t = 0$, we have

$$I = I_0 = \frac{V}{R} \text{ (maximum)}$$

Hence, (B) → (q, s)
Now, when the capacitor is fully charged, energy stored is

$$U = \frac{1}{2}CV^2$$

$$\Rightarrow \quad U = \frac{1}{2}C\left[V_0\left(1 - e^{-t/RC}\right)\right]^2$$

$$\Rightarrow \quad U = \frac{1}{2}CV_0^2\left(1 - e^{-t/RC}\right)^2$$

Hence, U is an exponential function of t
So, (C) → (p, s)
Energy dissipated as heat is

$$dH = I^2 R\,dt$$

$$\Rightarrow \quad \int dH = \int_0^\infty \left(I_0 e^{-t/RC}\right)^2 R\,dt$$

$$\Rightarrow \quad H = \frac{1}{2}CV^2$$

Hence, (D) → (p, s)

4. A → (q)
B → (s)
C → (q)
D → (s)

Let potential of point e is V volts. Then,

$$I_{ae} + I_{be} + I_{ce} + I_{de} = 0$$

$$\Rightarrow \quad \left(\frac{2-V}{1}\right) + \left(\frac{4-V}{2}\right) + \left(\frac{6-V}{1}\right) + \left(\frac{4-V}{2}\right) = 0$$

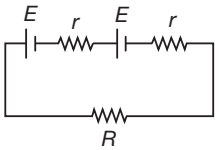
$$\Rightarrow \quad V = 4 \text{ V}$$

Now current through any wire can be obtained by the equation,

$$I = \frac{\Delta V}{R}$$

5. A → (q)
B → (q)
C → (s)
D → (r)
For series combination

$$R_{ext} = R,\ R_{int} = 2r \text{ and } E_{net} = 2E$$



Acc to Maximum Power Transfer Theorem

$$R = 2r$$

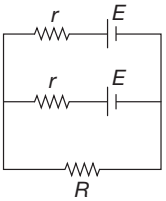
$$\Rightarrow \quad P = \left[\frac{4E^2}{(R+2r)^2}\right]R = \frac{4E^2}{4R^2}R = \frac{E^2}{R} = \frac{E^2}{2r}$$

For parallel combination

$$E_{net} = E,\ R_{ext} = R,\ R_{int} = \frac{r}{2}$$

Acc to Maximum Power Transfer Theorem,

$$R_{ext} = R_{int}$$



$$\Rightarrow \quad R = \frac{r}{2}$$

$$\Rightarrow \quad r = 2R$$

Now $P = \dfrac{E^2}{\left(R + \dfrac{r}{2}\right)^2}R = \left(\dfrac{E^2}{4R^2}\right)R$

$$\Rightarrow \quad P = \frac{E^2}{4R} = \frac{E^2}{4\left(\frac{r}{2}\right)} = \frac{E^2}{2r}$$

6. A → (p)
B → (p)
C → (p)
D → (p)

$i_1 = i_2$ or i is same at both sections.

$A_1 < A_2$

(A) Current density $= \frac{i}{A} \propto \frac{1}{A}$

(C) $\frac{\text{Resistance}}{\text{length}} = \frac{\rho}{A} \propto \frac{1}{A}$

(D) and (B) E or potential difference per unit length $= (i)$ (Resistance per unit length)

$\Rightarrow \quad \frac{P.D.}{\ell} = (i)\left(\frac{\rho}{A}\right) \propto \frac{1}{A}$

7. A → (q)
B → (r)
C → (t)
D → (p)

As 3 Ω and 7 Ω are in series across 6 V and in series potential divides in proportion to resistance

So, $V_A - V_D = \frac{R(V)}{(R+S)} = \frac{3\times 6}{3+7} = \frac{18}{10} = 1.8$ V

Hence (A) → (q)

Similarly, $V_A - V_B = \frac{X(V)}{(X+Y)} = \frac{2\times 6}{2+4} = 2$ V

$(V_A - V_B) - (V_A - V_D) = V_D - V_B = 2 - 1.8 = 0.2$ V

Hence (B) → (r)
Energy stored in the capacitor will be zero if $V_B = V_D$.
So for condition of a balanced wheatstone bridge, we have

$\frac{2}{Y} = \frac{3}{7}$

$\Rightarrow \quad Y = \frac{14}{3}\ \Omega$

So (C) → (t)
Also, in steady state no current flows through the branch containing the capacitor. Hence (D) → (p)

8. A → (r)
B → (s)
C → (q)
D → (p)

9. A → (p, q)
B → (p, q)
C → (r, s)
D → (r, s)

Resistance of bulbs A and B are

$R_A = \frac{V_A^2}{P}$

and $R_B = \frac{V_B^2}{P}$

$\Rightarrow \quad \frac{R_A}{R_B} = \frac{V_A^2}{V_B^2}$

When the bulbs are connected in series, current through them will be same. Hence, potential difference across them will be proportional to their resistances. Therefore, ratio of potential difference across them will be equal to ratio of their resistances, $R_A : R_B$ which is $V_A^2 : V_B^2$

So, (A) → (p, q)
Power consumed will be

$P_A = I^2 R_A$

and $P_B = I^2 R_B$

$\Rightarrow \quad \frac{P_A}{P_B} = \frac{R_A}{R_B} = \frac{V_A^2}{V_B^2}$

Hence, (B) → (p, q)
If bulbs are connected in parallel, then

$I_1 R_A = I_2 R_B$

$\Rightarrow \quad \frac{I_1}{I_2} = \frac{R_B}{R_A} = \frac{V_B^2}{V_A^2}$

So, (C) → (r, s)
and power consumed is given by

$$\frac{P_A}{P_B} = \frac{\frac{V^2}{R_A}}{\frac{V^2}{R_B}} = \frac{R_B}{R_A} = \frac{V_B^2}{V_A^2}$$

Hence, (D) → (r, s)

10. A → (s)
B → (r)
C → (p)
D → (q)

11. A → (s)
B → (r)
C → (s)
D → (r)

$i = \frac{4-1}{1+1+1} = 1$ A (anti-clockwise)

(A) $V_A = E - ir = 4 - 1\times 1 = 3$ V

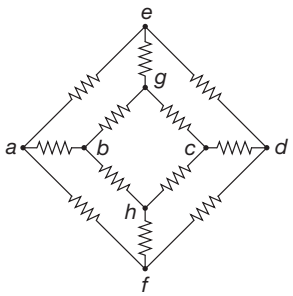
(B) $V_B = E + ir = 1 + 1\times 1 = 2$ V

CHAPTER 3

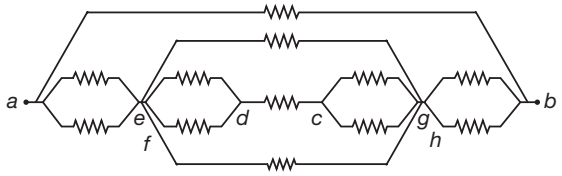
(C) $|P_A| = Ei - i^2 r = (4\times1) - (1)^2(1) = 3$ W

(D) $|P_B| = Ei + i^2 r = (1)(1) + (1)^2(1) = 2$ W

12. A → (s)
B → (q)
C → (r)
D → (p)



Symmetry of circuit shows that e and f are at same potential and g and h are another same potential. So, this circuit can be redrawn as shown.



Equivalent resistance in the middle line between e and h is

$$\frac{R}{2} + R + \frac{R}{2} = 2R$$

The total equivalent resistance between e and h is R' such that

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}$$

$$\Rightarrow \quad R' = \frac{2R}{5}$$

The equivalent resistance between a and b along path aeb

$$\frac{R}{2} + \frac{2R}{5} + \frac{R}{2} = \frac{7R}{5}$$

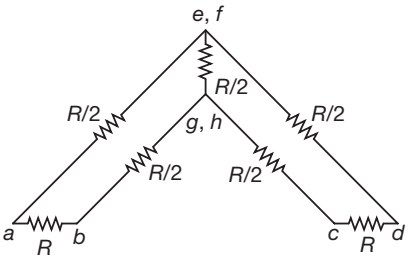
The total equivalent resistance between a and b is the resistance of R and $\frac{7R}{5}$ in parallel.

$$\Rightarrow \quad R_{ab} = \frac{R\times\frac{7R}{5}}{R+\frac{7R}{5}} = \frac{\frac{7R^2}{5}}{\frac{12R}{5}} = \frac{7R}{12}$$

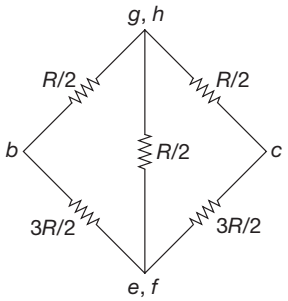
$$\Rightarrow \quad R_{ab} = \frac{7R}{12}$$

So, (A) → (s)

Resistance between b and c can be calculated as shown. Due to symmetry condition, network can be folded along the line bc. The circuit looks like



Finally it becomes balanced wheatsone bridge, so resistance of $\frac{R}{2}$ between g and e can be neglected. Equivalent resistance between b and c is the parallel combination of R and $3R$



So, $R_{bc} = \frac{R\times3R}{4R} = \frac{3R}{4}$

So (B) → (q)

Similarly, we can get $R_{ac} = \frac{5R}{6}$

So, (C) → (r)

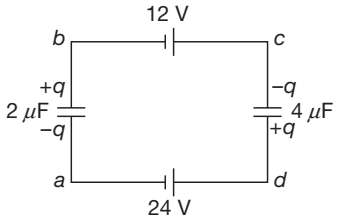
when c and h an shorted, $R_{ch} = 0$

So, (D) → (p)

13. A → (q)
B → (p)
C → (r)
D → (s)

Apply Kirchhoff's Law

$$V_a = \frac{q}{2} + 12 + \frac{q}{4} - 24 = V_a$$



$$\frac{3}{4}q = 12$$

$$\Rightarrow \quad q = 16\ \mu C$$

$$V_a + \frac{16}{2} = V_b$$

$$\Rightarrow \quad V_a - V_b = -8V$$

Similarly $V_d - V_c = -4$ V

Work done by cell 12 V is $-12 \times 16 = 192\ \mu J$

Work done by cell 24 V is $24 \times 16 = 384\ \mu J$

Integer/Numerical Answer Type Questions

1. Since, $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminium, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27 g. Thus, the mass per atom is

$$\frac{27\text{ g}}{N_A} = \frac{27\text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23}\ \text{gatom}^{-1}$$

Thus, $n = \dfrac{\text{density of aluminium}}{\text{mass per atom}} = \dfrac{2.7\ \text{gcm}^{-3}}{4.49 \times 10^{-23}\ \text{gatom}^{-1}}$

$$\Rightarrow \quad n = 6.02 \times 10^{22}\ \text{atomscm}^{-3} = 6.02 \times 10^{28}\ \text{atomsm}^{-3}$$

Therefore,

$$v_d = \frac{I}{nqA} = \frac{5\text{ A}}{(6.02 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(4 \times 10^{-6}\ \text{m}^2)}$$

$$\Rightarrow \quad v_d = 1.3 \times 10^{-4}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_d = 130 \times 10^{-6}\ \text{ms}^{-1} = 130\ \mu\text{ms}^{-1}$$

2. $R = R_c + R_n = R_c\left[1 + \alpha_c(T - T_0)\right] + R_n\left[1 + \alpha_n(T - T_0)\right]$

$$0 = R_c\alpha_c(T - T_0) + R_n\alpha_n(T - T_0)$$

$$\Rightarrow \quad R_c = -R_n\frac{\alpha_n}{\alpha_c}$$

So, $R = R_c + R_n = -R_n\dfrac{\alpha_n}{\alpha_c} + R_n$

$$\Rightarrow \quad R_n = R\left(1 - \frac{\alpha_n}{\alpha_c}\right)^{-1}$$

$$\Rightarrow \quad R_c = R\left(1 - \frac{\alpha_c}{\alpha_n}\right)^{-1}$$

$$\Rightarrow \quad R_n = 10\ \text{k}\Omega\left[1 - \frac{(0.4 \times 10^{-3}\ ^\circ\text{C}^{-1})}{(-0.5 \times 10^{-3}\ ^\circ\text{C}^{-1})}\right]^{-1}$$

$$\Rightarrow \quad R_n \cong 5560\ \Omega \text{ and } R_c = 4440\ \Omega$$

3. The drift velocity is $I = nqv_dA = nqv_d(\pi r^2)$

$$\Rightarrow v_d = \frac{I}{nq\pi r^2} = \frac{1000\text{ A}}{8.49 \times 10^{28}\ \text{m}^{-3}(1.6 \times 10^{-19}\ \text{C})\pi(10^{-2}\ \text{m})^2}$$

$$\Rightarrow \quad v_d = 2.34 \times 10^{-4}\ \text{ms}^{-1}$$

Since $v = \dfrac{x}{t}$

$$\Rightarrow \quad t = \frac{x}{v} = \frac{200 \times 10^3\ \text{m}}{2.34 \times 10^{-4}\ \text{ms}^{-1}} = 8.54 \times 10^8\ \text{s} = 27\ \text{yr}$$

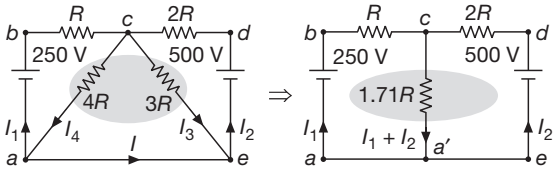
4. $R = R_0\left[1 + \alpha(T - T_0)\right]$

$$\Rightarrow \quad T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0} - 1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I} - 1\right]$$

In this case, $I = \dfrac{I_0}{10}$

$$\Rightarrow \quad T = T_0 + \frac{1}{\alpha}(9) = 20^\circ + \frac{9}{0.00450\ ^\circ\text{C}^{-1}} = 2020\ ^\circ\text{C}$$

5. Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure, we get the resistance in the middle branch as (1.71) R.



Apply Kirchhoff's Loop Rule to both loops in figure (b) to obtain

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $(1.71R)I_1 + (3.71R)I_2 = 500$

With $R = 1000\ \Omega$, simultaneous solution of these equations yields

$$I_1 = 10\text{ mA}$$

and $I_2 = 130$ mA

From figure (b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240$ V

CHAPTER 3

Thus, from figure (a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\text{ V}}{4000\ \Omega} = 60\text{ mA}$

Finally, Applying Kirchhoff's Point Rule at point a in figure (a) gives :

$I = I_4 - I_1 = 60\text{ mA} - 10\text{ mA} = +50\text{ mA},$

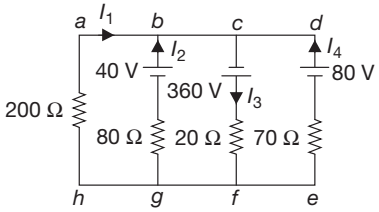
$\Rightarrow$ $I = 50\text{ mA}$ from point a to point e.

6. Name the currents as shown in the figure. Then $I_1 + I_2 + I_4 = I_3$. Loop equations are

Loop $abgha$ $\quad -200I_1 - 40 + 80I_2 = 0$

Loop $bcfgb$ $\quad -80I_2 + 40 + 360 - 20I_3 = 0$

Loop $cfedc$ $\quad +360 - 20I_3 - 70I_4 + 80 = 0$



Eliminate I_3 by substitution

$$\begin{cases} I_2 = 2.5I_1 + 0.5 \\ 400 - 100I_2 - 20I_1 - 200I_4 = 0 \\ 440 - 20I_1 - 20I_2 - 90I_4 = 0 \end{cases}$$

Eliminate I_2 . $\begin{cases} 350 - 270I_1 - 20I_4 = 0 \\ 430 - 70I_1 - 90I_4 = 0 \end{cases}$

Eliminate $I_4 = 17.5 - 13.5I_1$ to obtain

$430 - 70I_1 - 1575 + 1215I_1 = 0$

$I_1 = \dfrac{70}{70} = 1\text{ A}$ upward in $200\ \Omega$

Now $I_4 = 4\text{ A}$ upward in $70\ \Omega$

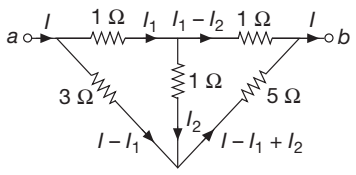
$I_2 = 3\text{ A}$ upward in $80\ \Omega$

$I_3 = 8\text{ A}$ downward in $20\ \Omega$

and for the $200\ \Omega$, $\Delta V = IR = (1\text{ A})(200\ \Omega) = 200\text{ V}$

7. $\Delta V_{ab} = (1)I_1 + (1)(I_1 - I_2)$

$\Delta V_{ab} = (1)I_1 + (1)I_2 + (5)(I_1 - I_1 + I_2)$



$\Delta V_{ab} = (3)(I - I_1) + (5)(I - I_1 + I_2)$

Let $I = 1\text{ A}$, $I_1 = x$, and $I_2 = y$

Then, the three equations become

$\Delta V_{ab} = 2x - y\ ,$

$\Rightarrow$ $y = 2x - \Delta V_{ab}$

$\Delta V_{ab} = -4x + 6y + 5$

$\Rightarrow$ $\Delta V_{ab} = 8 - 8x + 5y$

Substituting the first into the last two gives

$7\ \Delta V_{ab} = 8x + 5$

$\Rightarrow$ $6\ \Delta V_{ab} = 2x + 8$

Solving these simultaneously yields $\Delta V_{ab} = \dfrac{27}{17}\text{ V}$

Then, $R_{ab} = \dfrac{\Delta V_{ab}}{I} = \dfrac{\frac{27}{17}\text{ V}}{1\text{ A}}$

$\Rightarrow$ $R_{ab} = \dfrac{27}{17}\ \Omega = 1 + \dfrac{x}{17}$

$\Rightarrow$ $x = 10$

8. (a) $RC = (1\times10^6\ \Omega)(5\times10^{-6}\text{ F}) = 5\text{ s}$

(b) $Q = CE = (5\times10^{-6}\text{ C})(30\text{ V}) = 150\ \mu\text{C}$

(c) $I(t) = \dfrac{E}{R}e^{-\frac{t}{RC}} = \left(\dfrac{30}{1\times10^6}\right)\exp\left[\dfrac{-10}{(1\times10^6)(5\times10^{-6})}\right]$

$= 4.06\ \mu\text{A} \simeq 4\ \mu\text{A}$

9. $q(t) = Q\left[1 - e^{-\frac{t}{RC}}\right]$

$\Rightarrow$ $\dfrac{q(t)}{Q} = 1 - e^{-\frac{t}{RC}}$

$\Rightarrow$ $0.6 = 1 - e^{-\frac{0.9}{RC}}$

$\Rightarrow$ $e^{-\frac{0.9}{RC}} = 1 - 0.6 = 0.4$

$\Rightarrow$ $-\dfrac{0.9}{RC} = \log_e(0.4)$

$\Rightarrow$ $RC = \dfrac{-0.9}{\log_e(0.4)} = 0.982\text{ s} \simeq 1\text{ s}$

10. The potential difference across the capacitor

$\Delta V(t) = \Delta V_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$

$\Rightarrow$ $4\text{ V} = (10\text{ V})\left[1 - e^{\frac{-(3\text{ s})}{\left[R(10\times10^{-6}\ \text{s}\Omega^{-1})\right]}}\right]$

Therefore, $0.4 = 1 - e^{-\frac{(3\times10^5\ \Omega)}{R}}$

$\Rightarrow$ $e^{-\frac{(3\times10^5\ \Omega)}{R}} = 0.6$

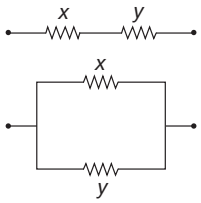
Taking the natural logarithm of both sides,

$$-\frac{3\times10^5\ \Omega}{R}=\log_e(0.6)$$

$$\Rightarrow\quad R=-\frac{3\times10^5\ \Omega}{\log_e(0.6)}=+\,5.87\times10^5\ \Omega=587\ \text{k}\Omega$$

11. Let the two resistances be x and y.

Then, $R_s=x+y=\dfrac{P_s}{I^2}=\dfrac{225\ \text{W}}{(5\ \text{A})^2}=9\ \Omega$



$$y=9\ \Omega-x$$

$$\Rightarrow\quad R_P=\frac{xy}{x+y}=\frac{P_p}{I^2}=\frac{50\ \text{W}}{(5\ \text{A})^2}=2\ \Omega$$

$$\Rightarrow\quad \frac{x\ (9\ \Omega-\text{x})}{x+(9\ \Omega-\text{x})}=2\ \Omega$$

$$x^2-9x+18=0$$

Factoring the second equation, $(x-6)(x-3)=0$

$$\Rightarrow\quad x=6\ \Omega$$

$$\Rightarrow\quad x=3\ \Omega$$

Then, $y=9\ \Omega-x$ gives $y=3\ \Omega$

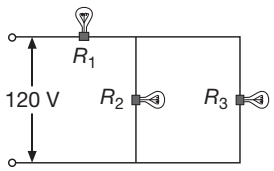
$$\Rightarrow\quad y=6\ \Omega$$

The two resistances are found to be $6\ \Omega$ and $3\ \Omega$

12. (a) First determine the resistance of each light bulb:

$$P=\frac{(\Delta V)^2}{R}$$

$$R=\frac{(\Delta V)^2}{P}=\frac{(120\ \text{V})^2}{60\ \text{W}}=240\ \Omega$$



We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances

$$R_{eq}=R_1+\frac{1}{\left(\frac{1}{R_2}\right)+\left(\frac{1}{R_3}\right)}=240\ \Omega+120\ \Omega=360\ \Omega$$

The total power dissipated in the $360\ \Omega$ is

$$P=\frac{(\Delta V)^2}{R_{eq}}=\frac{(120\ \text{V})^2}{360\ \Omega}=40\ \text{W}$$

(b) The current through the network is given by $P=I^2R_{eq}$

$$\Rightarrow\quad I=\sqrt{\frac{P}{R_{eq}}}=\sqrt{\frac{40\ \text{W}}{360\ \Omega}}=\frac{1}{3}\ \text{A}$$

The potential difference across R_1 is

$$\Delta V_1=IR_1=\left(\frac{1}{3}\ \text{A}\right)(240\ \Omega)=80\ \text{V}$$

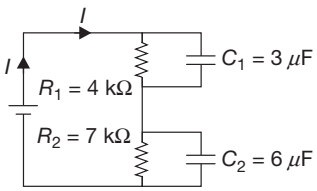
The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23}=IR_{23}=\left(\frac{1}{3}\ \text{A}\right)\left(\frac{1}{\frac{1}{240\ \Omega}+\frac{1}{240\ \Omega}}\right)=40\ \text{V}$$

13. (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

$$P=I^2R_2$$

$$\Rightarrow\quad I=\sqrt{\frac{P}{R_2}}=\sqrt{\frac{2.4\ \text{VA}}{7000\ \frac{V}{A}}}=18.5\ \text{mA}$$



The potential difference across R_1 and C_1 is

$$\Delta V=IR_1=(1.85\times10^{-2}\ \text{A})\left(4000\ \frac{V}{A}\right)=74.1\ \text{V}$$

The charge on C_1 is

$$Q_1=C_1\,\Delta V=(3\times10^{-6}\ \text{CV}^{-1})(74.1\ \text{V})=222\ \mu\text{C}$$

The potential difference across R_2 and C_2 is

$$\Delta V=IR_2=(1.85\times10^{-2}\ \text{A})(7000\ \Omega)=130\ \text{V}$$

The charge on C_2 is

$$Q_2=C_2\,\Delta V=\left(6\times10^{-6}\ \frac{C}{V}\right)(130\ \text{V})=778\ \mu\text{C}$$

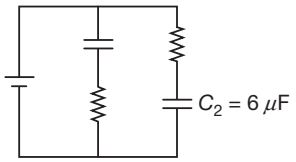
The battery emf is given by

$$IR_{eq}=I(R_1+R_2)$$

$$\Rightarrow\quad IR_{eq}=1.85\times10^{-2}(4000+7000)=204\ \text{V}$$

(b) In equilibrium, after the switch has been opened for a long time, no current can flow in the circuit. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge C_2 is

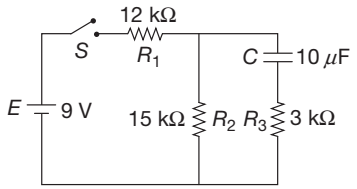
$$Q_2' = C_2\,\Delta V = (6\times10^{-6}\ \text{CV}^{-1})(204\ \text{V}) = 1222\ \mu\text{C}$$



Change in the value of charge on Q_2 is given by

$$\Delta Q_2 = Q_2' - Q_2 = 1222\ \mu\text{C} - 778\ \mu\text{C} = 444\ \mu\text{C}$$

14. (a) After steady state conditions have been reached, there is no DC current through the capacitor
Thus, for R_3, we have $I_{R_3} = 0$ (steady state)



For the other two resistors, the steady state current is simply determined by the 9 V emf across the 12 kΩ and 15 kΩ resistors in series
For R_1 and R_2, we have

$$I_{(R_1+R_2)} = \frac{E}{R_1+R_2} = \frac{9\ \text{V}}{(12\ \text{k}\Omega + 15\ \text{k}\Omega)} = 333\ \mu\text{A}$$

(steady state)

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10\ \mu\text{F})(333\ \mu\text{A})(15\ \text{k}\Omega)$$

$$\Rightarrow \quad Q = 50\ \mu\text{C}$$

(c) When the switch is opened, the branch containing R_1 is no longer a part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of

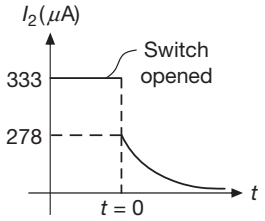
$$(R_2+R_3)C = (15\ \text{k}\Omega + 3\ \text{k}\Omega)(10\ \mu\text{F}) = 0.18\ \text{s}$$

The initial current I_0 in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series. So,

$$I_0 = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)}$$

$$\Rightarrow \quad I_0 = \frac{(333\ \mu\text{A})(15\ \text{k}\Omega)}{(15\ \text{k}\Omega + 3\ \text{k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph.



Afterwards, it decays according to

$$I = I_0 e^{-t/(R_2+R_3)C} = (278\ \mu\text{A})e^{-t/(0.18\ \text{s})}\ (\text{for } t > 0)$$

(d) The charge q on the capacitor decays from Q_0 to $\frac{Q_0}{5}$ according to

$$q = Q_0 e^{-t/(R_2+R_3)C}$$

$$\Rightarrow \quad \frac{Q_0}{5} = Q_0 e^{(-t/0.18\ \text{s})}$$

$$\Rightarrow \quad 5 = e^{t/0.18\ \text{s}}$$

$$\Rightarrow \quad \log_e 5 = \frac{t}{180\ \text{ms}}$$

$$\Rightarrow \quad t = (0.18\ \text{s})(\log_e 5) = 290\ \text{ms}$$

15. Since α can vary at different temperatures, so it is advisable to make a rough estimate

$$R = R_{20}(1 + \alpha\Delta T)$$

$$\Rightarrow \quad \Delta T = \frac{R - R_{20}}{\alpha R_{20}}$$

$$\Rightarrow \quad T - 20\ ^\circ\text{C} = \frac{(190-15)}{(4.5\times10^{-3})(15)} = 2592\ ^\circ\text{C}$$

$$\Rightarrow \quad T = 2612\ ^\circ\text{C}$$

$$\Rightarrow \quad T = 2612 + 273 = 2885\ \text{K}$$

16. Let R_x and R_y be the respective resistances at a given temperature. The total resistance after a charge ΔT becomes

$$R = R_x(1 + 0.0025\,\Delta T) + R_y(1 + 0.00075\,\Delta T) \quad \ldots(1)$$

As $0.001(°C)^{-1}$ is the temperature coefficient of the combination, we also have

$$R = (R_x + R_y)(1 + 0.001\,\Delta T) \quad \ldots(2)$$

From equations (1) and (2), we get

$$R_x(1 + 0.0025\,\Delta T) + R_y(1 + 0.00075\,\Delta T)$$
$$= (R_x + R_y)(1 + 0.001\,\Delta T)$$
$$\Rightarrow \quad R_x(0.0015\,\Delta T) = R_y(0.00025\,\Delta T)$$

Thus we get

$$R_x = \frac{5}{3}R_y \quad \ldots(3)$$

Also, $R_x + R_y = 1000\ \Omega$ $\ldots(4)$

On solving equations (3) and (4), we get

$$R_x = 625\ \Omega \text{ and } R_y = 375\ \Omega$$

The respective lengths are

$$L_x = \frac{1}{100} \times 625 = 6.25 \text{ km} = 6250 \text{ m}$$

and $L_y = \frac{1}{50} \times 375 = 7.5 \text{ km} = 7500 \text{ m}$

17. The power consumed (P) in time t is given by

$$P = \frac{dU}{dt} = \frac{V^2}{R(t)}$$
$$\Rightarrow \quad dU = \frac{V^2}{R(t)}dt$$
$$\Rightarrow \quad U = \int_0^t \frac{V^2}{R(t)}dt$$
$$\Rightarrow \quad U = \frac{(110)^2}{0.5}\int_0^t e^{-2t}dt$$
$$\Rightarrow \quad U = \frac{(110)^2}{2 \times 0.5}\left(e^{-2t}\right)_0^t$$
$$\Rightarrow \quad U = (110)^2(1 - e^{-2t})\ \text{J}$$

According to problem,

$$U = 7644\ \text{J}$$
$$\Rightarrow \quad 1 - e^{-2t} = \frac{7644}{(110)^2} = 0.632$$
$$\Rightarrow \quad e^{-2t} = 0.367$$
$$\Rightarrow \quad -2t \log_e e = \log_e 0.367$$
$$\Rightarrow \quad -2t = -1$$
$$\Rightarrow \quad t = 0.5\ \text{s}$$
$$\Rightarrow \quad t = 500\ \text{ms}$$

18. Heat produced in a resistance R in time t is

$$H = Pt = \frac{V^2}{R}\,t$$

For coil 1,

$$H_1 = \frac{V^2}{R_1}(15 \times 60) \quad \ldots(1)$$

For coil 2,

$$H_2 = \frac{V^2}{R_2}(30 \times 60) \quad \ldots(2)$$

But according to given problem $H_1 = H_2 = H$ i.e.,

$$\frac{15}{R_1} = \frac{30}{R_2}$$
$$\Rightarrow \quad R_2 = 2R_1 \quad \ldots(3)$$

(a) When both the coils are used in series, then

$$H_s = \frac{V^2}{(R_1 + R_2)}t_s = \frac{V^2}{3R_1} \times t_s \quad \{\because\ R_2 = 2R_1\}$$

But as here, $H_s = H_1 = H_2 = H$

$$\Rightarrow \quad \frac{V^2}{R_1}(15 \times 60) = \frac{V^2}{3R_1}t_s$$
$$\Rightarrow \quad t_s = (45 \times 60)\text{s} = 45 \text{ min}$$

(b) When both the coils are used in parallel, then

$$H_P = H = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right)t_p = \left(\frac{3V^2}{2R_1}\right)t_p \ \{\because\ R_2 = 2R_1\}$$
$$\Rightarrow \quad \frac{3V^2}{2R_1} \times t_p = \frac{V^2}{R_1} \times (15 \times 30)$$
$$\Rightarrow \quad t_p = (10 \times 60)\text{s} = 10 \text{ min}$$

19. Resistance of the wire at 300 K is

$$R_0 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\pi^2 \times 10^{-8} \times 1}{\pi \times (10^{-3})^2} = \pi \times 10^{-2}\ \Omega$$

Resistance at 900 K is

$$R_t = R_0(1 + \alpha t)$$
$$\Rightarrow \quad R_t = \pi \times 10^{-2}\left[1 + 7.8 \times 10^{-3} \times (900 - 300)\right]$$
$$\Rightarrow \quad R_t = 5.68\ \pi \times 10^{-2}\ \Omega$$

The rate at which the wire radiates energy is

$$W = P = I^2R_t = 5.68\pi \times 10^{-2}I^2 \text{ watt} \quad \ldots(1)$$

The rate at which a black body radiates energy is given by Stefan's Law

$$W = \sigma A_s\left(T^4 - T_0^4\right)$$

where A_s = surface area of the wire = $(2\pi r)L$, $T = 900$ K, $T_0 = 300$ K and $\sigma = 5.68 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$

Thus $W = 5.68\times10^{-8}\times\left(2\pi\times10^{-3}\right)\times1\left[(900)^4-(300)^4\right]$

$\Rightarrow \quad W = 5.68\times12.96\ \pi$ watt ...(2)

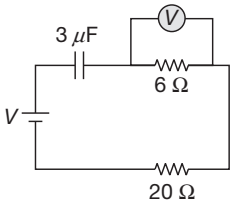
Equating (1) and (2), we get

$5.68\ \pi\times10^{-2}\times I^2 = 5.68\times12.96\ \pi$

$\Rightarrow \quad I^2 = 1296$

$\Rightarrow \quad I = 36$ A

20. (a) Since, a capacitor is an open circuit to DC in steady state. The effective circuit through which current will pass initially after closing switch S_1 is shown. When steady state is reached the capacitor will not allow current. Thus there is no current from battery and hence the reading of voltmeter is zero.



(b) Without current the resistors act as conducting wires. Hence potential difference across capacitor is 12 V .

Charge on capacitor, 3 μF, is

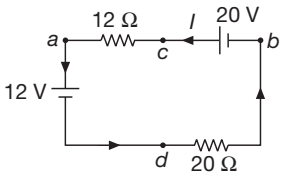
$Q = (3\times12)\,\mu\text{C} = 36\ \mu\text{C}$

(c) When all the switches are closed, the branch containing capacitor will not allow current after steady state is reached. The equivalent circuit from point of view of current is redrawn. Let the current in the single loop circuit be I. We begin at left lower corner and traverse the circuit anticlockwise while applying KVL. We get

$-20I + 20 - 12I - 12 = 0$

$\Rightarrow \quad I = \dfrac{8}{32}$

$\Rightarrow \quad I = \dfrac{1}{4}$ A



In the steady state power is consumed only in 12 Ω and 20 Ω resistors. All the resistors in a single loop circuit are in series, thus power consumed is

$$P = I^2R_{\text{eq}} = \left(\frac{1}{4}\right)^2(32) = 2\ \text{W}$$

(d) The capacitor is connected between points c and d. In order to determine the potential difference across it we begin at point c and traverse along any path to d, to get

$V_c - 12I - 12 = V_d$

$V_c - V_d = 12I + 12 = 3 + 12 = 15$ V

Charge on 2 μF capacitor $= (15)\times2 = 30\ \mu$C

21. (a) If there are N resistors of resistance R each, connected in parallel, the net effective R_e of such a combination is given by

$$\frac{1}{R_e} = \frac{1}{R} + \frac{1}{R} + \ldots N \text{ terms}$$

$$\Rightarrow \quad R_e = \frac{R}{N}$$

Now, this combination is connected to a source of emf E and internal resistance r. Hence current in the main circuit would be

$$I = \frac{E}{r + R_e}$$

This current will be equally divided between all the N resistors (so that potential drop across each resistor is same). Hence power generated in any single resistor will be

$$P = \left(\frac{I}{N}\right)^2 R = \frac{I^2R}{N^2} = \frac{E^2R}{N^2\left(r + \dfrac{R}{N}\right)^2}$$

$$\Rightarrow \quad P = \frac{(200)^2(300)}{(200)^2\left(0.5 + \dfrac{300}{200}\right)^2} = \frac{(200)^2(300)}{(200)^2(2)^2}$$

$$\Rightarrow \quad P = \frac{300}{4} = 75\ \text{W}$$

(b) In the next stage, one of the bulb burns out, i.e., now only $(N-1)$ resistors are left out. Power generated by any single resistor can be obtained by just replacing N by $(N-1)$ in the previous equation. So, we get

$$P' = \frac{E^2}{(N-1)^2}\frac{R}{\left(r + \dfrac{R}{N-1}\right)^2} = \frac{E^2R}{(N-1)^2\left(r + \dfrac{R}{N-1}\right)^2}$$

The relative change f, by definition, is given by

$$f = \frac{P' - P}{P}$$

$$\Rightarrow \quad f = \left[\frac{(rN+R)^2}{(r(N-1)+R)^2} - 1\right]$$

$$\Rightarrow \quad f=\frac{1}{\left(1-\frac{r}{rN+R}\right)^2}-1$$

Now, $(Nr+R)\gg r$, since there are many bulbs $(N=200)$. Hence, we can approximate,

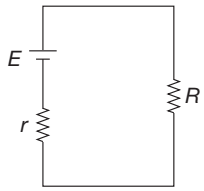
$$\left(1-\frac{r}{rN+R}\right)^{-2}\approx\left(1+\frac{2r}{rN+R}\right)$$

Substituting $r=0.5\ \Omega$, $R=300\ \Omega$ and $N=200$, we get

$$f=\frac{2(0.5)}{(0.5)(200)+300}=\frac{1}{400}=0.0025$$

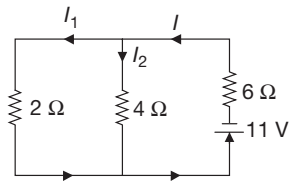
$$\Rightarrow \quad 10000f=25$$

22. Suppose, we insert the battery with $2\ \Omega$ resistance. Then we can take $2\ \Omega$ as the internal resistance (r) of the battery and combined resistance of the other two as the external resistance (R). The circuit in that case shown in figure.



Now power, $P=\frac{E^2}{R+r}$

This power will be minimum where $(R+r)$ is maximum and we observe that $(R+r)$ will be maximum only when the battery is inserted with $6\ \Omega$ resistance as shown in figure.



Net resistance in this case is

$$R_{eq}=6+\frac{2\times4}{2+4}=\frac{22}{3}\ \Omega$$

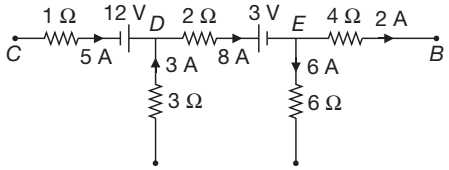
$$\Rightarrow \quad I=\frac{11}{22/3}=1.5\ \text{A}$$

This current will be distributed in $2\ \Omega$ and $4\ \Omega$ in the inverse ratio of their resistances.

$$\Rightarrow \quad \frac{I_1}{I_2}=\frac{4}{2}=2$$

$$\Rightarrow \quad I_1=\left(\frac{2}{2+1}\right)(1.5)=1\ \text{A}$$

23. Applying Kirchhoff's Junction Law at E current in wire DE is 8 A from D to E. Now further applying Junction Law at D. The current in $3\ \Omega$ resistance will be 3 A towards D.



Power dissipated in $3\ \Omega$ resistance

$$=I^2R=(3)^2(3)=27\ \text{W}$$

For path $CDEB$, we have

$$V_C-(5)(1)+12-(8)(2)-3-(4)(2)=V_B$$

$$\Rightarrow \quad V_C-V_B=5-12+16+3+8$$

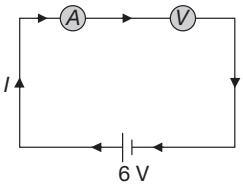
$$\Rightarrow \quad V_C-V_B=20\ \text{V}$$

24. Let R_1 = resistance of ammeter and R_2 = combined resistance of ammeter and voltmeter

In the first case, current in the circuit is given by

$$I=\frac{6}{R_2} \qquad \ldots(1)$$

and voltage across voltmeter is given by



$$V=6-(\text{voltage across ammeter})$$

$$\Rightarrow \quad V=6-IR_1$$

$$\Rightarrow \quad V=6-\frac{6R_1}{R_2} \qquad \ldots(2)$$

In the second case reading of ammeter becomes two times, i.e., the total resistance becomes half whereas the resistance of ammeter remains unchanged. Hence,

$$I'=\frac{6}{R_2/2}=\frac{12}{R_2} \qquad \ldots(3)$$

and $V'=6-(I')R_1$

$$\Rightarrow \quad V'=6-\frac{12R_1}{R_2} \qquad \ldots(4)$$

Further, it is given that $V'=\frac{V}{2}$

CHAPTER 3

$$\Rightarrow \quad 6-\frac{12R_1}{R_2}=3-\frac{3R_1}{R_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{1}{3}$$

Substituting this value in equation (4), we have

$$V'=6-(12)\left(\frac{1}{3}\right)$$

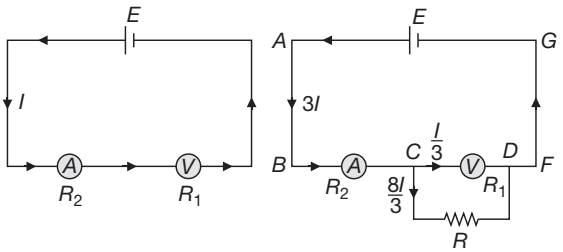
$$\Rightarrow \quad V'=2\text{ V}$$

25. Let E be the emf of the battery

In the first case let I be the current in the circuit, then

$$E=I(R_1+R_2) \qquad \ldots(1)$$

In the second case main current increases three times while current through voltmeter will reduce to $\frac{I}{3}$. Hence, the remaining $3I-\frac{I}{3}=\frac{8I}{3}$ passes through R as shown in figure.



$$V_C-V_D=\left(\frac{I}{3}\right)R_1=\left(\frac{8I}{3}\right)R$$

$$\Rightarrow \quad R_1=8R=8(3\ \Omega)=24\ \Omega$$

Applying Kirchhoff's Second Law in loop $ABFGA$,

$$E=3I(R_2)+\left(\frac{I}{3}\right)(R_1)=I\left(3R_2+\frac{R_1}{3}\right) \qquad \ldots(2)$$

From equations (1) and (2),

$$R_1+R_2=3R_2+\frac{R_1}{3}$$

$$\Rightarrow \quad 2R_2=\frac{2R_1}{3}$$

$$\Rightarrow \quad R_2=\frac{R_1}{3}$$

$$\Rightarrow \quad R_2=\frac{8R}{3}=\frac{8}{3}(3\ \Omega)=8\ \Omega$$

26. The voltage supplied by the charging plant is here constant which is equal to,

$$V=E_i+I_ir=(90)+(10)(2)$$

$$V=110\text{ V}$$

Let I_f be the current at the end of charging process. Then,

$$V=E_f+I_fr$$

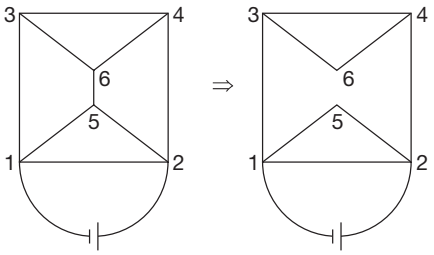
$$\Rightarrow \quad I_f=\frac{V-E_f}{r}=\frac{110-100}{2}$$

$$\Rightarrow \quad I_f=5\text{ A}$$

27. Let us represent the central junction of wires in the form of two junctions connected by the wire 5-6 as shown in figure. Then it follows from symmetry that there is no current through this wire. Therefore, the central junction can be removed from the initial circuit. Further,

$$R_{12}=R_{13}=R_{34}=R_{24}=r$$

and $R_{15}=R_{25}=R_{36}=R_{46}=\frac{r}{\sqrt{2}}$



Let V be the voltage between 1 and 2. Then the amount of heat liberated in conductor 1-2.

$$H_{12}=\frac{V^2}{r} \qquad \ldots(1)$$

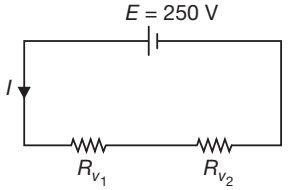
Current through 3-4, $I_{3-4}=\frac{V}{r(\sqrt{2}+3)}$

$$\Rightarrow \quad H_{3-4}=I_{3-4}^2r=\frac{V^2}{r(\sqrt{2}+3)^2}$$

$$\Rightarrow \quad \frac{H_{1-2}}{H_{3-4}}=(\sqrt{2}+3)^2=11+6\sqrt{2}$$

Comparing with $11x+y\sqrt{2x}$, we get $x=1,\ y=6$

28. (a) When the switch is open, no current flows through the branch that contains R_1 and R_2. Hence



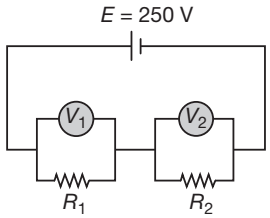
$$I = \frac{250}{10000} = \frac{1}{40} \text{ A}$$

According to Ohm's Law

$$V_1 = \frac{1}{40}(6000) = 150 \text{ V} \quad \text{and}$$

$$V_2 = (250 - 150) \text{ V} = 100 \text{ V}$$

(b) When the switch is closed the equivalent circuit is shown.



Since we observe that V_1 and R_1 are in parallel and V_2 and R_2 are in parallel and both in series and also simultaneously we have

$$\frac{R_1(R_{V_1})}{R_1 + R_{V_1}} = \frac{R_2(R_{V_2})}{R_2 + R_{V_2}} = 2.4 \text{ k}\Omega$$

Hence the potential of 250 V must be equally divided. So, reading of both the voltmeters is

$$V_1 = V_2 = \frac{E}{2} = 125 \text{ V}$$

29. Let N be the total number of cells required to be grouped in m rows, each row carrying n cells, then $N = mn$

We have, $I = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr} = \dfrac{NE}{mR + nr}$

$$\Rightarrow \quad 8 = \frac{2N}{5m + \dfrac{0.5N}{m}}$$

$$\Rightarrow \quad 20m^2 + 2N = Nm \qquad \ldots(1)$$

For least value of N, $\dfrac{dN}{dm} = 0$,

Differentiating equation (1), we have

$$40m + 2\frac{dN}{dm} = m\frac{dN}{dm} + N$$

$$\Rightarrow \quad N = 40 \text{ m} \qquad \left\{\because \frac{dN}{dm} = 0\right\} \qquad \ldots(2)$$

But $N = mn$ $\ldots(3)$

Solving equations (1), (2) and (3), we get

$n = 40$ and $m = 4$

$$\Rightarrow \quad N = mn = 160$$

30. Before connecting B with D and C with E,

$$R_{AF} = 5R = 5(2\ \Omega) = 10\ \Omega$$

After connecting B with D and C with E, a balanced Wheatstone bridge is formed between B and E.

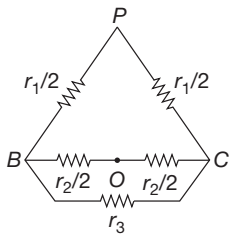
So, $R_{BE} = R$

and $R'_{AF} = 3R = 3(2\ \Omega) = 6\ \Omega$

$$\Rightarrow \quad \Delta R = 10 - 6 = 4\ \Omega$$

i.e., the new resistance decreases by $4\ \Omega$

31. Points (A and C) and (D and B) are symmetrically located with respect to points O and P. Hence, the circuit can be drawn as shown in figure.



This is a balanced Wheatstone bridge between P and O Hence, r_3 can be removed. And,

$$R_{PO} = \frac{r_1 + r_2}{4}$$

Here $r_1 = R_{PB} = R_{PD} = \dfrac{(\pi a)\lambda}{2}$

and $r_2 = R_{OB} = (a)\lambda$

$$\Rightarrow \quad R_{PO} = \frac{(2+\pi)a\lambda}{8} = \left(\frac{2+\pi}{8}\right)\cancel{a}\left(\frac{64}{2+\pi}\right)\frac{1}{\cancel{a}} = 8\ \Omega$$

32. $v_d = \dfrac{I}{neA} = \dfrac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$

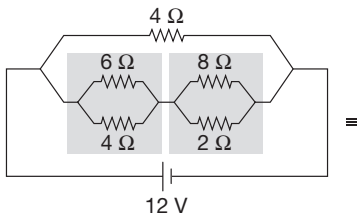
$$\Rightarrow \quad v_d = 0.735 \times 10^{-6} \text{ ms}^{-1}$$

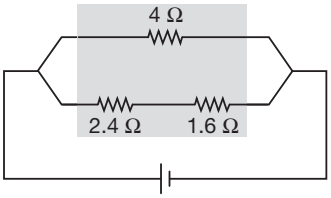
$$\Rightarrow \quad v_d = 0.735\ \mu\text{ms}^{-1}$$

Since, $t = \dfrac{\ell}{v_d} = \dfrac{10^4}{0.735 \times 10^{-6} \times 365 \times 24 \times 3600}$ year

$$\Rightarrow \quad t = 431 \text{ year}$$

33. The equivalent circuit is as shown in figure.





$$\Rightarrow \quad R_{\text{series}} = \frac{6\times 4}{6+2} + \frac{8\times 2}{8+2} = 4\ \Omega$$

$$R_{\text{total}} = \frac{4(2.4+1.6)}{4+2.4+1.6} = \frac{(4)(4)}{4+4} = 2\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{12}{2} = 6\ \text{A}$$

34. (a) Current in the circuit,

$$I = \frac{\text{net emf}}{\text{net resistance}} = \frac{6+12}{1+2+3} = 3\ \text{A}$$

Now, $V_G = 0$ V

$$V_A = 12\ \text{V},\ V_B = 12 - 1\times 3 = 9\ \text{V},$$
$$V_C = 12 - 3\times 3 = 3\ \text{V}$$

and $V_D = 12 - 3\times 6 = -6$ V

(b) $I = \dfrac{12-6}{1+2+3} = 1$ A

$V_G = 0$

$\Rightarrow \quad V_A = 12$ V, $V_B = 12 - 1\times 1 = 11$ V,

$V_C = 12 - 3\times 1 = 9$ V and $V_D = 12 - 6\times 1 = 6$ V

35. Let n cells be wrongly connected in the battery. Let e.m.f. of each cell be E and R be the total resistance of the circuit

$$(12-n)E - nE = (12-2n)E$$

When the cells aid the battery, the net e.m.f. is

$$E_1 = (12-2n)E + 2E$$

So, current in circuit is

$$I_1 = 3 = \frac{(12-2n)E + 2E}{R} \qquad \ldots(1)$$

When the cells oppose the battery, the net e.m.f. is

$$E_2 = (12-2n)E - 2E$$

So, current in circuit is

$$I_2 = 2 = \frac{(12-2n)E - 2E}{R} \qquad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{3}{2} = \frac{(12-2n)+2}{(12-2n)-2}$$

Solving, we get $n = 1$

Hence one cell of the battery is wrongly connected.

36. Leakage current I is given by

$$I = \frac{CV}{K\rho\varepsilon_0} = \frac{\left(\dfrac{K\varepsilon_0 A}{d}\right)V}{K\varepsilon_0\rho}$$

$$\Rightarrow \quad I = \frac{AV}{\rho d} = \frac{(10\times 10^{-4})(6)}{(10^{13})(1\times 10^{-3})}$$

$$\Rightarrow \quad I = 6\times 10^{-13}\ \text{A}$$

So, $* = 6$

37. The resistance of a layer of medium of thickness dx at a distance x from the first plate of capacitor is

$$dR = \frac{\rho(x)dx}{A} = \frac{1}{\sigma(x)}\frac{dx}{A}$$

Since σ varies linearly with distance from the first plate, so we have

$$\sigma = \sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{d}\right)x \ \Rightarrow\ dR = \left[\frac{1}{\sigma_1 + \left(\dfrac{\sigma_2-\sigma_1}{d}\right)x}\right]\frac{dx}{A}$$

Hence, total resistance R is given by

$$R = \frac{1}{A}\int_0^d \frac{dx}{\sigma_1 + \left(\dfrac{\sigma_2-\sigma_1}{d}\right)x}$$

$$\Rightarrow \quad R = \frac{d}{A(\sigma_2-\sigma_1)}\log_e\left(\frac{\sigma_2}{\sigma_1}\right)$$

The current I is given by

$$I = \frac{V}{R} = \frac{AV(\sigma_2-\sigma_1)}{d\log_e\left(\dfrac{\sigma_2}{\sigma_1}\right)} = \frac{AV}{d}\left[\frac{\sigma_2-\sigma_1}{\log_e\left(\dfrac{\sigma_2}{\sigma_1}\right)}\right]$$

$$\Rightarrow \quad I = \frac{(230\times 10^{-4})(300)(2-1)\times 10^{-12}}{(2\times 10^{-3})(2.3\log_{10}2)}$$

$$\Rightarrow \quad I = 5\times 10^{-9}\ \text{A} = 5\ \text{nA}$$

38. Resistance of bulb is given by

$$R_0 = \frac{V_0^2}{P} = \frac{100\times 100}{500} = 20\ \Omega$$

Current in bulb is $I = \dfrac{V_0}{R_0} = \dfrac{100}{20} = 5$ A

For same power dissipation in bulb, the current in circuit must be 5 A

When bulb is in a circuit having supply voltage 220 V, the safe resistance of the circuit

$$R' = \frac{V'}{I} = \frac{200}{5} = 40\ \Omega$$

Hence, required resistance in series is

$$R = R' - R_0 = 40 - 20 = 20\ \Omega$$

39. The resistance R of fuse wire of length ℓ, cross sectional area A and resistivity ρ is

$$R = \frac{\rho\ell}{A}$$

Here $\rho = 22\times10^{-6}\ \Omega\text{cm} = 22\times10^{-8}\ \Omega\text{m}$

$$A = 0.2\ \text{mm}^2 = 0.2\times10^{-6}\ \text{m}^2 = 2\times10^{-7}\ \text{m}^2$$

Since, $Q = I^2Rt$ (in joule)

$$Q = I^2\left(\frac{\rho\ell}{A}\right)t \qquad \ldots(1)$$

Again if m is mass of wire, d is its density, c its specific heat and ΔT, the rise of temperature, then

$$Q = mc\Delta T$$

$$\Rightarrow\quad Q = (\text{Volume}\times\text{density})\times c\times(327-20)$$

$$\Rightarrow\quad Q = (A\ell dc)(307) \qquad \ldots(2)$$

From (1) and (2), we get

$$(A\ell d)c\times307 = I^2\left(\frac{\rho\ell}{AJ}\right)t$$

$$\Rightarrow\quad t = \frac{(A^2cd)(307)J}{I^2\rho} \qquad \ldots(3)$$

Here density $d = 11.34\ \text{gcm}^{-3} = 11.34\times10^3\ \text{kgm}^{-3}$, current $I = 20\ \text{A}$, $\rho = 22\times10^{-6}\ \Omega\text{cm} = 22\times10^{-8}\ \Omega\text{m}$ and specific heat $c = 0.032\ \text{calg}^{-1}(^\circ\text{C})^{-1}$

$$\Rightarrow\quad c = (0.032)(4200)\ \text{Jkg}^{-1}(^\circ\text{C})^{-1},$$

$$A = 0.2\ \text{mm}^2 = 2\times10^{-7}\ \text{m}^2$$

Substituting given values in (3), we get

$$t = \frac{(2\times10^{-7})^2\times4.2\times10^3\times11.34\times10^3\times0.032\times307}{(20)^2\times22\times10^{-8}}$$

$$\Rightarrow\quad t = 0.095\ \text{s} = 95\times10^{-3}\ \text{s} = 95\ \text{ms}$$

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1. From Colour Code of Carbon Resistors table, we get

$$200\ \Omega = \text{Red} + \text{Black} + \text{Brown}$$

Since, $\text{Green} \equiv 5$

$$\Rightarrow\quad \text{Green} + \text{Black} + \text{Brown} \equiv 500\ \Omega$$

Hence, the correct answer is (B).

2. $$E_{eq} = \frac{\frac{E_1}{2R_1} + \frac{E_2}{R_2} + \frac{E_3}{2R_1}}{\frac{1}{2R_1} + \frac{1}{R_2} + \frac{1}{2R_1}}$$

$$\Rightarrow\quad E_{eq} = \frac{\frac{2}{2} + \frac{4}{2} + \frac{4}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$$

$$\Rightarrow\quad E_{eq} = \frac{5}{\frac{3}{2}} = \frac{10}{3} = 3.3\ \text{V}$$

Hence, the correct answer is (D).

3. For maximum power consumption in external resistance

Internal resistance = External resistance

$$\Rightarrow\quad R = r$$

Hence, the correct answer is (B).

4. Resistance of potentiometer wire is

$$R_p = 400\times0.01$$

$$\Rightarrow\quad R_p = 4\ \Omega$$

$$\Rightarrow\quad I = \frac{3}{6} = 0.5\ \text{A}$$

Reading of voltmeter is

$$V_{AJ} = IR_{AJ}$$

$$\Rightarrow\quad V_{AJ} = 0.5\times50\times0.01$$

$$\Rightarrow\quad V_{AJ} = 0.25\ \text{V}$$

Hence, the correct answer is (D).

5. Equivalent resistance of the given circuit is

$$R_{eq} = 45 + \frac{10\times50}{10+50} = \left(45 + \frac{50}{6}\right)\Omega = \frac{160}{3}\ \Omega$$

$$\Rightarrow\quad I = \frac{15}{\frac{160}{3}} = \frac{9}{32}\ \text{A}$$

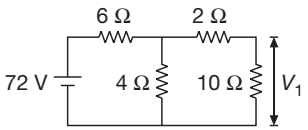
Hence, the correct answer is (B).

CHAPTER 3

6. At steady state, current through capacitor is zero.

$\Rightarrow \quad V_C = V_1$

where, $V_1 = \frac{5}{6} \times V_0 \times \frac{3}{9}$



$\Rightarrow \quad V_1 = \frac{5 \times 72 \times 3}{6 \times 9} = 20 \text{ V}$

Since $Q_1 = CV_1$

$\Rightarrow \quad Q_1 = 200 \ \mu\text{C}$

Hence, the correct answer is (A).

7. Between D and C, the resistance is

$R_1 = \frac{R}{8}$

and between E and C, the resistance is

$R_2 = \frac{7R}{8}$

$\Rightarrow \quad \frac{1}{R_{\text{eq}}} = \frac{8}{R} + \frac{8}{7R}$

$\Rightarrow \quad R_{\text{eq}} = \frac{7R}{64}$

Hence, the correct answer is (D).

8. $V = I_g \left(R_s + R_g \right)$

$\Rightarrow \quad V = 4 \times 10^{-3} [5050]$

$\Rightarrow \quad V \approx 20 \text{ V}$

Hence, the correct answer is (B).

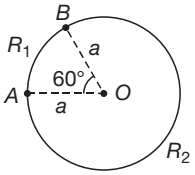
9. $R_0 = \rho \frac{\ell}{A} = 3 \ \Omega$

When wire is stretched to twice its original length, then

$R' = n^2 R_0$

$\Rightarrow \quad R' = (2)^2 (3) = 12 \ \Omega$

We have to find resistance between A and B.



$\angle AOB = 60° = \frac{\pi}{3}$

$\Rightarrow \quad R_1 = \left(\frac{12}{2\pi a} \right) a \left(\frac{\pi}{3} \right)$

$\Rightarrow \quad R_1 = 2 \ \Omega$

and $R_2 = \left(\frac{12}{2\pi a} \right) a \left(\frac{5\pi}{3} \right)$

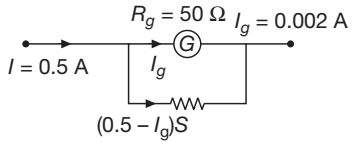
$\Rightarrow \quad R_2 = 10 \ \Omega$

$\Rightarrow \quad \frac{1}{R_{\text{eq}}} = \frac{1}{2} + \frac{1}{10} = \frac{6}{10}$

$\Rightarrow \quad R_{\text{eq}} = \frac{5}{3} \ \Omega$

Hence, the correct answer is (A).

10.



Since, $I_g R_g = \left(0.5 - I_g \right) \text{S}$

$\Rightarrow \quad (0.002)(50) = \left(0.5 - I_g \right) \text{S}$

$\Rightarrow \quad S \approx \frac{0.002 \times 50}{0.5} = 0.2 \ \Omega$

Hence, the correct answer is (A).

11. Since $\rho = \frac{m}{ne^2 \tau}$

$\Rightarrow \quad \rho = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}} \ \Omega \text{ m}$

$\Rightarrow \quad \rho = \frac{9.1}{8.5 \times 2.56 \times 25} \times 10^{(-59+53)}$

$\Rightarrow \quad \rho = 1.67 \times 10^{-8} \ \Omega \text{ m}$

Hence, the correct answer is (D).

12. $\ell n R(T) = a - \frac{a}{b} - \frac{1}{T^2}$

a, b are constant

$\Rightarrow \quad R(T) = R_0 e^{\frac{-T_0^2}{T^2}}$

Hence, the correct answer is (B).

13. $I_g = 10^{-4} \text{ A}$, $R = 2 \times 10^6 \ \Omega$

and $V = 5$ volt

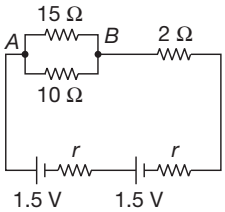
Since $V = I_g (R + G)$

$\Rightarrow \quad 5 = 10^{-4}(2\times10^{6} + G)$

$\Rightarrow \quad G =$ NEGATIVE

Which is NOT POSSIBLE.
Data Contradictory.
*No given option is correct.

14. For the given circuit, it is given that



$V_{AB} = 2\ V$

$\Rightarrow \quad I = \frac{2}{15} + \frac{2}{10} = \frac{1}{3}$ A

Also $I(2r+2) = 1.5 + 1.5 - V_{AB}$

$\Rightarrow \quad 2r + 2 = (3-2)3$

$\Rightarrow \quad r = \frac{1}{2}\ \Omega$

Hence, the correct answer is (A).

15. $\frac{R}{X} = \frac{\ell}{100-\ell}$

$\Rightarrow \quad X = \frac{R(100-\ell)}{\ell}$

For 1, $X = \frac{100\times40}{60} = \frac{2000}{3}\ \Omega = 667\ \Omega$

For 2, $X = \frac{100\times87}{13} = \frac{8700}{13}\ \Omega = 669\ \Omega$

For 3, $X = \frac{10\times98.5}{1.5} = \frac{1970}{3}\ \Omega = 656\ \Omega$

For 4, $X = \frac{1\times99}{1} = 99\ \Omega$

Clearly, we can see that the value of X calculated in CASE-4 is inconsistent with the other values.
Hence, the correct answer is (D).

16. Mobility is $\mu = \frac{v_d}{E}$

and resistivity $\rho = \frac{E}{j} = \frac{EA}{i}$

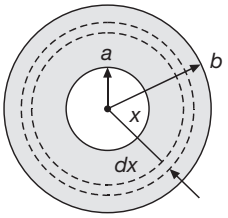
$\Rightarrow \quad \mu = \frac{v_d A}{i\rho}$

$\Rightarrow \quad \mu = \frac{1.1\times10^{-3}\times\pi\times(5\times10^{-3})^2}{5\times1.7\times10^{-8}}$

$\Rightarrow \quad \mu = 1.0\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$

Hence, the correct answer is (D).

17.



Resistance of infinitesimal element is given by

$dR = \frac{\rho dx}{4\pi x^2}$

$\Rightarrow \quad R = \int dR$

$\Rightarrow \quad R = \left(\frac{\rho}{4\pi}\right)\int_a^b \frac{dx}{x^2}$

$\Rightarrow \quad R = \left(\frac{\rho}{4\pi}\right)\left(\frac{1}{a} - \frac{1}{b}\right)$

Hence, the correct answer is (A).

18. $R_{eq} = 2R + R + 4R + R = 8R$

Since, $P = \frac{E^2}{R_{eq}}$

$\Rightarrow \quad \frac{16\times16}{8R} = 4$ watt

$\Rightarrow \quad R = 8\ \Omega$

Hence, the correct answer is (A).

19. **For** R_1,

Since $I_g = 10^{-3}$ Amp

$\Rightarrow \quad 10^{-3}(R_1 + 100) = 2\ \text{V}$

$\Rightarrow \quad R_1 = 1900\ \Omega$

For R_2,

$10^{-3}(R_1 + R_2 + 100) = 10\ \text{V}$

$\Rightarrow \quad R_1 + R_2 + 100 = 10000$

$\Rightarrow \quad R_2 = 8000\ \Omega$

For R_3,

$10^{-3}(R_1 + R_2 + R_3 + 100) = 20\ \text{V}$

$\Rightarrow \quad R_1 + R_2 + R_3 + 100 = 20\times1000$

$\Rightarrow \quad R_3 = 10000\ \Omega$

Hence, the correct answer is (A).

CHAPTER 3

20. $V = \dfrac{ER}{R+r}$

For $R \to \infty$, $V = E = 1.5$ V

For $R = 0$, $I = \dfrac{E}{r} = 1$

$\Rightarrow \quad r = 1.5\ \Omega$

Hence, the correct answer is (A).

21. $V = I_g(R_V + G) = GI_0$ …(1)

Also, $(I_0 - I_g)R_A = I_g G$ …(2)

From (1), we get

$$R_V = \frac{G(I_0 - I_g)}{I_g}$$

From (2), we get

$$R_A = \frac{I_g G}{I_0 - I_g}$$

$\Rightarrow \quad R_A R_V = G^2$

$$\Rightarrow \quad \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$$

Hence, the correct answer is (A).

22. Since $A\ell = \text{Constant}$

and $R = \rho\dfrac{\ell}{A}$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} + \frac{\Delta A}{A}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell} = 2 \times 0.5\%$$

$$\Rightarrow \quad \frac{\Delta R}{R}\% = 1\%$$

Hence, the correct answer is (D).

23.

R	V	O	S
↓	↓	↓	↓
2	7	3	±10%

$\Rightarrow \quad R = 27 \times 10^3 \pm 10\%$

Hence, the correct answer is (C).

24. Applying Kirchhoff's Loop Law, we get

$$20 - 2i_1 - 2(i_1 + i_2) = 0$$

$\Rightarrow \quad 20 - 2i_1 = 10 - 4i_2$ …(1)

Also, $10 - 4i_1 - 2(i_1 + i_2) = 0$

$$\Rightarrow \quad \frac{10 - 2i_1}{4} = i_2$$ …(2)

From (1) and (2), we get

$$20 = 2i_1 + 2i_1 + 2i_2$$

$\Rightarrow \quad 20 = 4i_1 + 5 - i_1$

$\Rightarrow \quad 3i_1 = 15$

$\Rightarrow \quad i_1 = 5$ and $i_2 = 0$

$\Rightarrow \quad i = i_1 + i_2 = 5$ A

Hence, the correct answer is (B).

25. $v_d = \dfrac{I}{neA}$

$$\Rightarrow \quad v_d = \frac{1.5}{(9 \times 10^{28})(1.6 \times 10^{-19})(5 \times 10^{-6})}$$

$\Rightarrow \quad v_d = 2.08 \times 10^{-5}\ \text{ms}^{-1}$

$\Rightarrow \quad v_d = 0.02\ \text{mms}^{-1}$

Hence, the correct answer is (A).

26. G O Y Golden

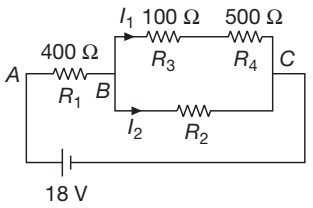
Since $G = 5$, $O = 3$, $Y = 4$ and Golden $= 5\%$

$\Rightarrow \quad R = 53 \times 10^4 \pm 5\%$

$\Rightarrow \quad R = (530\ \text{k}\Omega \pm 5\%)$

Hence, the correct answer is (D).

27.



Since voltmeter is ideal, so

$$I_1 = \frac{V}{R_4} = \frac{5}{500} = 0.01\ \text{A}$$

$\Rightarrow \quad V_B - V_C = 600 I_1 = 6$ V

Since $V_A - V_C = 18$ V

$\Rightarrow \quad (V_A - V_B) + (V_B - V_C) = 18$ V

$\Rightarrow \quad V_A - V_B = 12$ V

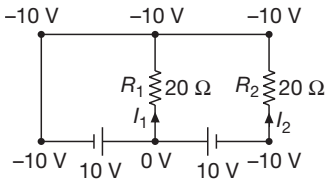
So, $I_1 + I_2 = \dfrac{12}{400} = 0.03$ A

$\Rightarrow \quad I_2 = 0.03 - 0.01 = 0.02$ A

$$\Rightarrow \quad R_2 = \frac{6}{0.02} = 300\ \Omega$$

Hence, the correct answer is (D).

28.



The potentials of various points is shown in figure after assigning 0 V potential to the junction. So,

$$I_1 = \frac{0-(-10)}{20} = \frac{1}{2}\ A = 0.5\ A \text{ and}$$

$$I_2 = \frac{-10-(-10)}{20} = 0\ A$$

Hence, the correct answer is (C).

29. G B R Br

Resistance $= 50 \times 10^2 \pm 1\%$

$$P = I^2R$$

$$\Rightarrow \quad 2 = (5000)I^2$$

$$\Rightarrow \quad \frac{4}{10000} = I^2$$

$$\Rightarrow \quad I = \frac{2}{100}$$

$$\Rightarrow \quad I = 20\ mA$$

Hence, the correct answer is (A).

30. $R_{AB} = \dfrac{12 \times 6}{12+6} = 4\ \Omega$

Hence, the correct answer is (A).

31. For no deflection, $I = \dfrac{\varepsilon}{r+12r} = \dfrac{\varepsilon}{13r}$

$$\Rightarrow \quad V_{AB} = IR_{AB} = \left(\frac{\varepsilon}{13r}\right)12r = \frac{12\varepsilon}{13r}$$

So, $\dfrac{12\varepsilon}{13}$ potential is developed across L

1 unit potential is developed across $\dfrac{L}{\left(\dfrac{12\varepsilon}{13}\right)} = \dfrac{13L}{12\varepsilon}$

So, potential of $\dfrac{\varepsilon}{2}$ is developed

across a length given by

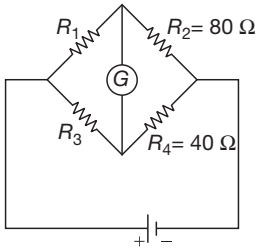
$$\ell_{AJ} = \left(\frac{13L}{12\varepsilon}\right)\frac{\varepsilon}{2}$$

$$\Rightarrow \quad \ell_{AJ} = \frac{13L}{24}$$

Hence, the correct answer is (D).

32. From colour code table, we get

R_1 (O, R, B) $= 320\ \Omega$



Since $R_2 = 80\ \Omega$

$R_4 = 40\ \Omega$

For a Balanced Wheatstone Bridge, we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\Rightarrow \quad R_3 = \frac{320 \times 40}{80}$$

$$\Rightarrow \quad R_3 = 160\ \Omega = 16 \times 10^1\ \Omega$$

So, colour for R_3 is Brown, Blue, Brown

Hence, the correct answer is (D).

33. $R' = \dfrac{RR_V}{R+R_V}$

Since, $R' = \dfrac{95}{100} \times 30 = 28.5\ \Omega$

$$\Rightarrow \quad 28.5 = \frac{30R_V}{30+R_V}$$

$$\Rightarrow \quad 855 + 28.5R_V = 30R_V$$

$$\Rightarrow \quad R_V = \frac{855}{1.5}$$

$$\Rightarrow \quad R_V = 570\ \Omega$$

Hence, the correct answer is (A).

34. $I^2R = 4.4$ watt

$$\Rightarrow \quad R = \frac{4.4}{4 \times 10^{-6}}\ \Omega$$

Since $P_{(\text{dissipated})}$ at voltage V is 11 volt

$$\Rightarrow \quad P = \frac{V^2}{R} = \frac{11 \times 11 \times 4 \times 10^{-6}}{4.4}$$

$$\Rightarrow \quad P = \frac{11 \times 11 \times 4 \times 10^{-6} \times 10}{44}$$

$$\Rightarrow \quad P = 1.1 \times 10^{-4}\ \text{watt}$$

$$\Rightarrow \quad P = 11 \times 10^{-5}\ \text{watt}$$

Hence, the correct answer is (A).

35. When in series, $P_S = \dfrac{P_1P_2}{P_1+P_2} = 60$ W

Since $P_1 = P_2 = P$

$\Rightarrow \quad P_S = \dfrac{P}{2}$

$\Rightarrow \quad P = 120$ watt

When in parallel,

$P_P = 2P = 2\times 120 = 240$ W

Hence, the correct answer is (D).

36. $\dfrac{P}{R} = \dfrac{Q}{X}$

$\Rightarrow \quad \dfrac{P}{400} = \dfrac{Q}{X}$ …(1)

On interchanging P and Q, we have

$\dfrac{Q}{405} = \dfrac{P}{X}$

$\Rightarrow \quad P = \dfrac{QX}{405}$

Substitute in (1), we get

$\dfrac{QX}{400\times 405} = \dfrac{Q}{X}$

$\Rightarrow \quad X = \sqrt{400\times 405}$

$\Rightarrow \quad X = 402.5\ \Omega$

Hence, the correct answer is (C).

37. At null point, $E = V_{AJ} = IR_{AJ}$

where $I = \left(\dfrac{6}{R_{AB}+R_H}\right)R_{AJ}$

Since AB is uniform wire, so if λ is resistance per unit length of AB, then $R_{AJ} = 4\lambda$

$\Rightarrow \quad E_1 = 0.5 = \left(\dfrac{6}{4+2}\right)(4\lambda)$

Also, $E_2 = \left(\dfrac{6}{4+6}\right)(4\lambda)$

$\Rightarrow \quad \dfrac{E_2}{0.5} = \dfrac{6}{10}$

$\Rightarrow \quad E_2 = 0.3$ V

Hence, the correct answer is (C).

38. Full scale deflection current is

$I_g = 30\times 0.005 = 0.15$ A

Since $\gamma = I_g(G+R)$

$\Rightarrow \quad 15 = 0.15(20+R)$

$\Rightarrow \quad R = 80\ \Omega$

Hence, the correct answer is (C).

39. Since $\dfrac{1}{r_{eq}} = 3$

$\Rightarrow \quad r_{eq} = \dfrac{1}{3}\ \Omega$

Also, $\dfrac{E_{eq}}{r_{eq}} = \dfrac{E_1}{r_1}+\dfrac{E_2}{r_2}+\dfrac{E_3}{r_3}$

$\Rightarrow \quad \dfrac{E_{eq}\times 3}{1} = \dfrac{1}{1}+\dfrac{2}{1}+\dfrac{3}{1} = 6$

$\Rightarrow \quad E_{eq} = 2$ V

Hence, the correct answer is (C).

40. For Condition of Balance, we have

$\dfrac{R_1}{R_2} = \dfrac{40}{60} = \dfrac{2}{3}$ …(1)

Now, $\dfrac{R_1+10}{R_2} = 1$ …(2)

From (1) and (2), we get

$\dfrac{R_1}{R_1+10} = \dfrac{2}{3}$

$\Rightarrow \quad R_1 = 20\ \Omega$ and $R_2 = 30\ \Omega$

Let resistance to be connected in parallel to

R_1+10 i.e., $30\ \Omega$ be R,

then for balance point to shift to the initial situation, we have

$\dfrac{\dfrac{30R}{30+R}}{30} = \dfrac{40}{60}$

$\Rightarrow \quad \dfrac{30\times R}{30+R} = 20$

$\Rightarrow \quad 30R = 600+20R$

$\Rightarrow \quad R = 60\ \Omega$

Hence, the correct answer is (A).

41. Since deflection θ is proportional to i, so

$\theta \propto i$

Let $R_G = R$

$\Rightarrow \quad i_1 = \dfrac{V}{220+R} = k\theta_0$ …(1)

$i_2 = \dfrac{V}{220+\dfrac{5\times R}{5+R}}\times\dfrac{5}{R+5} = k\times\dfrac{\theta_0}{5}$ …(2)

From (1) and (2), we get

$$\frac{25}{220\times(5+R)+5R}=\frac{1}{(220+R)}$$

$$\Rightarrow \quad \frac{1}{45R+220}=\frac{1}{5\times(220+R)}$$

$$\Rightarrow \quad R=22\ \Omega$$

Hence, the correct answer is (A).

42. Since $\frac{dR}{d\ell}=\frac{K}{\sqrt{\ell}}$

$$\Rightarrow \quad \int_0^R dR=K\int_0^{\ell}\frac{d\ell}{\sqrt{\ell}}$$

$$\Rightarrow \quad R=2K\sqrt{\ell}$$

$$\Rightarrow \quad \frac{R}{R'}=\frac{2K\sqrt{\ell}}{2K\left(1-\sqrt{\ell}\right)}=\frac{\sqrt{\ell}}{1-\sqrt{\ell}}$$

Since $R=R'$

$$\Rightarrow \quad \frac{\sqrt{\ell}}{1-\sqrt{\ell}}=1$$

$$\Rightarrow \quad 2\sqrt{\ell}=1$$

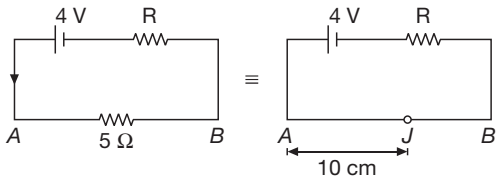
$$\Rightarrow \quad \ell=\frac{1}{4}=0.25\ \text{m}$$

Hence, the correct answer is (C).

43. $V_{AJ}=5\times10^{-3}$

$$R_{AJ}=\frac{5}{1}\times\frac{10}{100}=0.5\ \Omega$$

$$\Rightarrow \quad i=\frac{V_{AJ}}{R_{AJ}}=10^{-2}\ \text{A}$$



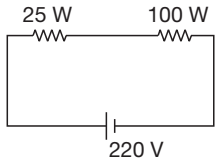
Since, $i=\frac{4}{R+5}=10^{-2}$

$$\Rightarrow \quad R+5=400\ \Omega$$

$$\Rightarrow \quad R=395\ \Omega$$

Hence, the correct answer is (D).

44. $\frac{1}{P}=\frac{1}{25}+\frac{1}{100}$



$$\Rightarrow \quad P=20\ \text{W}$$

Since Power $\propto R$

$$\Rightarrow \quad P_1=\frac{PR_1}{R_1+R_2}=16\ \text{W}$$

$$\Rightarrow \quad P_2=4\ \text{W}$$

Hence, the correct answer is (D).

45. $I_g=25\times4\times10^{-4}$

$$\Rightarrow \quad I_g=10^{-2}\ \text{A}$$

Since $V=I_g(R+50)$

$$\Rightarrow \quad R=200\ \Omega$$

Hence, the correct answer is (C).

46. At Junction S, we have

$$I_4=I_3+I_5$$

$$\Rightarrow \quad I_4=I_3+0.4$$

$$\Rightarrow \quad 0.8-0.4=I_3$$

$$\Rightarrow \quad I_3=0.4\ \text{A}$$

At Junction R, we have

$$I_1+I_2=I_4$$

$$\Rightarrow \quad -0.3+I_2=0.8$$

$$\Rightarrow \quad I_2=1.1\ \text{A}$$

At Junction Q, we have

$$I_3+I_6=I_1+I_2$$

$$\Rightarrow \quad 0.4+I_6=-0.3+1.1$$

$$\Rightarrow \quad I_6=0.4\ \text{A}$$

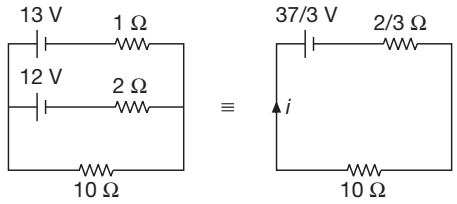
Hence, the correct answer is (A).

47. Equivalent e.m.f. of parallel batteries

$$E=\frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}=\frac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}}=\frac{37}{3}\ \text{V}$$

CHAPTER 3

Equivalent resistance of parallel batteries,



$$r_{eq} = \frac{2\times 1}{2+1} = \frac{2}{3}\ \Omega$$

Now, its equivalent circuit is as drawn.

Current in the circuit, $i = \dfrac{\frac{37}{3}}{10+\left(\frac{2}{3}\right)} = \dfrac{37}{32}$

Voltage across the load,

$$V_{10\,\Omega} = i\times 10 = \frac{37}{32}\times 10 = \frac{370}{32} = 11.56\ \text{V}$$

Hence, the correct answer is (B).

48. Let R_1 (left slot) and R_2 (right slot) be two resistances in two slots of a meter bridge.
Initially ℓ be the balancing length

Then, $\dfrac{R_1}{R_2} = \dfrac{\ell}{(100-\ell)}$ …(1)

Also, $R_1 + R_2 = 1000\ \Omega$ …(2)

On interchanging the resistances, balancing length becomes $(\ell - 10)$, so

$$\frac{R_2}{R_1} = \frac{\ell-10}{100-(\ell-10)} = \frac{\ell-10}{110-\ell}$$

$\Rightarrow \quad \dfrac{100-\ell}{\ell} = \dfrac{\ell-10}{110-\ell}$ (From equation (1))

$\Rightarrow \quad 11000 + \ell^2 - 210\ell = \ell^2 - 10\ell$

$\Rightarrow \quad 200\ell = 11000$

$\Rightarrow \quad \ell = 55\ \text{cm}$

From equation (1), we get

$$\frac{R_1}{R_2} = \frac{55}{45}$$

$\Rightarrow \quad R_1 = \dfrac{55}{45}R_2$

$\Rightarrow \quad R_1 = \dfrac{55}{45}(1000 - R_1)$ (From equation (2))

$\Rightarrow \quad R_1 + \dfrac{55}{45}R_1 = 1000\times\dfrac{55}{45}$

$\Rightarrow \quad 100R_1 = 1000\times 55$

$\Rightarrow \quad R_1 = 550\ \Omega$

Hence, the correct answer is (C).

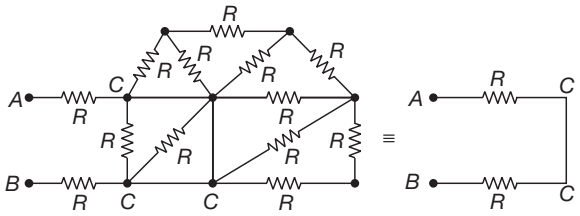
49. For a balanced meter bridge

$$Y(39.5) = X(100-39.5)$$

$\Rightarrow \quad X = \dfrac{12.5\times 39.5}{60.5} = 8.16\ \Omega$

When X and Y are interchanged so ℓ_1 and $(100-\ell_1)$ will also interchange and hence $\ell_2 = 60.5$ cm.
Hence, the correct answer is (B).

50.



$$R_{AB} = R + R = 2R$$

Hence, the correct answer is (A).

51. Rate of heat developed, $P = \dfrac{V^2}{R}$

For a given V, we have

$$P \propto \frac{1}{R} = \frac{A}{\rho\ell} = \frac{\pi r^2}{\rho\ell}$$

$\Rightarrow \quad \dfrac{P_1}{P_2} = \left(\dfrac{r_1^2}{r_2^2}\right)\left(\dfrac{\ell_2}{\ell_1}\right)$

Since it is given that, $\ell_2 = \dfrac{\ell_1}{2}$ and $r_2 = 2r_1$

$\Rightarrow \quad \dfrac{P_1}{P_2} = \dfrac{1}{4}\times\dfrac{1}{2} = \dfrac{1}{8}$

$\Rightarrow \quad P_2 = 8P_1$

Hence, the correct answer is (A).

52. Current flowing through copper rod is given by

$$I = neAv_d = \rho A v_d \qquad \{\because\ \rho = ne\}$$

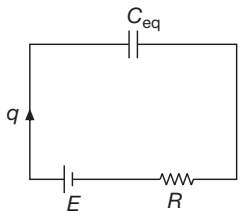
$\Rightarrow \quad v_d = \dfrac{I}{\rho A}$

Time taken by charges to travel distanced d is

$$t = \frac{d}{v_d} = \frac{d}{\left(\frac{I}{\rho A}\right)} = \frac{\rho A d}{I}$$

Hence, the correct answer is (A).

53. Equivalent circuit is shown in figure. Charging of capacitor is given by



$$q = q_{max}\left(1 - e^{-\frac{t}{RC_{eq}}}\right), \text{ where } q_{max} = C_{eq}E$$

$$\Rightarrow \quad q = C_{eq}E\left(1 - e^{-\frac{t}{RC_{eq}}}\right)$$

Both capacitors will have same charge as they are connected in series.

Hence, the correct answer is (A).

54. Resistance of element at $500\ °C$ is

$$R_T = \frac{\text{Voltage applied}}{\text{Current}} = \frac{220}{2} = 110\ \Omega$$

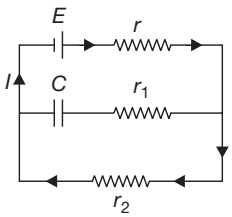
Since $R_T = R_0(1 + \alpha\Delta T)$

$$\Rightarrow \quad 110 = 100(1 + \alpha \times 500)$$

$$\Rightarrow \quad \alpha = \frac{10}{100 \times 500} = 2 \times 10^{-4}\ °C^{-1}$$

Hence, the correct answer is (B).

55. In steady state, flow of current through capacitor will be zero.



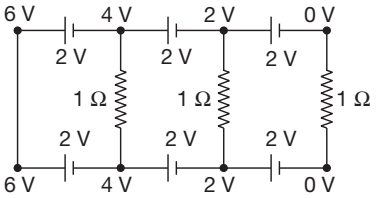
$$I = \frac{E}{r + r_2}$$

$$V_C = Ir_2 = \frac{Er_2}{r + r_2}$$

$$\Rightarrow \quad q_C = CV_C = CE\frac{r_2}{r + r_2}$$

Hence, the correct answer is (C).

56. Using Nodal Analysis method, we observe that the potential difference across each resistor is zero.



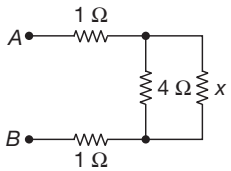
So, no current will flow through each resistor.

Hence, the correct answer is (D).

57. In a balanced Wheatstone bridge, the null point remains unchanged even if cell and galvanometer are interchanged.

Hence, the correct answer is (B).

58. Let equivalent resistance of the infinite network be x. Then, equivalent resistance between points A and B is also x.



$$\Rightarrow \quad x = \frac{4x}{4 + x} + 2$$

$$\Rightarrow \quad x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-8)}}{2} = \frac{2 \pm \sqrt{36}}{2}$$

$$\Rightarrow \quad x = \frac{2 \pm 6}{2} = 4\ \Omega$$

(Since negative value of resistance is not accepted)

$$\Rightarrow \quad I_1 = \frac{9}{4 + 0.5} = 2\ \text{A}$$

So, reading of A_1 is $2\ \text{A}$.

Hence, the correct answer is (B).

59. When key is plugged between 2 and 1, then

$$V_1 = iR_1 = k\ell_1 \qquad \ldots(1)$$

where k is the potential gradient of wire PQ

When key is plugged between 3 and 1, then

$$V_2 = i(R_1 + R_2) = k\ell_2 \qquad \ldots(2)$$

On dividing equation (2) by equation (1), we get

$$\frac{R_1}{R_1 + R_2} = \frac{\ell_1}{\ell_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\ell_1}{\ell_2 - \ell_1}$$

Hence, the correct answer is (B).

60. For $P = 4\ \Omega$, $\ell_1 = 60$ cm

$$\Rightarrow \quad \frac{P}{Q} = \frac{\ell_1}{100 - \ell_1} = \frac{60}{40} = \frac{3}{2}$$

$$\Rightarrow \quad Q = \frac{2}{3}P = \frac{8}{3}\ \Omega$$

Now, $P' = P + R$ and $\ell_1' = 80$ cm

$$\Rightarrow \quad \frac{P'}{Q} = \frac{\ell_1'}{100 - \ell_1'} = \frac{80}{20} = 4$$

$$\Rightarrow \quad \frac{P + R}{Q} = 4$$

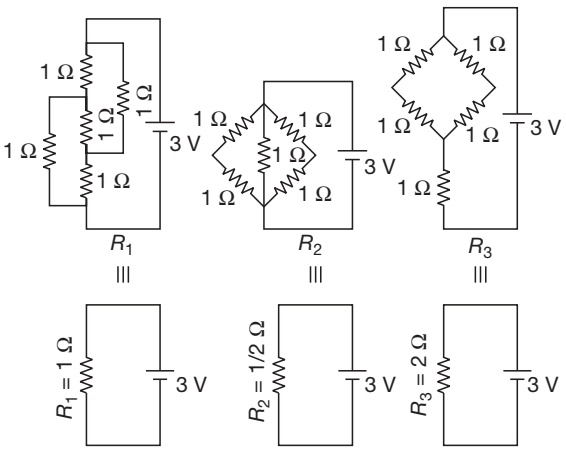
$$\Rightarrow \quad \frac{4 + R}{\frac{8}{3}} = 4$$

$$\Rightarrow \quad 4 + R = \frac{32}{3}$$

$$\Rightarrow \quad R = \frac{32}{3} - 4 = \frac{20}{3}\ \Omega$$

Hence, the correct answer is (A).

61. The given three circuits are equivalent to the following three simpler circuits.



So, $P_1 = \frac{3^2}{1} = 9$ W

$$P_2 = \frac{3^2}{\frac{1}{2}} = 18 \text{ W} \text{ and}$$

$$P_3 = \frac{3^2}{2} = 4.5 \text{ W}$$

$$\Rightarrow \quad P_2 > P_1 > P_3$$

Hence, the correct answer is (B).

62. Resistance of a wire of length ℓ and radius r is given by

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{A} \times \frac{A}{A} = \frac{\rho V}{A^2} = \frac{\rho V}{\pi^2 r^4} \qquad \{\because\ V = A\ell\}$$

$$\Rightarrow \quad R \propto \frac{1}{r^4}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$$

According to the problem, we have

$$R_1 = 100\ \Omega,\ r_1 = r,\ r_2 = \frac{r}{2},\ R_2 = ?$$

$$\Rightarrow \quad R_2 = R_1\left(\frac{r_1}{r_2}\right)^4 = 16R_1 = 1600\ \Omega$$

Hence, the correct answer is (D).

63. *Cu* is conductor, so with increase in temperature, resistance will increase.
Si is semiconductor, so with increase in temperature resistance will decrease.
Hence, the correct answer is (D).

64. Let the source voltage be V. Then, equivalent resistance of the circuit when $r = fR$ is

$$R_{eq} = R + \frac{r \times R}{r + R} = R + \frac{fR}{f + 1} = \frac{(2f + 1)R}{(f + 1)}$$

$$\Rightarrow \quad \text{Current in the circuit is}$$

$$I = \frac{V}{R_{eq}} = \frac{V(f + 1)}{R(2f + 1)}$$

Current through the resistance $r(= fR)$ is

$$I_2 = \frac{I}{f + 1} = \frac{V}{R(2f + 1)}$$

So, heat generated per unit time in the resistor r is

$$H = I_2^2 r = \frac{V^2 f}{R(2f + 1)^2}$$

For maximum H, we have $\frac{dH}{df} = 0$

$$\Rightarrow \quad \frac{V^2}{R}\left(\frac{1}{(2f + 1)^2} - \frac{4f}{(2f + 1)^3}\right) = 0$$

$$\Rightarrow \quad 2f + 1 - 4f = 0$$

$$\Rightarrow \quad f = \frac{1}{2}$$

Hence, the correct answer is (A).

65. Here, $R(T) = R_0\left[1+\alpha(T-T_0)\right]$

At $T_0 = 300$ K, $R_0 = 100\ \Omega$ and

at $T = 500$ K, $R = 120\ \Omega$

$\Rightarrow \quad 120 = 100(1+\alpha(200))$

$\Rightarrow \quad 200\alpha = \frac{6}{5} - 1 = \frac{1}{5}$

$\Rightarrow \quad \alpha = 10^{-3}\ ^\circ C^{-1}$

Temperature of the toaster is raised at constant rate from 300 K to 500 K is 30 s.

So, increment in the temperature in time t is

$$\Delta T = \frac{(500-300)}{30}t$$

$$\Delta T = \frac{20}{3}t$$

Total work done during this time in raising the temperature is

$$W = \int_0^t \frac{V^2 dt}{R(t)} = \int_0^t \frac{V^2 dt}{R_0(1+\alpha\Delta T)}$$

$$\Rightarrow \quad W = \int_0^{30} \frac{(200)^2\, dt}{100\left(1+10^{-3}\times\frac{20}{3}t\right)} = 400\int_0^{30}\frac{dt}{\left(1+\frac{t}{150}\right)}$$

$$\Rightarrow \quad W = 400\times 150\left[\ell n\left(1+\frac{t}{150}\right)\right]_0^{30}$$

$$\Rightarrow \quad W = 60000\left[\ell n\left(1+\frac{30}{150}\right) - \ell n 1\right] = 60000\ell n\left(\frac{6}{5}\right)\text{ J}$$

$$\Rightarrow \quad W = 60\ell n\left(\frac{6}{5}\right)\text{ kJ}$$

Hence, the correct answer is (C).

66. $v_d = 2.5\times 10^{-4}\ \text{ms}^{-1}$, $n = 8\times 10^{28}\ \text{m}^{-3}$

Since, $I = neAv_d$

$$\Rightarrow \quad \frac{V}{R} = neAv_d$$

$$\Rightarrow \quad \frac{V}{\left(\frac{\rho\ell}{A}\right)} = neAv_d$$

$$\Rightarrow \quad \rho = \frac{V}{nev_d\ell}$$

Substituting values, we get

$$\rho = 1.6\times 10^{-5}\ \Omega\text{m}$$

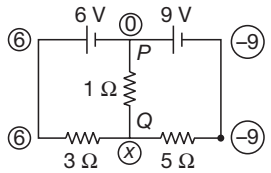
Hence, the correct answer is (D).

67. Let us redraw the circuit and assign potential at P to be zero and that at Q to be x. Then at the junction Q, $\Sigma I_Q = 0$

$$\Rightarrow \quad \frac{x-6}{3} + \frac{x-0}{1} + \frac{x+9}{5} = 0$$

$$\Rightarrow \quad x\left(\frac{5+15+3}{15}\right) = 2 - \frac{9}{5}$$

$$\Rightarrow \quad x = \frac{3}{23}\text{ V}$$



$$\Rightarrow \quad I = \frac{x-0}{1} = \frac{3}{23}\text{ A}$$

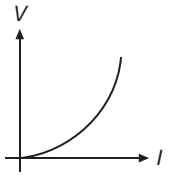
$\Rightarrow \quad I = 0.13$ A from Q to P

Hence, the correct answer is (C).

68. Given, $v_d \propto \sqrt{E}$

Since, $I = neAv_d$

$$\Rightarrow \quad I \propto \sqrt{E} \qquad \ldots(1)$$



Since $E = \frac{V}{\ell}$

$\Rightarrow \quad E \propto V$

So from (1), we get

$$I \propto \sqrt{V}$$

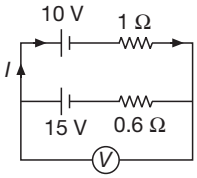
$$\Rightarrow \quad I^2 \propto V$$

Hence, the correct answer is (C).

69. Current in the circuit, $I = \frac{5}{1.6}$

$$\Rightarrow \quad I = \frac{50}{16} = \frac{25}{8}\text{ A}$$

Reading of the voltmeter is

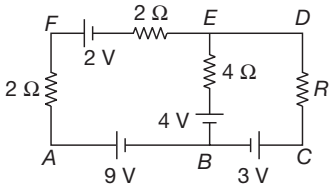


$$V = 15 - \frac{25}{8} \times 0.6$$

$$\Rightarrow \quad V = 15 - \frac{15}{8} = 13.1 \text{ V}$$

Hence, the correct answer is (C).

70.



Current in $4\ \Omega$ is zero.

Start from C and go to D (via B and E), we get

$$V_C - 3 + 4 - V_D = 0$$

$$\Rightarrow \quad V_C - V_D = -1 \text{ V} \qquad \ldots(1)$$

Starting from A and going to C (via B), we get

$$V_A - 9 + 3 - V_C = 0$$

$$\Rightarrow \quad V_A - V_C = 6 \text{ V} \qquad \ldots(2)$$

Add (1) and (2), we get

$$V_A - V_D = 5 \text{ V}$$

Hence, the correct answer is (C).

71. Total power P is given by

$$P = (15)(40) + (5)(100) + (5)(80) + (1)(1000)$$

$$\Rightarrow \quad P = 600 + 500 + 400 + 1000 = 2500\ W$$

Since, $P = VI$

$$\Rightarrow \quad I = \frac{2500}{220} A$$

$$\Rightarrow \quad I = \frac{125}{11} A = 11.3 \text{ A}$$

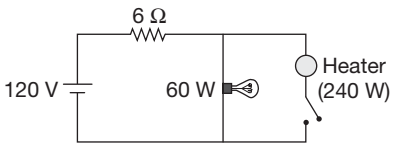
So, minimum capacity of main fuse of building must be 12 A.

Hence, the correct answer is (C).

72. Assuming that both bulb and the heater have a rating of 120 V, then

$$R_{\text{bulb}} = \frac{120 \times 120}{60} = 240\ \Omega$$

and $R_{\text{heater}} = 60\ \Omega$



Initial current is $i = \frac{120}{240 + 6} = \frac{120}{246}$

New current is $i' = \frac{120}{48 + 6} \times \frac{60}{60 + 240} = \frac{120}{54} \times \frac{1}{5} = \frac{24}{54}$

Decrease in voltage $= 240 \times \left(\frac{120}{246} - \frac{24}{54} \right) = 10.4 \text{ V}$

So the nearest approximate answer is (D)

Hence, the correct answer is (D).

73. During discharging of a capacitor, we have $V = V_0 e^{-t/\tau}$ where $\tau (= RC)$ is the time constant of RC circuit. At $t = \tau$, we have $V = \frac{V_0}{e} = 0.37\ V_0$

From the graph, $t = 0$, $V_0 = 25$ V

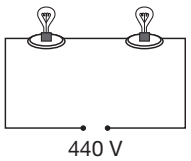
$$\Rightarrow \quad V = 0.37 \times 25$$

$$\Rightarrow \quad \text{V} = 9.25 \text{ V}$$

This voltage occurs at time that lies between 100 sec and 500 sec. Hence, time constant τ of this circuit lies between 100 sec and 150 sec.

Hence, the correct answer is (C).

74. 25 W – 220 V 100 W – 220 V



Since $R = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$

So, the resistance of 25 W-220 V bulb is

$$R_1 = \frac{(220)^2}{25}\ \Omega$$

and resistance of 100 W-200 V bulb is

$$R_2 = \frac{(220)^2}{100}\ \Omega$$

When these two bulbs are connected in series, the total resistance is

$$R_S = R_1 + R_2 = (220)^2 \left(\frac{1}{25} + \frac{1}{100} \right) = \frac{(220)^2}{20}\ \Omega$$

So, the current, $I = \frac{440}{\frac{(220)^2}{20}} = \frac{2}{11}$ A

Potential difference across 25 W bulb is

$$V_1 = IR_1 = \frac{2}{11} \times \frac{(220)^2}{25} = 352 \text{ V}$$

Potential difference across 100 W bulb is

$$V_2 = IR_2 = \frac{2}{11} \times \frac{(220)^2}{100} = 88 \text{ V}$$

Thus the bulb 25 W will be fused, because it can tolerate only 220 V while the voltage across it is 352 V.

Hence, the correct answer is (B).

75. Resistance of wire $R = \dfrac{\rho \ell}{A}$ …(1)

On stretching, volume (V) remains constant.

So $V = A\ell$

$$\Rightarrow \quad A = \frac{V}{\ell}$$

$$\Rightarrow \quad R = \frac{\rho \ell^2}{V} \qquad \{\text{From (1)}\}$$

$$\Rightarrow \quad R \propto \ell^2$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell} \qquad \{\because V \text{ and } \rho \text{ are constants}\}$$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

Hence, the correct answer is (B).

76. Since, by definition $\Delta R = R\alpha\Delta T$

For series combination, we have

$$R_s = R_1 + R_2$$

$$\Rightarrow \quad \Delta R_s = \Delta R_1 + \Delta R_2$$

$$\Rightarrow \quad 2R_0\alpha_s\Delta T = R_0\alpha_1\Delta T + R_0\alpha_2\Delta T$$

$$\Rightarrow \quad \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination, we have

$$R_P = \frac{R_1R_2}{R_1 + R_2} = \frac{R_1R_2}{R_s}$$

$$\frac{\Delta R_P}{R_p} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_s}{R_s}$$

$$\Rightarrow \quad \alpha_P\Delta T = \alpha_1\Delta T + \alpha_2\Delta T - \alpha_s\Delta T$$

$$\Rightarrow \quad \alpha_P = \alpha_1 + \alpha_2 - \alpha_s$$

$$\Rightarrow \quad \alpha_P = \alpha_1 + \alpha_2 - \left(\frac{\alpha_1 + \alpha_2}{2}\right)$$

$$\Rightarrow \quad \alpha_P = \frac{\alpha_1 + \alpha_2}{2}$$

Hence, the correct answer is (A).

77. From the statement given, $\alpha = 2.5 \times 10^{-3} {}^\circ C^{-1}$.

The resistance of a wire change from 100 Ω to 150 Ω when the temperature is increased from 27 °C to 227 °C.

Hence, the correct answer is (A).

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Single Correct Choice Type Problems

1. For infinite line charge, we have

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

Also, we know that

$$j = \sigma E$$

$$\Rightarrow \quad j = \sigma\left(\frac{\lambda}{2\pi\varepsilon r}\right)$$

But $i = jA = j(2\pi r\ell)$

$$\Rightarrow \quad \frac{dq}{dt} = j(2\pi r\ell) = \frac{\sigma\lambda}{2\pi\varepsilon r}(2\pi r\ell)$$

$$\Rightarrow \quad i = -\frac{\sigma\lambda\ell}{\varepsilon}$$

(Negative sign because i is flowing radially outwards).

$$\Rightarrow \quad \frac{dq}{dt} = -\frac{\sigma\lambda\ell}{\varepsilon}$$

$$\Rightarrow \quad \frac{d(\lambda\ell)}{dt} = -\frac{\sigma\lambda\ell}{\varepsilon}$$

$$\Rightarrow \quad \int_{\lambda_0}^{\lambda} \frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\int_0^t dt$$

$$\log_e \lambda\Big|_{\lambda_0}^{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right)t$$

$$\Rightarrow \quad \lambda = \lambda_0 e^{-\left(\frac{\sigma}{\varepsilon}t\right)}$$

Since, $j = \dfrac{\sigma\lambda}{2\pi\varepsilon r}$

$$\Rightarrow \quad j = \left(\frac{\lambda_0\sigma}{2\pi\varepsilon r}\right)e^{-\left(\frac{\sigma}{\varepsilon}t\right)}$$

Hence, the correct answer is (C).

2. $$\frac{1}{R} = \frac{1}{R_{Al}} + \frac{1}{R_{Fe}} = \left(\frac{A_{Al}}{\rho_{Al}} + \frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{\ell}$$

$$= \left[\frac{(7^2 - 2^2)}{27} + \frac{2^2}{10}\right]\frac{10^{-6}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

CHAPTER 3

Solving we get,

$$R = \frac{1875}{64} \times 10^{-6}\ \Omega$$

$$\Rightarrow \quad R = \frac{1875}{64}\ \mu\Omega$$

Hence, the correct answer is (B).

3. For balanced meter bridge

$$\frac{X}{R} = \frac{\ell}{(100-\ell)} \qquad \{\text{where, } R = 90\ \Omega\}$$

$$\Rightarrow \quad \frac{X}{90} = \frac{40}{100-40}$$

$$\Rightarrow \quad X = 60\ \Omega$$

$$\Rightarrow \quad X = R\frac{\ell}{(100-\ell)}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta \ell}{\ell} + \frac{\Delta \ell}{100-\ell}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{0.1}{40} + \frac{0.1}{60}$$

$$\Rightarrow \quad \Delta X = 0.25$$

So, $X = (60 \pm 0.25)\ \Omega$

Hence, the correct answer is (C).

4. Using the concept of balanced Wheat stone bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \quad \frac{X}{(52+1)} = \frac{10}{(48+2)}$$

$$\Rightarrow \quad X = \frac{10 \times 53}{50} = 10.6\ \Omega$$

Hence, the correct answer is (B).

5. We will require a voltmeter, an ammeter, a test resistor and a variable battery to verify Ohm's law.
Voltmeter which is made by connecting a high resistance with a galvanometer is connected in parallel with the test resistor.
Further, an ammeter which is formed by connecting a low resistance in parallel with galvanometer is required to measure the current through test resistor.
Hence, the correct answer is (C).

6. $R = \frac{\rho \ell}{A}$

$$\Rightarrow \quad R = \frac{\rho L}{tL} = \frac{\rho}{t}$$

Independent of L
Hence, the correct answer is (C).

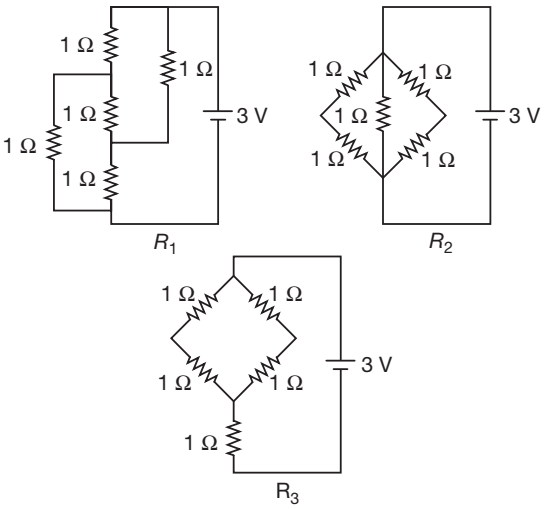
7. $R = \frac{V^2}{P}$

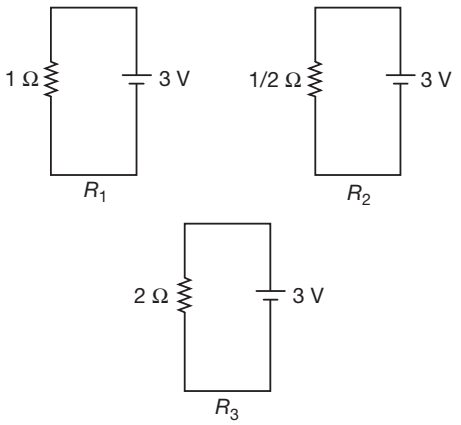
$$\Rightarrow \quad R \propto \frac{1}{P}$$

$$\Rightarrow \quad \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$$

Hence, the correct answer is (D).

8. The given three circuits R_1, R_2 and R_3 are equivalent to the following three circuits.





$$P_1 = \frac{3^2}{1} = 9\ \text{W}$$

$$P_2 = \frac{3^2}{\frac{1}{2}} = 18\ \text{W}$$

$$P_3 = \frac{3^2}{2} = 4.5\ \text{W}$$

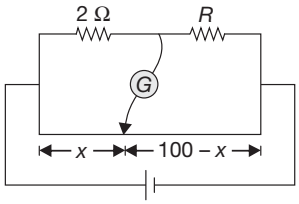
$P_2 > P_1 > P_3$

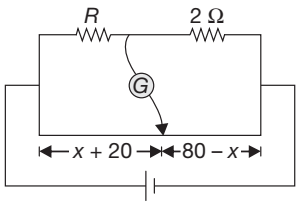
Hence, the correct answer is (C).

9. $R > 2\ \Omega$

$\Rightarrow \quad 100 - x > x$

Applying $\frac{P}{Q} = \frac{R}{S}$





We have $\frac{2}{R} = \frac{x}{100 - x}$ …(1)

$\frac{R}{2} = \frac{x + 20}{80 - x}$ …(2)

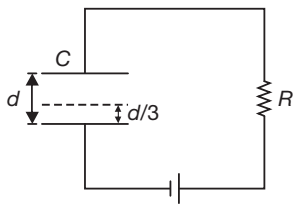
Solving equations (1) and (2), we get

$R = 3\ \Omega$

Hence, the correct answer is (A).

10. Time constant $= \tau = RC$, where

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{A\varepsilon_0}{d - x}\right)\left(\frac{KA\varepsilon_0}{x}\right)}{\frac{A\varepsilon_0}{d - x} + \frac{A\varepsilon_0}{x}}$$



$\Rightarrow \quad C = \frac{KA\varepsilon_0}{x + K(d - x)}$ where $x = \frac{d}{3} - Vt$

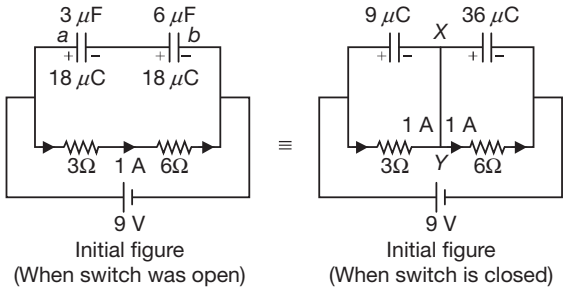
$\Rightarrow \quad \tau = \frac{RKA\varepsilon_0}{\frac{d}{3} - Vt + K\left(d - \frac{d}{3} + Vt\right)}$ where $A = 1\ \text{m}^2$ and $K = 2$

$\Rightarrow \quad \tau = \frac{3(2R\varepsilon_0)}{d - 3Vt + 6d - 2d + 6Vt} = \frac{6R\varepsilon_0}{5d + 3Vt}$

Hence, the correct answer is (A).

11. From Y to X charge flows to plates a and b

$(q_a + q_b)_i = 0$, $(q_a + q_b)_f = 27\ \mu\text{C}$



$\therefore \quad 27\ \mu\text{C}$ charge flows from Y to X

Hence, the correct answer is (C).

12. Current flowing through both the bars is equal.
Now, the heat produced is given by

$H = I^2 Rt$

$\Rightarrow \quad H \propto R$

$\Rightarrow \quad \frac{H_{AB}}{H_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{(1/2r)^2}{(1/r)^2} = \frac{1}{4} \qquad \left\{\text{as } R \propto \frac{1}{A} \propto \frac{1}{r^2}\right\}$

$\Rightarrow \quad H_{BC} = 4H_{AB}$

Hence, the correct answer is (A).

13. $\tau = CR$

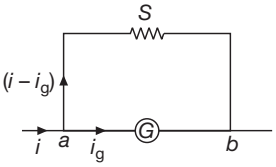
$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\ \mu s$

$\tau_2 = \left(\frac{C_1 C_2}{C_1 + C_2}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\ \mu s$

$\tau_3 = (C_1 + C_2)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = (6)\left(\frac{2}{3}\right) = 4\ \mu s$

Hence, the correct answer is (B).

14.



$V_{ab} = i_g.G = (i - i_g)S$

$\therefore \quad i = \left(1 + \frac{G}{S}\right) i_g$

Substituting the values, we get

$i = 100.1$ mA

Hence, the correct answer is (A).

15. $W = 0$. Therefore, from First Law of Thermodynamics,

$\Delta U = \Delta Q = I^2Rt = (1)^2(100)(5 \times 60)$ J $= 30$ kJ

Hence, the correct answer is (D).

16. Current in the respective loop will remain confined in the loop itself.

Therefore, current through 2 Ω resistance $= 0$

Hence, the correct answer is (C).

17. Given: $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$\Rightarrow \quad V_C = \frac{3}{4}V$

$\Rightarrow \quad V\left(1 - e^{-t/\tau_C}\right) = \frac{3}{4}V$

$\Rightarrow \quad e^{-t/\tau_C} = \frac{1}{4}$ …(1)

Here, $\tau_C = cR = 10$ sec

Substituting this value of τ_C in equation (1) and solving for t we get

$t = 13.86$ sec

Hence, the correct answer is (A).

18. $R_{PQ} = \frac{5}{11}r$, $R_{QR} = \frac{4}{11}r$ and $R_{PR} = \frac{3}{11}r$

$\Rightarrow \quad R_{PQ}$ is maximum.

Hence, the correct answer is (A).

19. BC, CD and BA are known resistance.

The unknown resistance is connected between A and D.

Hence, the correct answer is (C).

20. Charging current, $I = \frac{E}{R}e^{-\frac{t}{RC}}$

Taking log both sides, $\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$

When R is doubled, slope of curve increases. Also at $t = 0$, the current will be less. Graph Q represents the best.

Hence, the correct answer is (B).

21. Ammeter is always connected in series and voltmeter in parallel.

Hence, the correct answer is (A).

22. The ratio $\frac{AC}{CB}$ will remain unchanged.

Hence, the correct answer is (A).

23. $P = I^2R$

Current is same, so $P \propto R$.

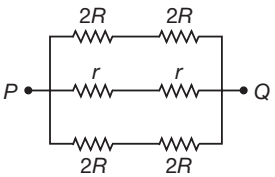
In the first case it is $3r$, in second case it is $\frac{2}{3}r$, in third case it is $\frac{r}{3}$ and in fourth case the net resistance is $\frac{3r}{2}$.

$R_{III} < R_{II} < R_{IV} < R_I$

$\Rightarrow \quad P_{III} < P_{II} < P_{IV} < P_I$

Hence, the correct answer is (A).

24.



Hence, the correct answer is (A).

25. $P = \frac{V^2}{R}$ so, $R = \frac{V^2}{P}$

$\Rightarrow \quad R_1 = \frac{V^2}{100}$ and $R_2 = R_3 = \frac{V^2}{60}$

Now, $W_1 = \frac{(250)^2}{(R_1 + R_2)^2}R_1$, $W_2 = \frac{(250)^2}{(R_1 + R_2)^2}R_2$

and $W_3 = \frac{(250)^2}{R_3}$

$W_1 : W_2 : W_3 = 15 : 25 : 64$

$\Rightarrow \quad W_1 < W_2 < W_3$

Hence, the correct answer is (D).

26. Current I can be independent of R_6 only when R_1, R_2, R_3, R_4 and R_6 from a balanced Wheatstone bridge.

Therefore, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ or $R_1R_4 = R_2R_3$.

Hence, the correct answer is (C).

27. $H = \frac{V^2}{R}t$

In the first case (wire of length ℓ)

$mc\Delta T = \frac{(3E)^2}{R}t$ …(1)

In the second case (wire of length 2ℓ)

$$(2m)c\Delta T = \frac{(nE)^2}{2R}t \qquad \ldots(2)$$

{Because doubling the length will simply double the mass and the resistance of the wire} so, from (1) and (2) we get,

$$n^2 = 36$$

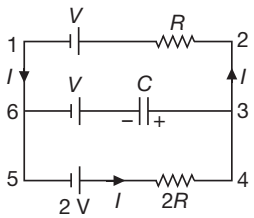
$$\Rightarrow \quad n = 6$$

Hence, the correct answer is (B).

28. In steady state the branch containing the capacitor can be omitted and hence current in the circuit is

$$I = \frac{2V - V}{R + 2R}$$

$$\Rightarrow \quad I = \frac{V}{3R}$$



For loop 36543

$$-V_C - V + 2V - I(2R) = 0$$

$$\Rightarrow \quad V_C = -V + 2V - \frac{V}{3R}(2R)$$

$$\Rightarrow \quad V_C = V - \frac{2V}{3} = \frac{V}{3}$$

Hence, the correct answer is (C).

29. Reading of Galvanometer remains same whether S is opened or closed i.e. no current must be flowing through this branch. Hence P and Q are in series and R and G are in series too.

So, $I_R = I_G$

Hence, the correct answer is (A).

30. $\Rightarrow \quad I = \frac{9}{9} = 1\text{ A}$

At A a current of 1 A divides into 0.5 A and 0.5 A.

At B the current of 0.5 A divides into 0.25 A and 0.25 A

Hence, the correct answer is (D).

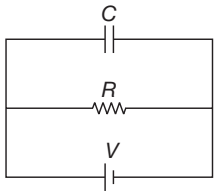
31. $i = neAv_d$

Drift speed, $v_d = \frac{1}{neA} \propto \frac{1}{A}$

Therefore, for non-uniform cross-section (different values of A) drift speed will be different at different sections. Only current (or rate of flow of charge) will be same.

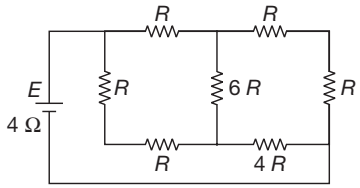
Hence, the correct answer is (D).

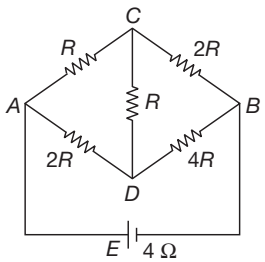
32. Since, the capacitor plates are directly connected to the battery, it will take no time in charging.



Hence, the correct answer is (D).

33. The given circuit is that of a Wheatstone bridge.

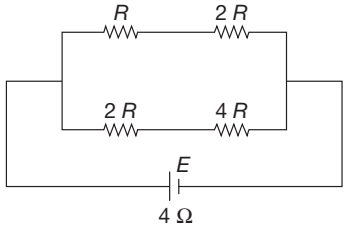




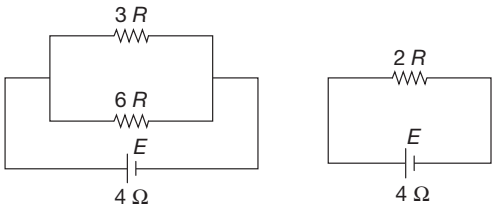
The circuit is a balanced one since,

$$\frac{\text{Resistance across } AC}{\text{resistance across } AD} = \frac{\text{resistance across } CB}{\text{resistance across } BD}$$

Thus, no current will flow across $6R$ of the side CD. The given circuit will now be equivalent to



CHAPTER 3



For maximum power, net external resistance = Total internal resistance.

$\Rightarrow \quad 2R = 4$

$\Rightarrow \quad R = 2\ \Omega$

Hence, the correct answer is (B).

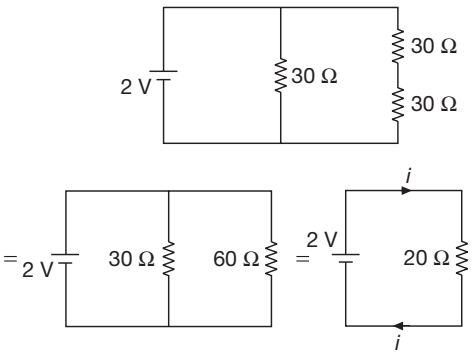
34. Resistivity of conductors increases with increase in temperature because rate of collisions between free electrons and ions increase with increase of temperature. However, the resistivity of semiconductors decreases with increase in temperature, because more and more covalent bonds are broken at higher temperatures.

Hence, the correct answer is (C).

35. Cooling increases the resistance of a semiconductor and decreases the resistance of a conductor.

Hence, the correct answer is (D).

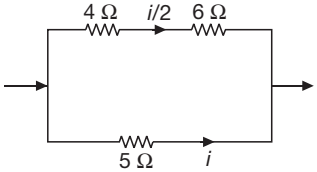
36. The simplified circuit is shown in the figure.



Therefore, current $i = \dfrac{2}{20} = \dfrac{1}{10}\ A$

Hence, the correct answer is (C).

37. Since, resistance in upper branch of the circuit is twice the resistance in lower branch. Hence, current there will be half.



Now, $P_4 = \left(\dfrac{i}{2}\right)^2 (4)$ $\qquad \left(P = i^2R\right)$

$P_5 = (i)^2 (5)$

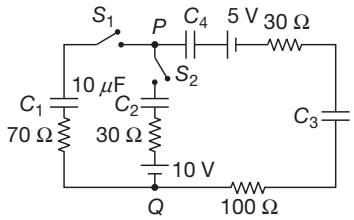
$\Rightarrow \quad \dfrac{P_4}{P_5} = \dfrac{1}{5}$

$\Rightarrow \quad P_4 = \dfrac{P_5}{5} = \dfrac{10}{5} = 2\ \text{cals}^{-1}$

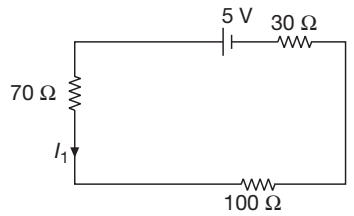
Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1.



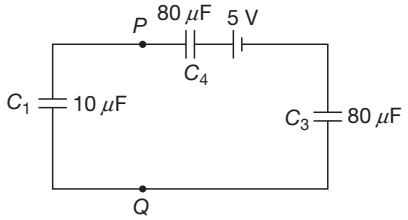
When switch S_1 is closed, so the equivalent circuit at $t = 0$ is shown in figure.



$\Rightarrow \quad I_1 = \dfrac{5}{200} = 25\ \text{mA}$

At steady state, for S_1 closed, we have

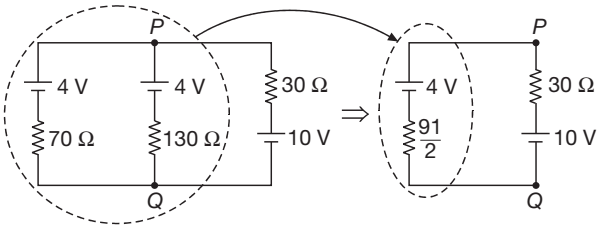
$C_{eq} = 8\ \mu\text{F}$ and $Q = 40\ \mu\text{C}$



So, $V(C_1) = 4$ volt and $V(C_3) = V(C_4) = 0.5$ volt

At steady state potential difference between P and Q is 4 volt.

Now when switch S_2 is closed, then equivalent circuit is shown in figure.



$$\Rightarrow \quad I_{PQ} = \frac{6 \times 2}{151} = 0.08 \text{ A}$$

Hence, (A) and (C) are correct.

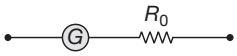
2. Maximum coil current is

$I_c = 2 \times 10^{-6}$ A and coil resistance is

$R_c = 10\ \Omega$

So, maximum potential difference across coil is

$V_{max} = 2 \times 10^{-6} \times 10 = 2 \times 10^{-5}$ volt

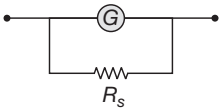


When converted into voltmeter of range 100 mV, then we have

$$(10 + R_0) \times 2 \times 10^{-6} = 100 \times 10^{-3}$$

$$\Rightarrow \quad R_0 = 49990\ \Omega$$

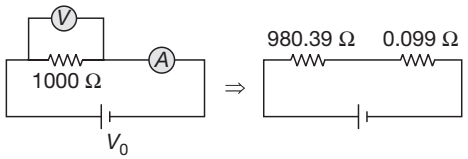
When converted into an ammeter of range 1 mA, then we have



$$R_s \times 9.98 \times 10^{-4} = 2 \times 10^{-5}\ \Omega$$

$$\Rightarrow \quad R_s = 0.02\ \Omega$$

Now, Ohm's Law setup is shown in the figure.



If R_{eq} is the equivalent resistance of the circuit, then

$R_{eq} = 980.48\ \Omega$

$$\Rightarrow \quad I = \frac{V_0}{980.48}$$

If V' be the reading of the voltmeter, then

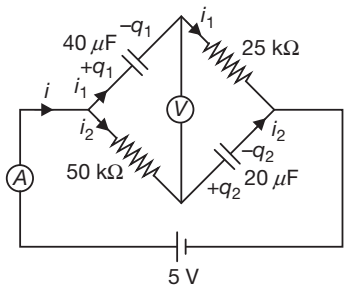
$$V' = \frac{V_0 \times 50000}{980.48 \times 51}$$

So, the measured value of resistance is

$$R_{(\text{measured})} = \frac{V'}{I} = \frac{50000}{51} = 980.4\ \Omega$$

Hence, (A) and (B) are correct.

3.



Just after pressing key,

$5 - 25000 i_1 = 0$

$5 - 50000 i_2 = 0$

{As charge on both capacitors is zero}

$$\Rightarrow \quad i_1 = 0.2 \text{ mA} \text{ and } i_2 = 0.1 \text{ mA}$$

Also, $V_B + 25000 i_1 = V_A$

$$\Rightarrow \quad V_B - V_A = -5V$$

After a long time, steady state is achieved, so i_1 and $i_2 = 0$

$$\Rightarrow \quad 5 - \frac{q_1}{40} = 0$$

$$\Rightarrow \quad q_1 = 200\ \mu\text{C}$$

and $5 - \frac{q_2}{20} = 0$

$$\Rightarrow \quad q_2 = 100\ \mu\text{C}$$

So, $V_B - \frac{q_2}{20} = V_A$

$$\Rightarrow \quad V_B - V_A = +5 \text{ V}$$

Hence, (A) is correct

For capacitor 1, $q_1 = 200\left(1 - e^{-t/1}\right)\ \mu\text{C}$

$$\Rightarrow \quad i_1 = \frac{dq_1}{dt} = \frac{1}{5} e^{-t/1} \text{ mA}$$

For capacitor 2, $q_2 = 100\left[1 - e^{-t/1}\right]\ \mu\text{C}$

$$\Rightarrow \quad i_2 = \frac{dq_2}{dt} = \frac{1}{10} e^{-t/1} \text{ mA}$$

$$V_B - \frac{q_2}{20} + i_1 \times 25 - V_A = 0$$

CHAPTER 3

$\Rightarrow \quad V_B - V_A = 5(1-e^{-t}) - 5e^{-t} = 5(1-2e^{-t})$

At $t = \ln 2$, $V_B - V_A = 5(1-1) = 0 \qquad \left\{\because e^{-\ln 2} = -\frac{1}{2}\right\}$

So, (B) is correct

At $t = 1$, $i = i_1 + i_2 = \frac{1}{5}e^{-1} + \frac{1}{10}e^{-1} = \left(\frac{3}{10}\right)\frac{1}{e}$

At $t = 0$, $i = i_1 + i_2 = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

So, (C) is correct

After a long time, $q_1 = q_2 = \text{constant}$

So, $i_1 = i_2 = 0$

Hence, (D) is correct.

Hence, (A),(B), (C) and (D) are correct.

4. When the filament breaks, the temperature of the filament will be high. Since according to Wein's Law, $\lambda_m \propto \frac{1}{T}$, so

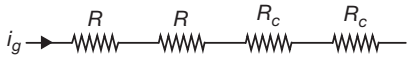
$$\nu_m \propto T \qquad \left\{\because \nu_m \propto \frac{1}{\lambda_m}\right\}$$

Hence the filament emits more light at higher band of frequencies.

Towards the end of life of the bulb, R (resistance) increases as temperature increases. Since $V = \text{constant}$, so $P = \frac{V^2}{R}$. So, less power is consumed towards the end of life of bulb.

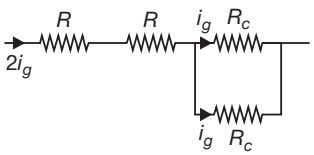
Hence, (C) and (D) are correct.

5. **For (A),**



$V_1 = 2i_gR + 2i_gR_C$

For (B),



$V_2 = 4i_gR + i_gR_C = 2i_gR + 2i_gR_C + 2i_gR - i_gR_C$

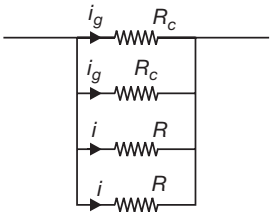
$V_2 = V_1 + i_g(2R - R_C)$

$\Rightarrow \quad V_2 > V_1$ as $\frac{R}{2} > R_C$

Hence, $2R > R_C$

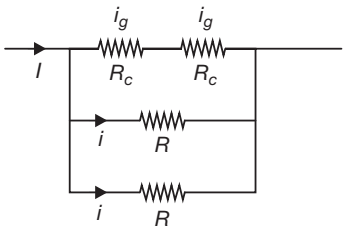
So, (B) is correct

For (C)



$I_1 = 2i_g + 2i = 2i_g + 2i_g\frac{R_C}{R}$

For (D)



$I_2 = i_g + 2i = i_g + 2\times\frac{2i_gR_C}{R}$

$\Rightarrow \quad I_2 = 2i_g + 2i_g\frac{R_C}{R} + 2i_g\frac{R_C}{R} - i_g$

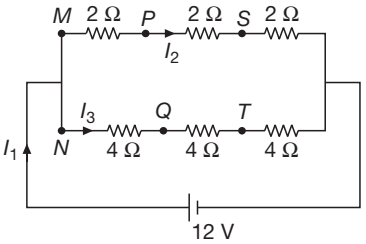
$\Rightarrow \quad I_2 = I_1 + \frac{2i_g}{R}\left(R_C - \frac{R}{2}\right)$

$\Rightarrow \quad I_2 < I_1$ as $R_C < \frac{R}{2}$

So, (C) is correct

Hence, (B) and (C) are correct.

6. Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below.



$I_2 = \frac{12}{2+2+2} = 2\text{ A}$

$\Rightarrow \quad I_3 = \frac{12}{4+4+4} = 1\text{ A}$

$\Rightarrow \quad I_1 = I_2 + I_3 = 3\text{ A}$

$\Rightarrow \quad I_{PQ} = 0$

$\Rightarrow \quad V_P = V_Q$

Potential drop (from left to right) across each resistance is

$$\frac{12}{3} = 4 \text{ V}$$

$\Rightarrow \quad V_{MS} = 2 \times 4 = 8 \text{ V}$

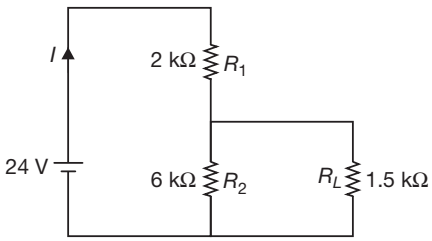
$\Rightarrow \quad V_{NQ} = 1 \times 4 = 4 \text{ V}$

$\Rightarrow \quad V_S < V_Q$

Hence, (A), (B), (C) and (D) are correct.

7. $R_{eq} = \left(\frac{6 \times 1.5}{7.5} + 2 \right) = \frac{6}{5} + 2 = \frac{16}{5} \text{ k}\Omega$

$$I = \frac{V}{R_{eq}} = \frac{24}{16} \times 5 \text{ mA} = \frac{3}{2} \times 5 = 7.5 \text{ mA}$$



for potential difference across R_1

$\Rightarrow \quad V_1 = 7.5 \times 2 = 15 \text{ V}$

for potential difference across R_2

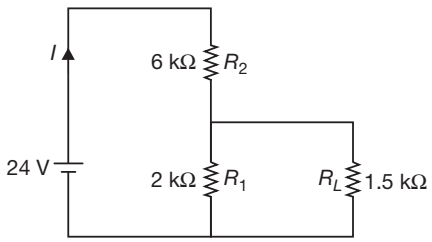
$\Rightarrow \quad V_2 = 24 - 15 = 9 \text{ V}$

for power

$$P_{R_1} : P_{R_2} = \frac{V_1^2}{R_1} : \frac{V_2^2}{R_2} = \frac{(15)^2}{2} : \frac{9^2}{6} = \frac{225}{2} \times \frac{6}{81} = \frac{25}{3}$$

$$P_{R_L} = \frac{V_2^2}{R_L} = \frac{9^2}{1.5} = 54 \text{ mW}$$

If R_1 and R_2 are interchanged



$$R' = R_1 \| R_2 = \left[\frac{(2)(1.5)}{2 + 1.5} \right] = \frac{3}{3.5}$$

$$V_L' = \frac{R'}{R_2 + R'} \times 24 \text{ V} = 3 \text{ V}$$

Now power dissipated in R_L is

$$P_L' = \frac{V_L'^2}{R_L} = \frac{3^2}{1.5} = 6 \text{ mW}$$

Hence, (A) and (D) are correct.

8. At 0 K, a semiconductor becomes a perfect insulator. Therefore, at 0 K, if some potential difference is applied across an insulator or semiconductor, current is zero. But a conductor will become a super conductors at 0 K. Therefore, current will be infinite. In reverse biasing at 300 K through a p-n junction diode, a small finite current flows due to minority charge carriers.
Hence, (A), (B) and (D) are correct.

9. To increase the range of ammeter a parallel resistance (called shunt) is required which is given by

$$S = \left(\frac{i_g}{i - i_g} \right) G$$

For option (C): $S = \left(\frac{50 \times 10^{-6}}{5 \times 10^{-3} - 50 \times 10^{-6}} \right) (100) \approx 1 \ \Omega$

To change it in voltmeter, a high resistance R is put in series, where R is given by

$$R = \frac{V}{i_g} - G$$

For option (B):

$$R = \frac{10}{50 \times 10^{-6}} - 100 \approx 200 \ k\Omega$$

Hence, (B) and (C) are correct.

10. $q = q_0 e^{-\frac{t}{RC}}$

$\Rightarrow \quad q = (CV_0) e^{-\frac{t}{RC}}$

$$I = \frac{dq}{dt} = -\frac{CV_0}{RC} e^{-\frac{t}{RC}}$$

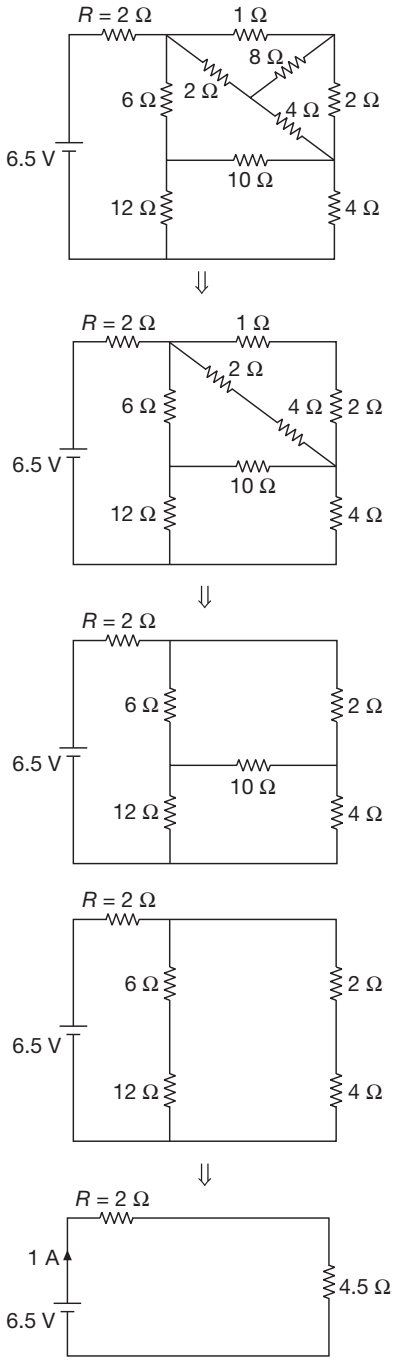
$$I(t = 0) = -\frac{V_0}{R}$$

which we observe is independent of the capacitance and only depends upon resistance.
Hence, the correct answer is (B).

CHAPTER 3

Integer/Numerical Answer Type Questions

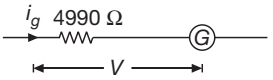
1.



2. V_{AB} = Equivalent emf of two batteries in parallel

$$= \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = 5 \text{ V}$$

3.



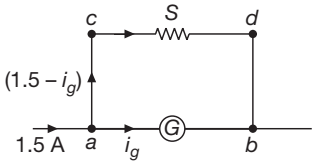
$$i_g(G+4990) = V$$

$$\Rightarrow \quad \frac{6}{1000}(G+4990) = 30$$

$$\Rightarrow \quad G+4990 = \frac{30000}{6} = 5000$$

$$\Rightarrow \quad G = 10\ \Omega$$

$$V_{ab} = V_{cd}$$



$$\Rightarrow \quad i_g G = (1.5 - i_g)S$$

$$\Rightarrow \quad \frac{6}{1000} \times 10 = \left(1.5 - \frac{6}{1000}\right)S$$

$$\Rightarrow \quad S = \frac{60}{1494} = \frac{2n}{249}$$

$$\Rightarrow \quad n = \frac{249 \times 30}{1494}$$

$$\Rightarrow \quad n = \frac{2490}{498} = 5$$

4. In series, $i = \dfrac{2E}{2+R}$

$$\Rightarrow \quad J_1 = i^2R = \left(\frac{2E}{2+R}\right)^2 \cdot R$$

In parallel, $i = \dfrac{E}{0.5+R}$

$$\Rightarrow \quad J_2 = i^2R = \left(\frac{E}{0.5+R}\right)^2 \cdot R$$

Since, $\dfrac{J_1}{J_2} = 2.25$

$$\Rightarrow \quad \frac{J_1}{J_2} = \frac{4(0.5+R)^2}{(2+R)^2}$$

$$\Rightarrow \quad 1.5 = \frac{2(0.5+R)}{(2+R)}$$

Solving we get, $R = 4\ \Omega$

5. Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time t will be given by

$$V = 10\left(1 - e^{\frac{-t}{\tau}C}\right)$$

Here $\tau_c = C_{net}R_{net}$

$$= (1\times10^{6})(4\times10^{-6}) = 4s$$

$\therefore \quad V = 10\left(1 - e^{\frac{-t}{4}}\right)$

Substituting $V = 4$ volt , we have

$$4 = 10\left(1 - e^{\frac{-t}{4}}\right) \text{ or } e^{\frac{-t}{4}} = 0.6 = \frac{3}{5}$$

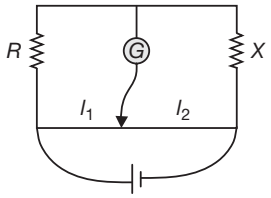
Taking log both sides we have,

$$-\frac{t}{4} = \text{In } 3 - \text{In } 5$$

or $t = 4(\text{In } 5 - \text{In } 3) = 2\ s$

Assertion and Reasoning Type Problems

1. $R\ell_2 = \ell_1 x$



To get null point at the same position, means ℓ_1 and ℓ_2 are still the same. As temperature increases, value of unknown resistance increases. To get the same null point, R must be increased. So Statement-1 is wrong. Statement-2 is True.

Hence, the correct answer is (D).

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